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Fostering Within-Family Human Capital Investment: An Intragenerational Insurance Perspective of Social Security

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An Intragenerational Insurance Perspective of Social
Security**

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ABSTRACT

Fostering Within-Family Human Capital Investment: An Intragenerational Insurance Perspective of Social Security

We develop a general equilibrium stochastic OLG model with heterogeneous households. Households differ with respect to their productivity. Productivity depends stochastically on parents' unobservable investment in their child's human capital and an aggregate productivity shock. We introduce a PAYG social security system that conditions benefits on the aggregate wage sum and on the wage of one's child. We analyze the effects of such a social security system on the endogenous distribution of human capital and compare it to real world systems, which typically do not condition benefits on the wages of one's children. We decompose the effects of social security on the investment in human capital into an incentive effect, an insurance effect, a redistributive effect and a general equilibrium effect. Furthermore, we discuss the effects of social security on the long run distribution of human capital. Our approach suggests a novel role for a well-designed social security system: it can foster human capital accumulation and act as intragenerational insurance against human capital risk.

JEL Classification: J24, H55, D61

Keywords: human capital formation, social security, intragenerational insurance, heterogeneous households

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1 Introduction

Most economists considering insurance aspects of social security have stressed the issue of intergenerational risk sharing.¹ In a representative consumer framework, this is the obvious kind of risk management to consider. In such a framework, social security can act as insurance against factor share risk (Merton (1983)), as insurance against the state in which individuals are born (Gordon and Varian (1988), Bohn (1998)), as insurance against demographic risk (Smith (1982), Demange and Laroque (1999), Bohn (2000)) and as insurance against aggregate productivity risk during old age (Barbie, Hagedorn and Kaul (2000)). Most of the discussion on social security reform has focused on the role of social security for improving intergenerational risk sharing and the implications for intergenerational risk sharing of various social security reform proposals.²

There is only one paper, by Robert Shiller (1999), mentioning that social security can also have important functions in the pooling of risks among individuals in a single generation, i.e. the role for social security as intragenerational insurance. Shiller (1999) discusses the role of social Security as insurance against income risk in a heterogeneous population. In spite of practical limitations, however, this kind of risk sharing can at least partly be done by individual themselves by investing in a diversified portfolio. Furthermore, the government already provides insurance against income risk by a progressive income tax system.

In this paper, we develop a highly stylized reform proposal that highlights new role of social security as intragenerational insurance. A well-designed social security system can act as insurance against the human capital risk of one's children. Real world social security systems can be interpreted as the full insurance solution to a moral hazard problem that has to trade off incentives to invest in the education of one's children against the risk that arises from the fact that there is a stochastic relationship between investment in education and realized human capital (earnings ability, productivity) and thus wage. We develop a general equilibrium stochastic OLG model with heterogeneous households. There is intra-generational heterogeneity in the initial endowment (wage) an individual starts with. This initial endowment can be used for consumption or as investment in the education of one's offspring. The model is highly stylized because it excludes other forms of savings than the education of one's own offspring. The crucial assumption is that this

¹The early literature on intergenerational risk sharing and social security includes Enders and Lapan (1982), Smith (1982), Gordon and Varian (1988) and Gale (1990). More recent work includes Bohn (1998) and Abel (1999).

²See Shiller (1999) for an excellent discussion. Diamond (1997) gives an overview of the recent social security reform debate, including insurance aspects of social security.

investment in education is not observable/contractable. One interpretation is that this investment is done within the family in non-pecuniary form. Another interpretation is simply that not all forms of monetary investments in education can easily be subsidized by a Pigouvian kind of tax-transfer schemes. Our model builds on the empirical literature on early learning which stresses the role of families in fostering skill [e.g. Heckman (1999)]. Parents' investment in education determines the human capital (or earnings ability) of a child and thus his wage, but the relationship is stochastic. Thus the distribution of investments of the heterogeneous (parent) population generates a new distribution of children's initial endowments which in turn is the basis for the next generation's investment decision. Social security enters the picture as follows. As we have assumed that saving in the form of investment in education of one's own child is the only possible saving taking place, households rely on some kind of transfer in the second period of their lives (old age). At the one extreme is a within-family transfer that specifies a payment from the child to the parent, depending on the income of the child. This kind of transfer has the advantage of retaining full investment incentives for parents as they are claimants on a fraction of their child's income which in turn depends (stochastically) on the parents' investment in education. The obvious disadvantage of this within-family transfer is that it provides no insurance for the parents against the income risk of their child. This income risk of the child, however, directly translates into a retirement income risk of the parents. At the other extreme is a full insurance solution that resembles real world social security systems. The retirement income of an individual is independent of the earnings of one's own children but depends on one's income when young and the average labor income of the next generation. This provides insurance against the human capital risk of one's children, but erodes investment incentives in the education of the children. In fact, in our model this solution would lead to an immediate breakdown of the economy as nobody would invest in education and thus human capital and wages would be zero.

This interpretation of social security as a corner solution in an insurance-incentive tradeoff is the centerpoint of our paper. It allows us to gain some new insights about an optimal social security benefit formula in a dynamic context (with endogenous human capital distribution). Furthermore, it sheds some light on popular proposals in European countries to link social security benefits to the number of children in order to provide incentives to have more children and thus weaken the demographic problems social security faces. It becomes clear that what matters for the viability of social security systems is not the size of the population, but the wage sum that is earned by the working population. Unemployed or low-income (low human capital) individuals are thus not of much help in

curing the social security crisis. The important point we make is that better incentives for investment in education *within* the social security system may be a policy measure to achieve a more favorable long run distribution of human capital endowments and provide appropriate insurance against human capital risk of one's children. Our analysis suggests that parents' social security benefits should be conditioned (among other individual characteristics) on the contributions of one's children in order to establish optimal intragenerational risk sharing.

The paper is organized as follows. Section 2 introduces our stochastic OLG economy with heterogeneous households. We also propose a social security system in which benefits are parameterized in a weight that determines how much of the benefits depends on the average wage and how much depends on the wage of one's child. We interpret this parameter as social insurance incentive parameter. Section 3 justifies our modeling of social security from a contract-theoretic perspective. In section 4 we prove that under weak assumptions a unique equilibrium exists in the static household problem. We consider the dynamic problem in section 5 and show the existence of an invariant distribution of human capital endowments. Furthermore, we give a sufficient condition for uniqueness of the invariant distribution. As the transition matrix in our problem is not monotone, uniqueness cannot be guaranteed in general. We derive some comparative static results in the static problem in section 6 and discuss the tradeoff between insurance and incentives in our general equilibrium framework. We identify four channels through which an adjustment in the social security incentive parameter affects households' investment decisions: an incentive effect, a redistributive effect, a general equilibrium effect and an insurance effect. We show that the sign of the overall effect depends on the income position of the household. Section 7 illustrates the assumptions and their implications of the model through an example. Section 8 concludes.

