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The convergence of international interest rates prior to Monetary Union

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DISCUSSION PAPER

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Bernd Wilfling

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Bernd Wilfling

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Summary

The process of international interest rate convergence for arbitrary terms (represented by the term structure of interest rate differentials) is derived in a model of a small open economy which faces a purely time-contingent exchange rate regime switch from flexible to fixed rates. Special attention is paid to a situation in which financial markets deem a delay in the regime switch beyond the publicly announced fixing date possible. The closed-form solution of the term structure allows us to analyze the volatility of interest rate differentials thus providing a useful tool for interest-rate-sensitive security valuation and other risk management applications. Furthermore, the model demonstrates that the economy under consideration has to pay for the exchange rate stabilization triggered by the mere announcement of future regime switching by a higher uncertainty in the evolution of domestic interest rates.

Zusammenfassung

Diese Arbeit beschreibt den Konvergenzprozeß internationaler Zinssätze beliebiger Fristigkeit (repräsentiert durch die Zeitstruktur von Zinsdifferentialen) unter der Modellannahme einer kleinen offenen Volkswirtschaft, die sich einem zeitlich fest terminierten Übergang von flexiblen Wechselkursen in ein Festkurssystem gegenübersteht. Besondere Berücksichtigung findet dabei eine Situation, in der die Finanzmärkte eine Verzögerung des Regimewechsels über das von den wirtschaftspolitischen Instanzen angekündigte Regimewechseldatum hinaus für möglich halten. Die analytisch geschlossene Gleichgewichtslösung für die Zeitstruktur ermöglicht eine exakte Analyse der Volatilität der Zinsdifferentialen. Diese Ergebnisse sind von hoher praktischer Relevanz für die Bewertung zinssensitiver Finanzanlagen bzw. für anderweitige Anwendungen im Bereich des Zinsrisikomanagements. Ein weiteres zentrales Ergebnis des theoretischen Modells besteht darin, daß die betrachtete Volkswirtschaft die Wechselkursstabilisierung, welche durch die bloße politische Ankündigung der zukünftigen Implementierung eines Festkurssystems induziert wird, mit einer erhöhten Unsicherheit im Hinblick auf die inländische Zinsentwicklung bezahlen muß.

JEL classifications: E43, F31, F33

Key words: Exchange rate regime switches, interest rates, term structure, stochastic processes, uncertainty

1 Introduction

Following the Maastricht timetable, Stage 3 of the European Monetary Union (EMU) started on January 1, 1999. In line with former decisions taken by the European Council in Brussels (Belgium) in early May 1998, the EMU started on that day with a core group of 11 countries, namely Austria, Belgium, Finland, France, Germany, Ireland, Italy, Luxembourg, the Netherlands, Portugal and Spain. An important stipulation accompanying the introduction of the euro as the single EMU-currency was the irreversible fixing of the EMU-currencies' bilateral exchange rates on January 1, 1999 at their central parities from the European Exchange Rate Mechanism (ERM). This episode of fixing a presently floating exchange rate at a known future date and at a publicly announced parity provides an example of a purely time-contingent exchange rate regime switch.¹

Although the recent currency crises in Asia have led many economists and policy advisors to advocate for more flexible exchange rate arrangements, it seems plausible for at least two reasons that time-contingent regime switches from flexible to fixed rates will command some interest in the future. The first reason from an EMU-perspective is that four 'ECU-countries'—Denmark, England, Greece and Sweden—do not belong to the first wave of EMU-Ins. It is very likely that these countries—and more generally all later EMU-entrants—will face dated pegs of their currencies to the euro in the next decade. Apart from that there currently is some political activity to propagate the creation of a single currency for the Mercosur countries or the Southeast Asian Nations (see e.g. Eichengreen, 1999, p. 95).

On the road towards Stage 3 of EMU there was no doubt among economists that the interest rates of the 'In-countries' would broadly converge during the interim period (i.e. the time between the date at which the future regime switch is publicly announced and the date at which the exchange rates are actually fixed). In fact, from a theoretical point of view, in a fixed exchange rate system with free capital mobility the uncovered interest parity condition implies zero differentials between home and foreign interest rates in the absence of devaluation risks and if risk premiums between 'similar' domestic and foreign bonds can be disregarded. But how do interest rates converge when foreign exchange markets anticipate a time-contingent regime switch from floating to fixed exchange rates? It is the aim of this paper to analyze the process of interest rate

¹In the literature on foreign exchange, rates under a credible target zone or currency band (like the ERM) are often considered as fixed if the band width is sufficiently small. After the speculative turmoil in 1992/93 all ERM bands—except that between the DM and the Dutch Guilder—were widened to $\pm 15\%$. This wide range of possible variation permits to consider the ERM exchange rates as quasi-flexible or at least as managed-float rates.

convergence during this period. To assess the dynamics of convergence, the interest rate differentials for arbitrary terms are derived in a stylized model, i.e. a closed-form solution of the complete term structure of interest rate differentials is given and its main properties will be examined.

In order to obtain the full term structure of interest rate differentials, a free-float system prior to the regime switch to fixed rates is assumed, the reason for this assumption being a gain in expositional clarity. However, the techniques used in this paper are general enough to be directly applied to other pre-switch regimes, e.g. a managed-float system. Moreover, for various kinds of macroeconomic reasoning, the model presented in the next sections explicitly takes into account that financial markets may be uncertain about strict adherence to the announced fixing-date, i.e. agents deem a delay in the exchange rate regime switch possible.

The setup used in this paper is in several respects similar to that of Svensson (1991) who analyzes the term structure of interest rate differentials in a target zone. While the theory of and the empirical findings on the *term structure of interest rates* have been (and are still) widely discussed (see e.g. Cox, Ingersoll and Ross, 1985; Singleton, 1989; Cuthbertson, 1996; Estrella and Mishkin, 1997) there are only relatively few results on the *term structure of interest rate differentials* scattered in the literature. This seems all the more surprising since for a small open economy facing a given term structure of world interest rates, the domestic term structure of interest rates follows the term structure of interest rate differentials when capital markets are sufficiently integrated (cf. Svensson, 1991, p. 90). Therefore, the analysis of the term structure of interest rate differentials yields a shortcut to exploring the term structure of domestic interest rates (see Kempa et al. 1999, p. 818).

The paper is organized as follows. After a short description of the underlying monetary exchange rate model with flexible prices, Section 2 presents the exchange rate dynamics during the transition from flexible to fixed rates thereby taking into account a possible delay in the regime switch. Based on these results, Section 3 derives a complete term structure of interest rate differentials during the interim period. Section 4 is two-split, elaborating the main features of the process of interest rate convergence first under certainty and second under uncertainty about the punctuality of the regime switch. Section 5 offers some concluding remarks.

2 Preliminaries and previous results

To derive the term structure of interest rate differentials in the presence of an announced (purely) time-contingent future exchange rate regime switch from flexible to fixed rates, it is convenient to recall the exchange rate dynamics during this period. For notational convenience, let t_A denote the date at which the authorities announce the future regime switch (the date of announcement) whereas the date at which the exchange rate is planned to be irrevocably fixed (the fixing date, the switching date) will be denoted by t_S . Furthermore, let \bar{x} symbolize the specific parity at which the exchange rate will be fixed at t_S . It is assumed that the monetary authorities will inform the market of the fixing parity \bar{x} at the date of announcement. Hence, agents are perfectly informed about all modalities of fixing from t_A onwards.

To assess the exchange rate dynamics during the interim period $[t_A, t_S)$ consider the well-known stochastic version of the (continuous-time) monetary-flex-price exchange rate model which has become the standard setting in recent target zone literature (cf. Froot & Obstfeld, 1992). There, the logarithmic spot rate—measured as the domestic-currency price for foreign exchange—at time t , $x(t)$, equals the sum of a ‘fundamental’, $k(t)$, and a speculative term proportional to the expected (instantaneous) rate of change in the exchange rate:

$$x(t) = k(t) + \alpha \cdot \frac{E[dx(t)|\phi(t)]}{dt}, \quad \alpha > 0. \quad (1)$$

In Eq. (1), $E[\cdot|\phi(t)]$ denotes the expectation operator conditional on the information set $\phi(t)$ containing all information available to market participants at time t . Rational expectations are assumed.²

The fundamental k is an aggregate of given macroeconomic variables—such as the domestic and foreign money supplies and outputs—as well as stochastic shocks to money demand and, via the money supplies, under direct control of a monetary authority. Prior to the fixing date t_S , k should follow a continuous-time stochastic process. Throughout this paper, the evolution of k over time (up to t_S) is modelled by a driftless Brownian motion, i.e.

$$dk(t) = \sigma \cdot dw(t), \quad t < t_S, \quad (2)$$

with infinitesimal standard deviation $\sigma > 0$ and $dw(t)$ the increment of a standard Wiener process. This process is particularly useful when modelling a situation in which the central banks refrain from interventions in the foreign exchange market prior to the

²In Eq. (1), $E[dx(t)|\phi(t)]/dt$ is an abbreviation of $\lim_{s \downarrow 0} \{E[x(t+s)|\phi(t)] - x(t)\} / s$.

regime switch. Therefore, underlying the Brownian motion (2) for the fundamental k up to t_S will subsequently be referred to as a free-float pre-switch regime.³

In order to state explicit results on the exchange rate dynamics prior to and after the regime switch, it proves convenient to split the time domain into the mutually exclusive intervals $[0, t_A)$, $[t_A, t_S)$ and $[t_S, \infty)$. For $t \in [0, t_A)$ the regime switch has not yet been officially announced by the authorities. For simplicity, it is therefore assumed that agents expect the current free-float regime to last forever. It is well-known that the (bubble-free) solution of the flex-price monetary exchange rate equation (1) then obtains as

$$x(t) = k(t), \quad (3)$$

i.e. the free-float equilibrium exchange rate $x(t)$ coincides with the current value of the fundamental, $k(t)$.

