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Taylor Rules and Macroeconomic Instability or How the Central Bank Can Pre-empt Sunspot Expectations

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Taylor Rules and Macroeconomic Instability or How the Central Bank Can Pre-empt Sunspot Expectations

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Abstract

This paper derives new results on the effects of employing Taylor rules in economies that are subject to real market imperfections such as production externalities. Taylor rules that aggressively respond to output can eliminate sunspot equilibria that arise from the increasing returns. The paper also finds that rules which should be chosen (avoided) in perfect market environments often yield (ensure) multiple (unique) rational expectations solutions in alternative settings. Therefore, exact knowledge on the degree of market imperfection may be pivotal for robust policy advice.

*I would like to thank Michael Burda, Roger Farmer, Andreas Schabert and two anonymous referees for helpful comments. Remaining errors are, of course, my own. I would also like to thank the UCLA Economics Department for its hospitality. This paper was written while Weder was a *DFG Heisenberg Fellow*. *Keywords*: Indeterminacy, Increasing returns to scale, Taylor Rules, Cash-in-advance economies. *JEL classification*: **E32**, **E52**.

1 Introduction

The Taylor (1993) rule provides a good description of how many central banks attempt to set interest rates in order to achieve stable prices while avoiding large fluctuations in output and employment. There is increasing evidence, however, that Taylor rules can be a source of economic instability by themselves. For example, Benhabib et al. (2001) demonstrate that steering under such policy may introduce real indeterminacy in an otherwise determinate economy. As a consequence, the Taylor rule debate frequently advises the monetary authority to assign aggressive backward-looking principles in which interest rates respond to predetermined variables, in particular to inflation (see for example Carlstrom and Fuerst, 1999, and Benhabib et al., 2003).

The present paper qualifies this assertion by suggesting that before spelling out concrete policy rules, the monetary authority must first be *au courant* with the specific economic environment. Carlstrom and Fuerst (2000, 2001) have demonstrated how monetary policy should depend on the timing convention in monetary models. The present paper adds another dimension to the discussion: the real side of the economy. In particular, it demonstrates that the presence of mild forms of market imperfections – modelled as arising from production externalities – may have fundamental consequences on how monetary policy should be conducted.

The motivation for the current research stems from the insights of recently formulated non-monetary dynamic general equilibrium models with sunspot equilibria and self-fulfilling prophecies.[1] In these models the possibility of a continuum of equilibria is the consequence of empirically plausible market imperfections – therefore, sunspot equilibria are more than theoretical curiosities. The present paper combines the two branches of the indeterminacy literature by money-augmenting these real models in order to examine the effects of monetary policy on indeterminacy as well as to assess monetary policy recommendations in suboptimal economies. The framework I will draw on is Wen (1998) which is currently the most successful attempt in terms of obtaining sunspot equilibria at small increasing returns and in generating realistic business cycles (see also Benhabib and Wen, 2003).

[1] A partial list includes Benhabib and Nishimura (1998), Christiano and Harrison (1999), Farmer and Guo (1994), Perli (1998), Schmitt-Grohé and Uribe (1997), Weder (1998, 2001), and Wen (1998).

1.1 Main results

Given its relation to recent policy-proposals (i.e. Carlstrom and Fuerst, 1999, 2000, and Benhabib et al., 2003), the current paper is primarily concerned with backward-looking Taylor rules. The main findings can be stated as follows: by responding sufficiently to (past) output movements in setting the nominal interest rate, the monetary authority can stabilize the (sunspot-driven) economy. The reasoning is that the nominal interest rate operates like an inflation tax – it distorts market outcomes. By the central bank fighting output fluctuations, sunspot blips will be dimmed: when the central bank builds up the costs of buoyant expectations these will no longer be sustainable in the first place. Phrased alternatively, the inflation tax-distortions operate by defeating the effects of high increasing returns to scale and non-fundamental equilibria can be removed.

I also find that Taylor rules work quite differently depending on the fundamentals of the economy. In fact, it appears to be the case that strategies which should be chosen (avoided) in perfect markets environments do in fact yield multiple (unique) rational expectations solutions in alternative settings. For example, backward-looking policy settings that ensure unique rational expectations in cases of constant returns to scale (aggressive with respect to inflation and modestly passive with respect to output) are connected to determinacy at moderate imperfections and *vice versa*. On the other hand, current-looking rules which always create indeterminacy under constant returns are a vehicle to remove technology-based sunspot equilibria. Consequently, the central bank should have a clear picture of market imperfections before setting policy rules. Existing empirical studies do not provide an unambiguous answer on the extent of the imperfections.

1.2 Related work

The argument which is developed in the current paper is framed within a fully specified cash-in-advance environment which has been shown by Carlstrom and Fuerst (2000) to have fundamentally different policy implications than New-IS-LM or money-in-utility-frameworks.[2] My study differs from theirs in three key aspects, however. First, their production technology is constant returns to scale. By contrast, I allow empirically plausible production externalities which lead to increasing returns. Second, in their model the central bank's nominal interest rate targets inflation only. The current paper considers versions of the original Taylor rule in which the interest rate is increased or decreased according to what is happening to both real GDP

[2] King (2000) is a good review of the New-IS-LM model.

and to inflation – as it turns out, output-targeting becomes essential in eliminating sunspot equilibria in imperfect economies. Furthermore, I present new results on interest smoothing.

The work here also shares similarities with Christiano (2000). Nevertheless, it differs from his analysis in several important ways. Christiano introduces money into the Christiano and Harrison (1999) model which assumes increasing returns to scale equal the inverse of the capital share. As a consequence, the model has two stationary states. In his model, the monetary authority sets the nominal interest rate proportional to the contemporaneous employment level and is thereby able to eliminate endogenous sunspot cycles in the neighborhood of one of the steady states. The present paper does not rest on scale economies that fault on empirical findings: the economy does not move between two steady states therefore the choice and also the optimal set of policy differs. More concretely, the monetary rule which is proposed by Christiano does not generally deliver determinacy at modest increasing returns – the present model puts a more stringent cap on policy.

The paper is organized as follows. The next section presents the model economy. Section 3 discusses the connection of monetary policy, market imperfections and sunspot equilibria. Section 4 addresses issues of interest smoothing and alternative monetary modelling. Section 5 concludes.

2 The economy

The physical setup of the economy's real part is a standard real business cycle model augmented by production complementarities. Currency is introduced by imposing restrictions on the timing of exchanges.

2.1 Preferences and technologies

The representative household seeks at time $t = 0$ to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t (\ln c_t - \eta l_t) \qquad 0 < \beta < 1,\ \eta > 0$$

where β, c_t and l_t are the discount factor, consumption and labor during t. The household rents labor and capital services to firms. All markets are perfectly competitive. The household's budget constraint can be stated as

$$M_{t+1} + P_t k_{t+1} = M_t + \Pi_t + P_t w_t l_t + P_t (r_t + 1 - \delta_t) k_t - P_t c_t + N_t (R_t - 1)$$

where P_t is the price level, M_t are the cash balances at the beginning of t, w_t is the real wage and r_t is the real rental rate of capital, k_t. The variable u_t denotes the degree of capital utilization. The depreciation rate of installed capital, δ_t, is increasing in utilization

$$\delta_t = \frac{1}{\theta} u_t^{\theta} \qquad \theta > 1.$$

N_t stands for one-period bank deposits which pay a short-term nominal interest given by R_t. Π_t is the profit flow from firms and intermediaries. A positive value is assigned to the inconvertible currency by assuming that during the shopping session the household is subject to the cash-in-advance-restriction

$$M_t + P_t w_t l_t \geq P_t c_t + N_t$$

that is, households circulate all their money (plus wage payments) to firms by consumption purchases and loans to the financial intermediaries.

Output is produced by a large number of competitive firms with identical technologies. The economy as a whole is affected by organizational synergies that cause the output of an individual firm to be higher if all other firms in the economy are producing more. The term A_t stands for these aggregate externalities. The production complementarities are taken as given for the individual optimizer and they cannot be priced or traded. Departures from constant returns to scale are measured by $\gamma > 0$.[3] A firm of type i has technology

$$y_{i,t} = A_t^{\gamma} (u_t k_{i,t})^{\alpha} l_{i,t}^{1-\alpha} \qquad A_t = (u_t k_t)^{\alpha} l_t^{1-\alpha} \quad \text{and} \quad 0 < \alpha < 1.$$

Here, k_t (l_t) denotes – by way of normalization – the economy-wide average capital (labor) input. Before hiring workers, firms must borrow cash at the short term rate from the financial intermediaries. This is because they start the period without sector to finance their wage bills. This is the second source of money demand.