2 The Model

The Basic Setup. Consider the following OLG model. Assume there are two generations alive at each period of time, old and young, and that there are $i = 1, \dots, N$ households in every generation. Every household has exactly one child at the end of the first period and lives for two periods. Time extends from zero to infinity ($t = 0, 1, \dots$). Saving is possible from first period net income only through investment in education e_i of one's child. Old age consumption is financed by social security as described below.

Uncertainty. There are N sources of uncertainty in our model, given by N idiosyncratic

individual shocks $\omega_i \in Y_i$ ($i = 1, \dots, N$). Let $\Omega \equiv Y_1 \times \dots \times Y_N$ be a product of finite sets. Ω is endowed with its power set as σ -algebra. We assume that shocks are independent and identically distributed over time according to a probability measure P on $(\Omega, \mathcal{P}(\Omega))$ with $\text{supp } P = \Omega$. This stochastic component is unknown to the household when investing in education. The economy wide shock can be interpreted as aggregate productivity shock, the individual shocks can be interpreted as genetic shocks.

Human capital The investment in education e_i in a child influences the human capital endowment h_i of the child according to a function $h_i = h(\cdot)$, where $h : \mathbb{R}_+ \times Y_i \rightarrow \mathbb{R}_+$. This function $h(e, \omega_{t,i})$ maps the amount e of the single consumption good invested in human capital and the individual shock $\omega_{t,i}$ into human capital (or equivalently in our model: efficiency units of labor) with $h(0, \omega_{t,i}) = 0$. Therefore, this function is a random variable that is parametrized by investment in education. In addition, this function is strictly concave and twice continuously differentiable in its first argument. Let the range of each $h(\cdot, \omega_{t,i})$ contain $[0, \bar{h}]$ and let it satisfy $\lim_{e \rightarrow 0} h_1(e, \omega_{t,i}) \rightarrow \infty$, where $h_1(\cdot)$ denotes the partial derivative with respect to e . Aggregate human capital is $H := \sum h_i$. A crucial assumption is that the investment in education is not contractable in our model. The interpretation is that at least parts of the investment in education take place in childhood within a family, for example in the form of time spent with the child. For simplicity we did not model this kind of cost as explicit time cost (time which cannot be spent earning wage income) but as direct monetary cost. Alternatively, we could simply assume that not all forms of investment in education can be monitored at a reasonable cost. Thus we exclude subsidies for these kinds of unobservable investments.

Production. The production function $F(H)$ is assumed to be linear, where H are efficiency units of labor, measured in human capital. So, $F(H) = w \cdot H$ for some unskilled wage $w > 0$.

Labor Market and Firms. We assume a competitive labor market. On the competitive labor market, since individuals are assumed to have no preferences for leisure, there will be an inelastic supply of efficiency units of labor, i.e. certain amounts of human capital. When young, a household receives gross wage income that depends multiplicatively on the unskilled wage w and its skill level $h_{t,i}$. The wage sum in the economy is $W := wH = w \sum h_i$

Social security and saving. We assume that there is a pay-as-you-go social security system in the economy. From the gross wage $wh(\cdot)$, a fraction τ is collected as social security payroll tax. The household i born at time t can spend the net income when young on consumption $c_{t,i}^y$ or on investment in education of its only child $e_{t,i}$. We consider

a social security benefit function that consists of two parts. The first part is a fraction $1 - \alpha$ of an equal share of the wage sum in the economy, W/N . The second part is a fraction α of the wage of the household's own child, $wh()$. The first part is exogenous for the household because it is assumed that the wage sum is perceived as fixed for the individual household. The second part can be influenced by the household through the investment decision.

Note that for $\alpha = 0$ the social security benefit depends only on the wage sum and is independent of the individual investment decision. This system insures the individuals completely against the human capital risk of their children. Real world social security systems typically offer this kind of full insurance. For $\alpha = 1$ the social security system conditions solely on the child's wage, but not on the wage sum. This system induces full incentives for investment in education, but supplies the individual with no insurance against human capital risk. This kind of system is a pure within-family transfer system. In reality, this system still exists in rural areas of less developed countries.

Household Income. The household income can be summarized as follows. When young, a household receives a net wage income of $V_i := wh_{t-1,i}(1 - \tau)$. When old, a household receives a two part social security benefit and an interest income.

Household preferences. Household preferences are assumed to be represented by the time separable utility function $u(c_{t,i}^y) + Ev(c_{t,i}^o)$ where $c_{t,i}^y$ denotes consumption when young and $c_{t,i}^o$ denotes consumption when old. u and v are assumed to be twice continuously differentiable, strictly increasing and strictly concave. Assume that $\lim_{b \rightarrow 0} u'(b) \rightarrow \infty$ and $\lim_{b \rightarrow 0} v'(b) \rightarrow \infty$. Furthermore assume that $v''' \geq 0$.

Individual problem. The individual has to decide how much to invest in the education of its child e_i . The individual's problem can be written as follows:

$$\begin{aligned} & \max [u(c_{t,i}^y) + Ev(c_{t,i}^o)] \\ \text{s.t.} \quad & c_{t,i}^y + e_{t,i} = w \cdot h(e_{t-1,i}, \omega_{t,i}) \cdot (1 - \tau), \\ & c_{t,i}^o = \tau \cdot \left[(1 - \alpha) \cdot \frac{1}{N} \cdot w \cdot \sum_{j=1}^N h(e_{t,j}, \omega_{t+1,j}) + \right. \\ & \quad \left. \alpha \cdot w \cdot h(e_{t,i}, \omega_{t+1,i}) \right] \end{aligned}$$

$$0 \leq \alpha \leq 1, 0 \leq \tau \leq 1.$$

The problem can be transformed by substituting the expressions for c^y and c^o into the objective function. Furthermore, individuals take prices (wages w and interest rates R) and repayments from the wage sum $W = w \cdot \sum h_i$ as given. We obtain the following problem for a period t born individual i , where we have skipped indices and omegas and slightly abused the notation. In addition, we have used the definition $V := w_{t-1}h_{t-1,i}(1 - \tau)$. V can be interpreted as inheritance. It is stochastic in our model.

$$\max u(V - e) + Ev \left(\tau \cdot \left[(1 - \alpha) \cdot \frac{1}{N} \cdot W + \alpha \cdot w \cdot h(e, \cdot) \right] \right) \quad (1)$$

$$s.t. \quad 0 \leq e \leq V$$

In summary, the individual when young invests part of his initial endowment V in the education e of his child. The payoff of this investment is received during old age. It consists of a fraction of the wage sum and a fraction of the wage of his child. Furthermore the individual receives a fraction of the aggregate capital income because it owns part of the capital stock.

