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Since when have FOREX markets incorporated EMU into currency pricing? Evidence from four exchange rate series

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DISCUSSION PAPER

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# **Since when have FOREX markets incorporated EMU into currency pricing? Evidence from four exchange rate series**

**Bernd Wilfling**

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## **HWWA DISCUSSION PAPER**

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## Summary

Recent theory on exchange rate dynamics suggests that the mere announcement of regime switching from floating to fixed rates at a given future date triggers a reduction in exchange rate volatility during the interim period. Using a Markov-switching GARCH model this paper estimates the volatility processes of four EMU exchange rate returns *vis-à-vis* the German mark using daily data for the time prior to Stage III of EMU. Statistical inference yields the dates at which financial markets began to incorporate the expected EMU participation of each country into currency pricing. The data exhibits strong econometric evidence for two distinct views concerning the ultimate EMU membership: (1) Finland and France were considered irrefutable EMU members long before any official announcements. (2) At first, the markets did not reckon with the participation of Italy and Portugal for a long time, but then suddenly reversed their assessment more or less at a stroke.

## Zusammenfassung

Neuere Theorien zur Wechselkursdynamik implizieren, daß bereits die bloße Ankündigung eines Regimewechsels von flexiblen zu festen Kursen zu einem vorgegebenen Zukunftsdatum eine Verringerung der Wechselkursvolatilität während der Interimsphase bewirkt. Auf der Basis eines Markov-Switching GARCH Modells schätzt diese Arbeit die Volatilitätsprozesse von vier EWU-Wechselkursrendite-Zeitreihen gegenüber der DM für die Zeit vor der 3. Stufe der Europäischen Währungsunion. Hieraus lassen sich durch statistische Inferenz die Zeitpunkte ermitteln, ab denen die Finanzmärkte begannen, die erwartete EWU-Teilnahme der betreffenden Staaten in ihrer Kursbildung zu berücksichtigen. Die Daten liefern starke ökonometrische Evidenz für zwei qualitativ unterschiedliche Marktbeurteilungen im Hinblick auf die letztendliche EWU-Teilnahme: (1) Finnland und Frankreich wurden bereits weit vor jeglicher offiziellen Ankündigung als definitive EWU-Teilnehmer eingestuft. (2) Zunächst rechneten die Märkte für lange Zeit nicht mit einer Teilnahme Italiens und Portugals, um diese Einschätzung dann jedoch schlagartig zu revidieren.

**JEL classifications:** F31, F33, C51

**Key words:** EMU, exchange rate policy, volatility, regime-switching GARCH models

# 1 Introduction

Following the Maastricht timetable and various decisions taken at several meetings of the European Council, Stage III of the European Monetary Union (EMU) started on January 1, 1999 with a core group of the following 11 countries: Austria, Belgium, Finland, France, Germany, Ireland, Italy, Luxembourg, the Netherlands, Portugal and Spain. An important stipulation accompanying the introduction of the euro was the irreversible fixing of the bilateral EMU exchange rates from that day onwards at their central parities from the European Exchange Rate Mechanism (ERM).

In several recent papers on exchange rate dynamics such a transition from floating to fixed rates is often referred to as a *time-contingent* switch of exchange rate regime (see e.g. Sutherland 1995, De Grauwe et al. 1999).<sup>1</sup> The reason for this classification is obvious: the date of fixing is exogenously given and conveyed to financial markets by a more or less credible announcement of the authorities prior to the switch. The papers mentioned above investigate different aspects of exchange rate dynamics before and during the interim period (i.e. the time between the date at which the authorities announce their aim of future regime-switching and the eventual fixing date).

However, all these theoretical models have one restrictive and unrealistic assumption in common: there is a clear-cut announcement date from which on rational market participants incorporate all information about future regime-switching into current exchange rate valuation. But bearing in mind the political and institutional realities on the road to Stage III of EMU, the *ad-hoc*-determination of such an announcement date is likely to be burdensome. Furthermore, it is very unlikely to find a unique and common date for all EMU currencies marking the counterpart of the announcement date from the theoretical models. This last conjecture seems all the more justified when recalling political debates from the years 1996/97 on the question of which currencies should belong to the first wave of EMU Ins.

It is the aim of this paper to identify (*ex post*) the alternative points in time (or at least preferably short time intervals) at which markets began to incorporate the EMU participation of Finland, France, Italy and Portugal into currency pricing *vis-a-vis* the German mark. Technically, this is achieved by analyzing the volatility structure of exchange rate returns. Following a theoretical time-contingent exchange rate model, the returns should reveal a significant regime shift in conditional variances at the an-

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<sup>1</sup>In the literature, exchange rates under a credible target zone (like the ERM) are often considered as fixed if the band width is sufficiently small. After the speculative turmoil in 1992/93, all ERM bands—except that between the German mark and the Dutch guilder—were widened to  $\pm 15\%$ . This relatively wide range of possible variation allows us to consider the ERM exchange rates as flexible or at least as managed-float rates.



nouncement date, namely from a regime of constantly high volatility to a regime of continuously declining conditional variances. In Section 3 it is shown that this structural change is well captured by an appropriately specified Markov-switching GARCH model. Using these estimation results and efficient filter techniques then leads to the computation of the so called *smoothed probabilities*. These quantities represent the *smoothed inference* about the volatility regime the exchange rate process was in at any arbitrary date from the sampling period, thus providing econometric evidence for currency valuation in foreign exchange markets on the road to Stage III of EMU.

The remainder of this paper is organized as follows. Section 2 briefly summarizes the theoretical exchange rate model and elaborates its main implications on the volatility of exchange rate returns. Section 3 presents the econometric methodology, estimation results, and statistical inference. Section 4 offers some concluding remarks.

## 2 Theoretical model and empirical implications

### 2.1 The theoretical exchange rate model

To elaborate some testable hypotheses on the evolution of exchange rate volatility in the presence of an announced time-contingent regime switch from floating to fixed rates, it is convenient to recall the dynamics during this period. For this, let the authorities announce at date  $t_A$  that they plan to fix the currently floating exchange rate permanently from the future date  $t_S$  onwards at the parity  $\bar{x}$ . As a basis for general exchange rate behaviour, consider the well-known stochastic version of the (continuous-time) monetary flex-price model from recent literature on state-contingent policy shifts (for an overview, see Bertola 1994). In this model, the logarithmic spot rate—measured as the domestic-currency price for foreign exchange—at time  $t$ ,  $x(t)$ , equals the sum of a 'fundamental',  $k(t)$ , and a speculative term proportional to the expected (instantaneous) rate of change in the exchange rate:

$$x(t) = k(t) + \alpha \cdot \frac{E [dx(t)|\phi(t)]}{dt}, \quad \alpha > 0. \quad (1)$$

In Eq. (1),  $E[\cdot|\cdot]$  denotes the expectation operator conditional on the present time- $t$  information set  $\phi(t)$  which includes all information available to rational market participants at time  $t$ . The composite fundamental  $k$  consists of several economic variables such as domestic and foreign money supplies and outputs. More generally,  $k$  can be thought of as a collection of all economic and/or political factors which markets deem to be important for the determination of exchange rates.

Prior to the potential fixing date  $t_S$ ,  $k$  should follow a continuous-time stochastic process. Since the paper analyzes ERM currencies, it is reasonable to consider a stochastic process which allows for central bank interventions that aim at keeping the exchange rate  $x$  near its ERM central parity. According to Eq. (1) a consistent behaviour to achieve this is to prevent the fundamental  $k$  from wandering too far away from the exchange rate target value  $\bar{x}$ .<sup>2</sup> This behaviour will subsequently be modelled by a mean-reverting Ornstein-Uhlenbeck process with stochastic differential

$$dk(t) = \eta \cdot [\bar{x} - k(t)] \cdot dt + \sigma \cdot dw(t), \quad (2)$$

with  $\sigma > 0$  denoting the infinitesimal standard deviation and  $dw(t)$  the increment of a standard Wiener process. The parameter  $\eta \geq 0$  represents the intensity with which the fundamental  $k$  tends to revert towards the parity  $\bar{x}$  after a temporary deviation. It is most intuitive to interpret  $\eta$  as a measure of the willingness and/or the capability of central banks to stabilize the exchange rate  $x$  via the fundamental  $k$  near the target level  $\bar{x}$  by appropriate interventions.

It should be noted that modelling the fundamental  $k$  as an unregulated Ornstein-Uhlenbeck process—i.e. without any further restriction on  $k$  with respect to interventions at the edges of the ERM bands—is a technical simplification. Letting  $k$  evolve according to Eq. (2) rather models a managed-float pre-switch regime than a strict ‘target zone’ system as formalized by Krugman (1991). For a justification of this simplifying view see Svensson (1992, pp. 134).

To derive closed-form solutions of the exchange rate path before and during the interim period, it is convenient to consider the successive time intervals  $[0, t_A)$ ,  $[t_A, t_S)$  and  $[t_S, \infty)$ . For  $t \in [0, t_A)$ , the regime switch has not yet been announced. For simplicity, assume for a moment that agents therefore expect the current managed-float system to hold forever. The (bubble-free) solution of the exchange rate equation (1) is then given by

$$x(t) = \bar{x} + \frac{k(t) - \bar{x}}{1 + \alpha\eta} = \frac{1}{1 + \alpha\eta} \cdot k(t) + \frac{\alpha\eta}{1 + \alpha\eta} \cdot \bar{x}, \quad (3)$$

(for details, cf. Wilfling 2000, pp. 90).

