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## Credit Risk, Credit Rationing, and the Role of Banks: The Case of Risk Averse Lenders

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Institut für Volkswirtschaftslehre

Universität Augsburg

## Volkswirtschaftliche Diskussionsreihe

**Credit Risk, Credit Rationing, and the  
Role of Banks:  
The Case of Risk Averse Lenders**

**Thilo Pausch**

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# Credit Risk, Credit Rationing, and the Role of Banks: The Case of Risk Averse Lenders

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February 2005

## Abstract

The standard situation of ex post information asymmetry between borrowers and lenders is extended by risk aversion and heterogenous levels of reservation utility of lenders. In a situation of direct contracting optimal incentive compatible contracts are valuable for both, borrowers and lenders. However, there may appear credit rationing as a consequence of borrowers optimal decision making. Introducing a bank into the market increases total wealth due to the appearance of a portfolio effect in the sense of first order stochastic dominance. It can be shown that this effect may even reduce the problem of credit rationing provided it is sufficiently strong.

*Keywords:* risk aversion, costly state verification, credit rationing, bank

*JEL classification:* D82, G21, L22

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# Credit Risk, Credit Rationing, and the Role of Banks: The Case of Risk Averse Lenders

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The standard situation of ex post information asymmetry between borrowers and lenders is extended by risk aversion and heterogeneous levels of reservation utility of lenders. In a situation of direct contracting optimal incentive compatible contracts are valuable for both, borrowers and lenders. However, there may appear credit rationing as a consequence of borrowers optimal decision making. Introducing a bank into the market increases total wealth due to the appearance of a portfolio effect in the sense of first order stochastic dominance. It can be shown that this effect may even reduce the problem of credit rationing provided it is sufficiently strong.

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## 1 Introduction

The analysis of the reasons for formation and viability of financial intermediaries has been subject to a large number of research papers during the last 25 years. In particular the seminal works of Diamond (1984), Williamson (1986), and Krasa and Villamil (1992) argued that banks as the most common kind of financial intermediary exist due to their ability to exploit economies of scope in performing monitoring of borrowers on behalf of lenders. In this regard banks collect funds from lenders which they employ to finance a large number of borrowers. This, in turn, reduces average costs of monitoring borrowers and in this way makes banks valuable institutions from the lender's point of view.

However, the assumption of risk neutral borrowers, banks, and lenders made in these papers has often been criticized in the literature. In particular Hellwig (2000) and Hellwig (2001) allow for risk aversion of borrowers and intermediaries as well while lenders basically are considered risk neutral. In the light of the recent literature on firms' behavior in risky situations these assumptions are quite reasonable.

For example Froot et al. (1993) figure out that in general per se risk neutral firms may behave as if they are risk averse in case of the existence of risky investment opportunities for which outside finance is necessary. Their argument is based on the observation of Jensen and Meckling (1976) and Myers and Majluf (1984) that outside finance is more expensive to firms than internally generated funds. A similar argument is presented in Froot and Stein (1998) in the context of the banking industry. Furthermore, Pausch and Welzel (2002) show that even regulatory constraints as suggested by the Basel Capital Accords make risk neutral banks to decide in a risk averse way.

Nevertheless, Hellwig's assumptions on his agents' attitudes towards risk imply that lenders behave less risk averse than banks and borrowers which is not that clear. On the one hand, banks and borrowers (entrepreneurs) usually enjoy limited liability when investment projects fail. That is, firms' managers are not liable for the debts of the firm with the full extent of their private wealth. Hence limited liability weakens incentives of managers to care about risk of investment opportunities. (cf. Jensen and Meckling, 1976, p. 331) Particularly in the case of banks a similar reasoning can be applied: in addition to limited liability banks are well diversified in their loan portfolios which may reduce their overall exposure to risk and in this way the level of risk aversion. On the other hand, a typical lender has to be considered a private person or private household with individual wealth based on a single source of earned income. Moreover, compared to firms' need of finance a lender's funds available for investment are small which prevents the latter from building a well diversified portfolio of investment projects.

From the above arguments it seems obvious that lenders may behave more risk averse than banks and borrowers which provides the starting point for the analysis of the present paper. With ex post information asymmetry between borrowers and lenders as well as allowing for different levels of reservation utilities of lenders – i.e., considering a kind of market supply for financial funds – we widen the scope of the traditional analysis of beneficial financial intermediation to questions regarding the optimal level of finance contracts concluded in the market. With viable financial intermediation our model therefore allows to derive results regarding the optimal size of a financial intermediary. A financial intermediary in this context is considered a traditional bank taking deposits and issuing loans.

As Hellwig (2000, p. 720f) points out, with risk aversion in the model the arguments using the large numbers arguments of Diamond (1984) are no longer feasible. But in contrast to Hellwig (2000) who makes up a situation in which the law of large number is still valid, we show that in the context of risk averse lenders the concept of first order stochastic dominance can be applied to derive interesting results. In particular, in addition to the effect figured out by Diamond (1984) and Williamson (1986) that occurrence of a bank reduces lenders' expected costs of monitoring the borrower there appears a second effect in the present model. We show that a bank

with a well diversified loan portfolio offers an investment opportunity to lenders which they consider less risky than a single borrower's investment project. As a result, banks need to pay lower risk premia than single borrowers which, in turn, makes financial intermediation a viable business. Moreover, in a financial market without a bank there appears some kind of credit rationing as a result of borrowers' optimal decision making. With the existence of a bank this problem may be reduced and additional profits may be realized. Profits, however, are shared among banks and lenders while borrowers still achieve their reservation utility.

The remainder of the paper is organized as follows: section 2 sets up the model and analyzes the equilibrium in the financial market without intermediation. Section 3 considers occurrence of a bank and derives the corresponding market equilibrium. In section 4 the results of the previous sections are compared and the optimal size of the bank in case of viability is determined. Section 6 concludes.

## 2 Base Model without Bank

Consider a one period setting which may be interpreted as a single representative situation out of an infinitely sequence of equal decisions in a capital market. There are two kinds of actors involved:  $i = 1, \dots, n$  borrowers each endowed with an investment project for which one unit of outside finance is needed – in fact we assume that borrowers do not have any own funds – and  $j = 1, \dots, m$  lenders each equipped with one unit of capital appropriations. Hence, for realizing her investment project a borrower needs to contract with one lender. Moreover, let  $n < \infty$ ,  $m < \infty$  and  $n \leq m$ , i.e. the numbers of borrowers and lenders are finite but sufficiently large to treat them as (quasi) continuous and there exists enough capital in the market to realize all investment projects.

Investment projects are identical and stochastically independent among borrowers. In this regard let  $y_i \in [0; \bar{y}]$  represent the payoff of an arbitrary borrower  $i$  and assume  $E(y_i) > 1$ . At the beginning of the period – when financing contracts may be negotiated – borrowers as well as lenders do not know the actual outcome from the project at the end of the period. However, the probability distributions – represented by density  $f(y_i)$  and corresponding cumulative distribution function (cdf)  $F(y_i)$  – are common knowledge.<sup>1</sup> Nevertheless, at the end of the period under consideration there appears information asymmetry since the borrowers are able to observe the actual outcome from their respective investment projects while lenders are not. The latter can find out the actual outcome only when spending some fixed cost of  $c > 0$ , i.e. we are concerned with a situation of costly state verification.

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<sup>1</sup>Note, due to the assumption of identical and stochastically independent investment projects these functions are the same for every single project.

So far the model of the present paper is quite similar to the set-ups used in Diamond (1984), Williamson (1986), and Krasa and Villamil (1992). Differences, however, appear with assumptions on the agents' attitudes towards risk and lenders' outside options to invest funds besides financial contracts.

As explained in the introduction it is reasonable considering lenders more risk averse than borrowers. The most simple way to catch this in the model of the present paper is to assume that borrowers are risk neutral and lenders to behave risk averse. Therefore the formers' objective concerns expected profit from realizing investment projects and the latters' concern is in expected utilities of repayments from financing contracts. In this regard all lenders evaluate payments with the same strictly increasing and concave von-Neumann-Morgenstern utility function:  $U(\cdot)$  with  $U'(\cdot) > 0$  and  $U''(\cdot) < 0$ . In case of lenders and borrowers fail to negotiate a financing contract, borrowers are not able to conduct the investment project and realize a payoff of zero. The lenders, however, may invest their funds into an investment other than the borrowers' projects which guarantees lender  $j$  a reservation utility of  $U_{Rj} \in [\underline{U}_R; \overline{U}_R] \forall j$  with  $\underline{U}_R > 0$ . The intuition behind this assumption is that lenders may have different sources of working income which grant them access to heterogeneous alternative investments. As a result reservation utilities may differ between lenders and altogether there exists a minimum ( $\underline{U}_R$ ) and a maximum ( $\overline{U}_R$ ) level of reservation utility. Furthermore, borrowers do not know the reservation utility of a certain lender  $j$ . They just know the distribution of reservation utility represented by the density function  $g(U_{Rj})$  and the corresponding cumulative distribution function  $G(U_{Rj})$ .