3 Contract-theoretic considerations

We briefly discuss our modeling of the social security system from an contract theoretic perspective. We explain why we only use one incentive parameter α instead of adopting a standard approach.

When considering unobserved actions in a complete contracting framework with uncertainty (hidden action setting) there are two main questions concerning the optimal incentive contract. On the one hand one is interested in the set of observables the contract has to be depend on. On the other hand one tries to determine the optimal contract or at least to make some structural statements.

A first natural approach, in particular from a growth theory or OLG perspective, is to model the return of an investment decision e as a function $f(e, \omega)$ where ω is a stochastic shock to the production function and $f'(e, \cdot) > 0$ for all e and all ω . A convenient representation would be an additive form as e.g. $f(e) + \epsilon(\omega)$. But with this specification the hidden action problem can be solved without efficiency losses or at least with only arbitrarily small efficiency losses. If the domain of the random variable ϵ is compact than the first best can easily be reached by punishing the deviator heavily if an non-justifiable outcome realizes. Non-justifiable means that an outcome x realizes which is not feasible

with the e which is to be implemented. If the domain of ϵ is the set of real numbers, the outcome x is drawn from a continuum. Therefore the so-called Mirrless problem emerges. Mirrless (1974) has shown that a first best can (for certain probability distributions) be arbitrarily approximated. This works through stochastic punishment. Therefore we will have to restrict to a different approach if we want to choose a contract theoretic approach. The candidate for this is the standard modeling approach to hidden action in contract theory. One assumes that there are n possible outcomes and a positive probability of reaching each outcome for every investment level. Thereby investment influences the outcome in a stochastic manner. A higher level of investment means that the agent is faced with a probability distribution which stochastically dominates one with a lower level of investment. Given this setting, it is not difficult to find out the observables the contract has to depend on. For reference see the papers of Holmström (1979) and Mookherjee (1984) in a many agents setting. We argue now why this is not possible in our model. The essential difference between our model and this standard models is that they use a partial equilibrium approach whereas we use a general equilibrium approach. In both models all agents are risk averse but in partial equilibrium models there is a risk neutral profit maximizing principal. In our model the optimality criterion is the maximization of a welfare criterion which is a weighted sum of the agents' utilities. Therefore there is no "risk-absorbing" party in the model which bears risk in exchange for monetary compensation. Nevertheless, because of risk aversion, there is a demand for insurance. With an idiosyncratic shock, the risk could in principle (in a complete market setting) be completely diversified. But this makes it necessary for generic distributions that the remuneration of an agent depends on the realization of all outcomes of all other agents/consumers in the economy. However, this makes a general formulation of the problem not manageable. Above all the well known problems of the standard approach remain present. In particular it is typically neither possible to make strong statements about the shape of the optimal incentive contract nor to make structural predictions at all. Furthermore, one can argue that such elaborated designed institutions are not feasible in reality either because of high administrative/transaction costs or because of a too high dependence on the distribution function which can not be assumed to be time independent. Thus as in the incomplete contracting literature we restrict to simple rules and institutions. This results in our modeling of the social security system with only one (incentive) parameter.

4 Existence and uniqueness of equilibrium in the static problem

The first-order conditions (FOC) for the individual problem are as follows (using the boundary behavior and the strict concavity of the utility functions u, v and h):

$$u'(V - e) = E \left\{ v' \left(\tau \cdot \left[(1 - \alpha) \cdot \frac{1}{N} \cdot W + \alpha \cdot w \cdot h(e, \cdot) \right] \right) \cdot \tau \cdot \alpha \cdot w \cdot h'(e, \cdot) \right\}.$$

A perfect foresight competitive equilibrium is therefore characterized by the following equations:

$$u'(V_i - e_i) = E \left\{ v' \left[\tau \cdot w \cdot \left(\alpha \cdot h(e_i, \omega_i) + (1 - \alpha) \cdot \frac{1}{N} \sum_{j=1}^N h(e_j, \omega_j) \right) \right] \right. \quad (2)$$

$$\left. \cdot \tau \cdot \alpha \cdot w \cdot h_1(e_i, \omega_i) \right\} \quad \forall i = 1, \dots, N.$$

To simplify notation we will refer to the FOC as follows:

$$u'(V_i - e_i) = RHS(e_i, \alpha)$$

The following two assumptions together with the previous assumptions are a sufficient condition for the existence of a unique fixed point of the first order equation system above for a given (V_1, \dots, V_N) .

- Assumption 1: For $\hat{e} \neq e$, we have $\forall \omega, \omega' \in Y_1 \times \dots \times Y_N$:

$$\sum_{j=1}^N h(e_j, \omega_j) \geq \sum_{j=1}^N h(\hat{e}_j, \omega_j) \iff \sum_{j=1}^N h(e_j, \omega'_j) \geq \sum_{j=1}^N h(\hat{e}_j, \omega'_j).$$

Assumption 1 compares aggregate human capital as a result of two different education profiles. It imposes the following: if in one state aggregate human capital resulting from a certain investment profile is higher than that resulting from another investment profile, then this should hold uniformly in all states. Assumption 1 is e.g. satisfied if shocks are additive. Although these conditions are not very strong, it can be seen from the following proof that they could be weakened further without affecting the existence result.

Proposition 1 *Under assumptions 1, for any given profile of after tax incomes (V_1, \dots, V_N) a solution to the static household problem exists and is unique.*

Proof: *Uniqueness.* To see uniqueness suppose there were two or more fixed points. Then three cases are possible:

- $0 < e_i \leq \widehat{e}_i \quad \forall i = 1, \dots, N$ with strict inequality for some i

From the first order condition (2) the left-hand side of all equations does not decrease by replacing e with \widehat{e} . Using the strict concavity of $h(\cdot, \omega_i)$, for some individual i the right-hand side is decreasing. To see this, note first, that $w \cdot \frac{1}{N} \sum_{j=1}^N h(e_j, \omega_j)$ is increasing for i . This fact together with the concavity of v and h imply the claim. Thus no other fixed point with this property can exist.

