

von Döllen, Andreas; Requate, Till

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Environmental Policy and Incentives to Invest in Advanced Abatement Technology if Arrival of Future Technology is Uncertain - Extended Version

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Environmental Policy and Incentives to Invest in Advanced Abatement Technology if Arrival of Future Technology is Uncertain - Extended Version ¹

by Andreas von Döllen and Till Requate

Department of Economics

University of Kiel

24118 Kiel

Germany

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Abstract

We study long-term incentives for polluting and regulated firms to invest in advanced abatement technologies, when some new technology is available but an even better technology will be expected in the future. Firms can invest only once. We find that depending on the adoption fixed costs all possible combinations of investment patterns can occur in social optimum. Further we show that if the regulator anticipates the arrival of the new technology he can decentralize socially optimal allocation by charging Pigouvian tax or issuing tradable permits through announcing his policy and setting ex post optimal policy levels, after firms have invested.

Keywords: Emission taxes, tradable permits, option value theory, uncertainty, Poisson distribution

JEL Classification: O31, L5, Q55

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1 Introduction

In the last decade the impact of environmental policy instruments on both adoption and the development of advanced abatement technology has been a major issue of interest in environmental economics. Since the early papers by Malueg [1989], Milliman and Prince [1989], Jung et al. [1996] and others, more recently researchers such as Kennedy and Laplante [1999], Deniccolo [1999], Montero [2002a, 2002b], Parry [1995,1998,2003], Petrakis and Xepapadeas [1999] Requate and Unold [2001,2003], and Requate [2005] have investigated how different policy instruments but also different timing and commitment structures influence the incentives to adopt advanced abatement technology. Except van Soest [2005] and Bulte and van Soest [2001], in all these models the investment decision is once and for all decision in a certain environment. In reality, however, the development of further, even better technologies is to be expected. Hence firms have an option value to postpone their investment decision and wait until an even better technology is available. Dixit and Pindyck [1994] have developed an analytical tool box to analyze problems like these.

In this paper we study a model where some advanced abatement technology a is available, but an even better technology b may be available in the future. To simplify the analysis we adopt some modeling features from endogenous growth theory (see Aghion and Howitt, 1998) by assuming that the future technology is exogenous but the arrival time is subject to uncertainty and is distributed according to a Poisson process. In a first step we characterize the socially optimal pattern of investment given the investment cost of both technologies, the one available immediately and the one coming up in the future. We find that the distribution of the three different technologies depends crucially on the (relative) size of the fixed investment costs. surprisingly, all combinations of different technologies may occur: No firm should invest in any technology if the fixed investment costs of both technologies are sufficiently high, or all firms should adopt the available technology a immediately, or all firms should wait and adopt technology b . It also may happen that some firms should adopt technology a , some firms should keep the conventional technology, but no firm should adopt technology b . It may happen that some firms should keep the conventional technology, and

all remaining firms should wait and adopt the future technology b , and finally it may be optimal that eventually all three technologies will be employed at the same time.

In a second step we investigate decentralized decision making by firms under different regimes of regulation, in particular Pigouvian taxation and tradable permits. Here we start to study investment behavior under long-term commitment policies. I.e. the regulator makes a long term commitment to a particular tax rate or a particular quantity of permits to be issued. Such commitment can be found in many European countries employing emission taxes, and in the US under Title IV of the Clean Air Act Amendments 1990 for the case of tradable permits.

In a third step we assume that the regulator anticipates arrival of new technology and is able to optimally react on it by adjusting his policy instrument. In particular we ask the question what policy, and what timing and commitment structure is necessary in order to decentralize the social optimum. We discuss both ex ante commitment and ex post optimal policies. Ex ante commitment means that the regulator commits to implement a particular tax rate, or to issue a particular amount of permits as soon the advanced technology b is available and before the firms have adopted that technology. Ex post optimal policy means that the regulator does not only observe the arrival date of the new technology b but also observes how many firms have adopted the advanced technology. Then, given the number of firms having invested in technology a and technology b , respectively, he sets the optimal level of his policy instrument. We show that ex post optimal policy induces the social optimum. This is so because the firms anticipate the behavior of the regulator and since they are small and therefore are unable to act strategically. Under ex ante optimal policy the first best allocation can be implemented by auctioning off tradable permits. Under the tax policy, by contrast, multiple equilibria exist, one of which is optimal but many of them are non-optimal. These results are in line with the findings of Requate and Unold (2003), who studied the case where only one advanced technology is available.

The paper is organized as follows. In the next section we present the model. In section 4 we characterize socially optimal allocations. In section 5.1 we investigate the firms' investment

decisions under different policy instruments, in particular we study the firms' behavior if the regulator has made a long term commitment either to a tax policy or to a fixed number of permits. In section 7 we investigate how the socially optimal allocations can be decentralized. The final section concludes. The proofs are given in the appendix.

2 The Model

We consider a competitive industry consisting of a continuum of ex ante symmetric firms represented by the interval $[0, 1]$. All firms emit a homogenous pollutant which can be abated. The opportunity cost of abatement is represented by a cost function $C(e, \theta)$, which depends on the emission level e , chosen by the firm, and a technology parameter θ . We make the following assumptions about C :

- Assumption 2.1.** i) For each θ there exists a unique *laisser-faire* abatement level $e^{max}(\theta)$, characterized by $\frac{\partial C}{\partial e}(e^{max}(\theta), \theta) = 0$. For each emission level $e < e^{max}(\theta)$ we have $C(e, \theta) > 0$, $-\frac{\partial C}{\partial e} > 0$ and $\frac{\partial^2 C}{\partial e^2} > 0$, i.e. both abatement and marginal abatement costs are positive in the relevant range, and marginal abatement costs are increasing with more abatement.
- ii) A higher technology parameter leads to a decrease of both the variable and the marginal abatement costs , i.e. $\frac{\partial C}{\partial \theta} < 0$ and $-\frac{\partial^2 C}{\partial e \partial \theta} < 0$ for $e \leq e^{max}(\theta)$. Moreover, costs fall at a decreasing rate i.e. $\frac{\partial^2 C}{\partial \theta^2} > 0$.

In the following, we will assume that there are three exogenously given technologies 0, a and b , represented by their corresponding technology parameters θ_0, θ_a and θ_b with $\theta_0 < \theta_a < \theta_b$. To simplify notation, we will write $C_i(\cdot)$ instead of $C(\cdot, \theta_i)$ for $i = 0, a, b$.

Initially all firms start with technology 0, referred to as the *conventional* technology. Advanced technology a is available yet and can in principle be adopted immediately. Buying and installing this technology causes a fixed cost $F_a > 0$. The even better technology b will be available in the future with a certain probability. Its arrival time is Poisson-distributed with exogenous arrival parameter λ . Buying and installing that technology costs $F_b > 0$.

Investment in one of these technologies is irreversible. Moreover we assume that if a firm invests in technology a , it is prohibitively expensive to adopt technology b if technology a has already been installed. In other words: investment in both technologies is not feasible. This assumption generates an option value to postpone the decision to adopt advanced technology a at time $t = 0$.

Further we denote total emissions of industry by $E = \int_0^1 e_i di$, where e_i is the i -th firm emission level. The damage born from pollution depends on aggregate emissions only and is evaluated by a social damage function $D(E)$ which is increasing and convex in E , i.e. $D'(E) > 0$ and $D''(E) > 0$. Finally, we will assume that both the social planner and the firms discount the future at a constant discount rate r . Moreover we will refer to the "first stage" as the time interval where only technologies 0 and a are available. In particular the date of first decision making $t = 0$ is called the first stage. By contrast, the "second stage" is referred to as the time interval when the advanced technology b is available.

3 Expected Net Present Costs of an Investment Decision

In this section we provide a formula for the net present value of total cost incurred by an economic agent who - deviating from the assumption of irreversible investment for a moment - can invest twice, at time 0 and time t when the advanced technology b is available, respectively. This formula is very general and does not only refer to the model of this paper.

Lemma 3.1. *Let F_a and F_b denote the fixed cost incurred when investing in technology a or b , respectively. Further let C_0 , C_a and C_b denote the current values of the cost flow resulting from not investing, investing in technology a , and investing in technology b , respectively. If the agent invests immediately in technology a and substitutes technology a by technology b as*

soon as it is available, the present value of total cost is given by:

$$\begin{aligned} & F_a + \int_0^\infty \left(\int_0^t C_a \cdot e^{-rs} ds + \int_t^\infty C_b \cdot e^{-rs} ds + F_b \cdot e^{-rt} \right) \lambda e^{-\lambda t} dt \\ & = F_a + \frac{1}{r + \lambda} C_1 + \frac{\lambda}{r + \lambda} \left(\frac{C_b}{r} + F_b \right) \end{aligned} \quad (1)$$

If the agent does not invest in technology a , but adopts technology b , as soon as that is available, the net present value of total costs is given by

$$\frac{1}{r + \lambda} C_0 + \frac{\lambda}{r + \lambda} \left(\frac{C_b}{r} + F_b \right). \quad (2)$$

4 The Social Optimum

Before considering regulation and the regulated firms' behavior it is useful to study the socially optimal investment pattern. The social planner's problem is to minimize the expected social costs by balancing industry's total abatement costs against the damage caused by the pollution. To do so he decides on both, each firm's emission level at each point of time, and the shares of firms which should either adopt technology a , technology b , or none of both. Note that social costs will be constant over time in the two stages before and after technology b is available. Note further that the emission level of a firm using technology $i = 0, a$ in the first stage may differ from its emission level in the second stage even if the firm does not change technology. Hence we denote the emission level of a firm at the first stage by e_i and the level of a firm using technology $i = 0, a, b$ at the second stage by e_i^2 .

Thus, using Lemma 3.1 the social planner minimizes

$$\begin{aligned} & \min_{\{e_0, e_a, e_0^2, e_a^2, e_b^2, n_a, n_b\}} \left\{ \frac{1}{\lambda + r} [(1 - n_a)C_0(e_0) + n_a C_a(e_a) + D((1 - n_a)e_0 + n_a e_a)] + n_a F_a \right. \\ & + \frac{\lambda}{\lambda + r} \left[\frac{1}{r} [(1 - n_a - n_b)C_0(e_0^2) + n_a C_a(e_a^2) + n_b C_b(e_b^2) + D((1 - n_a - n_b)e_0^2 + n_a e_a^2 + n_b e_b^2)] \right. \\ & \left. \left. + \frac{\lambda}{\lambda + r} n_b F_b \right] \right\}, \end{aligned} \quad (3)$$

subject $n_0 = 1 - n_a - n_b$.

In the following we will show that depending on F_a and F_b every possible adoption scenario can indeed be socially optimal i.e. no adoption of one or both of the technologies or partial adoption of one or both technologies. We will characterize the optimal pattern of technology adoption contingent on the size of F_a and F_b . Before we state our results formally, it is useful to illustrate the forthcoming results with the help of Figure 1, which displays the different patterns of investment for all combinations of fixed investment costs.

- Figure 1 about here -

Line $\overline{AA'}$ is the locus of all pairs (F_a, F_b) such that $n_a = 1$, i.e. all firms should adopt technology a but the social planner is indifferent about the last firm to adopt or to wait for the arrival of technology b . The part \overline{AC} of $\overline{AA'}$ is increasing since a higher F_a requires a higher F_b to keep n_b equal to zero. In the area bounded by \overline{ACHBA} , n_a is strictly smaller than 1, but all the remaining firms $1 - n_a$ adopt technology b , as soon as that is available. By contrast in the area bounded by $\overline{A'CGB'}$, we have also $n_a < 1$, but the remaining firms do not adopt any of the new technologies (because F_b is too high). Therefore the curve $\overline{CA'}$ is vertical.

Similarly, the line $\overline{BB'}$ represents the locus of all pairs (F_a, F_b) where no firm should adopt technology a , but the social planner is just indifferent about having the marginal firm to adopt technology a or not. Below the dotted line Z' , the adoption cost F_b is so low that left of the branch \overline{BH} all the remaining firms $1 - n_a$ should adopt technology b , while to the right of \overline{BH} all should firms wait for technology b . Above Z'' , no firm should ever wait for technology b , no matter how large F_a , because F_b is too large. In that area to the left of $\overline{GB'}$ some firms should adopt technology a , while to the right of $\overline{GB'}$ none of the two technologies should ever been adopted, because costs of both are too high. Finally, along the branch \overline{GH} always some firms should wait for technology b . On that branch and on its right n_a is zero, while to its left n_a is positive. Again \overline{GH} is increasing since a higher F_b has to be compensated by a higher F_a to leave it unattractive for the social planner to let some firms adopt technology a . The branch \overline{CH} is the boundary where some firms adopt technology

a and the remaining firms wait for technology b . Note that a higher F_a makes technology a less attractive. Instead of adopting technology b there are two alternatives: waiting for technology b , or not investing at all. In order to wait for technology b , a higher F_a requires a lower F_b . The opposite holds for \overline{CG} . Above \overline{CG} some but not all firms adopt technology a , while no firm is waiting for technology b . Here a higher F_a requires a higher F_b to make the employment of technology b non-optimal.

We now state our results formally. For this purpose, we start backwards. For a given number of firms n_a which have adopted the new technology in the first stage, we determine both the optimal number of firms n_b which should adopt the latest technology and the optimal emission levels e_0^2 , e_a^2 , and e_b^2 for each technology. Thus in the second stage the social planner's problem can be written as:

$$\min_{\{n_b, e_0^2, e_a^2, e_b^2\}} \left\{ \frac{1}{r} [(1-n_a-n_b)C_0(e_0^2) + n_a C_a(e_a^2) + n_b C_b(e_b^2) + D((1-n_a-n_b)e_0^2 + n_a e_a^2 + n_b e_b^2)] + n_b F_b \right\} \quad (4)$$

The following result characterizes the optimal rate of adoption of technology b given that a share of n_a has already adopted technology a .

Proposition 4.1. Let the share of firms n_a , having adopted technology a , be given. Then there exists an interval of fixed costs $[\underline{F}_b(n_a), \overline{F}_b(n_a)]$ of technology b , such that

1. No firm should adopt technology b for $F_b > \overline{F}_b(n_a)$.
2. All firms should adopt it for $F_b < \underline{F}_b(n_a)$.
3. For $F_b \in [\underline{F}_b(n_a), \overline{F}_b(n_a)]$ partial adoption of technology b is optimal. The optimal number of firms adopting technology b is decreasing in F_b while the optimal marginal damage at the second stage is increasing in F_b .
4. If n_a increases, the lower bound $\underline{F}_b(n_a)$ increases while the upper bound $\overline{F}_b(n_a)$ decreases and both converge to a cost level F_M as n_a goes to 1. Moreover, the optimal number of firms $n_b^*(n_a)$ adopting technology b is decreasing in n_a .

5. For all F_b for which an n_a exists such that $F_b \in (\underline{F}_b(n_a), \overline{F}_b(n_a))$, both the socially optimal emission levels e_i^2 of a firm using technology $i = 0, a, b$ and the socially optimal marginal damage are independent of n_a .

Note that part 1. - 3. are similar to the result of Requate and Unold [2003]. For $n_a = 0$, their result follows as a special case.

Results 4 says that the interval of investment costs for which partial adoption of technology b is optimal shrinks as n_a increases. The intuition is that the more firms have already adopted technology a , the fewer firms are necessary to adopt the even better technology b . The reason why the upper bound of fixed costs $\overline{F}_b(n_a)$ for which partial adoption is optimal is decreasing is similar: The more firms have already adopted technology a , the less it pays for the remaining firms to adopt the latest technology b , and thus the threshold of adoption costs, for which no further firm should adopt the latest technology b , falls. By contrast, it may be surprising at first glance that the lower bound $\underline{F}_b(n_a)$ for which partial adoption is optimal is increasing with n_a . However, the more firms have already adopted the less advanced technology a , the fewer firms should adopt the latest technology at a given fixed costs. This in turn implies that the threshold $\underline{F}_b(n_a)$ for which all the firms should adopt the latest technology increases. The intuition for result 5 is that in the optimum each firm operates at its minimal average cost including social damage. Therefore a change in n_a can be met by purely adjusting n_b , the optimal number of firms investing in technology b , without affecting the firms' optimal emissions and the socially optimal marginal damage.

Knowing which investment pattern for the latest technology is optimal for given n_a , we will now investigate what number of firms n_a is optimal to adopt technology a at time zero, when technology a is available but technology b is not. Following the same argument as above there will be some interval of investment costs for which partial adoption of the first technology is optimal.

Proposition 4.2. Let the fixed investment cost for the latest technology F_b be given. Then there exists an interval $[\underline{F}_a(F_b), \overline{F}_a(F_b)]$ of investment cost to adopt technology a such that:

1. No firm should adopt technology a for $F_a \geq \overline{F}_a(F_b)$, implying that for $F_b \geq \overline{F}_b(0)$ no

adoption of any technology is optimal, and for $F_b \leq \underline{F}_b(0)$ full adoption of technology b is optimal.

2. All firms should adopt the first technology for $F_a \leq \underline{F}_a(F_b)$.
3. For $F_a \in [\underline{F}_a(F_b), \overline{F}_a(F_b)[$ a share $0 < n_a^* < 1$ of firms should adopt technology a . Moreover, n_a^* falls and optimal marginal damage at the first stage increases as F_a increases.
4. For all values $F_a \in]\underline{F}_a, \overline{F}_a[$, where for the corresponding n_a partial or full adoption at the second stage is optimal we have $\frac{\partial n_a^*}{\partial \lambda} < 0$ and $\frac{\partial n_a^*}{\partial F_b} > 0$.
5. For $F_b < \underline{F}_b(0)$ we get $\frac{\partial F_a}{\partial F_b} = \frac{\partial \overline{F}_a}{\partial F_b} > 0$, while for $\underline{F}_b(0) < F_b < F_M$ we obtain $\frac{\partial F_a}{\partial F_b} > \frac{\partial \overline{F}_a}{\partial F_b} > 0$. In case of $F_M < F_b < \overline{F}_b(0)$ we get $\frac{\partial F_a}{\partial F_b} = 0$ and $\frac{\partial \overline{F}_a}{\partial F_b} > 0$. Finally for $F_M < F_b < \overline{F}_b(0)$ we have $\frac{\partial F_a}{\partial F_b} = \frac{\partial \overline{F}_a}{\partial F_b} = 0$.
6. Whenever $n_b^* > 0$ given $n_a = 0$ $\partial \overline{F}_a / \partial \lambda > 0$ holds. If given F_b full adoption of technology b is efficient for large values of n_a then also $\partial \underline{F}_a / \partial \lambda > 0$ is the case. Otherwise \underline{F}_a is independent of λ .

