

Cervellati, Matteo; Sunde, Uwe

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## Human Capital Formation, Life Expectancy and the Process of Economic Development

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Matteo Cervellati  
Uwe Sunde

September 2002

# Human Capital Formation, Life Expectancy and the Process of Economic Development

**Matteo Cervellati**

*Universitat Pompeu Fabra, Barcelona*

**Uwe Sunde**

*IZA Bonn and University of Bonn*

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IZA

P.O. Box 7240  
D-53072 Bonn  
Germany

Tel.: +49-228-3894-0  
Fax: +49-228-3894-210  
Email: [iza@iza.org](mailto:iza@iza.org)

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## ABSTRACT

### Human Capital Formation, Life Expectancy and the Process of Economic Development\*

This paper presents a non-Malthusian theory of long-term development. We model the interplay between the process of human capital formation, technological progress, and the biological constraint of finite lifetime expectancy. All these processes are interdependent and determined endogenously. The model is analytically solved and simulated for illustrative purposes. The resulting dynamics reproduce a long period of stagnant growth as well as an endogenous and rapid transition to a situation characterized by permanent growth. This transition can be interpreted as industrial revolution. Historical and empirical evidence is discussed and shown to be in line with the predictions of the model.

JEL Classification: E10, J10, O10, O40, O41

Keywords: long-term development, endogenous life expectancy, heterogeneous human capital, technological progress, industrial revolution

Corresponding author:

Uwe Sunde  
Institute for the Study of Labor (IZA)  
PO Box 7240  
53072 Bonn  
Germany  
Tel.: +49 228 3894 221  
Fax: +49 228 3894 180  
Email: [sunde@iza.org](mailto:sunde@iza.org)

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# 1 Introduction

Economists have always had a great interest in understanding the determinants and the mechanics of the dramatic economic and demographic changes that accompanied the transition to modern society since the onset of the Industrial Revolution. After stagnant development for most of the history, from the second half of the 18th century onwards, aggregate and per capita income displayed a virtual explosion, as depicted in Figure 1 by data from the United Kingdom.<sup>1</sup> However, this transition also involved simultaneous changes in other important dimensions of the human environment and indicators of general living conditions such as life expectancy, population density, and education. At the same time as the economic transition took off, from the 18th century onwards, also the biological environment sharply changed. Mortality fell significantly and average life expectancy at birth as well as at later ages, which had virtually been unchanged for millennia, increased sharply within just a few generations, as illustrated in Figure 2.<sup>2</sup> Even though fertility declined substantially during the second half of the 19th century, see Galor and Weil (2000), the size of population started to increase substantially in European countries, as illustrated in in Figure 3 for English data. The increase in population size even after the decrease of fertility suggests that the reduction in reproduction is more than compensated by an increase in lifetime duration. A long era of stagnant growth of both output and population size was followed by an acceleration in the development of both variables during the second half of the 18th century. While GDP grows unboundedly ever since, population growth eventually dips after the 1950s.

Some of these dramatic changes have previously been addressed in the context of savings and population growth, see Komlos and Artzrouni (1990) and Kremer (1993), and and specialization and technological change, see Goodfriend and McDermott (1995). Unified theories of the transition from Malthusian stagnation to growth, starting with the work by Galor and Weil (2000), focus on the quantity-quality trade-off between fertility and education of offspring, or on the accumulation of factors like capital, see Hansen and Prescott (2002), or knowledge, see Jones (2001). Compare also Lucas (2002), and Galor and Moav (2002a, 2002b) and the references therein.

Simultaneously to these early developments during the 18th century, the traditional social environment changed profoundly, as the vast majority of the population became educated, and acquired knowledge beyond the working knowledge of performing a few manual tasks inherited by previous generations. Literacy, which used to be the privilege of a little elite, became

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<sup>1</sup>The data are taken from Maddison (1991) and exclude South Ireland. Missing intermediate values are obtained by linear interpolation. Data for other European countries exhibit similar patterns.

<sup>2</sup>Data are taken from Www.Mortality.Org (2002) and Floud and McCloskey (1994).

widespread among the population, as is illustrated in Figure 4 for England and Wales by the ability to sign documents.<sup>3</sup> The process of human capital accumulation accelerated as more and more people acquired the ability to innovate, and to use innovations. On the other hand, the spread of new technologies in turn made it more profitable to acquire knowledge.

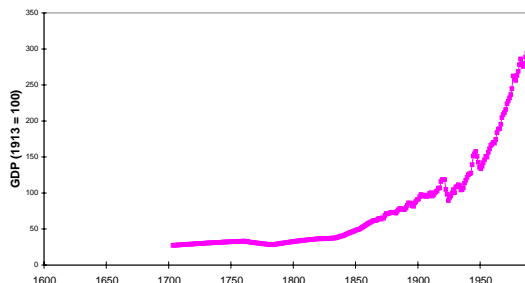


Figure 1: GDP per capita (U.K.)

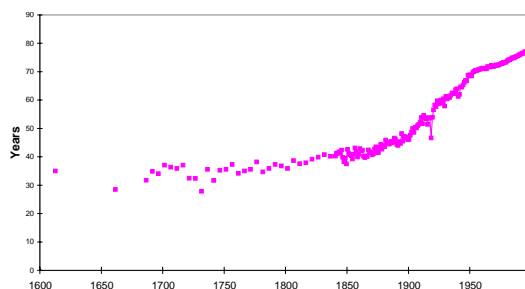


Figure 2: Average Life Expectancy (England and Wales)

The correlation in the timing and magnitude of these different dimensions of development has triggered substantial efforts devoted to analyzing the mechanisms at work. But, as Mokyr (1993) already pointed out, two separate strands of the literature, one about the causes and mechanics of the industrial revolution, and another about the decline in mortality, largely co-exist without any obvious connection or compatibility between the two. On the one hand, empirical evidence suggests that life expectancy affects the

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<sup>3</sup>The data reflect the ability to sign marriage documents and are taken from Schofield (1973) and West (1978). This measure of literacy has the advantage of being direct and reflecting an intermediate level of literacy skills since “the proportion of the population able to sign was less than the proportion able to read and greater than the proportion able to write” (Schofield, 1973, p. 440), and roughly corresponds to the proportion of the population able to read fluently. Moreover, writing skills are essential for acquiring other skills like arithmetic or other substantive knowledge. For similar evidence concerning France see Cipolla (1969) and Floud and McCloskey (1994).

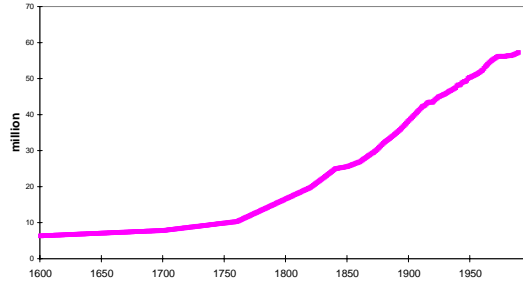


Figure 3: Population Size (England with Wales)

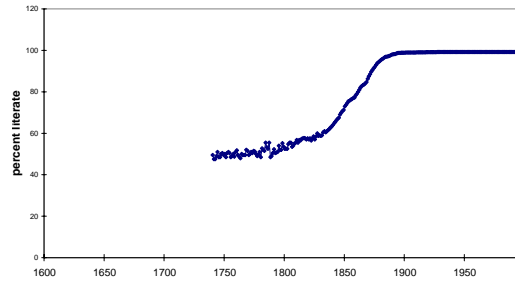


Figure 4: Average Literacy Levels in England and Wales

accumulation of human capital, which in turn determines growth. Swanson and Kopecky (1999) and Kalemli-Ozcan (2002) present evidence for the effect of life expectancy on educational attainment, growth and fertility choice.<sup>4</sup> On the other hand, a large body of historical and demographic evidence suggests that economic development and the level of human capital profoundly affect life expectancy. This evidence suggests that traditionally little education and knowledge about health and means to avoid illness supported the outbreak, propagation and mal-treatment of diseases and ultimately led to high mortality. However, an increasing popular knowledge of the treatment of common diseases and about the importance of hygiene and sanitation, as well as the availability of respective technologies, helped to increase life expectancy over time, see e.g. Mokyr (1993) and Easterlin (1999). There is also evidence that children's life expectancy increases in parents' human capital or education (Schultz, 1993), and that the human capital intensive invention of new drugs increased life expectancy (see Lichtenberg, 1998).<sup>5</sup>

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<sup>4</sup>See also Reis-Soares (2001).

<sup>5</sup>Blackburn and Cipriani (2002) cite further empirical evidence for the view that life expectancy depends on economic conditions. Moreover, the dissemination of knowledge about hygiene was one of the purposes of the widespread introduction of voluntary or mandatory home economics courses for women, see Huls (1993, ch. 7).

Hence, there is a consensus in the literature that life expectancy is a crucial determinant of human capital accumulation and economic development, and that the level of human capital and development in general affects lifetime duration. However, the mechanisms at work during the early stages of the industrial revolution are more difficult to explore. There is disagreement among economic historians, see Riley (2001) and Easterlin (2002), about whether the onset of increases in life expectancy can be precisely dated for different countries. A similar disagreement concerns the question whether this onset coincided with the beginning of the industrial revolution and the transition to a faster regime of growth, or whether changes in life expectancy preceded or followed changes in the economic environment. The question, which factor was causally responsible for all these profound changes, is still the object of a lively discussion.<sup>6</sup>

As the evidence suggests, the relation between demographic variables and human capital is crucial for understanding the patterns of long-term development. An influential strand of literature concentrated attention on the link between fertility choice and human capital formation by analyzing the quantity-quality trade-off faced by parents regarding their offspring. Kalemli-Ozcan *et al.* (2000), Kalemli-Ozcan (2002), De la Croix and Licandro (1999), Boucekkine *et al.* (2002a, 2002b) and Blackburn and Cipriani (2002) explicitly consider mortality in dynamic models of fertility choice.

This paper provides a unified framework complementary to models of fertility to analyze the endogenous interactions between human capital formation, technological progress and life expectancy in the context of long term development. We provide a microfounded theory of human capital formation, which focuses on the quantity-quality trade-off arising from the availability of heterogeneous types of human capital and the corresponding education technologies. The model has three basic building blocks. The first is a microfounded theory of human capital formation in which overlapping generations of heterogeneous individuals decide upon the type and the amount of human capital to acquire during their lives. With this choice, individuals maximize lifetime utility, taking life expectancy and the state

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<sup>6</sup>Some authors explain the decline in mortality and the increase in life expectancy by increases in household incomes and technological progress (see e. g. McKeown, 1977). However, this view has been criticized on the basis of the empirical evidence, which suggests that technological (medical) progress took off too late to explain early increases in lifetime duration. Moreover, by and large, the standard of living in terms of income, housing and nutrition of the majority of the population hardly changed before 1850, indicating that this explanation does not tell the entire story, see Mokyr (1993). Others, like Boucekkine, de la Croix, and Licandro (2002b) and the references therein, argue that at the dawn of the industrial revolution mortality declined exogenously. They cite evidence from life tables and parish registers from Geneva and Venice, which show that life expectancy as measured at age ten already increased between 1640 and 1740 in these urban centers. Moreover, adult mortality seems to have fallen before child mortality declined substantially. However, this line of argument leaves the cause of the industrial revolution essentially unexplained.



of technology into account. The second block reflects the idea that human capital acquired by a given generation facilitates the formation of human capital for future generations. However, what matters is not only the stock of knowledge, but also the type of knowledge accumulated in the population. The third block is motivated by the historical and demographic evidence mentioned above and concerns the effects of the economic and social environment on life expectancy.