For  $t \in [t_A, t_S)$ , market participants seem to be perfectly informed about all modalities of future exchange rate fixing. Now, it should be taken into account that agents

---

<sup>2</sup>Recall that all EMU currencies were irrevocably fixed at their central ERM parities at January 1, 1999. Therefore, in the above model  $\bar{x}$  denotes both, the ERM parities before the switch as well as the fixing parities during the fixed-rate system.

may be uncertain about strict adherence to the announced fixing date  $t_S$  in that they deem a delay in the regime switch beyond  $t_S$  possible. Generally, there are a variety of ways to model market uncertainty about the punctuality of the regime switch. A fruitful approach is that of Wilfling and Maennig (2001) who assume that market participants—based on their present date- $t$  information set  $\phi(t)$ —associate a specific probability distribution function with the lifetime of the pre-switch managed-float system. Denoting this lifetime by the random variable  $Z$ , the probability that  $Z$  does not exceed the future date  $s > t$  is assumed given by

$$F_Z(s; p, \lambda) \equiv \Pr \{Z \leq s | \phi(t)\} = \begin{cases} 0 & \text{for } s < t_S \\ 1 - p \cdot e^{\lambda(t_S - s)} & \text{for } s \geq t_S, p \in [0, 1], \lambda \geq 0 \end{cases} \cdot (4)$$

The parameters  $p$  and  $\lambda$  in the distribution function (4) have neat economic interpretations. First,  $p$  represents the probability of the switch not occurring punctually at  $t_S$ . In other words,  $1 - p$  is the (unconditional) probability which agents assign to the event that the switch takes place exactly at  $t_S$ . Second, conditional on a delayed regime switch,  $\lambda$  is the (constant) proportional hazard rate that the regime switch takes place in the infinitesimal time period following any date  $s$  beyond  $t_S$ . It is important for further considerations to note two special cases included in (4):

- (a) For  $p = 0$  ( $\lambda$  arbitrary), the fixing date  $t_S$  is considered fully credible by the market. Formally, the same is true for  $p > 0, \lambda \rightarrow \infty$ . In this latter case the regime switch is not expected to take place at  $t_S$  with probability 1, but the delay is considered to be infinitesimally short.
- (b) For  $(p, \lambda) = (1, 0)$ , agents believe that the regime switch will never take place.

Taking into account the uncertainty structure (4) and using the same forward integration technique as in Wilfling and Maennig (2001), it is straightforward to derive the equilibrium exchange rate path for  $t \in [t_A, t_S)$ :

$$x(t) = \bar{x} + \frac{k(t) - \bar{x}}{1 + \alpha\eta} \cdot \left[ 1 - \left( 1 - \frac{p \cdot (1 + \alpha\eta)}{1 + \alpha\eta + \alpha\lambda} \right) \cdot e^{(1 + \alpha\eta) \cdot (t - t_S) / \alpha} \right]. \quad (5)$$

The exact form of the equilibrium exchange rate for the time after  $t_S$  crucially hinges on the specific policy action taken at the potential fixing date  $t_S$ . The simplest scenario, correctly reflecting the entrance into Stage III of EMU, is that the authorities—in spite of potential market uncertainty during the interim period about the exact timing of the regime switch—indeed fix the spot rate from  $t_S$  onwards at the parity  $\bar{x}$ . Under

this setup, it clearly follows that

$$x(t) = \bar{x} \tag{6}$$

for all  $t \geq t_S$ .

## 2.2 Empirical implications

The equilibrium exchange rate path consisting of the sequences (3), (5) and (6) yields at least three implications which should be observable empirically:

- (a) the effects on the exchange rate induced by the announcement at  $t_A$  of future regime switching at  $t_S$ ,
- (b) the 'smooth' exchange rate convergence towards the fixing parity  $\bar{x}$  at the end of the interim period  $[t_A, t_S)$  and
- (c) the evolution of the (conditional) variances of exchange rate returns during the interim period.

First, let us address the announcement effects at date  $t_A$ . Before tracking down an explicit formula of the exchange rate jump, it is important to note the following: The above model assumes that, prior to  $t_A$ , agents are not aware that an announcement will be made. Hence, the announcement itself is news inducing an exchange rate reaction at  $t_A$ . With reference to Stage III of EMU, such a clear-cut and exogenously given announcement date did not exist and Section 3 will take up this point at greater detail.

Formally, the height  $H$  of the jump at  $t_A$  may be defined from the Eqs. (3) and (5) as

$$\begin{aligned} H &\equiv \lim_{t \downarrow t_A} x(t) - \lim_{t \uparrow t_A} x(t) \\ &= \left[ 1 - \frac{p \cdot (1 + \alpha\eta)}{1 + \alpha\eta + \alpha\lambda} \right] \cdot e^{-[(1+\alpha\eta)/\alpha] \cdot u} \cdot v, \end{aligned} \tag{7}$$

where

$$u \equiv t_S - t_A \quad \text{and} \quad v \equiv \bar{x} - \lim_{t \uparrow t_A} x(t)$$

denote the most intuitive components of the jump: the length of the interim period  $[t_A, t_S)$ , and the distance between the announced fixing parity  $\bar{x}$  and the spot rate which is to prevail at  $t_A$  along the managed-float path (3). Since  $\alpha > 0$ ,  $p \in [0, 1]$  and  $\eta, \lambda \geq 0$ , it follows that—except for one case—the jump direction of  $H$  coincides with

the sign of  $v$ .<sup>3</sup> It is easy to check from Eq. (7) that the absolute height  $|H|$  is *ceteris paribus* decreasing in  $p$  and  $u$  and increasing in  $\lambda$  and  $|v|$ .

Second, let us turn to the convergence of the exchange rate towards the fixing parity  $\bar{x}$  at the end of the interim period. Formally, the path (5) yields

$$\lim_{t \uparrow t_S} x(t) = \bar{x} + p \cdot \frac{\lim_{t \uparrow t_S} k(t) - \bar{x}}{1 + \alpha\eta + \alpha\lambda}. \quad (8)$$

It is evident from Eq. (8) that the exchange rate will approach the fixing parity  $\bar{x}$  'smoothly', if the regime switch at  $t_S$  is considered fully credible by the market (i.e. for  $p = 0$  or  $\lambda \rightarrow \infty$ ):

$$\lim_{t \uparrow t_S} x(t) = \bar{x}.^4$$

As mentioned in Section 2.1, the uncertainty parameters  $p$  and  $\lambda$  are set due to the agents' current knowledge represented by the time- $t$  information set  $\phi(t)$ . Consequently, from a theoretical point of view, both parameters are principally free to vary during the interim period, if market participants revise—for whatever reason—their assessment of the punctuality of the regime switch. In conjunction with (8) the following result obtains: The only reason for the exchange rate  $x$  not to converge towards the fixing parity  $\bar{x}$  is that market uncertainty about the punctuality of the switch will last until  $t_S$  (i.e.  $p \neq 0, \lambda < \infty$  at all dates  $t < t_S$ ). Clearly, this scenario is very unrealistic for most real world situations in which the regime switch is finally implemented punctually at  $t_S$ . In this case agents will anticipate the punctuality of the switch in advance and consequently set  $p = 0$  and/or  $\lambda \rightarrow \infty$  early enough to ensure an arbitrage-free exchange rate convergence towards the fixing parity.

Finally, let us address the variability of exchange rates during the interim period. For practical purposes it is most convenient to draw on the concept of the infinitesimal variance of  $x$  denoted by  $\nu_{\{x\}}^2(x(t), t)$ . This function, explicitly depending on the current exchange rate  $x(t)$  and the time index  $t$ , adequately approximates the conditional variance of the increment in the exchange rate  $x$  accrued over a sufficiently small time interval of length  $h > 0$ :<sup>5</sup>

$$\text{Var}[x(t+h) - x(t)|\phi(t)] = \nu_{\{x\}}^2(x(t), t) \cdot h + o(h), \quad (9)$$

---

<sup>3</sup>Only if  $(p, \lambda) = (1, 0)$ , i.e. market participants assume that the regime switch will never take place, the jump  $H$  equals 0 and is then independent of  $v$ .

<sup>4</sup>To be mathematically precise: The exchange rate  $x$  converges towards  $\bar{x}$  *with probability 1*. Throughout this paper, all mathematical limits of the stochastic processes  $k$  and  $x$  draw on the concept of 'almost sure convergence', i.e. convergence with probability 1.

<sup>5</sup>See, among others, Karlin and Taylor (1981, pp. 159).

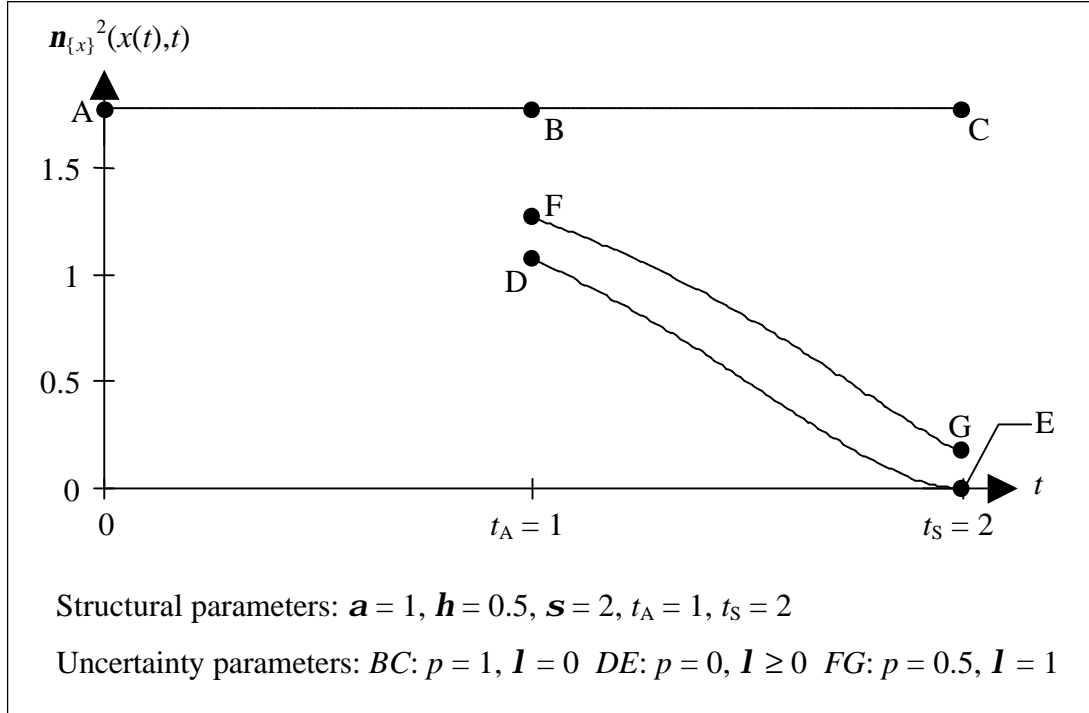


Figure 1: Instantaneous variance-paths of the logarithmic exchange rate

where  $o(h)$  is a remainder term of order smaller than  $h$ .