In words the result states the following. If the adoption cost of technology a is sufficiently low compared to those of technology b , it is optimal that all firms immediately adopt technology a , which is intuitive (part 1). If conversely, the adoption cost of technology a is sufficiently high, it is optimal that all firms wait for the arrival of technology b (part 2). For intermediate values of technology a 's adoption cost some firms should adopt immediately technology a while some should wait for the arrival of technology b . In that case the lower the adoption cost of technology a the more firms should adopt technology a (part 3). In part 4 and 5 we look at the comparative statics effects of increasing either λ or F_b on both the optimal number of firms n_a^* adopting technology a , and the boundaries $\underline{F}_a(F_b)$ and $\overline{F}_a(F_b)$. The directions are as we would expect: If the mean arrival time becomes shorter (i.e. λ rises) we want less firms to invest in the available technology a but rather wait for the even better technology b . If by contrast technology b becomes more expensive, we want more firms to

invest in the less advanced technology a . This also implies that the interval of costs where partial adoption of technology a is optimal is shifted to the right.

By the first proposition, for a given n_a we can find a value F_b such that the share of firms adopting technology b equals a given n_b . Now we can consider this F_b as given. The second proposition now allows us to find an F_a for which a share of n_a firms adopts the first technology. This leads us to the following result:

Corollar 4.3. *For all n_a, n_b where $n_a + n_b \leq 1$ we find F_a, F_b so that n_a and n_b are the corresponding socially optimal shares.*

Proposition 4.2 and the corollary can best be illustrated by Figure 1.

5 The Investment Decision of the Firms

5.1 General Decision Rules

After having characterized the socially optimal investment pattern depending on the relationship of the investment costs, we will now study decentralized decision investment decisions by firms under different policy instruments, in particular emission taxes and tradable permits. Before studying the behaviour under long term tax or permit policies we derive some general decision rules which are in common for both policies. In each regime there are two policy levels, i.e. prices for emissions in the time period between the adoption of the first and arrival of the second technology, denoted by σ^1 and σ^2 , respectively ($\sigma^1 = \sigma^2$ being a special case). Since the arrival time of the new technology b is stochastic, the length of the time period where the firms face σ^1 is uncertain. A firm's emission level in period $i = 1, 2$, where the firm uses technology 0 or a in the first stage and 0, a , or b in the second, is given by the following first-order condition:

$$\frac{\partial C}{\partial e}(\theta_j, e) = \sigma^i \tag{5}$$

the solution being denoted by $e(\theta_j, \sigma^i)$.

For short we will write e_i , $i = 0, a$ for a firm's emission level in the first stage when technology i is used, and e_i^2 , $i = 0, a, b$ for the firm's emission level in the second stage when technology θ_i is used. Given the different technologies and their (uncertain) availability, the firms have the choice between three strategies:

- strategy 0: stay with technology 0 forever,
- strategy 0a: adopt technology a immediately,
- strategy 0b: stay with technology 0 in stage 1 and adopt technology b as soon as that is available.

We start by calculating the expected net present cost for each strategy: Using Lemma 3.1, we obtain the following expected net present cost of the different possible strategies in the first stage:

$$\frac{1}{r + \lambda} [C_j(e_j) + \sigma^1 e_j] + \delta_{j1} F_a + \frac{\lambda}{r + \lambda} \frac{1}{r} [C_k(e_k^2) + \sigma^2 e_k^2] + \delta_{k2} \frac{\lambda}{\lambda + r} F_b \quad (6)$$

where $j = 0, 1$, $k = 0, a, b$, $\delta_{lm} = 1$ if $l = m$ and $\delta_{lm} = 0$ otherwise. $j = k = 0$ corresponds to strategy 0 (never invest at all), $j = k = 1$ corresponds to strategy 0a (adopt technology a immediately), $j = 0$, $k = b$ corresponds to strategy 0b.

At the beginning of the second stage the net present cost of the relevant strategies is determined by

$$\frac{1}{r} [C_k(e_k^2) + \sigma^2 e_k^2] + \delta_{k2} F_b \quad (7)$$

where $k = 0, b$, $\delta_{lb} = 1$ if $l = b$ and $\delta_{lb} = 0$ otherwise. For $k = 0$ we obtain the net present cost of strategy 0 at the second stage while for $k = b$ we get the net present cost of strategy 0b at the second stage.

To Figure out the best strategy, the firms look at the cost difference ΔC_{ij} resulting from any two strategies i and j . The term

$$\Delta C_{0b}(\sigma^2) = \frac{1}{r} [C_b(e_b^2) - C_0(e_0^2) + \sigma^2(e_b^2 - e_0^2)] + F_b \quad (8)$$

compares the two feasible strategies in the second stage. If $\Delta C_{0b}(\sigma^2)$ is negative, the firm decides to adopt technology b .

The next term represents the cost difference between strategy $0a$ and strategy $0b$ at stage 1:

$$\begin{aligned} \Delta C_{ab}(\sigma^1, \sigma^2) = & \frac{1}{r + \lambda} [C_a(e_a) - C_0(e_0) + \sigma^1(e_a - e_0)] + F_a + \\ & \frac{\lambda}{r + \lambda} \frac{1}{r} [C_a(e_a^2) - C_b(e_b^2)] + \sigma^2(e_a^2 - e_b^2) + \frac{\lambda}{r + \lambda} F_b, \end{aligned} \quad (9)$$

Finally at stage 1 the cost difference between strategy 0 and strategy $0a$ is given by:

$$\begin{aligned} \Delta C_{0a}(\sigma^1, \sigma^2) = & \frac{1}{r + \lambda} [C_a(e_a) - C_0(e_0) + \sigma^1(e_a - e_0)] + \\ & \frac{\lambda}{r + \lambda} \frac{1}{r} [C_a(e_a^2) - C_0(e_0^2)] + \sigma^2(e_a^2 - e_0^2) + F_a. \end{aligned} \quad (10)$$

For both terms a negative sign induces the firm to adopt technology a .

5.2 Long Term Tax Commitment

In this subsection we will assume that the regulator has moved first and has made a long term commitment to the level of his tax rate τ . Thus according to the last section 5.1, we have the special case where $\sigma^1 = \sigma^2 = \tau$. To abbreviate notation we denote by e_i a firm's emission level if it uses technology θ_i , $i = 0, a, b$. Thus, abusing notation slightly, we can rewrite (10), (8), and (9), respectively, as

$$\begin{aligned} \Delta C_{0a}(\tau) &\equiv \frac{1}{r} [C_a(e_a) - C_0(e_0) + \tau(e_a - e_0)] + F_a, \\ \Delta C_{0b}(\tau) &\equiv \frac{1}{r} [C_b(e_b) - C_0(e_0) + \tau(e_b - e_0)] + F_b, \end{aligned}$$

and

$$\begin{aligned} \Delta C_{ab}(\tau) &\equiv \frac{1}{r + \lambda} [C_a(e_a) - C_0(e_0) + \tau(e_a - e_0)] \\ &+ \frac{\lambda}{r + \lambda} \frac{1}{r} [C_a(e_a) - C_b(e_b) + \tau(e_a - e_b)] + F_a - \frac{\lambda}{r + \lambda} F_b. \end{aligned}$$

When these expressions are zero, the firm is indifferent between the two technologies 0 and a , or 0 and b , or a and b , respectively. The following lemma characterizes the roots of these expressions.

- Lemma 5.1.** 1. For $i = a, b$, the function ΔC_{0i} is decreasing in τ . If F_i is not too large, there exists a unique solution $\bar{\tau}_{0i}$ of $\Delta C_{0i}(\tau) = 0$ such that firms are indifferent between technologies 0 and a . If the solution exists, staying with the conventional technology 0 is more profitable for $\tau < \bar{\tau}_{0i}$, while for $\tau > \bar{\tau}_{0i}$ the firm is better off by adopting technology i . Furthermore, $\frac{\partial \bar{\tau}_{0i}}{\partial F_i} > 0$.
2. The slope of ΔC_{ab} is ambiguous. If ΔC_{ab} is monotonic in τ , then, if a solution $\bar{\tau}_{ab}$ of $\Delta C_{ab}(\tau) = 0$ exists, it is unique.² If ΔC_{ab} is decreasing (increasing), the firm prefers to invest into technology a if $\tau < \bar{\tau}_{ab}$ (if $\tau > \bar{\tau}_{ab}$), and the firm prefers to postpone the investment and to adopt technology b as soon as the latter is available if $\tau > \bar{\tau}_{ab}$ (if $\tau < \bar{\tau}_{ab}$).

For the proof see the appendix. The intuition is as follows. The greater τ , the higher the cost difference and the more it matters what technology a firm uses. Thus, if the investment costs F_i are not too large, there will always be a tax rate sufficiently high such that both technologies a and b become more attractive compared to the conventional technology 0. The impact of τ on the decision between technology a and b is less clear-cut as can be seen from the ambiguity of the slope of ΔC_{ab} : investing in the first technology may lead to a decrease of the abatement costs as long as technology b is not available. As soon as technology b is available, however, the firm faces opportunity costs of using a less efficient technology. It is not clear, which of the two technologies leads to the larger cut of the present value of total future costs. The following example shows that this ambiguity can arise even for a simple quadratic cost function:

Example 5.2. Let $C(\theta, e) := (e^{-\alpha\theta} + A - \beta e)^2/2\beta$. Then one can easily verify that given a tax rate τ , the cost minimizing emission level of a firm with technology θ , is given by $e(\theta, \tau) = (A + e^{-\alpha\theta} - \tau)/\beta$. This yields

$$\frac{\partial \Delta C_{ab}}{\partial \tau} = \frac{1}{\lambda + r} \frac{e^{-\alpha\theta_a} - e^{-\alpha\theta_b}}{\beta} + \frac{\lambda}{r(\lambda + r)} \frac{e^{-\alpha\theta_a} - e^{-\alpha\theta_b}}{\beta}$$

Therefore we get $\Delta C_{ab} \geq (\leq) 0$ if and only if $r(e^{-\theta_b} - e^{-\theta_a}) \leq (\geq) \lambda(e^{-\theta_a} - e^{-\theta_b})$.

²Whether a solution exists depends on the cost difference $F_a - \frac{\lambda}{r+\lambda} F_b$.

The slope of ΔC_{ab} basically depends on both, the efficiency parameter θ_i and the parameters λ and r .

Lemma 5.3. *Assume that $\Delta C_{ab}(\tau)$ is monotonic in τ and a (unique) tax rate $\bar{\tau}_{ab}$ exists, leaving the firm indifferent between technology a and b . Then the following holds: If $\Delta C_{ab}(\tau)$ is decreasing (increasing), then $\partial\bar{\tau}_{ab}/\partial F_a < (>)0$ and $\partial\bar{\tau}_{ab}/\partial F_b > (<)0$*

Intuition suggests that a firm's decision depends on the order of these three threshold tax rates. However, not each combination of this three values is feasible. In particular we will prove the following result:

Proposition 5.4. If F_a and F_b are such that all three threshold taxes exist, then:

1. For every $\bar{\tau}$ satisfying $\Delta C_{ab}(\bar{\tau}) = 0$ we get $\bar{\tau} \notin [\bar{\tau}_i, \bar{\tau}_j]$ for $i, j = a, b$ and $i \neq j$. Thus if $\bar{\tau}_a < \bar{\tau}_b$ then $\Delta C_{ab} < 0$ for all $\tau \in [\bar{\tau}_a, \bar{\tau}_b]$. If $\bar{\tau}_b < \bar{\tau}_a$ then $\Delta C_{ab} > 0$ for all $\tau \in [\bar{\tau}_b, \bar{\tau}_a]$.
2. If ΔC_{ab} is monotonically decreasing, then $\bar{\tau}_{ab} < \bar{\tau}_a < \bar{\tau}_b$ or $\bar{\tau}_b < \bar{\tau}_a < \bar{\tau}_{ab}$.
3. If ΔC_{ab} is monotonically increasing, then $\bar{\tau}_{ab} < \bar{\tau}_b < \bar{\tau}_a$ or $\bar{\tau}_a < \bar{\tau}_b < \bar{\tau}_{ab}$.

The last two results can be illustrated as follows

1. If ΔC_{ab} is decreasing, we obtain:

$$\begin{array}{c} \text{no inv.} \quad | \quad \text{no inv.} \quad | \quad \text{inv. in } a \quad | \quad \text{inv. in } a \\ \hline \bar{\tau}_{ab} \quad \quad \quad \bar{\tau}_a \quad \quad \quad \bar{\tau}_b \end{array} \quad \text{OR} \quad \begin{array}{c} \text{no inv.} \quad | \quad \text{inv. in } b \quad | \quad \text{inv. in } b \quad | \quad \text{inv. in } a \\ \hline \bar{\tau}_b \quad \quad \quad \bar{\tau}_a \quad \quad \quad \bar{\tau}_{ab} \end{array}$$

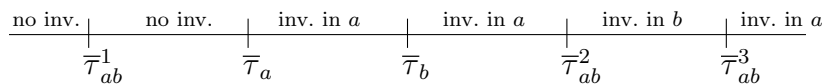
2. If ΔC_{ab} is increasing, we obtain:

$$\begin{array}{c} \text{no inv.} \quad | \quad \text{no inv.} \quad | \quad \text{inv. in } b \quad | \quad \text{inv. in } b \\ \hline \bar{\tau}_{ab} \quad \quad \quad \bar{\tau}_b \quad \quad \quad \bar{\tau}_a \end{array} \quad \text{OR} \quad \begin{array}{c} \text{no inv.} \quad | \quad \text{inv. in } a \quad | \quad \text{inv. in } a \quad | \quad \text{inv. in } b \\ \hline \bar{\tau}_a \quad \quad \quad \bar{\tau}_b \quad \quad \quad \bar{\tau}_{ab} \end{array}$$

The intuition is straightforward. If ΔC_{ab} decreases, and $\bar{\tau}_b < \tau$, a firm's optimal decision in the second stage would be to adopt technology b . Therefore, in the first stage, the relevant decision has to be made between adopting technology a , on the one hand, and waiting for

technology b , on the other. Thus the relevant threshold tax rate is $\bar{\tau}_{ab}$, and thus e.g. $\bar{\tau}_{ab} < \tau$ would induce investment in technology a .

If we do not assume ΔC_{ab} to be monotonic, then there may be multiple switches between intervals where all firms adopt technology a and intervals where all firms adopt technology b for $\tau > \max\{\bar{\tau}_a, \bar{\tau}_b\}$. Note, however, that there are no switches between $\bar{\tau}_a$ and $\bar{\tau}_b$ and we therefore only need to know the sign of ΔC_{ab} either at $\bar{\tau}_a$ or $\bar{\tau}_b$. The next figure illustrates how in that case the investment pattern may look like:



5.3 Long Term Commitment to a Quantity of Permits

In this section we analyze how the firms make their investment decisions when they are regulated by tradable permits and the regulator has made a long term commitment to a constant quantity of permits being issued. Analogously to the tax case we can define price thresholds for permits where a firm is indifferent between any two strategies. According to the different strategies these threshold prices can be derived from (8), (9) or (10) as the prices where these terms are equal to zero. Thus we define $\bar{\sigma}_{0a}(\sigma^2)$ by $\Delta_{0a}(\bar{\sigma}_{0a}(\sigma^2), \sigma^2) = 0$, $\bar{\sigma}_{ab}(\sigma^2)$ by $\Delta_{ab}(\bar{\sigma}_{ab}(\sigma^2), \sigma^2) = 0$ and $\bar{\sigma}_{0b}$ by $\Delta_{0b}(\bar{\sigma}_{0b}) = 0$, where $\sigma^2 > 0$ is arbitrary. Thus the threshold prices at the first stage depend on the expected permit price at the second stage. By backward induction we will analyze the second stage first where the share of firms n_a adopting the technology a is given. Having accomplished this we can write the threshold prices at the first stage as a function of n_a since we know the permit price at the second stage corresponding to n_a .

With these preparations we are ready to characterize the permit price in the second stage depending on the supply of permits:

Proposition 5.5. Let L be the amount of permits issued by the regulator. Let $0 \leq n_a < 1$ be given and let σ denote the market price for permits in the second stage. Then the following holds:

1. If $L \geq \bar{E}^2(n_a)$, then $\sigma < \bar{\sigma}_{0b}$ and none of the remaining firms invests.
2. If $L \leq \underline{E}^2(n_a)$, then $\sigma > \bar{\sigma}_{0b}$ and all of the remaining firms invest.
3. If $L \in]\underline{E}^2(n_a), \bar{E}^2(n_a)[$, then $\sigma = \bar{\sigma}_{0b}$ and a share of firms

$$n_b = \frac{e_0(\bar{\sigma}_{0b}) + n_a(e_a(\bar{\sigma}_{0b}) - e_0(\bar{\sigma}_{0b})) - L}{e_0(\bar{\sigma}_{0b}) - e_b(\bar{\sigma}_{0b})}$$

will adopt the second technology.

4. If n_a increases, $\underline{E}^2(n_a)$ increases and $\bar{E}^2(n_a)$ decreases. Furthermore both \underline{E}^2 and \bar{E}^2 converge to $E_M = e_a(\bar{\sigma}_{0b})$ as n_a goes to 1.

The proof is given in the appendix. Note that for $n_a = 0$ the claims 1.)-3.) corresponds to Lemma 2 in Requate and Unold [2003]. The reason for 4.) is that with a greater number n_a a greater abatement level has already been achieved. We can now determine both the firms' decision and the pattern of investment in the first stage. For this purpose it is useful to first investigate the relationship between the number of firms n_a investing in technology a and the first stage equilibrium price for permits. Since for given L and n_a the equilibrium permit price $\sigma^2(n_a)$ in the second stage is uniquely determined, we write $\bar{\sigma}^1(n_a)$ as the relevant threshold price at the first stage where the remaining $1 - n_a$ firms are indifferent between adopting technology a , on the one hand, and not adopting it and behaving optimally at the second stage, on the other. Thus if no firm wants to adopt the second technology in the second stage, $\bar{\sigma}^1(n_a)$ is given by $\bar{\sigma}_{0a}(\sigma^2(n_a))$. Otherwise $\bar{\sigma}^1(n_a)$ is given by $\bar{\sigma}_{ab}(\sigma^2(n_a))$. If there is partial adoption in the second stage we obtain $\bar{\sigma}_{ab}(\sigma^2(n_a)) = \bar{\sigma}_{0a}(\sigma^2(n_a))$. The next result establishes how both the threshold price $\bar{\sigma}^1$ and the equilibrium price σ^1 depend on n_a .