If there does not exist a bank in the contracting situation just set up, borrowers and lenders will have to directly negotiate financing contracts. Such contracts commit lenders to pay out one unit of funds to borrowers at the beginning of the period. Furthermore, borrowers agree to meet a repayment  $t(y_i)$  at the end of the period which may depend on the realization of the payoff of their investment project. Since lenders can not observe the actual realization without cost, they have to rely on the payoff the borrower claims to be realized. Hence both parties define a set  $S \subset [0; \bar{y}]$  which specifies all claimed payoffs for which lenders spend verification cost  $c$  and observe the actual outcome of the investment project.

In the process of negotiation borrowers are assumed to enjoy all the bargaining power. One may argue that due to the sufficiently large number of lenders they are able to enter into a contract with the lender who accepts most favorable conditions from the borrower's point of view. However, since the level of reservation utility of a certain lender is unknown to the borrower, she can only determine the probability for a successful contractual agreement whereby this probability rises as the contract provides more beneficial terms for the lender – this is a direct result from the assumptions on the distribution of reservation utility. Moreover, due to the asymmetry of information between contracting parties the contract to be met has

to create incentives to the borrower to correctly report outcome of the investment project.

Due to the assumptions of the model for the analysis to follow it is sufficient to consider a representative pairing of any borrower  $i$  and any lender  $j$  and determine the optimal terms of the contract they enter. The decision process as a whole can be represented as three-stage game: on the first stage borrower  $i$  offers a financing contract  $(t(y_i), S)$  which maximizes her expected profit. Thereafter, on stage 2, a lender decides whether to accept the contract which is the case if and only if the expected utility generated by the contract is at least the same as the lender's reservation utility  $U_{Rj}$ . Since the borrower does not know the lender's reservation utility definitely, she can only determine probability for a successful contract. Stage 3 considers the borrower's incentives for truthfully reporting ex post realized payoffs from the project.

To solve this game we apply the concept of Subgame Perfect Nash Equilibrium which can be found by using backward induction. (cf. Rasmusen, 1989, p. 85.) That is, we start on stage 3 to determine the incentive compatible structure of the contract. Afterwards we have a look at stage 2 to find out when a lender accepts the contract offered. At the end stage one is solved and optimal terms of the financing contract are determined.

As explained earlier, due to ex post asymmetric information between borrowers and lenders with respect to the actual realization of the payoff from the investment project, the borrower may have an incentive to claim another outcome in order to reduce the repayment to the lender. Therefore the threat of the lender to monitor the borrower is used to give the borrower incentives for truthfully reporting outcome of the investment project. However, with fixed cost  $c$  this is exactly a situation of costly state verification which has been first analyzed by Townsend (1979) and applied to the borrower-lender relationship in Gale and Hellwig (1985). From these papers it is well known that an incentive compatible financing contract needs to feature the following repayment function depending on project outcome:

$$t(y_i) = \begin{cases} t_0 & ; y_i \geq t_0 \\ \overline{t(y_i)} < t_0 - c & ; y_i < t_0 \end{cases} \quad (1)$$

The interpretation is straight forward: the contracting parties agree on a constant repayment  $t_0$  as long as the claimed outcome from the investment project is at least at the same level. In this situation the lender does not perform any monitoring activities. In case of the borrower's claim not being able to pay  $t_0$  the lender spends verification cost  $c$ , observes the actual realization and enforces the pre-specified repayment in this situation  $\overline{t(y_i)}$  – of course he will additionally try to cover  $c$ . It is obvious that in the latter case repayment depends on actual  $y_i$  since verification provides the necessary information to lenders. But in the former case without monitoring lenders do not know the actual realization of  $y_i$  and therefore will be paid the



constant minimum repayment without verification which is  $t_0$ . It should be noted that due to the assumption that borrowers do not have any own funds, they are able to repay  $t_0$  just as long as  $y_i \geq t_0$  holds.<sup>2</sup> As a result repayment function (1) represents the solution to stage 3 of the contracting game.

Turning now to stage 2 one has to find out, in which situations a lender accepts a financing contract offered. In this regard it has been already mentioned that some contract with a repayment function that conforms to equation (1) will be accepted when the level of expected utility generated by this contract – denoted henceforth  $EU([t(y_i)])$  – is at least the same as a lender’s reservation utility. Taking into account the borrowers uncertainty with respect to a certain lender’s reservation utility the probability for entering a contract is

$$Prob(U_{Rj} \leq EU([t(y_i)])) = G( EU([t(y_i)]) ). \quad (2)$$

From (2) it is obvious that increasing the expected utility from the financing contract increases probability of successful contracting. Hence, equation (2) represents the lender’s participation constraint in which the expected utility from the financing contract can be written as

$$EU([t(y_i)]) = \int_0^{t_0} U(\overline{t(y_i)}) dF(y_i) + U(t_0)(1 - F(t_0)).$$

Taking into account the results for stages 2 and 3 of the contracting game, one can now state the borrower’s optimization problem to be solved at stage 1:

$$\max_{\overline{t(y_i)}, t_0} \frac{m}{n} G( EU([t(y_i)]) ) \left[ \int_0^{t_0} (y_i - \overline{t(y_i)} - c) dF(y_i) + \int_{t_0}^{\overline{y}} (y_i - t_0) dF(y_i) \right] \quad (3)$$

$$\text{s.t. } EU([t(y_i)]) = \int_0^{t_0} U(\overline{t(y_i)}) dF(y_i) + U(t_0)(1 - F(t_0)) \quad (4)$$

$$\overline{t(y_i)} + c \leq y_i \quad \forall y_i < t_0 \quad (5)$$

$$\overline{t(y_i)}, t_0 > 0.$$

Equation (3) states the borrower’s expected profit depending on uncertainty concerning the payoff from the investment project as well as concerning the success of negotiating a contract. With respect to the latter source of uncertainty it should be noted that  $G( EU([t(y_i)]) )$  marks the probability for a single financing contract. Since there are  $m$  lenders the total number of contracts in the market is  $mG( EU([t(y_i)]) )$ . Moreover, there may be less borrowers than lenders ( $n \leq m$ ) which creates a probability for a successful contract in the market of  $\frac{m}{n} G( EU([t(y_i)]) )$ . Equation (4) is the

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<sup>2</sup>Note, the assumption of risk aversion does not affect the incentive structure of the problem. Therefore, the structure of the incentive compatible financing contract remains the same regardless the contracting parties attitudes towards risk. (cf. Hellwig, 2001, p.719) See also Pausch (2003) for a more formal proof of this result.

well known expression to calculate the lender's expected utility from the financing contract. Relation (5) accounts for the assumption that borrowers do not have any own funds. Therefore they can meet a repayment of at most  $y_i$ , i.e. the actual repayment from the investment project. Of course, when optimally deciding on  $\overline{t(y_i)}$  and  $t_0$  we only consider strictly positive values since with  $\overline{t(y_i)}, t_0 = 0$  there would not exist a financing contract that is beneficial to both, borrowers and lenders.

Applying the Kuhn-Tucker-Theorem one can now derive the first order necessary conditions for the optimal financing contract between borrower and lender:

$$\begin{aligned} & g(\text{EU}([t(y_i)])) \left[ \int_0^{t_0} (y_i - \overline{t(y_i)} - c) dF(y_i) + \int_{t_0}^{\overline{y}} (y_i - t_0) dF(y_i) \right] = \\ &= \frac{G(\text{EU}([t(y_i)])) f(y_i) + \mu}{U'(\overline{t(y_i)}) f(y_i)} \quad \forall y_i < t_0 \end{aligned} \quad (6)$$

$$\begin{aligned} & g(\text{EU}([t(y_i)])) \left[ \int_0^{t_0} (y_i - \overline{t(y_i)} - c) dF(y_i) + \int_{t_0}^{\overline{y}} (y_i - t_0) dF(y_i) \right] = \\ &= \frac{(1 - F(t_0)) - (y_i - \overline{t(y_i)} - c) f(t_0)}{U'(t_0) (1 - F(t_0)) - (U(t_0) - U(\overline{t(t_0)})) f(t_0)} G(\text{EU}([t(y_i)])) \end{aligned} \quad (7)$$

$$y_i - \overline{t(y_i)} - c \geq 0 ; \mu \geq 0 ; (y_i - \overline{t(y_i)} - c) \mu = 0 \quad \forall y_i < t_0. \quad (8)$$

Equations (6) and (7) specify the optimal repayment in case of verification by the lender  $\overline{t(y_i)}$  and in case of no verification  $t_0$ , respectively. Conditions (8) define any situation when the borrower has to utilize all payoff from the investment project for repayment to the lender.