For $t \in [t_A, t_S)$, participants seem to be perfectly informed about all modalities of future exchange rate regime switching. In particular, the authorities have announced the fixing parity \bar{x} and the fixing date t_S . Now, let us take into account that agents may be uncertain about strict adherence to the preannounced fixing date t_S . In what follows, the notion 'uncertainty' is always used in the sense of 'uncertainty about the punctuality of the regime switch'. In this context, uncertainty characterizes any situation in which agents impute a non-zero probability to the delay in the regime switch beyond the proclaimed date t_S .

In general, there may be a variety of ways of how to model this kind of market uncertainty. Here, let us adopt the approach of Wilfling and Maennig (2001) who suppose that market participants—based on their present date- t -information set $\phi(t), t \in [t_A, t_S)$ —associate a specific probability measure with the future lifetime of the pre-switch free-float system. In particular, denoting this lifetime by the random variable Z , the cumulative distribution function of Z , F_Z , which represents the probability that Z does not exceed the future date $s > t$, is assumed to have the parametric form

$$F_Z(s; p, \lambda) \equiv \Pr \{Z \leq s | \phi(t)\} = \begin{cases} 0 & \text{for } s < t_S \\ 1 - p \cdot e^{\lambda(t_S - s)} & \text{for } s \geq t_S, p \in [0, 1], \lambda \geq 0 \end{cases} \cdot (4)$$

³Alternatively, a free-float pre-switch regime could more generally be modelled by a Brownian motion with arbitrary (deterministic) drift $\theta \in \mathbb{R}$, i.e. by the stochastic differential

$$dk(t) = \theta \cdot dt + \sigma \cdot dw(t) \quad (2a)$$

for the fundamental k . Setting the deterministic trend $\theta = 0$ as in Eq. (2) does not affect the qualitative nature of the results.

This specification of F_Z induces several features of the agents' uncertainty assessment. First, the probability of the switch taking place before t_S is zero. Second, the parameters p and λ have the following neat interpretations: p represents the probability of the switch not occurring at the officially announced fixing date t_S so that $1 - p$ is the (unconditional) probability which agents assign to the event that the regime switch will take place exactly at t_S . Conditional on a delayed regime switch, the parameter λ represents the (constant) proportional hazard rate that the regime switch will take place in the infinitesimal time period following any date $s \geq t_S$. In other words, for sufficiently small Δs , $\lambda \cdot \Delta s$ approximates the probability of the regime switch taking place in the interval $[s, s + \Delta s]$ provided that the switch has not occurred up to date s inclusively. It is important to note two special cases included in Eq. (4):

- (a) For $p = 0$ (λ arbitrary) the switching date t_S is considered fully credible by the market. Formally the same is true for $p > 0, \lambda \rightarrow \infty$. In this case the regime switch is not expected to take place at t_S with probability 1 but the delay is considered to be infinitesimally short.
- (b) For $p = 1, \lambda = 0$, agents believe that the switch will never take place.

Another two properties of the random lifetime Z and its probability distribution (4) will be of interest for further considerations. First, it is easy to check that the conditional distribution function of Z provided that the pre-switch free-float system has 'survived' up to some date $z \geq t_S$ is given by

$$\Pr\{Z \leq s | Z > z \geq t_S\} = 1 - e^{\lambda \cdot (z-s)}. \quad (5)$$

Obviously, this conditional distribution is independent of p and (for $z > t_S$) also of the switching date t_S . Second, representing the market's present date- t -assessment of the timing of the regime switch by the distribution function (4) implies that agents expect the switch to take place at

$$E[Z | \phi(t)] = t_S + \frac{p}{\lambda}. \quad (6)$$

Note that the expected time of fixing in Eq. (6) is strictly increasing in p and strictly decreasing λ .

Now, taking into account the uncertainty structure from Eq. (4) and using a forward integration technique Wilfling and Maennig (2001) derive the exchange rate path for the interim period $t \in [t_A, t_S)$ as

$$x(t) = \left[1 - \left(1 - \frac{p}{1 + \alpha\lambda} \right) \cdot e^{(t-t_S)/\alpha} \right] \cdot k(t) + \left(1 - \frac{p}{1 + \alpha\lambda} \right) \cdot e^{(t-t_S)/\alpha} \cdot \bar{x}. \quad (7)$$

Furthermore, it is essential for the computations in the next section to evaluate the exchange rate path for the time beyond the officially announced fixing date t_S . The exact specification of the saddlepath $x(t)$ for $t \geq t_S$ crucially hinges on the policy action taken at date t_S . Here, it suffices to consider the following two scenarios:

- (a) The authorities indeed fix the exchange rate irreversibly from t_S onwards at the parity \bar{x} . In this case the rate clearly fulfills

$$x(t) = \bar{x} \quad \text{for all } t \geq t_S. \quad (8)$$

- (b) The authorities refrain from fixing at t_S and the rate is still floating at present date $t \geq t_S$ while the market still believes the regime switch to the fixed parity \bar{x} to be executed in the future and that—in line with Eq. (5)—the probability of the switch occurring in any future interval $(t, s]$ is given by

$$\Pr\{Z \leq s | Z > t \geq t_S\} = 1 - e^{\lambda \cdot (t-s)}.^4 \quad (9)$$

Under this setup, the exchange rate path for $t \geq t_S$ can be computed as

$$x(t) = \frac{1}{1 + \alpha\lambda} \cdot k(t) + \left(1 - \frac{1}{1 + \alpha\lambda}\right) \cdot \bar{x} \quad (10)$$

(cf. Wilfling and Maennig, 2001, Eq. 21).⁵

In the following sections, the partial exchange rate paths (3), (7), (8) and (10) will provide the key ingredients for the derivation of a closed-form solution of the term structure of interest rate differentials under a free-float system when the market anticipates a time-contingent, but possibly delayed regime switch to fixed rates.

3 The interest rate differential

For what follows, let us adopt the same setting as Svensson (1991) who analyzes the term structure of interest rate differentials in target zones. Formally, let $i^*(t, \tau)$ symbolize the foreign nominal interest rate on a pure discount bond (denominated in foreign currency) purchased at time t with $\tau > 0$ denoting the time to maturity (term). The

⁴Using the conditional distribution from Eq. (9) as the waiting time for the regime switch to fixed rates is equivalent to modelling the timing of the switch as (the first occurrence of) a Poisson process (starting at present date $t \geq t_S$) with arrival rate λ .

⁵The formulation of the exchange rate path (10) implicitly assumes that the fundamental k continues to evolve along the Brownian motion (2) up to the present date $t \geq t_S$ inclusively.

open domestic economy is assumed to be small and the foreign interest rate $i^*(t, \tau)$ is regarded as exogenously given.⁶ Accordingly, the nominal interest rate on a home currency pure discount bond purchased at time t and maturing at date $t + \tau$ will be denoted by $i(k(t), t, \tau)$. Note that the realization of the fundamental k at present date t , $k(t)$, is among the arguments of the domestic interest rate i . Neglecting any risk premium between home and foreign nominal bonds, the following form of the uncovered interest parity condition is supposed to hold at all times:

$$ID(k(t), t, \tau) \equiv i(k(t), t, \tau) - i^*(t, \tau) = \frac{E[x(t + \tau)|\phi(t)] - x(t)}{\tau}. \quad (11)$$

From Eq. (11) the differential for instantaneous interest rates, i.e. rates on bonds with infinitesimally short time to maturity (say overnight interest rates) obtains as

$$ID(k(t), t, 0) \equiv \lim_{\tau \downarrow 0} ID(k(t), t, \tau) = \frac{E[dx(t)|\phi(t)]}{dt} \quad (12)$$

(cf. Footnote 2).