Since the spot rate  $x$  from the Eqs. (3) and (5) is a linear function (with time-varying coefficients) of the stochastic fundamental  $k$ , which itself evolves over time according to the Ornstein-Uhlenbeck process (2), Ito's lemma yields the following instantaneous variances of the exchange rate:

$$\nu_{\{x\}}^2(x(t), t) = \left[ \frac{\sigma}{1 + \alpha\eta} \right]^2 \quad \text{for } t < t_A, \quad (10)$$

and

$$\nu_{\{x\}}^2(x(t), t) = \left[ \frac{\sigma}{1 + \alpha\eta} \right]^2 \cdot \left[ 1 - \left( 1 - \frac{p \cdot (1 + \alpha\eta)}{1 + \alpha\eta + \alpha\lambda} \right) \cdot e^{(1 + \alpha\eta) \cdot (t - t_S) / \alpha} \right]^2 \quad (11)$$

for  $t \in [t_A, t_S)$ .

Figure 1 displays some instantaneous variance-paths of the forms (10) and (11) over time. For further interpretations recall that in the exchange-rate Eq. (1)  $x$  represents the logarithmic spot rate, i.e.  $x(t) \equiv \ln[X(t)]$  with  $X$  denoting the non-logarithmic, nominal exchange rate. In conjunction with Eq. (9), the infinitesimal variance  $\nu_{\{x\}}^2(x(t), t)$  approximates the conditional variance of changes in the logarithmic rates over a time interval of length  $h = 1$ . In other words,  $\nu_{\{x\}}^2(x(t), t)$  approximates

the conditional variance of one-step-ahead exchange rate returns with an error of order 1:

$$\nu_{\{x\}}^2(x(t), t) \approx \text{Var} \{ \ln[X(t+1)] - \ln[X(t)] | \phi(t) \}. \quad (12)$$

In this sense, the segment  $AB$  represents the (constant) instantaneous variance-path (10) while the segments  $BC$ ,  $DE$  and  $FG$  represent variance-paths from Eq. (11) under alternative uncertainty scenarios.

It is obvious from Figure 1 that—except for  $(p, \lambda) = (1, 0)$ —the variance-path (11) lies strictly below the constant path (10). Formally, this is evident from a direct comparison of (10) and (11) and the fact that

$$\left[ 1 - \left( 1 - \frac{p \cdot (1 + \alpha\eta)}{1 + \alpha\eta + \alpha\lambda} \right) \cdot e^{(1+\alpha\eta) \cdot (t-t_S)/\alpha} \right]^2 < 1 \quad (13)$$

for all  $(p, \lambda) \neq (1, 0)$ . Moreover, the following relations provide deeper insights into the qualitative nature of instantaneous variances (regarded as proxies of the conditional variances of one-step-ahead exchange rate returns) during the interim period:

$$\frac{\partial \nu_{\{x\}}^2(x(t), t)}{\partial t} < 0 \quad \text{for all } t \in [t_A, t_S), (p, \lambda) \neq (1, 0), \quad (14)$$

$$\lim_{t \uparrow t_S} \nu_{\{x\}}^2(x(t), t) = \left[ \frac{\sigma \cdot p}{1 + \alpha\eta + \alpha\lambda} \right]^2, \quad (15)$$

$$\frac{\partial \nu_{\{x\}}^2(x(t), t)}{\partial p} > 0 \quad \text{for all } t \in [t_A, t_S), p \in [0, 1], \lambda \geq 0, \quad (16)$$

$$\frac{\partial \nu_{\{x\}}^2(x(t), t)}{\partial \lambda} < 0 \quad \text{for all } t \in [t_A, t_S), p \in (0, 1], \lambda \geq 0. \quad (17)$$

Due to the Eqs. (14) and (15) the variances are strictly decreasing during the interim period and, for  $p = 0$ , vanish completely as  $t$  tends to  $t_S$  (i.e. if agents are absolutely convinced of the punctuality of the regime switch at  $t_S$ ). The precise impact of variations in the uncertainty parameters  $p$  and  $\lambda$  on the variances are obvious from the Eqs. (16) and (17). Increases (decreases) in  $p$  and/or decreases (increases) in  $\lambda$  lead to upward (downward) shifts of the variance-paths.

The above relations give rise to a specific volatility pattern of exchange rate returns before and during the interim period. The conditional variances  $\text{Var}[\ln(X(t+1)) - \ln(X(t)) | \phi(t)]$  should evolve over time within two successive regimes which are separated from each other by the announcement date  $t_A$ . For  $t \in [0, t_A)$  the variances should be high and fluctuate around a constant level. During the interim period  $[t_A, t_S)$  the variances should be uniformly lower and monotone decreasing over time. And finally

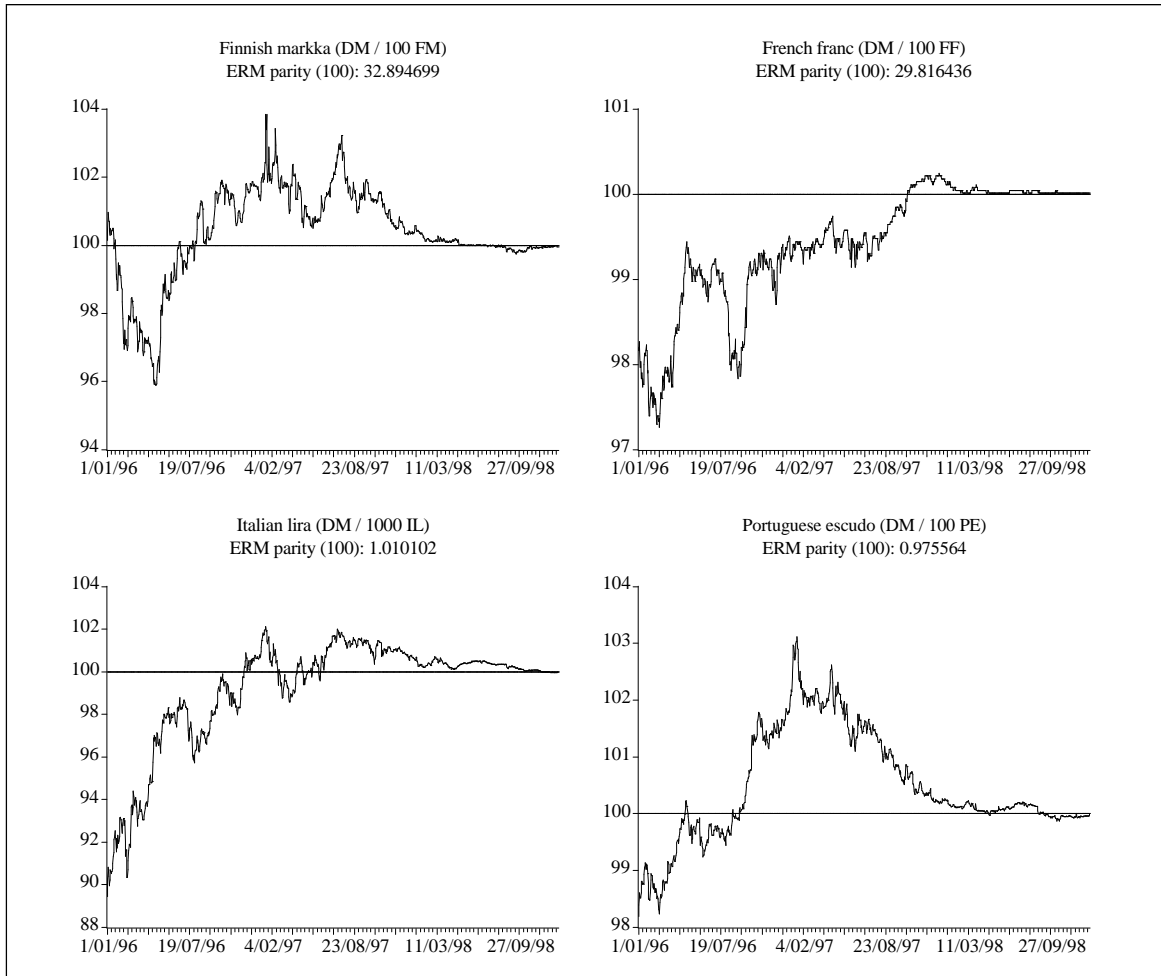


Figure 2: Nominal EMU exchange rates

it follows from the Eqs. (16) and (17), that the extent of the difference in volatility between the two regimes is highest under the uncertainty scenario  $p = 0$  and/or  $\lambda \rightarrow \infty$ , i.e. if agents consider the fixing date  $t_S$  fully credible.