In the human capital formation process at the center of the paper, human capital is viewed as knowledge embodied in people, which they optimally acquire in order to generate income. Accumulating human capital means getting to know, and being able to use, production technologies. In this sense, human capital formation is not a modern phenomenon, since throughout human history individuals devoted part of their lifetime to its acquisition. Human capital is not a homogeneous production factor, but captures multiple abilities ranging from the use of simple techniques to the application of abstract knowledge in solving problems never faced before. Correspondingly, the acquisition of different types of human capital requires different education processes. In particular, for some types of human capital, it is sufficient to observe and imitate tasks, to learn by doing, whereas other, more abstract types of knowledge require a more formal, and more time-consuming, education process.<sup>7</sup> Therefore, the profitability of the acquisition of different types of human capital depends, apart from individual attitudes, on the technological and biological environment: a larger life horizon facilitates the acquisition of any type of human capital, but it particularly favors the formation of human capital, which is time-intensive to acquire. The observed changes in the patterns of human capital acquisition resemble these arguments. The substantial increases in life expectancy shown above were accompanied by ever higher average numbers of years of schooling in England and Wales: from 2.3 years for children born around 1800 schooling increased to 5.2 years for children born around 1850 and to 9.1 years for children born around 1900, and reached more than eleven years in the 1980s (see Maddison, 1991, and Galor and Weil, 2000). Moreover, while almost all human capital was acquired through informal on-the-job learning in apprenticeships before the industrial revolution, formal schooling represents the main channel of human capital formation afterwards (see Cipolla, 1976).<sup>8</sup>

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<sup>7</sup>This reflects the mastery learning theory, according to which learning complicated materials builds on the mastering of the elementary concepts, which in turn take time to be understood, see Becker *et al.* (1990).

<sup>8</sup>Time devoted to learning in apprenticeships before the industrial revolution (Venice, early 17th century) was substantial, involving 3 to 5 years with minimum starting ages of between 7 to 10 years. Consequently, people completed education and started working at 10 to 15 years of age, while after the industrial revolution the starting age lied above 17 years, see Cipolla (1976).

The main mechanism is as follows. Individuals' decisions on human capital formation shape the structure of the economy and affect productivity and life expectancy for future generations. This leads to a potential virtuous circle of more human capital formation, higher life expectancy and growth. However, as long as the biological barrier of low life expectancy is binding, the economy is trapped on a stagnant growth path. For a long time, the economy develops with positive and almost negligible increments. Once life expectancy is large enough and the level of technology is sufficiently advanced, however, ever larger proportions of the population acquire growth enhancing human capital, and development takes off. A phase of fast growth and a profound change in the structure of economy starts, and the economy converges within a few generations to a sustained growth path. As a consequence of the increase in life expectancy, population size grows even though fertility behavior is unchanged. The mechanism underlying long-term development does not depend on Malthusian features like land as fixed factor of production, subsistence levels of consumption, an assumed positive correlation between per capita income and fertility, or on any scale effects. Instead, we emphasize the importance of life expectancy on individual education decisions: only when it is individually optimal for a sufficiently large share of the population to acquire growth enhancing human capital, an endogenous transition occurs. Moreover, the same mechanism works throughout history, although largely undetected at the early stages, without the need for any external shift to trigger a transition towards different growth regime.

The paper is organized as follows. In section 2 we describe the economic environment, we state and solve the individual problem of human capital formation, and describe the dynamic links between generations of individuals through technology and life expectancy. Section 3 presents a characterization of the development process. Section 4 contains an illustrative simulation of the model, and section 5 concludes. All proofs are collected in the appendix.

## 2 The Model

The economy is populated by an infinite sequence of overlapping generations of individuals. Generations will be denoted with subscript  $t$ . Every generation is born  $l_t = l$  periods after the birth of the respective previous generation, and there is no fertility decision to be made.<sup>9</sup> A generation consists of a continuum of agents with population size normalized to one. Individuals face a life expectancy  $T_t$  specific to their generation  $t$ , the determinants of which will be discussed below. Every individual is endowed with

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<sup>9</sup>Instead of assuming a fixed frequency of births, one could alternatively model the length of the time spell between the births of two successive generations, hence the timing of fertility, as a function of the life expectancy of the previous generation.

ability  $a \in [\underline{a}, \bar{a}]$ , which is distributed with density  $f(a)$ .<sup>10</sup>

## A Production of Human Capital

In order to make an income to be able to consume, individuals have to spend their ability and some fraction of their living time to form some human capital, which they can then supply on the labor market. Every generation has to build up the stock of human capital from zero, since the peculiar characteristic of human capital is that it is embodied in people (even if the production can be easier if the previous generation had a lot of it). In order to isolate the development effects related to life expectancy and human capital accumulation, any links between generations through savings or bequests are excluded. We abstract from real resources as input for the human capital formation process, as well as issues related to capital market development and public provision of education. Instead, we focus on changes in the economic and biological environment creating the necessary and sufficient conditions for large parts of the population to acquire human capital.<sup>11</sup>

The heterogeneity of human capital discussed above, is reflected by the consideration of two types of human capital, which differ with respect to their production process and the returns they generate. The first type is interpreted as high-quality, and growth enhancing. It is characterized by a high content of abstract knowledge, and facilitates innovation and development of new ideas. We refer to this as *theoretical* human capital and denote it by  $h$ . The second type is labeled *applied* human capital, denoted by  $p$ , and can be interpreted as the ability of using some existing technologies. It contains less intellectual quality, but more manual and practical skills that are important in performing tasks related to existing technologies.<sup>12</sup>

Time  $e$  and individual ability  $a$  are the only inputs of human capital production:  $p = p(e, a)$ , and  $h = h(e, a)$ . In line with the previous discussion, these production processes are inherently different with respect to the effectiveness of time in the education process. To acquire  $h$ , it is necessary to first spend time on the building blocks of the elementary concepts without being productive in the narrow sense. Once the basic concepts are mastered, the remaining time spent on education is very productive. On the other hand, the time devoted to acquire  $p$  is immediately effective, al-

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<sup>10</sup>We assume that the *ex ante* distribution of innate ability or intelligence does not change over the course of generations.

<sup>11</sup>Some recent contributions study the emergence of sufficient conditions for development in the presence of market imperfections, and unequal distribution of economic and political power, see e.g. Galor, Moav, and Vollrath (2002), and Galor and Moav (2002a).

<sup>12</sup>Hassler and Rodriguez-Mora (2000) use a related perception of abstract versus applied knowledge. In the language of growth economics, theoretical human capital is the growth enhancing type of labor, while applied human capital can be associated with the raw labor input. In labor economics,  $h$  would be labeled as skilled labor,  $p$  as unskilled labor.

beit with a lower overall productivity. This characteristic is captured by a fixed cost  $\underline{e}$  measured in time units, which an agent needs to pay when acquiring  $h$ , but not when acquiring  $p$ .<sup>13</sup> Personal ability is relatively more important in acquiring theoretical human capital. These characteristics of the education processes are formalized as:

$$h = \begin{cases} \alpha(e - \underline{e})a & \text{if } e \geq \underline{e} \\ 0 & \text{if } e < \underline{e} \end{cases} \quad (1)$$

and

$$p = \beta e. \quad (2)$$

Any unit of time produces  $\alpha a$  units of  $h$  and  $\beta$  units of  $p$  with  $\alpha \geq \beta$ . This formulation captures two crucial features of the human capital formation process: A larger life expectancy induces individuals to acquire more of any type of human capital and makes theoretical, high quality human capital relatively more attractive for individuals of any level of ability. Any alternative model of human capital formation reproducing these two features would be entirely equivalent for the purpose of this paper. Alternative settings like learning on-the-job could similarly be used to illustrate the importance of lifetime duration for human capital formation.

## B Aggregate Production

A unique final consumption good is produced by multiple sectors, in which new technological vintages become available overtime. The stocks of human capital of both types available in the economy at any moment in time, i.e. embodied in all generations alive at that date, are the only factors of production. We model, along the line of Hansen and Prescott (2002), a one-good-two-sectors economy.<sup>14</sup> Sectors structurally differ with respect to the intensity with which they use different human capital. Denote as  $\mathcal{P}$  the sector using  $p$  relatively more intensively and  $\mathcal{H}$  the sector using  $h$  relatively more intensively. Technological process takes place in both sectors in the form of new production technologies characterized by a larger total factor productivity becoming available over time. Technological improvements are modeled as vintages in the sense that older production functions are still available in each sector and can potentially be used along with the

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<sup>13</sup>A sufficient condition for the results below is that for applied human capital  $p$  the fixed cost in terms of time is smaller, so for simplicity it is normalized to zero. We abstract from other costs of education, like tuition fees etc. Moreover, the fixed cost is assumed to be constant and the same for every generation. Costs that increase or decrease along the evolution of generations would leave the qualitative results of the paper unchanged.

<sup>14</sup>The focus of the paper is not on the macroeconomic role of demand for different consumption goods. Equivalently, one could model different sectors as producing differentiated intermediate goods to be used in the production of a unique final good. The role of different income elasticities for different goods for structural change from agriculture to industry has been studied by Laitner (2000).

newest ones.<sup>15</sup> Denote by  $A_{H,v}(\tau)$  (respectively  $A_{P,v}(\tau)$ ) the total factor productivity and by  $Y_v^{\mathcal{P}}(\tau)$  (respectively  $Y_v^{\mathcal{H}}(\tau)$ ) the production realized in sector  $\mathcal{P}$  (respectively  $\mathcal{H}$ ) using vintage technology  $v$  at time  $\tau$ . Then total production at time  $\tau$  is given by<sup>16</sup>

$$Y(\tau) = \sum_v Y_v^{\mathcal{P}}(\tau) + \sum_v Y_v^{\mathcal{H}}(\tau). \quad (3)$$

Human capital is inherently heterogenous across generations, because individuals acquire it in an environment characterized by the availability of different vintages of technologies. Human capital acquired by agents of a generation allows them to use technologies up to the latest available vintage. Human capital is thus characteristic for a generation. This implies that a generation's stock of human capital of either type is not a perfect substitute of that acquired by older and younger generations, and is sold at its own price. Let the respective aggregate amounts of human capital acquired by generation  $t$  be

$$P_t = \int_{\underline{a}}^{\bar{a}} p_t(a) f(a) da, \quad (4)$$

$$H_t = \int_{\underline{a}}^{\bar{a}} h_t(a) f(a) da. \quad (5)$$

Wage rates are determined in the macroeconomic competitive labor market and equal marginal productivities.<sup>17</sup> Denote by  $w_t^h(\tau)$  and  $w_t^p(\tau)$  the wage rate paid at any moment in time  $\tau$  to every unit of human capital of type  $h$  or  $p$ , respectively, acquired by generation  $t$ . These instantaneous wage rates are given by

$$w_t^h(\tau) = \frac{\delta Y(\tau)}{\delta H_t}, \text{ and } w_t^p(\tau) = \frac{\delta Y(\tau)}{\delta P_t}. \quad (6)$$

To make the model analytically tractable, we consider a Cobb-Douglas specification of the production function. Moreover, we assume that every vintage of human capital fully specializes in the respective latest vintage of technology, so that  $t = v$ .<sup>18</sup> As a benchmark, we consider the extreme case in which

<sup>15</sup>This means that different technologies of productions are available at any moment in time. If we interpret the different sectors e.g. as agricultural and industrial, the production of corn can then take place using donkeys or tractors.

<sup>16</sup>The specification used by Hansen and Prescott (2002) is contained as the special case when only the latest vintage can be used.

<sup>17</sup>Empirical evidence supports the view that different sectors competed for labor, and wage payments reflected productivities even at early stages of industrial development, see e.g. Magnac and Postel-Vinay (1997).

<sup>18</sup>This specialization can be seen as the outcome of an optimization problem: A technological vintage goes out of use and becomes passive once the individuals working on it die.

every sector uses only one type of human capital. The production functions are thus:

$$Y_t^P = A_{P,t}P_t^\gamma, \text{ and } Y_t^H = A_{H,t}H_t^\gamma, \quad (7)$$

respectively, with  $\gamma \in (0, 1)$  and  $A_v^P(\tau), A_v^H(\tau) \in \mathcal{R}^+$ .<sup>19</sup> The corresponding instantaneous wage rates are given by:<sup>20</sup>

$$w_t^h(\tau) = A_{H,t}\gamma H_t^{\gamma-1}, \quad (8)$$

$$w_t^p(\tau) = A_{P,t}\gamma P_t^{\gamma-1}. \quad (9)$$

## C The Individual Optimization Problem

Consider the decision problem for members of a given generation  $t$  of individuals. Agents face an intertemporal problem of maximizing their lifetime utility. Individuals have to choose both the optimal quantity and quality of human capital they want to acquire. The first decision regards the optimal allocation of available lifetime between education and work. The second decision is about which type of human capital to acquire.<sup>21</sup> Utility is linear in consumption and there is no discounting. Hence, utility maximization implies maximization of total lifetime earnings.<sup>22</sup> Moreover, while acquiring human capital, an agent cannot work and therefore earns no income. The

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Subsequent generations will find it more profitable to use new, more productive vintages of technology than to revive old ones. In other words, this implies that e.g. a mechanic in the late 20th century learns how to repair a common rail diesel engine, but not a steam engine. However, as will become clear below, vintages build upon the advances of previous vintages, e.g. common rail diesel engines incorporate technological principles that partly derive from the use of steam engines.