The uncovered interest parity conditions (11), (12) and the exchange rate paths from Section 2 now provide the main tool for computing the interest rate differentials for arbitrary terms τ on the whole time domain. Before the date of announcement (i.e. for $t \in [0, t_A)$), the market believes the free-float regime to hold for the indefinite future. Thus, invoking Eq. (3) the interest rate differential is obtained as

$$ID(k(t), t, \tau) = \frac{E[x(t + \tau)|\phi(t)] - x(t)}{\tau} = \frac{k(t) - k(t)}{\tau} = 0. \quad (13)$$

Obviously, the interest rate differential under a free-float regime (which is assumed to hold forever) is independent of the purchase-time t , the term τ , and the current state of the fundamental $k(t)$.⁷

Next, consider the interim period $[t_A, t_S)$. For reasons of argument, the derivation starts with the interest rate differential for instantaneous bonds. For this, recall the exchange rate path (7) which takes into account the uncertainty structure F_Z from

⁶Note that no assumption is made on the dynamics of the foreign interest rate $i^*(t, \tau)$. This interest rate may evolve according to an appropriate stochastic process (see Chan et al., 1992, or Dewachter, 1996, for an overview of suitable stochastic processes used to model the dynamics of interest rates). However, the only assumption made here is that $i^*(t, \tau)$ is exogenously given to the small domestic economy.

⁷The reason for the interest rate differential (11) under a permanent free-float to equal zero on the whole time domain lies in the assumption of the driftless Brownian motion (2) for the fundamental k . If the dynamics of k is modelled by a Brownian motion with deterministic trend θ as in Eq. (2a) (see Footnote 3), the corresponding interest rate differential will equal the drift rate θ for all t , i.e. the constant expected change in the spot rate.

Eq. (4). According to (12), the instantaneous interest rate differential $ID(k(t), t, 0)$ essentially consists of $E[dx(t)|\phi(t)]$, i.e. the expected infinitesimal change of the (log-) spot rate. The most direct way to compute this magnitude is to interpret the exchange rate x in Eq. (7) as a function of the stochastic fundamental k and time t . The application of Ito's lemma then yields the stochastic differential of x , $dx(t)$. Taking expectations conditional on time t and dividing by dt finally provides the instantaneous interest rate differential for $t \in [t_A, t_S)$:

$$ID(k(t), t, 0) = \left(1 - \frac{p}{1 + \alpha \cdot \lambda}\right) \cdot e^{(t-t_S)/\alpha} \cdot \frac{\bar{x} - k(t)}{\alpha}. \quad (14)$$

In contrast to the case of instantaneous bonds just described, the computation of the interest rate differentials for strictly positive terms $\tau > 0$ turns out to be more complicated. The reason is the expected exchange rate at the time of maturity, $E[x(t + \tau)|\phi(t)]$. Its determination at present date $t \in [t_A, t_S)$ depends on whether or not the time to maturity $t + \tau$ falls before or beyond the announced switching date t_S . Distinguishing between the cases $t + \tau < t_S$ and $t + \tau \geq t_S$ entails a two-branched interest rate differential path during the interim period, namely on the subintervals $[t_A, t_S - \tau)$ and $[t_S - \tau, t_S)$, respectively. For $t \in [t_A, t_S - \tau)$ the computation of $E[x(t + \tau)|\phi(t)]$ is straightforward. Explicitly, due to the rational expectation hypothesis agents form their exchange rate forecasts by evaluating $x(t + \tau)$ according to Eq. (7) and then take expectations conditional on the information set $\phi(t)$. Proceeding in this fashion and applying the uncovered interest parity condition (11) then leads after some algebra to the interest rate differential

$$ID_1(k(t), t, \tau) = \left(1 - \frac{p}{1 + \alpha \cdot \lambda}\right) \cdot e^{(t-t_S)/\alpha} \cdot \frac{1 - e^{\tau/\alpha}}{\tau} \cdot [k(t) - \bar{x}].^8 \quad (15)$$

The case $t \in [t_S - \tau, t_S)$ requires a more laborious technique to derive $E[x(t + \tau)|\phi(t)]$. Its determination hinges on the more general question how market participants form their expectations of the future exchange rate $x(u)$, $u \geq t_S$, on the basis of their present knowledge represented by $\phi(t)$. First recall, that according to (4) agents assess at present date $t \in [t_A, t_S)$ the exchange rate to be fixed at t_S with probability $1 - p$ whereas with probability p the pre-switch free-float regime will remain unaffected at t_S . Thus, for $u \geq t_S$

$$E[x(u)|\phi(t)] = (1 - p) \cdot \bar{x} + p \cdot E[x_1(u)|\phi(t)]. \quad (16)$$

⁸Observe that the interest rate differential path (14) may be rederived by letting $\tau \rightarrow 0$ in Eq. (15).

In Eq. (16), $x_1(u)$ denotes the equilibrium exchange rate that will prevail if the regime switch will not have occurred at t_S . The structure of the expectation $E[x_1(u)|\phi(t)]$, $u \geq t_S$, results from the following reflection: Since the exchange rate has not been fixed at t_S , Eq. (5) implies that the probability of the switch occurring between t_S and u is given by $[1 - e^{\lambda \cdot (t_S - u)}]$, whereas with probability $[e^{\lambda \cdot (t_S - u)}]$ the switch does not take place up to u inclusively. Therefore,

$$E[x_1(u)|\phi(t)] = [1 - e^{\lambda \cdot (t_S - u)}] \cdot \bar{x} + e^{\lambda \cdot (t_S - u)} \cdot E[x_2(u)|\phi(t)], \quad (17)$$

where $x_2(u)$ represents the exchange rate path which will prevail if the regime switch will not have occurred up to date $u \geq t_S$ inclusively. But based on their present date- t -information set $\phi(t)$, $t \in [t_A, t_S)$, and due to the rational expectation hypothesis, the market *ex ante* assigns—in line with the conditional distribution (5)—the probability $[1 - e^{\lambda \cdot (u - s)}]$ to the event that the switch will be executed in the future interval $(u, s]$. Hence, according to Eqs. (9) and (10), the exchange rate path $x_2(u)$ is unambiguously identified as

$$x_2(u) = \frac{1}{1 + \alpha \cdot \lambda} \cdot k(u) + \left(1 - \frac{1}{1 + \alpha \cdot \lambda}\right) \cdot \bar{x}.$$

Now, taking the expectation of $x_2(u)$ conditional on $\phi(t)$ and backward substitution into the Eqs. (17) and (16) leads to

$$E[x(u)|\phi(t)] = \frac{p}{1 + \alpha \cdot \lambda} \cdot e^{\lambda \cdot (t_S - u)} \cdot k(t) + \left[1 - \frac{p}{1 + \alpha \cdot \lambda} \cdot e^{\lambda \cdot (t_S - u)}\right] \cdot \bar{x} \quad (18)$$

for $u \geq t_S$.

Formula (18) now allows us to compute $E[x(t + \tau)|\phi(t)]$ for $t \in [t_S - \tau, t_S)$. Using the interest parity condition (11), the second branch of the interest rate differential is after some rearrangements found to be

$$ID_2(k(t), t, \tau) = \left[1 - \frac{p}{1 + \alpha \cdot \lambda} \cdot e^{\lambda \cdot (t_S - t - \tau)} - \left(1 - \frac{p}{1 + \alpha \cdot \lambda}\right) \cdot e^{(t - t_S)/\alpha}\right] \cdot \left[\frac{\bar{x} - k(t)}{\tau}\right]. \quad (19)$$

Note from the partial differential paths (15) and (19) that with probability 1

$$\lim_{t \uparrow t_S - \tau} ID_1(k(t), t, \tau) = \lim_{t \downarrow t_S - \tau} ID_2(k(t), t, \tau)$$

ensuring a smooth transition between the successive interest-rate-differential branches.⁹

⁹Subsequently, all mathematical limits of the stochastic processes x or k draw on the concept of

Finally, it remains to specify the dynamics of interest rate differentials for the time after the official fixing date t_S . For this, the analysis restricts to the same two scenarios which lead to the exchange rate equations (8) and (10). If the authorities in fact implement the irreversible regime switch to fixed rates from t_S onwards then the expectation about the future exchange rate $E[x(t+\tau)|\phi(t)]$ equals \bar{x} for every $t \geq t_S$ and arbitrary term $\tau > 0$. Hence, for all $t \geq t_S$ the Eqs. (11) and (12) provide the constant interest rate differentials

$$ID(k(t), t, \tau) = 0 \quad \text{and} \quad ID(k(t), t, 0) = 0 \quad (20)$$

for all strictly positive terms $\tau > 0$ and instantaneous bonds, respectively.