2.2 Intermediaries and the central bank

The monetary branch of the economy comes in two parts: the intermediary sector and the central bank. The perfectly competitive intermediaries have two sources of cash. They accept loans from the households, N_t, which are

[3] See Benhabib and Farmer (1994) and others for an alternative (and in reduced-form) equivalent formulation that incorporates internal increasing returns at the intermediate-firm level in an imperfectly competitive market structure without free entry. In that case, the parameter γ would (also) relate to the monopoly markup.

repaid at the gross rate of interest R_t. Intermediaries also receive new cash injections, $M_{t+1}^s - M_t^s$, from the economy's monetary authority. This money is loaned to firms. The intermediaries' constraint is

$$N_t + M_{t+1}^s - M_t^s \geq P_t w_t l_t.$$

Firms' loans must be repaid at the end of the period, in time for the financial intermediaries to use the proceeds to repay the households.

Most central banks implement monetary policy by controlling a short-term nominal interest rate. Accordingly, it has become standard to represent monetary policy in terms of commitment to a rule for the nominal rate of interest. In the present paper, the monetary authority sets the short-run nominal interest rate based on what is happening to both real GDP and inflation. For example, a backward-looking rule is given by

$$R_{t+1} = R\left(\frac{\pi_t}{\pi}\right)^{\tau}\left(\frac{y_t}{y}\right)^{\omega} \qquad \tau \geq 0 \quad \omega \geq 0$$

or in linearized from

$$\widehat{R}_{t+1} = \tau\widehat{\pi}_t + \omega\widehat{y}_t$$

in which the variables appear as percentage deviations from their stationary states R, π and y. We denote rules with $\tau < 1$ ($\tau > 1$) as passive (aggressive) since the nominal interest rate moves less (more) than one-for-one with past inflation. Term ω refers to the weight given to deviations of real GDP from the target level. Since the general equilibrium setting imposes a money demand relationship (that is, the cash-in-advance setup), the interest rate rule implies that the money supply is endogenous.

2.3 Dynamics and calibration

In what follows, I restrict the analysis to a symmetric equilibrium in which $u_{i,t} = u_t$, $k_{i,t} = k_t$ and $l_{i,t} = l_t$. The aggregate production technology becomes

$$y_t = (u_t k_t)^{\alpha(1+\gamma)} l_t^{(1-\alpha)(1+\gamma)} \tag{1}$$

which exhibits returns to scale equal to $1+\gamma$. The economy's dynamics are given by

$$\begin{aligned}
\eta c_t &= \frac{w_t}{R_t} = \frac{\alpha y_t l_t^{-1}}{R_t} && (2)\\
u_t^{\theta} &= \alpha \frac{y_t}{k_t} && (3)\\
E_t \frac{\beta}{c_{t+1}\pi_{t+1}} &= \frac{1}{c_t R_t} && (4)\\
\frac{1}{c_t R_t} &= E_t \frac{1}{c_{t+1} R_{t+1}} \beta [\alpha \frac{y_{t+1}}{k_{t+1}} + 1 - \frac{1}{\theta} u_{t+1}^{\theta}] && (5)\\
c_t + k_{t+1} &= y_t + (1 - \frac{1}{\theta} u_t^{\theta}) k_t. && (6)
\end{aligned}$$

Equation (2) describes the leisure-consumption trade-off and (3) pins down the optimal utilization rate of capital. Equations (4) and (5) are the usual Fisher and Euler conditions. (6) repeats the intertemporal constraint. No closed-form solution exists, thus the model must be approximated. In log-linearized form, the dynamics boil down to (the Appendix discerns details)

$$\begin{bmatrix} E_t \widehat{c}_{t+1} \\ \widehat{R}_{t+1} \\ \widehat{k}_{t+1} \\ E_t \widehat{R}_{t+2} \end{bmatrix} = \mathbf{M} \begin{bmatrix} \widehat{c}_t \\ \widehat{R}_t \\ \widehat{k}_t \\ \widehat{R}_{t+1} \end{bmatrix}.$$

The dynamical system contains two non-predetermined (or jump) variables: $\widehat{c}_{t+1}$ and $\widehat{R}_{t+2}$. Therefore, indeterminacy requires that at most one eigenvalue of the 4×4-matrix $\mathbf{M}$ is outside the unit circle. Two eigenvalues larger than one (and two smaller than one) imply determinacy. If, say, exactly three eigenvalues have modulus less than one, then there are multiple rational expectations solutions which take on the form

$$\begin{bmatrix} \widehat{c}_{t+1} \\ \widehat{R}_{t+1} \\ \widehat{k}_{t+1} \end{bmatrix} = \widetilde{\mathbf{M}} \begin{bmatrix} \widehat{c}_t \\ \widehat{R}_t \\ \widehat{k}_t \end{bmatrix} + \begin{bmatrix} \zeta_{t+1} \\ 0 \\ 0 \end{bmatrix}. \qquad (7)$$

Here ζ_{t+1} is an arbitrary random variable with $E_t \zeta_{t+1} = 0$.

Table 1: Calibration

α	β	δ
0.30	0.99	0.025

I will now assign parameter values and demonstrate the empirical plausibility of sunspot equilibria. The time unit is taken to be a quarter of a year. The calibration is based on empirical observations on post-war U.S. data. The capital share, α, is set equal to 30 percent, the discount factor, β, is chosen to be 0.99 and the steady state rate of capital depreciation, δ, is 2.5 percent. The parameter θ can then be derived from steady state conditions:

$$\theta = \frac{1/\beta - 1 + \delta}{\delta} = 1.4.$$

When abstracting from the monetary side, the calibration implies a minimum degree of externalities, $\gamma^{\min}$, needed for indeterminacy that amount to 0.1037. The value is reasonable given empirical findings. For example, Caballero and Lyons (1992) obtain increasing returns estimates in the order of 1.26 to 1.56. Baxter and King (1991) find returns to scale of 1.53, however, combined with a standard error of 0.56. Burnside, Eichenbaum and Rebelo (1995) report a point estimate of 0.98. Again their standard error of 0.34 is large. Basu and Fernald (1997) also find close to constant returns, however, the imparted estimation-uncertainty is significant again.

The reasoning for multiplicity in the real economy is as follows. Equations (1) and (3) entail the reduced form-output

$$y_t = \text{const} * k_t^{\frac{\alpha(1+\gamma)(\theta-1)}{\theta-\alpha(1+\gamma)}} l_t^{\frac{(1-\alpha)(1+\gamma)\theta}{\theta-\alpha(1+\gamma)}}.$$

Thus, the effective labor-output elasticity is larger than unity for[4]

$$\gamma > \gamma^{up} = \frac{\alpha(\theta-1)}{\alpha + (1-\alpha)\theta} > 0.$$

Accordingly, the reduced-form labor demand curve is upward sloping at mild increasing returns.[5]

Now, how do sunspot equilibria come about? Suppose people suddenly have pessimistic expectations and expect lower future income. The permanent income motive will reduce today's consumption. The static-first order

[4]If the depreciation costs are high ($\theta \to \infty$) and accordingly capital utilization is set constant by agents, the condition reduces to that found in Harrison and Weder (2002); the minimum increasing returns fall outside of the plausible region. However, the condition also implies an upward-sloping labor demand curve and all of the below qualitative results can be replicated in such an environment.

[5]Upward sloping labor demand arises for $\gamma \geq \gamma^{\min} \geq \gamma^{up} = 0.0945$. The reason for the gap between $\gamma^{\min}$ and γ^{up} and the gap's positive dependence on the time period's length comes from discounting future benefits (costs) at the relevant intertemporal margins. When formulating the model in continuous time, the two thresholds are identical – the same sort of gap arises in the original Benhabib and Farmer (1994) model (see Salyer, 1995).

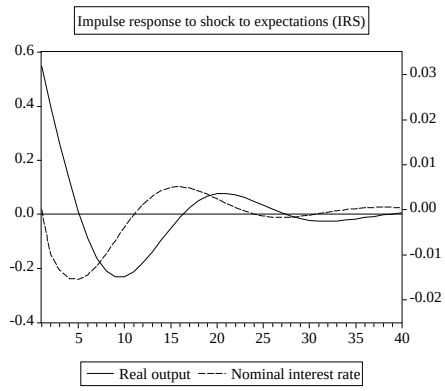

Figure 1: *Shock to consumption demand*

condition (2) implies that the labor supply schedule moves outwards. Given the upward sloping equilibrium labor demand curve, employment and investment will actually both fall today. As a consequence, the future capital stock, output and consumption will all be low, and in sum, the initially pessimistic expectations will be self-fulfilled. The sunspot circle is completed.