Proposition 5.6. The threshold price $\bar{\sigma}^1(n_a)$ is continuous and non-decreasing in n_a while the market price σ^1 is decreasing and continuous in n_a .

To determine the pattern of investment, given the supply of permits L , we proceed similarly as in the second stage. We define $\bar{E}^1 := e_0(\bar{\sigma}^1(0))$ and $\underline{E}^1 := e_a(\bar{\sigma}^1(1))$. \bar{E}^1 is

the aggregate emission level which induces no firm to adopt technology a , but each firm is indifferent between adopting and not adopting technology a and which leads to a permit price equal to $\bar{\sigma}^1(0)$. Analogously \underline{E}^1 is the aggregate emission level which induces an outcome where all firms adopt technology a but each firm is indifferent between adopting and not adopting technology a and which thus leads to a the permit price equal to $\bar{\sigma}^1(1)$. As we will see in the next result, $(\underline{E}^1, \bar{E}^1)$ is the interval of permits where partial adoption of technology a occurs.

Proposition 5.7. 1. For $L > \bar{E}^1$ none of the firms will adopt the first technology and $\sigma^1 < \bar{\sigma}^1(0)$.

2. If $L < \underline{E}^1$ all firms will adopt technology a and $\sigma^1 > \bar{\sigma}^1(1)$.

3. If $L \in]\underline{E}^1, \bar{E}^1[$ then a share $0 < n_a < 1$ of firms will adopt technology a and the permit price is equal to $\bar{\sigma}^1(n_a)$. Moreover n_a is given by the solution of the equation $L = (1 - n_a)e_0(\bar{\sigma}^1(n_a)) + n_a e_a(\bar{\sigma}^1(n_a))$.

The intuition for the result is as follows: At the threshold price $\bar{\sigma}^1(0)$ firms are indifferent between adopting and not adopting technology a , but no firm in fact adopts that technology such that total emissions amount to \bar{E}^1 . If the number of permits L exceeds \bar{E}^1 , the price for permits must therefore be lower than $\bar{\sigma}^1(0)$, and all firms strictly prefer to not adopt technology a . At the threshold price $\bar{\sigma}^1(1)$ firms are indifferent between adopting and not adopting technology a , too, but all firms in fact adopt technology a such that total emissions amount to \underline{E}^1 . If the number of permits L is smaller than \underline{E}^1 , the price for permits must exceed $\bar{\sigma}^1(1)$, and all firms strictly prefer to adopt technology a . If L is between \underline{E}^1 and \bar{E}^1 , a share $0 < n_a < 1$ adopts the new technology, and the permit price is $\sigma^1(n_a) = \bar{\sigma}^1(n_a)$ with $\bar{\sigma}^1(0) < \bar{\sigma}^1(n_a) < \bar{\sigma}^1(1)$. Moreover, $\bar{\sigma}^1(n_a)$ is increasing in n_a .

- Figure 2 and Figure 3 about here -

Figure 2 illustrates the relationship between the threshold price and n_a , for the cases where either full or no adoption of technology b occurs at the second stage. The curves AA ,

BB , and CC each depict the equilibrium price for permits if we vary n_a but treat it as an exogenous variable. Along each curve L is fixed. If we increase L , we move from AA to CC . Along AA , the quantity of permits is larger than \bar{E}^1 , and the equilibrium prices for each fixed n_a are everywhere larger than the threshold price $\bar{\sigma}^1(n_a)$. Hence full adoption will occur. Along CC , the quantity of permits is smaller than \underline{E}^1 , and the equilibrium prices for each fixed n_a are everywhere smaller than the threshold price $\bar{\sigma}^1(n_a)$. Hence no adoption will occur. Along BB , where $\underline{E}^1 < L < \bar{E}^1$, the equilibrium price curve for any exogenously given n_a , i.e. $\sigma^1(n_a)$, intersects the threshold price curve $\bar{\sigma}^1(n_a)$. At this point the equilibrium price for fixed n_a is exactly equal to the threshold price where firms are indifferent between adopting and not adopting technology a . This intersection determines the equilibrium number of adopting firms.

Why is the curve $\bar{\sigma}^1(n_a)$ increasing? Recall that Figure 2 covers the case where either full or no adoption of technology b occurs at the second stage. Consider first the case where none of the firms adopt technology b . Here a higher n_a induces less demand for permits and thus a lower price at stage 2, making technology a less attractive at stage 1. Therefore the threshold price $\bar{\sigma}^1(n_a)$ for which it begins to be attractive to invest in technology a must be increasing in n_a .

Consider now the case where all the remaining firms $1 - n_a$ adopt technology b at stage 2. Here a higher n_a triggers more demand for permits and thus induces the permit price to rise at stage 2, which makes it more attractive to wait for technology b instead of adopting technology a at stage 1. Put differently, a higher n_a raises the option value to postpone the investment decision. Therefore again the threshold price $\bar{\sigma}^1(n_a)$ must increase with n_a to compensate for that effect. Note the fundamental difference to the case where there is no second technology in the future (the Requate-Unold [2003] model) where the threshold price is always constant in n_a .

Figure 3 captures the case where partial adoption may occur in the second stage. Again the curves AA through DD depict the equilibrium prices if we vary n_a but treat it as an exogenous variable. Along each curve L is constant, and if we increase L , we move from

AA to DD. If n_a is sufficiently large, say greater than some \hat{n}_a as depicted in the picture, we know from Proposition 5.5 (part 4), that either no or full adoption will occur at stage 2. Therefore to the right of \hat{n}_a we obtain the same picture as in Figure 2. To the left of \hat{n}_a partial adoption of technology b will occur at stage 2. In this case a higher n_a will be compensated by a lower n_b at the second stage such that the threshold price stays constant.

6 Welfare Analysis in Cases of Myopic Regulation

In the section we study the allocations derived in the last two sections, assuming that the regulator has made a long-term commitment to the optimal policy with respect to the conventional technology 0. In case of a tax policy, the regulator has set a single tax rate τ_0 equal to the socially optimal marginal damage $D'(E_0^*)$. In the case of permits he has issued an amount of permits L_0 equal to E_0^* . This may for example occur if the regulator did not anticipate the new technologies for reason of insufficient information.

Firstly we will examine the case of taxes. From the results we have derived before we know that in general only one of both technology will be adopted by all firms under a regulation by a long term tax. The choice depends on the relative difference between the installment costs of both technologies. To derive the result, define $\hat{F}_a(F_b)$ as the installment cost level where for the corresponding socially optimal share n_1 the relationship $\bar{F}_b(n_a) = F_b$ holds. I.e. exactly for this cost level the socially optimal share of firms adopting technology a get that large that no adoption of technology 2 gets socially optimal in the second stage. Thus the graph of $\hat{F}_a(F_b)$ is identical to the one of $\bar{F}_b(F_a)$ in Figure 1. Mathematically $\hat{F}_a(F_b)$ is the inverse of $\bar{F}_b(F_a)$. Trivially $\hat{F}_a(F_b) \in [\underline{F}_a, \bar{F}_a]$. First we state the result in case of a tax regulation.

Proposition 6.1 (Myopic Regulation, Tax Case). Assume that the regulator commits the tax rate τ_0 for once and forever. Then, given installment cost F_b of technology b , there exists a threshold level $F_a^t(F_b)$ for the installment cost of technology 1 in such way that irrespective to the optimal decision at the second stage we get:

1. If $F_a > F_a^t(F_b)$, then no firms will adopt technology a . All firms take the optimal decision in the second stage.
2. If $F_a < F_a^t(F_b)$, then all firms will adopt the first technology. No firms take the optimal decision in the second stage.
3. If $F_a = F_a^t(F_b)$, the share of firms which will adopt the first technology is ambiguous. The remaining firm take the optimal decision in the second stage.

For the allocation of both technology this yields:

- a.) If $F_b > \bar{F}_b(0)$, then for each firm the optimal decision in the second stage will be to adopt not technology b . It is $F_a^t = \bar{F}_a$ and F_a^t is independent from F_b .
- b.) For $F_b = \bar{F}_b(0)$, each firm, which has not invested in the first stage is indifferent between not investing and the adoption of the second technology. It is $F_a^t = \bar{F}_a$. This implies for example that for $F_a = F_a^t$ the outcome is complete ambiguous.
- c.) If $F_M < F_b < \bar{F}_b(0)$, for each firm that still uses the conventional technology 0 the adoption of technology b is optimal in the second stage. $F_a^t(F_b) \leq \bar{F}_a(F_b)$. More exactly if ΔC_{ab} decreases, we have $F_a^t(F_b) \in [\hat{F}_a(F_b), \bar{F}_a]$ whereas if ΔC_{ab} increases we have $F_a^t(F_b) < \hat{F}_a(F_b)$, but not necessarily $F_a^t(F_b) < \underline{F}_a$.
- d.) If $F_b \leq F_M$, again for each firm that still uses the conventional technology 0 the adoption of technology b is optimal in the second stage. Again $F_a^t(F_b) \leq \bar{F}_a$. But now if the function ΔC_{ab} decreases, only $\underline{F}_a < F_a^t(F_b)$ holds whereas if the function ΔC_{ab} increases, now $F_a^t(F_b) < \underline{F}_a$ has to be the case.

The resulting exact allocation for cost pair (F_a, F_b) can be best seen in the Figures 4 and 5.

- Figure 4 and Figure 5 about here

By and large this is the result would have been expected in the light of Requate and Unold [2003]. But there is a little but important difference we have to explain. If ΔC_{ab} decreases, there may also occur under-investment e.g. for $F_b < \underline{F}_b(0)$ or at least full adoption of the first technology where partial adoption of the second technology is socially optimal because for $\underline{F}_b(0) < F_b < F_M$ it may also be socially optimal that some firms do not invest. In this sense we have under-investment for this case. Figure 4 visualizes the result. The dotted lines represent the border lines for the socially optimal allocation (compare to Figure 1). The thick vertical line represents $F_a^t(F_b)$. It is easy to check that the slope is equal to the one of $\underline{F}_a(F_b)$ for $F_b < F^M$. Thus for both cases, the case that ΔC_{ab} decreases (Figure 4 and Figure 5) and the one that ΔC_{ab} increases (Figure 5) the F_a, F_b plane is separated by $F_a^t(F_b)$ and $\overline{F}_2(0)$ into four areas where either none of the technology or one of the new technologies is adopted by all firms. If ΔC_{ab} decreases $F_a^t(F_b)$ lies right from $\underline{F}_a(F_b)$, which, as mentioned above induces under-investment. For the case that ΔC_{ab} increases over-investment is induced in all cases where the market outcome does not coincide with the social optimal allocation.

Thus the result shows that the decision between the two new technologies has a direct impact on the allocation. To illuminate that recall that $\partial \Delta C_{ab} \partial \tau > 0$ implies that for a high tax rate the firm would prefer technology a . In that context, technology b may be interpreted as a small enhancement of the technology a which can be developed quickly. Now if τ is small the firms will wait for the enhancement. Otherwise they would adopt the basic technology. But thinking in that way it seems plausible that for even some very high costs F_a the firms will still adopt the first technology facing τ_0 while the regulator may prefer that some firms would adopt the second technology to compensate the cost disadvantage.

Now we do the same thing as above for the permit case. Again we start by analyzing the second stage. In the one technology case (Requate and Unold [2003]), there exist an interval $[F^u, F^o]$ of installment cost where given $L_0 = E_0^*$ partial adoption is the market outcome. Furthermore from the social planner's point of view for all $F \in]F^u, F^o[$ under-investment occur. The intuition behind that is straight-forward. Given the installment cost we will find a market where the adoption of technology gets attractive for some firms. Since there is an

upper and lower bound for the market price - corresponding to the cases that all and no firm invests respectively - for very high installment cost the market price is always too low to induce an incentive to adopt technology while for low installment even if all firms adopt the technology the market price will exceed this threshold price. For a moderate level of the installment cost the market price may equal the price where a firm is indifferent between adopting the technology or not if some firms have adopted the new technology.

In principle we will expect that these results will also hold for this two technology scenario. So given a share $n_a > 0$ it is intuitive that there will exist an interval $[F_b^u(n_a), F_b^o(n_a)]$ where partial adoption of technology 2 is the market outcome. Since the amount of permit is not flexible there will be a lower incentive to adopt technology if some firms have already invested into technology a (i.e. $n_a > 0$) or technology b as if the regulator will change the amount of permit to the socially optimal emission level. Thus we will expect that in general $F_b^u(n_a)$ and $F_b^o(n_a)$ will be smaller than \underline{F}_b and \overline{F}_b respectively. That is what we will show in the next proposition.

Proposition 6.2 (Myopic Regulation, Permit Case, Second Stage). Assume the amount of permits is equal to L_0 and a share $0 \leq n_a < 1$ of firms have adopted technology a . Then there exist an interval $[F_b^o(n_a), F_b^u(n_a)]$ of installment costs with the following properties

1. For all $n_a \geq 0$ it is $F_b^u(n_a) < \underline{F}_b(n_a)$. Furthermore we get $F_b^o(0) = \overline{F}_b(0)$ and $F_b^o(n_a) < \overline{F}_b(n_a)$ for all $n_a > 0$.
2. If $F_b > F_b^o(n_a)$ none of the remaining firms using the conventional technology will adopt technology b .
3. If $F_b < F_b^u(n_a)$ all $1 - n_a$ firms will adopt technology b .
4. If $F_b \in]F_b^u(n_a), F_b^o(n_a)[$ a partial share $0 < n_b < 1 - n_a$ of firms adopts technology b . This share is always smaller than the socially optimal share $n_b^*(F_b)$.
5. If n_a increases, then also $F_b^u(n_a)$ increases while $F_b^o(n_a)$ decreases. Both tend to a cost level $\tilde{F}_M < F_M$ as n_a goes to 1.

In principle this is an analog to proposition 4.1. Also the intuition of this result is similar to the one of proposition (4.1). As explained above the intuition why all these bounds are smaller than the corresponding socially optimal ones is the lower incentive to invest due to the fixed amount of permits which is larger than every socially optimal level if any adoption has occurred.

As we know the market outcome in the second stage we can analyze the adoption of technology a in the decentralized case. In principle we get an analog to proposition 5.

Proposition 6.3 (Myopic Regulation, Permit Case, First Stage). Assume that the amount of permits is equal to L_0 . Consider the installment cost F_b of technology b to be given. Then there exist an interval $[F_a^o(F_b), F_a^u(F_b)]$ in such way that for $F_a \geq F_1^o(F_b)$ no adoption firm will adopt technology a , for $F_a \leq F_a^u$ all firms will adopt technology and for $F_a \in]F_a^u, F_a^o[$ a partial share n_a of firms adopts technology a .

With these preparations we can compare the resulting allocation (n_a, n_b) of both technologies corresponding to a cost pair (F_a, F_b) to the socially optimal one allocation (n_a^*, n_b^*) which corresponds to (F_a, F_b) . We will see that analogously to the tax case the slope of ΔC_{ab} as a function of the permit price σ plays an important role to the question whether under-investment or over-investment occurs. Note as we have explained above that this slope can be interpreted as a measure whether the first or the second technology is a greater improvement.

Corollar 6.4. Consider the same assumptions as in proposition 6.3.

1. $F_b > F_b^o(0)$ implies $F_a^o(F_b) = \bar{F}_a(F_b)$ and $F_a^u(F_b) < \underline{F}_a(F_b)$. For $F_a \in]F_a^u(F_b), F_a^o(F_b)[$ the share of firms adopting technology 1 is smaller than socially optimal one.
2. For $\tilde{F}_M < F_b < F_b^o(0)$ we have $F_a^o(F_b) = \bar{F}_a(F_b)$ for $F_b \geq \underline{F}_b(0)$ and $F_a^o(F_b) > \bar{F}_a(F_b)$ for $F_b < \underline{F}_b(0)$. For $F_b > F^M$ it is $F_1^u(F_2) < \underline{F}_a(F_b)$ while for $F_b < F^M$ this only holds in ΔC_{ab} decreases. If ΔC_{ab} increases it is ambiguous whether $F_a^u(F_b) < \underline{F}_a(F_b)$ or $F_a^u(F_b) > \underline{F}_a(F_b)$. For $F_a \in]F_a^u(F_b), F_a^o(F_b)[$ the share n_a of firms that adopt technology

1 is always smaller than the socially optimal share n_a^* . If $n_b(n_a) > 0$ and $n_b(n_a) + n_a < 1$ then $n_a + n_b(n_a) < n_a^* + n_b^*(n_a^*)$.

3. In case of $F_b^u(0) < F_b < \tilde{F}_M$ and $F_b \geq \underline{F}_b(0)$ it is $F_a^o(F_b) = \overline{F}_a(F_b)$ while in case of $F_b^u(0) < F_b < \tilde{F}_M$ and $F_b < \underline{F}_b(0)$ it is $F_a^o(F_b) > \overline{F}_a(F_b)$. If $\Delta C_{ab}(\sigma)$ increases in the permit price σ then $F_a^u(F_b) > \underline{F}_a(F_b)$ while if $\Delta C_{ab}(\sigma)$ decreases in σ then $F_a^u(F_b) < \underline{F}_a(F_b)$ is induced. This implies that for $F_a \in [F_a^u(F_b), F_a^o(F_b)]$ that the share n_a as well as the total share $n_a + n_b(n_a)$ of firms that invest in any new technology is smaller than the corresponding socially optimal shares as long as $n_a + n_b(n_a) < 1$. If $n_a + n_b(n_a) = 1$ then it is ambiguous whether n_a is smaller, equal or greater than the socially optimal share n_a^* .
4. $F_b < F_b^u(0)$ implies $F_a^o(F_b) > \overline{F}_a$. If $\Delta C_{ab}(\sigma)$ increases in σ then $F_a^u(F_b) > \underline{F}_a(F_b)$ while $F_a^u(F_b) < \underline{F}_a(F_b)$ if $\Delta C_{ab}(\sigma)$ decreases in σ . In that case for $F_a \in]F_a^u(F_b), F_a^o(F_b)[$ it is always ambiguous whether the share n_a is smaller, equal or greater than the corresponding socially optimal share n_a^* . If $\Delta C_{ab}(\sigma)$ increases for $F_a \in]\underline{F}_a(F_b), F_a^u(F_b)[$ we get $n_a > n_a^*$ and $n_b > n_b^*$. In contrast to that if $\Delta C_{ab}(\sigma)$ decreases for $F_a \in]F_a^u(F_b), \underline{F}_a(F_b)[$ we get $n_a < n_a^*$ and $n_b > n_b^*$. In both cases for $F_a \in]\overline{F}_a(F_b), F_a^o(F_b)[$ we get $n_a > n_a^*$ and $n_b > n_b^*$.