Next, assume that the authorities refrain from fixing at t_S and give rise to market participants to associate the timing of the regime switch with the probability distribution (9). Then, for every $t \geq t_S$, the expected exchange rate at the time of maturity $t + \tau$ can be derived as follows: According to Eq. (9) agents impute the probability $[1 - e^{-\lambda\tau}]$ of the switch occurring in the interval $(t, t + \tau]$. On the other hand, the probability of the switch not happening in that interval so that the exchange rate continues to evolve along the path (10) until date $t + \tau$ is given by $[e^{-\lambda\tau}]$. Consequently,

$$E[x(t+\tau)|\phi(t)] = [1 - e^{-\lambda\tau}] \cdot \bar{x} + e^{-\lambda\tau} \cdot E[x_3(t+\tau)|\phi(t)], \quad (21)$$

with $x_3(\cdot)$ on the right hand side of (21) representing the exchange rate path (10). Taking the expectation conditional upon $\phi(t)$ then transforms Eq. (21) into

$$E[x(t+\tau)|\phi(t)] = \frac{e^{-\lambda\tau}}{1 + \alpha \cdot \lambda} \cdot k(t) + \left[1 - \frac{e^{-\lambda\tau}}{1 + \alpha \cdot \lambda}\right] \cdot \bar{x}. \quad (22)$$

Inserting the expected value (22) and the exchange rate path (10) into Eq. (11) yields the interest rate differential for all $t \geq t_S$ and strictly positive terms $\tau > 0$:

$$ID(k(t), t, \tau) = \frac{1 - e^{-\lambda\tau}}{\tau} \cdot \frac{\bar{x} - k(t)}{1 + \alpha \cdot \lambda}. \quad (23)$$

At last, the corresponding instantaneous interest rate differential may be derived by letting $\tau \rightarrow 0$ in Eq. (23):

$$ID(k(t), t, 0) = \frac{\lambda}{1 + \alpha \cdot \lambda} \cdot [\bar{x} - k(t)]. \quad (24)$$

'convergence with probability 1'. The above equality of the stated ID_1 - and ID_2 -limits results from the fact that the trajectories of any Brownian motion (used to model the macroeconomic fundamental k) are almost always (i.e. with probability 1) continuous functions of t .

4 Features of the interest rate differentials

The main properties of the interest rate differentials so far derived will now be elaborated in greater detail. For this, the section is split into two parts: Subsection 4.1 considers the situation in which agents are sure at any interim-date that the exchange rate regime switch will occur punctually at t_S . Subsection 4.2 then takes into account a possible delay in the regime switch beyond t_S and analyzes the impact of this kind of uncertainty on the process of interest rate convergence.

4.1 Certainty about the punctuality of the regime switch

As discussed in Section 2, a fully credible switching date t_S is represented by the uncertainty parameters $p = 0$, λ arbitrarily chosen. Hence for this subsection, the interest rate differentials during the interim period $[t_A, t_S)$ for instantaneous bonds and strictly positive terms follow from Eqs. (14), (15) and (19) with $p = 0$:

$$ID(k(t), t, 0) = e^{(t-t_S)/\alpha} \cdot \frac{\bar{x} - k(t)}{\alpha}, \quad (25)$$

$$ID_1(k(t), t, \tau) = e^{(t-t_S)/\alpha} \cdot \frac{1 - e^{\tau/\alpha}}{\tau} \cdot [k(t) - \bar{x}] \quad \text{for } t \in [t_A, t_S - \tau), \quad (26)$$

$$ID_2(k(t), t, \tau) = \left[1 - e^{(t-t_S)/\alpha}\right] \cdot \left[\frac{\bar{x} - k(t)}{\tau}\right] \quad \text{for } t \in [t_S - \tau, t_S]. \quad (27)$$

A first impression of the interest rate differential dynamics under certainty is provided by Figure 1 which depicts the evolution of the differentials for given structural parameters $\sigma, t_A, t_S, \bar{x}, \alpha$ and alternative terms $\tau = 0, 0.5, 1$. For the time after t_S , Figure 1 assumes that the authorities implement the announced regime switch punctually at t_S thus materializing the agents' expectations what—according to Eq. (20)—leads to differentials of height zero for arbitrary terms.

Three characteristics of the ID-paths are striking:

- (a) At t_A the interest rate differentials jump from their pure free-float path (13) on the ID_1 -path given in Eq. (26).¹⁰ The jump at t_A is a direct consequence of the fact that the announcement at t_A of future exchange rate regime switching at t_S is modelled as news in the chosen setup. This news affects both, the current exchange rate path $x(t)$ and—at least as important—the agents' expectations

¹⁰To be precise: At t_A the interest rate differential for instantaneous bonds jumps on the path (25). Moreover, if the term τ exceeds the length of the interim period ($t_S - t_A$) then the differential jumps directly from the path (13) on the ID_2 -path (27).

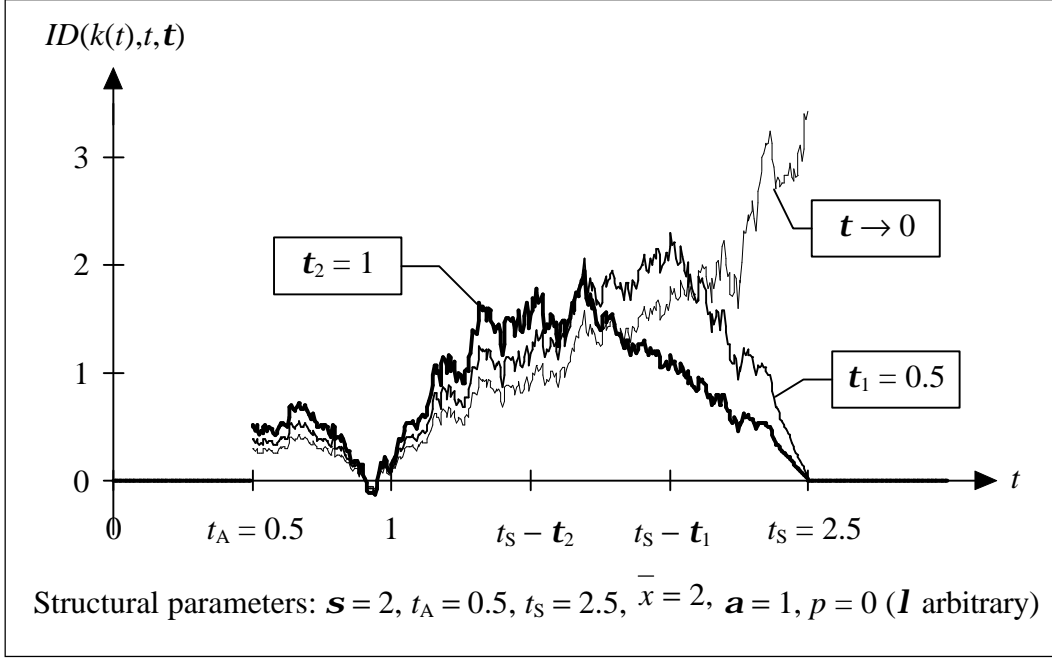


Figure 1: Interest rate differentials for alternative terms

about future exchange rates. This new exchange rate environment is immediately transferred to the dynamics of the interest rates differentials via the uncovered interest parity condition (11). It is straightforward to analyze the absolute height as well as the direction of the ID-jumps at t_A by means of standard calculus.

- (b) Intuitively, one might expect the absolute value of interest rate differentials for longer terms to be less than for shorter terms. As shown in Figure 1, this need not be the case during the whole interim period. In fact, it turns out to be cumbersome to derive a clear analytic relationship between the absolute value of interest rate differentials and their terms with respect to time.
- (c) The interest rate differentials for strictly positive terms $\tau > 0$ converge to zero as t tends to t_S , ensuring a smooth adjustment of domestic and foreign interest rates during the interim period. Formally, this follows from Eq. (27) which implies

$$\lim_{t \uparrow t_S} ID_2(k(t), t, \tau) = 0$$

with probability 1. In contrast to this, the interest rates for instantaneous bonds ($\tau \rightarrow 0$) do in general not equalize until t_S since Eq. (25) yields

$$\lim_{t \uparrow t_S} ID(k(t), t, 0) = \frac{1}{\alpha} \cdot \left(\bar{x} - \lim_{t \uparrow t_S} k(t) \right)$$

which is almost always different from zero. Why do strictly positive maturity interest rate differentials converge to zero for $t \rightarrow t_S$ while instantaneous differentials exhibit a jump at t_S ? One explanation is as follows: If domestic and foreign interest rates for bonds with strictly positive terms $\tau > 0$ did not adjust smoothly, riskless profits would be possible by buying the domestic and selling the foreign bond (or vice versa) infinitesimally shortly before t_S . These arbitrage opportunities can only be ruled out by a smooth adjustment of domestic and foreign interest rates. But for instantaneous bonds, no corresponding arbitrage removing transactions can be done since the time to maturity always falls before t_S for all $t \in [t_A, t_S)$.

Next, we address the *variability of interest rate differentials* during the interim period $[t_A, t_S)$. The economic significance of this topic arises from the key role that the *volatility of interest rates* plays in the valuation of interest-rate-sensitive securities or for the selection of optimal hedging strategies for risk averse investors. But again—as argued in the introduction—if one considers a small economy under perfect capital mobility facing an exogenous term structure of world interest rates, then from the perspective of domestic investors the volatility of interest rate differentials serves as an important predictor of the volatility of domestic interest rates.