- $e_i \geq \widehat{e}_i > 0 \quad \forall i = 1, \dots, N$ with strict inequality for some i

The same argument as before applies with reversed inequalities.

- $e_i < \widehat{e}_i$ for some i and $e_j > \widehat{e}_j$ for some j

Two further subcases have to be distinguished. First assume that $\sum_{j=1}^N h(e_j, \omega_j) \leq \sum_{j=1}^N h(\widehat{e}_j, \omega_j)$. By assumption 1, this expression holds for all $(\omega_1, \dots, \omega_N)$ if it holds for one such tuple. Consider an individual i for which e_j increases. Then the left-hand side in (2) increases, while $w \cdot \alpha \cdot h(e_i, \omega_i)$ increases and also $w \cdot \frac{1}{N} \sum_{j=1}^N h(e_j, \omega_j)$ increases. Thus $v'(\cdot)$ decreases and due to the strict concavity of f and $h(\cdot, \omega_i)$ the expression outside $v'(\cdot)$ decreases too. The reverse subcase is handled similarly.

Existence. It remains to be shown that a fixed point for the equation system (2) exists. This will be proved by using the Brouwer fixed point theorem. Consider a notationally cleaned up first order condition from individual i 's point of view

$$u'(V_i - e_i) = E \left\{ v' \left(\tau \cdot \left((1 - \alpha) \frac{W}{N} + \alpha \cdot w \cdot h(e_i) \right) \right) \cdot \tau \cdot \alpha \cdot w \cdot h'(e_i) \right\}. \quad (3)$$

Given the boundary behavior of u, v and h , the solution to the individual optimization problem is interior, i.e. $0 < e_i < V_i$ (which also justifies the use of first-order conditions), since the given wage w is positive. This can be seen as follows. If e_i were equal to V_i , the left-hand side would be infinity while the right hand side would be a finite number (using the full support assumption on P). If e_i were equal to zero the right hand side would be infinity because of the Inada condition on e and the left hand side would be finite. Also, by the strict concavity of the problem, the solution will be unique for given W .

Now recall that W is a function depending on the shocks ω . By the maximum theorem the maximizer is a continuous function of W (where W is viewed as a vector in \mathbb{R}^L with

L being the cardinality of Ω). Thus we have a continuous function $k_i : \mathbb{R}_+^L \rightarrow [0, V_i]$ for each individual i with $k_i(W) = \arg \max (1)$. To determine prices which are consistent with the choice of the individuals, i.e. in order to have a perfect foresight equilibrium, we introduce an artificial market agent who chooses the wage sum corresponding to the choice of individuals. Viewed as a function from \mathbb{R}^N to \mathbb{R}^L , this map is continuous.

Thus

$$W : \times_{i=1}^N [0, V_i] \rightarrow [0, \bar{F}]$$

where $\bar{F} = \bar{F}(\omega) = \max_{\omega \in \Omega} F \left(\sum_{i=1}^N h(V_i, \omega_i), 1, \omega_0 \right)$ is an upper bound for the wage sum.

To be able to obtain a fixed point, we thus consider a map from

$$[0, \bar{F}] \times [0, w_n] \times [0, \bar{R}] \times_{i=1}^N [0, V_i]$$

into itself defined as

$$\Phi(W, e) = \times_{i=1}^N k_i(W, w, R) \times W(e).$$

So by Brouwer's fixed point theorem, there exists a fixed point.

qed

5 Dynamic Problem

In this section we show the existence and give a sufficient condition for the uniqueness of an invariant distribution of the transition probabilities of the vector e_t in the dynamic setup.

Existence. Assume that the Jacobian matrix of (2) with respect to h is nonsingular. Now apply the implicit function theorem as stated in Hildenbrand (1974) to the equation system of first order conditions (2) if $e_{t-1} \gg 0$ after writing V_i as a continuous function of ω_t, e_{t-1} with $V_i \left[\left(h(e_{t-1,j}, \omega_{tj}) \right)_{j=1}^N \right] = w \cdot h(e_{t-1,i}, \omega_{t,i}) \cdot (1 - \tau)$ and endowing Ω with the metric for the discrete topology. For given ω_t, e_{t-1} this gives a unique continuous function $g_{e_{t-1}, \omega_t}(\cdot, \cdot)$ with $e_t = g_{e_{t-1}, \omega_t}(e_{t-1}, \omega_t)$ in a neighborhood of ω_t, e_{t-1} in the product topology on $[0, \bar{h}]^N$ and the discrete metric space Ω . By putting these local functions together, we find a stochastic difference equation $e_t = g(e_{t-1}, \omega_t)$, continuous in e_{t-1} , defined for $e_{t-1} \gg 0$ which determines the human capital investment in period t given the human capital investment in period $t - 1$ and the shock realization in period t . For $e_{t-1} = 0$, we know that $e_t = 0$ for all ω_t . Since as $e_{t-1} \rightarrow 0$ in some components also $g(e_{t-1}, \omega_t) \rightarrow 0$ in the

same components, the function g can be continuously extended to the boundary. Then by exercise 8.10 in Stokey and Lucas (1989), the transition P generated by g according to Theorem 8.9 in Stokey and Lucas (1989) has the Feller property. Thus, since the state space $S = [0, \bar{h}]^N$ is compact, by Theorem 12.10 in Stokey and Lucas (1989), an invariant distribution of P exists.

Uniqueness. To apply Theorem 12.12 from Stokey and Lucas (1989), the mixing condition given in assumption 12.1 in Stokey and Lucas (1989) and the monotonicity of P have to be checked. Another way of establishing uniqueness is to use a relatively recent result from probability theory literature.

Theorem [Theorem 5.2 in Lasserre (1996)]:

A unique invariant probability measure exists if and only if for every continuous function $a \neq 0$ defined on $[0, \bar{h}]^N$ with $0 \leq a \leq 1$ and $a \equiv 0$ on some nonempty compact set $K_a \subseteq [0, \bar{h}]^N$ and every positive scalar M

$$\exists \gamma \geq 0, \exists b \in C\left([0, \bar{h}]^N\right) \text{ with } \int b(y) P(dy, x) \leq b(x) + 1 - \gamma a(x) \quad \forall x \in [0, \bar{h}]^N \text{ and } \gamma > M$$

or

$$\exists b \in C\left([0, \bar{h}]^N\right), \exists \gamma \geq 0 \text{ s.t. } \int b(y) P(dy, x) \leq b(x) - 1 + \gamma a(x) \quad \forall x \in [0, \bar{h}]^N.$$

It is, however, only possible to check this condition from case to case, since it cannot be verified to hold in general and will probably not do so. But since the condition is necessary and sufficient, it is at least in principle possible to test a given transition function for uniqueness of the stationary distribution.