### 3 Econometric analysis

#### 3.1 The data

The data used in this study are daily spot rates of four EMU currencies—the Finnish markka (FM), the French franc (FF), the Italian lira (IL), the Portuguese escudo (PE)—*vis-a-vis* the German mark (DM) covering the period from January 1, 1996 to December 31, 1998.<sup>6</sup> The rates—expressed as DM-prices of foreign currency—were

<sup>6</sup>In principle, any other EMU currency can be analyzed. The subsequent investigation is restricted to the four currencies for a practical reason: As is shown in Section 3.3, the Markov-switching GARCH



Table 1: Summary statistics of the nominal EMU exchange rate series

Series	Quotation	Mean	Std. Dev.	Skewness	Kurtosis	Dev. in (% <sup>a</sup> )
FM	DM / 100 FM	33.0134	0.4398	-0.9055	4.2902	0.0478
FF	DM / 100 FF	29.6476	0.2113	-1.1363	3.5491	-0.0119
IL	DM / 1000 IL	1.0020	0.0257	-1.8230	5.6794	0.0002
PE	DM / 100 PE	0.9802	0.0094	0.3272	2.5931	0.0065

<sup>a</sup>) Percentage deviation of last-day (December 31, 1998) exchange rate from central parity ( $100 \cdot \ln[\text{central parity}/X_{1096}]$ ).

provided by the *Vereins- und Westbank* (Hamburg) and are daily averages of interbank rates recorded at seven days per week (1096 observations per currency).

The data set was checked in a number of ways, including testing for outliers as well as visual inspection of other anomalies. Additionally, the data was compared with alternative data sets from *Reuter's Wirtschaftsdienst* and corresponding time series in the *Frankfurter Allgemeine Zeitung*. All these check ups did not give rise to any serious doubts about the validity of the data. Consequently, all 1096 observations per currency were used in the analysis.

Figure 2 depicts the four nominal EMU exchange rates along with the quotations used in the subsequent analysis and their ERM central parities which—for comparative reasons—were scaled to 100. Table 1 displays some summary statistics of the series. Except for the Portuguese escudo the skewness statistics are uniformly negative and the distributions are highly kurtotic. These findings are consistent with observations from earlier empirical studies on ERM exchange rates (see e.g. Neely, 1999).

The last column of Table 1 contains the percentage deviations of the last-day exchange rates (obs.  $X_{1096}$  from December 31, 1998) from their central (fixing) parities. All deviations are much lower than 1% with the Finnish markka revealing the highest absolute discrepancy of about 0.05 %. It is very likely that these deviations are due to rounding or to the condensation of high-frequency rates to daily averages.

Figure 2 clearly indicates that for all currencies the convergence towards the fixing parities began quite early. At first glance, this empirical phenomenon seems to be in line with the theoretical model from Section 2 which predicts exchange rate convergence towards  $\bar{x}$  for  $t \rightarrow t_S$  if agents are absolutely convinced of the punctuality of the regime switch. Nevertheless, there may be different economic reasons for the smooth approach to the fixing parity. To be more explicit, recall the exchange rate path (5) during the interim period with  $p = 0$ :

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models have to be estimated by numerical methods. Except for these four exchange rates, the applied methods did not converge or produced implausible results. Partial results for the other EMU currencies are available upon request.

- (a) The structure of the path (5) shows that the convergence towards  $\bar{x}$  for  $t \rightarrow t_S$  will be achieved, if the parameter  $\eta$  (representing the intervention activities of the monetary authorities) remains constant during the interim period. In this case the convergence is due to market's removal of arbitrage opportunities.
- (b) Eq. (5) may equivalently be written as

$$x(t) - \bar{x} = \frac{k(t) - \bar{x}}{1 + \alpha\eta} \cdot \left[ 1 - e^{(1+\alpha\eta) \cdot (t-t_S)/\alpha} \right].$$

It follows that at every date  $t \in [t_A, t_S)$  the rate  $x(t)$  may be pushed arbitrarily close towards  $\bar{x}$  by an appropriately chosen value of  $\eta$ , in other words, by a suitable degree of central bank intervention.

Without further empirical investigation it is impossible to specify *ex post* the exact reason for the convergence shown in Figure 2.

Next, let us turn to the following question: Is it possible to identify an exogenous announcement date matching the effects on the exchange rate induced by the jump  $H$  from Eq. (7)? Two such potential dates were the meetings of the European Council in Mondorf (Luxembourg) on September 13/14, 1997, and in Brussels on May 2/3, 1998, respectively. At the Mondorf-meeting the Council settled the procedure of exchange rate fixing between the EMU Ins to be used on January 1, 1999, but did not decide upon the 'In countries' themselves.<sup>7</sup> This latter decision was officially taken at the meeting in Brussels in early May 1998. However, an analysis of the corresponding exchange rate jumps does not exhibit any statistical significance that one these dates may be viewed as the counterpart of the theoretical announcement date  $t_A$ . Most of the jumps go into the 'wrong' direction and the jump heights do not differ significantly from other daily exchange rate jumps taken from adequately chosen time intervals around these dates.<sup>8</sup>

As a consequence of the failure to determine announcement dates  $t_A$  by means of political or institutional decisions, the volatility of exchange rate returns will now be used to identify *ex post* the separation date between the alternative volatility regimes.

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<sup>7</sup>At this meeting, the so called fixed-conversion rule was chosen, i.e. the fixing of exchange rates at a preannounced parity  $\bar{x}$  as formalized in Section 2. Although nothing was said about explicit values of the parities  $\bar{x}$ , the later use of the ERM central parities was already broadly expected by financial markets at that time.

<sup>8</sup>It is important to note that a 'small' jump at  $t_A$  may be consistent with the theoretical model from Section 2. For example, if the spot rate  $x$  is very close to  $\bar{x}$  (or even equals  $\bar{x}$ ) shortly before  $t_A$ , it follows from (7) that the jump height at  $t_A$  will be small (or even equal zero). Details of the above-mentioned analysis are available upon request.

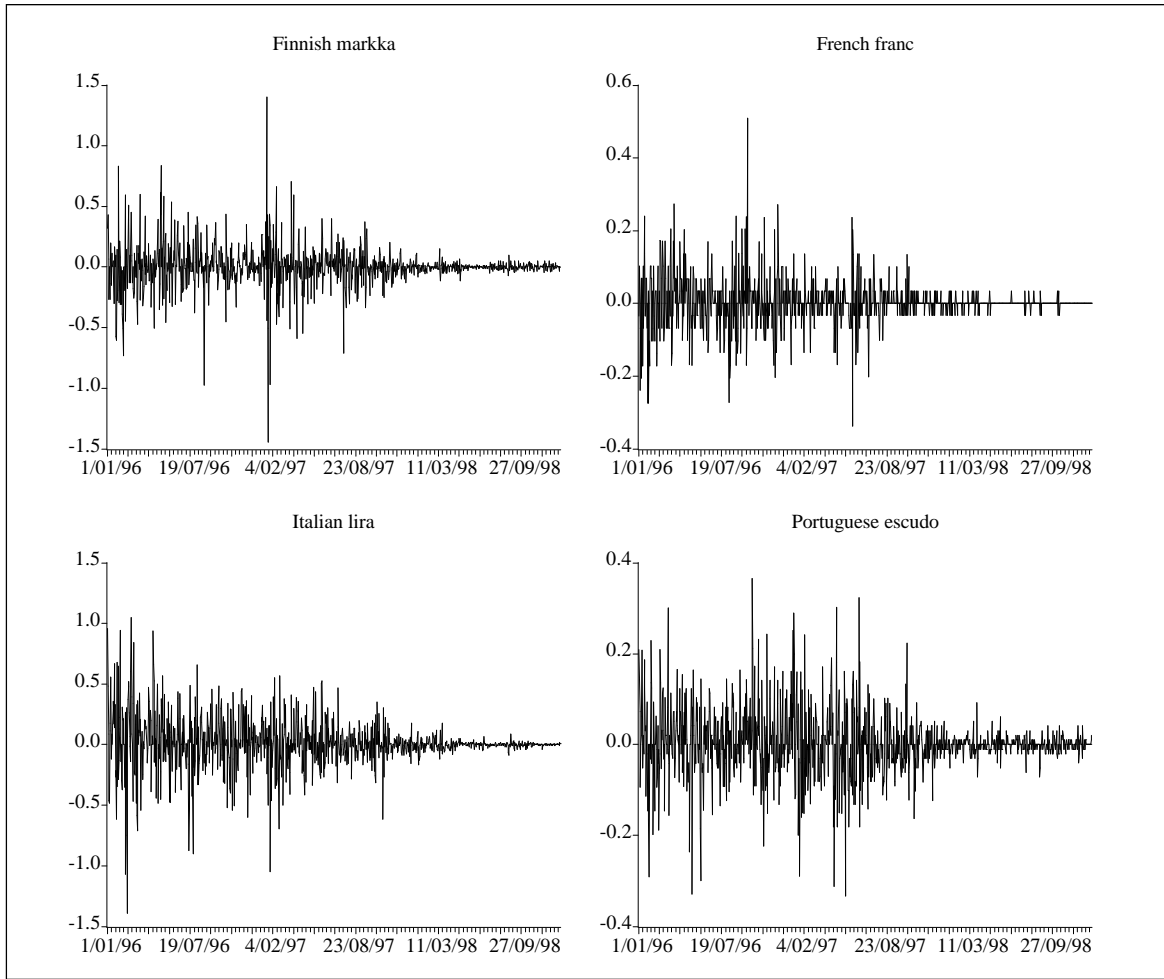


Figure 3: EMU exchange rate returns

Formally, the exchange rate returns may be defined as

$$R_t \equiv 100 \cdot [\ln(X_t) - \ln(X_{t-1})]. \quad (18)$$

Figure 3 depicts the heteroscedastic nature of the four return series. The variances tend to decline at the end of the interim period as predicted by the theoretical model from Section 2. Although this decline is clearly visible, the Figures 2 and 3 do not give any hint as to when the shift between the two volatility regimes occurred explicitly. The question of when FOREX markets began to incorporate Stage III of EMU into currency pricing will be answered in the following sections by more sophisticated econometric methods. But before applying these techniques, it is necessary to test for unit roots in the returns.