In what follows, the variability will be measured by the variance of the interest rate differential for some future date s conditional on the information given at present date t , i.e.

$$\text{Var} [ID(k(s), s, \tau) | \phi(t)]$$

where $t_A \leq t < s \leq t_S$. In particular, consider the ID-paths (25) for instantaneous bonds and the two branches ID_1, ID_2 for strictly positive terms from the Eqs. (26) and (27). It is obvious from these formulae that every ID-path is a linear function (with time-dependent coefficients) of the stochastic fundamental k . As k evolves according to the Brownian motion (2) until t_S , it follows directly that the probability distribution of each interest rate differential conditional upon $\phi(t)$ is normal with variances

$$\text{Var} [ID(k(s), s, 0) | \phi(t)] = \frac{1}{\alpha^2} \cdot [e^{(s-t_S)/\alpha}]^2 \cdot \sigma^2 \cdot (s - t), \quad (28)$$

$$\text{Var} [ID_1(k(s), s, \tau) | \phi(t)] = [e^{(s-t_S)/\alpha}]^2 \cdot \left[\frac{1 - e^{\tau/\alpha}}{\tau} \right]^2 \cdot \sigma^2 \cdot (s - t), \quad (29)$$

$$\text{Var} [ID_2(k(s), s, \tau) | \phi(t)] = \frac{1}{\tau^2} \cdot [1 - e^{(s-t_S)/\alpha}]^2 \cdot \sigma^2 \cdot (s - t) \quad (30)$$

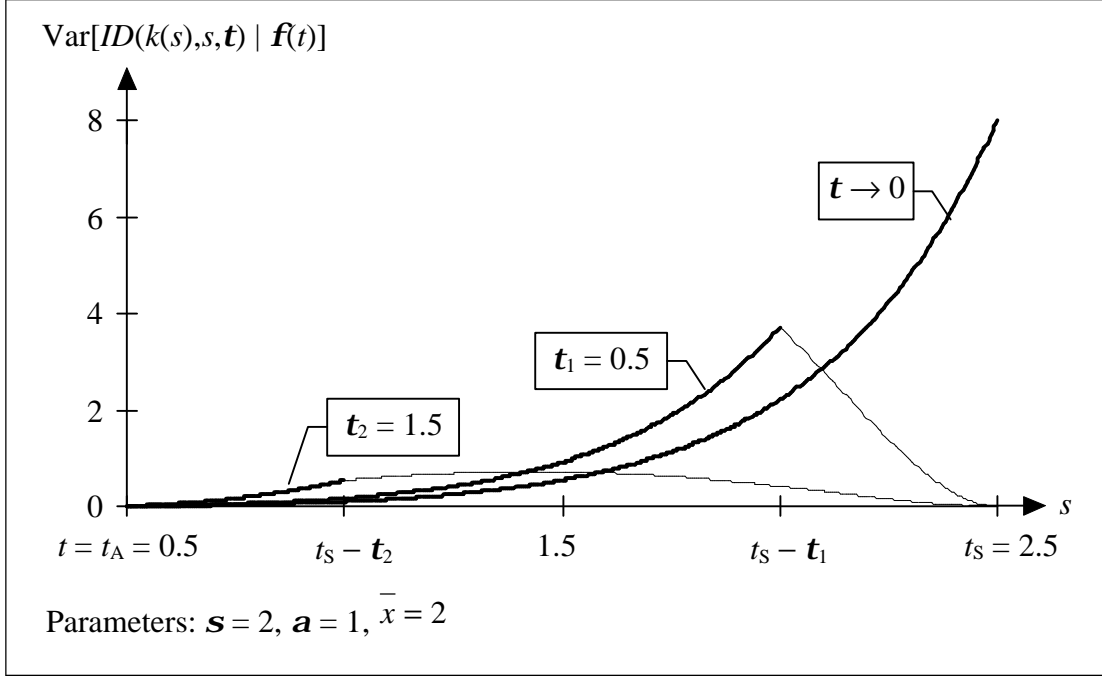


Figure 2: Variance paths of interest rate differentials with alternative terms

for instantaneous bonds and the two ID-branches for strictly positive terms, respectively.

Figure 2 displays the variance paths for the alternative terms $\tau \rightarrow 0$, $\tau_1 = 0.5$, $\tau_2 = 1.5$ for given structural parameters. For the strictly positive terms τ_1 and τ_2 the variance paths switch from the branches (29) (the thick lines in Figure 2) to (30) (the thin lines) at the corresponding branching points $t_S - \tau_1$ and $t_S - \tau_2$, respectively. For a more formal analysis, consider the variance path for instantaneous bonds in Figure 2. Obviously, looking into the future from present date t onwards, the variability of the corresponding interest rate differential increases with time s . This qualitative feature can be confirmed analytically by the derivative of (28) with respect to s :

$$\frac{\partial \text{Var}[ID(k(s), s, 0) | \phi(t)]}{\partial s} = \frac{\sigma^2}{\alpha^2} \cdot [e^{(s-t_S)/\alpha}]^2 \cdot \left[\frac{2 \cdot (s-t)}{\alpha} + 1 \right] > 0. \quad (31)$$

Similarly, the variability of the ID_1 -branch is also strictly increasing in s , since

$$\frac{\partial \text{Var}[ID_1(k(s), s, \tau) | \phi(t)]}{\partial s} = \sigma^2 \cdot \left[\frac{1 - e^{\tau/\alpha}}{\tau} \right]^2 \cdot [e^{(s-t_S)/\alpha}]^2 \cdot \left[\frac{2 \cdot (s-t)}{\alpha} + 1 \right] > 0. \quad (32)$$

In contrast to this, the variability of the ID_2 -branch must not be strictly monotonic in s . Figure 2 shows that the corresponding variance path may be strictly decreasing in

s (the thin line for $\tau_1 = 0.5$) but it also possible that it increases first up to some point and then decreases with s tending towards t_S (the thin line for term $\tau_2 = 1.5$). It is indeed difficult to assess by means of standard calculus the exact monotonic behaviour of $\text{Var}[ID_2(k(s), s, \tau)|\phi(t)]$ with respect to s on the domain $s \in (t_S - \tau, t_S)$. But it follows directly from Eq. (30) that the variability must be decreasing for s tending towards t_S since

$$\lim_{s \uparrow t_S} \text{Var}[ID_2(k(s), s, \tau)|\phi(t)] = 0. \quad (33)$$

Eq. (33) confirms that the variability of interest rate differentials for strictly positive terms $\tau > 0$ vanishes for s infinitesimally close to t_S . Furthermore, it follows from Eqs. (29) and (30) that

$$\lim_{s \uparrow t_S - \tau} \text{Var}[ID_1(k(s), s, \tau)|\phi(t)] = \lim_{s \downarrow t_S - \tau} \text{Var}[ID_2(k(s), s, \tau)|\phi(t)] \quad (34)$$

ensuring a continuous transition of the ID_1 - and ID_2 -variance paths at $t_S - \tau$.¹¹

Next, we turn to the following question: How does the variability of interest rate differentials for a given future date s depend on the term length τ ? Obviously from Figure 2, there is a region at the left edge of the interval (t, t_S) in which the variance for longer-term bonds is higher than for bonds with shorter time to maturity. For notational convenience, define the following auxiliary function of the term τ :

$$f(\tau) \equiv \frac{1 - e^{\tau/\alpha}}{\tau}. \quad (35)$$

Note the following properties of f :

- (a) $f(\tau) < 0$ for all $\tau > 0$,
- (b) $f'(\tau) \equiv df(\tau)/d\tau < 0$ for all $\tau > 0$.

From these properties of f one easily finds

$$\frac{\partial \text{Var}[ID_1(k(s), s, \tau)|\phi(t)]}{\partial \tau} = 2 \cdot [e^{(s-t_S)/\alpha}]^2 \cdot \sigma^2 \cdot (s-t) \cdot f(\tau) \cdot f'(\tau) > 0, \quad (36)$$

$$\frac{\partial^2 \text{Var}[ID_1(k(s), s, \tau)|\phi(t)]}{\partial s \partial \tau} = 2\sigma^2 \cdot f(\tau) \cdot f'(\tau) \cdot [e^{(s-t_S)/\alpha}]^2 \cdot \left[\frac{2(s-t)}{\alpha} + 1 \right] > 0. \quad (37)$$

(36) establishes that the variability of interest rate differentials on the ID_1 -branch is *ceteris paribus* higher for longer terms than for shorter terms for every future date s .

¹¹Although always continuous at the branching point $t_S - \tau$, the whole variance path for strictly positive term $\tau > 0$ represented by the successive $\text{Var}[ID_1(k(s), s, \tau)|\phi(t)]$ - and $\text{Var}[ID_2(k(s), s, \tau)|\phi(t)]$ -curves is in general not differentiable at $s = t_S - \tau$ (see the variance path for τ_1 in Figure 2).

Additionally, it follows from (37) that the ID_1 -variance-path is steeper for longer than for shorter terms. For the ID_2 -branch we have

$$\frac{\partial \text{Var}[ID_2(k(s), s, \tau)|\phi(t)]}{\partial \tau} = -\frac{2}{\tau^3} \cdot \left[1 - e^{(s-t_S)/\alpha}\right]^2 \cdot \sigma^2 \cdot (s-t) < 0, \quad (38)$$

i.e. the variability of interest rate differentials on the ID_2 -branch is *ceteris paribus* lower for longer-term bonds than for shorter-term bonds. In contrast to (37), the sign of the mixed partial derivative

$$\frac{\partial^2 \text{Var}[ID_2(k(s), s, \tau)|\phi(t)]}{\partial s \partial \tau}$$

is undetermined so that no general statement is possible about the steepness of the ID_2 -variance paths with respect to the term length.