<sup>19</sup>In principle, both sectors could be characterized by different productivity parameters  $\gamma_H$  and  $\gamma_P$ . This case will be illustrated in the simulations below. However, while the main results remain unaffected by asserting a common value to both sectors, it simplifies the exposition of the model considerably. Incorporating both types of human capital in both sectors of production does not alter the results as long as the difference in the relative intensities of their use in the respective sector is maintained and no input is indispensable.

<sup>20</sup>Decreasing marginal productivity of human capital of any type insures interior equilibria. In the benchmark case with only one factor in each sector, this assumption implies decreasing returns to scale with factor payments not exhausting income. Since the crucial feature for individual human capital decisions is that wages somehow reflect the productivity of the respective type of human capital, we simply assume a uniform distribution of non-wage income across the population to close the economy. Equivalently one could set wages to *average* productivity (since the wage ratio would just be scaled by a fixed factor) or additionally introduce fixed factors in both sectors (which would ensure constant returns to scale and appropriate all rents without affecting education decisions) to obtain the same results.

<sup>21</sup>We abstract from decisions about retirement and leisure. Allowing individuals to acquire both types of human capital would not change the formal arguments, but would imply a somewhat different interpretation of human capital.

<sup>22</sup>Concave utility, discounting and perfect capital markets could be introduced to model life cycle considerations. Without affecting the main results, these issues are beyond the scope of the current analysis.

problem of an agent with ability  $a$  born in generation  $t$  is therefore to choose the type of human capital  $i \in \{h, p\}$  and the optimal education time spent on its accumulation,  $e^i$ , given life expectancy  $T_t$  and the wage  $w_t^i$  such that

$$\{i, e^i\} = \arg \max V^i(a, i, e^i, T_t, w_t^i) = \int_{e^i}^{T_t} i w_t^i(\tau) d\tau \quad (10)$$

subject to  $i \in \{h, p\}$  and  $0 \leq e^i \leq T_t$ .

For any individual of ability  $a$ , there is a unique time investment which maximizes lifetime earnings from any type of human capital,

$$e_t^{h*} = \arg \max (T_t - e^h) \alpha (e^h - \underline{e}) a w_t^h = \frac{T_t + \underline{e}}{2}, \quad (11)$$

$$e_t^{p*} = \arg \max (T_t - e^p) \beta e^p w_t^p = \frac{T_t}{2}, \quad (12)$$

respectively. The type of human capital an individual chooses to acquire consequently depends on:

$$V^{p*}(e_t^{p*}, a, w_t^p) \gtrless V^{h*}(e_t^{h*}, a, w_t^h).$$

Using (11) and (12), the respective levels of human capital in the two cases are

$$h_t^*(T_t, a) = \alpha \frac{T_t - \underline{e}}{2} a, \quad (13)$$

and

$$p_t^*(T_t, a) = \beta \frac{T_t}{2}. \quad (14)$$

Accordingly, the respective indirect lifetime utilities are given by:

$$V^{p*}(p_t^*, a, w_t^p) = \frac{T_t^2}{4} \beta w_t^p, \quad (15)$$

and

$$V^{h*}(h_t^*, a, w_t^h) = \alpha a \frac{(T_t - \underline{e})^2}{4} w_t^h. \quad (16)$$

Agents with higher ability have a comparative advantage in the acquiring  $h$ , and the lifetime utility for those investing in  $h$  increases monotonically in the ability parameter. An agent is indifferent between acquiring  $h$  or  $p$  if and only if

$$V_t^{p*}(e_t^{p*}, a, w_t^p) = V_t^{h*}(e_t^{h*}, a, w_t^h). \quad (17)$$

For every vector of wage rates there is only one level of ability  $\tilde{a}_t$  for which the indirect utilities are equal,

$$\tilde{a}_t = \frac{w_t^p}{w_t^h} \left[ \left( \frac{\beta}{\alpha} \right) \frac{T_t^2}{(T_t - \underline{e})^2} \right]. \quad (18)$$

Due to the monotonicity of  $V^{h^*}$  in ability, all agents with  $a < \tilde{a}$  will optimally choose to acquire human capital  $p$ , while those with ability  $a > \tilde{a}$  will optimally choose to obtain  $h$ . The corresponding aggregate levels of human capital of either type are then given by

$$H_t = \int_{\tilde{a}}^{\bar{a}} h_t(T_t, a) f(a) da, \quad (19)$$

and

$$P_t = \int_{\underline{a}}^{\tilde{a}} p_t(T_t, a) f(a) da, \quad (20)$$

respectively. All individuals with higher ability than  $\tilde{a}_t$  choosing to acquire theoretical human capital actually enjoy larger lifetime earnings than those endowed with an ability smaller than  $\tilde{a}_t$  acquiring  $p$ .

## D Equilibrium on Factor Markets

Denote by  $\lambda(\tilde{a}_t)$  the fraction of the population acquiring human capital  $p$ , and by  $(1 - \lambda(\tilde{a}_t))$  the fraction of the population acquiring human capital  $h$ :

$$\lambda(\tilde{a}_t) := \int_{\underline{a}}^{\tilde{a}_t} f(a) da, \quad (21)$$

$$1 - \lambda(\tilde{a}_t) := \int_{\tilde{a}_t}^{\bar{a}} f(a) da. \quad (22)$$

By equation (18) and since  $T_t - \underline{e} > 0$ , the fraction  $[1 - \lambda(\tilde{a}_t)]$  increases with lifetime duration  $T_t$ , with the relative wage  $w_t^h/w_t^p$  and with  $\alpha/\beta$ .

The markets for human capital are in equilibrium when individually optimal education decisions and the respective wages determined on the macroeconomic level are mutually compatible:

**Definition 1.** *The factor market equilibrium for generation  $t$  is characterized by a vector*

$$\left\{ \{h_t^*(T_t, a_t)\}_{a \in [\underline{a}, \bar{a}]}, \{p_t^*(T_t, a_t)\}_{a \in [\underline{a}, \bar{a}]}, H_t^*, P_t^*, w_t^{h^*}, w_t^{p^*}, \tilde{a}_t^* \right\}$$

*such that, for any given  $T_t$  and distribution  $f(a)$ , conditions (13), (14), (19), (20), (8), (9), and (18) are simultaneously satisfied.*

An equilibrium defines an implicit function in  $(\tilde{a}_t^*, T_t)$  linking the equilibrium cut-off level of ability  $\tilde{a}_t^*$  to lifetime duration  $T_t$ . For computational convenience, we assume uniform distribution of abilities on the support  $[0, 1]$ .<sup>23</sup>

<sup>23</sup>In fact, the results can be generated in the model with any distribution of abilities including a degenerate distribution with just one ability level for all members of the population. However, the process of how individuals sort into equilibrium in this case would be less clear, since there would be no ability cut-off separating the population. The decomposition of the population into the two groups would just be given by equilibrium conditions.



In this case the aggregate levels of human capital can be explicitly computed:

$$P_t(\tilde{a}_t^*) = \int_0^{\tilde{a}_t^*} p_t(T_t, a) da = \tilde{a}_t^* \beta \frac{T_t}{2}, \quad (23)$$

$$H_t(\tilde{a}_t^*) = \int_{\tilde{a}_t^*}^1 h_t(T_t, a) da = \left( \frac{1 - \tilde{a}_t^{*2}}{2} \right) \alpha \frac{T_t - \underline{e}}{2}. \quad (24)$$

Substituting the equilibrium wage ratio resulting from Equations (8) and (9) using  $P_t(\tilde{a}_t^*)$  and  $H_t(\tilde{a}_t^*)$  from Equation (23) and (24), respectively, into the equilibrium ability threshold given by equation (18), we have, for any given generation  $t$ :

$$\tilde{a}_t^* \left( \frac{(1 - \tilde{a}_t^{*2})}{2\tilde{a}_t^*} \right)^{\gamma-1} = \frac{A_{P,t}}{A_{H,t}} \left( \frac{\beta}{\alpha} \right)^\gamma \left( \frac{T_t}{T_t - \underline{e}} \right)^{\gamma+1}. \quad (25)$$

For notational convenience, we reformulate equation (25) by solving for lifetime expectancy as a function of the ability threshold:

$$T_t(\tilde{a}_t^*) = \frac{\underline{e}}{1 - \frac{g(\tilde{a}_t^*)}{\Omega_t}}, \quad (26)$$

with

$$g(\tilde{a}_t^*) \equiv \frac{(1 - \tilde{a}_t^{*2})^{\frac{1-\gamma}{1+\gamma}}}{\tilde{a}_t^{* \frac{2-\gamma}{1+\gamma}}} k, \quad (27)$$

$$k \equiv 2^{-\frac{1-\gamma}{1+\gamma}}, \text{ and}$$

$$\Omega_t \equiv \left[ \left( \frac{A_{H,t}}{A_{P,t}} \right) \left( \frac{\alpha}{\beta} \right)^\gamma \right]^{\frac{1}{1+\gamma}}, \quad (28)$$

where  $g(\tilde{a}_t^*) > 0$ ,  $\forall \tilde{a}_t^* \in [0, 1]$ , where  $T_t(\tilde{a}_t^*)$  is defined for all  $\tilde{a}_t^* \in [\underline{\tilde{a}}_t^*, 1]$ , with  $\underline{\tilde{a}}_t^*$  such that  $g(\underline{\tilde{a}}_t^*) = \Omega_t \Leftrightarrow \lim_{\tilde{a}_t^* \rightarrow \underline{\tilde{a}}_t^*} T_t(\tilde{a}_t^*) = \infty$ , and that  $\forall \tilde{a}_t^* \in [\underline{\tilde{a}}_t^*, 1] : 1 - \frac{g(\tilde{a}_t^*)}{\Omega_t} > 0$ . The value  $\underline{\tilde{a}}_t^* > 0$  represents the maximum fraction of the population that would optimally choose to acquire human capital  $h$  for a given level of relative productivity  $A_H/A_P$ . This maximum fraction cannot be exceeded, even if the biological constraint of finite lifetime duration would disappear (i. e. if  $T \rightarrow \infty$ ).