The determinacy properties of the model do not change when money is introduced and the central bank pegs the nominal rate one-for-one with past inflation and it does not react to output movements ($\tau = 1$ and $\omega = 0$). The minimum increasing returns are 1.1037; the specific interest rate policy causes the economy to behave identically to the model that only includes the real sector.

It is useful to plot impulse response functions for real output and the nominal interest rate (which is proportional to one-period-delayed inflation) in the presence of production externalities. I set $\tau = 1$, $\omega = 0$, assume mild externalities ($\gamma = 0.11$) and hit the artificial economy by a one-time positive shock to consumption, i.e. the $\zeta_{t+1}-$shock to expectations as in equation (7).[6] Output increases at the demand shocks's impact – the presence of aggregate externalities increases labor input such that an expansionary effect is possible for a given capital stock. Moreover, (i) inflation falls in the initial stages of the economic boom and (ii) the contemporaneous correlation of output and the short-run nominal interest rate is slightly positive at 0.24. This is similar to the number for the U.S. economy as reported in Cooley and Hansen (1995, Table 7.1.).

[6]To avoid a unit eigenvalue, I calibrate $\tau = 1.001$.

3 How should monetary policy be conducted?

This Section discusses the effects of various versions of the Taylor rule on the qualitative dynamics of the artificial economy. It opens by assuming that the central bank sets nominal interest rates after having observed (past) inflation and output (Section 3.1.). This is followed by discussions of forward-looking and current-looking rules (Sections 3.2. and 3.3.).

3.1 Indeterminacy zones with backward-looking rules

This Subsection will examine various versions of backward-looking Taylor rules in economies with constant and increasing returns to scale. I will start combing for parametric indeterminacy zones by considering a constant returns to scale technology ($\gamma = 0$) which will help in understanding the increasing returns cases.

3.1.1 Constant returns to scale and $\omega = 0$

When I set $\omega = 0$, the four eigenvalues of $\mathbf{M}$ are

$$\left\{ \frac{1-\beta(1-\alpha)(1-\delta)}{\alpha\beta}, \frac{\alpha}{1-\beta(1-\alpha)(1-\delta)}, \tau, 0 \right\}.$$

The eigenvalues are the same as those reported in Carlstrom and Fuerst (2000) despite the absence of variable capital utilization and variable capital depreciation in their model. The third eigenvalue exposes the policy's direct impact on dynamics: policy-induced indeterminacy can be avoided simply by responding aggressively to past inflation whereas passive responses lead to multiplicity. The reasoning for the occurrence of sunspot equilibria can be understood as follows: suppose that current inflation increases by one percent, which given an aggressive policy, implies that the $t+1$ nominal rate goes up by more than one percent. The inflation tax depresses future consumption and lifts current consumption – the real rate goes down. This, however, is only possible when the rate of inflation, π_{t+1}, increases. As a result, the central bank's target R_{t+2} rises by even more and policy induces an unsustainable explosive inflation-pattern. The initial rise in inflation is not supported and consequently the sunspot cycle is stopped. In contrast, if the bank follows a passive policy, the chain of events remains stationary and sunspot expectations are self-fulfilled.

Figure 2 shows the impulse response dynamics of real output and of the nominal interest rate after a one-time positive shock to consumption. The calibration uses $\tau = 1/2$, $\omega = 0$ and $\gamma = 0$, thus, the passive response of

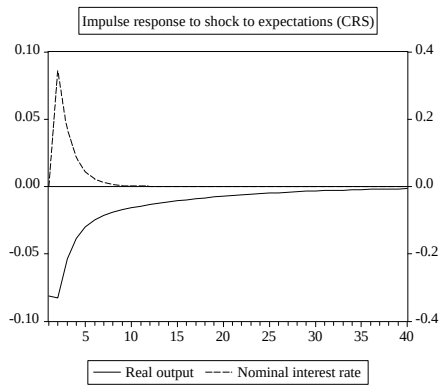

Figure 2: *Shock to consumption demand*

monetary policy is the only source of indeterminacy. The positive shock to consumption demand induces a contractionary output response. Economic activity falls at impact; the direction of output's response is not surprising given the assumptions of market-clearing, perfectly flexible prices and constant returns to scale. Labor demand is downward-sloping, thus, the positive consumption shock decreases labor input. Furthermore, the transitory consumption boom is inflationary and the next period's nominal rate goes up. Indeterminacy when arising from the central bank's policy appears not to generate an empirically plausible output response to demand innovations (see for example Blanchard and Quah, 1989). At least for the specific model considered here, the analysis suggests that increasing returns (or sticky prices) are required to produce a (more) realistic output reaction to real-side demand shocks.[7]

3.1.2 Constant returns to scale and $\omega > 0$

Carlstrom and Fuerst (2000) do not consider the effects of output-targeting on the economy's dynamics. This will be done here next as a further step towards an understanding of Taylor rules in suboptimal equilibria. Suppose that the central bank reacts in part to past movements in output, $\omega > 0$.

Dynamics can be derived analytically within two special cases. The first assumes a simplified economy without capital and in which output is produced with the linear technology

$$y_t = Al_t$$

[7]The same picture arises when the interest rate is shocked so as to mimic changes in monetary policy.

or simply $\alpha \to 0$. The model reduces to the scalar equation

$$E_t \widehat{R}_{t+2} = (\tau - \omega)\widehat{R}_{t+1}.$$

Indeterminacy arises for parameter constellations that satisfy

$$\omega - 1 < \tau < 1 + \omega. \tag{8}$$

The right-hand-side inequality in (8) repeats the aforementioned result. Namely that active inflation-fighting policies eliminate sunspot equilibria. More generally, the central bank must simultaneously select both policy parameters because an intermediate range of τ values that generates indeterminacy for given ω exists. Put in another way, letting for example $\tau = 1.5$, the bank's output-coupled response must fall outside $1/2 < \omega < 2.5$ otherwise policy-induced cycles crop up.

The picture becomes slightly more complex once the complete model is considered. By employing another special case, namely $\delta \to 1$ as in McCallum (1989), the basic insights can be derived from tractable analytical expressions.[8] The four eigenvalues of $\mathbf{M}$ are

$$\left\{\alpha, \frac{1 + \alpha\beta \pm \sqrt{4\alpha\beta(\omega(1-\alpha\beta) - \tau) + (1+\alpha\beta\tau)^2}}{2\alpha\beta}, 0\right\}.$$

It is quite instructive to consider $\tau = 1$, in which the central bank follows neither a passive nor an active policy. We can then ask what is the separate effect of output-targeting, that is $\omega > 0$? Starting at $\omega = 0$, the eigenvalues are

$$\left\{\alpha, \frac{1}{\alpha\beta}, \tau, 0\right\}$$

and the economy would drift into the indeterminacy zone by decreasing τ. It is easy to see that by increasing ω, the third eigenvalue

$$\frac{1 + \alpha\beta - \sqrt{(1-\alpha\beta)(1-\alpha\beta(1-4\omega))}}{2\alpha\beta}$$

will be pushed inside the unit circle. Indeterminacy arises and analogously to the above simplified model, there is also an upper level of ω

$$\omega > \frac{2(1+\alpha\beta)}{1-\alpha\beta} > 0$$

[8] By allowing $0 < \delta < 1$, the same results apply but for presentational ease, I report these cases in numerical fashion only.

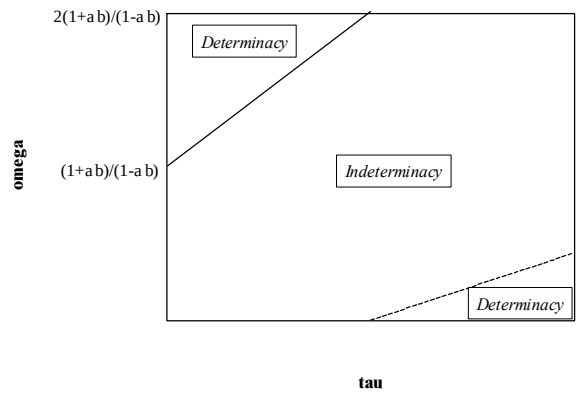

Figure 3: *Sunspot zones when returns to scales are constant; backward-looking rules.*

at which determinacy enters again since two eigenvalues will have modulus larger than one.[9] If on the other hand, $\tau \to 0$, determinacy is obtained for

$$\omega > \frac{1+\alpha\beta}{1-\alpha\beta} > 0.$$

Here the result from the capital-free version of the economy repeated: output-targeting alone can produce determinacy. However, it should not be chosen in a "medium range" otherwise sunspot cycles arise. Figure 3 summarizes.