This result may seem surprising at first glance since we have argued above that a decreasing ΔC_{ab} could be interpreted as a preference to adopt the first technology. It seems more plausible that there would be over-investment, meaning there is too much investment into the second technology, if ΔC_{ab} is increasing. But note, however, that a decreasing ΔC_{ab} means also that the firms would prefer the second technology if the tax is low. Or more generally speaking the firms prefer the second technology when the incentive to adopt a new technology is rather low. But this is the case here because too many permits have been issued.

The Figures 6 and 7 visualize the result for both the case that ΔC_{ab} is decreasing (Figures 6) and the case that ΔC_{ab} is increasing (Figures 7). Again as in Figures 4 and 5 the dotted lines represent the border lines of the socially optimal allocation. The thick vertical lines represent $F_a^o(F_b)$ and $F_a^u(F_b)$ while the upward and the downward slopping horizontal lines represent F_b^u and F_b^o as a function of F_a . It is easy to show that for both F_a^o and F_a^u the slope equals $-\frac{\lambda}{\lambda+r}$ which is for very small F_b and thus both lines are a parallel shift of \underline{F}_a and \overline{F}_a . By comparing the allocations we see that if ΔC_{ab} increases we have always under-investment as in the one stage case when the allocation does not coincide with the socially optimal one. But if ΔC_{ab} decreases there is also a small area (Figure 6) where over-investment is induced. It is the area where only the adoption of technology a would be socially optimal while the market outcome is such that a partial share $0 < n_a < 1$ adopts technology a and the remaining firms adopt technology b .

7 Optimal Policy in Anticipation of the New Technology

In the last section we have investigated the firms' behavior for the case where the regulator has made a long term commitment to one of the policies. In this section we assume that the regulator anticipates arrival of new technology and reacts optimally on arrival of new technology. In this case we can think of three scenarios:

1) The regulator anticipates the expected arrival time of the latest technology, denoted by say \bar{t}_2 , and makes an ex ante commitment to set the tax rate at $\tau = \tau^{2*}$ (or $L = E_2^*$) for $t \geq \bar{t}_2$. The tax rate τ^{2*} corresponds the optimal marginal damage if the optimal number of firms has adopted technology b . Alternatively he may issue is the corresponding optimal number of permits $L = E^{2*}$. This policy, however, will not be optimal in general, since the realization of the arrival time is stochastic and may be smaller or larger than \bar{t}_2 . Hence we will not further investigate second best optimal policies of this kind.

2) *Ex ante contingent commitment.* The regulator knows technology a and anticipates

arrival of technology b , but he does not anticipate the exact arrival date t_2 . He sets the optimal level of his policy instrument τ^1 or L^1 , in case of taxes or permits, respectively, for $t < t_2$ when only technologies 0 and a are available, and he makes a contingent commitment to implement the optimal levels of his policy instruments τ^2 or L^2 , for $t \geq t_2$, when all three technologies are available. Then some firms may adopt technology a immediately at time $t_1 = 0$ while some other firms may decide to adopt technology b , as soon as that technology is available. The timing of this game is depicted in Figure 8 for the case of a tax policy. Note that the points of time t_1 and t_2 are artificially split up into further virtual points of time t_{10} , t_{11} and t_{12} , where the new technology a becomes available at t_{10} , then the regulator makes his commitment at time t_{11} , and finally the firms make their decision at time t_{12} . The time intervals $t_{12} - t_{11}$ and $t_{11} - t_{10}$ are assumed to be infinitesimally small. The case of t_{20}, t_{21} , and t_{22} is similar except that at t_{21} the regulator implements his policy τ^2 or L^2 according to his commitment. This virtual timing structure can be considered as a proxy of the real world's if the time lag $t_2 - t_1$ is large compared to the lags $t_{12} - t_{11}$ and $t_{11} - t_{10}$.

- Figure 8 about here -

3) *Ex post regulation.* In the final scenario the firms move first by deciding whether or not to adopt technology a at the (virtual) point of time t_{11} . Then the regulator observes how many firms have adopted technology 1 and then sets his tax rate τ^1 , or issues his number of permits L^1 , respectively, at the virtual point of time t_{12} . As soon as the new technology becomes available at time t_2 , (more exactly at the virtual point of time t_{20}), those firms which have not adopted technology a at time t_1 , now decide whether or not to adopt technology b at the virtual point of time t_{21} . Then the regulator again observes how many firms have adopted the latest technology b and sets the optimal level of his policy instrument at virtual point of time t_{22} . The timing of this game is depicted in Figure 9 for the case of a tax policy. Note that we assume that the regulator maximizes welfare, and the firms, knowing this, anticipate the investment equilibrium and thus the level of the policy instrument in the time intervals $[t_1, t_2)$ and $[t_2, \infty)$.

- Figure 9 about here -

First we will show that under ex ante contingent commitment permits outperform taxes because the efficient allocation can always be implemented through a unique equilibrium under permits while under taxes multiple equilibria may occur only one being the efficient.

Proposition 7.1 (Ex ante commitment). Assume the welfare-maximizing regulator moves first and announces his full policy setting.

1. (*Ex ante optimal tax policy*) Let τ^1 be the emission tax level relevant for the time period $[t_1, t_2[$ and τ^2 . If it is socially optimal that either none of the firms adopt any of the new technology or all firms adopt either technology a or technology b there exist a unique subgame perfect equilibrium where the regulator in both stages charges a tax equal to the corresponding socially optimal damage and also in both stages the social optimal share of firms adopts the corresponding technologies. Otherwise there always exists an infinity number of equilibria where only one is the efficient one.
2. (*Ex ante optimal permit policy*) There exists a unique subgame perfect equilibrium where the regulator issues the socially optimal number of permits $L^1 = E^{1*}$ for the time period $[t_1, t_2[$ and also the socially optimal number of permits $L^2 = E^{2*}$ for $t \geq t_2$ when technology b is available. Furthermore in both stages the social optimal share of firms adopts the corresponding technologies.

The intuition behind the result is simple. Since an ex ante committed tax is not flexible in principle the decision of other firms does not affect the equilibrium tax level. Thus by the results of section 5.2 either all firms take the same decision or each firm is indifferent between different strategies which induces that the choice is ambiguous. In the proof of this proposition we see that the latter is always the case if the social optimal allocation consist partial adoption of at least one technology.

However optimal ex post regulation induces the social optimum with both, taxes or permits. The next proposition establish the result.

Proposition 7.2 (Ex post optimal policy). Let F_a, F_b be given and assume that a welfare-maximizing regulator observes the shares of investing firms at each stage. Then under both, taxes and permits, there exists a unique subgame perfect equilibrium where at each stage the optimal number of firms invest and the regulator sets the tax rates τ^1 and τ^2 equal to the corresponding socially marginal damage, or issues the corresponding number of permits, respectively.

Note that this result generalizes a similar result of Requate and Unold [2003] for the case where only technologies 0 and a exist. The nice feature about proposition 7.2 is that despite the uncertainty about the future it is sufficient for the regulator to stick to the Pigouvian rule. Thus the regulator does not have to solve complex commitment problems and problems of time-(in-)consistency. The optimality of ex-post regulation however only works if the firms are small and cannot influence the regulator's decision by their investment decision. In other words their impact on marginal damage must be sufficiently small.

8 Conclusion

The aim of this paper was to investigate incentives to adopt new technology when some new technology is available and some even better (but exogenously given) technology will be available in the future but the arrival time is subject to uncertainty. This creates an option value to postpone the decision to adopt new technology. Our analysis drew on Requate and Unold [2003] who considered a deterministic framework where one new was available with certainty. We have shown that, depending on the fixed investment cost of both technology a and technology b , all constellations are possible: All firms adopt technology a , but none adopts technology b , and vice versa; some firms adopt technology a but some firms wait and adopt technology b if that becomes available. It may even be optimal that all three technologies, the conventional technology 0, technology a , and technology b will be employed simultaneously. Moreover we have shown that the socially optimal outcome can be decentralized by ex post optimal policy. This means that the firms move first by deciding whether

or not to adopt technology a immediately, or to wait for technology b , and in a second step the regulator implements the ex post optimal level of his policy instrument. The reason why this policy is socially optimal is the fact that the firms are many and cannot strategically influence the regulator's decision. If firms were few, the optimality result would, of course, not hold. Ex ante commitment, by contrast leads to the first best outcome only in the case of permits, whereas this was not the case for taxes due to multiplicity of equilibria.

We have assumed that firms are symmetric ex ante in this paper. Despite optimality of equilibrium, firms would have to solve a coordination problem because ex ante symmetric firms may be different ex post. The coordination problem may be solved by introducing an exogenously given order of investment. It may also be solved by introducing some asymmetry among the firms along the lines of Requate and Unold [2001] which we here have to relegate for further research. A further issue of research will be to endogenize the new technology since the analysis becomes more complicated if the technology jumps are endogenous.

A Appendix

Proof of Proposition 4.1: Ad 1.-3.: Given n_a at stage 2 the social planner solves

$$\min_{n_b, n_0, e_0^2, e_a^2, e_b^2} \left\{ \frac{1}{r} (n_0 C_0(e_0^2) + n_a C_a(e_a^2) + n_b C_b(e_b^2)) + D(n_0 e_0^2 + n_a e_a^2 + n_b e_b^2) + n_b F_b \right\} \quad (11)$$

subject to the constraints $n_0 \geq 0$, $n_b \geq 0$ and $n_0 + n_a + n_b = 1$ with corresponding Kuhn-Tucker multipliers μ_i for the non-negative constraint for n_i for $i = 0, b$ and Lagrange multiplier ν w.r.t. $n_0 = 1 - n_a - n_b$. For simplicity we write e_i instead of e_i^2 for $i = 0, a, b$. The first order conditions w.r.t. e_i , n_0 and n_b are given by

$$C'_i(e_i) + D'(E^2) = 0, \quad i = 0, a, b \quad (12)$$

$$\frac{1}{r} (C_0(e_0) + e_0 D'(E^2)) - \mu_0 - \nu = 0 \quad (13)$$

$$\frac{1}{r} (C_b(e_b) + e_b D'(E^2)) + F_b - \mu_b - \nu = 0 \quad (14)$$

Eliminating ν from the equations (13) and (14) yields

$$\frac{1}{r} (C_b(e_b) - C_0(e_0) + (e_b - e_0) D'(E^2)) + F_b - \mu_b + \mu_0 = 0 \quad (15)$$

Considering first the interior solutions (i.e. $\mu_0 = \mu_b = 0$), we differentiate the equation system (12) and (15) with respect to F_b . Employing the Envelope Theorem, we obtain:

$$C_i''(e_i) \frac{\partial e_i}{\partial F_b} + D''(E) \frac{\partial E^2}{\partial F_b} = 0, \quad i = 0, a, b \quad (16)$$

and

$$\frac{1}{r} D''(E) \frac{\partial E^2}{\partial F_b} (e_b - e_0) + 1 = 0 \quad (17)$$

Solving for $\partial E^2 / \partial F_b$ yields:

$$\frac{\partial E^2}{\partial F_b} = \frac{r}{D''(E)(e_0 - e_b)} > 0$$

Substituting this equation into (16) yields $\partial e_i / \partial F_b < 0$ for each type of firm. We can write the total emissions as $E(F_b) = (1 - n_a - n_b)e_0 + n_a e_a + n_b e_b$. Differentiating this equation with respect to F_b and solving the result for $\partial n_b / \partial F_b$ yields

$$\frac{\partial n_b}{\partial F_b} = \frac{\frac{\partial E^2}{\partial F_b} - (1 - n_a - n_b) \frac{\partial e_0}{\partial F_b} - n_a \frac{\partial e_a}{\partial F_b} - n_b \frac{\partial e_b}{\partial F_b}}{e_b - e_0} < 0.$$

Note now that the expression $C_b(e_b) - C_0(e_0) + (e_b - e_0)D'(E^2)$ is decreasing in E . Let us denote by \bar{E}^2 and \underline{E}^2 the maximum and the minimum feasible values of emissions in the second stage, respectively, occurring when no firm or all remaining firms adopt the technology b , respectively. Furthermore let $\underline{F}_b(n_a)$ and $\bar{F}_b(n_a)$ be the values of the fixed investment costs for which complete or no adoption of technology b by the remaining $1 - n_a$ firms is optimal. Then for each $F \in]\underline{F}_b(n_a), \bar{F}_b(n_a)[$ we find E such that $C_b(e_b) - C_0(e_0) + (e_b - e_0)D'(E) + F = 0$ and a corresponding n_b with $0 < n_b < 1 - n_a$ such that $E = n_a e_a + n_b e_b + (1 - n_a - n_b)e_0$. Conversely, every F_b for which there is an interior solution n_b of (11) must satisfy $F_b \in]\underline{F}_b(n_a), \bar{F}_b(n_a)[$ since for the corresponding E^2 we have $E^2 \in]\bar{E}^2, \underline{E}^2[$.

ad 4.: For the maximum feasible aggregate emission level $\bar{E}^2 = n_a e_a + (1 - n_a)e_0$, where e_i satisfies (12) we obtain

$$\frac{\partial \bar{E}^2}{\partial n_a} = (e_a - e_0) / \left[1 + D'' \left(\frac{n_a}{\frac{\partial^2 C_a}{\partial e^2}} + \frac{1 - n_a}{\frac{\partial^2 C_0}{\partial e^2}} \right) \right] < 0.$$

Analogously we get for the minimal feasible emission level

$$\frac{\partial \underline{E}^2}{\partial n_a} = (e_a - e_b) / \left[1 + D'' \left(\frac{n_a}{\frac{\partial^2 C_a}{\partial e^2}} + \frac{1 - n_a}{\frac{\partial^2 C_b}{\partial e^2}} \right) \right] > 0.$$

Since $\bar{E}^2 = n_a e_a + (1 - n_a) e_0$ and $\underline{E}^2 = n_a e_a + (1 - n_a) e_b$ both tend to the level $E_M = e_a^*(1)$ as n_a tends to 1, such that all firms use technology a . Now observe that $\underline{F}_b(n_a)$ and $\bar{b}_2(n_a)$ are given by $-[C_b(e_b) - C_0(e_0) + (e_b - e_0)D'(\bar{E}_2)]/r = \bar{F}_b(n_a)$ and $-[C_b(e_b) - C_0(e_0) + (e_b - e_0)D'(\underline{E}^2)]/r = \underline{F}_b(n_a)$, respectively. By differentiating both equations with respect to n_a we can easily derive that $\underline{F}_b(n_a)$ increases and $\bar{F}_b(n_a)$ decreases in n_a . Because obviously the LHSs of both equations tend to the same value as n_a tends to 1, both $\underline{F}_b(n_a)$ and $\bar{F}_b(n_a)$ tend to the same level F_M .

ad 5.: Let $F_b \in (\underline{F}_b(n_a), \bar{F}_b(n_a))$ be fixed. Differentiating (12) and (15) with respect to n_a and applying the Envelope Theorem we obtain

$$C_i''(e_i) \frac{\partial e_i}{\partial n_a} + D''(E^2) \frac{\partial E^2}{\partial n_a} = 0, \quad i = 0, a, b \quad (18)$$

and

$$\frac{1}{r} D''(E^2) \frac{\partial E^2}{\partial n_a} (e_b - e_0) = 0 \quad (19)$$

Hence $\partial E^2 / \partial n_a = 0$. Substituting into (18) yields $\partial e_i / \partial n_a = 0$ for $i = 0, a, b$.

Proof of Proposition 4.2 Part 1-3: First note that for given F_b there are four general possible scenarios. If $F_b \geq \bar{F}_b(0)$ for all n_a , no adoption of technology b will be socially optimal in the second stage since \bar{F}_b is decreasing in n_a . If $F_b \leq \underline{F}_b(0)$ for all n_a , full adoption of technology b by the remaining will be socially optimal in the second stage. If $F_b \in [F_M, \bar{F}_b(0))$ by Proposition 4.1, there exists a unique $\hat{n}_a \leq 1$ such that partial adoption of technology b is socially optimal for $n_a < \hat{n}_a$ and no adoption is optimal for $n_a \geq \hat{n}_a$. Conversely, if $F_b \in (\underline{F}_b(0), F_M]$ by Proposition 4.1, there exists a unique $\hat{n}_a \leq 1$ such that partial adoption of technology b is socially optimal for $n_a < \hat{n}_a$, and full adoption of technology b by the remaining firms is optimal for $n_a \geq \hat{n}_a$. For the latter two cases we now solve the minimization problem under the restrictions $n_a \in (0, \hat{n}_a)$ and $n_a \in (\hat{n}_a, 1)$ and then add up the results. Since obviously optimal total emissions $E^2(n_a)$ are continuous in n_a we conclude that $n_a^*(F_a)$ is continuous. First we consider the case $F_b > \bar{F}_b(0)$: Hence for any n_a adoption of technology b is never socially optimal. Therefore we can apply the proof Proposition (4.1) i.) - iii.) in the case of $n_a = 0$.