The problem of determining the equilibrium vector is well defined, and all variables characterizing the equilibrium human capital formation of each generation are uniquely identified, since the implicit function relating the cut off  $\tilde{a}_t^*$  to life expectancy  $T_t$  is monotonically decreasing in  $T_t$ . Formally:

**Proposition 1.** *For any generation  $t$ , there exists exactly one factor market equilibrium characterized by the a pair  $(\tilde{a}_t^*, T_t^*)$ , with  $\tilde{a}_t^* \in [\underline{\tilde{a}}_t^*, 1]$  and  $T_t \in [\underline{e}, \infty)$ , which satisfies condition (25).*

In this context, it is worth noting that the maximum proportion of the population that would acquire  $h$  in the absence of biological constraints,  $(1 - \tilde{a}_t^*)$ , is increasing with the relative productivity of the sector using theoretical human capital intensively,  $\frac{A_H}{A_P}$ . This observation will prove useful later on and is therefore summarized in:

**Lemma 1.** *The lower bound on the support of ability thresholds decreases as  $\Omega_t$  increases, that is  $\frac{\partial \tilde{a}_t^*}{\partial \Omega_t} < 0$ .*

The ability cut-off  $\tilde{a}^*$  identifying the indifferent agent is lower for higher expected lifetime duration, which reflects the fact that *ceteris paribus* more individuals decide to obtain human capital of type  $h$  if they expect to live longer. With  $a$  being uniformly distributed on the support  $[0, 1]$ , The threshold  $\tilde{a}^*$  is identical to the share of population acquiring  $p$ ,  $\lambda(\tilde{a}^*)$ . Moreover, the function  $\tilde{a}^*(T)$ , representing the threshold ability defining the proportion of the population acquiring human capital  $h$ , is S-shaped:

**Proposition 2.** *The cut-off level  $\tilde{a}_t^*(T)$ , which identifies the equilibrium fraction of members of a generation  $t$  acquiring human capital  $h$ , is a decreasing, S-shaped function of expected lifetime duration  $T$  of this generation, with zero slope for  $T \rightarrow 0$  and  $T \rightarrow \infty$ , and exactly one inflection point.*

The S-shape relation between life expectancy  $T$  and the fraction of population acquiring  $h$ ,  $\lambda$ , is a first central result. From Proposition 2 it is clear that the higher the life expectancy, the more people will invest in the time-consuming human capital acquisition of  $h$ . However, this relation is stronger and more pronounced for intermediate values of  $T$  and  $\lambda$ . For low levels of life expectancy, the share of population investing in  $h$  is small due to the fixed cost involved with acquiring  $h$ , which prevents a large part of the population to receive sufficient lifetime earnings to be worth the effort. The larger the fixed cost, the more pronounced is the concavity of the equilibrium locus. In this situation, it takes sufficiently large increases in life expectancy to incentivate a noticeable fraction of individuals to switch from  $p$ -acquisition to  $h$ -acquisition. On the other hand, when the ability threshold is very low, and a substantial share of the population is engaged in  $h$ , very large increases in  $T$  are necessary to make even more individuals acquire  $h$  instead of  $p$ : Due to decreasing returns in both sectors, the relative wage  $w_h/w_p$  is very low when only few individuals decide to invest in  $p$ . This dampens the attractiveness of investing in  $h$  for the individuals with low ability, even though life expectancy is very high, rendering the equilibrium locus convex.

In what follows, denote the equilibrium relation between  $\tilde{a}_t^*$  and  $T_t$  implicitly defined in equation (25) characterizing the process of human capital formation of a given generation  $t$  by

$$\tilde{a}_t^* = \Lambda(T_t, A_t). \quad (29)$$

## E Life Expectancy

The empirical evidence discussed above suggests a positive relation between the amount of human capital embodied in a generation, reflecting its level of development and knowledge, and the life expectancy of future generations.<sup>24</sup> With the available evidence in mind, we formalize this positive externality by making the simple assumption that life expectancy of generation  $t$  increases in the fraction of population the previous generation ( $t - 1$ ) that acquired human capital of type  $h$ :<sup>25</sup>

$$T_t = \Upsilon(\lambda_{t-1}) = \underline{T} + \rho(1 - \lambda_{t-1}), \quad (30)$$

where  $(1 - \lambda_{t-1}) = 1 - \lambda(\tilde{a}_{t-1}^*) = \int_{\tilde{a}_{t-1}^*}^{\bar{a}} f(a)da$  is the fraction of generation ( $t - 1$ ) that has acquired human capital of type  $h$ , and  $\rho > 0$  is a parameter describing the extent of the positive externality. This formulation implies that the positive link and the dynamic process does not rely on scale effects. Note also that by the definition of  $\lambda$ , life expectancy is a function of the threshold ability level for the decision to acquire general human capital  $h$  of the respective generation:

$$T_t = \Upsilon(\tilde{a}_{t-1}^*), \quad (31)$$

There is a biological limit to extending life expectancy implicitly contained in the specification of equation (30) since, by definition of  $\lambda$  as a fraction, the lifetime duration is bounded from above and thus cannot be increased beyond a certain level. We take this as a commonly agreed empirical regularity (see also Vaupel, 1998). The minimum lifetime duration without any human capital of type  $h$  is given by  $\underline{T}$ . The precise functional form of this relation entails no consequences for the main results, and a (potentially more intuitive) concave relationship would not change the main argument.

## F Technological Progress

The second dynamic element concerns the notion that larger stocks of human capital acquired by a generation facilitates the accumulation of human

<sup>24</sup>Admittedly, this is only true to a certain extent. Of course, individuals can effectively influence their life expectancy by their life style, smoking habits, drug and alcohol consumption, sports and fitness behavior health care expenditures etc. However, during early phases of development, individuals lacked a detailed knowledge about which factors and activities are detrimental or advantageous for average life duration. Moreover, beneficial factors, such as leisure, were simply not available. An explicit consideration of positive correlation between life expectancy and the level of education would reinforce the results.

<sup>25</sup>Equivalently, life expectancy could be related to average or total human capital, see Boucekkine, de la Croix, and Licandro (2002a), or income, see Blackburn and Cipriani (2002), of the previous generation(s). If one accepts a positive effect of the level of human capital on aggregate income, this assumption is also consistent with evidence indicating that the aggregate income share spent on health care increases with aggregate income levels, see Getzen (2000) and Gerdtham and Jönsson (2000) and the references there for respective evidence.

capital for future generations. This positive externality has been extensively studied in the literature and can be formalized in several ways. The level of human capital available in the economy may effect individual human capital acquisition directly. This channel has been considered by Becker *et al.* (1990). Alternatively, the available human capital can indirectly make human capital more profitable to acquire in the future. This is the case if, for example, human capital exerts an externality on productivity along Lucas (1988) and Romer (1990), or when human capital induces a non-neutral technological process, as studied e.g. by Nelson and Phelps (1966), Acemoglu (1998), and Galor and Moav (2000), among others. For the purpose of this paper, these mechanisms are equivalent in terms of generating the central results. Since the vintage structure of the production technology described above is particularly suited to explicitly adopt technological change, we formalize the positive externality of human capital along the this line.

The level of human capital acquired by a given generation increases total factor productivity for subsequent technological vintages.<sup>26</sup> This interpretation is similar to the idea that the stock of ideas transfers into the productivity of future generations suggested by Jones (2001). In the model, we adopt Jones' specification, which is a generalization of the original contribution of Romer (1990). By its nature, theoretical human capital  $h$  is relatively more productivity enhancing than practical human capital  $p$ . Moreover, the positive effect is stronger in the sector  $\mathcal{H}$  that uses theoretical human capital more intensively, since it is the more innovative sector, applying and implementing new and innovative technologies faster. Consequently, total factor productivity (TFP) growth the sector  $\mathcal{H}$  is a function of  $H$  and the level of productivity already achieved in this sector.<sup>27</sup> Advances in technology are embodied in the latest vintage according to:

$$\dot{A}_{H,t} = \frac{A_{H,t} - A_{H,t-1}}{A_{H,t-1}} = \delta H_{t-1}^\phi A_{H,t-1}^\chi, \quad (32)$$

where  $\delta > 0$ ,  $\phi > 0$ , and  $\chi > 0$ . This can be re-written to:

$$A_{H,t} = \left( \delta H_{t-1}^\phi A_{H,t-1}^\chi + 1 \right) A_{H,t-1}. \quad (33)$$

What is important for the argument of the paper is the relative strength of these impacts, so there is no loss in constraining the productivity effect to  $A_H$  only. Thus, for simplicity we assume  $\dot{A}_{P,t} = 0$  so that total factor

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<sup>26</sup>This formulation follows the conventional argument frequently made in growth theory, see Nelson and Phelps (1966). Empirical evidence, see Doms, Dunne, and Troske (1997) supports this view.

<sup>27</sup>In the specification used, this function exhibits decreasing returns, while Romer (1990) assumed constant returns. The advantage of the present specification is that it is less rigid and more realistic. It is important to note that there are no scale effects involved in this specification. In fact, the crucial assumption for everything that follows is the relation between TFP and the *share* of the previous generation  $(1 - \lambda_{t-1})$  investing in  $h$ .

productivity in the first sector is constant and can be normalized to 1:  $A_{P,t} = A_{P,0} = 1 \forall t \in [0, \infty)$ .<sup>28</sup> For notational simplicity, we will denote the relative total factor productivity of the two sectors as

$$A_t \equiv \frac{A_{H,t}}{A_{P,t}} \text{ for every } t \in \{0, \infty\} . \quad (34)$$

With a uniform ability distribution, we can substitute  $H_{t-1} = \frac{\alpha}{2} (T_{t-1} - e) (1 - \lambda_{t-1})$  from Equation (24) into (33), and obtain an explicit expression for the dynamic evolution of relative productivity:

$$A_t = \left\{ \delta \left[ \frac{\alpha}{2} (T_{t-1} - e) (1 - \lambda_{t-1}) \right]^\phi A_{t-1}^\chi + 1 \right\} A_{t-1} \equiv F(A_{t-1}, T_{t-1}, \lambda_{t-1}) . \quad (35)$$

This specification emphasizes the particular role of human capital  $h$  in the accumulation of knowledge, and subsequently for technological progress. The specific functional form has little impact. In fact every, functional specification alternative to (32), which implies a positive correlation between  $\dot{A}_t$  and  $H_t$  would yield qualitatively identical results. It is also worthwhile noting that the qualitative features of the model are unaltered if technological process is taken to be exogenous, that is if  $\dot{A}_t = \varepsilon > 0$ .<sup>29</sup> These dynamic links close the model.

### 3 The Process of Economic Development

This section analyzes the dynamic evolution of the economy. The solution of the model allows to characterize the process of development as an interplay of individually rational behavior and macroeconomic externalities. The global dynamics of the economy are fully described by the trajectories of lifetime duration  $T_t$ , the fraction of the population acquiring human capital  $\lambda_t$ , and relative productivity  $A_t$ . We therefore characterize the dynamic development of the economy over time by studying the evolution of the key variables over generations.<sup>30</sup> For notational simplicity, denote  $\tilde{a}^*$  simply as  $a$ . Taking into consideration the one-to-one relationship between  $\lambda_{t-1}$  and  $a_{t-1}$ , the

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<sup>28</sup>In general, both types of human capital can have a positive intertemporal effect on total factor productivity of both sectors, as long as the technological externality is biased towards  $H$ -type human capital. In the simulations presented below, we actually allow total factor productivity in the sector using practical human capital intensively to grow according to:

$$A_{P,t} = \left( \delta_P H_{t-1}^{\phi_P} A_{P,t-1}^{\chi_P} + 1 \right) A_{P,t-1} .$$

This reflects the historical fact that agricultural productivity also increased as productivity in other sectors went up, e. g. during the industrial revolution, see Streeten (1994).

<sup>29</sup>As will become clearer below, the only consequence of an exogenous change in relative productivity  $\dot{A}$  is the missing re-inforcing feedback effect of endogenous technological progress after the industrial revolution.