Table 2: ADF- ( $\tau_\mu$ ) and PP- tests for a unit root in exchange rate returns

Series	ADF-test			PP-test		
	$\hat{b}^a)$	Std. Err. <sup>b)</sup>	ADF-stat. <sup>c)</sup>	$\hat{b}^a)$	Std. Err. <sup>b)</sup>	PP-stat. <sup>c)</sup>
FM	-1.102	0.059	-18.715[3]***	-0.958	0.030	-31.722***
FF	-0.965	0.030	-31.898[0]***	-0.965	0.030	-31.878***
IL	-0.962	0.040	-23.889[1]***	-0.901	0.030	-30.160***
PE	-1.044	0.081	-12.937[7]***	-0.891	0.030	-29.635***

a) Estimate of parameter  $b$  from (19).

b) Standard error of estimator  $\hat{b}$ .

c) \*, \*\*, \*\*\* denote significance at 10%, 5% and 1% levels using MacKinnon's (1991) critical values. Number of lags used for ADF-tests are given in squared brackets.

To test the null hypothesis of a unit root consider the following regression equation:

$$\Delta R_t = a_0 + a_1 \cdot t + b \cdot R_{t-1} + \sum_{j=1}^s c_j \cdot \Delta R_{t-j} + \epsilon_t, \quad (19)$$

where  $\Delta$  denotes the difference operator,  $a_0, a_1, b, c_1 \dots c_s$  represent constant parameters while  $t$  denotes a time trend and  $\epsilon_t$  a white noise error process. Testing for a unit root in exchange rate returns by the well-known Augmented Dickey-Fuller test (ADF-test, see Dickey and Fuller 1979, 1981) is then equivalent to testing the hypothesis  $b = 0$  in (19). For this, the standard  $t$ -value is used as the ADF-test statistic which has to be compared to the critical values of MacKinnon (1991).

The statistical performance of ADF-tests crucially hinges on two specification problems. The first concerns the inclusion of the constant  $a_0$  and/or the time trend  $t$  in Eq. (19). This leads to the two respective notations  $\tau_\mu$  for the ADF-test, if only  $a_0$  is included in Eq. (19), and  $\tau_\tau$ , if both,  $a_0$  and the time trend  $t$ , are included (cf. Banerjee et al. 1993, Chapter 4). Although there exist explicit strategies for handling this specification problem (see e.g. Dolado et al. 1990), it is common in the empirical literature to use the  $\tau_\mu$ -test for data from exchange rate bands like the ERM. This is usually justified by the assumption that exchange rates in bands should be stationary around the central parity so that the deterministic time trend  $t$  may be omitted (see Anthony and MacDonald 1998). In this analysis both,  $\tau_\tau$ - and  $\tau_\mu$ -tests have been applied to the EMU returns, although Table 2 only reports the  $\tau_\mu$ -results.<sup>9</sup>

The second specification problem affecting the goodness of ADF-tests concerns the number  $s$  of lagged differences  $\Delta R_{t-1}, \dots, \Delta R_{t-s}$  in Eq. (19). For the ADF-tests from Table 2 the number  $s$  of lagged differences was chosen by a procedure described in Campbell and Perron (1991, pp. 155). According to Table 2, the ADF-tests reject the

<sup>9</sup>The  $\tau_\tau$ -tests yield exactly the same statistical conclusions.

null hypothesis of a unit root for all series at the 1% level. There is, however, one further aspect which still needs some attention. A critical assumption of the ADF-test is that the errors  $\{\epsilon_t, t = 1, \dots, T\}$  are independent and have the same variance for all  $t$ . While the Ljung-Box-Q-tests do not reveal any significant autocorrelation among the estimated residuals  $\hat{\epsilon}_t$ , there is clear evidence of heteroscedasticity (declining variances of  $\hat{\epsilon}_t$ ). To circumvent this problem, Phillipps-Perron-tests (PP-tests) were performed.<sup>10</sup> These tests, also based on the regression (19), allow for autocorrelation and/or heteroscedasticity in the distribution of the error process (see Hamilton 1994, pp. 506). In accordance with the ADF-tests, the PP-tests also reject the null hypothesis of a unit root for all series. Consequently, the EMU exchange rate returns do not reveal any statistical significance of a stochastic trend and will be used in the autoregressive models of the following sections without further differencing.

### 3.2 A conventional GARCH model

As outlined in Section 3.1, the determination of an exact announcement date which separates the two volatility regimes of exchange rate returns prior to Stage III of EMU turns out to be difficult. From a political perspective it rather seems justified to ask whether the separation date  $t_A$  necessarily lies within the data range from January 1, 1996 to December 31, 1998. The reason for this question is that the exact scheduling of EMU (and in particular the begin of Stage III on January 1, 1999) had already been suggested by the Maastricht Treaty in December 1991, so that—in a strict sense—this early date may be viewed as the announcement date.

Assuming the entire data range as part of the interim period suggests the use of conventional GARCH models to recursively estimate the process of conditional variances  $\{\text{Var}[R_t|\phi_{t-1}], t = 1, \dots, T\}$ .<sup>11</sup> For this, let the returns  $R_t$  from Eq. (18) be modelled as

$$R_t = \boldsymbol{\varphi}_{t-1}^\top \mathbf{a} + \epsilon_t, \quad t = 1, \dots, T. \quad (20)$$

In Eq. (20)  $\boldsymbol{\varphi}_{t-1}$  denotes a  $(q \times 1)$  vector of explanatory variables whose values are included in the information set  $\phi_{t-1}$  and which may include lagged values of the returns  $R_t$ .  $\mathbf{a}$  is a  $(q \times 1)$  vector of unknown parameters. The disturbance  $\epsilon_t$  is said to follow a GARCH( $u, v$ ) process, if the distribution of  $\epsilon_t$  conditional upon  $\phi_{t-1}$  is normal and given by

$$\epsilon_t|\phi_{t-1} \sim N(0, h_t) \quad (21)$$

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<sup>10</sup>See Phillipps and Perron (1988)

<sup>11</sup>For an introduction and early overview of GARCH models see Bollerslev et al. (1992). To be in line with the standard GARCH literature, the conditional variances of one-step-ahead future returns will subsequently be denoted by  $\text{Var}[R_t|\phi_{t-1}]$  rather than by  $\text{Var}[R_{t+1}|\phi_t]$ .

with

$$h_t = b_0 + \sum_{i=1}^v b_i \cdot \epsilon_{t-i}^2 + \sum_{i=1}^u c_i \cdot h_{t-i}, \quad (22)$$

where  $u, v \geq 0$  represent the order of the GARCH process and the parameters  $b_i, i = 0, \dots, v$  and  $c_i, i = 1, \dots, u$  have to be chosen such that the corresponding variances  $h_t$  are positive.

The first practical problem in modelling exchange rate returns is to specify the conditional mean  $\varphi_{t-1}^\top \mathbf{a}$  in Eq. (20). In many financial applications the mean is modelled by an appropriate autoregressive pattern (AR processes). Here,  $\varphi_{t-1}^\top \mathbf{a}$  will be represented by an AR(1) process, i.e.

$$R_t = a_0 + a_1 \cdot R_{t-1} + \epsilon_t, \quad t = 1, \dots, T. \quad (23)$$

The use of this parsimonious AR(1) scheme is twofold. First, the inclusion of higher autoregressive and/or additional moving-average components in the mean specification did not improve statistical results significantly. Second, due to their complicated probabilistic nature, the Markov-switching GARCH models in the next subsection are difficult to estimate for highly parametrized mean specifications. For comparative reasons it therefore seems appropriate to use the same parsimonious AR(1) scheme in both models.

Next, the orders  $u$  and  $v$  of the GARCH process (21) and (22) have to be specified. In many empirical studies a parsimonious GARCH(1,1)-specification has been applied successfully.<sup>12</sup> The use of this structure reduces (22) to the form

$$h_t = b_0 + b_1 \cdot \epsilon_{t-1}^2 + b_2 \cdot h_{t-1}, \quad (24)$$

where, for notational convenience, the parameter  $c_1$  from (22) has been replaced by  $b_2$ .

Table 3 reports the estimation results of the AR(1)-GARCH(1,1) specifications along with some diagnostic statistics. The parameters were estimated by (quasi) maximum likelihood methods using the Berndt, Hall, Hall, and Hausman (1974) algorithm as implemented in the statistical software package Econometric Views. Heteroscedasticity-consistent standard errors were used to compute  $t$ -statistics and corresponding  $p$ -values (see Bollerslev and Wooldridge 1992).

The estimates reveal that for the returns of the French franc and the Finnish markka none of the AR parameters  $a_0$  and  $a_1$  reach statistical significance while at least  $a_1$  is statistically significant for the Italian lira and the Portuguese escudo at 1% and 10%

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<sup>12</sup>For theoretical arguments in favour of a simple GARCH(1,1)-specification see Bollerslev et al. (1992, p. 10) and the literature cited there.