Indeterminacy from output-targeting arises as follows. Suppose again that current inflation increases. Future nominal interest rates will rise which will increase today's consumption (see also Figure 2). Therefore, the labor supply curve shifts inward which, since the capital stock is given, lowers current output. Accordingly, the future interest rate will rise by less than under the pure inflation-peg:

$$\widehat{R}_{t+1} = \tau \underset{(\uparrow)}{\widehat{\pi}_t} + \omega \underset{(\downarrow)}{\widehat{y}_t}.$$

As a consequence, stationary sunspot sequences become more likely when the central bank targets output. These sunspot sequences are only possible when the ω-weights fall into a certain range. Very small values of ω – in combination with aggressive inflation-targeting – deliver determinacy. For very large ω-values, the output-related movements will be too strong to preserve stationarity and sunspots can be ruled out again.[10]

[9] Another way to see this is by noting that as $\omega \to \infty$, the expressions under the square roots go to infinity.

[10] Numerically, in the full model with $0 < \delta < 1$, ω must be larger than 30 (see also

3.1.3 Increasing returns to scale

Next, I turn to cases in which the economy is imperfect and subject to real indeterminacy stemming from production externalities. That is, I will discuss how the determinacy properties of the Wen (1998) model change when a monetary policy rule is introduced. The question I ask is: can the central bank stamp out externality-generated sunspot equilibria by choosing an appropriate Taylor-design?

When the central bank reacts to inflationary movements only, indeterminacy arising from mild production externalities cannot be eliminated. This can easily be seen by numerically checking for the minimum increasing returns for alternative τ-values (other parameters are as in Table 1). These returns to scale are 1 at $\tau \to 0$, they jump to 1.103708061 at $\tau = 1$ and are 1.103734395 at $\tau = 1000000$. I therefore shift attention to cases involving $\omega > 0$.

Generally, once $\gamma > 0$, analytical versions of the eigenvalues become highly non-linear in γ (and the other key parameters) and, thus offer only limited insights. Yet by assuming $\delta \to 1$, one can show how monetary policy must include targeting output in order to eliminate indeterminacy. Consider the case in which inflation-targeting is neither active nor passive, $\tau = 1$ (to simplify notation). Then the matrix $\mathbf{M}$'s eigenvalues at $\omega = 0$ are

$$\left\{0, 1, \frac{(-1+\beta(1+\alpha(1+\gamma)(\alpha(1+\beta)-1)) \pm \sqrt{\Gamma^2 - 4\alpha\beta\Delta}}{2\alpha\beta(\beta(1+\gamma)-1)}\right\}$$

$$\begin{aligned}\Gamma &\equiv 1-\beta(1-\alpha)(1+\alpha(1+\gamma)) - \alpha\beta^2(\alpha+\alpha(1+\gamma)) \\ \Delta &\equiv (1-\beta(1+\gamma))(\alpha(1-\beta)+(1+\alpha-\beta(1+3\alpha+\alpha\beta))\gamma).\end{aligned}$$

Simple algebra delivers the minimum increasing returns

$$\gamma^* = \frac{(1+\alpha)(1-\beta)(1+\alpha\beta)}{1-\beta+\alpha^2\beta(1-\beta)+\alpha(4\beta+\beta-1)}$$

at which indeterminacy kicks in. At this point, the four eigenvalues are

$$\left\{0, 1, -1, \frac{1+\alpha-\beta-2\alpha\beta-\alpha^3\beta^2(2-\beta)+\alpha^3\beta(3-\beta(2-\beta))}{\alpha\beta(1+\alpha(1-\beta(3-\alpha))}\right\}$$

and the third eigenvalue crosses into the unit circle for increasing returns higher than γ^*.[11] While holding fixed the threshold level of increasing re-

Figure 5).

[11] The fourth eigenvalue has modulus less than one for

$$\frac{1-2\beta-\beta^2-\sqrt{1-16\beta+18\beta^2+\beta^4}}{2\beta(\beta-3)} < \alpha < \frac{1-2\beta-\beta^2+\sqrt{1-16\beta+18\beta^2+\beta^4}}{2\beta(\beta-3)}$$

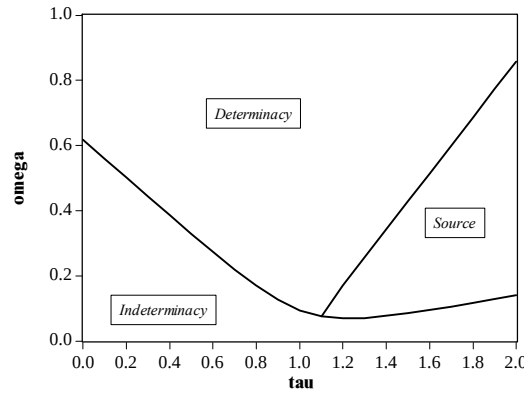

Figure 4: *Inflation and output targeting under increasing returns to scale ($\gamma = 0.11$), backward-looking rules.*

turns – numerically, the value is 1.0144 – I introduce output-targeting. In particular, by boosting $\omega \to \infty$, the eigenvalues become

$$\{0, \alpha, \infty, \infty\}.$$

That is, policies that are sufficiently offensive in countering output movements help to rule out indeterminacy (numerically, determinacy already arises for $\omega > 0.03834$).

The availability of tractable analytical expressions is highly dependent on specific parameter assumptions; a way of circumventing this is by employing numerical solutions. Let us step up γ to 0.11 so that production complementarities induce sunspot equilibria in environments without money (the other parameters are as in Table 1). The specific value draws on Benhabib and Wen (2003) who propose it by observing that the model's cycle frequency matches that of U.S. output. Furthermore, the magnitude of scale economies falls into the scope of the studies mentioned in Section 2.3.

Figure 4 pins down the policy advice under the assumption of $\gamma = 0.11$. The only way for the central bank to tackle endogenous cycles that arise from the production externalities is by working against output movements: if $\omega = 0$, indeterminacy always occurs. The Figure also shows that for any given τ, the output-related response must be sufficiently large to obtain unique solutions: even when ω takes on strictly positive values, the specific choice of τ may imply any of the three possible regimes (indeterminacy,

which is plausible (for example, when setting the discount factor at 0.99, the capital share is restricted to be between 0.005 and 0.978).

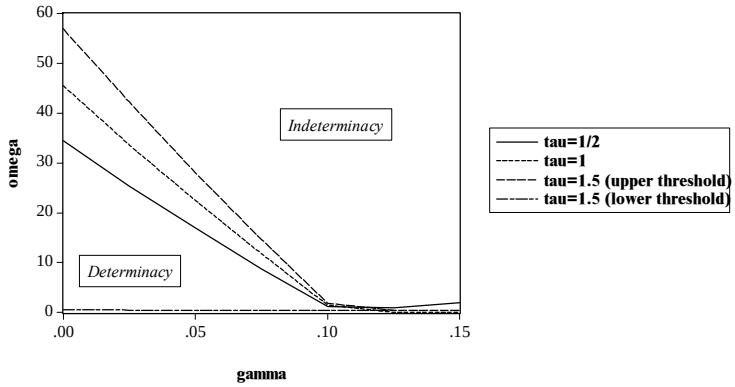

Figure 5: *Output-targeting versus minimum increasing returns to scale (at alternative values for τ and calibration as in Table 1); backward-looking rules.*

determinacy or source – the latter region may imply endogenous cycles on its own). In agreement with the above analytical experiment, monetary policies that are very offensive with respect to output fluctuations (i.e. $\omega \to \infty$) are generally a good insurance against indeterminacy.