<sup>30</sup>In the next section we simulate the model to illustrate the implied dynamics.

dynamic path is fully described by the infinite sequence  $\{a_t, T_t, A_t\}_{t \in [0, \infty)}$ , resulting from the evolution of the three dimensional, nonlinear first-order dynamic system derived from equations (29), (31) and (33):

$$\begin{cases} a_t &= \Lambda(T_t, A_t) \\ T_t &= \Upsilon(a_{t-1}) \\ A_t &= F(A_{t-1}, T_{t-1}, a_{t-1}) \end{cases} . \quad (36)$$

For illustrative purposes, we analyze the behavior of the economy by looking at the dynamic adjustment of human capital and lifetime duration *conditional* on the value of the relative productivity. We consider the system:

$$\begin{cases} a_t &= \Lambda(T_t, A) \\ T_t &= \Upsilon(a_{t-1}) \end{cases} , \quad (37)$$

which delivers the dynamics of human capital formation and life expectancy for *any given* level of technology  $A > 0$ . From the previous discussion we know that the first equation of the conditional system is defined for  $a_t \in (\underline{a}_t(A), 1]$  and  $T \in [\underline{e}, \infty)$ . Denote by  $HH(A)$  the S-shaped locus  $T_t = \Lambda^{-1}(a_t, A)$  in the space  $\{T, a\}$  resulting from the factor market equilibrium equilibrium, and by  $TT$  the locus  $T_t = \Upsilon(a_{t-1})$  representing the inter-generational externality on lifetime duration. Any steady state of the conditional system is characterized by the intersection of the two loci  $HH(A)$  and  $TT$ :

**Definition 2.** *A steady state equilibrium of the dynamic system (37) is a vector  $\{a^C, T^C\}$  with  $a^C \in (\underline{a}(A), 1]$  and  $T^C \in [\underline{e}, \infty)$ , such that, for any  $A \in (0, \infty)$ :*

$$\begin{cases} a^C &= \Lambda(T^C, A) \\ T^C &= \Upsilon(a^C) \end{cases} .$$

The system (37) displays steady state equilibria of different types  $j$  with different properties. The set of equilibria  $E^j(A) \equiv \{a^j(A), T^j(A)\}$  can be characterized as:

**Proposition 3.** *For any  $A \in (0, \infty)$ ,  $a \in (\underline{a}(A), 1)$ , and  $T \in (\underline{e}, \infty)$ , the conditional dynamic system (37) is characterized by:*

- (i) *At least one steady state equilibrium;*
- (ii)  *$H(A) > 0$  and  $P(A) > 0$  in any steady state;*
- (iii) *At most three steady states denoted by  $E^H(A)$ ,  $E^u(A)$ , and  $E^L(A)$  with the following properties:*
  - (a)  *$a^H(A) \leq a^u(A) \leq a^L(A)$  and  $T^H(A) \geq T^u(A) \geq T^L(A)$ ;*
  - (b)  *$E^H(A)$  and  $E^L(A)$  are locally stable;*

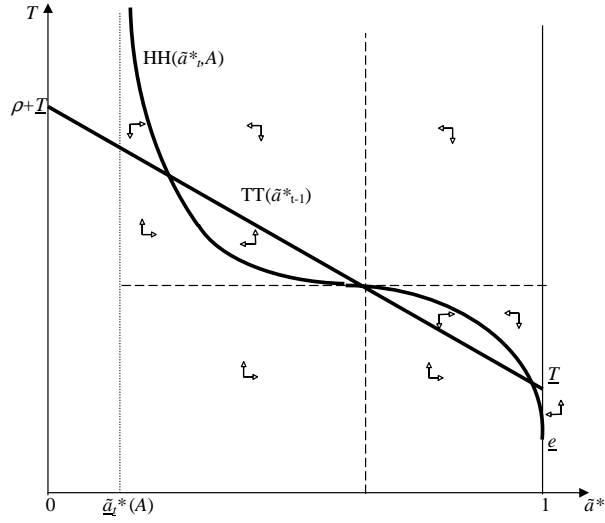


Figure 5: Phase Diagram of the Conditional Dynamic System

- (c)  $E^u(A)$  is locally unstable;
- (d) if there is a unique steady state, it is globally stable and it is of type  $H$  or  $L$ , respectively, depending on whether  $HH(A)$  is locally convex or concave in the steady state.

Hence, there exists at least one dynamic equilibrium while, due to the S-shape of  $HH(A)$ , there are at most three steady states, with the intermediate one being unstable. Strictly positive amounts of both types of aggregate human capital are acquired in any steady state. The *High*-type equilibria are characterized by a relatively large fraction of the population acquiring  $h$ , large lifetime expectancy, and the locus  $HH(A)$  being locally convex at  $a^H$ . The *Low*-type equilibria exhibit little lifetime duration, a small share of the population acquiring  $h$ , and the locus  $HH(A)$  is locally concave at  $a^L$ . Figure 5 illustrates the system (37) in the case of three equilibria.

The analysis of the full dynamic system must account for the evolution of all the variables. Human capital  $h$  helps in adopting new ideas and technologies, and thus creates higher productivity gains than practical human capital  $p$ . This means that in the long run relative productivity  $A_t$  will tend to increase. This result is summarized by

**Lemma 2.** *Relative Productivity  $A_t$  increases monotonically over generations with  $\lim_{t \rightarrow \infty} A_t = +\infty$ .*

The strict monotonicity of  $A_t$  over generations depends on the assumption  $\dot{A}_{P,t} = 0$ . However, this assumption is not necessary for the main

argument. What is crucial is that relative productivity will *eventually* be increasing once a sufficiently large fraction of the population acquires  $h$ .<sup>31</sup> As  $A_t$  increases, the fraction of the population investing in  $h$  also increases. The levels of life expectancy necessary to make an agent of ability  $a$  indifferent between acquiring either types of human capital tend to decrease and the locus  $HH(A)$  shifts down for any  $a$  (excluding the extremes):

**Proposition 4.** *The life expectancy required for any given level of ability to be indifferent between acquiring  $h$  or  $p$  decreases, as relative productivity  $A$  increases:  $\left. \frac{\partial T(a,A)}{\partial A} \right|_{HH(A)} < 0, \forall a \in (0, 1)$ .*

Thus, the more productive theoretical human capital  $h$  becomes relatively to applied human capital  $p$ , the less restrictive is the fixed cost requirement of acquiring it, as the break-even of the investment in education is attained at a lower age.

Consider a non-developed economy in which life expectancy at birth is low, as for example during the middle ages.<sup>32</sup> Since  $A$  is low, investing in  $h$  is relatively costly for a large part of the population as the importance of the fixed cost for education,  $e$ , is large. This means that the concave part of the  $HH(A)$ -locus is large and the conditional system is characterized by a unique dynamic equilibrium of type  $\{a^L(A), T^L(A)\}$ , exhibiting low life expectancy and a little class of individuals deciding to acquire theoretical human capital. This situation is depicted in panel (1) of Figure 6. During this early stage of development, the feedback effects on lifetime duration and productivity are close to negligible, but just not quite negligible.

Over time, productivity growth makes investing in  $h$  more profitable for everybody, and life expectancy increases slowly. Graphically, the locus  $HH(A)$  shifts downwards as time passes, and the importance of the concave part decreases. After a sufficiently long period of this early stage of development,  $HH(A)$  exhibits a tangency point, and eventually three intersections with  $TT$ . From this moment onwards, in addition to  $E^L$ , also steady states of type  $E^u$  and  $E^H$  emerge. Since the intermediate equilibrium is locally unstable, the economy remains trapped in the area of attraction of the  $L$ -type equilibria, as depicted in panel (2) of Figure 6.

<sup>31</sup>In the simulations below, we allow  $\dot{A}_{P,t} > 0$  starting from large  $A_{P,0}$  and small  $A_{H,0}$ . Relative productivity  $A_t$  initially decreases, reflecting the larger innovative dynamics of sector  $P$  during early stages of development. Since  $h$  is relatively more important for technological progress,  $A_H$  eventually leapfrogs  $A_P$ . Therefore,  $A_t$  is eventually increasing and keeps increasing from this point on. The qualitative prediction is unchanged, but during early stages of development the high productivity in the  $P$ -sector induces the acquisition of  $p$  and delays a widespread acquisition of  $h$ .

<sup>32</sup>As will become clear below, starting from this point is without loss of generality. However, even though the model is also capable of demonstrating the situation of developed economies, the main contribution lies in the illustration of the transition from low to high levels of development.



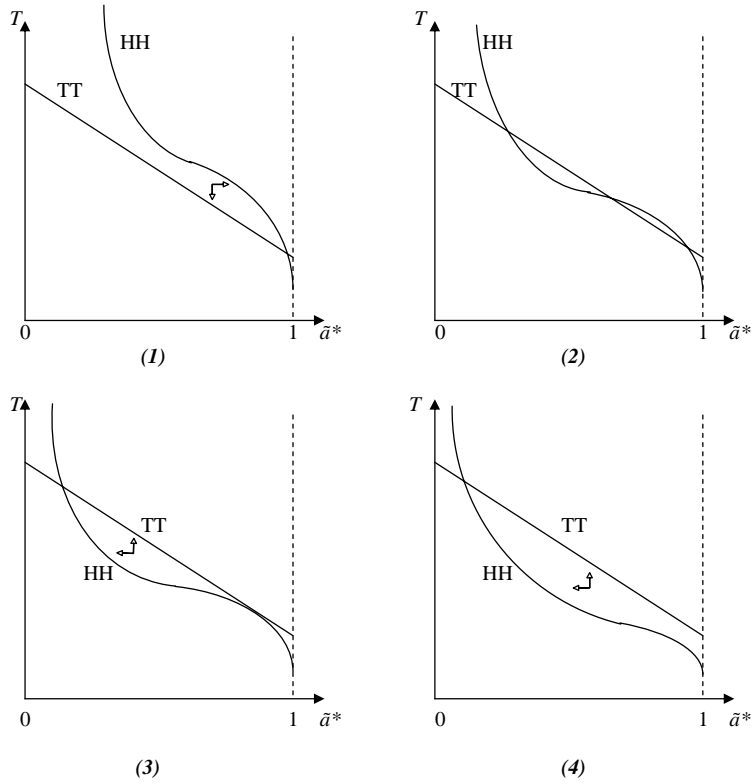


Figure 6: The Process of Development

As generations pass, the dynamic equilibrium induced by initially low life expectancy moves along  $TT$ . The consecutive downward shifts of  $HH(A)$ , however, eventually lead to a situation in which the initial dynamic equilibrium lies in the tangency of the two curves, as shown in panel (3) of Figure 6. In the neighborhood of this tangency, the static equilibrium locus  $HH(A)$  lies below the linear  $TT$ -locus, such that the equilibrium is not anymore stable. Already the following generation faces a life expectancy that is high enough to induce a substantially larger fraction to acquire human capital  $h$  than in the previous generation. At this point a unique  $E^H$  steady state exists, as is shown in panel (4) of Figure 6. A period of extremely rapid development is triggered, during which life expectancy virtually explodes, and the human capital structure of the population changes dramatically towards theoretical,  $h$ -type education. This phase of rapid change in general living conditions and the economic environment reflects what happened during the industrial revolution. This phase of fast development lasts for a few consecutive generations. After this transition, life expectancy converges slowly to its (biologically determined) upper bound  $\rho + \underline{T}$ , which is never

achieved. Even though the fraction of the population acquiring human capital  $h$  keeps growing, there will always be some fraction of the population acquiring applied knowledge  $p$ .

In the following proposition, we summarize these global dynamics. The evolution of the system is given by the sequence of ability thresholds, life expectancies and relative productivities  $\{a_t, T_t, A_t\}_{t \in [0, \infty)}$ , starting in a situation of an undeveloped economy:

**Proposition 5.** (Development Path of the Economy) *A stagnant economy with sufficiently large costs  $\underline{e}$  of human capital acquisition and sufficiently small initial life expectancy  $T$ , which is trapped in a sequence of L-type equilibria, passes through the following phases of development:*

1. *Initially, the economy exhibits a sequence of unique L-type steady states with low, but monotonically increasing, levels of life expectancy  $T$  and shares of the population acquiring human capital  $(1 - \lambda)$ .*
2. *H-type steady states, exhibiting larger  $T$  and  $(1 - \lambda)$ , emerge, while the economy remains trapped in the area of attraction of L-type steady states.*
3. *Eventually, the L-type steady state becomes unstable and disappears. Growth accelerates, life expectancy  $T$  and human capital acquisition  $(1 - \lambda)$  increase substantially as the economy converges towards a sequence of H-type equilibria.*
4. *The monotonic growth of  $T$  and  $(1 - \lambda)$  slows down as the economy converges to the H-type steady states.*

It is important to note that the actual trajectory of the system depends on the initial conditions and cannot be precisely identified in general. Proposition 5 in fact states that the system moves generation by generation in the area of attraction of the locally stable conditional state  $E^L$  during phases (1) to (3). In phase (4), the system converges to a series of globally stable steady states  $E^H$ . In historical terms, the model therefore exemplifies the different stages of development.<sup>33</sup>

Note that the inevitability of the transition to  $E^H$ -equilibria is driven by the formulation of technological progress in the tradition of endogenous growth theory. An alternative view of technological progress with stochastic

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<sup>33</sup>Europe could be thought of as being trapped in a sequence of  $E^L$  equilibria during ancient times and the middle ages. At some point during the late 18th century development took off, as the multiplicity of equilibria vanished. However, one could also think that e.g. African economies are still trapped today in dynamic equilibria characterized by low life expectancy and little theoretical knowledge (like literacy).

elements, as destruction of knowledge, forgetting and non-continuous, periodic improvements, could imply different predictions about the inevitability of the industrial revolution.<sup>34</sup>

## 4 A Simulation of the Development Process

This section presents a simulation of the model to illustrate the mechanism and its capability to replicate the patterns of long-term development. We simulate the model using parameters reflecting empirical findings where possible. However, note that these simulations do not claim utmost realism, and we do not calibrate and fine-tune the model in order to achieve an optimal fit with real world data. Table 1 contains the values of the parameters and initial conditions used in the simulation.