Table 3: Parameter estimates and related statistics for AR(1)-GARCH(1,1) models

Parameter/ Statistic	Estimates <sup>a)</sup>			
	FM	FF	IL	PE
$a_0$	-0.0005 (-0.3779)	0.0003 (0.5658)	-0.0014 (-1.2610)	-0.0002 (-0.1651)
$a_1$	0.0290 (0.7871)	-0.0443 (-0.9697)	0.1018*** (2.9510)	0.0587* (1.6988)
$b_0$	$6.5 \times 10^{-6}$ (0.6932)	$2.3 \times 10^{-6}$ (1.1034)	$2.6 \times 10^{-6}$ (0.7781)	$3.7 \times 10^{-6}$ (0.8945)
$b_1$	0.1124** (2.5308)	0.0952*** (3.6870)	0.1128*** (4.7284)	0.0612*** (4.0922)
$b_2$	0.9042*** (32.2374)	0.9082*** (43.1640)	0.8969*** (49.4416)	0.9379*** (77.7520)
Log-Likelihood	1012.3020	2132.4550	946.7104	1754.2840
$LB_1^{2b)}$	0.0047 (0.9453)	7.7829*** (0.0053)	1.6467 (0.1994)	2.4653 (0.1164)
$LB_2^2$	2.7468 (0.2532)	8.3165** (0.0156)	1.6481 (0.4387)	3.4800 (0.1755)
$LB_3^2$	2.8306 (0.4185)	9.7288** (0.0210)	2.3393 (0.5050)	5.4563 (0.1413)
$LB_5^2$	3.4497 (0.6310)	10.6100* (0.0597)	3.1606 (0.6752)	6.3771 (0.2712)
$LB_{10}^2$	5.9517 (0.8193)	12.1960 (0.2722)	8.3730 (0.5925)	10.1170 (0.4303)
$LB_{15}^2$	6.6797 (0.9659)	15.5260 (0.4142)	9.9043 (0.8257)	16.9090 (0.3243)

<sup>a)</sup> Estimates for parameters from the Eqs. (23), (21) and (24). \*, \*\*, \*\*\* denote significance at 10%, 5% and 1% levels.  $t$ -statistics are based on heteroscedasticity-consistent standard errors.

<sup>b)</sup>  $LB_i^2$  denotes the Ljung-Box-Q-statistic for serial correlation of the squared standardized residuals out to lag  $i$ .  $p$ -values are in parentheses.

levels, respectively. The GARCH parameters  $b_1$  and  $b_2$  are highly significant for all currencies.

On the whole, the results from Table 3 give two concrete hints that a structural break in the evolution of the variances indeed occurred during the sampling period. The first indicator refers to the sums of the estimated GARCH parameters  $\hat{b}_1 + \hat{b}_2$  which equal 1.0166 for the Finnish markka, 1.0034 for the French franc, 1.0097 for the Italian lira and 0.9991 for the Portuguese escudo. For all currencies these sums are greater or at least very close to unity. This phenomenon, known as 'persistence in volatility', is typical of financial time series (cf. Bollerslev et al. 1992, pp. 14) and, according to Gray (1996b, p. 31), may arise from a misspecified model neglecting structural breaks.

A second indicator of a structural break in the volatility process refers to the squared standardized residuals. If the GARCH models were correctly specified, the (non-squared) standardized residuals should be mean zero and variance one series. Moreover, the standardized residuals should be independently distributed implying independent and hence uncorrelated squared standardized residuals. The lower part of Table 3 reveals significant autocorrelation in the squared residuals for the French franc. One possibility of removing this serial correlation is to use a more complex mean specification. But Gray (1996b, p. 43) shows that this serial correlation may also be due to the neglect of a structural break.

Obviously, there is some statistical evidence against a simple AR(1)-GARCH(1,1) specification of EMU exchange rate returns over the whole sampling period. The results rather point at the existence of two alternative volatility regimes within this period. These regimes will now be identified by Markov-switching GARCH models.

### 3.3 The Markov-switching GARCH model

Markov-switching (or regime-switching) models, which allow for endogenous specifications of stochastic regime shifts, were popularized at the end of the 80s (see e.g. Hamilton 1990) and have mainly been used thereafter for modelling and predicting volatility of interest rates (e.g. Cai 1994, Hamilton and Susmel 1994, Dewachter 1996). The idea of a univariate regime-switching model is that the data generating process of the variable of interest is affected by a non-observable random variable  $S_t$  representing the state the data generating process is in at time  $t$ . For the purpose of this paper, assume the two distinct regimes 1 and 2 at any point in time so that either  $S_t = 1$  or  $S_t = 2$  for all  $t = 0, \dots, T$ . Regime 1 is to represent the state in which market participants have not yet incorporated Stage III of EMU into currency pricing so that exchange rate returns should be in the high-volatility regime. By contrast, Regime 2 is to characterize the situation in which agents already anticipate future exchange rate fixing so that returns belong to the low-volatility regime.

To formalize assume as in Gray (1996b) that each parameter specifying the conditional mean or the conditional variance of the return  $R_t$  may take on two distinct values depending on the regime indicator  $S_t = i, i = 1, 2$ . Denoting the mean and the variance in Regime  $i$  by  $\mu_{it}$  and  $h_{it}$ , respectively, and further assuming normality in each regime, the conditional distribution of the return may be represented as a mixture



of two distributions:

$$R_t|\phi_{t-1} \sim \begin{cases} N(\mu_{1t}, h_{1t}) & \text{with probability } p_{1t} \\ N(\mu_{2t}, h_{2t}) & \text{with probability } (1 - p_{1t}) \end{cases}, \quad (25)$$

with  $p_{1t} \equiv \Pr\{S_t = 1|\phi_{t-1}\}$  denoting the so called *ex-ante* probability of being in Regime 1 at time  $t$ .

On the analogy of the conventional GARCH model it is convenient to assume a parsimonious AR(1) process for the conditional mean of the returns in each regime, i.e.

$$\mu_{it} = a_{0i} + a_{1i} \cdot R_{t-1} \quad \text{for } i = 1, 2. \quad (26)$$

While the specification (26) is straightforward, the explicit modelling of the conditional variance process is slightly more problematic. The reason lies in a phenomenon known as path dependence which, if not carefully handled, may entail severe estimation problems (see Cai 1994, Hamilton and Susmel 1994). Gray (1996b) solves this problem by using the fact that the returns follow a mixture of distributions with time-varying coefficients. From Eq. (25) the variance of the returns at time  $t$  is given by

$$\begin{aligned} h_t &= E[R_t^2|\phi_{t-1}] - \{E[R_t|\phi_{t-1}]\}^2 \\ &= p_{1t} \cdot (\mu_{1t}^2 + h_{1t}) + (1 - p_{1t}) \cdot (\mu_{2t}^2 + h_{2t}) - [p_{1t} \cdot \mu_{1t} + (1 - p_{1t}) \cdot \mu_{2t}]^2. \end{aligned} \quad (27)$$

The variance  $h_t$  represents an aggregate of conditional variances from both regimes and can now be used to specify the conditional variances  $h_{1t+1}$  and  $h_{2t+1}$  for each regime in a GARCH(1,1) model. Accordingly, the variance processes within each regime at time  $t$  may be expressed as

$$h_{it} = b_{0i} + b_{1i} \cdot \epsilon_{t-1}^2 + b_{2i} \cdot h_{t-1}, \quad (28)$$

where, from Eq. (27),  $h_{t-1}$  is given by

$$\begin{aligned} h_{t-1} &= p_{1t-1} \cdot (\mu_{1t-1}^2 + h_{1t-1}) + (1 - p_{1t-1}) \cdot (\mu_{2t-1}^2 + h_{2t-1}) \\ &\quad - [p_{1t-1} \cdot \mu_{1t-1} + (1 - p_{1t-1}) \cdot \mu_{2t-1}]^2 \end{aligned} \quad (29)$$

and

$$\begin{aligned} \epsilon_{t-1} &= R_{t-1} - E[R_{t-1}|\phi_{t-2}] \\ &= R_{t-1} - [p_{1t-1} \cdot \mu_{1t-1} + (1 - p_{1t-1}) \cdot \mu_{2t-1}]. \end{aligned} \quad (30)$$

Finally, it remains to specify the probabilistic nature of the regime indicator  $S_t$ . To keep the analysis simple,  $S_t$  will be modelled as a first order Markov process with constant transition probabilities  $P$  and  $Q$ , i.e.

$$\begin{aligned}
\Pr \{S_t = 1|S_{t-1} = 1\} &= P, \\
\Pr \{S_t = 2|S_{t-1} = 1\} &= 1 - P, \\
\Pr \{S_t = 2|S_{t-1} = 2\} &= Q, \\
\Pr \{S_t = 1|S_{t-1} = 2\} &= 1 - Q.
\end{aligned}
\tag{31}$$

Arguing along the same lines as Gray (1996b, pp. 58), the specifications from the Eqs. (26) to (31) lead to the log-likelihood function

$$\begin{aligned}
\Lambda = \sum_{t=1}^T \ln &\left[ \frac{p_{1t}}{\sqrt{2\pi h_{1t}}} \cdot \exp \left\{ -\frac{(R_t - \mu_{1t})^2}{2h_{1t}} \right\} \right. \\
&\left. + \frac{1 - p_{1t}}{\sqrt{2\pi h_{2t}}} \cdot \exp \left\{ -\frac{(R_t - \mu_{2t})^2}{2h_{2t}} \right\} \right].
\end{aligned}
\tag{32}$$

The whole series of *ex-ante* probabilities  $p_{1t} \equiv \Pr\{S_t = 1|\phi_{t-1}\}$  can then be estimated recursively by

$$p_{1t} = P \cdot \frac{f_{1t-1}p_{1t-1}}{f_{1t-1}p_{1t-1} + f_{2t-1}(1 - p_{1t-1})} + (1 - Q) \cdot \frac{f_{2t-1}(1 - p_{1t-1})}{f_{1t-1}p_{1t-1} + f_{2t-1}(1 - p_{1t-1})}, \tag{33}$$

where  $f_{1t}$  and  $f_{2t}$  denote the  $N(\mu_{1t}, h_{1t})$ - and  $N(\mu_{2t}, h_{2t})$ -normal density functions, respectively (see Gray 1996b, pp. 37).