Next consider $F_b < \underline{F}_b(0)$. In this case, for any n_a it is socially optimal that the remaining firms will adopt technology b . Therefore at the first stage the social planner solves the problem

$$\begin{aligned} \min_{\{e_0, e_a, n_0, n_a\}} & \frac{1}{\lambda + r} [n_0 C_0(e_0) + n_a C_a(e_a) + D(n_0 e_0 + n_a e_a)] + n_a F_a \\ & + \frac{\lambda}{\lambda + r} \frac{1}{r} [n_0 C_b(e_b^2(n_a)) + n_a C_a(e_a^2(n_a)) + D(n_0 e_b^2(n_a) + n_a e_a^2(n_a))] + \frac{\lambda}{\lambda + r} n_0 F_b \end{aligned} \quad (20)$$

under the constraints $n_0 \geq 0$, $n_a \geq 0$ and $n_0 + n_a = 1$. Set $E^1 = n_0 e_0 + n_a e_a$ and $E^2 = n_0 e_b^2(n_a) + n_a e_a^2(n_a)$. Again μ_i , $i = 0, a$, is the Kuhn-Tucker multiplier for $n_i \geq 0$ and ν the Lagrange multiplier for $n_0 + n_a = 1$. Following the same procedure as in the proof of Proposition 4.1 i.e. differentiating equation (20) with respect to e_i and n_i , $i = 0, a$, applying the Envelope Theorem, and eliminating ν we obtain:

$$\begin{aligned} F_a = & \frac{1}{\lambda + r} (C_0(e_0) - C_a(e_a) + (e_0 - e_a) D'(E^1)) \\ & + \frac{\lambda}{\lambda + r} \frac{1}{r} (C_b(e_b^2) - C_a(e_a^2)) + (e_b^2 - e_a^2) D'(E^2) + \frac{\lambda}{\lambda + r} F_b + \mu_a - \mu_0 \end{aligned} \quad (21)$$

Considering first the interior solutions (i.e. $\mu_0 = \mu_a = 0$) and multiplying the last equation by $r(\lambda + r)$, we obtain:

$$\begin{aligned} r(\lambda + r) F_a = & r(C_0(e_0) - C_a(e_a)) + (e_0 - e_a) D'(E^1) \\ & + \lambda(C_b(e_b^2) - C_a(e_a^2)) + (e_b^2 - e_a^2) D'(E^2) + r\lambda F_b \end{aligned} \quad (22)$$

Furthermore, in the second period

$$C'_i(e_i^2) + D'(E^2) = 0, \quad i = a, b \quad (23)$$

still holds. Differentiating (23) with respect to n_a , we obtain

$$-C''_i(e_i^2) \frac{\partial e_i^2}{\partial n_a} = D''(E^2) [e_a^2 - e_b^2 + n_a \frac{\partial e_a^2}{\partial n_a} + (1 - n_a) \frac{\partial e_b^2}{\partial n_a}], \quad i = a, b.$$

Thus $\partial e_a^2 / \partial n_a = [C''_b(e_b^2) / C''_a(e_a^2)] \cdot \partial e_b^2 / \partial n_a$. Substituting this expression into (22) we obtain:

$$\frac{\partial e_b^2}{\partial n_a} = \frac{D''(E^2)(e_a^2 - e_b^2)}{-C''_i(e_i^2) - D''(E^2) n_a \frac{C''_b(e_b^2)}{C''_a(e_a^2)} - D''(E^2)(1 - n_a)} < 0$$

implying $\partial e_a^2 / \partial n_a < 0$. Since $D''(E^2) \cdot \partial E^2 / \partial n_a = \partial^2 C / \partial e^2 \cdot \partial e_a^2 / \partial n_a$ we obtain $\partial E^2 / \partial n_a > 0$.

Note that we can reinterpret the maximization problem above as a two stage procedure,

where we first determine e_a, e_0 for given n_a . These values will be delivered by (23) together with $E^1 = n_a e_a + (1 - n_a) e_0$. Now doing the analogous calculations as above, we derive $\partial e_i / \partial n_a > 0$, $i = 0, a$, and $\partial E^1 / \partial n_a < 0$. With these results we are able to calculate $\partial E^1 / \partial F_a$ and $\partial n_a / \partial F_a$. For that purpose we differentiate (22) and (23) w. r. t. F_a yielding

$$C_i''(e_i) \frac{\partial e_i}{\partial F_a} + D''(E^1) \frac{\partial E^1}{\partial F_a} = 0, \quad i = 0, a,$$

and

$$r(\lambda + r) = r(e_0 - e_a) D''(E^1) \frac{\partial E^1}{\partial F_a} + \lambda(e_b^2 - e_a^2) D''(E^2) \frac{\partial E^2}{\partial F_a}.$$

Using $\partial E^1 / \partial F_a = \partial E^1 / \partial n_a \cdot \partial n_a / \partial F_a$ and solving for $\partial n_a / \partial F_a$ yields

$$\frac{\partial n_a}{\partial F_a} = \frac{r(\lambda + r)}{r(e_0 - e_a) D''(E^1) \frac{\partial E^1}{\partial n_a} + \lambda(e_b^2 - e_a^2) D''(E^2) \frac{\partial E^2}{\partial n_a}} < 0$$

from which we can derive $\partial e_i / \partial F_a < 0$, $\partial e_i^2 / \partial F_a > 0$, $\partial E^1 / \partial F_a > 0$ and $\partial E^2 / \partial F_a < 0$.

Furthermore by differentiating the RHS of equation (21) with respect to n_a , still assuming an interior solution (i.e. $\mu_0 = \mu_a = 0$), we verify that the RHS of 21) decreases in n_a . Thus analogously to the proof of Proposition 4.1 we can show that there exists $[\underline{F}_a, \overline{F}_a]$ such that $\mu_0 = \mu_a = 0$ for every $F_a \in [\underline{F}_a, \overline{F}_a]$, and such that $\mu_0 > 0$ if $F_a > \overline{F}_a$ and $\mu_a > 0$ if $F_a < \underline{F}_a$ respectively

Finally let $F_b \in [\underline{F}_b(0), \overline{F}_b(0)]$. Depending on whether $F_b > F_M$ or $F_b < F_M$, there exists an $\bar{n}_a < 1$ such that $F_b = \underline{F}_b(\bar{n}_a)$ or $F_b = \overline{F}_b(\bar{n}_a)$. The social planner's problem can then be written as

$$\begin{aligned} & \min_{e_0, e_a, n_0, n_a} \frac{1}{\lambda + r} (n_0 C_0(e_0) + n_a C_a(e_a) + D(n_0 e_0 + n_a e_a)) + n_a F_a \\ & + \frac{\lambda}{\lambda + r} \frac{1}{r} ((1 - n_a - n_b(n_a)) C_0(e_0^2(n_a)) + n_b(n_a) C_b(e_b^2(n_a)) + n_a C_a(e_a^2(n_a))) \\ & + D((1 - n_a - n_b(n_a)) e_0^2(n_a) + n_b(n_a) e_b^2(n_a) + n_a e_a^2(n_a)) + \frac{\lambda}{\lambda + r} n_b(n_a) F_b \end{aligned}$$

subject to the constraints $0 \leq n_a \leq \bar{n}_a$ and $n_0 + n_a = 1$. Set $E^1 := n_0 e_0 + n_a e_a$ and $E^2 := (1 - n_a - n_b(n_a)) e_0^2(n_a) + n_b(n_a) e_b^2(n_a) + n_a e_a^2(n_a)$. Analogously as in the case above from the FOCs we can derive

$$\begin{aligned} 0 &= \frac{1}{\lambda + r} (C_a(e_a) - C_0(e_0) + (e_a - e_0) D'(E^1)) \\ &+ \frac{\lambda}{\lambda + r} \frac{1}{r} (C_a(e_a^2) - C_0(e_0^2) + (e_a^2 - e_0^2) D'(E^2)) + F_a \end{aligned} \quad (24)$$

Now we can apply Proposition 4.1 iii.) to the case of partial adoption of technology b . Hence both e_i^2 , $i = 0, a, b$, and E^2 only depend on F_a but neither on n_a nor on F_a . Thus, by differentiating (23) and (24) w. r. t. F_a , we obtain :

$$\frac{\partial^2 C_i}{\partial e^2} \frac{\partial e_i}{\partial F_a} + D''(E^1) \frac{\partial E^1}{\partial F_a} = 0, \quad i = 0, a,$$

and

$$\frac{1}{\lambda + r} ((e_a - e_0) D''(E^1) \frac{\partial E^1}{\partial F_a}) + 1 = 0$$

Therefore

$$\frac{\partial E^1}{\partial F_a} = \frac{\lambda + r}{(e_0 - e_a) D''(E^1)} > 0$$

Following the proof of Proposition 4.1 we derive $\partial e^i / \partial F_a < 0$, for $i = 0, a$ and $\partial n_a / \partial F_a < 0$. Thus we find a minimum value F_{min} and a maximum value F^{max} such that for each $F_a \in]F_{min}, F^{max}[$ there is an interior solution for n_a . Now if $F_b = F_M$ then obviously $\underline{F}_a = F_{min}$ and $\overline{F}_a = F^{max}$. For the case that no or full adoption of technology b is optimal, the result immediately follows. Otherwise as derived above there are two intervals $[\underline{F}, \overline{F}]$ and $[\underline{F}', \overline{F}']$ such that for all $F_a \in (\underline{F}, \overline{F})$ we have $0 < n_a < \hat{n}_a$ and for $F_a \in (\underline{F}', \overline{F}')$ we have $\hat{n}_a < n_a < 1$ and either no ($F_b > F_M$) or full ($F_b < F_M$) adoption of technology b follows. Obviously $\overline{F} = \underline{F}'$. Thus $[\underline{F}, \overline{F}']$ is the interval we are looking for. q.e.d.

Proof of Proposition 4.2 4.-5.: ad 4.: Let us first consider the case where partial adoption is socially optimal at the second stage. Thus (24) is relevant. If we differentiate (24) with respect to λ , solve for $\partial n_a / \partial \lambda$ and apply that E^2 is independent of n_a , we obtain

$$\frac{\partial n_a}{\partial \lambda} = \frac{(C_a(e_a) - C_0(e_0) + (e_a - e_0) D'(E^1) - (C_a(e_a^2) - C_0(e_0^2) + (e_a^2 - e_0^2) D'(E^2)))}{(\lambda + r)(e_a - e_0) D''(E^1) \frac{\partial E^1}{\partial n_a}}$$

The second part of the numerator, representing the second stage, is smaller than the first part in absolute values. Thus the numerator is negative. The denominator is clearly positive.

If we differentiate (24) with respect to F_b and solve for $\partial n_a / \partial F_b$ we obtain:

$$\frac{\partial n_a}{\partial F_b} = \frac{\frac{\lambda}{r} (e_a^2 - e_0^2) D''(E^2) \frac{\partial E^2}{\partial F_b}}{(e_0 - e_a) D''(E^1) \frac{\partial E^1}{\partial n_a}}$$

Both the numerator and the denominator are negative, since $\partial E^1/\partial n_a < 0$ and $\partial E^2/\partial F_b > 0$. In case of full adoption at the second stage, we differentiate (21) with respect to λ and again solve for $\partial n_a/\partial \lambda$ to obtain:

$$\begin{aligned} \frac{\partial n_a}{\partial \lambda} = & \frac{C_a(e_a) - C_0(e_0) + (e_a - e_0)D'(E^1)}{(\lambda + r)[(e_a - e_0)D''(E^1)\frac{\partial E^1}{\partial n_a} + \frac{\lambda}{r}(e_a^2 - e_b^2)D''(E^2)\frac{\partial E^2}{\partial n_a}]} \\ & - \frac{C_a(e_a^2) - C_0(e_0^2) + (e_a^2 - e_0^2)D'(E^2)}{(\lambda + r)[(e_a - e_0)D''(E^1)\frac{\partial E^1}{\partial n_a} + \frac{\lambda}{r}(e_a^2 - e_b^2)D''(E^2)\frac{\partial E^2}{\partial n_a}]} \\ & + \frac{(C_b(e_b^2) - C_0(e_0^2) + (e_b^2 - e_0^2)D'(E^2) + rF_b)}{(\lambda + r)[(e_a - e_0)D''(E^1)\frac{\partial E^1}{\partial n_a} + \frac{\lambda}{r}(e_a^2 - e_b^2)D''(E^2)\frac{\partial E^2}{\partial n_a}]} \end{aligned}$$

The denominator is clearly positive. The difference of the first two parts of the numerator is negative for the same reasons as in case of partial adoption at the second stage. The third part in principle represents the first order condition with respect to n_b . Since, however, the optimum is adopted at the upper boundary for n_b (complete adoption), this term must be negative. Analogously we get:

$$\frac{\partial n_a}{\partial F_b} = \frac{\lambda}{(e_a - e_0)D''(E^1)\frac{\partial E^1}{\partial n_a} + \lambda\frac{1}{r}(e_a^2 - e_b^2)D''(E^2)\frac{\partial E^2}{\partial n_a}} < 0$$

ad 5.: It is obvious that neither \underline{F}_a nor \overline{F}_a depends on F_b if no adoption of technology b is optimal at the second stage, which for \underline{F}_a is the case if and only if $F_b > F_M$, while for \overline{F}_a this is the case if and only if $F_b > \overline{F}_b b(0)$. In case of partial adoption at the second stage we only have to consider \overline{F}_a . Partial adoption is optimal if and only if $\underline{F}_b(0) < F_b < \overline{F}_b(0)$. In that case $\overline{F}_a(F_b)$ is given by equation (24) where $E^1 = e_0$ and $E^2 = n_b e_2 + (1 - n_b)e_0$, where n_b is the optimal share of firms using technology b . From these equations we obtain

$$= \frac{\lambda}{(\lambda + r)r}(e_a^2 - e_0^2)D''(E^2)\frac{\partial E^2}{\partial F_b} + \frac{\partial \overline{F}_a}{\partial F_b}.$$

Since $\partial E^2/\partial F_b = r/(D''(E^2)(e_b^2 - e_0^2))$ we get $\partial \overline{F}_a/\partial F_b = \lambda/(\lambda + r) \cdot [(e_a^2 - e_0^2)/(e_b^2 - e_0^2)]$. For $\overline{F}_a(F_b)$ full adoption of technology b can only occur if and only if $F_b < F_M$. For $\overline{F}_a(F_b)$ full adoption of technology b is the relevant scenario if and only if $F_b < \underline{F}_b(0)$. Thus to derive the effect on $\overline{F}_a(F_b)$ we have to differentiate (21) considering $E^1 = e_0$ and $E^2 = e_2$. Thus we obtain:

$$0 = \frac{\partial \overline{F}_a}{\partial F_b} - \frac{\lambda}{(\lambda + r)}.$$

Analogously we can compute that $\frac{\partial F_a}{\partial F_b} = \frac{\lambda}{(\lambda+r)}$. Finally note that on $[F_b(0), F_M]$ we have $\partial \underline{F}_a / \partial F_b > \partial \bar{F}_a / \partial F_b$ since $\frac{(e_a^2 - e_0^2)}{(e_b^2 - e_0^2)} < 1$.

Proof of Lemma (5.1): *ad 1.* : Differentiating ΔC_{0i} with respect to τ and using the Envelope Theorem yields

$$\frac{\partial \Delta C_{0i}}{\partial \tau} = \frac{1}{r}(e_i - e_0) < 0.$$

For $\tau = 0$ we have $\Delta C_{0i} = F_i$. If the term $C_i(e_i) - C_0(e_0)$ is bounded (which in particular is the case if C is bounded), a sufficient condition for a solution of $\Delta C_{0i}(\tau) = 0$ to exist is that F_i is smaller than $\frac{1}{r}[C_i(0) - C_0(0)]$. If $C_i(e_i) - C_0(e_0)$ goes to $-\infty$ as e_i, e_0 tend to 0, a root for $\Delta C_{0i}(\tau) = 0$ exists for each F_i . Differentiating $\Delta C_{0i}(\tau) = 0$ implicitly with respect of F_i , using the Envelope Theorem, and solving for $\partial \bar{\tau}_i / \partial F_i$ yields:

$$\frac{\partial \bar{\tau}_i}{\partial F_i} = \frac{r}{(e_0 - e_i)} > 0.$$

ad 2. : Differentiating $\Delta C_{ab}(\tau) = 0$ with respect to τ and using the Envelope Theorem yields

$$\frac{\partial \Delta C_{ab}}{\partial \tau} = \frac{1}{\lambda + r}(e_a(\tau) - e_0(\tau)) + \frac{\lambda}{\lambda + r} \frac{1}{r}(e_a(\tau) - e_b(\tau)).$$

Obviously, the sign and it's development is ambiguous without further restrictions. So let us assume that the sign is unique and let us consider that ΔC_{ab} increases. Now observe that $\Delta C_{ab}(0) = F_a - \frac{\lambda}{r+\lambda} F_b$. Thus if F_a and F_b are such that $\Delta C_{ab}(0) < 0$ holds and such that $\Delta C_{ab}(\tau)$ gets positive for τ sufficiently large, by reasons of continuity there must exist a tax rate $\bar{\tau}$ such that $\Delta C_{ab}(\bar{\tau}) = 0$. Since Δ_{ab} is monotonic, this value is unique.

Proof of Lemma (5.3): Differentiating $\Delta C_{ab}(\tau) = 0$ with respect to F_a and using the Envelope Theorem, we obtain:

$$\frac{\partial \bar{\tau}^{ab}}{\partial F_a} \left(\frac{1}{\lambda + r}(e_a - e_0) + \frac{\lambda}{\lambda + r} \frac{1}{r}(e_a - e_b) \right) + 1 = 0.$$

This yields

$$\frac{\partial \bar{\tau}_{ab}}{\partial F_a} = - \frac{r(\lambda + r)}{r(e_a - e_0) + \lambda(e_a - e_b)} = \frac{1}{\frac{\partial \Delta C_{ab}}{\partial \tau}}.$$

Analogous we can derive

$$\frac{\partial \bar{\tau}_{ab}}{\partial F_b} = \frac{r\lambda}{r(e_a - e_0) + \lambda(e_a - e_b)} = \frac{\lambda}{\lambda + r} \frac{1}{\frac{\partial \Delta C_{ab}}{\partial \tau}}.$$

Proof of Proposition (5.4) ad 1.: First we consider the case $\bar{\tau}_a < \bar{\tau}_b$. Assume first $\tau \in]\bar{\tau}_a, \bar{\tau}_b[$. Because ΔC_i , $i = a, b$ decreases, we obtain $\Delta C_a(\tau) = \frac{1}{r}[C_a(e_a) - C_0(e_0)] + \tau(e_a - e_0) + F_a < 0$ and $\Delta C_b(\tau) = \frac{1}{r}[C_b(e_b) - C_0(e_0)] + \tau(e_b - e_0) + F_b > 0$. This gives us the following inequality chain:

$$\begin{aligned}
0 &> \frac{1}{r}[C_a(e_a) - C_0(e_0)] + \tau(e_a - e_0) + F_a \\
&= \frac{1}{r + \lambda}[C_a(e_a) - C_0(e_0)] + \tau(e_a - e_0) \\
&\quad + \frac{\lambda}{r + \lambda} \frac{1}{r}[C_a(e_a) - C_0(e_0)] + \tau(e_a - e_0) + F_a \\
&> \frac{1}{r + \lambda}[C_a(e_a) - C_0(e_0)] + \tau(e_a - e_0) \\
&\quad + \frac{\lambda}{r + \lambda} \frac{1}{r}[C_a(e_a) - C_b(e_b)] + \tau(e_a - e_b) + F_a \\
&\quad - [\frac{\lambda}{r + \lambda} \frac{1}{r}[C_b(e_b) - C_0(e_0)] + \tau(e_b - e_0)] + \frac{\lambda}{r + \lambda} F_b \\
&= \Delta C_{ab}(\tau).
\end{aligned}$$

Next we consider the case $\bar{\tau}_b < \bar{\tau}_a$. Assume that $\tau \in]\bar{\tau}_b, \bar{\tau}_a[$. Following the same argument as above this implies $\Delta C_a(\tau) = \frac{1}{r}[C_a(e_a) - C_0(e_0)] + \tau(e_a - e_0) + F_a > 0$ and $\Delta C_b(\tau) = \frac{1}{r}[C_b(e_b) - C_0(e_0)] + \tau(e_b - e_0) + F_b < 0$. A similar inequality chain yields $0 < \Delta C_{ab}(\tau)$.

ad 2.: First assume $\bar{\tau}_a < \bar{\tau}_b$. Since ΔC_{ab} decreases, $\Delta C_{ab}(\tau) < 0$ for each $\tau \in [\bar{\tau}_a, \bar{\tau}_b]$ and $\bar{\tau}_{ab}$ is given by $\Delta C_{ab}(\bar{\tau}_{ab}) = 0$, $\bar{\tau}_{ab} < \bar{\tau}_a$ must be the case. Now if $\bar{\tau}_a > \bar{\tau}_b$ analogously $\bar{\tau}_{ab} > \bar{\tau}_a$ follows.

ad 3.: Analogously.