Table 1: Parameter Values Used for Simulation

$\alpha$	0.5	$\delta_P$	0.05	$\rho$	75.0	$A_P(0)$	1.6
$\beta$	0.5	$\phi_H$	0.95	$\underline{e}$	15.0	$\tilde{a}(0)$	0.9911
$\gamma$	0.6	$\phi_P$	0.95	$\underline{T}$	25.0		
$\delta_H$	0.11	$\chi$	0.75	$A_H(0)$	1.0		

Marginal productivity of time spent in education, given a specific level of ability, is assumed to be the same in the production of both types of human capital. The assumption  $\delta_H > \delta_P$  implies that TFP grows relatively faster in the  $\mathcal{H}$ -sector. Both sectors exhibit the same extent of decreasing returns to this stock of human capital  $\gamma$ . A maximal life expectancy of 100 years cannot be exceeded, while the minimum life expectancy is assumed to be 25 years.<sup>35</sup> The assumptions imply also that the total scope of extending life expectancy by research, medical inventions and the like is 75 years ( $\rho$ ). The fixed cost of acquiring theoretical human capital  $h$ ,  $\underline{e}$ , is 15 years. Initially, TFP in the  $\mathcal{P}$ -sector is 1.6 times higher than in the  $\mathcal{H}$ -sector.<sup>36</sup> Clearly, the model is capable of producing a deliberately long stagnancy period before the transition. For the illustration, we simulate the economy over 250

<sup>34</sup>For example, one could easily introduce random shocks affecting life expectancy and/or the stock of theoretical human capital in the economy, representing events exogenous to the economic system such as wars. In this case the links between generations through human capital are weakened or broken, which might prolong or even completely prevent the economic and biological transitions characterized above.

<sup>35</sup>This is in line with Streeten (1994) who cites evidence that average life expectancy in central Europe was even lower than 25 before 1650.

<sup>36</sup>This reflects the fact that at this point in time already a large number of generations has acquired applied knowledge that has increased TFP over time. Initially, 0.89 percent of the population acquire  $h$ .

generations.<sup>37</sup>

Simulation results for life expectancy and the fraction of the population acquiring theoretical human capital are depicted in Figure 7. Initially, life expectancy is quite low for many generations, but it increases over time with very little increments over the generations. At a certain point (around 1760) a period of rapid growth in average lifetime duration begins. Within just a few generations, life expectancy increases from mid-20 to over 60, then the growth of life expectancy slows down again. Just when life expectancy starts to take off, also the social structure of the economy starts changing rapidly, as ever larger proportions of the population acquire theoretical human capital.<sup>38</sup> However, also this evolution slows down from its initial rapidness, as the share of educated people exceeds roughly three quarters of the population. Nevertheless, due to the permanent growth in TFP, the aggregate stock of theoretical human capital keeps increasing, even after the transition, albeit at a somewhat slower rate. Simulation results for aggre-

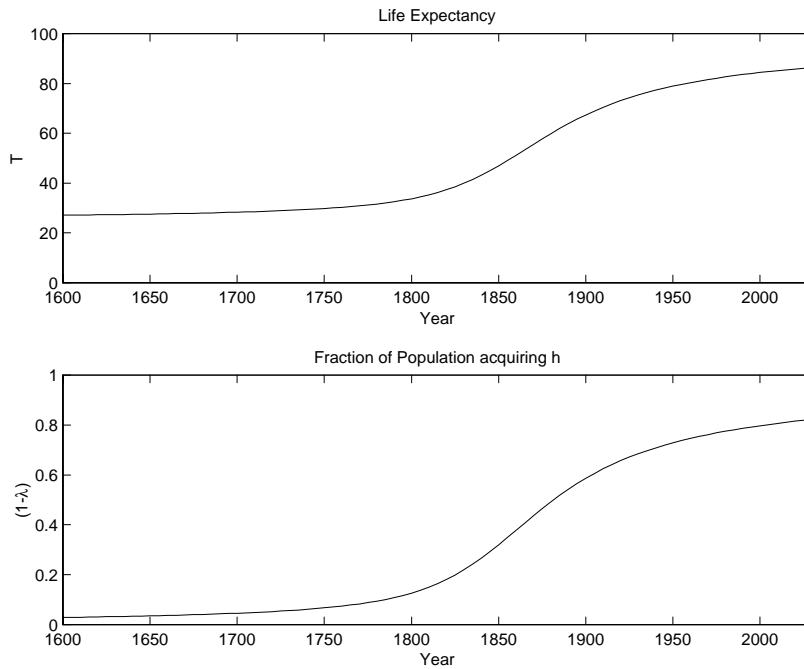


Figure 7: Life Expectancy  $T$  and the Proportion of the Population acquiring Human Capital  $h$ ,  $(1 - \lambda)$

gate income, income created in the  $\mathcal{P}$ -sector, and population size are shown in Figure 8. After having grown only very slowly aggregate income virtually

<sup>37</sup>Interpreting every 5 years as the arrival of a new generation, this reflects roughly a horizon from year 1000 to 2250, which includes the industrial revolution.

<sup>38</sup>This reflects in a rapid decrease of the ability threshold for abstract education.

explodes and keeps growing rapidly, even when growth in life expectancy and the fraction  $1 - \lambda$  ebbs away. Despite permanent growth in income generated in sector  $\mathcal{P}$ , development is mainly driven by progress in sector  $\mathcal{H}$ .<sup>39</sup>

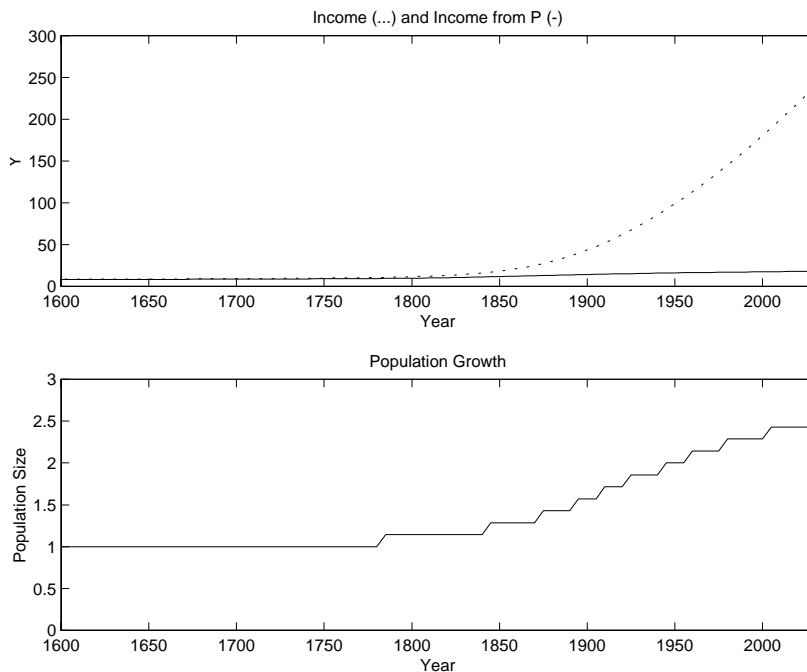


Figure 8: Income, Income from  $\mathcal{P}$ -sector, and Population Size

As life expectancy increases, more and more generations populate the economy at the same time: the population grows and almost triples even though individual fertility behavior is assumed to be constant and the same throughout generations. This is illustrated in the lower panel of Figure 8. Eventually, population size stabilizes.<sup>40</sup> A final observation is the endogenous structural transition from sector  $\mathcal{P}$  to sector  $\mathcal{H}$ , as is illustrated by Figure 9.

## 5 Concluding Remarks

The process of long-term development of the Western world was characterized by a lengthy period of stagnancy of economic conditions and life

<sup>39</sup>The simulations also reveal that about 250 years after the transition take-off, TFP in the  $\mathcal{H}$ -sector is about ten times higher than before the transition, while TFP in the  $\mathcal{P}$ -sector is about three to four times larger.

<sup>40</sup>The non-smooth, jagged development of the population size follows from the fact that the number of populations alive at each point in time is an integer.

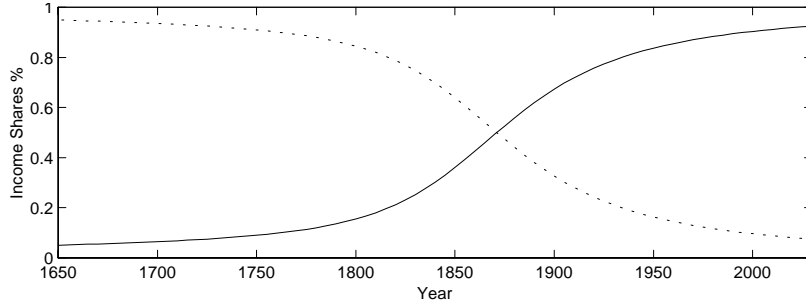


Figure 9: Structural Change: Income Shares of  $\mathcal{P}$ - ( $Y_P/Y$  (...)) and  $\mathcal{H}$ -sector ( $Y_H/Y$  (-))

expectancy. This was suddenly followed by a period of fast and dramatic changes in both these dimensions. What eventually triggered this rapid transition is the topic of a lively discussion within the profession. This paper presents a simple microfoundation of human capital formation, which allows to explain the historical patterns by explicitly taking complex interactions between economic, social and biological factors into account. Both, economic development and changes in life expectancy are modeled as endogenous processes. An implication of this view is that even during the apparently stagnant environment before the industrial revolution, economic and biological factors affected each other.

Life expectancy is the crucial state variable in the individual education decision. In turn, this education decision has implications for the education decision of future generations, both through life expectancy and productivity changes. Thus, advances in technological progress, human capital formation and lifetime duration reinforce each other. However, the peculiarity of human capital is that every generation has to acquire it anew. But the costs for human capital formation are prohibitively high for large parts of the population when the level of development is still low and when life expectancy is low. At a certain point in time the entire system is sufficiently developed so that the positive feedback loop has enough momentum to overcome the retarding effects of costs for human capital formation. We analytically characterize the resulting development path, which exhibits an S-shape with a long period of economic and biological stagnation, followed by a relatively short period of dramatic change in living conditions and the economic and social environment.

In order to isolate the role of the individual human capital investment problem for the dynamics of the system, we explicitly rule out Malthusian features like scale effects related to population size or the stock of human capital, fertility-education trade-offs, the presence of fixed factors of production, like land, or the existence of consumption subsistency levels. Moreover,

the mechanism presented in this paper is able to reproduce the observed patterns of long-term economic development without the need of relying on some exogenous events and strict temporal causalities. There is thus no need for identifying a driving shock that triggered the transition. By simulating the model for illustration purposes, we show that the long-run behavior of key indicators of development like income, income growth, productivity, lifetime duration, and population size implied by the model is in line with empirical evidence and stylized facts.