Figure 5 generates determinacy-indeterminacy-loci in the $\gamma - \omega-$space for alternative degrees of inflation-targeting. The graph shows that (i) the minimum degree of output-targeting falls when increasing returns , (ii) it rises with the degree of inflation-targeting and (iii) it must be substantial to generate determinacy at constant returns and increasing returns ($\omega > 30$).[12]

The result that output-targeting eliminates sunspot equilibria is reminiscent of Guo and Lansing (1998) and Christiano and Harrison (1999) who find that progressive tax systems effectively eliminate indeterminacy by taxing away increasing returns in real economies. The Taylor policy suggested here generates a similar distortionary effect. The economic reasoning is easy to see: if ω is sufficiently large, then output fluctuations (i.e. production bunching) that arise from believing in them simply become too costly. Let us walk through a sunspot sequence that is stopped by the central bank for further understanding of the result. Suppose that people embellish optimistic expectations without any real cause. By projecting high future income, they will ratchet up today's consumption expenditures. The high increasing re-

[12]There exists a lower bound for every τ-value under which determinacy holds. For $\tau = 1/2$ ($\tau = 1$) the threshold does not involve positive values of ω and therefore was not plotted. Moreover, for $\tau < 1$, the minimum ω increases again after passing through the threshold level $\gamma^{\min}$.

turns will increase today's employment and output as a result of the upward sloping equilibrium labor demand curve. Now, if the output response of the central bank is strong enough, then R_{t+1} will increase and the initially sanguine expectations will be preempted (i) by increasing the costs of hiring labor (Equation 3) and (ii) by reducing $t+1$ consumption demand (Equation 5) which will lower $t+1$ employment. The initially optimistic expectations are not fulfilled and the sunspot cycle is broken.

To conclude, the analysis of backward-looking rules suggests that central banks should be careful about the specific economic environment when setting policy rules. I find that Carlstrom and Fuerst's (2000) advocated backward-looking inflation-targeting rule no longer guarantees uniqueness. It is imperative for the interest rate to intercept output fluctuations. Furthermore, the (numerical) analysis suggests that Taylor's (1993) original principle ($\tau = 1.5$ and $\omega = 0.5$) constitutes a successful code of stabilizing sunspot fluctuations at $\gamma = 0.11$. However, if increasing returns are slightly smaller – say, at $\gamma = 0.10$ – or constant then the economy skids into a regime under the clout of sunspots. A decidedly aggressive output-targeting is only successful preemptive strategy that comes out of both technological regimes' analyses.

Proposition 1 *(Backward-looking rules) The central bank can rule out indeterminacy by aggressively targeting inflation when returns to scale are constant. However, the presence of mild production externalities requires a tough stand on output in order to eliminate indeterminacy. Only policies that very aggressively target (past) output have the potential of ruling out indeterminacy independent of the degree of market imperfections.*

3.2 Indeterminacy zones with forward looking rules

It may be suspected that the result is dependent on assuming a backward-looking policy rule. Yet, the general picture that Taylor rule settings should consider technology does not change when the central bank pays attention to expected values such as in

$$\widehat{R}_t = \tau E_t \widehat{\pi}_{t+1} + \omega E_t \widehat{y}_{t+1} \qquad \tau \geq 0 \quad \omega \geq 0. \tag{9}$$

The Appendix describes the solution of the complete model which boils down to

$$\begin{bmatrix} E_t \widehat{c}_{t+1} \\ E_t \widehat{R}_{t+1} \\ \widehat{k}_{t+1} \end{bmatrix} = \mathbf{J} \begin{bmatrix} \widehat{c}_t \\ \widehat{R}_t \\ \widehat{k}_t \end{bmatrix}$$

which involves the two jump-variables $\widehat{c}_{t+1}$ and $\widehat{R}_{t+1}$.

3.2.1 Constant returns to scale and $\omega = 0$

Beginning with the case of $\omega = \gamma = 0$, the three eigenvalues of $\mathbf{J}$ are

$$\left\{ \frac{1-\beta(1-\alpha)(1-\delta)}{\alpha\beta}, \frac{\alpha}{1-\beta(1-\alpha)(1-\delta)}, \frac{1}{\tau} \right\}$$

which reverses the results obtained under backward-looking rules: all policy responses $\tau > 1$ are precluded, so it is necessary for policy to be passive in Taylor's sense. The underlying economics are easy to grasp. The real rate is given by (using equation 8)

$$r_t = R_t - E_t \pi_{t+1} = \frac{\tau - 1}{\tau} R_t.$$

It decreases when current (sunspot-driven) consumption rises. For $\tau > 1$, the fall in the real rate translates into a fall of the nominal rate. Thus, the inflation-consumption distortion declines and the sunspot cycle is completed.

3.2.2 Constant returns to scale and $\omega > 0$

Let us turn to cases involving $\omega > 0$. Resorting again to a linear production technology without capital leads to the simplified model

$$E_t \widehat{R}_{t+2} = \frac{1}{\tau - \omega} \widehat{R}_{t+1}$$

from which more general Taylor formulas can be discussed analytically: determinacy requires

$$\tau > \omega - 1 \qquad \text{for} \qquad \tau < \omega$$

or

$$\tau < \omega + 1 \qquad \text{for} \qquad \tau > \omega.$$

Now there are two zones of indeterminacy. Again, if $\omega = 0$, the monetary authority should be passive when responding to expected inflation. If $\tau \to 0$, the monetary authority should be passive when responding to expected output. Moreover, monetary policy should respond in tendency equally to output and to inflation. Asymmetric responses will likely drive the economy into sunspot districts.

Consider next the more general case with capital accumulation (and $\delta \to 1$). The three eigenvalues of $\mathbf{J}$ are

$$\{\alpha, \frac{\tau - \omega + \alpha\beta(1+\omega) \pm \sqrt{(\alpha\beta(1+\omega) + \tau - \omega)^2 - 4\alpha\beta\tau}}{2\alpha\beta\tau}\}.$$

Four special cases will demonstrate the effect of policy. If $\tau \to 0$, it is easy to see that the second and third eigenvalues approach infinity and sunspot equilibria are removed. Furthermore, if $\tau \to \infty$ the eigenvalues are

$$\{\alpha, 0, \frac{1}{\alpha\beta}\}$$

and contrary to the backward-looking case, active inflation-targeting produces indeterminacy. Of course, this mimics the simplified version's policy proposals. If output-targeting is very passive, $\omega \to 0$, then the eigenvalues are given by

$$\{\alpha, \frac{1}{\alpha\beta}, \frac{1}{\tau}\}$$

and dynamics are determined by $\tau \lesseqgtr 1$. The eigenvalues are

$$\{\alpha, 0, \infty\}$$

if $\omega \to \infty$ which again suggests a passive output-response. In particular, there is an upper level, ω^*,

$$\omega > \omega^* = \frac{1 + \tau + \alpha\beta(1+\tau)}{1 - \alpha\beta} > 0$$

for which indeterminacy always applies. In sum, policy should not be too aggressive. The economic reasoning for this indeterminacy is easy to understand. Suppose policy is aggressive. A sunspot-driven increase in consumption lowers the real rate and given $\tau > 1$ it also lowers the nominal rate. The rise in consumption will decrease labor supply and therefore output. Current investment falls and next period's capital stock will be smaller. Thus, it is likely that future output is lower as well – there will be downward pressure on the current nominal rate. If the fall of R_t is sufficient because policy reacts sharply to the output fluctuations, the sunspot cycle cannot be broken.

3.2.3 Increasing returns to scale

Let us now go through the increasing returns to scale scenario. Beginning with the case $\tau = 1$ and $\omega = 0$, the eigenvalues are

$$\{1, \frac{-1 + \gamma + \alpha^2\beta^2(1+\gamma) + \beta\left(1 + (1+2\alpha)\gamma - \alpha^2(1+\gamma)\right) \pm \sqrt{\Theta + \Lambda^2}}{2(\alpha\beta^2(1+\gamma) - \alpha\beta)}\}$$

$$\begin{aligned}
\Theta &\equiv 4\alpha^2\beta(1 - (1+\gamma)\beta)(\beta - 1 + (2\beta - 1)\gamma) \\
\Lambda &\equiv 1 + \gamma - \alpha^2\beta^2(1+\gamma) - \beta(1 + (1+2\alpha)\gamma - \alpha^2(1+\gamma)).
\end{aligned}$$

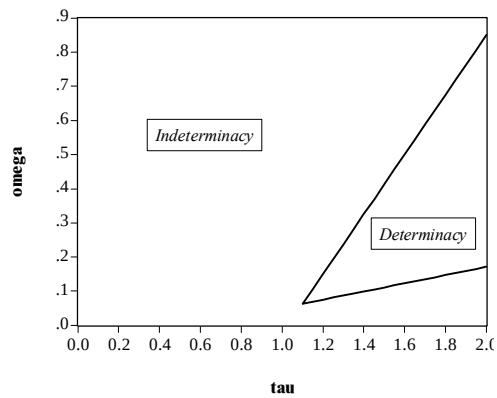

Figure 6: *Indeterminacy zones with increasing returns to scale ($\gamma = 0.11$); forward-looking rules.*

Indeterminacy arises for

$$\gamma > \gamma^*$$

and eigenvalues at γ^* and $\omega = 0$ are

$$\{-1, 1, -\frac{1 - \alpha(3 - \beta) + \alpha\beta}{1 + \alpha - 3\alpha\beta + \alpha^2\beta}\}$$

where the third eigenvalue's modulus is smaller than one. Now, by holding everything else constant and by letting ω go to infinity, the eigenvalues become

$$\{0, \alpha, \infty\}.$$

This implies that simply following (very) aggressive inflation-targeting does not bring about determinacy. This is the antithesis of the backward-looking policy's proposal.