Taking a closer look at the first order necessary conditions one can state and prove

**Proposition 1** *An optimal financing contract between borrower and lender – in case of coming about – exhibits the repayment function*

$$t(y_i) = \begin{cases} t_0 & \text{if } y_i \geq t_0 \\ y_i - c & \text{if } y_i < t_0 \end{cases}$$

and creates a strictly positive expected profit for the borrower.

**Proof:** See the Appendix.

A comparison of the optimal contract above to the one derived by Gale and Hellwig (1985) for the case of risk neutral lenders and borrowers shows that they

are identical. That is, the optimal financing contract between risk neutral borrowers and risk averse lenders is still a standard debt contract in the sense of Gale and Hellwig.

However, having a closer look at the model of the present paper shows that our result is much more general than those of Gale and Hellwig (1985). Note, by allowing for different levels of lenders' reservation utilities the standard debt contract does not accomplish with certainty as long as it generates a given level of expected utility. Rather a borrower has to trade-off that a higher expected utility level from the contract increases the probability of its acceptance but also calls for a higher repayment to lenders and hence reduces borrowers' expected profits in case of success. As a result borrowers will choose the repayment such that  $\underline{U}_R < EU([t(y_i)]) \leq \overline{U}_R$  holds. That is, on the one hand the standard debt contract will never generate a level of expected utility that is below the lenders' minimum reservation utility because in this situation the contract will not be accepted and the borrower loses a positive expected profit. On the other hand the borrower should not rise the expected utility of the contract beyond the maximum reservation utility of lenders since this would only decrease her expected profit without increasing the probability of success.

But with an optimal expected utility of the standard debt contract which is between the lenders' minimum and maximum levels of reservation utility it is obvious that there may be situations where some borrowers are not successful in acquiring outside finance for their investment project. Therefore, the present model provides another explanation for some kind of credit rationing in the market equilibrium. However, when this kind of credit rationing happens depends on the specific characterization of the probability distributions incorporated in the model.

### 3 Intermediation by a Bank

We now introduce a financial intermediary in form of a monopolistic bank into the model. Despite the fact that the bank's business is simply to take deposits from lenders and issue loans to borrowers her situation is very similar to the one of borrowers. The bank is assumed to behave risk neutral, i.e. maximizes expected profit, and does not have any own funds. At the moment we neglect any costs of operating the bank. Hence, to issue loans to the amount of  $k$  the bank needs to take deposits to the amount  $d$  for which  $k \leq d$  holds. However, since taking deposits is costly to the bank the objective of cost minimization included in maximization of the expected profit implies  $k = d$ .

Concerning the bank's level of information it is assumed that on the one hand she is not able to observe the actual realizations the investment projects of her

borrowers. To get this information the bank has to spend fixed cost  $c$  per borrower.<sup>3</sup> On the other hand only the distribution of lenders' reservation utilities is known to the bank. As a result, for a certain lender she can just determine a probability for accepting a offer for a deposit contract. The assumptions regarding borrowers' investment projects and lenders' reservation utilities remain the same. Of course lenders are not able to observe the bank's earnings from issuing loans without cost. Thus, just like in the previous section they each have to spend fixed cost  $c$  to get this information.

From the bank's point of view due to the above assumptions borrowers and lenders are identical each. Therefore deposits and loan contracts do not differ among lenders and borrowers, respectively. As a result, when the probability to enter into a deposit contracts with a lender is  $G(EU([t_D(Y_k)]))$  just like in the previous section, the overall amount of deposits is  $mG(EU([t_D(Y_k)]))$ . In this regard  $t_D(Y_k)$  denotes the bank's repayment to a single lender at the end of the period depending on revenue per loan the bank earns in its loan business. The situation of contracting with borrowers is quite similar. Since borrowers' investment projects are identical, loan contracts will be too. Hence, when the bank offers  $k$  loan contracts the probability of a certain borrower to enter into one of these is  $\frac{k}{n}$ .

Let  $t_K(y_i)$  denote a borrower's repayment to the bank under any loan contract, then the bank earns  $Y = \sum_{i=0}^k t_K(y_i)$  in its loan business. The bank's revenue per loan will then be denoted by  $Y_k = \frac{1}{k}Y$ .

For the analysis of the viability of a bank we will proceed as follows: first we determine a potential market equilibrium when there exists a bank. Thereafter we compare equilibria with and without bank to derive conclusions on the role of a bank in the present context. In this regard the determination of a market equilibrium when there exists a bank can be modeled as a three-stage game very similar to the one of the previous section: on the first stage the bank designs deposit and loan contracts such that her expected profit gets maximized. Here we assume that the bank enjoys all the bargaining power. However, designing optimal contracts in the present situation includes two aspects. On the one hand the structure of loan an deposit contracts has to be profit maximizing. That is, there mustn't be an opportunity to increase expected profit simply by modifying the shape of the respective repayment functions. On the other hand the level of utility generated by the contracts – and hence the level of repayment – has to be set optimally. To decompose the two aspects we split stage one in two parts: stage 1a determines the optimal level of repayments by choosing  $k = d$ . That is, the optimal decision on the amount of loans – and hence by assumption the optimal amount of deposits – implies an optimal decision on the probability of a certain lender to accept a deposit

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<sup>3</sup>Note, verification cost  $c$  in this section are the same as the lenders' verification cost  $c$  in the previous section. Hence we assume that there is no systematic cost advantage in verification for the bank.

contract. In this regard the bank sets the optimal level of expected utility generated from the deposit contract. In Stage 1b the bank chooses the structure of the loan an deposit contract such that the optimal level of  $k = d$  from stage 1a is met at minimal cost and with maximal revenues from loans. Then, on stage 2 lenders as well as borrowers decide whether to enter into contracts with the bank or to stay in the situation of direct contracting of the previous section. Stage 3 considers incentives of the contracts for borrowers and bank to truthfully report outcomes from investment projects and loan business, respectively, at the end of the period.

For solving the game the concept of the Subgame Perfect Nash Equilibrium using the technique of backward induction is applied, again. Therefore we start with stage 3 to analyze incentives and incentive compatibility of loan and deposit contracts. Considering the former at first, one can observe an incentive problem which is well known in the literature: risk neutral borrowers get outside finance to realize investment projects and observe the corresponding outcome. The risk neutral lender – i.e. the bank in our model – can not observe the actual payoff of the project without incurring cost  $c$  to monitor the borrower. Hence as long as the bank does not monitor the borrower will always claim that she can not repay anything because of the bad performance of the project. This, however, is exactly the situation which has been analyzed in Gale and Hellwig (1985) resulting in a contract which provides a fixed payment – say  $t_{K0}$  – whenever the bank does not perform any monitoring activities – i.e.  $y_i \geq t_{K0} \forall i$  – and a outcome dependent repayment  $\overline{t_K(y_i)} < t_{K0} - c$  in case of verification taking place – i.e.  $y_i < t_{K0} \forall i$ .

In addition there appears a second incentive problem between the bank and its lenders which is basically the same as the incentive problem of the former section. The bank receives an outcome  $Y_k$  per loan at the end of the period which is not observable by lenders. Thus the bank may have an incentive to misreport on the realized outcome as long as there is no verification by lenders. Since lenders are risk averse and the bank behaves risk neutral, the arguments presented in the former section are still valid. However, to derive the exact structure of deposit contracts one needs to adjust arguments for the present situation: since deposit contracts are identical for all lenders the bank's repayment to lenders is all in all  $dt_D(Y_k)$  where  $t_D(Y_k)$  denotes the repayment to any lender if the bank reports an outcome from the loan business of  $Y_k$  per loan. Due to the assumption that the bank does not have any own funds, the bank can repay at most  $kY_k$  – i.e. the total outcome of granting loans. Taking into account that  $d = k$  was assumed earlier, the bank's limited liability can be represented as

$$Y_k \geq t_D(Y_k).$$

Using these modifications the arguments of the previous section can be directly applied to see that incentive compatibility of deposit contracts implies a constant repayment of the bank – say  $t_{D0}$  – when there is no verification and an outcome dependent repayment  $\overline{t_D(Y_k)} \leq t_{D0} - c$  when there appears verification by the lenders.