## A Appendix

### Proof of Proposition 1:

*Proof.* Consider Equation (26). For notational simplicity, denote  $\tilde{a}^*$  simply as  $a$ ,  $\underline{e}$  as  $e$ . By standard calculus,

$$T'(a) = \frac{e \frac{g'(a)}{\Omega}}{\left[1 - \frac{g(a)}{\Omega}\right]^2} < 0, \quad (38)$$

since  $g'(a) = -\frac{g(a)}{1+\gamma} \left[ \frac{2-\gamma-\gamma a^2}{2a^2(1-a^2)} \right] < 0$ ,  $\forall a \in [0, 1]$ . Therefore, we conclude that for a given set of parameters  $\gamma \in (0, 1)$ ,  $A = \frac{A_H}{A_P}$ , for every  $a \in [\underline{a}, 1]$  there is one and only one  $T > 0$  such that (26) is satisfied.  $\square$

### Proof of Lemma 1:

*Proof.* The claim follows from the definition of  $\tilde{a}^*(\Omega)$ , and the fact that  $g(\tilde{a}^*)$  is strictly decreasing in  $\tilde{a}^*$ .  $\square$

### Proof of Proposition 2:

The intuition of the proof proceeds as follows: We solve equilibrium condition (25) for  $T$  as a function of  $\tilde{a}^*$  and investigate the behavior of this function. Due to the fact that  $T(\tilde{a}^*)$  is strictly monotonically decreasing within the admissible support the function is invertible within this range of support. We then show that there exists one and only one  $\tilde{a}^*$  for which the second derivative of this function equals zero. Since the condition for the second derivative to equal zero cannot readily be solved for  $\tilde{a}^*$ , we decompose it into two components and show that one is strictly monotonically increasing within the support while the other is strictly monotonically decreasing, such that there must exist one and only one  $\tilde{a}^*$  for which the condition is satisfied by the intermediate value theorem. But if  $T(a)$  has a single inflection point and is invertible, also  $a(T)$  has a single inflection point and is therefore S-shaped.

*Proof.* Consider again Equation (26). We use the notational shorthands as in proof of Proposition 1. Using standard calculus, one can now show that:

$$T'(a) = \frac{e \frac{g'(a)}{\Omega}}{\left[1 - \frac{g(a)}{\Omega}\right]^2}, \quad (39)$$

and

$$T''(a) = \frac{e \frac{g''(a)}{\Omega} \left[1 - \frac{g(a)}{\Omega}\right] + 2e \frac{[g'(a)]^2}{\Omega^2}}{\left[1 - \frac{g(a)}{\Omega}\right]^3}. \quad (40)$$

Due to the fact that  $T'(a) < 0 \forall a \in [\underline{a}, 1]$ , we note that the function  $T(a)$  is invertible in the range  $a \in [\underline{a}, 1]$  of the support. Note also that  $T(a) \geq \underline{T} \forall a \in [0, 1]$ , so the inverse function  $a(T)$  is strictly monotonically decreasing for all positive  $T$ .

It will prove useful to substitute  $a^2$  with  $b$  and to re-write  $g(a) \equiv h(b) = \frac{(1-b)^{\frac{1-\gamma}{1+\gamma}}}{b^{\frac{2-\gamma}{2(1+\gamma)}}} k$ ,  $g'(a) \equiv h'(b)$ , and  $g''(a) \equiv h''(b)$ , where  $\underline{b} = (\underline{a})^2$ . Thus define  $T(a) =$



$\mathcal{T}(b)$ , so the derivatives  $T'(a) = \mathcal{T}'(b)$  and  $T''(a) = \mathcal{T}''(b)$  can be re-written in terms of  $b$ :

$$\mathcal{T}'(b) = -\frac{[\mathcal{T}(b)]^2 h'(b)}{e \Omega}$$

Existence of an inflection point can already be inferred from a closer examination. Since

$$h'(b) = -\frac{k}{2(1+\gamma)}(1-b)^{\frac{-2\gamma}{1+\gamma}} b^{\frac{-1}{1+\gamma}}(2-\gamma-\gamma b) = -\frac{k}{1+\gamma}h(b)B(b) < 0 \quad \forall b \in [\underline{b}, 1]$$

(where  $B(b) = \frac{1-\gamma}{1-b} \frac{2-\gamma}{2b}$ ), we know that also  $\mathcal{T}'(b) < 0 \quad \forall b \in [\underline{b}, 1]$ . Moreover, one immediately sees that  $\lim_{b \rightarrow 1} h'(b) = -\infty \Leftrightarrow \lim_{a \rightarrow 1} T'(a) = -\infty$ , such that  $T$  has infinitely negative slope at both boundaries of the admissible support, suggesting that there must exist at least one inflection point. From these arguments it is also clear that the slope of the inverse function,  $a'(T)$ , converges to zero at both boundaries of the support.

Analysis of the second derivative  $\mathcal{T}''(b)$  allows to show existence and uniqueness of an inflection point. In particular,  $\mathcal{T}''(b) = 0$  requires:

$$\begin{aligned} h''(b) \left(1 - \frac{h(b)}{\Omega}\right) &= -\frac{2}{\Omega} (h'(b))^2 \\ \Leftrightarrow \frac{kh(b)}{1+\gamma} \left[ \frac{B^2(b)}{1+\gamma} - B'(b) \right] \left(1 - \frac{h(b)}{\Omega}\right) &= -\frac{2kh(b)}{\Omega(1+\gamma)} B^2(b) \\ \Leftrightarrow \left( \frac{-1}{1+\gamma} + \frac{B'(b)}{B^2(b)} \right) &= \frac{2k}{\Omega(1+\gamma)} \left( \frac{h(b)}{1 - \frac{h(b)}{\Omega}} \right). \quad (41) \\ (LHS) &= (RHS) \end{aligned}$$

Noting that

$$\frac{B'(b)}{B^2(b)} = \frac{-2\gamma b^2 + 4b(2-\gamma) + 2(\gamma-2)}{(2-\gamma-\gamma b)^2},$$

one finds that

$$\frac{\partial \left( \frac{B'(b)}{B^2(b)} \right)}{\partial b} = \frac{8(2-\gamma)(1-\gamma)}{(2-\gamma-\gamma b)^3} > 0, \forall \gamma \in (0, 1), b \in [0, 1].$$

This implies that the LHS of the condition for an inflection point ( $\mathcal{T}''(b) = 0$ ), equation (41), is strictly monotonically increasing in  $b$ . Furthermore, applying calculus one can also verify that the RHS of condition (41) is strictly monotonically decreasing in  $b$  on the support  $[0, 1]$ :

$$\frac{\partial \left( \frac{h(b)}{1 - \frac{h(b)}{\Omega}} \right)}{\partial b} = \frac{h'(b)}{\left(1 - \frac{h(b)}{\Omega}\right)^2} < 0, \forall b \in [\underline{b}, 1].$$

In order to ensure that there is a value of  $b$  for which (41) is satisfied, it remains to be shown that the value of the LHS is smaller than that of the RHS for  $b = \underline{b}$  and larger for  $a = b = 1$ . Noting that  $LHS(b = 1) = \frac{-1}{1+\gamma} + \frac{1}{1-\gamma} > 0$  and that  $RHS(b = 1) = 0$  since  $h(1) = 0$ , one sees that the latter claim is true. The facts

that  $h'(b) < 0 \forall b \in [0, 1]$ , and that  $\lim_{b \downarrow \underline{b}} h(b) = \infty$  indicate that  $h(b)$  exhibits a saltus at  $b = \underline{b}$ . Since  $LHS(0) = \frac{-1}{1+\gamma} - \frac{2}{2-\gamma} < 0$  and due to the fact that the LHS is strictly monotonically increasing  $\forall b \in [0, 1]$ , the values of LHS and RHS can only be equal for one single value of  $a$ . These arguments are illustrated in Figure 10. This means that there exists one and only one level of  $b \in [\underline{b}, 1]$  such that  $T''(b) = 0$ .

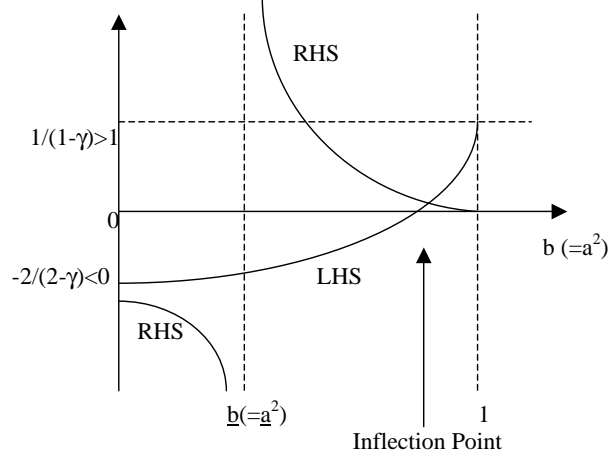


Figure 10: Existence and Uniqueness of an Inflection Point

From the fact that the function is invertible in this range of the support, and since there is a one-to-one relationship between  $a$  and  $b$ , we conclude that the function  $a(T)$  also exhibits exactly one inflection point.  $\square$

### Proof of Proposition 3:

*Proof.* Note: As long as there is no danger of confusion, we suppress the subscripts ' $t$ ' for generation  $t$  for notational convenience (e. g.  $T_t(a_t) = T(a)$ , etc.).

*(i):* Existence of a dynamic equilibrium for the conditional system. Recall that the locus  $TT$  is linear with slope  $-\rho$  and values  $T(a = 0) = \underline{T} + \rho$  and  $T(a = 1) = \underline{T}$ . From the proof of Proposition 2 we know that, for any  $A > 0$ , the locus  $HH(A)$  is such that  $\lim_{a \downarrow \underline{a}(A)} T_t(a, A) = \infty$ , and that its value is monotonically decreasing  $\forall a > \underline{a}(A)$ . Hence, if the value of this non-linear relation at  $a = 1$  is smaller than that of the linear relation of the intergenerational externality, there must exist at least one intersection by the intermediate value theorem. However, note that  $T(1) = \underline{e} \forall t$ , and that by assumption  $\underline{e} < \underline{T}$ . That means the fixed cost for theoretical education is always lower than any minimum life expectancy, otherwise theoretical education would never be an alternative, not even for the most able individual in the world. Hence a dynamic equilibrium exists for every generation  $t$ .

*(ii):* From the proof of *(i)* and noting that any steady state is characterized by an interior solution with  $a < 1$ , since  $T(a = 1) = \underline{T} > \underline{e}$ , which in turn implies that  $H_t > 0$  and  $P_t > 0$  for any  $t > 0$ .

*(iii):* The claims follow from Proposition 2: We know that  $HH(A)$  has always a unique turning point and takes values above and below  $TT$  at the extremes  $\underline{a}(A)$

and 1. Hence the two curves can intersect *at most* three times, while they intersect at least once by (i). Claim (a) follows from the negative slopes of both loci that allow to rank steady states. Claims (b) and (c) are true since, in the extreme equilibria  $E^H$  and  $E^L$ ,  $HH(A)$  intersects  $TT$  from above, which means that the system is locally stable, while the opposite happens in the intermediate equilibrium  $E^u$ , since  $HH(A)$  must cut  $TT$  from below. Thus  $E^u$  is locally unstable. Claim (d) follows from the fact that if only one steady state exists it must be stable since  $HH(A)$  starts above  $TT$  and ends below so it must cut from above. The concavity/convexity of  $HH(A)$  in the stable equilibria is used to identify them since in case of multiplicity one must be in the concave and the other in the convex part.  $\square$

### Proof of Lemma 2

*Proof.* By assumption,  $\delta > 0$ ,  $\phi > 0$ , and  $\chi > 0$  in equation (33), such that  $\dot{A}_{H,t} > 0$ , and  $A_{H,t} > A_{H,t-1} \forall t$ .  $A_{t-1}$  and  $A_t$  are linked in an autoregressive way, and equation (35) is of the form  $A_t = (c_{t-1} + 1) A_{t-1} = d_{t-1} A_{t-1}$ , where  $d_{t-1} = \delta H_t^\phi A_{t-1}^\chi + 1 > 1$  for any  $t$ , since from Proposition 3  $H_t > 0$  for any  $t$  and  $\delta > 0$ . This means that the process is positive monotonous and non stationary. Starting with any  $A_0 > 0$  we can rewrite  $A_t = \left(\prod_{i=1}^t d_{i-1}\right) A_0$ , where  $\left(\prod_{i=1}^t d_{i-1}\right) > 1$  and  $\lim_{t \rightarrow \infty} \left(\prod_{i=1}^t d_{i-1}\right) = \infty$ .