Tables 4 and 5 display the (quasi) maximum likelihood estimates of the regime-switching AR(1)-GARCH(1,1) models (25) to (33). The log-likelihood function (32) was optimized using the BFGS-algorithm as implemented in the 'maximize'-routine from the software package RATS 4.2. Standard errors and  $t$ -values were computed from the diagonal of the heteroscedasticity-consistent covariance matrix (see White 1980). For each series, the unconditional probability  $(1 - Q)/(2 - P - Q)$  of being in Regime 1 for all  $t = 1, \dots, T$  was chosen as the starting value for  $p_{1t}$  with  $P$  and  $Q$  taking on the values 0.9 and 0.95, respectively.

The estimates from Tables 4 and 5 show that 'more' AR parameters (in both regimes) are statistically significant at conventional levels than in the single regime GARCH model from Section 3.2. Another interesting observation refers to the sum of the GARCH parameters  $b_{1i} + b_{2i}$  indicating the degree of volatility persistence. First recall

Table 4: Estimates and related statistics for regime-switching GARCH models

Param./Stat.	FM		FF	
	Estimate <sup>a)</sup>	t-/(p-value)	Estimate <sup>a)</sup>	t-/(p-value)
Regime 1:				
$a_{01}$	-0.0029**	-2.1236	0.0046*	1.8144
$a_{11}$	0.1429***	5.7028	0.0990***	2.8715
$b_{01}$	0.0061***	14.9332	0.0004***	10.9393
$b_{11}$	0.1365***	28.0955	0.2188***	5.7722
$b_{21}$	0.9817***	31.0676	0.7745***	13.2939
Regime 2:				
$a_{02}$	-0.0011	-1.4486	-0.0003	-0.0265
$a_{12}$	-0.0225	-0.2514	-0.2395***	-6.0738
$b_{02}$	$7.1 \times 10^{-5}$ ***	10.8095	$-3.2 \times 10^{-6}$	-0.1797
$b_{12}$	0.0345	0.3561	0.1098***	4.0207
$b_{22}$	0.1986***	16.6276	0.1721**	2.4178
Transition prob.:				
$P$	0.9397***	100.5814	0.9077***	49.4524
$Q$	0.8874***	55.8349	0.9588***	40.6817
Log-likelihood	1027.7638		2602.0894	
$LB_1^{2b)}$	0.0777	(0.7805)	0.3387	(0.5606)
$LB_2^2$	0.0818	(0.9599)	1.2356	(0.5391)
$LB_3^2$	0.9586	(0.8113)	1.8760	(0.5985)
$LB_5^2$	1.0964	(0.9544)	2.9158	(0.7130)
$LB_{10}^2$	7.7733	(0.6510)	3.7676	(0.9572)
$LB_{15}^2$	11.6604	(0.7045)	5.7904	(0.9831)

a) Estimates for parameters from the Eqs. (26) to (32). \*, \*\*, \*\*\* denote significance at 10%, 5% and 1% levels.  $t$ -statistics are based on heteroscedasticity-consistent standard errors.

b)  $LB_i^2$  denotes the Ljung-Box-Q-statistic for serial correlation of the squared standardized residuals out to lag  $i$ .  $p$ -values are in parenthesis.

that in the conventional GARCH model the sums  $\hat{b}_1 + \hat{b}_2$  are all above or at least very close to unity. In contrast to this, the estimates from Tables 4 and 5 reveal a clear reduction in most of these sums. To be explicit, the values of  $\hat{b}_{1i} + \hat{b}_{2i}$  from Tables 4 and 5 are clearly lower than the corresponding sums from Table 3 in six out of eight regimes. Only in two cases (Regime 1 for the Finnish markka, Regime 1 for the French franc) there is no (significant) reduction in volatility persistence.<sup>13</sup> It is remarkable that in four regimes the sums  $\hat{b}_{1i} + \hat{b}_{2i}$  are lower than 0.5.

Two further results may be obtained by a comparison of the corresponding GARCH parameters between the Regimes 1 and 2. First note that—except for the Portuguese escudo—one finds  $\hat{b}_{11} > \hat{b}_{12}$  indicating that conditional variances of returns in Regime

<sup>13</sup>The sums are given by  $\hat{b}_{11} + \hat{b}_{21} = 1.1182$  (FM, Regime 1) and  $\hat{b}_{11} + \hat{b}_{21} = 0.9933$  (FF, Regime 1).

Table 5: Estimates and related statistics for regime-switching GARCH models

Param./Stat.	IL		PE	
	Estimate <sup>a)</sup>	t-/(p-value)	Estimate <sup>a)</sup>	t-/(p-value)
Regime 1:				
$a_{01}$	0.0098**	2.3250	0.0019	0.3490
$a_{11}$	0.1254***	4.8743	0.1168*	1.8690
$b_{01}$	0.0107***	115.0006	0.0048***	11.7445
$b_{11}$	0.2104***	21.7184	0.1159***	3.3799
$b_{21}$	0.7042***	36.7540	0.2759	1.4994
Regime 2:				
$a_{02}$	-0.0016***	-3.9064	-0.0005	-0.4497
$a_{12}$	0.1400***	5.3874	-0.0483	-0.7248
$b_{02}$	$-7.2 \times 10^{-5}$ ***	-13.7205	0.0002**	2.4723
$b_{12}$	0.2065***	4.8677	0.1626***	2.8860
$b_{22}$	0.5153***	27.9263	0.0408	1.0788
Transition prob.:				
$P$	0.9884***	1086.3518	0.9872***	36.0008
$Q$	0.9775***	555.7165	0.9717***	13.2565
Log-Likelihood	925.8092		1765.2595	
$LB_1^{2b)}$	0.1015	(0.7500)	0.0026	(0.9591)
$LB_2^2$	0.2348	(0.8892)	0.0240	(0.9881)
$LB_3^2$	1.3273	(0.7227)	0.6380	(0.8877)
$LB_5^2$	1.6320	(0.8974)	0.8200	(0.9757)
$LB_{10}^2$	3.0197	(0.9810)	4.7826	(0.9052)
$LB_{15}^2$	4.1711	(0.9971)	13.2252	(0.5849)

a) Estimates for parameters from the Eqs. (26) to (32). \*, \*\*, \*\*\* denote significance at 10%, 5% and 1% levels.  $t$ -statistics are based on heteroscedasticity-consistent standard errors.

b)  $LB_i^2$  denotes the Ljung-Box-Q-statistic for serial correlation of the squared standardized residuals out to lag  $i$ .  $p$ -values are in parenthesis.

1 are characterized by higher sensitivity to recent shocks than in Regime 2. Apart from that we have  $\hat{b}_{21} > \hat{b}_{22}$  for all currencies implying a higher persistence in conditional variances for Regime 1.

Finally, it remains to interpret the estimates of  $P$  and  $Q$ . These parameters represent the probabilities that the data generating process stays in the same regime during the transition from date  $t - 1$  to  $t$ , in other words, the probabilities of no structural break between  $t - 1$  and  $t$ . The estimates  $\hat{P}$  and  $\hat{Q}$  are always higher than 0.88 for all currencies. In five out of eight cases these probabilities are even greater than 0.95 indicating a high degree of persistence for each of the Regimes 1 and 2.

The lower parts of Tables 4 and 5 contain Ljung-Box-statistics for serial correlation of squared (standardized) residuals out to lag  $i$ . All squared residuals are free of autocor-

relation giving further econometric evidence for the adequacy of the regime-switching GARCH models. Note that in contrast to the conventional GARCH model from Section 3.2 the residuals of the returns of the French franc are free of autocorrelation now.

In regime-switching models, two further alternative probabilities are of interest. On the one hand, the *ex-ante* probabilities  $p_{1t} \equiv \Pr\{S_t = 1|\phi_{t-1}\}, t = 2, \dots, T$ , which can be estimated recursively by (33), are an important tool for forecasting. On the other hand, the *smoothed* probabilities,  $\Pr\{S_t = 1|\phi_T\}, t = 1, \dots, T$ , may be used to find out *ex post* if and when regime switches have occurred in the sample. In general, the smoothed probabilities may be computed by alternative filter techniques. The calculation of the smoothed Regime-1 probabilities in this paper makes use of an algorithm from Gray (1996a).

Figures 4 and 5 display the smoothed Regime-1 probabilities and the conditional variances  $\text{Var}[R_t|\phi_{t-1}]$  of exchange rate returns over the whole sample. According to the theoretical results from Section 2.2 the smoothed probabilities should ideally equal 1 at the beginning of the sample (representing the high-volatility Regime 1) and then suddenly drop to zero at date  $t_A$  (the beginning of the low-volatility Regime 2) for the rest of the sampling period. For each currency such a tentative date signifying the structural break is indicated in the figures.

The evolution of the smoothed probabilities for the Finnish markka and the French franc exhibit a striking similarity. At the beginning of the sampling period the probabilities are close to unity but interrupted by frequent downturns. From the dates '25-DEC-1997' and '5-NOV-1997' onwards, the probabilities change their baseline to zero but with more or less frequent upturns. An appealing interpretation is that financial markets considered both countries as irrefutable EMU members long before any official announcement, but that it was not before the beginning of 1998 that markets became more and more confident of the punctual implementation of Stage III of EMU. From a theoretical point of view this early lack of confidence can be represented by frequent changes in the uncertainty parameters  $p$  and  $\lambda$  in the equilibrium exchange rate path (5). According to the Eqs. (16) and (17) these changes may temporarily reduce or increase the variances of exchange rate returns thus possibly implying a statistically significant switch between the alternative volatility regimes. This could explain the frequent up- and downturns of the smoothed probabilities in Figure 4.