Nevertheless, it is possible to indicate for which type of "test" function a which of the conditions are likely to hold. Suppose that the set K_a is relatively "small" (in the sense that for every starting point $x \in K_a$, there is a uniformly bounded positive probability of leaving K_a). Then by choosing the values of b sufficiently high outside K_a , the second condition is fulfilled for $x \in K_a$. To see that it is fulfilled for $x \in [0, \bar{h}]^N \setminus K_a$, note that γ can be chosen arbitrarily large. If K_a is "big" in the sense that there is a uniformly bounded positive probability for all $x \in [0, \bar{h}]^N \setminus K_a$ of reaching K_a in the next period and the probability of leaving K_a if starting there is low, b can be chosen to be very negative on K_a , so that the first condition holds.

6 Comparative Statics in the Static Problem

In this section, we analyze how changes in the social security incentive parameter α affect households' consumption and investment decisions e_i . We first describe how households' consumption and investment decisions depend on the distribution of wealth V_i and then turn to the comparative statics with respect to α . In the following we will assume (dropping indices) that $h(e, \omega) = h(e) + \epsilon(\omega)$, where $\epsilon(\omega)$ is a random variable with zero mean and finite variance.

Lemma 1 (*The rich invest more*):

If $V_1 < \dots < V_N$, then $e_1 < \dots < e_N$.

Proof: Consider the FOC of the static problem, $u'(V_i - e_i) = RHS(e_i, \alpha)$, where we recall that $RHS(\cdot) := E[v'(\cdot)]\alpha wh'(e)\tau$ denotes the right hand side of the FOC as a function of α and e_i . Assume $V_i > V_j$ and $e_i \leq e_j$. It follows that $V_i - e_i > V_j - e_j$. Thus $u'(V_i - e_i) > u'(V_j - e_j)$ by concavity of $u(\cdot)$.

Differentiating $RHS(\cdot)$ yields:

$$\frac{\partial RHS(\cdot)}{\partial e} = E[v''(\cdot)] \cdot [\alpha wh'(e)\tau]^2 + E[v'(\cdot)]\alpha wh''(e)\tau < 0$$

This gives $RHS(e_j, \alpha) < RHS(e_i, \alpha) = u'(V_i - e_i) < u'(V_j - e_j)$, where the first inequality follows from $\partial RHS/\partial e < 0$, the equality follows from the FOC for household i and the last inequality follows from the concavity of $u(\cdot)$. But this contradicts $RHS(e_j, \alpha) = u'(V_j - e_j)$ by the FOC of household j .qed

Lemma 2 (*The rich consume more*):

If $V_1 < \dots < V_N$ then $c_1^y = V_1 - e_1 < \dots < V_N - e_N = c_N^y$

Proof: Assume $V_i > V_j$. By Lemma 1, $e_i > e_j$. Thus $RHS(e_j, \alpha) > RHS(e_i, \alpha)$. By the FOC this gives $u'(V_j - e_j) > u'(V_i - e_i)$.qed

Next we consider how changes in α affect the expected marginal revenue of investment of the households which is given by $RHS(\cdot)$. We know from the FOC that investment e_i rises if $RHS(\cdot)$ rises.

$$\begin{aligned} \frac{\partial RHS(\cdot)}{\partial \alpha} &= E\left\{v''(\cdot) \cdot \tau \cdot \left[-\frac{W(\alpha)}{N} + w \cdot [h(e) + \epsilon] + (1 - \alpha)\frac{W'(\alpha)}{N}\right]\right\} \cdot \alpha w h'(e) \tau \\ &\quad + E[v'(\cdot)] w h'(e) \tau \end{aligned}$$

The marginal revenue changes due to two effects. The first summand is an indirect effect, the second summand a direct effect. The direct effect is positive because a higher incentive parameter raises the individual's incentives to invest for a given expected wealth in the second period of life (incentive effect). The sign of the indirect effect depends on the term in braces. The indirect effect accounts for three changes due to a change in α . First, a change in α entails changes in the redistributive properties of our social security system (redistributive effect). Second, changes in α will feed back through changes in investment incentives on the realization of the aggregate wage sum $W(\alpha)$ (general equilibrium effect). Third, the insurance arrangement of our social security system is affected (insurance effect). We will neglect for a moment the general equilibrium effect of a change in α , $W'(\alpha)$. The redistributive effect is described by the term $-W(\alpha)/N + w \cdot h(e)$. Its sign depends on the income position of the household. If the household income is below the average income, $W(\alpha)/N > w \cdot h(e)$, this effect is also positive. It is negative for above average income households, since a higher α implies less redistribution via the social security system. This benefits the high income households whose marginal revenue in old age thus falls due to higher after social security tax income. A similar argument shows that a higher a hurts the poor. Regarding investment incentives, this implies less investment by the rich and more investment by the poor. The insurance effect is described by the term $E[v''(\cdot) \cdot \tau \cdot w \cdot \epsilon]$. Under our assumptions $E[\epsilon] = 0$ and $v''' > 0$ this term can be shown to be positive. A proof and a detailed discussion of this effect will be given below. The interpretation is as follows: a rise in α reduces the insurance coverage of the households (recall that $\alpha = 0$ is the full insurance case) and this in turn lowers the second period expected utility, thus raising expected marginal utility in period 2 as described by $RHS(\cdot)$. As a consequence, the insurance effect of a higher α will raise investment.

Thus, neglecting the general equilibrium effect, a higher incentive parameter has an unambiguously positive effect on the investment of the poor, but an ambiguous effect on the rich. Since the effect of a change in α on investment behavior is generally ambiguous, we give a sufficient condition under which more can be said about the changes in the investment behavior of the households.

- Assumption 3: $\frac{\partial}{\partial e} \frac{\partial RHS(e, \alpha)}{\partial \alpha} < 0$

Assumption 3 says that the changes (due to higher incentives) in marginal revenue of an investment should fall monotonically in the level of investment e . This assumption is consistent with our previous finding that $\partial RHS/\partial\alpha > 0$ for small e , i.e. for poor households, and $\partial RHS/\partial\alpha < 0$ for large e , i.e. for rich households. It strengthens this finding by imposing that this fall in $\partial RHS/\partial\alpha$ should be monotonic in e .