Proof of Proposition 5.5 We will prove the first three claims via contradiction:

ad 1. : Let $L > \bar{E}^2(n_a)$ and suppose $\sigma > \bar{\sigma}^2$. Hence, all $(1 - n_a)$ firms using the old technology, would adopt technology b and aggregate emissions will fall below $\underline{E}^2(n_a)$. Therefore the permit market would not be cleared because $\underline{E}^2(n_a) < L$, a contradiction. Now suppose $\sigma = \bar{\sigma}^2$. In this case the emission level could not be larger than $\bar{E}^2(n_a)$. But $\bar{E}^2(n_a) < L$. Analogous we can prove 2. and 3. . The share of firms investing if $L \in [\underline{E}^2(n_a), \bar{E}^2(n_a)]$ can easily be derived from $n_a e_a + n_b e_b + (1 - n_a - n_b) e_0 = L$.

ad 4. : It is obvious that $\bar{E}^2(n_a)$ is strictly decreasing in n_a and tends to $e_a(\bar{\sigma}^2)$. Similary

$\underline{E}_2(n_a)$ is strictly increasing in n_a and also tends to $e_a(\bar{\sigma}^2)$. q.e.d.

To prove Proposition 5.7 we use the following Lemma which characterizes the market price at the first stage:

Lemma A.1. *The market price for permits in the first stage $\sigma^1(n_a)$ is strictly decreasing in n_a .*

Afterwards we will prove that the threshold price $\bar{\sigma}^1(n_a)$ is non-decreasing and continuous in n_a respectively. For this purpose we will prove the following Lemma:

Lemma A.2. *The threshold price $\bar{\sigma}^1(n_a)$ is characterized as follows:*

1. *If $L > \bar{E}^2(0)$, then $\bar{\sigma}^1(n_a)$ equals $\bar{\sigma}^{0a}(\sigma^2(n_a))$ for all n_a and is increasing in n_a .*
2. *If $L < \underline{E}^2(0)$, then $\bar{\sigma}^1(n_a)$ equals $\bar{\sigma}^{ab}(\sigma^2(n_a))$ for all n_a and is increasing in n_a .*
3. *If $L \in [\underline{E}^2(0), \bar{E}^2(0)]$, we have the following three subcases:*

- (a) *For $L = E_M$, $\bar{\sigma}^1(n_a)$ equals $\bar{\sigma}^{ab}(\bar{\sigma}^2)$ for all n_a and is therefore constant.*
- (b) *For $L < E_M$ let \bar{n}_a be the share of firms where $L = \underline{E}^2(\bar{n}_a)$. For $n_a \leq \bar{n}_a$ the threshold price $\bar{\sigma}^1(n_a)$ equals $\bar{\sigma}^{ab}(\bar{\sigma}^2)$ and is therefore constant in n_a . For $n_a > \bar{n}_a$ the price $\bar{\sigma}^1(n_a)$ equals $\bar{\sigma}^{ab}(\sigma^2(n_a))$ and is increasing. For $n_a = \bar{n}_a$ we have $\bar{\sigma}^1(n_a) = \bar{\sigma}^{ab}(\bar{\sigma}^2) = \bar{\sigma}^{ab}(\sigma^2(\bar{n}_a))$.*
- (c) *If $L > E_M$, let \bar{n}_a be the share of firms where $L = \bar{E}^2(\bar{n}_a)$. For $n_a \leq \bar{n}_a$ the threshold price $\bar{\sigma}^1(n_a)$ equals $\bar{\sigma}^{ab}(\bar{\sigma}^2)$ and is therefore constant in n_a . For $n_a > \bar{n}_a$ the price $\bar{\sigma}^1(n_a)$ equals $\bar{\sigma}^{0<}(\sigma^2(n_a))$ and is increasing. For $n_a = \bar{n}_a$ we have $\bar{\sigma}^1(n_a) = \bar{\sigma}^{ab}(\bar{\sigma}^2) = \bar{\sigma}^{0a}(\sigma^2(\bar{n}_a))$.*

Proof of Lemma A.1: We differentiate the two market equilibrium equations, $-\frac{\partial C_i}{\partial e} = \sigma^1$, $i = 0$, and $L = n_a e_a + (1 - n_a) e_0$ implicitly with respect to n_a . This yields $-\partial^2 C_i / \partial e^2 \cdot \partial e_i / n_a = \partial \sigma^1 / \partial n_a$, $i = 0, a$, and $0 = e_a - e_0 + n_a \partial e_a / \partial n_a + (1 - n_a) \frac{\partial e_0}{\partial n_a}$. Thus we can derive

$$\frac{\frac{\partial^2 C_a}{\partial e^2} \partial e_a}{\frac{\partial^2 C_0}{\partial e^2} \partial n_a} = \frac{\partial e_0}{\partial n_a},$$

and therefore

$$\frac{\partial e_a}{\partial n_a} = \frac{e_0 - e_a}{n_a + (1 - n_a) \frac{\frac{\partial^2 C_a}{\partial e^2}}{\frac{\partial^2 C_0}{\partial e^2}}} > 0.$$

Obviously this implies $\partial \sigma^1 \partial n_a < 0$. q.e.d.

Proof of Lemma A.2: *ad 1.:* In that case, given each n_a it is optimal for all remaining firms not to adopt technology b in the second stage. Moreover, the market price in the second stage equals the one in the first stage since none of the remaining firm adopts technology b . Differentiating $\Delta C_{0a}(\bar{\sigma}^1(n_a), \sigma^1) = 0$ (Note that $\bar{\sigma}^1(n_a) = \bar{\sigma}^{0a}(\sigma^1)$) with respect to n_a , employing both the Envelope Theorem and Lemma A.1, we obtain:

$$\frac{\partial \bar{\sigma}^1}{\partial n_a} = \frac{\lambda \frac{\partial \sigma^1}{\partial n_a} (e_a(\sigma^1) - e_0(\sigma^1))}{r(e_0(\bar{\sigma}^1) - e_a(\bar{\sigma}^1))} > 0.$$

ad 2.: In that case, given each n_a it is optimal for the remaining firms to adopt technology b in the second stage. Thus in the first stage each firm decides between adopting technology a immediately and adopting technology b in the second stage. Thus $\bar{\sigma}^1(n_a) = \bar{\sigma}^{ab}(\sigma^2)$. Differentiating $\Delta C_{ab}(\bar{\sigma}^1(n_a), \sigma^2) = 0$ with respect to n_a , and using the Envelope Theorem, we obtain:

$$\frac{\partial \bar{\sigma}^1}{\partial n_a} = \frac{\lambda \frac{\partial \sigma^2}{\partial n_a} (e_a^2(\sigma^2) - e_b(\sigma^2))}{r(e_0(\bar{\sigma}^1) - e_a(\bar{\sigma}^1))}.$$

Thus to prove $\partial \bar{\sigma}^1 / \partial n_a > 0$ it suffies to show $\partial \sigma^2 / \partial n_a > 0$. Differentiating the market equilibrium equations $-\frac{\partial C_i}{\partial e} = \sigma^2$, $i = a, b$, and $L = n_a e_a^2 + (1 - n_a) e_b^2$ with respect to n_a we obtain $-\partial^2 C_i / \partial e^2 \cdot \partial e_i^2 / \partial n_a = \partial \sigma^2 / \partial n_a$, $i = a, b$, and $0 = e_a^2 - e_b^2 + n_a \partial e_a^2 / \partial n_a + (1 - n_a) \partial e_b^2 / \partial n_a$, yielding $[\frac{\partial^2 C_a}{\partial e^2} / \frac{\partial^2 C_b}{\partial e^2}] \cdot [\partial e_a^2 / \partial n_a] = \partial e_b^2 / \partial n_a$ and therefore

$$\frac{\partial e_a^2}{\partial n_a} = \frac{e_b^2 - e_a^2}{n_a + (1 - n_a) \frac{\frac{\partial^2 C_a}{\partial e^2}}{\frac{\partial^2 C_b}{\partial e^2}}} < 0.$$

Obviously this implies $\partial \sigma^2 / \partial n_a > 0$.

ad 3.: (a) By Proposition 5.5 we know that if $L = E_M$ then for all $n_a < 1$ partial adoption of technology b occurs in the second stage. This implies that in equilibrium each firm that has not adopted technology a in the first stage is indifferent between adopting technology b and not adopting it in the second stage. Therefore $\sigma^2 = \bar{\sigma}^2$. Therefore in the first stage

$\bar{\sigma}^1 = \bar{\sigma}^{ab}(\bar{\sigma}^2) = \bar{\sigma}^{0a}(\bar{\sigma}^2)$ must be the case in equilibrium.

(b) By Proposition 5.5 we know that \bar{n}_a satisfying $L = \underline{E}(\bar{n}_a)$ exists since $L < E_M$. Furthermore for all $n_a < \bar{n}_a$ partial adoption in the second case occurs while for all $n_a \geq \bar{n}_a$ full adoption of technology b is the equilibrium outcome. Thus in each case we can apply the arguments above and therefore we only need to show that $\bar{\sigma}^1(n_a)$ is continuous at \bar{n}_a . But by Proposition 5.5 we know that all remaining $(1 - \bar{n}_a)$ firms invest in technology b and σ^2 must be equal to $\bar{\sigma}^2$. That is how \bar{n}_a is defined. Thus all these $(1 - \bar{n}_a)$ firms are indifferent between investing into technology b and not investing at all in the second stage. It therefore directly follows from $\Delta C_{0b}(\bar{\sigma}^2) = 0$ that for $\bar{\sigma}^1$ and $\bar{\sigma}^1$ satisfying $\Delta C_{0b}(\bar{\sigma}^1, \bar{\sigma}^2) = 0$ and $\Delta C_{ab}(\bar{\sigma}^1, \bar{\sigma}^2) = 0$ the equality $\bar{\sigma}^1 = \bar{\sigma}^1$ must hold. Hence the threshold price $\bar{\sigma}^1$ is continuous in \bar{n}_a .

(c) Analogous to (b).

Proof of Proposition 5.7: First note that in case of an equilibrium with partial adoption, the threshold price $\bar{\sigma}$ must equal the market price.

ad 1.: If $L > \bar{E}^1$ the market price is lower than the lowest possible threshold price. Thus no firm will adopt technology a .

ad 2.: If $L < \underline{E}^1$ the market price is higher than the highest possible threshold price. Therefore all firms will adopt technology a .

ad 3.: In this case we find n_a such that $E(\sigma^1(n_a)) = L = E(\bar{\sigma}(n_a))$. For this n_a the permit market clears and there is no incentive for any firm to take another decision. q.e.d.

Proof of proposition (6.1): In the following E^1 and E^2 denote the socially optimal aggregated emission levels corresponding to (F_a, F_b) . To shorten the notation in general we will only write \hat{F}_a and \tilde{F}_a .

First note that irrespective to the share of firms which have adopted technology a in the first stage all firms with the conventional technology will adopt technology b if $F_b > \bar{F}_2(0)$ since for $F_b = \bar{F}_2(0)$ we have $\bar{\tau}_{0b} = \tau_0$. Furthermore note that for $F_b = \bar{F}_b(0)$ the equality $E^2 = E_0^*$ holds. Since $\tau_0 = D'(E_0^*) = \bar{\tau}^2$ for $F_b = \bar{F}_b(0)$ the first order condition of the social planner coincides with (8). Moreover for $F_a = \bar{F}_a(\bar{F}_b(0))$ obviously $E^1 = E_0^*$ holds

(see Figure 4 and 5). Now since for $F_a = \bar{F}_a(\bar{F}_b(0))$ the first order condition of the social planner coincides with (9) and (10), we get $\tau_0 = D'(E_0^*) = \bar{\tau}_{0a} = \bar{\tau}_{ab}$.

a.) : In this case $\bar{\tau}_{0b} > D'(E_0^*)$ since $\frac{\partial \bar{\tau}_{0b}}{\partial F_b} > 0$. Hence no firm would like to invest into the second technology irrespective to what happened in the first stage. If $F_a < \bar{F}_a$ then $\bar{\tau}_{0a} < \tau_0$ has to follow which implies complete adoption technology 1. As mentioned above $F_a = \bar{F}_a$ yields $\bar{\tau}_{0a} = D'(E_0^*)$. Thus every firm is indifferent between adopting technology a and doing not so. At last $F_a > \bar{F}_a$ yields $\bar{\tau}_{0a} > \tau_0$. Therefrom no firm would adopt technology a .

b.) : For $F_a = \underline{F}_a$ by (9) and the corresponding first order condition of the social planner (22) $\bar{\tau}_{ab} = D'(E^1)$ has to hold. Obviously $D'(E^1) < \tau_0$. If ΔC_{ab} decreases it must be $\Delta C_{ab}(\tau_0) < 0$. This implies $\tilde{F}_a > \underline{F}_a$ since \tilde{F}_a is defined by $\Delta C_{ab}(\tau_0) = 0$ given F_b . If ΔC_{ab} increases, $\Delta C_{ab}(\tau_0) > 0$, and analogously $\tilde{F}_a < \underline{F}_a$ follows. *c.)* : First remember that $\hat{F}_a(F_b)$ is the cost level where $\bar{F}_b(\hat{F}_a(F_b)) = F_b$ holds. Thus $\tau_{ab} = D'(E^1)$ for $F_a = \hat{F}_a$ since it is socially optimal for this installment cost pair that a share \hat{n}_a of firms adopt technology a and no firm adopts technology b . Again note that $D'(E^1) < \tau_0$. Again if ΔC_{ab} decreases, $\Delta C_{ab}(\tau_0) < 0$ holds and therefore $\tilde{F}_a > \hat{F}_a$. Analogously follows that if ΔC_{ab} increases $\tilde{F}_a < \hat{F}_a$ has to hold.

d.) : This is obvious.

q.e.d.

Proof of proposition (6.2): We denote with E^2 the socially optimal aggregated emission level which corresponding to a installment cost of F_b . First note that given F_b it is socially optimal that a partial share of firms adopt technology b always $\bar{\sigma}_{02} = D'(E^2)$ has to hold.

1.) : For $n_a = 0$ in case that no firm adopts technology b the permit price σ^2 equals $D'(E_0^*)$. Furthermore from the first order condition of the social optimal solution for $F_b = \bar{F}_b(0)$ we have $\bar{\sigma}_{0b} = D'(E_0^*)$ follows. If $n_a > 0$ then obviously none of the firms using still the conventional technology will adopt technology b for $F_b = \bar{F}_b(0)$ since $\sigma^2(0) < D'(E_0^*) = \bar{\sigma}_{0b}$. But since $F_b^o(n_a)$ is determined by $\Delta C_{0b}(\sigma^2(0)) = 0$ it follows that $F_b^o(n_a) < \bar{F}_b(0)$.

For $F_b = \underline{F}_b(n_a)$ we have $\bar{\sigma}_{0b} = D'(E^2)$. Obviously $D'(E^2) > \sigma^2(1 - n_a)$, since $D'(E^2) = \sigma^2(1 - n_a)$ is only the case if the socially optimal number of permits has been issued. Since

$\Delta C_{0b}(\sigma^2(1 - n_a)) = 0$ determines $F_b^u(n_a)$ the relationship $F_b^u(n_a) < \underline{F}_b(n_a)$ must hold.

2.) and 3.) follow immediately.

4.) : We only need to consider the case $F_b \in]\underline{F}_b(n_a), F_b^o(n_a)[$. First note that $\underline{F}_b(n_a) < F_b^o(n_a)$ must not necessarily be the case. But for $\underline{F}_b(n_a) > F_b^o(n_a)$ our claim is obviously true. So assume the opposite. But obviously for $F_2 \in [\underline{F}_b(n_a), F_b^o(n_a)]$ as well $D'(E^2) = \bar{\sigma}_{0b}$ as $\sigma^2(n_b^*) < \bar{\sigma}_{0b}$ have to hold if the socially optimal share n_b^* of firms invests into technology b since $L_0 = E_0^* > E_2^*$. Therefore $\Delta C_{0b}(\sigma^2(n_b^*)) > 0$. Therefore the permit price must decrease which induces that the share of firms adopting technology b has to be smaller than the socially optimal one.

5.) : For $n'_a > n_a$ we obviously get $\sigma^2(0)|_{n_a} > \sigma^2(0)|_{n'_a}$ and $\sigma^2(1 - n'_a) > \sigma^2(1 - n_a)$. Since given a share \tilde{n}_a the upper bound $F_b^o(\tilde{n}_a)$ is defined by $\Delta C_{0b}(\sigma^2(0)) = 0$ and the lower bound $F_b^u(\tilde{n}_a)$ by $\Delta C_{0b}(\sigma^2(1 - \tilde{n}_a)) = 0$ we get $F_b^o(n'_a) < F_b^o(n_a)$ and $F_2^u(n'_a) > F_2^u(n_a)$. The level \tilde{F}_M is given by $\Delta C_{0b}(\sigma^2|_{n_a=1}) = 0$ and it is obvious that both bounds tend to this level. And since for $n_a = 1$ the permit price $\sigma^2 < D(E^2)$ also $\tilde{F}_M < F_M$ follows immediately.