The result can also be derived by turning to numerical solutions. Suppose that production externalities are $\gamma = 0.11$. Figure 6 plots determinacy regions in the $\tau - \omega$-space. Uniqueness requires a tough policy with respect to inflation which must be backed up by a *relatively* mild response to output. Moreover, the constant returns proposal ($\tau < 1$) always creates sunspot equilibria now, thus, the policy advice is different across technological regimes. For example, if the central bank sets $\tau = 1.5$, then sunspot cycles are ostracized for weak output-targeting $0.12 < \omega < 0.42$. However, at slightly

smaller externalities at which indeterminacy does not arise from technology, say at $\gamma = 0.10$, that very policy creates indeterminacy.

The economics behind the result are parallel to those in Section 3.1.: active interest rate policy can be used to counter the benefits of production bunching and is therefore capable to automatically stabilizing the economy.

Proposition 2 *(Forward-looking rules) At constant returns to scale, passive policies rule out sunspot equilibria. In the presence of mild production externalities, output-targeting does not create determinacy alone and it must be supported by aggressive inflation-targeting.*

3.3 Indeterminacy with current-looking rules

Let us next consider current-looking rules of the form

$$\widehat{R}_t = \tau \widehat{\pi}_t + \omega \widehat{y}_t \qquad \tau \geq 0 \quad \omega \geq 0.$$

The dynamics of the complete model are given by

$$\begin{bmatrix} E_t \widehat{c}_{t+1} \\ E_t \widehat{R}_{t+1} \\ \widehat{k}_{t+1} \end{bmatrix} = \mathbf{W} \begin{bmatrix} \widehat{c}_t \\ \widehat{R}_t \\ \widehat{k}_t \end{bmatrix}.$$

3.3.1 Constant returns to scale and $\omega = 0$

When $\omega = 0$ and when returns to scale are constant, the three eigenvalues of $\mathbf{W}$ are

$$\left\{ 0, \frac{\alpha}{1 - \beta(1-\alpha)(1-\delta)}, \frac{1 + \tau(1 + (1-\alpha)(1-\delta)\beta) + \beta(1-\alpha)\delta}{\alpha\beta\tau} \right\}.$$

It is easy to show that the first two eigenvalues are always inside the unit circle. Since the underlying dynamical system involves two jump variables, the economy is always subject to indeterminacy.

3.3.2 Constant returns to scale and $\omega > 0$

There is always indeterminacy even when output is targeted by the central bank. This can be seen by inspecting the eigenvalues of $\mathbf{W}$:

$$\left\{ 0, \frac{\alpha}{1 - (1-\alpha)\beta(1-\delta)}, \frac{1 + \tau - \omega - \beta(1 - \omega - (1-\alpha)\delta(1 + \tau - \omega) - \tau(1-\alpha)}{\alpha\beta\tau} \right\}.$$

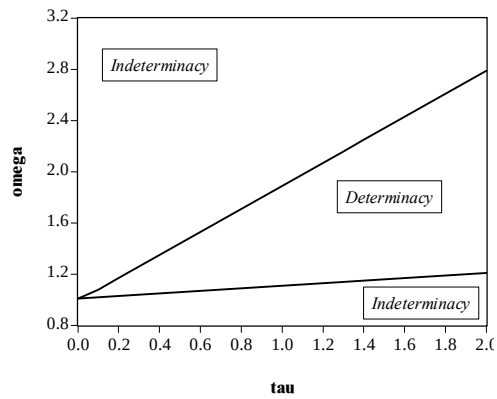

Figure 7: *Indeterminacy and determinacy regions when mild production externalities ($\gamma = 0.11$) are present; current-looking rules.*

The first two eigenvalues are always inside the unit circle. In fact, they do not even contain the parameter ω. Therefore, the upshot appears to be a clear case against using current-looking rules either with or without output-targeting. This is easy to understand from the simplified version of the model. The dynamics boil down to

$$0 = \frac{1 + \omega - \tau}{1 + \omega} E_t \widehat{\pi}_{t+1}$$

which implies that future rates of inflation are pinned down. However, the initial rate, $\widehat{\pi}_t$, is not pinned down. The same holds for period t's nominal interest rate. Real indeterminacy arises.

3.3.3 Increasing returns

The picture changes, however, when $\gamma > \gamma^{\min}$. Figure 7 plots determinacy regions while setting $\gamma = 0.11$. Now there exists an area in which applying a current-looking Taylor rule can eliminate sunspot equilibria. In a nutshell, the central bank must react sufficiently to output and (to a lesser extent) to inflation.

Current-looking rules may therefore be a channel that eliminates sunspot fluctuations that arise from scale economies. In a sense, the result parallels Christiano (2000). He finds that – in the neighborhood of the high level steady state – a policy that sets the nominal interest rate proportional to current employment can stabilize the economy. The current economy with a more realistic departure from constant returns to scale places a much more

stringent requirement on policy. In fact, output-targeting must sufficiently *lean-against-the-wind* and always be supported by inflation-targeting.

Proposition 3 *(Current-looking rules) Current-looking rules always imply indeterminacy at constant returns to scale. Indeterminacy arising from mild externalities can be eliminated by aggressive output-targeting (in combination with inflation-targeting).*

To conclude, Section 3 has shown that various formulations of the Taylor rule generate very different economic dynamics depending on the economic environment. In particular, the presence of market imperfections (production externalities or monopolistic competition) inverts the policy recommendations that apply when these market imperfections are not present. This implies that pinning down the empirical degree of imperfections in actual economies may turn out to be an integral part of designing monetary policy.

4 Extensions

This section presents extensions on recently promoted interest rate smoothing policies as well as policy-results using a money-in-utility framework.

4.1 Nominal interest rate smoothing

Empirical studies on the Taylor rule generally include the lagged interest rate. In what follows I will explore determinacy properties of interest smoothing in a forward-looking and a backward-looking Taylor rule. The motivation arises from recent theoretical work that has suggested that indeterminacy can be relinquished by adding lagged values of the nominal interest rate to the standard Taylor-rule (see for example Rotemberg and Woodford, 1999, Giannoni and Woodford, 2002, and Benhabib et al., 2003).

4.1.1 Nominal interest rate smoothing and forward-looking inflation-targeting

Let us first consider hybrid Taylor-type rules such as

$$\widehat{R}_t = \rho \widehat{R}_{t-1} + \tau E_t \widehat{\pi}_{t+1} + \omega \widehat{y}_t \tag{10}$$

in which the parameter ρ stands for interest rate smoothing (the formulation applies Clarida, Gali and Gertler's, 1998, baseline case which they estimate for serveral central banks). Generally, estimates of (10) find a high degree of

inertia and slightly greater than one-for-one increases in the nominal rate in response to inflation. Furthermore, the response to the output gap is mostly found to be small for the U.S. post-1980 period. Therefore, let us consider the case $\omega = 0$. Under constant returns to scale, the following four eigenvalues depict the dynamics

$$\left\{\frac{1-\beta(1-\alpha)(1-\delta)}{\alpha\beta}, \frac{\alpha}{1-\beta(1-\alpha)(1-\delta)}, \frac{1-\sqrt{1-4\rho\tau}}{2\tau}, \frac{1+\sqrt{1-4\rho\tau}}{2\tau}\right\}$$

(which involves two jump variables). The first two eigenvalues split around the unit circle. If $\rho = 0$, the third eigenvalue is zero and the policy should be passive; of course, the argument simply repeats the finding for forward-looking rules (Section 3.2.1.). By making ρ positive, the last two eigenvalues become complex at

$$\rho' = \frac{1}{4\tau}$$

and cross the unit circle at

$$\rho'' = 1 - \tau.$$

We see from the ρ''-condition, that depending purely on a passive rule no longer guarantees determinacy since, *ceteris paribus*, large values of ρ push the economy out of the determinacy region. Empirical evidence points to central bank policies such as $\rho \approx 0.85$ and $\tau \approx 1.1$ – my analysis suggests that banks should avoid these policies to not create endogenous fluctuations. Figure 8 shows the determinacy regions for (i) constant returns to scale and (ii) $\gamma = 0.11$. The two areas do not overlap which underlines the central bank's dilemma. Taylor-rule prescriptions concerning the smoothing parameter differ across technological regimes: under constant returns the policy's response should be passive whereas strong interest rate smoothing will eliminate sunspot equilibria when increasing returns to scale are operating.