When comparing the variability of two alternative interest rate differentials with strictly positive but different terms, one has to bear in mind that there is a time interval during the interim period on which the differential with smaller term is still on its ID_1 -path while the differential with longer term has already reached its ID_2 -branch (see region $(t_S - \tau_2, t_S - \tau_1)$ in Figure 2). Formally, consider the variability of N interest rate differentials with terms $\tau_1 < \tau_2 < \dots < \tau_N$. All interest rate differentials will be on their ID_1 -variance path for $s \in (t, t_S - \tau_N)$. According to (36) one finds for this region

$$\begin{aligned} \text{Var}[ID_1(k(s), s, \tau_1)|\phi(t)] &< \text{Var}[ID_1(k(s), s, \tau_2)|\phi(t)] \\ &< \dots < \text{Var}[ID_1(k(s), s, \tau_N)|\phi(t)]. \end{aligned}$$

On the other hand, all interest rate differentials will be on their ID_2 -variance path for $s \in (t_S - \tau_1, t_S)$. Then (38) establishes

$$\begin{aligned} \text{Var}[ID_2(k(s), s, \tau_1)|\phi(t)] &> \text{Var}[ID_2(k(s), s, \tau_2)|\phi(t)] \\ &> \dots > \text{Var}[ID_2(k(s), s, \tau_N)|\phi(t)]. \end{aligned}$$

Except for the edges $(t, t_S - \tau_N)$ and $(t_S - \tau_1, t_S)$, where the interest rate differentials for longer terms exhibit higher variability at the left and lower variability at the right edge, the ranking of ID-variability with respect to the term τ crucially hinges on the specific structural parameters $\sigma, \alpha, t, t_S, \tau_1, \dots, \tau_N$.

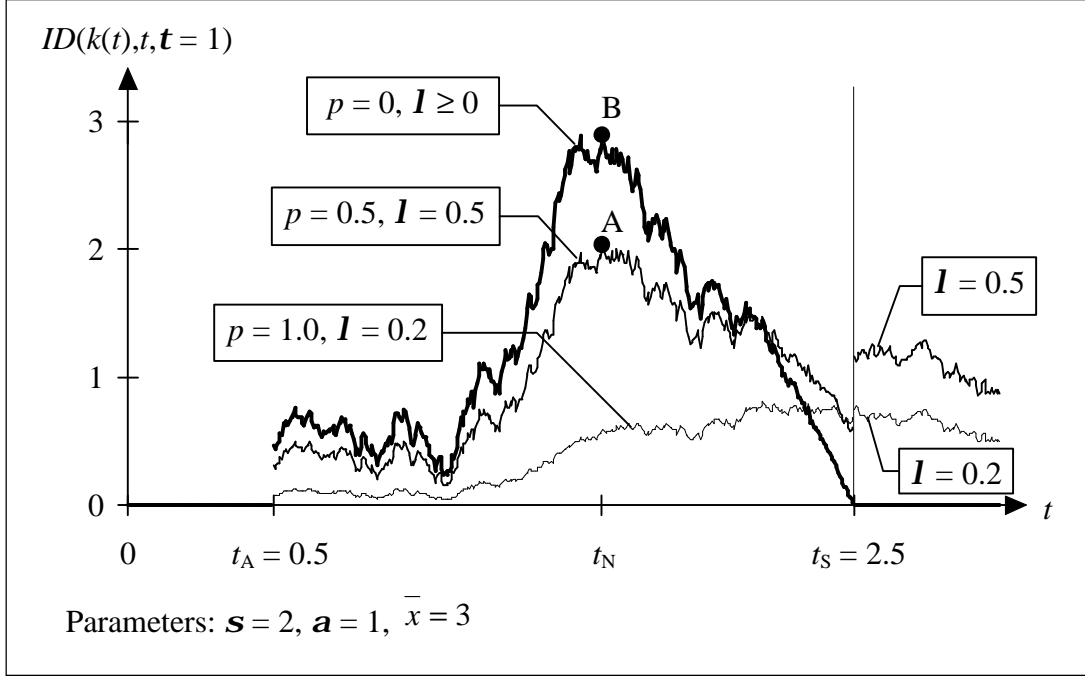


Figure 3: Interest rate differentials for the term $\tau = 1$ under uncertainty

4.2 Uncertainty about punctual regime switching

In contrast to the preceding subsection, it is now assumed that the fixing date t_S is possibly not considered fully credible by the market in the sense that agents take into account a delay in the regime switch beyond t_S . Their uncertainty assessment is represented by the parameters p and λ in the cumulative distribution function $F_Z(s; p, \lambda)$ for the lifetime of the pre-switch free-float regime from Eq. (4). The relevant interest rate differentials are thus given by the Eqs. (14), (15) and (19).

Figure 3 displays the interest rate differential paths before, during, and after the interim period $[t_A, t_S]$ for the term $\tau = 1$ where all paths were generated with the same structural parameters σ, α, \bar{x} but under alternative assessments about the punctuality of the switch. The parameter constellation $(p = 0, \lambda \geq 0)$ represents the situation of Subsection 4.1 with the market assessing the regime switch to occur punctually at t_S . For the two other (p, λ) -constellations market participants reckon with a more or less pronounced delay while —according to Eq. (6)—they expect the switch to take place at $t_S + p/\lambda$.

First, it is striking that—in contrast to the certainty-situation from Section 4.1—the interest rate differentials for strictly positive terms under uncertainty do not necessarily have to converge to zero at the end of the interim period $[t_A, t_S]$. If—infinitesimally

shortly before t_S —the market deems a delay in the regime switch possible (i.e. $p \neq 0$ shortly before t_S), domestic and foreign interest rates will (almost surely) not adjust smoothly at the end of the interim period. In fact, one finds from Eq. (19)

$$\lim_{t \uparrow t_S} ID_2(k(t), t, \tau) = \frac{p}{1 + \alpha \cdot \lambda} \cdot \left[\frac{1 - e^{-\lambda \cdot \tau}}{\tau} \right] \cdot \left[\bar{x} - \lim_{t \uparrow t_S} k(t) \right]$$

which is different from zero with probability 1. Figure 3 further illustrates that the differentials will necessarily exhibit a jump at t_S if $p \neq 0$ infinitesimally shortly before t_S . For this, consider the differential for the constellation $(p, \lambda) = (0.5, 0.5)$. At t_S there are two possible jumps each being explicable by the arrival of news at t_S :

- (a) First, let the authorities in fact implement the regime switch punctually at t_S . In this case it is news for the market that the switch was actually executed at t_S , an event that—due to the agents’ uncertainty assessment—happens somewhat unexpectedly. The differential jumps immediately on the zero-line at t_S because of the prompt equalization of domestic and foreign interest rates. However, it is important to note that this scenario is not very realistic. If the authorities in fact implement the regime switch punctually at t_S , market participants will be most likely to anticipate this far before t_S and therefore revise their uncertainty assessment by a corresponding change in the parameters p and λ . Such a situation is visualized in Figure 3. For all dates $t < t_N$ the agents’ uncertainty assessment is given by $(p, \lambda) = (0.5, 0.5)$. At date t_N news are arriving on the scene inducing market participants homogeneously to believe that the switch will happen punctually at t_S . Consequently they set $(p = 0, \lambda \geq 0)$ and the differential jumps at t_N from point A to point B on the certainty-differential. Provided that the market does not revise this last assessment any further, domestic and foreign interest rates will eventually smoothly adjust at the end of the interim period.
- (b) A second possible jump of the differential at date t_S takes place if the authorities do not fix the exchange rate at t_S while market participants associate the probability distribution (9) with the future timing of the regime switch. In this case the news consists of the information that the switch in fact happened to delay. At t_S the interest rate differential for strictly positive terms then jumps from the ID_2 -path (19) on the path (23).¹² Figure 3 displays two alternative in-

¹²For the sake of completeness it should be noted that this jump only occurs if $p \neq 1$ infinitesimally shortly before t_S . This may be ascertained formally by verifying that for $p = 1$ there will be an almost sure continuous transition from the differential (19) to the path (23).

terest rate differentials of the form (23) for the time after t_S (labelled by $\lambda = 0.2$ and $\lambda = 0.5$) each representing a possible continuation of the uncertainty paths $(p, \lambda) = (1.0, 0.2)$ and $(p, \lambda) = (0.5, 0.5)$.