Proof of proposition 6.3 and of the subsequent corollar: In the following E^1 and E^2 denote the socially optimal aggregated emission levels corresponding to a installation cost pair (F_a, F_b) .

For each $0 \leq n_b < 1 - n_a$ the corresponding F_a depends on the market price and the market outcome in the second stage. If all remaining firms or even a partial share of them adopt technology b , then the cost level F_a which corresponds to n_a is given by $\Delta C_{ab}(n_a) = 0$. Furthermore by proposition 6.2 one of the four possibilities may be the case: For all n_a none of the remaining firms adopt technology b , for all n_a all of the remaining firms adopt technology b , there exists \hat{n}_a such that partial adoption of technology b is the market outcome for $n_a < \hat{n}_a$ and none of the remaining firms adopt technology b for $n_a > \hat{n}_a$ and there exists \hat{n}_a such that partial adoption of technology b is the market outcome for $n_a < \hat{n}_a$ and all of the remaining firms adopt technology b for $n_a > \hat{n}_a$. It is easy to show that n_a decreases in F_a .

ad corollar :

1. : In this case, no firm would adopt technology b at any circumstances. Thus the proof is similar to the proof of proposition 6.2 where $n_a = 0$.

2. : For $F_a = F_a^o(F_b)$ partial adoption of technology b is the market outcome in the second stage. By the FOC of the social optimum and proposition 5.5 the permit price in the second stage must equalize the socially optimal marginal damage $D'(E^2)$ as long partial adoption of technology b is also socially optimal for $n_a = 0$, i.e. $F_b > \underline{F}_b$. This implies, since in the first stage $\sigma^1 = D'(E_0^*)$ holds, that $\bar{F}_a(F_b) = F_b^o(F_b)$ since both are given by $\Delta C_{ab}(D'(E_0^*), D'(E_b^*)) = 0$. If $F_b < \bar{F}_b$ full adoption is socially optimal, i.e. $D'(E^2) > \bar{\sigma}^2 = \sigma^2$. Since for $F_a = \bar{F}_a(F_b)$ we have $\Delta C_{ab}(D'(E_0^*), D'(E_2^*)) = 0 > \Delta C_{ab}(D'(E_0^*), \sigma^2) = 0$ the relationship $F_a^o(F_b) > \bar{F}_a(F_b)$ must hold. For $F_a = F_a^u(F_b)$ we get $\sigma^1 < D'(E^1)$ and $\sigma^1 = \sigma^2$, since all firms adopt technology a , and for no adoption of technology b would be the optimal alternative in the second stage in the decentralized case, since $F_2 > \tilde{F}^M$. For $F_2 > F^M$ no adoption is also optimal from the social planners point of view. Since $\Delta C_{0a}(D'(E^1), D'(E^1)) = 0$ for $\underline{F}_a(F_b)$ we get $F_a^u(F_b) < \underline{F}_a(F_b)$ since $\Delta C_{0a}(\tau)$ decreases in τ in that case. For $F_b < F_M$ we have adoption of technology b as socially optimal alternative while no adoption is the optimal alternative in the decentralized situation. Thus for $F_a = \bar{F}_a(F_b)$ we have $\Delta C_{0a}(D'(E^1), D'(E^1)) < \Delta C_{ab}(D'(E^1), D'(E^1)) = 0$. Thus if ΔC_{ab} increases it is ambiguous whether $F_a^u(F_b) < \underline{F}_a(F_b)$ or $F_a^u(F_b) > \underline{F}_a(F_b)$ (see Figure 7). But if ΔC_{ab} decreases $\Delta C_{0a}(D'(E^1), D'(E^1)) < \Delta C_{ab}(D'(E^1), D'(E^1)) = 0$ and $0 = \Delta C_{0a}(\sigma^1, \sigma^1) > \Delta C_{ab}(\sigma^1, \sigma^1)$ can only be the case if $F_a^u(F_b) < \underline{F}_a(F_b)$ since ΔC_{0a} is steeper than ΔC_{ab} .

$n_a^* + n_b^* < n_a + n_b$: If both $n_a^* = n_a = 0$ the result follows from 6.2. Thus consider a cost pair (F_a, F_b) where $1 > n_a^* > 0$ and $1 > n_a > 0$ and where $n_b^* > 0$ and $n_b > 0$ and where $n_a^* + n_b^* < 1$. First note that by the FOC of the social optimal solution and proposition 5.5 both, the socially optimal damage and the permit price, must equal the threshold price $\bar{\sigma}^2$ which only depends on F_b . Note also that if $n_a > 0$ and $n_a^* > 0$ also $\sigma^1 = \bar{\sigma}^1 = D'(E^1)$ have to hold. Now consider $n_a^* + n_b^* \geq n_a + n_b$: Note that in our case only $n_a + n_b < 1$ is possible. Now since $L > E^1$ we must get $n_a < n_a^*$. But this implies $n_b^* + n_a^* - n_a \geq n_b$. Even if only equality would hold we would get $E < E^2 < L$ since the permit price equals $D'(E^2)$. Now

assume $n_a + n_b = 1$. This would imply that $\bar{\sigma}^2 \leq D'(E^2)$ and thus $\sigma^2 \leq D'(E^2)$.

3. : The result about $F_a^o(F_b)$ can be derived as in the previous case. So consider $F_a = F_a^u(F_b)$. For $F_a = F_a^u(F_b)$ we get $\sigma^1 < D'(E^1)$ and $\sigma^1 = \sigma^2$, since all firms adopt technology a , and adoption of technology b would be the optimal alternative in the second stage, since $F_b < \tilde{F}_M$. Thus ΔC_{ab} is the relevant function we have distinguish between the case that ΔC_{ab} decreases and ΔC_{ab} increases. Now $\bar{F}_a(F_b)$ is characterized by $\Delta C_{ab}(D'(E^1), D'(E^1)) = 0$ while $F_a^u(F_b)$ is characterized by $\Delta C_{ab}(\sigma^1, \sigma^1) = 0$. Thus if ΔC_{ab} increases then this implies $F_a^u > \bar{F}_a(F_b)$. Analogously if ΔC_{ab} decreases $F_a^u < \bar{F}_a$ follows. The claim about $n_a + n_b$ can be proven analogously to the previous case.

4. : If $F_b \leq F_b^u(0)$ always full adoption of technology 2 by the remaining firms is the market outcome in the second stage. Since for $F_b \leq F_b^u$ also $F_b^u(0) < \underline{F}_b(0)$ holds for $n_a = 0$ the market price σ^2 will be smaller than the socially optimal marginal damage $D'(E^2)$. Thus as above $F_a^o(F_b) > \bar{F}_a(F_b)$ follows. and again as above also $F_a^u(F_b) < \underline{F}_a(F_b)$ follows if ΔC_{ab} decreases and $F_a^u(F_b) < \underline{F}_a(F_b)$ follows if ΔC_{ab} increases. The rest follows immediately.

Proof of proposition (7.1) permit case: In principle we apply the same arguments as in the proof of the next proposition, proposition 7.2. As we will do there we can define a threshold price which is non-decreasing in the share of technology for each stage while the market price is strictly decreasing. Thus we can prove that there is always a unique equilibrium and we can easily see - like in that proof - that the efficient allocation is an equilibrium. *tax case:* First we consider all cases where it is socially optimal that all firms adopt the same technology. If for example $F_b > \bar{F}_b(0)$ and $F_a > \bar{F}_a(F_b)$, then by the proofs of proposition 4.1 and 4.2 $D'(E^1) < \bar{\tau}_{0a}$ and $D'(E^2) < \bar{\tau}_{0b}$. Thus no firm will invest. If $F_b < \underline{F}_b(0)$ and $F_a > \bar{F}_a(F_b)$, $D'(E^1) < \bar{\tau}_{0a}$ and $D'(E^2) > \bar{\tau}_{0b}$ and therefore all firms will adopt technology b . If $F_b > \bar{F}_b(0)$ and $F_a < \underline{F}_a(F_b)$, $D'(E^1) > \bar{\tau}_{0a}$ and $D'(E^2) < \bar{\tau}_{0b}$ and therefore all firms will adopt technology a . If $F_b < \underline{F}_b(0)$ and $F_a < \underline{F}_a(F_b)$, $D'(E^1) > \bar{\tau}_{0a}$ and $D'(E^2) > \bar{\tau}_{0b}$. Furthermore $\tau = D'(E^1) = D'(E^2)$ is such that all firms will prefer to adopt technology a . In all other cases at least one of the equalities $D'(E^1) = \bar{\tau}_{0a}$, $D'(E^1) = \bar{\tau}_{0b}$ or $D'(E^2) = \bar{\tau}_{0b}$ will hold. Thus all firms are indifferent between at least two strategies.

Proof of Proposition 7.2: *tax case:* First we will analyze the second stage, where n_a is given: The regulator will set $\tau^2 = D'(E^{2*}(n_b))$, where $E^{2*}(n_b)$ is the socially optimal aggregate emission level corresponding to n_b . To find the equilibrium value for n_b , we have to compare τ to the threshold level $\bar{\tau}_b$, i.e. the tax level where a firm would be indifferent between adoption and no adoption of technology b . This level is given by $\Delta_{0b}(\bar{\tau}_b) = 0$. Note that by the proof of Proposition 4.1 this is also nothing else than the first order condition for the socially optimal n_b if we replace $\bar{\tau}_b$ by $D'(E^2)$. Therefore $\bar{\tau}_b = D'(E^{2*}(n_b^*))$ holds if for the socially optimal share n_b^* the inequality $0 < n_b^* < 1 - n_a$ holds. In this case the resulting share of firms adopting technology b must be the socially optimal. Otherwise by the Ex-Post regulation rule $\tau > D'(E^{2*}(n_b^*))$ or $\tau < D'(E^{2*}(n_b^*))$ must hold since $E^{2*}(n_b)$ decreases in n_b by proposition 4.1. If $n_b^* = 0$ by the proof of proposition (4.1) $D'(E^{2*}(n_b^*)) \leq \bar{\tau}_b$ holds. Thus for each share $n_b > 0$ we must have $D'(E^{2*}(n_b)) < \bar{\tau}_b$ since E^{2*} decreases in n_b . Thus partial or full adoption cannot be an equilibrium outcome. An analogous argument applies to the case $n_b^* = 1 - n_a$.

Now we analyze the first stage. First for each n_a we define the threshold tax rate $\bar{\tau}(n_a)$. If no adoption of technology b is the result in the second stage under ex post regulation, $\bar{\tau}(n_a)$ is given by $\Delta_{0a}(\bar{\tau}(n_a), D'(E^{2*}(0))) = 0$. If partial adoption of technology b is the outcome, $\bar{\tau}(n_a)$ is given by $\Delta_{ab}(\bar{\tau}(n_a), D'(E^{2*}(n_b^*(n_a)))) = 0$ while if full adoption of technology b follows, the threshold tax is given by $\Delta_{ab}(\bar{\tau}(n_a), D'(E^{2*}(1 - n_a))) = 0$. Relating these equations to proof of Proposition 4.2 we see that these equations correspond to the first order condition of the socially optimal share at the first stage. If in particular partial adoption of both technologies is the social optimal outcome $\Delta_{ab}(D'(E^{1*}), D'(E^{2*})) = 0$ holds. Thus $\bar{\tau}(n_a^*) = D'(E^{1*}(n_a^*))$ holds if for given F_a and F_b $0 < n_a^* < 1$ holds. Furthermore $n_a^* = 1$ is equivalent to $\bar{\tau}(1) \leq D'(E^{1*}(1))$ and $\bar{\tau}(0) \geq D'(E^{1*}(0))$ is equivalent to $n_a^* = 0$. Thus the social optimal outcome is also an equilibrium with the optimal tax rate. We still need to show that it is unique. thus we assume first that $\bar{\tau}(n_a)$ is non-decreasing in n_a : First consider the case $0 < n_a^* < 1$. Since E^{1*} decreases in n_a for all $n_a < n_a^*$ we must have $\bar{\tau}(n_a) < D'(E^{1*}(n_a))$, and for all $n_a > n_a^*$ we must have $\bar{\tau}(n_a) > D'(E^{1*}(n_a))$. Therefore in the first case there is a further

incentive for the remaining firms to adopt technology a while in the second case, too many firms would have adopted technology a . In case of $n_a^* = 0$, for each share $n_a > 0$ we would have $\bar{\tau}(n_a) > D'(E^{1*}(n_a))$. Thus partial or full adoption of technology a cannot be an market equilibrium. An analogous argument applies to the case of $n_a^* = 1$. Thus it only remains to show that indeed $\bar{\tau}(n_a)$ is non-decreasing. We distinguish three cases:

Case 1: No adoption will be expected in equilibrium in the second stage: Then $\bar{\tau}(n_a)$ is given by $\Delta_{0a}(\bar{\tau}(n_a), D'(E^{2*}(0))) = 0$ as mentioned above. Obviously $\partial E^{2*}/\partial n_a < 0$. Thus

$$\frac{\partial \bar{\tau}}{\partial n_a} = \frac{(e_a^2 - e_0^2)D''(E^{2*})\frac{\partial E^{2*}}{\partial n_a}}{e_0 - e_a} > 0.$$

Case 2: Partial adoption in the second stage: By Proposition 4.1 the socially optimal emission level in the second stage is independent of n_a . Since the socially optimal emission level is also the market outcome in the second stage by differentiating $\Delta C_{ab}(\bar{\tau}(n_a), D'(E^{2*}(n_b^*(n_a)))) = 0$ w.r.t. n_a we obtain $\partial \bar{\tau}/\partial n_a = 0$.

Case 3: Complete adoption will be expected in equilibrium in the second stage: In this case $\bar{\tau}(n_a)$ is given by $\Delta C_{ab}(\bar{\tau}(n_a), D'(E^{2*}(1 - n_a))) = 0$. But it immediately follows that $\partial E^{2*}(1 - n_a)/\partial n_a > 0$ and thus

$$\frac{\partial \bar{\tau}}{\partial n_a} = \frac{(e_a^2 - e_b^2)D''(E^{2*})\frac{\partial E^{2*}(1-n_a)}{\partial n_a}}{e_0 - e_a} > 0.$$

permit case: Analogously if we substitute the tax rate by the permit price. q.e.d.

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C Figures

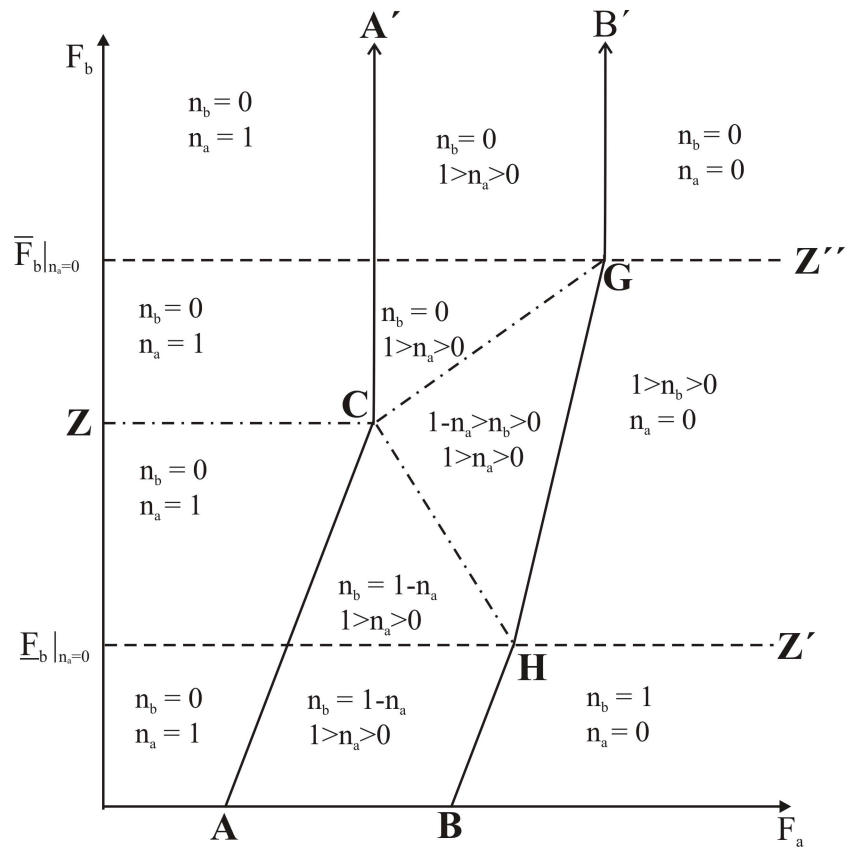


Figure 1. Socially optimal allocation with respect to (F_a, F_b)

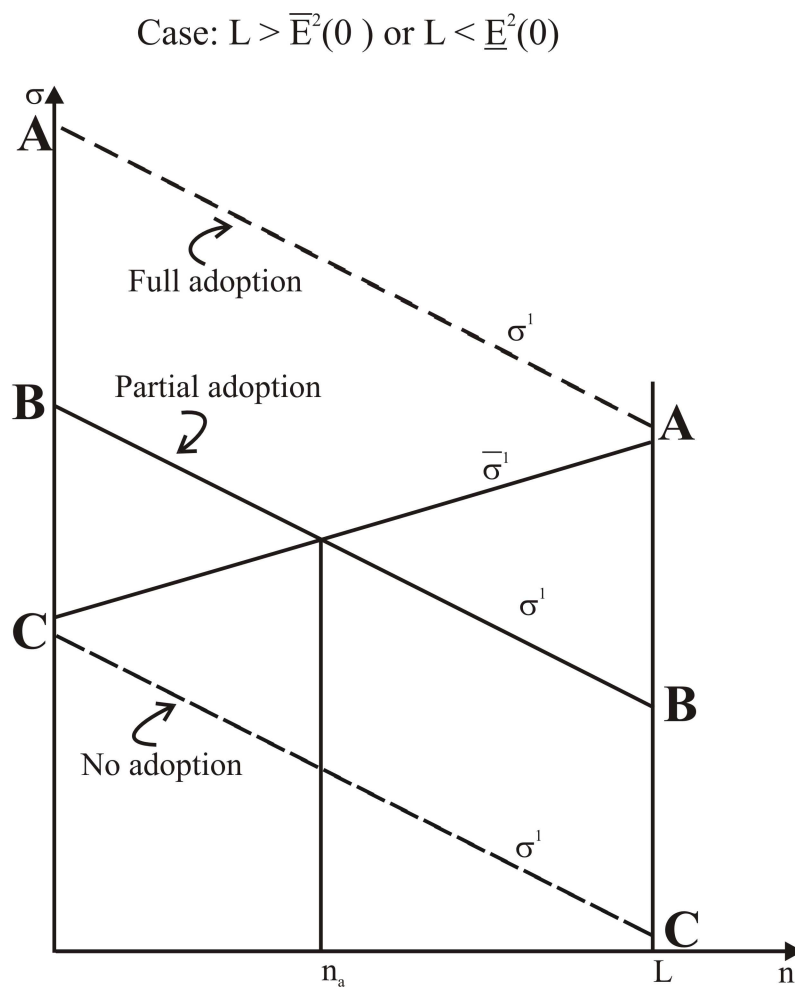


Figure 2. Permit market (first stage) in case of no or full adoption in the second stage.