With summing up the above arguments one can state the incentive compatible structure of loan and deposit contracts, respectively, as follows:

$$t_K(y_i) = \begin{cases} t_{K0} & ; y_i \geq t_{K0} \\ \overline{t_K(y_i)} & ; y_i < t_{K0} \end{cases} \quad \text{and} \quad t_D(Y_k) = \begin{cases} t_{D0} & ; Y_k \geq t_{D0} \\ \overline{t_D(Y_k)} & ; Y_k < t_{D0} \end{cases}. \quad (9)$$

Having derived the results for stage 3 of the game we can now go on with an analysis of stage 2. In this regard borrowers as well as lenders will accept the bank's loan and deposit contract if and only if both do not suffer a loss in expected profits and utilities, respectively. From a borrower's point of view this may be stated formally as

$$\frac{k}{n} \left[ \int_0^{t_{K0}} (y_i - \overline{t_K(y_i)} - c) dF(y_i) - \int_{t_{K0}}^{\bar{y}} (y_i - t_{K0}) dF(y_i) \right] \geq E(\Pi_{direct}). \quad (10)$$

Note, relation (10) represents a borrower's participation constraint where  $E(\Pi_{direct})$  denotes the expected profit a borrower realizes in case of entering into a direct financing contract with a lender. In the previous section this expected profit was shown to be strictly positive and is exogenous for the bank. Furthermore notice, that the number of loan contracts offered ( $k$ ) positively affects the borrower's expected profit since a higher level of  $k$  increases the probability of receiving a loan.

In the analysis of the previous section it was found out that with direct contracting of borrowers and lenders there appears a certain optimal level  $t_0$  for the financing contract. As a consequence lenders are offered contracts which generate a certain level of expected utility  $EU(t_0)$  and a certain probability of success of  $G(EU(t_0))$ .<sup>4</sup> This, however may be considered the lenders' alternative utility when rejecting the bank's deposit contract. Hence the bank's deposit contracts have to generate at least this level of expected utility which we will denote  $EU_{direct}$  in the following. Thus, the following condition must hold to ensure participation of lenders:

$$G(EU([t_D(Y_k)])) \geq G(EU_{direct}).$$

Moreover, when the bank decides to offer deposit contracts to the amount of  $d = k$ , it defines the level of expected utility a deposit contract must generate at the same time, i.e.

$$mG(EU([t_D(Y_k)])) = d$$

must hold where we have used the fact that multiplying the probability of success  $G(\cdot)$  by the total number of lenders  $m$  yields the total number of deposit contracts in the market. Hence a lender's participation constraint can be stated as follows:

$$G(EU([t_D(Y_k)])) = \frac{d}{m} \geq G(EU_{direct}). \quad (11)$$

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<sup>4</sup>Note, from Proposition 1 it is obvious that the optimal financing contract is completely characterized by the repayment without verification ( $t_0$ ) since in case of verification borrowers have to hand over total project outcome to lenders.

Note, this formulation of the lender's participation constraint accomplishes for the two-stage process in designing the optimal deposit contracts. In order to determine the optimal structure of the contract – stage 1b – the first part of (11), i.e. the equation, is relevant. For the analysis of the optimal level of expected utility of the deposit contract – stage 1a – the second part of (11), i.e. the inequation, has to be used.

With the results derived so far, we are now able to have a closer look at stage 1b of the game which is concerned with the optimal structure of deposits and loan contracts. In this regard assume that the bank has already chosen die optimal amounts of deposits and loans  $d$  and  $k$ , respectively, which means that the lenders' optimal level of expected utility has been already specified. Hence, to find the optimal structure of deposits and loan contracts we have to find the solution of the following maximization problem:

$$\max_{[\overline{t_K(y_i)}, t_{K0}], [\overline{t_D(Y_k)}, t_{D0}]} \left[ \int_0^{t_{K0}} \overline{t_K(y_i)} dF(y_i) - t_{K0} (1 - F(t_{K0})) \right] - \left[ \int_0^{t_{D0}} (\overline{t_D(Y_k)} + c) dH(Y_k) - t_{D0} (1 - H(t_{D0})) \right] \quad (12)$$

$$\text{s.t. } G \left( \int_0^{t_{D0}} U(\overline{t_D(Y_k)}) dH(Y_k) + U(t_{D0}) (1 - H(t_{D0})) \right) = \frac{d}{m} \quad (13)$$

$$\frac{k}{n} \left[ \int_0^{t_{K0}} (y_i - \overline{t_K(y_i)} - c) dF(y_i) - \int_{t_{K0}}^{\bar{y}} (y_i - t_{K0}) dF(y_i) \right] \geq E(\Pi_{direct}) \quad (14)$$

$$\overline{t_D(Y_k)} + c \leq Y_k \quad \forall Y_k < t_{D0} \quad (15)$$

$$\overline{t_K(y_i)} + c \leq y_i \quad \forall y_i < t_{K0} \quad (16)$$

$$\overline{t_K(y_i)}, t_{K0}, \overline{t_D(Y_k)}, t_{D0} > 0.$$

Equation (12) represents the bank's expected profit per unit of loan which it will maximize by deciding on the details of deposit and loan contracts. Here the term in first squared brackets is the expected outcome per unit of loan which is identical for any single loan in the bank's portfolio due to identical loan contracts to fund identical investment projects. The second term in squared brackets displays the bank's expected cost per unit of loan, i.e. its expected repayment to lenders per unit of loan. In computing this latter part of the bank's expected profit we have used the fact that repayment to lenders is based on average outcome  $Y_k$  from the loan portfolio and  $d = k$ . In this regard the function  $H(Y_k)$  denotes the cumulative distribution function of the random variable  $Y_k$ . Moreover, we have used the fact that  $Y_k \in [0; t_{K0}]$  since any loan contract can repay at least 0 due to the borrowers' limited liability and at most  $t_{K0}$  due to incentive compatibility. As a result it must be true that  $H(0) = 0$  and  $H(t_{K0}) = 1$ .

Conditions (13) and (14) represent the participation constraints of lenders and borrowers, respectively. In (13) we used the arguments above and the consideration

with respect to the probability distribution  $H(Y_k)$  to state the lender's expected utility from a deposit contract as follows:

$$EU([t_D(Y_k)]) = \int_0^{t_{D0}} U(\overline{t_D(Y_k)}) dH(Y_k) + U(t_{D0})(1 - H(t_{D0})).$$

The participation constraint of borrowers (14) has been already explained earlier.

Constraints (15) and (16) account for the part of considerations with respect to incentive compatibility of deposit and loan contracts, respectively, which refer to the sets of outcome when verification takes place. Note, the the remaining part of incentive compatibility restraints has already been used in the formulation of expected profits and utilities above. In addition, we assume that contract details are strictly positive since optimal repayments of zero imply that lenders and borrowers do not enter into deposit and loan contracts respectively. In this case, however, a bank does not exist and there is no need to think about deposit and loan contracts.

Applying the Kuhn-Tucker-Theorem, one can now derive the first order necessary conditions for the maximization problem above:

$$\lambda_{KN} + \frac{\mu_{KN}}{\frac{k}{n}f(y_i)} = \frac{1}{\frac{k}{n}} \forall y_i < t_{K0} \quad (17)$$

$$\lambda_{KN} = \frac{(\overline{t_K(t_{K0})} - t_{K0})f(t_{K0}) + (1 - F(t_{K0}))}{\frac{k}{n} \left( (1 - F(t_{K0})) - (t_{K0} - \overline{t_K(t_{K0})} - c)f(t_{K0}) \right)} \quad (18)$$

$$\lambda_{KG} = \frac{h(Y_k) + \mu_{KG}}{g(EU(t_D(Y_k)))U'(\overline{t_D(Y_k)})h(Y_k)} \forall Y_k < t_{D0} \quad (19)$$

$$\lambda_{KG} = \frac{(1 - H(t_{D0})) - (t_{D0} - \overline{t_D(t_{D0})} - c)h(t_{D0})}{g(\cdot) \left( U'(t_{D0})(1 - H(t_{D0})) - (U(t_{D0}) - U(\overline{t_D(t_{D0})}))h(t_{D0}) \right)} \quad (20)$$

$$G(EU([t_D(Y_k)])) - \frac{d}{m} = 0 \quad (21)$$

$$\frac{k}{n} [E(\Pi([t_K(y_i)]))] - E(\Pi_{direct}) \geq 0 ; \lambda_{KG} \geq 0$$

$$\left( \frac{k}{n} [E(\Pi([t_K(y_i)]))] - E(\Pi_{direct}) \right) \lambda_{KG} = 0 \quad (22)$$

$$Y_k - \overline{t_D(Y_k)} - c \geq 0 ; \mu_{KG} \geq 0$$

$$(Y_k - \overline{t_D(Y_k)} - c) \mu_{KG} = 0 \forall Y_k < t_{D0} \quad (23)$$

$$y_i - \overline{t_K(y_i)} - c \geq 0 ; \mu_{KN} \geq 0$$

$$(y_i - \overline{t_K(y_i)} - c) \mu_{KN} = 0 \forall y_i < t_{K0}. \quad (24)$$