The next question is how uncertainty about the punctuality of the regime switch affects the variability of interest rate differentials during the interim period $[t_A, t_S]$. The forward-looking variance paths under uncertainty can be derived along the same arguments as their certainty counterparts from Section 4.1. The ID-variance paths follow from Eqs. (14), (15) and (19) and are found to be

$$\text{Var}[ID(k(s), s, 0)|\phi(t)] = \left[1 - \frac{p}{1 + \alpha \cdot \lambda}\right]^2 \cdot \frac{1}{\alpha^2} \cdot \left[e^{(s-t_S)/\alpha}\right]^2 \cdot \sigma^2 \cdot (s - t), \quad (39)$$

$$\begin{aligned} \text{Var}[ID_1(k(s), s, \tau)|\phi(t)] &= \left[1 - \frac{p}{1 + \alpha \cdot \lambda}\right]^2 \cdot \left[e^{(s-t_S)/\alpha}\right]^2 \\ &\quad \times \left[\frac{1 - e^{\tau/\alpha}}{\tau}\right]^2 \cdot \sigma^2 \cdot (s - t), \end{aligned} \quad (40)$$

$$\begin{aligned} \text{Var}[ID_2(k(s), s, \tau)|\phi(t)] &= \frac{1}{\tau^2} \cdot \left[1 - \frac{p}{1 + \alpha \cdot \lambda} \cdot e^{\lambda \cdot (t_S - s - \tau)}\right. \\ &\quad \left. - \left(1 - \frac{p}{1 + \alpha \cdot \lambda}\right) \cdot e^{(s-t_S)/\alpha}\right]^2 \cdot \sigma^2 \cdot (s - t) \end{aligned} \quad (41)$$

for instantaneous (zero term) differentials and the two ID-branches for strictly positive terms, respectively.

Figure 4 illustrates three ID-variance-paths for different terms under the uncertainty scenario ($p = 0.8, \lambda = 0.5$). First, note that in accordance with the certainty case there is a continuous (but in general non-differentiable) transition from the ID_1 - to the ID_2 -variance path at the branching-point $t_S - \tau$, i.e.

$$\lim_{s \uparrow t_S - \tau} \text{Var}[ID_1(k(s), s, \tau)|\phi(t)] = \lim_{s \downarrow t_S - \tau} \text{Var}[ID_2(k(s), s, \tau)|\phi(t)].$$

Second, observe that the variance paths (39) for zero term and the ID_1 -branch (40) under uncertainty only differ from their certainty counterparts (28) and (29) by the premultiplicative term

$$\left[1 - \frac{p}{1 + \alpha \cdot \lambda}\right]^2 \quad (42)$$

which is independent of both, the future date s and the term τ . Hence, it follows from

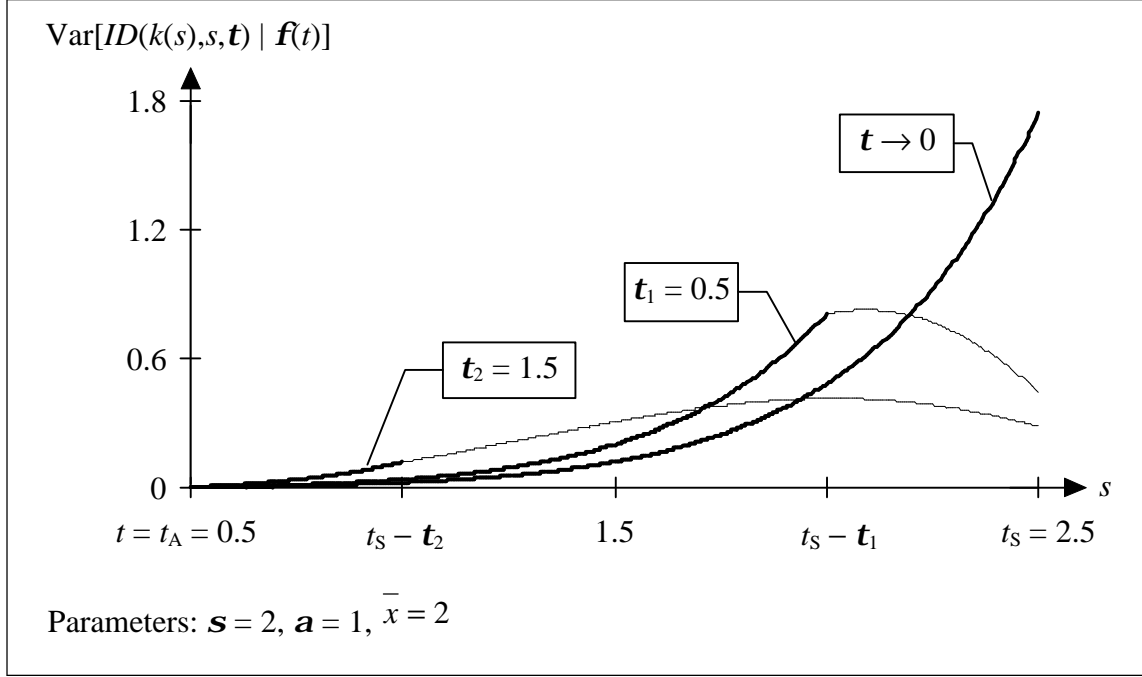


Figure 4: Variance paths under the uncertainty scenario ($p = 0.8, \lambda = 0.5$)

(31), (32), (36) and (37) that under uncertainty we have

$$\begin{aligned} \frac{\partial \text{Var}[ID(k(s), s, 0) | \phi(t)]}{\partial s} &> 0, \quad \frac{\partial \text{Var}[ID_1(k(s), s, \tau) | \phi(t)]}{\partial s} > 0, \\ \frac{\partial \text{Var}[ID_1(k(s), s, \tau) | \phi(t)]}{\partial \tau} &> 0, \quad \frac{\partial^2 \text{Var}[ID_1(k(s), s, \tau) | \phi(t)]}{\partial s \partial \tau} > 0 \end{aligned}$$

implying the same qualitative ID-variability patterns for zero-term and along the ID_1 -branch as under certainty (i.e. the variances increase with s and the ID_1 -variance paths are higher and steeper for longer than for shorter terms). In Subsection 4.1 it was proved that under certainty the variability of interest rate differentials along the ID_2 -branch is *ceteris paribus* lower for longer than for shorter terms. Although the same is shown in Figure 4 for $s > t_S - \tau_1$, this cannot be confirmed analytically as the sign of the partial derivative

$$\frac{\partial \text{Var}[ID_2(k(s), s, \tau) | \phi(t)]}{\partial \tau}$$

can in general change for s ranging on its admissible domain.

Another difference to the certainty case is that the ID-variance for strictly positive terms do not have to converge to zero for s tending towards t_S . According to Eq. (41)

it follows that

$$\begin{aligned} \lim_{s \rightarrow t_S} \text{Var} [ID_2(k(s), s, \tau) | \phi(t)] &= \frac{1}{\tau^2} \cdot \left[\frac{p}{1 + \alpha \cdot \lambda} \right]^2 \\ &\times \left[1 - e^{-\lambda \cdot \tau} \right]^2 \cdot \sigma^2 \cdot (t_S - t) \end{aligned} \quad (43)$$

which, in general, is greater than zero. Observe from Eq. (43) that the ID-variance for positive terms also vanishes for $s \rightarrow t_S$ under uncertainty whenever $\lambda = 0$. Although surprising at first glance, there is an appealing interpretation for this: $\lambda = 0$ means that agents believe the exchange rate regime switch to occur punctually at t_S with probability $1 - p$ but, if it does not happen at t_S , it will never be implemented. This implies two possible outcomes for any interest rate differential after t_S :

- (a) The irreversible regime switch happens at t_S (with probability $1 - p$) and according to Eq. (20) the interest rate differential will be zero for ever after.
- (b) The switch does not happen at t_S and the free-float regime will continue forever. In this case, according to Eq. (13), the arbitrarily termed differential will also equal zero.

In each case, the interest rate differential will be constant after t_S and hence its variance will be zero.

The final question is how changes in the market's uncertainty assessment about the timing of the exchange rate regime switch affect the variability of an interest rate differential for a given term. Figure 3 displays the dynamics of an interest rate differential for the term $\tau = 1$ under the alternative uncertainty scenarios $(p = 0, \lambda \geq 0)$, $(p = 0.5, \lambda = 0.5)$ and $(p = 1.0, \lambda = 0.2)$. Evidently, the differential associated with the constellation $(p = 1.0, \lambda = 0.2)$ exhibits the lowest oscillations at least on the interval $[t_A, t_S - \tau)$ implying the lowest variance on this region.

Figure 5 illustrates the variance-paths of the interest rate differentials for the term $\tau = 1$ and the uncertainty scenarios from Figure 3. The three curves give rise to the following conjecture: When comparing alternative uncertainty scenarios it is always the (p, λ) -constellation associated with the most distanced expected regime-switching date $t_S + p/\lambda$ which entails the lowest variance path at least on the interval $(t, t_S - \tau)$. To strengthen this conjecture further, consider the extreme market assessment $(p = 1, \lambda = 0)$ yielding an expected switching date $t_S + p/\lambda$ in the infinite future. It is easy to check that under this (p, λ) -representation the three variance paths (39), (40) and (41) all reduce to zero for every $s \in (t, t_S)$ and arbitrary term τ .