Proposition 2 *If assumption 3 holds then either a) or b) is true:*

- a) *Higher incentives (higher α) induce higher investment e_i for all households i .*
- b) *If there exists a household j for which investment e_j falls although the incentives rise, then e_k falls for all $k > j$. Thus household j is a cutoff household.*

Proof: We prove part b) of the proposition in four steps. Assume α is raised from α^1 to $\alpha^2 > \alpha^1$ and there exists some j such that $e_j^2 < e_j^1$, where e_j^z denotes the investment of household j under the incentive scheme α^z ($z = 1, 2$).

Step 1: $0 < RHS(e_j^1, \alpha^1) - RHS(e_j^1, \alpha^2)$

We have $RHS(e_j^1, \alpha^1) = u'(V - e_j^1) > u'(V - e_j^2) = RHS(e_j^2, \alpha^2) > RHS(e_j^1, \alpha^2)$, where the first and last equality follow from the FOC, the first inequality follows from $e_j^1 < e_j^2$, the last inequality follows from $e_j^1 < e_j^2$ and $\partial RHS/\partial e < 0$. Thus it follows $RHS(e_j^1, \alpha^1) > RHS(e_j^1, \alpha^2)$, as claimed.

Step 2: $RHS(e_j^1, \alpha^1) - RHS(e_j^1, \alpha^2) > RHS(e_j^2, \alpha^1) - RHS(e_j^2, \alpha^2) > 0$

We have $RHS(e_j^1, \alpha^1) - RHS(e_j^1, \alpha^2) = - \int_{\alpha^1}^{\alpha^2} \frac{\partial RHS(e_j^1, a)}{\partial a} da > - \int_{\alpha^1}^{\alpha^2} \frac{\partial RHS(e_j^2, a)}{\partial a} da = RHS(e_j^2, \alpha^1) - RHS(e_j^2, \alpha^2)$. The inequality follows from assumption 3 and $e_j^2 < e_j^1$. The left hand side is positive from step 1. The right hand side is also positive by replacing e_j^1 with e_j^2 in step 1.

Step 3: $RHS(e_k^1, \alpha^2) < RHS(e_k^1, \alpha^1)$ for $k > j$

We have $0 < RHS(e_j^1, \alpha^1) - RHS(e_j^1, \alpha^2) < RHS(e_k^1, \alpha^1) - RHS(e_k^1, \alpha^2)$, where the first inequality follows from step 1 and the second inequality from $e_k^1 > e_j^1$ and step 2.

Step 4: $e_k^1 < e_k^2$ implies $RHS(e_k^2, \alpha^2) < RHS(e_k^1, \alpha^2) < RHS(e_k^1, \alpha^1) = u'(V_k - e_k^1) < u'(V_k - e_k^2)$

The first inequality follows from $\partial RHS/\partial e < 0$, the second from step 3, the equality from the FOC and the last inequality from the concavity of $u(\cdot)$ and our assumption $e_k^1 < e_k^2$. But this is a contradiction. Hence $e_k^1 > e_k^2$, as claimed in part b) of the Proposition. Part a) is also a possible outcome of our model as will be shown in an example below. \square

The proposition shows that higher incentives to invest need not lead to higher investment over the whole range of households. In particular, those households who are rich gain by

the higher incentives through the redistributive effect and therefore might reduce their investment. The reason for this is, as pointed out above, that in our model higher incentives imply less redistribution, which makes the rich better off in expected utility terms. This lowers their expected marginal utility of second period consumption and consequently lowers investment.

To highlight how the insurance effect enters the model we will isolate this effect by holding constant the incentive effect, the redistributive effect and the general equilibrium effect. This can be achieved by assuming that all households invest exogenously the same amount e , $e_i = e \forall i$. It then follows that $h_i = h(e) + \varepsilon_i$ and thus $w_i = w \cdot h_i = wh(e) + w\varepsilon_i$ and $W = Nwh(e)$ from $\sum \varepsilon_i = 0$. We then have the following result.

Proposition 3 *Households gain by the isolated insurance effect so that in this case full insurance, $\alpha = 0$, would be optimal.*

Proof: We show that utility falls if α is raised for $\alpha \in [0, 1]$ in the case of exogenously fixed and identical investments e . Consider $\frac{\partial}{\partial \alpha} E[v(\cdot)]$. We have $\frac{\partial}{\partial \alpha} E[v[\tau((1-\alpha)W/N + \alpha wh(e) + \alpha w\varepsilon)]] = E\{v'(\tau W/N + \tau \alpha w\varepsilon) \cdot \tau \cdot [w(h(e) + \varepsilon) - W/N]\} = E\{v'(\tau W/N + \tau \alpha w\varepsilon) \cdot \tau w\varepsilon\}$. We will show that this term is negative. It suffices to show that $E[v'(c + d\varepsilon) \cdot \varepsilon] < 0$, where c, d are positive constants. We have $\sum_{\varepsilon > 0} v'(c + d\varepsilon)\varepsilon f(\varepsilon) < \sum_{\varepsilon > 0} v'(c)\varepsilon f(\varepsilon) = v'(c) \sum_{\varepsilon > 0} \varepsilon f(\varepsilon) = -v'(c) \sum_{\varepsilon < 0} \varepsilon f(\varepsilon) > -\sum_{\varepsilon < 0} v'(c + d\varepsilon)\varepsilon f(\varepsilon)$. The first and last inequality follow from $v'' < 0$, the second equality used $E[\varepsilon] = 0$. This implies $\sum_{\varepsilon > 0} v'(c + d\varepsilon)\varepsilon f(\varepsilon) + \sum_{\varepsilon < 0} v'(c + d\varepsilon)\varepsilon f(\varepsilon) < 0$ and thus the claim. \square

Note that it follows from this proof that $E[v''(\cdot) \cdot \tau \cdot \varepsilon] < 0$ under our assumptions $E[\varepsilon] = 0$ and $v''' > 0$. This was claimed above. Note further that the result only holds for the *isolated* insurance effect, but $\alpha = 0$ can never be optimal in the model. The reason is that $\alpha = 0$ would imply $e_i = 0$ for all households i , because under full insurance no investment incentive exists due to the possibility of free riding. We have avoided this in the proposition by separating insurance from incentive aspects through fixing $e_i = e$ exogenously.

In summary, we have identified four channels through which a change in the social security incentive parameter affects households' investment decisions, namely an incentive effect, a redistributive effect, a general equilibrium effect and an insurance effect. Under our assumptions we were able to sign the effects and we also derived some results about the overall effect. In particular, the sign of the overall effect depends on the income position of the household. For below average income households higher investment incentives will indeed induce higher investment. For above average income households this need not be

true.