Here, conditions (17) and (18) determine the optimal structure of the loan contract's repayment function  $\overline{t_K(y_i)}$  and  $t_{K0}$ , respectively, whereas (19) and (20) define the



optimal structure of the repayment function of a deposit contract – i.e.  $\overline{t_D(Y_k)}$  and  $t_{D0}$ , respectively. Conditions (21) and (22) or rather (23) and (24) represent the Kuhn-Tucker-conditions for the lender's and borrower's participation constraints or rather the lender's and borrower's incentive compatibility constraints, respectively. In this regard  $\lambda_{KG}$  and  $\lambda_{KN}$  are the Lagrange-Multipliers for the lender's and borrower's participation constraint and  $\mu_{KG}$  and  $\mu_{KN}$  are the Lagrange-Multipliers of the lender's and borrower's incentive compatibility constraints, respectively. Moreover,  $h(Y_k)$  represents the density function of the probability distribution of  $Y_k$  which corresponds to the cumulative distribution function  $H(Y_k)$ .

From the inspection of these first order necessary conditions we can now state and prove our

**Proposition 2** *The optimal structure of both the bank's deposit and loan contracts is equivalent to the structure of standard debt contracts. That is, the optimal repayment functions are*

$$t_D(Y_k) = \begin{cases} t_{D0} & ; Y_k \geq t_{D0} \\ Y_k - c & ; Y_k < t_{D0} \end{cases} \quad \text{and} \quad t_K(y_i) = \begin{cases} t_{K0} & ; y_i \geq t_{K0} \\ y_i - c & ; y_i < t_{K0} \end{cases},$$

respectively. In addition  $t_{K0} > t_{D0}$  holds.

**Proof:** See the Appendix.

The interpretation of this result is straightforward: the bank funds its granting of loans by borrowing from lenders. Due to the kind of information asymmetry on both sides of the bank's business and the assumption of limited liability, the optimal contracts have to provide incentives, too. These, however, are strongest with standard debt contracts since then the respective borrower is taken away all outcome from investment project or loan business, respectively. As a result, the constant repayment when there is no verification by the respective lender can be pushed to a minimum level which, in turn, reduces probability of verification and therefore expected cost of verification to a minimal level, too. In this regard it is necessary that the bank sets repayment from loan contracts  $t_{K0}$  above repayment to lenders  $t_{D0}$  since the bank's outcome per unit of loan has to cover at least expected cost per unit of loan of monitoring borrowers in addition repayment to lenders. In this way  $t_{K0} > t_{D0}$  is a necessary condition for a bank to be viable.

Moreover, in the proof of Proposition 2 it was argued that the borrower's participation constraint is binding in the optimum – i.e.  $\lambda_{KN} > 0$ . This result is also obvious: since we assumed that the bank enjoys all the bargaining power in our model, it will issue loan contracts which just ensure that borrowers will accept them.

Therefore borrowers realize the same expected profit as with direct contracting with lenders and the bank acquires all additional profits.

Before we can go on with the analysis of stage 1a it has to be recognized that on the one hand deposit and loan contracts are completely characterized by repayments  $t_{D0}$  and  $t_{K0}$ , respectively. On the other hand,  $t_{D0}$  and  $t_{K0}$  still depend on the size of the bank – i.e. on the total amount of loans granted  $k$  which we assumed to be the same as the amount of deposits taken  $d$ . Thus, we have to find out the interdependence between the level of repayment and the amount of loans (or deposits) first.<sup>5</sup>

Consider first the effect of changing  $k$  on the optimal level of  $t_{K0}$ . While having argued so far that the first order necessary condition (18) determines the optimal level of  $t_{K0}$  for the present part of the analysis the borrower's participation constraint (14) is considered. The reason is that during the derivation of the optimal structure of the loan contract we found out this constraint to be binding and therefore if the bank plans to increase the amount of loans granted it has to ensure that there is a sufficient number of borrowers to enter into loan contracts. Therefore, we can state and prove

**Proposition 3** *In the optimum the following relationship holds:*

$$\frac{dt_{K0}}{dk} > 0.$$

**Proof:** Applying the implicit function theorem<sup>6</sup> to the borrower's participation constraint (14) which binds in the optimum yields:

$$\frac{dt_{K0}}{dk} = \frac{\frac{1}{n} \int_{t_{K0}}^{\bar{y}} (y_i - t_{K0}) dF(y_i)}{\frac{k}{n} (1 - F(t_{K0}))}.$$

In this latter equation  $n > 0$ ,  $k > 0$  and  $1 - F(t_{K0}) > 0$  hold due to the assumptions of the model. For the numerator one derives by applying standard manipulations to (14):

$$\int_{t_{K0}}^{\bar{y}} (y_i - t_{K0}) dF(y_i) = \frac{n}{k} E(\Pi_{direct}) > 0$$

where the inequality follows from  $E(\Pi_{direct}) > 0$  as figured out in the proof of Proposition 1.  $\square$

<sup>5</sup>In the following we focus on the relationship between  $k$  and  $t_{D0}$  as well as  $t_{K0}$  since with the assumption  $d = k$  it is not necessary to perform a similar analysis with respect to  $d$  explicitly.

<sup>6</sup>See Chiang (1984, p. 208ff) for a detailed representation.

The interpretation of Proposition 3 is evident: from the bank's point of view it is beneficial to offer more loan contracts if the price – i.e. the repayment – per contract increases. However, one might ask why borrowers should be willing to accomplish higher repayments when the bank offers a larger amount of loan contracts. The reason is that increasing  $k$  increases a borrower's probability of success for getting a loan  $\frac{k}{n}$ . As a result ceteris paribus a borrower's expected profit rises. Since optimal loan contracts are designed such that borrowers just realize  $E(\Pi_{direct})$  the bank will raise  $t_{K0}$  at the same time as long as borrowers still continue to accept loan contracts.

For the analysis of the relationship between  $t_{D0}$  and  $k$  we proceed analogously as in the previous considerations. In this regard we have a closer look at the lender's participation constraint (13) since just like in the previous case this condition represents the binding restriction for the design of optimal deposit contracts. That is, the repayment  $t_{D0}$  has to be determined such that the resulting level of expected utility exactly ensures the desired amount of deposit and hence loan contracts to accomplish. However, there are two effects which have to be taken into consideration in this regard. First, increasing  $k$  requires increasing  $d$  by the same amount due to the assumption  $k = d$ . The second effect appears with the distribution of average outcome from the loan business  $Y_k$ . When  $k$  is changed the number of loans which contribute to the bank's total revenue changes, too. This, in turn, may affect the distribution of  $Y_k$  where  $Y_k \in [0; t_{K0}]$  still holds. This latter effect has to be figured out first before we can go on with the analysis.

Therefore we state and prove

**Lemma 1** *Increasing the amount of loans  $k$  yields first order stochastic dominance in the distribution of  $Y_k$ , formally:*

$$\frac{d}{dk}H(\delta) \leq 0 \quad \forall \delta \in [0; t_{K0}].$$

**Proof:** See the Appendix.

In fact, Lemma 1 is a formal representation of the portfolio-effect which appears when a bank grants more than just a single loan. In this regard any additional loan creates a non-negative extra revenue and hence raises the probability that the average outcome from the loan portfolio takes on higher realizations with higher probability. In other words: the risk of small repayments from the loan business decreases on average as the amount of granted loans rises.