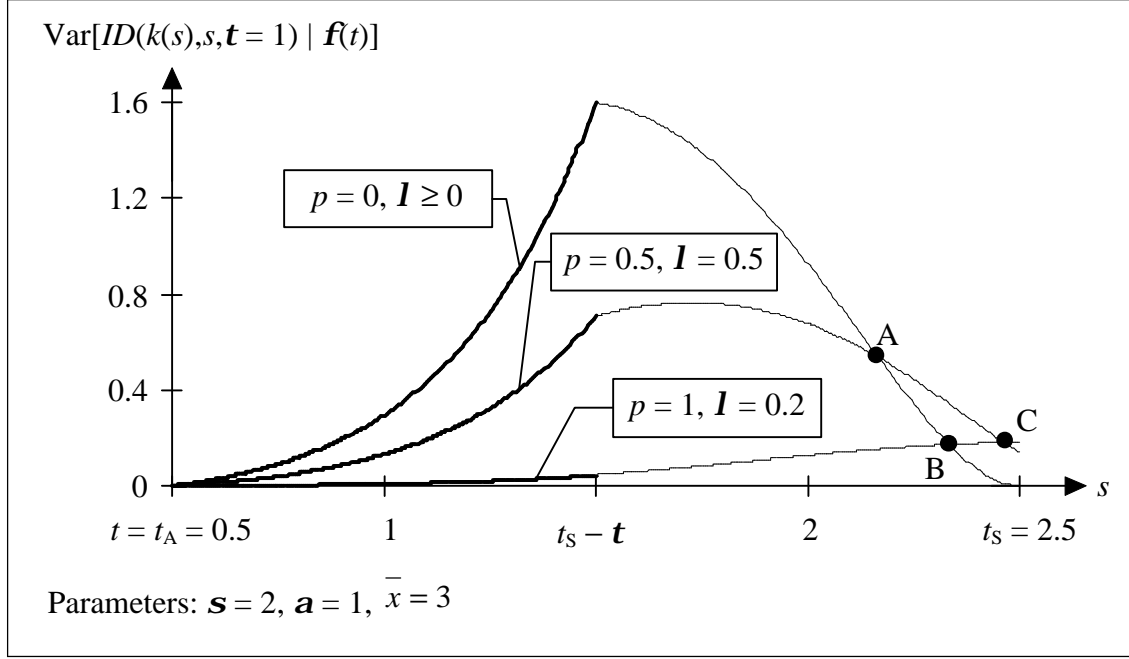


Figure 5: Variance paths for term $\tau = 1$ under alternative uncertainty scenarios

For a formal analysis of the responsiveness of the ID-variability to changes in the uncertainty parameters p and λ , observe that the variance paths (39) and (40) for zero term and along the ID_1 -branch only depend on p and λ through the factor (42), i.e.

$$h(p, \lambda) \equiv \left[1 - \frac{p}{1 + \alpha \cdot \lambda}\right]^2.$$

Hence, the signs of the partial derivatives of the variance paths (39) and (40) with respect to p and λ coincide with the signs of

$$\frac{\partial h(p, \lambda)}{\partial p} = -\frac{2}{1 + \alpha \cdot \lambda} \cdot \left[1 - \frac{p}{1 + \alpha \cdot \lambda}\right] < 0 \quad \text{for } (p, \lambda) \neq (1, 0), \quad (44)$$

and

$$\frac{\partial h(p, \lambda)}{\partial \lambda} = \frac{2 \cdot \alpha \cdot p}{(1 + \alpha \cdot \lambda)^2} \cdot \left[1 - \frac{p}{1 + \alpha \cdot \lambda}\right] > 0 \quad \text{for } p \neq 0, (p, \lambda) \neq (1, 0). \quad (45)$$

Obviously, the variance paths strictly decrease in p and strictly increase in λ for every admissible s . It is important to note that this result does not state an analytical confirmation of the above conjecture on the relationship between the level of the variance paths and the expected switching date $t_S + p/\lambda$.¹³ But the monotonic behaviour of

¹³On the contrary, concrete examples for two uncertainty constellations (p_1, λ_1) and (p_2, λ_2) with

the variance paths (39) and (40) implies that the ID-variance for every $s \in (t, t_S - \tau)$ and every term τ are monotonically decreasing (increasing) for increasing (decreasing) p -values while they are monotonically increasing (decreasing) for increasing (decreasing) λ -values. In all, under the free-float pre-switch regime the lowest variance path—namely that of a uniformly zero-variance for all $s \in (t, t_S)$ —is generated by the extreme uncertainty assessment $(p, \lambda) = (1, 0)$.¹⁴

Along the ID_2 -branch the variance path (41) cannot be uniformly ranked with respect to the uncertainty parameters p and λ in the sense just described. In general, the signs of the partial derivatives

$$\frac{\partial \text{Var}[ID_2(k(s), s, \tau)|\phi(t)]}{\partial p}, \quad \frac{\partial \text{Var}[ID_2(k(s), s, \tau)|\phi(t)]}{\partial \lambda}$$

depend on the parameters p and λ as well as on the explicit choice of the future date $s \in [t_S - \tau, t_S)$. This sign-dependence on s is visualized in Figure 5 where the ID_2 -variance paths (the thin lines) for alternative scenarios cross before t_S (see the points A, B, C).

5 Summary and conclusions

Based on a monetary flex-price exchange rate model this paper develops a complete term structure of interest rate differentials for a small open economy during the transition from flexible to fixed exchange rates. The closed-form solutions of the dynamics describing the international interest rate convergence for arbitrary terms enable an analytical investigation of several important aspects. So it is possible, for example, to fully determine the volatility processes of interest rate differentials during and after the interim period, a theoretical result useful in a variety of applications from financial economics (e.g. the valuation of interest-rate-sensitive securities or the selection of hedging strategies for domestic investors). Moreover, underlying a parametric uncertainty structure representing the financial markets' assessment of the exact timing of the exchange rate regime switch provides further insights into the equalization process of international interest rates. Clearly, frequent changes in the uncertainty assessment due to news (represented by exogenous revisions of the parameters p and λ) may introduce an additional source of volatility into the convergence process.

$t_S + p_1/\lambda_1 > t_S + p_2/\lambda_2$ may be found with the (p_1, λ_1) -variance path (40) lying completely above that for (p_2, λ_2) , so that the conjecture from above does not hold in general.

¹⁴The generality of this last statement for all $s \in (t, t_S)$ —and not only for $s \in (t, t_S - \tau)$ —results from the fact that the three variance paths (39), (40) and (41) all reduce constantly to zero for every s under the scenario $(p, \lambda) = (1, 0)$.

The paper exclusively focuses on a time-contingent exchange rate regime switch from a free-float system to completely fixed rates. Technically this is accomplished by letting the macroeconomic fundamental k evolve according to the (driftless) Brownian motion (2). From an EMU perspective it seems warranted to model a less flexible pre-switch exchange rate regime, e.g. a managed-float system. Representing the pre-switch dynamics of the fundamental by a mean-reverting Ornstein-Uhlenbeck process may portray such a more active intervention policy adequately. It is straightforward to derive the corresponding dynamics of the exchange rate and the interest rate differentials by means of stochastic calculus. While most qualitative features of the interest rate differential paths from the preceding sections remain valid under a transition from managed-float to fixed rates there are also some differences induced by the alternative pre-switch regimes. As the most striking and empirically most relevant dissimilarity it can be shown that under a managed-float pre-switch system a clear-cut relationship between the differential volatility along the ID_1 -path and the term τ as in Eq. (36) no longer holds (cf. Wilfling 2001).

A final important macroeconomic result holding under both pre-switch regimes may be stated most easily by considering the pre-switch free-float system from above and in particular the exchange rate path (7) during the interim period. It can be verified from Eq. (7) that under any uncertainty assessment $(p, \lambda) \neq (1, 0)$ the mere announcement of future exchange rate fixing triggers a significant reduction in the exchange rate variances—as compared to the volatility induced by the permanent free-float path (3)—during the interim period (see Wilfling and Maennig 2001, Subsection 4.3). In this context two points are important to note: (a) Exchange rate volatility is reduced even if markets deem a delay in the announced regime switch possible (except for $(p, \lambda) = (1, 0)$ where agents expect the pre-switch free-float system to hold permanently). (b) The highest reduction in exchange rate volatility is achieved if markets expect the regime switch to occur punctually at the fixing date preannounced by the authorities, i.e. for $(p = 0, \lambda \geq 0)$.

At first glance, the reduction in exchange rate volatility by the announcement of future regime switching alone—i.e. in the absence of any actual interventions in the foreign exchange market—arises the impression of a free lunch. But in conjunction with the relations (44) and (45) it is shown in Section 4.2 that any uncertainty assessment $(p, \lambda) \neq (1, 0)$ entails a higher variability of interest rate differentials for arbitrary terms as compared to the permanent free-float situation (cf. Footnote 14). Obviously, at least a certain amount of the reduced exchange rate variability goes into the volatility of interest rates. Or, interpreted more economically: During the interim period the

economy under consideration has to pay for the reduction in exchange rate volatility by an increase in the uncertainty about the evolution of domestic interest rates.

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