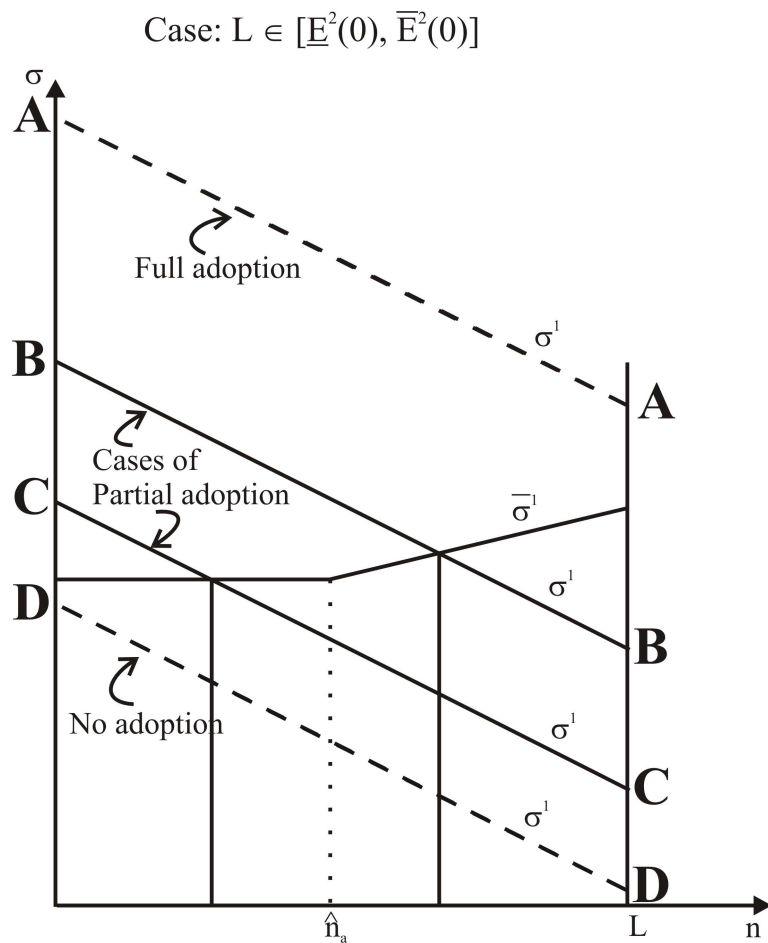


Figure 3. Permit market (first stage) in case of partial adoption in the second stage.

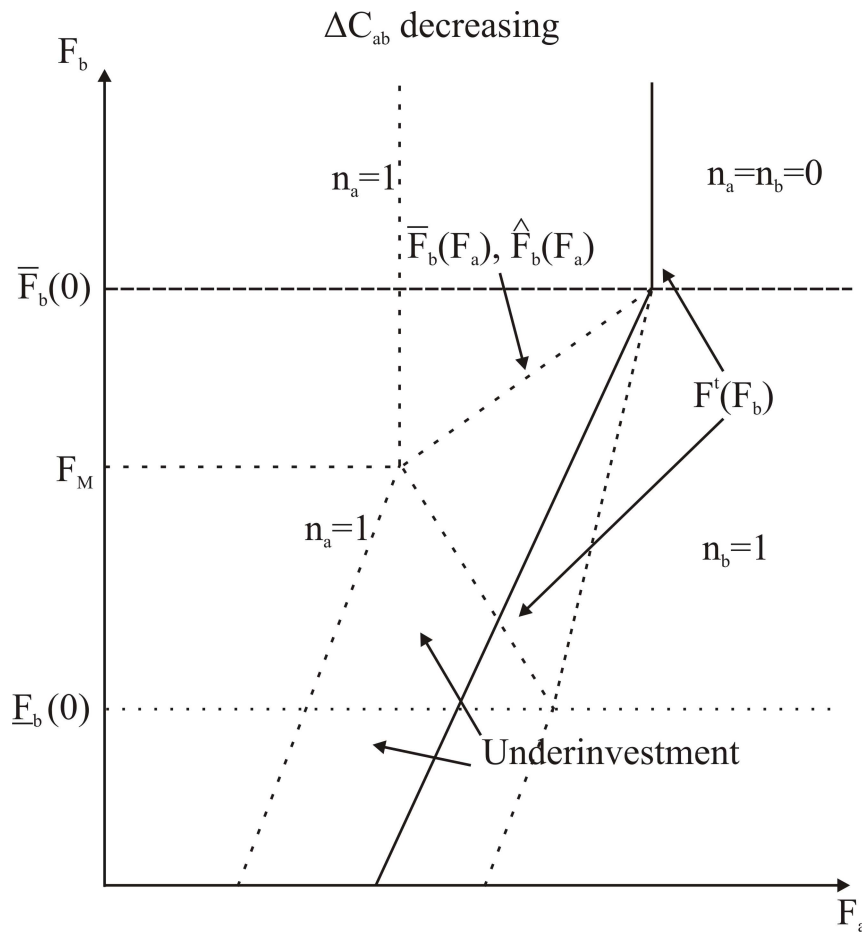


Figure 4. Allocation in case of myopic tax regulation, ΔC_{ab} decreases

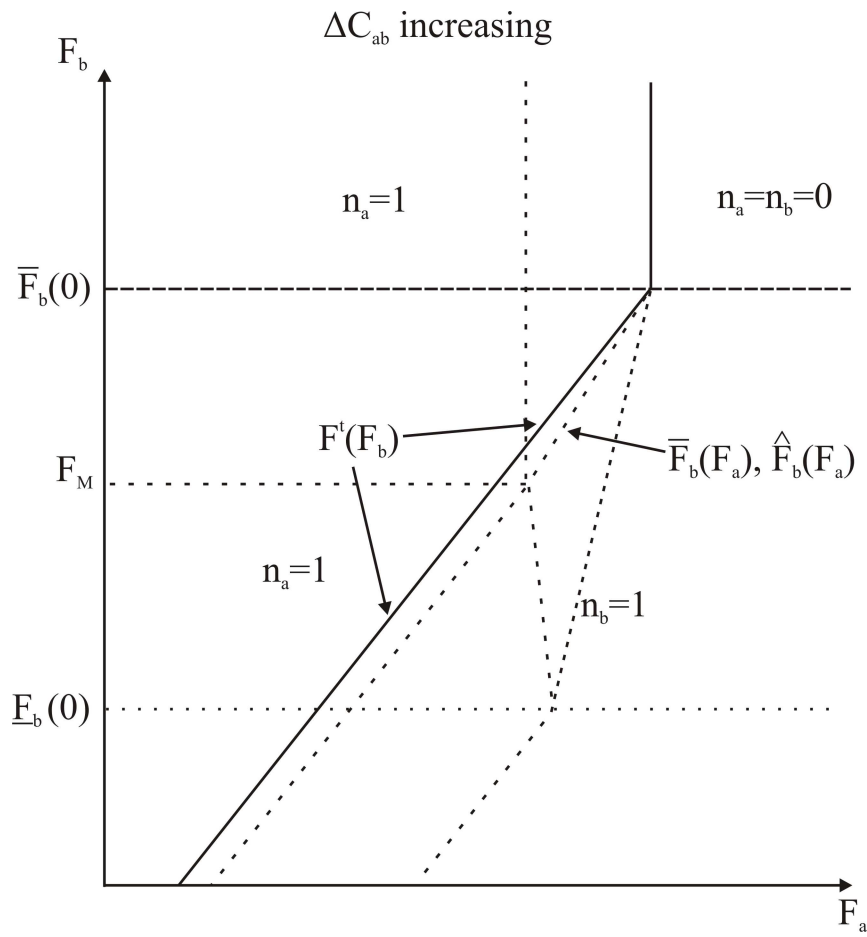


Figure 5. Allocation in case of myopic tax regulation, ΔC_{ab} increases

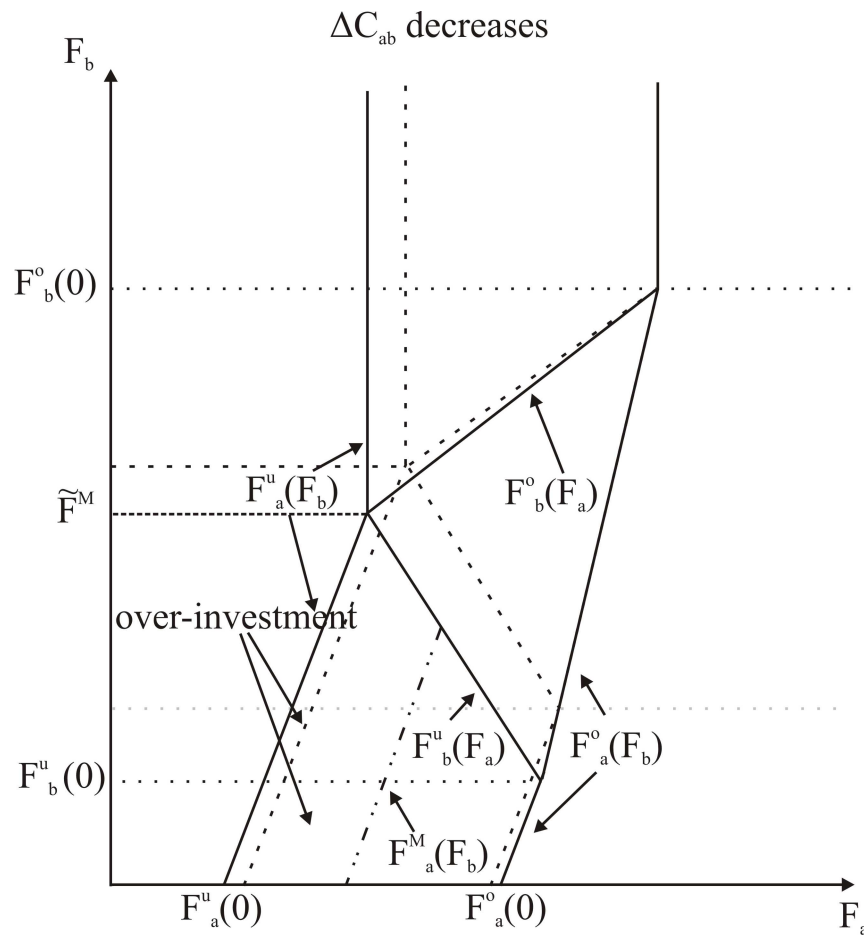


Figure 6. Allocation in case of myopic permit regulation, ΔC_{ab} decreases

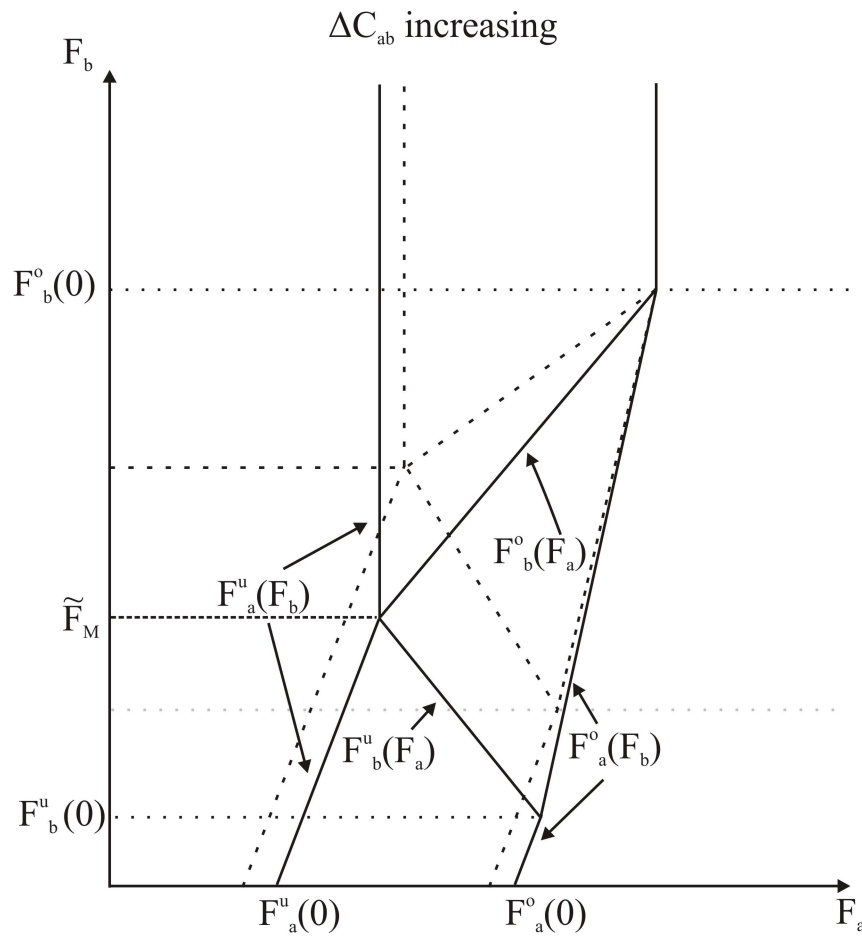


Figure 7. Allocation in case of myopic permit regulation, ΔC_{ab} increases

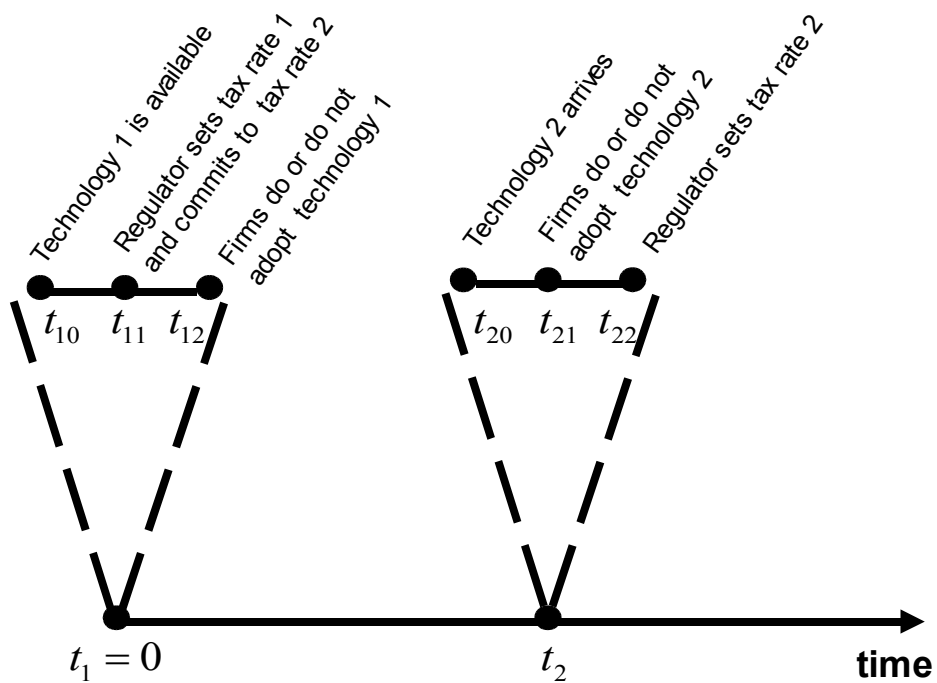


Figure 8. Ex-ante timing

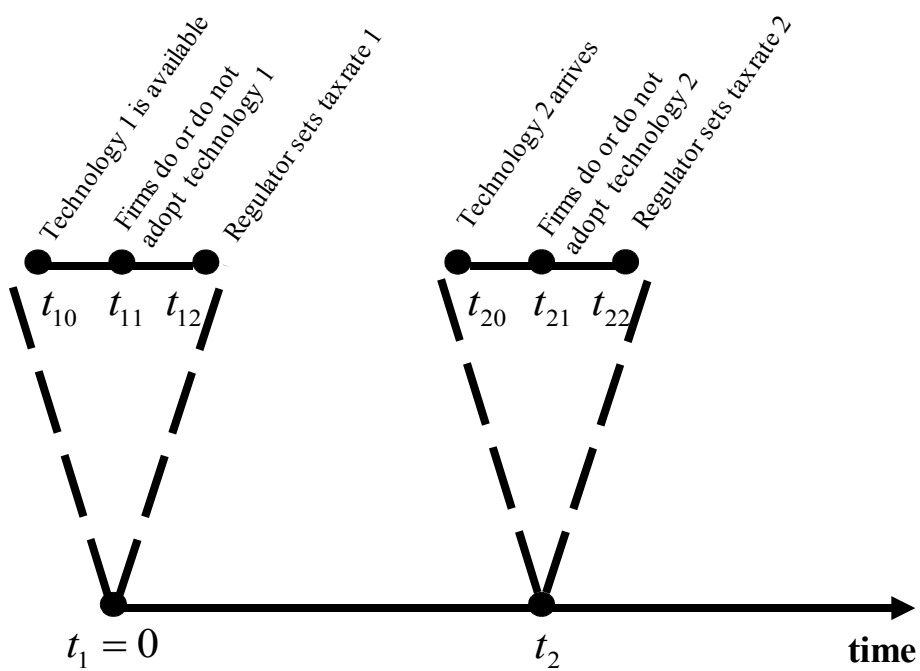


Figure 9. Ex-post timing