With the result of Lemma 1 we can now state and prove

**Proposition 4** *The effect of a change in the total amount of granted loans  $k$  on the optimal repayment of deposit contracts  $t_{D0}$  is ambiguous.*

**Proof:** Applying the implicit function theorem to the lender's participation constraint (13) yields

$$\frac{dt_{D0}}{dk} = - \frac{g(EU([t_D(Y_k)])) \left( \frac{\partial}{\partial k} EU([t_D(Y_k)]) \right) - \frac{1}{m}}{g(EU([t_D(Y_k)])) \left( \frac{\partial}{\partial t_{D0}} EU([t_D(Y_k)]) \right)}.$$

The denominator of this equation can be rewritten as

$$g(EU([t_D(Y_k)])) (U'(t_{D0}) (1 - H(t_{D0})) - (U(t_{D0}) - U(t_{D0} - c)) h(t_{D0}))$$

which is equivalent to the denominator of the first order necessary condition (20) which was shown to be strictly positive, earlier. Hence the sign of  $\frac{dt_{D0}}{dk}$  only depends on the sign of the numerator.

Form the argument of first order stochastic dominance of Lemma 1, it is well known in the literature that

$$\frac{\partial}{\partial k} EU([t_D(Y_k)]) \geq 0.^7$$

As a result, since  $m > 0$  and  $g(\cdot) > 0$  the sign of the numerator of the equation above and hence the sign of  $\frac{dt_{D0}}{dk}$  is ambiguous.  $\square$

The interpretation of Proposition 4 is straightforward. If the bank raises the amount of loans granted, it will simultaneously have to increase the amount of deposits taken. In this regard deposit contracts need to generate a level of expected utility to lenders which ensures that the required number of lenders will enter into deposit contracts. In this way the bank will have to raise  $t_{D0}$  – this is the interpretation of the second part ( $\frac{1}{m}$ ) of the numerator in the equation for  $\frac{dt_{D0}}{dk}$  above. However, at the same time there appears a further effect of increasing the amount of loans which is a rise in the lenders' expected utility due to the portfolio-effect of Lemma 1. That is, granting more loans reduces the risk for lower levels of outcome from the loan portfolio and in this way is considered beneficial from the lenders' point of view. This effect is represented by  $\frac{\partial}{\partial k} EU([t_D(Y_k)])$  in the equation for  $\frac{dt_{D0}}{dk}$  above which is negative and, therefore, allows the bank to reduce  $t_{D0}$ . In sum the actual sign of  $\frac{dt_{D0}}{dk}$  depends of the relative strength of both effects. In case of a relatively strong portfolio effect the bank may reduce the repayment to lenders whereas otherwise the repayment has to increase.

Bearing in mind these results, one can now go on with the analysis of stage 1a of the game, i.e. the optimal amounts of deposits and loans. For this purpose at first reformulate the bank's maximization problem as follows:

$$\max_k t_{K0}(k) - t_{D0}(k) - \int_{t_{D0}(k)}^{t_{K0}(k)} H(Y_k) dY_k \quad (25)$$

$$\text{s.t. } \frac{k}{m} \geq G(EU_{direct}). \quad (26)$$

<sup>7</sup>See e.g. Wolfstetter (1999, p. 137f).

In this regard (25) represents the bank's expected profit in terms of  $Y_k$ . That is, the bank's expected return per unit of loan from the loan portfolio – which was in (12) given as the expected repayment from a single loan – may be alternatively written as

$$\int_0^{t_{K0}} Y_k dH(Y_k)$$

and applying the optimal structure of the deposit contract the expected repayment to lenders can be rewritten as

$$\int_0^{t_{K0}} t_D(Y_k) dH(Y_k) = \int_0^{t_{D0}} Y_k dH(Y_k) + t_{D0} (1 - H(t_{D0})).$$

Integrating by parts both elements of the bank's expected profit and rearranging terms yields equation (25) which has a very nice interpretation: the bank's expected profit is equivalent to the spread between loan and deposit rates minus a kind of risk premium payed to lenders.

Equation (26) is the lenders' participation constraint. Note, lenders are willing to enter into deposit contracts only if the corresponding level of expected utility is at least the same as with direct contracting. In addition we have used the assumption of  $k = d$  and the result of Propositions 3 and 4 from which both  $t_{K0}$  and  $t_{D0}$  are affected by  $k$ . Moreover, due to  $G(EU_{direct}) > 0$  which was shown to hold in the previous section the participation constraint (26) ensures  $k > 0$ .

Once more applying the Kuhn-Tucker-theorem the following first order necessary conditions for the maximization problem can be found:

$$\frac{dt_{D0}}{dk} (1 - H(t_{D0})) + \int_{t_{D0}}^{t_{K0}} \frac{d}{dk} H(Y_k) dY_k = \frac{1}{m} \Lambda_{KG} \quad (27)$$

$$\frac{k}{m} - G(EU_{direct}) \geq 0 ; \Lambda_{KG} \geq 0 ; \left( \frac{k}{m} - G(EU_{direct}) \right) \Lambda_{KG} = 0. \quad (28)$$

In this context (27) determines the optimal level of loans  $k$  and because of  $k = d$  the optimal size of the bank. Conditions (28) are the Kuhn-Tucker-conditions for the lenders' participation constraint where  $\Lambda_{KG}$  is the corresponding Lagrange-Multiplier.

With first order necessary conditions (27) and (28) one can now state and prove

**Proposition 5** *If a bank enters the market the total number of loan and hence deposit contracts as at least the same as with direct contracting of borrowers and lenders.*

**Proof:** From (28) it is obvious that  $\Lambda_{KG} \geq 0$  must hold in the optimum. Hence

$$k = d \geq mG(EU_{direct})$$

completes the proof since the right hand side of the inequality represents the total number of directly negotiated financing contracts.  $\square$

It is not difficult to give an interpretation for this result. Lenders enter into deposit contracts if and only if they realize at least the same level of expected utility as with direct financing contracts. Thus, the probability of a single lender to accept the bank's offer is at least the same as with direct contracting resulting in a minimum amount of deposits – and hence loans due to the assumption – defined by the number of direct financing contracts. However, at a first glance this result is not surprising but a further analysis derives the following interesting

**Corollary 1** *For a bank to be viable in the optimum an increase in the amount of loans must not cause an decrease of  $t_{D0}$ , i.e.*

$$\frac{dt_{D0}}{dk} \geq 0.$$

**Proof:** Due to  $m > 0$  and  $\Lambda_{KG} \geq 0$  – see condition (28) – it follows from (27) that

$$\frac{dt_{D0}}{dk} (1 - H(t_{D0})) \geq - \int_{t_{D0}}^{t_{K0}} \frac{d}{dk} H(Y_k) dY_k$$

must hold in the optimum. In this regard  $(1 - H(t_{D0})) > 0$  and  $\frac{d}{dk} H(Y_k) \leq 0$  – see the proof of Lemma 1 – implies  $\frac{dt_{D0}}{dk} \geq 0$  as a necessary condition for an optimal bank size.  $\square$

To give an interpretation for Corollary 1 first note that changing  $t_{K0}$  does not affect the bank's expected profit – this is easily verified if one computes the derivative of the expected profit with respect to  $t_{K0}$ . But an increase of  $t_{D0}$  reduces the bank's expected utility since

$$\frac{d}{dt_{D0}} \left( t_{K0} - t_{D0} - \int_{t_{D0}}^{t_{K0}} H(Y_k) dY_k \right) = -(1 - H(t_{D0})) \leq 0$$

due to  $H(\cdot) \in [0; 1]$ . Furthermore, from Propositions 3 and 4 the effect of a change of  $k$  on  $t_{K0}$  and  $t_{D0}$ , respectively, is obvious. In this regard rising  $k$  causes an increase in  $t_{K0}$  which does not affect the bank's expected profit – see arguments above. However, as

$$\int_{t_{D0}}^{t_{K0}} \frac{d}{dk} H(Y_k) dY_k \leq 0$$

indicates for any realization of  $Y_k$  the corresponding value of  $H(Y_k)$  is the smaller the higher  $k$ . That is, the well known portfolio effect alters the probability distribution of  $Y_k$ . At the same time increasing  $k$  forces the bank to increase  $d$  which is included

in  $\frac{dt_{D0}}{dk}$ . Therefore, if  $\frac{dt_{D0}}{dk} < 0$  would be true the bank could go on increasing  $k$  and in this way decrease  $t_{D0}$  and increase expected profit. Hence, this situation can not be an optimum because then the impact of the portfolio effect on the bank's expected profit is not completely exploited. As a result the bank should increase  $k$  as long as the portfolio effect is not outweighed by the effect which is based on the necessary increase in expected utility of lenders to attract the optimal amount of deposit contracts.