7 Example

To gain some better understanding of the assumptions and their implications we illustrate our results with an example.

Assume $u(x) = v(x) = -\exp(-rx)$. Then $u'(x) = v'(x) = r \exp(-rx)$ and thus the FOC becomes

$$r \exp[-r(V - e)] = \int r \{ \exp[-r\tau((1 - \alpha)W/N + \alpha w(h(e) + \varepsilon(\omega)))] \} \alpha w h'(e) f(\omega) d\omega.$$

This is equivalent to the following expression, where the integral is the insurance effect discussed above.

$$\exp[-r(V - e)] = r \{ \exp[-r\tau((1 - \alpha)W/N + \alpha w(h(e)))] \} \alpha w h'(e) \tau \int \exp[-r\tau\alpha w \varepsilon(\omega)] f(\omega) d\omega$$

Now we further assume that $\varepsilon(\omega) \sim N(0, \sigma^2)$. This implies $r\tau\alpha w \varepsilon(\omega) \sim N(0, (r\tau\alpha w \sigma)^2)$. Furthermore assume $h(e) = \beta e$ for some $\beta > 0$. Using this and solving the integral, the FOC can be written as:

$$\exp[-r(V - e)] = r \{ \exp[-r\tau((1 - \alpha)W/N + \alpha w(h(e)))] \} \alpha w \beta \tau \exp[(r\tau\alpha w \sigma)^2 / 2]$$

which can be simplified using $k := \ln(r\alpha w \beta \tau)$ and $R := (r\tau\alpha w \sigma)^2 / 2$ and taking logs:

$$-r(V - e) = k - r\tau[(1 - \alpha)W/N + \alpha w \beta e] + R$$

Using $W = w(\sum e_i)\beta$ and taking care of the dropped indices yields:

$$r e_i (1 + \tau w \alpha \beta) = k + R + r \left[V_i - \tau w \beta (1 - \alpha) \frac{\sum e_i}{N} \right]$$

Using $c := r\tau w \beta (1 - \alpha) / N$ and $d := r(1 + \tau w \alpha \beta)$ gives:

$$e_i d = k + R + r V_i - c \sum_{j \neq i} e_j$$

In matrix notation:

$$\begin{pmatrix} d & c & & c \\ & c & & \\ & & & c \\ c & & c & d \end{pmatrix} \begin{pmatrix} e_1 \\ \\ \\ e_N \end{pmatrix} = \begin{pmatrix} k + R + rV_1 \\ \\ \\ k + R + rV_N \end{pmatrix}$$

The inverse of the matrix has $\frac{d+(N-2)c}{d^2+(N-2)cd-(N-1)c^2}$ on the diagonal and $\frac{-c}{d^2+(N-2)cd-(N-1)c^2}$ off the diagonal. Note that $d > 0, c > 0, d - c > 0$. Thus the denominator is positive. We can then solve for e_i :

$$e_i = (k + R) \frac{d - c}{d^2 + (N - 2)cd - (N - 1)c^2} + r \frac{d + (N - 2)c}{d^2 + (N - 2)cd - (N - 1)c^2} V_i - r \frac{c}{d^2 + (N - 2)cd - (N - 1)c^2} \sum_{j \neq i} V_j$$

This solution has the simple structure:

$$e_i = K_0(\alpha) + K_1(\alpha)V_i - K_2(\alpha) \sum_{j \neq i} V_j$$

Having explicitly calculated the solution for individual investments, we proceed as in the general case. We want to know how investment reacts if α rises from α^1 to $\alpha^2 > \alpha^1$. In particular, we want to replicate our cutoff result. Suppose investment falls for some j . We will check now whether it also falls for all $k > j$. Assume $\Delta e_j := e_j^2 - e_j^1 = \Delta_0 + \Delta_1 V_j - \Delta_2 \sum_{i \neq j} V_i < 0$, where $\Delta_s := K_s(\alpha^2) - K_s(\alpha^1)$. We want to show that for $k > j$ $\Delta e_k := e_k^2 - e_k^1 = \Delta_0 + \Delta_1 V_k - \Delta_2 \sum_{i \neq k} V_i < 0$. Equivalently, we want to derive conditions under which $\frac{\partial^2 e^*}{\partial V \partial \alpha} < 0$. Note that $W/w\beta = \sum e_i = \frac{N(k+R)}{den}(d-c) + \left[\frac{r[d+(N-2)c]}{den} - (N-1) \frac{rc}{den} \right] = \frac{d-c}{den} [N(k+R) + r \sum V_i] > 0$.

We have $\Delta e_k - \Delta e_j = \Delta_1 \cdot (V_k - V_j) - \Delta_2 \cdot (\sum_{i \neq k} V_i - \sum_{i \neq j} V_i) = \Delta_1 \cdot (V_k - V_j) - \Delta_2 \cdot (V_j - V_k)$. Since $V_j - V_k < 0$, this term will be negative if the coefficients Δ_1 and Δ_2 are both negative. We can rewrite $\Delta_i = \int \frac{\partial \Delta_i}{\partial a} da$ where the integration is from α^1 to α^2 . Tedious calculations reveal that indeed $\frac{\partial \Delta_1}{\partial \alpha} = -\frac{(N-1)\tau w \beta}{N(\tau \alpha \beta w + 1)^2} < 0$ and $\frac{\partial \Delta_2}{\partial \alpha} = -\frac{\tau w \beta}{N(\tau w \beta \alpha + 1)^2} < 0$. This proves $\Delta e_k - \Delta e_j < 0$ for $k > j$. This implies that if $\Delta e_j < 0$, then also $\Delta e_k < 0$, which is our cutoff result from the general case. Furthermore, this result implies that if $\Delta e_j > 0$ then either $\Delta e_j > \Delta e_k > 0$ or $\Delta e_j > 0 > \Delta e_k$. This result has the interpretation that richer households react less sensitively to higher investment incentives than poorer households:

if the poor invest more, then the rich either invest more or less. However, if they invest more, then this is smaller than the increase in the investment of the poor. This result could not be shown in the general case, but is a nice property of our example.