4.1.2 Nominal interest rate smoothing and backward-looking inflation-targeting

Rotemberg and Woodford (1999) and Giannoni and Woodford (2002) have suggested that backward-looking Taylor rules' performances can be improved by adding lagged values of the nominal interest rate. In particular, they consider the rule

$$\widehat{R}_{t+1} = \rho\widehat{R}_t + \tau\widehat{\pi}_t$$

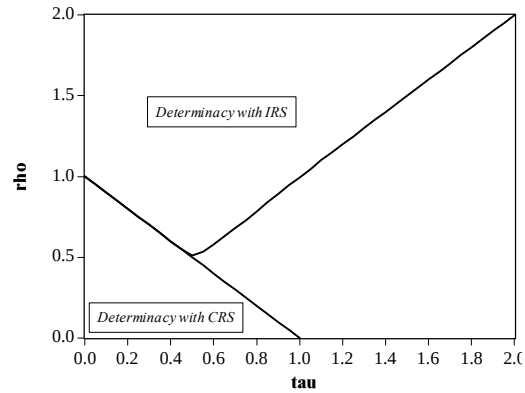

Figure 8: *The effect of interest smoothing: determinacy regions with constant returns to scale (CRS) and increasing returns to scale ($\gamma = 0.11$). The regions do not overlap.*

and find that a smoothing coefficient, ρ, greater than one guarantee unique equilibria. Let us again begin by assuming constant returns to scale and endogenous capital accumulation (the above mentioned papers by Giannoni, Rotemberg and Woodford abstract from capital). The dynamics are characterized by the four eigenvalues

$$\left\{\frac{1-\beta(1-\alpha)(1-\delta)}{\alpha\beta}, \frac{\alpha}{1-\beta(1-\alpha)(1-\delta)}, \tau+\rho, 0\right\}.$$

The parameters τ and ρ enter in complementary fashion and determinacy simply requires that $\tau + \rho > 1$ (the underlying dynamical system has two jump variables). Thus, $\rho > 1$ is indeed a sufficient condition for ruling out indeterminacy and the result carries over to the current perfectly flexible price, cash-in-advance model. However, the advice given is no longer valid once market imperfections are present: there are no (τ, ρ)-constellation that deliver determinacy at $\gamma = 0.11$. For example, with $\tau = \rho = 1000000$, the four eigenvalues are

$$\{0, 0.8625 \pm 0.2913i, 2 \cdot 10^6\}.$$

Once again, the presence of market imperfections has nontrivial effects on the design of monetary policy: pushing up the smoothing parameter, ρ, no longer guarantees the elimination of indeterminacy.

Proposition 4 *(Interest rate smoothing) The exact dose of interest rate smoothing needed to rule out sunspot fluctuations is highly dependent on the*

specific technological environment. Moreover, the analysis suggests that no prescription can be found that secures a stable economy under both constant and increasing returns to scale.

4.2 What if money enters the utility function?

One may argue that the preceding analysis has rested solely on the distortionary effects of inflation arising in cash-in-advance models. It is therefore a straightforward question to ask if the results carry over to money-in-utility frameworks (MIU).[13] For CIA and MIU classes of models, Carlstrom and Fuerst (2001) conclude:

> "[...] that central banks should use either current or backward–looking Taylor rules." [Carlstrom and Fuerst, 2001, p. 296]

They do not allow for increasing returns and assume that $\omega = \rho = 0$. These assumptions are relaxed here. In line with the standard timing convention on real balances – *cash-when-you-are-done* as coined by Carlstrom and Fuerst (2001) – the agents' period utility function is given by

$$\ln c_t + \ln\left(\frac{M_{t+1}}{P_t}\right) - \eta l_t.$$

To eliminate the other distortion as well, I no longer assume that the firms must acquire cash borrowed at the short term rate from an intermediate sector to finance their wage bills. I will stress on backward-looking Taylor rules only. The dynamics are given by[14]

$$\begin{bmatrix} E_t\widehat{c}_{t+1} \\ \widehat{R}_{t+1} \\ \widehat{k}_{t+1} \\ E_t\widehat{R}_{t+2} \end{bmatrix} = \mathbf{K} \begin{bmatrix} \widehat{c}_t \\ \widehat{R}_t \\ \widehat{k}_t \\ \widehat{R}_{t+1} \end{bmatrix}.$$

Let us begin with the simple case $\gamma = \rho = 0$. I also assume that $\delta \to 1$. Then, the eigenvalues are

$$\{\frac{1}{\alpha\beta}, \frac{\alpha(1+\tau+\rho) \pm \sqrt{4\tau + (1+\tau+\rho)^2\alpha^2 + 4\alpha(\rho+2\tau)}}{2}, 0\}.$$

[13] A complete discussion of Taylor rules, externalities and MIU is beyond the scope of this paper.

[14] The Appendix contains a description of the model equation.

It is easy to see that this parallels the cash-in-advance case: if the central bank raises the nominal interest rate more (less) than one-for-one with inflation, the model is determinate (indeterminate). Thus, an aggressive inflation-targeting rule is sufficient to stabilize the economy. Another way of fixing determinacy is resolute interest rate smoothing. It is straightforward to see that the second (third) eigenvalue approaches zero (infinity) as $\rho \to \infty$. That is, independently of the degree of inflation-targeting, unique equilibria arise.

These results change once increasing returns are introduced. By using $\gamma = 0.11$ and the other parameters taken from Table 1, determinacy requires that the inflation response falls into the "passive corridor" $0.3414 < \tau < 1$ (at $\omega = \rho = 0$). For less passive rules, indeterminacy arises and for aggressive rules three eigenvalues are outside the unit circle. In the latter case, the model becomes unstable and endogenous cycles may emerge.

A way of eradicating externality-based endogenous cycles can be constructed by smoothing the nominal interest rate. For

$$\gamma > \gamma^* = \frac{\frac{1}{\beta} - 1 + \alpha(1-\beta)}{3 - \frac{1}{\beta} - \alpha(1-\beta)}$$

(numerically, γ^* equals 0.0066) only one eigenvalue is strictly inside the unit circle.[15] Adding interest rate smoothing may reinstate unique dynamics. At γ^*, the eigenvalues are

$$\{0, -1, 1, \frac{1 - 3(1-\alpha)\beta - 2\alpha^2\beta^2 - \alpha^2\beta^3}{\alpha\beta(1-(2-\alpha)\beta} + \frac{\alpha\beta(1-\alpha\beta)}{\alpha\beta(1-(2-\alpha)\beta}\rho\}.$$

The last eigenvalue crosses into the unit circle at

$$\rho = \frac{\beta(3(1-\alpha) - \alpha\beta) - 1}{\alpha\beta}$$

(numerically $\rho = 4.623$) and moves out of the unit circle at

$$\rho = \frac{\alpha(2+\alpha)\beta^2 + (3-4\alpha)\beta - \alpha^2\beta^3 - 1}{\alpha\beta(1-\alpha\beta)}$$

(numerically $\rho = 6.5661$). That is, interest rate smoothing only helps to stabilize the economy if the central bank is able to identify the correct dose of the smoothing.

[15] At γ^* and $\tau = \delta = 1$, the eigenvalues are

$$\{0, -1, 1, \frac{1 - 3(1-\alpha)\beta - 2\alpha^2\beta^2 - \alpha^2\beta^3}{\alpha\beta(1-(2-\alpha)\beta)}\}.$$

5 Concluding remarks

Recent literature on Taylor rules has suggested that the monetary authority should adopt aggressive, backward-looking rules. For example, Carlstrom and Fuerst (2000) advise that

> "[t]o avoid doing harm, the central bank should place most weight on past movements in the inflation rate." [Carlstrom and Fuerst, 2000, p. 22]

The present paper has shown that we must be very careful with any generalized proposals. There are many dimensions, i.e. the modelling assumptions on the monetary side (Carlstrom and Fuerst, 2000, 2001) and the real side of the economy (the current paper), that have an impact on monetary policy's effects and that should accordingly be considered before spelling out a specific policy.