If in this situation the portfolio effect is very weak the lenders' participation constraint restricts  $t_{D0}$  downward. Thus one can observe, that the bank offers more deposit and loan contracts if and only if the risk reducing effect of building up a loan portfolio is sufficiently strong. Hence, even when there exists a bank in the market there may appear the same kind of credit rationing as with direct contracting between borrowers and lenders.

## 4 The Role of Intermediation by a Bank

To derive conclusions regarding the role of a bank in the context of the contracting situation between borrowers and lenders the present section studies if the bank can improve total wealth in the financial market of the present model. For this purpose optimal levels of expected profits and expected utilities derived in section 3 are compared to the corresponding values of section 2.

With direct contracts between borrowers and lenders it was figured out that borrowers realize an optimal and strictly positive level of expected profit  $E(\Pi_{direct}) > 0$ . At the same time lenders obtain a strictly positive level of expected utility  $EU_{direct} > 0$  from the direct financing contract. With financial intermediation by a monopolistic bank it was found out that borrowers still realize the expected profit  $E(\Pi_{direct}) > 0$ . Lenders, however realize a level of expected utility from deposit contracts which is at least the same as with direct contracting  $EU_{direct}$ . That is, depending on the bank's decision on the optimal amount of loans and hence deposits, lenders may even obtain a higher level of wealth. This is the case when the bank issues loans to a larger amount than arise in the direct market. Therefore, from the arguments of the previous sections borrowers as well as lenders obtain a level of wealth when there exists a bank which is at least the same as with direct contracting.

As a consequence, whether there exists a welfare increasing effect of the entry of a bank depends on its expected profit. In this regard we can state and prove

**Proposition 6** *The bank's expected profit is strictly positive in the optimum.*

**Proof:** Consider the representation of the bank's profit in (25). From Proposition 2 it is already known that  $t_{K0} > t_{D0}$  in the optimum. Furthermore, due to  $H(t_{K0}) >$

$H(t_{D0})$  and  $H(t_{K0}) = 1$  the following relation holds:

$$\int_{t_{D0}}^{t_{K0}} H(Y_k) dY_k < (t_{K0} - t_{D0}) H(t_{K0}) = t_{K0} - t_{D0}.$$

Hence, in the optimum it must be true that

$$t_{K0} - t_{D0} - \int_{t_{D0}}^{t_{K0}} H(Y_k) dY_k > 0.$$

□

As a result one can observe that the appearance of a bank which collects funds from lenders and grants loans to borrowers increases the welfare in the financial market of the present model.

Moreover, the existence of a bank may even weaken the problem of credit rationing. From the arguments above one can observe that the minimum level of bank's deposit and loan contracts is the same as the total amount of directly negotiated financing contracts; i.e.

$$k_{min} = d_{min} = mG(EU_{direct}).$$

In this case loan contracts and direct financing contracts exhibit exactly the same repayments for borrowers. Furthermore, lenders will receive a level of expected utility of  $G(EU_{direct})$  regardless of direct contracting or entering into deposit contracts. The difference between direct financing and deposit contracts is that  $t_0 = t_{K0} > t_{D0}$  holds which is possible because the bank is able to reduce the risk of financing borrowers' investment projects by building a large scale loan portfolio. However, in this case there still may appear credit rationing to the same amount as with direct contracting since profits from  $t_{K0} > t_{D0}$  completely remain with the bank.

But when the bank decides to fund a larger number of investment projects than would be the case with direct contracting – i.e. if  $k > k_{min}$  – credit rationing would be reduced. Note, in Proposition 3 it was figured out that borrowers willingness to pay for a loan increases if the bank offers a larger amount of loans. The monopolistic bank exploits this effect by leaving the borrowers at  $E(\Pi_{direct})$ . As a result, the bank's expected profits increase. However, in order to issue loans at the amount  $k > k_{min}$  it is necessary to raise the total amount of deposits, too, such that  $k = d$  still holds. For this purpose, the bank has to design deposit contracts in a way that resulting expected utilities for lenders are increased until  $mG(EU([t_D(Y_k)])) = d$ . Therefore, it is obvious that on the one hand in the optimum borrowers are willing to pay for a reduction of credit rationing which increases the bank's expected profit. On the other hand, however, the bank has to hand over parts of the additional profit from reducing credit rationing to lenders since at the end they have to be willing to carry the lower level of credit rationing. In this way they are payed an adequate compensation.



## 5 Conclusion

Recent literature on the theory of financial intermediation has figured out some important reasons for banks being valuable institution. The delegated monitoring argument of Diamond (1984), Williamson (1986), and Krasa and Villamil (1992) may be considered as the most important one in this regard. But while this argument is based on a cost advantage of banks in monitoring borrowers derived from the law of large numbers, the model of the present paper figures out another important reason for explaining the role of banks.

In a financial market with risk neutral borrowers who wish to realize risky investment projects using external finance and risk averse lenders who wish to invest funds, borrowers and lenders enter into direct financing contracts. These contracts solve both, the problem of ex post information asymmetry of lenders who are not able to observe the outcome of the borrowers' investment projects without cost and the borrowers information problem not to know a single lender's reservation utility. Therefore, borrowers have to trade off two effects: at first offering lenders a higher repayment increases the probability of the contract being accepted. But, at second, this reduces borrowers' expected profit. Therefore borrowers choose contracts with a strictly positive probability of being rejected by lenders. As a result, there appears some degree of credit rationing as a consequence of the borrowers' optimal decision making.

The appearance of a bank in this context undoubtedly enhances total welfare in the market, since borrowers as well as lenders enjoy at least the same level utility compared to the situation of direct contracting. The reason is that using the instrument of deposit contracts banks collect funds of a large number of lenders and are, in turn, able to grant a large number of loans to borrowers. In this way there appears a portfolio effect in the sense of first order stochastic dominance. That is, pooling a large number of identical independent risks – in particular risk regarding the repayments from borrowers investment projects – increases the bank's probability to realize higher levels of repayment from the loan business. From the lenders' point of view the investment activities of the bank are less risky than direct contracting with borrowers. As a result, the bank is able to impose a strictly positive spread between repayments from loan contracts and repayments to lenders as long as the portfolio effect is sufficiently strong. Moreover, with a strong portfolio effect the bank may also reduce the problem of credit rationing since then it may be optimal to fund a larger number of investment projects than would be the case in the direct market. However, the additional profit then has to be shared with lenders in order to acquire the sufficient amount of deposits in this situation.

Of course, one may argue that there are some assumptions in the model which appear not very realistic. For example usually there are more than a single bank in the market competing for borrowers and lenders. However, this does not affect the

main arguments of the present model. Just the allocation of wealth among borrowers, bank, and lenders is changed in this situation. The assumption of borrowers' identical investment projects is not as crucial as it may appear at the first glance. The argument of first order stochastic dominance still holds with different investment projects even when they are positively correlated. Of course, in the latter case the portfolio effect would be weakened and in this way operating a bank would be more complicated. But altogether the qualitative results of the present model appear quite robust against modifications in basic assumptions.

## Appendix

### Proof of Proposition 1

Consider equation (6) first. Due to  $f(y) > 0 \forall y_i \in [0; \bar{y}]$ ,  $\mu \geq 0$  because of the Kuhn-Tucker-Conditions (8),  $U'(\cdot) > 0$  and  $G(EU([t(y_i)])) \geq 0$  the right hand side of (6) is definitely non-negative. However, having a closer look at this term shows that it may equal zero if and only if  $G(EU([t(y_i)])) = 0$  and  $\mu = 0$  hold simultaneously. But  $G(EU([t(y_i)])) = 0$  implies that the contract offered will never be accepted by a lender and as a result one needs not care about the repayment function in particular. Therefore, for a finance contract to be accepted in the optimum it must be true that  $G(EU([t(y_i)])) > 0$  and hence

$$\frac{G(EU([t(y_i)]))f(y_i) + \mu}{U'(\overline{t(y_i)})f(y_i)} > 0 \forall y_i < t_0.$$

Thus, for (6) to hold the left hand side of this condition has to be strictly positive, too. Because of  $g(\cdot) > 0$  – by definition a feature of the density function of any random variable – this is the case if

$$\int_0^{t_0} (y_i - \overline{t(y_i)} - c) dF(y_i) + \int_{t_0}^{\bar{y}} (y_i - t_0) dF(y_i) > 0.$$

But this latter term is exactly the borrower's expected profit provided the financing contract accomplishes which proves the second part of the proposition.