**Note:** If there is TFP growth also in the  $P$ -sector, it is sufficient for the argument to hold to assume that  $\delta > \delta_P \geq 0$ ,  $\phi \geq \phi_P$  and  $\chi \geq \chi_P$  in equation (33) and footnote 28. Then, the relative increment to TFP each period is larger in the  $H$ -sector, and the claim holds for identical initial values. For higher initial values of  $A_P$  it only holds after sufficiently many periods (generations) have passed.  $\square$

### Proof of Proposition 4

*Proof.* As in the proof of Proposition 2, solve equation (25) for  $T(a)$  to get:

$$T_t(a_t) = \frac{e}{1 - \frac{g(a_t)}{\Omega_t}}. \quad (42)$$

The claim follows by partial derivation of equation (42),  $\frac{\partial}{\partial \Omega_t} T_t(a_t) = -\frac{g(a_t)e}{(\Omega_t - g(a_t))^2} < 0 \forall a_t \in [\underline{a}_t, 1]$ .  $\square$

### Proof of Proposition 5:

Consider Equation (42), and denote the function characterizing the slope of the  $HH(A)$  locus for any  $a$  by  $HH'(A) \equiv \frac{\partial T(a,A)}{\partial a}$ . Similarly, let  $HH''(A) \equiv \frac{\partial^2 T(a,A)}{\partial a^2}$  denote the second derivative of the  $HH(A)$  locus for any  $a$ . From proposition 2, we know that  $HH'(A)$  is U-shaped. It takes infinite value at the extremes of the support  $\{\underline{a}(A), 1\}$ , and exhibits a unique global minimum corresponding to the inflection point of  $HH(A_t)$ . In the following, we denote  $a_A^I$  as the level of  $a$  corresponding to the global minimum of the function  $HH'(A)$  (or, equivalently corresponding to the unique inflection point of the function  $HH(A)$ ), characterized by  $HH''(A, a_A^I) = 0$ . A useful intermediate result describes the effect of  $A$  on the slope of the  $HH$ -locus:

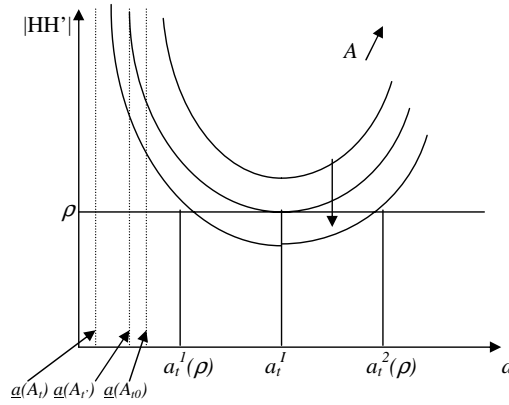
**Lemma 3.** *For any  $a \in [\underline{a}(A), 1]$ ,  $HH'(A)$  decreases as  $A$  increases.*

*Proof.* The result follows from

$$\frac{\partial}{\partial \Omega_t} \left| \frac{\partial T_t(a_t, A_t)}{\partial a_t} \right| = \frac{-\Omega_t + g(a_t) - 1}{\left(1 - \frac{g(a_t)}{\Omega_t}\right)^3} g'(a_t) e \frac{1}{\Omega_t} < 0 \quad \forall a_t \in [\underline{a}_t(A_t), 1],$$

and the definitions of  $A$  and  $\Omega$  in equation (28). Note also that  $\lim_{A \rightarrow \infty} \left( \frac{\partial T(a, A)}{\partial a} \right) = 0 \quad \forall a \neq \{0, 1\}$ ,  $\lim_{A \rightarrow 0} \left( \frac{\partial T(a, A)}{\partial a} \right) = +\infty$ ,  $\forall a \in [\underline{a}(A), 1]$  and  $\lim_{A \rightarrow 0} \underline{a}(A_0) = 1$ , therefore  $HH'(A)$  eventually take zero value in the interior of the bounded support as  $A \rightarrow \infty$ , and  $HH(A)$  is basically a vertical line at  $a = 1$  (with infinite slope) as  $A \rightarrow 0$ .  $\square$

Figure 11 plots  $HH'(A)$  for different  $A$ .



**Figure A1: Emergence of Multiple Equilibria**

Figure 11: Slope of the  $HH(A)$  Locus

Since  $HH'(A)$  shifts downwards monotonically as  $A$  increases, there exists a unique value  $A^0$ , and by lemma 2 also a unique  $t^0$ , such that  $HH'(A^0, a_{A^0}^I) = \rho$ . For this level of  $A = A^0$ ,  $HH'$  and  $TT'$  in Figure 11 are tangent. Hence,  $HH'(A, a_A^I) \geq \rho \iff A \leq A^0$ . Since  $HH'(A)$  is globally convex, and by definition of  $a_A^I$  as extremum (or from graphical inspection of Figure 11), for any  $t \geq t^0$  there exist exactly two levels of  $a$ ,  $a_{A_t}^1 \leq a_{A_t}^I \leq a_{A_t}^2$ , where  $a_{A_t}^1$  lies in the convex and  $a_{A_t}^2$  in the concave part of  $HH(A)$ , such that  $HH'(A_t, a_{A_t}^1) = \rho = HH'(A_t, a_{A_t}^2)$ .

Existence of at least one equilibrium of the conditional dynamic system (37) has been shown in Proposition 3. For any  $t < t^0$  the equilibrium is unique since  $HH'(A_t) > \rho \quad \forall a \in [\underline{a}(A), 1]$  and the loci  $HH(A_t)$  and  $TT$  necessarily intersect only once. For  $t \geq t^0$ , multiple equilibria may arise if  $HH(A)$  is flatter than  $TT$  in some range of the support. Thus, at  $t^0$  two scenarios are possible depending on the nature of the unique equilibrium. If  $HH(A^0, a_{A^0}^I) < TT(a_{A^0}^I)$ , the unique equilibrium is of type  $H$  since, by definition of  $a_A^I$  as inflection point, the two loci  $TT$  and  $HH$  intersect in the convex part of  $HH(A)$ . This is the case *if and only if* the concave part of  $HH(A)$  is sufficiently small, which is true if the fixed cost of acquiring human capital  $H$ ,  $\underline{e}$  is sufficiently small. In this case, nothing

prevents agents from acquiring high quality human capital from early stages on, and the economy develops smoothly as  $A$  increases overtime. In the other scenario,  $HH(A_0, a_{A_0}^I) \geq TT(a_{A_0}^I)$ , so the unique equilibrium at  $t^0$  is of type  $L$ . In this case, acquiring  $H$  is individually costly in an underdeveloped economy, or, equivalently,  $\underline{e}$  is sufficiently large to generate a development trap. The economy is characterized by a lengthy sequence of  $L$ -equilibria which is eventually followed by a development process as described in Proposition 5:

Given Lemma 2 on the monotonicity of  $A$  in  $t$ , Proposition 5 in the text is formally equivalent to:

**Proposition 5.** *Consider an economy characterized by a sufficiently large  $\underline{e}$ , such that the respective conditional system (37) is characterized by a sequence of unique steady state equilibria of type  $L$  as formalized in Proposition 3 for  $A \leq A^0$ . Then there exist two levels of productivity  $A^1$  and  $A^2$  with  $A^0 < A^1 < A^2 < \infty$  such that the dynamical system (37) is characterized by*

- (i) *a series of unique type  $L$  stationary equilibria with  $a^L(A_{t+1}) < a^L(A_t)$  and  $T^L(A_t) < T^L(A_{t+1}) \forall A_t \leq A^1$  (panel (a) in Figure 6);*
- (ii) *two steady states  $E^u(A^1)$  and  $E^L(A_1)$  at  $A_t = A^1$ ;*
- (iii) *three steady states:  $E^H(A_t)$ ,  $E^u(A_t)$  and  $E^L(A_t)$ , with the economy situated in the area of attraction of the  $L$ -equilibrium with  $a^L(A_{t+1}) < a^L(A_t)$  and  $T^L(A_{t+1}) > T^L(A_t) \forall A_t \in (A^1, A^2)$  (panel (b) Figure 6);*
- (iv) *two steady states  $E^H(A^2)$  and  $E^u(A^2)$  at  $A_t = A^2$  (panel (c) Figure 6);*
- (v) *a sequence of unique and globally stable  $H$ -type steady states with  $a^H(A_{t+1}) < a^H(A_t)$  and  $T^L(A_{t+1}) > T^L(A_t)$ ,  $\forall A_t > A^2$  (panel (d) Figure 6).*

*Proof.* Consider first claims (i) and (v). By construction, at  $t^0$  the steady state is  $L$ -type, so  $a_{A^0}^I = a_{A^0}^1 = a_{A^0}^2$  and  $HH(A^0, a_{A^0}^I) > TT(a_{A^0}^I)$ . Eventually there is a unique equilibrium with  $a_t^H$  close to zero since  $\lim_{A \rightarrow \infty} \underline{a}(A) = 0$  and  $HH(\underline{a}, A) = \infty$  for any  $A$ , so that the locus  $\lim_{A \rightarrow \infty} HH(A)$  exhibits infinite value at  $a = 0$  and value  $\underline{e}$  elsewhere. Hence, from a certain period onwards there must exist a unique  $H$  type equilibrium. This implies  $HH(\infty, a_\infty^1) = HH(\infty, a_\infty^2) = HH(\infty, a_\infty^2) = \underline{e} < TT(a)$  for any  $a \in (0, 1)$ . The dynamics of the system is determined by the position of  $HH(A_t, a_t^I)$ ,  $HH(A_t, a_t^1)$ , and  $HH(A_t, a_t^2)$  with respect to the corresponding values of the  $TT$ -locus. The system passes from a situation in which  $HH(A)$  lies above  $TT$  for a level of  $a$  for which both  $HH$  and  $TT$  have the same slope, to a situation in which  $HH$  lies below  $TT$  for the level of  $a$  for which both are parallel.

(ii), (iii) and (iv). From proposition 4 and lemma 3,  $HH(A)$  and  $HH'(A)$  decrease continuously and monotonically with  $A$ . By continuity, there exists a level  $A^1$  such that  $HH(A^1, a_{A^1}^1) = TT(a_{A^1}^1)$  but  $HH(A^1, a_{A^1}^2) > TT(a_{A^1}^2)$  since  $a_{A^1}^2$  lies in the concave part of  $HH(A)$ . For any  $A > A^1$ :  $HH(A, a_A^1) < TT(a_A^1)$  by proposition 4 and lemma 3. The same reasoning insures the existence of  $A^2$ :  $HH(A^2, a_{A^2}^2) = TT(a_{A^2}^2)$  and the fact that  $HH(A, a_A^2) < TT(a_A^2)$  for any  $A > A^2$ . Since  $HH(A, a)$  is continuous and monotonic in  $a$ : for any  $A \in (A^1, A^2)$   $HH(A, a_A^1) < TT(a_A^1)$  and  $HH(A^2, a_{A^2}^2) > TT(a_{A^2}^2)$  there exists a unique level  $a^u(A)$  which determines a locally unstable steady state of the system (37).

The levels of  $T$  and  $a$  associated to any locally stable steady state change monotonically as generations  $t$  pass because both loci  $HH(A)$  and  $TT$  increase in  $(1 - a)$ .

Therefore, by comparative statics in supermodular settings (or simple graphical inspection) an increase in  $A$  shifts down  $HH(A)$  and leads to an unambiguous increase (respectively decrease) in the level of  $T$  (respectively  $a$ ) associated to any locally stable equilibrium (the opposite is true for unstable ones).  $\square$

Note that, since  $A$  changes discretely as generations pass, it may be the case that not all the phases from  $(i)$  to  $(v)$  are exactly realized. In particular, stages  $(ii)$  and  $(iv)$  with the system displaying exact tangency and two equilibria, may not realize if the discrete change in  $A$  moves the system from one to three steady states within just one generation, namely if, for some  $t$ :  $A_t < A^1 < A_{t+1}$  or  $A_t < A^2 < A_{t+1}$ . Nonetheless, the global evolution of the dynamical system (36) necessarily follows the described phases with the full system evolving around an  $L$ -type locally stable steady state before an endogenous rapid transition to a globally stable steady state of type  $H$ .

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