To demonstrate this, I have added a cash-in-advance superstructure to an otherwise well specified dynamic general equilibrium model that generates indeterminacy by externalities. Once increasing returns are present, most of the policy proposals contained in the existing literature – such as the ones prescribed by Carlstrom and Fuerst – are flipped on its head. For example, when formulating the Taylor-policy on past observations, inflation-targeting will not eliminate sunspot equilibria and aggressive output-targeting is required. Furthermore, rules which should be avoided (chosen) in perfect market environments often ensure (yield) unique (multiple) rational expectations solutions in alternative settings.

To summarize, essential information on how monetary design should be framed in practice must be inferred from empirical estimates of market imperfections. Unfortunately, the existing work on the issue does not offer a clear cut answer – the measurement of the degree of increasing returns is simply too imprecise – which given my results poses a dilemma for the central bank. Cole and Ohanian (1999) suggest a basic problem for the ambiguity: insufficient variations in factor inputs. They conclude that currently available methods are not adequate to return estimates of scale economies such that we can eventually draw a conclusive diagnosis against or in favor of models with indeterminacy such as those summarized in footnote 1. I conclude that estimates on scale economies that are currently available are also not adequate to square conflicting Taylor-policy proposals.

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6 Appendix

The unique steady state is given by

$$\begin{aligned}
\eta\frac{c}{y} &= \frac{\alpha}{lR} \\
u^{\theta} &= \alpha\frac{y}{k} \\
\theta\delta &= 1/\beta - 1 + \delta \\
\beta R &= \pi \\
1 &= \beta[\alpha\frac{y}{k} + 1 - \delta] \\
\delta &= \frac{y}{k} - \frac{c}{k}.
\end{aligned}$$

Calibrating η, α, δ, β, l determines c/y, u, y/k, θ, R and π. Yet, not all these are needed in the approximated version of the model with is the collection

of seven equations

$$\widehat{y}_t = \alpha(1+\gamma)\widehat{u}_t + \alpha(1+\gamma)\widehat{k}_t + (1-\alpha)(1+\gamma)\widehat{l}_t$$

$$\widehat{l}_t = \widehat{y}_t - \widehat{c}_t + \widehat{R}_t$$

$$\theta\widehat{u}_t = \widehat{y}_t - \widehat{k}_t$$

$$\begin{aligned} &-\widehat{c}_t - \widehat{R}_t \\ &= -E_t(\widehat{c}_{t+1} + \widehat{R}_{t+1}) + \alpha\beta\frac{y}{k}\left[E_t\widehat{y}_{t+1} - \widehat{k}_{t+1}\right] - \beta\delta E_t\theta\widehat{u}_{t+1} \end{aligned}$$

$$\widehat{k}_{t+1} = (1-\delta)\widehat{k}_t - \delta\theta\widehat{u}_t + \frac{y}{k}\widehat{y}_t - \frac{c}{k}\widehat{c}_t$$

$$\widehat{R}_t + \widehat{c}_t = E_t\widehat{\pi}_{t+1} + E_t\widehat{c}_{t+1}$$

The backward-oriented rule

$$\widehat{R}_{t+1} = \tau\widehat{\pi}_t + \omega\widehat{y}_t$$

together with the Fisher-equation implies

$$\widehat{R}_t + \widehat{c}_t = \frac{1}{\tau}\widehat{R}_{t+2} - \frac{\omega}{\tau}\widehat{y}_{t+1} + E_t\widehat{c}_{t+1}.$$

The linear model can then be reduced to (from the first three equations)

$$\begin{bmatrix} \widehat{l}_t \\ \widehat{y}_t \\ \widehat{u}_t \end{bmatrix} = \mathbf{R} \begin{bmatrix} \widehat{c}_t \\ \widehat{R}_t \\ \widehat{k}_t \\ \widehat{R}_{t+1} \end{bmatrix}$$

and

$$\mathbf{M}_1 \begin{bmatrix} E_t\widehat{c}_{t+1} \\ \widehat{R}_{t+1} \\ \widehat{k}_{t+1} \\ E_t\widehat{R}_{t+2} \end{bmatrix} + \mathbf{M}_2 \begin{bmatrix} E_t\widehat{l}_{t+1} \\ E_t\widehat{y}_{t+1} \\ E_t\widehat{u}_{t+1} \end{bmatrix} = \mathbf{M}_3 \begin{bmatrix} \widehat{c}_t \\ \widehat{R}_t \\ \widehat{k}_t \\ \widehat{R}_{t+1} \end{bmatrix} + \mathbf{M}_4 \begin{bmatrix} \widehat{l}_t \\ \widehat{y}_t \\ \widehat{u}_t \end{bmatrix}$$

Eliminating the "$\widehat{l}$-vectors" yields

$$\begin{bmatrix} E_t\widehat{c}_{t+1} \\ \widehat{R}_{t+1} \\ \widehat{k}_{t+1} \\ E_t\widehat{R}_{t+2} \end{bmatrix} = \mathbf{M} \begin{bmatrix} \widehat{c}_t \\ \widehat{R}_t \\ \widehat{k}_t \\ \widehat{R}_{t+1} \end{bmatrix}$$

where

$$\mathbf{M} \equiv [\mathbf{M}_1 + \mathbf{M}_2\mathbf{R}]^{-1} [\mathbf{M}_3 + \mathbf{M}_4\mathbf{R}]$$

and $\mathbf{M}$ is 4×4. Forward-looking rules

$$\widehat{R}_t = \tau E_t\widehat{\pi}_{t+1} + \omega E_t\widehat{y}_{t+1}$$

imply the Fisher-equation

$$\widehat{R}_t + \widehat{c}_t = \frac{1}{\tau}\widehat{R}_t - \frac{\omega}{\tau}\widehat{y}_{t+1} + E_t\widehat{c}_{t+1}.$$

Dynamics are given by

$$\begin{bmatrix} E_t\widehat{c}_{t+1} \\ E_t\widehat{R}_{t+1} \\ \widehat{k}_{t+1} \end{bmatrix} = \mathbf{J} \begin{bmatrix} \widehat{c}_t \\ \widehat{R}_t \\ \widehat{k}_t \end{bmatrix}.$$

The matrix $\mathbf{J}$ is 3×3. A current-looking rule

$$\widehat{R}_t = \tau\widehat{\pi}_t + \omega\widehat{y}_t$$

implies the Fisher-equation

$$\widehat{R}_t + \widehat{c}_t = \frac{1}{\tau}\widehat{R}_{t+1} - \frac{\omega}{\tau}\widehat{y}_{t+1} + E_t\widehat{c}_{t+1}$$

and the model dynamics reduce to

$$\begin{bmatrix} E_t\widehat{c}_{t+1} \\ E_t\widehat{R}_{t+1} \\ \widehat{k}_{t+1} \end{bmatrix} = \mathbf{W} \begin{bmatrix} \widehat{c}_t \\ \widehat{R}_t \\ \widehat{k}_t \end{bmatrix}.$$

$\mathbf{W}$ is 3×3. Finally, the hybrid-rule

$$\widehat{R}_t = \rho\widehat{R}_{t-1} + \tau E_t\widehat{\pi}_{t+1} + \omega\widehat{y}_t$$

implies

$$\begin{bmatrix} E_t\widehat{c}_{t+1} \\ E_t\widehat{R}_{t+1} \\ \widehat{k}_{t+1} \\ \widehat{R}_t \end{bmatrix} = \mathbf{P} \begin{bmatrix} \widehat{c}_t \\ \widehat{R}_t \\ \widehat{k}_t \\ \widehat{R}_{t-1} \end{bmatrix}.$$

The simplified version of the money-in-utility version without capital is as follows. The labor market and Fisher equations imply

$$\frac{\eta}{c_t} = A \qquad \Rightarrow c_t = c \quad \nabla t$$

and

$$\frac{E_t\pi_{t+1}}{c_t} = \frac{\beta R_t}{E_t c_{t+1}} \qquad \Rightarrow \pi_{t+1} = \beta R_t.$$

Inserting the backward-looking Taylor rule

$$R_t = R\left(\pi_{t-1}/\pi\right)^{\tau}$$

yields the linear model

$$E_t\widehat{\pi}_{t+1} = \tau\widehat{\pi}_{t-1}.$$

The dynamics are determinate (indeterminate) for $\tau > 1$ $(0 < \tau < 1)$ as in the model with capital. The money demand function

$$\frac{c}{M_{t+1}/p_t} = \frac{R_t - 1}{R_t}$$

is – as a result of the separability in utility – redundant with respect to pin down the inflation-sequence. However, whenever indeterminacy arises, the nominal interest rate and therefore real cash-balances fluctuate with changes in non-fundamental expectations.