To prove the first part of Proposition 1 we now take a look at equation (7). Note, the term on the left hand side of (7) is the same as the term on the left hand side of (6) which has to be strictly positive in the optimum. Hence the term on the right hand side of (7) has to be also positive in the optimum, of course. However, as explained earlier, incentive compatibility of the financing contract demands  $t_0$  to be a constant. As a result, the right hand side of (7) does not change when  $y_i$  changes. With this result it is obvious that for the right hand side of (6) it must be true that

$$\frac{G(EU([t(y_i)]))f(y_i) + \mu}{U'(\overline{t(y_i)})f(y_i)} = \text{const.} \forall y_i < t_0.$$

This, however, can hold if and only if  $\mu > 0$ . Otherwise, i.e. with  $\mu = 0$ , optimality would require

$$\frac{G(EU(\overline{t(y_i)}))}{U'(\overline{t(y_i)})} = \text{const. } \forall y_i < t_0.$$

Due to  $G(\cdot) > 0$  in the optimum (see arguments above) and  $U''(\cdot) < 0$  by assumption this would imply

$$\overline{t(y_i)} = \text{const. } \forall y_i < t_0.$$

But this is impossible because by assumption a borrower can pay at most  $y_i$  to the lender since he does not have any own funds in addition to the payoff from the investment project. That is, a constant repayment in case of  $y_i < t_0$  would have to be zero which hurts incentive compatibility of the financing contract.<sup>8</sup>

At the end  $\mu > 0$  must be true in the optimum which implies – see the Kuhn-Tucker conditions (8) – the following repayment of the borrower in case of verification to the lender:

$$\overline{t(y_i)} = y_i - c \quad \forall y_i < t_0.$$

□

## Proof of Proposition 2

Consider the optimal structure of the deposit contract first. Since the situation is generally the same as with direct contracting of borrowers and lender, one can apply the arguments of the proof of Proposition 1 in perfect analogy to conditions (19) and (20): due to  $U'(\cdot) > 0$ ,  $h(\cdot) > 0$  – which is a feature of probability density functions – and  $\mu_{KG} \geq 0$  – which follows from the Kuhn-Tucker-Theorem it must be true from (19) that  $\lambda_{KG} > 0$ . In addition from (20) one can observe  $\lambda_{KG} = \text{const. } \forall Y_k$  because of  $t_{D0}$  being constant, too. Therefore, since  $U''(\cdot) < 0$  condition (19) holds if and only if

$$\overline{t_D(Y_k)} = Y_k - c \quad \forall Y_k < t_{D0}$$

which completes the proof of the first part of the proposition.

To prove the second part of Proposition 2 – i.e. the optimal structure of loan contracts – we have a look at the corresponding first order necessary conditions (17) and (18). In this regard inserting (18) into (17) and applying some standard manipulations yields

$$\mu_{KN} \frac{(1 - F(t_{K0})) - (t_{K0} - \overline{t_K(t_{K0})} - c) f(t_{K0})}{f(y_i)} = cf(t_{K0})$$

from which it is obvious that the right hand side of this latter equation is strictly positive since  $c > 0$  and  $f(\cdot) > 0$ . Therefore, the left hand side has to be positive,

<sup>8</sup>See Pausch (2003) for a detailed discussion of this argument.

too. Moreover from the Kuhn-Tucker-condition (24) it is apparent that  $\mu_{KN} \geq 0$  which requires that the numerator of the fraction on the left side of this equation is positive, too – i.e.

$$(1 - F(t_{K0})) - \left( t_{K0} - \overline{t_K(t_{K0})} - c \right) f(t_{K0}) > 0.$$

Hence for the optimal structure of the loan contract the following must hold:

$$\overline{t_K(y_i)} = y_i - c \quad \forall y_i < t_{K0}.$$

In addition, using the observation that the above expression is strictly positive in the optimum, one can show the borrower's participation constraint to be binding – i.e.  $\lambda_{KN} > 0$ . For this purpose note that the expression above is the same as the denominator on the right hand side of equation (18) except  $\frac{k}{n} > 0$ . Furthermore, due to  $c > 0$  it follows from the above relation that

$$(1 - F(t_{K0})) - \left( t_{K0} - \overline{t_K(t_{K0})} \right) f(t_{K0}) = \left( \overline{t_K(t_{K0})} - t_{K0} \right) f(t_{K0}) + (1 - F(t_{K0})) > 0$$

which is exactly the numerator of (18).

For the proof of the third part of the proposition rewrite the first order necessary condition (20) using the results derived so far to yield

$$\lambda_{KG} = \frac{1 - H(t_{D0})}{g(\text{EU}(\cdot)) (U'(t_{D0}) (1 - H(t_{D0})) - (U(t_{D0}) - U(t_{D0} - c)) h(t_{D0}))}.$$

From our earlier arguments it is already known that  $\lambda_{KG} > 0$ . If  $t_{D0} \geq t_{K0}$  would be true in the optimum, we would find  $1 - H(t_{D0}) = 0$  since  $Y_k \in [0; t_{K0}]$ . Hence, the bank can at most repay  $t_D = t_{K0}$  to lenders and  $t_{D0}$  will be payed at no time. Furthermore, due to  $U'(\cdot) > 0$ ,  $c > 0$  and  $h(\cdot) > 0$  the denominator would be strictly negative in this case. As a result we find  $\lambda_{KG} = 0$  in this situation which contradicts our observation that  $\lambda_{KG} > 0$  in the optimum. Hence, in the optimum  $t_{K0} > t_{D0}$  must hold.  $\square$

## Proof of Lemma 1

Consider the bank's total outcome from the loan business  $Y = \sum_{i=1}^k t_K(y_i)$  where  $t_K(y_i)$  are the borrowers' repayment functions which are identical due to identical and independent distributed random variables  $y_i$ . As a result all  $t_K(y_i)$  are identical and independent distributed, too. Let in the following  $f_K(t_K(y_i))$  denote the probability density function of any repayment function  $t_K(y_i)$  and  $\overline{H}(\gamma|k)$  the probability of  $Y \leq \gamma$  for a given amount of loans  $k$  which can be computed as follows<sup>9</sup>:

$$\overline{H}(\gamma|k) = \int_0^{t_{K0}} f_K(t_K(y_1)) \dots \int_0^{\gamma - \sum_{i=1}^{k-1} t_K(y_i)} f_K(t_K(y_k)) dt_K(y_k) \dots dt_K(y_1)$$

<sup>9</sup>See Larsen and Marx (1986, p. 142f).

where stochastic independence of the single borrower's repayments has been used.

The same can be done with respect to the probability of  $Y \leq \gamma$  when  $k + 1$  loans are granted:

$$\begin{aligned} \overline{H}(\gamma|k+1) &= \int_0^{t_{K0}} f_K(t_K(y_1)) \dots \cdot \\ &\cdot \int_0^{\gamma - \sum_{i=1}^{k-1} t_K(y_i)} f_K(t_K(y_k)) \int_0^{\gamma - \sum_{i=1}^k t_K(y_i)} f_K(t_K(y_{k+1})) \cdot \\ &\cdot dt_K(y_{k+1}) dt_K(y_k) \dots dt_K(y_1). \end{aligned}$$

Note, this latter expression differs from the former one only by the term

$$\int_0^{\gamma - \sum_{i=1}^k t_K(y_i)} f_K(t_K(y_{k+1})) dt_K(y_{k+1}) = F_K \left( \gamma - \sum_{i=1}^k t_K(y_i) \right) \in [0; 1].$$

Hence it must be true that

$$\overline{H}(\gamma|k+1) \leq \overline{H}(\gamma|k) \quad \forall \gamma.$$

If one considers that by definition  $Y_k = \frac{1}{k}Y$  holds, this result can be transferred to the probability distribution of  $Y_k$ . For this purpose we write<sup>10</sup>:

$$\overline{H}(\gamma|k) = P(Y \leq \gamma|k) = P\left(\frac{1}{k}Y \leq \frac{1}{k}\gamma|k\right) = P(Y_k \leq \frac{1}{k}\gamma|k) = H\left(\frac{1}{k}\gamma|k\right).$$

Now, define  $\frac{1}{k}\gamma = \delta$  and assume that  $k$  is sufficiently large, then the above relation can be rewritten as

$$\frac{d}{dk} H(\delta) \leq 0 \quad \forall \delta \in [0; t_{K0}]$$

which is a representation of first order stochastic dominance.<sup>11</sup>  $\square$

<sup>10</sup>See Larsen and Marx (1986, p. 134).

<sup>11</sup>See e.g. Wolfstetter (1999, p. 136f.).

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