The smoothed probabilities of the Italian lira and the Portuguese escudo evolve in closer line with the theoretical model thus identifying rather clear switching dates (or at least quite short switching intervals), namely around the '1-NOV-1997' and the

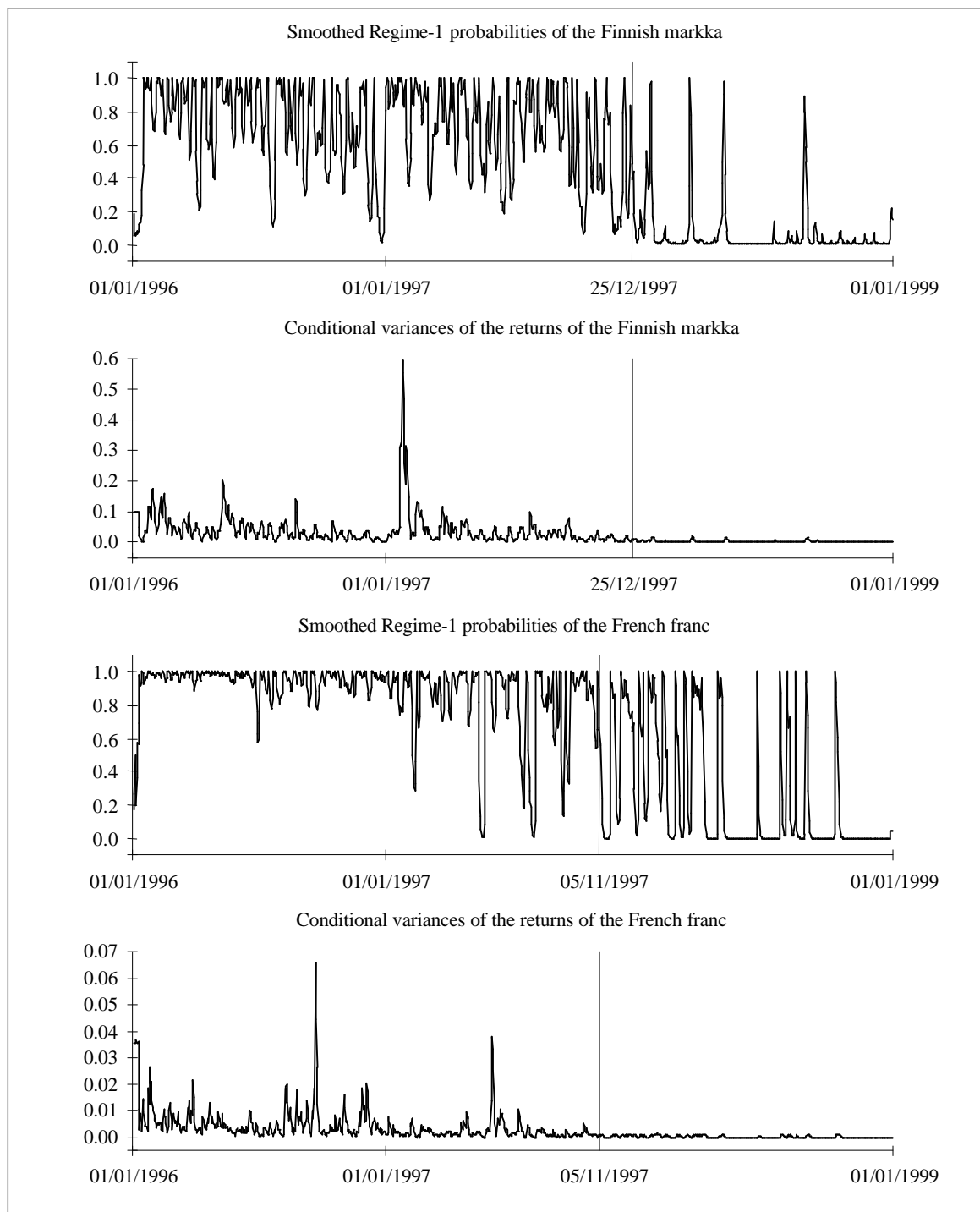


Figure 4: Smoothed Regime-1 probabilities and variances of returns

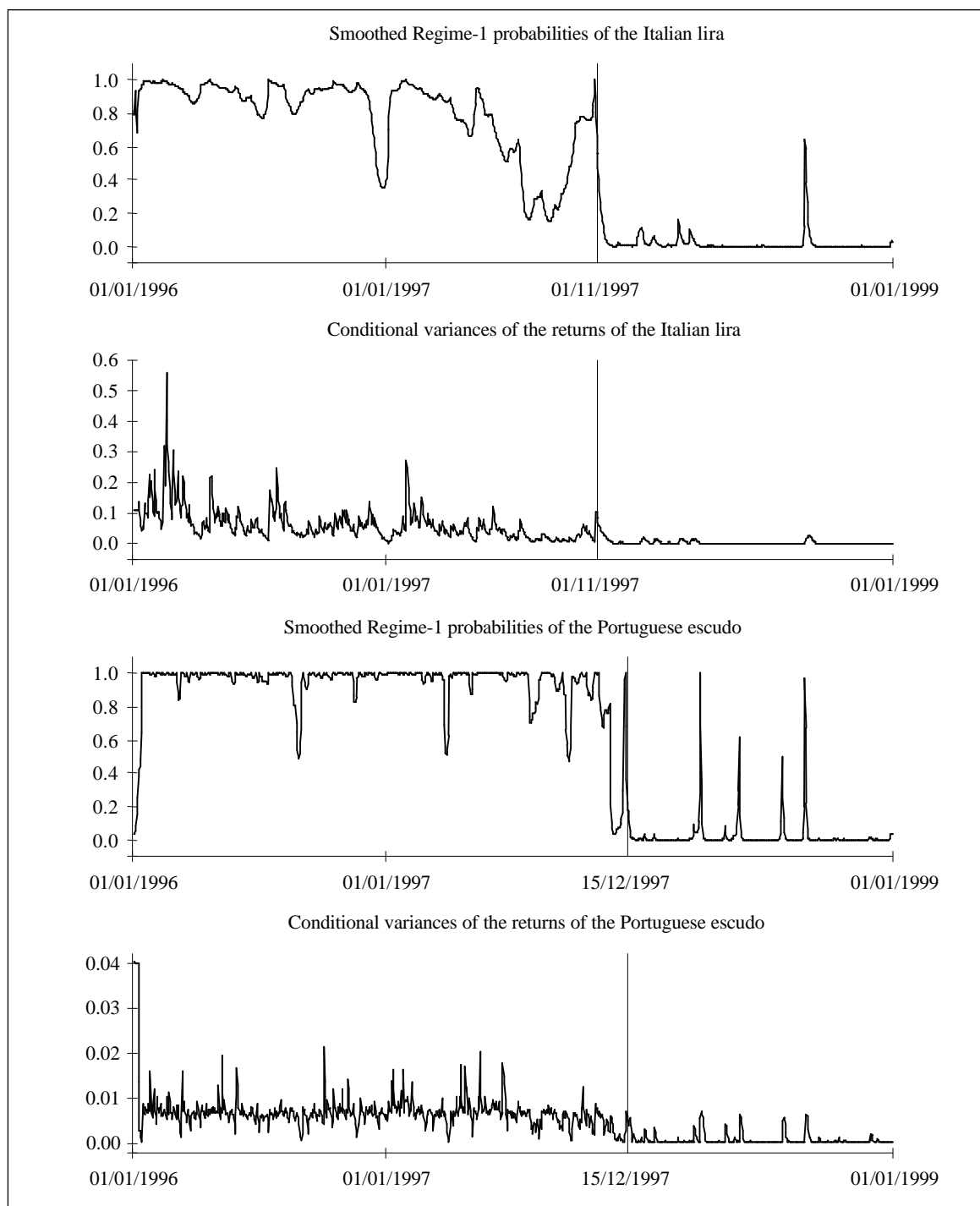


Figure 5: Smoothed Regime-1 probabilities and variances of returns

'15-DEC-1997', respectively. For these currencies the data suggests that financial markets did not reckon with the participation of Italy and Portugal until the end of 1997 when a drastic change in their assessment came about.

## 4 Concluding comments

This paper attempts to identify the dates from which on financial markets began to incorporate Stage III of EMU into currency valuation. Econometrically this problem is tackled by analyzing the volatility structure of exchange rate returns. According to a theoretical model the returns should undergo a regime shift in volatility at that moment when financial markets begin to consider a prospective entrance into EMU relevant for currency pricing.

From a statistical point of view the regime-switching GARCH models yield satisfactory results. The corresponding inference suggests two country-specific kinds of transitions into EMU. On the one hand, countries like France and Finland were considered as almost definite EMU candidates provided that Stage III would actually be implemented. The frequent switches between the high- and low-volatility regimes (as expressed by the up- and downturns of the smoothed Regime-1 probabilities from their baselines in Figure 4) may be viewed as a consequence of general doubts about whether Stage III would be implemented at all. On the other hand, for currencies like the Italian lira and the Portuguese escudo such general doubts apparently were not that important. These countries—not being considered as irrefutable EMU candidates—had to convince financial markets of their participation by the conduct of an appropriate economic policy and its acceptance by the political institutions in charge.

As far as the theoretical exchange rate model from Section 2 is concerned, one might argue that assuming a constant degree of central bank intervention during the interim period—represented by the (constant) parameter  $\eta$  in Eq. (2)—is not very realistic. In order to avoid speculative turmoil in the foreign exchange markets during the interim period it was necessary for the monetary authorities of the EMU countries to assure the credibility of their announcement of future EMU participation. A prominent advice to guarantee this was to be prepared to intervene in FOREX markets, particularly at the end of the interim period. Such an active policy can be modelled by a stochastic process with an intervention parameter that increases over time (e.g. a Brownian bridge). While this way of policy modelling introduces technical complexities into the analysis, it does not affect the main implication of the theoretical model from above, namely the existence of two successive volatility regimes for exchange rate returns.



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