Now we show that our sufficient condition about the cross partial derivative of RHS does not generally hold in our example. Calculate RHS and the relevant derivatives with respect to α and e and also the cross partial derivative:

$$\begin{aligned} RHS(e, \alpha) &= r\alpha w\beta\tau \exp\left\{-r\tau\left[(1-\alpha)\frac{W}{N} + \alpha w\beta e\right] + \frac{1}{2}(r\tau\alpha w\sigma)^2\right\} \\ &= r\alpha w\beta\tau \exp\left[-r\tau\frac{W}{N}\right] \exp\left\{-r\tau\alpha\left[-\frac{W}{N} + w\beta e - \frac{1}{2}r\tau\alpha w^2\sigma^2\right]\right\} \end{aligned}$$

$$\begin{aligned} \frac{\partial RHS}{\partial e} &= r\alpha w\beta\tau \exp\left[-r\tau\frac{W}{N}\right] \exp\left\{-r\tau\alpha\left[-\frac{W}{N} + w\beta e - \frac{1}{2}r\tau\alpha w^2\sigma^2\right]\right\} (-r\tau\alpha w\beta) \\ &= -(r\tau\alpha w\beta)^2 \exp\left[-r\tau\frac{W}{N}\right] \exp\left\{-r\tau\alpha\left[-\frac{W}{N} + w\beta e - \frac{1}{2}r\tau\alpha w^2\sigma^2\right]\right\} < 0 \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 RHS}{\partial \alpha \partial e} &= -2\alpha(rw\beta\tau)^2 \exp\left[-r\tau\frac{W}{N}\right] \exp\left\{-r\tau\alpha\left[-\frac{W}{N} + w\beta e - \frac{1}{2}r\tau\alpha w^2\sigma^2\right]\right\} \\ &\quad - (r\alpha w\beta\tau)^2 \exp\left[-r\tau\frac{W}{N}\right] \exp\left\{-r\tau\alpha\left[-\frac{W}{N} + w\beta e - \frac{1}{2}r\tau\alpha w^2\sigma^2\right]\right\} (-r\tau\alpha\left[-\frac{W}{N} + w\beta e - \frac{1}{2}r\tau\alpha w^2\sigma^2\right] + \alpha(r\tau w\sigma)^2) \\ &= -\alpha(rw\beta\tau)^2 \exp\left[-r\tau\frac{W}{N}\right] \exp\left\{-r\tau\alpha\left[-\frac{W}{N} + w\beta e - \frac{1}{2}r\tau\alpha w^2\sigma^2\right]\right\} \\ &= -\alpha(rw\beta\tau)^2 \exp\left[-r\tau\frac{W}{N}\right] \exp\left\{-r\tau\alpha\left[-\frac{W}{N} + w\beta e\right] + \frac{3}{2}(r\tau\alpha w\sigma)^2\right\} \end{aligned}$$

The sign of the last term is ambiguous. In the main text we assumed this mixed derivative to be negative. This holds in the example if the variance is sufficiently high so that the insurance effect dominates the redistributive effect. Recall, however, that it was shown above that the desired cutoff result holds in our example independent of the sign of this derivative.

Finally let us decompose the effect of a higher incentive parameter α on the expected marginal revenue of investment into the effects we found in the general case.

$$\begin{aligned}
\frac{\partial RHS}{\partial \alpha} &= rw\beta\tau \exp[-r\tau \frac{W}{N}] \exp \left\{ -r\tau\alpha \left[-\frac{W}{N} + w\beta e - \frac{1}{2}r\tau\alpha w^2\sigma^2 \right] \right\} \\
&\quad + r\alpha w\beta\tau \exp[-r\tau \frac{W}{N}] \exp \{ \} \left\{ -r\tau \left[-\frac{W}{N} + w\beta e - \frac{1}{2}r\tau\alpha w^2\sigma^2 \right] + (r\tau w\sigma)^2\alpha \right\} \\
&= rw\beta\tau \exp[-r\tau \frac{W}{N}] \exp \{ \} \left\{ 1 - r\tau \left[-\frac{W}{N} + w\beta e - \frac{1}{2}r\tau\alpha w^2\sigma^2 \right] + (r\tau\alpha w\sigma)^2 \right\} \\
&= rw\beta\tau \exp[-r\tau \frac{W}{N}] \exp \{ \} \left\{ 1 - r\tau\alpha \left[-\frac{W}{N} + w\beta e \right] + \frac{3}{2}(r\tau\alpha w\sigma)^2 \right\}
\end{aligned}$$

The term in the first line (which also shows up as a 1 in the brackets of the last line) is the positive incentive effect. The second term consists of the redistributive effect $\frac{W}{N} - w\beta e$ and the positive insurance effect $\frac{3}{2}(r\tau\alpha w\sigma)^2$. The general equilibrium effect was neglected. Note that the insurance effect is parameterized in the variance σ^2 .

8 Conclusion

We have developed a general equilibrium stochastic OLG model with intra-generational heterogeneity in human capital and endogenous human capital distribution. In this framework, we analyzed a novel insurance aspect of social security. We suggested that a well designed social security system can act as insurance against the human capital risk of one's children. An optimal social security scheme, however, has to tradeoff insurance provision against incentives for appropriate investment in education. This insurance-incentive aspect was built into the benefit formula of a proposed social security scheme. Our analysis allowed us to interpret real-world social security systems as corner solutions in this insurance-incentive tradeoff. Two assumptions were crucial for our results. First, we assumed non-observability of investment in education. Second, we assumed that the relationship between investment in education and realized human capital (wage) was stochastic.

We have identified four channels through which a change in a social security incentive parameter governing the insurance-incentive tradeoff affects households' investment decisions: an incentive effect, a redistributive effect, a general equilibrium effect and an insurance effect. Under our assumptions we were able to sign the effects and we also derived some results about the overall effect. In particular, the sign of the overall effect depends on the income position of the household. For below average income households, higher investment incentives will indeed induce higher investment. For above average income households this need not be true.

The discussion in this paper has focused on one narrow issue of social security reform. The analysis was carried out in a highly stylized model. Therefore the discussion should not be interpreted as a complete assessment of the suggested intragenerational insurance aspect. Let us point out some shortcomings of our analysis. One problem in implementing a social security benefit formula which conditions on social security contributions of one's children is that the number of children is assumed to be one in our model. In reality, however, some people may not be able to have children. Thus, one would additionally need an insurance against fertility risk, an obviously odd proposal. Second, it is unclear how the timing of the benefits and contributions would be dealt with. As our model is set up in discrete time, life periods are certain and transfers can easily be implemented. In reality, however, there is an additional problem of timing and spacing of births and the implied timing and spacing of the children being in the workforce. The question is then how the children's contributions should be weighted and when they should be considered when calculating retirement benefits of the parents.

Nevertheless we think our analysis highlights an important aspect which was neglected in the literature. The important point we make is that better incentives for investment in education *within* the social security system may be one policy measure to achieve a more favorable long run distribution of human capital endowments and provide appropriate insurance against human capital risk of one's children.

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