

Chen, Yu-Fu; Funke, Michael

**Working Paper**

## Exchange rate uncertainty and labour market adjustment under fixed and flexible exchange rates

HWWA Discussion Paper, No. 196

**Provided in Cooperation with:**

Hamburgisches Welt-Wirtschafts-Archiv (HWWA)

*Suggested Citation:* Chen, Yu-Fu; Funke, Michael (2002) : Exchange rate uncertainty and labour market adjustment under fixed and flexible exchange rates, HWWA Discussion Paper, No. 196, Hamburg Institute of International Economics (HWWA), Hamburg

This Version is available at:

<https://hdl.handle.net/10419/19357>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

# **Exchange Rate Uncertainty and Labour Market Adjustment under Fixed and Flexible Exchange Rates**

**Yu-Fu Chen  
Michael Funke**

HWWA DISCUSSION PAPER

**196**

Hamburgisches Welt-Wirtschafts-Archiv (HWWA)  
Hamburg Institute of International Economics

2002

ISSN 1616-4814

Das HWWA ist Mitglied der:

- Wissenschaftsgemeinschaft Gottfried Wilhelm Leibniz (WGL)
- Arbeitsgemeinschaft deutscher wirtschaftswissenschaftlicher Forschungsinstitute (ARGE)
- Association d'Instituts Européens de Conjoncture Economique (AIECE)

# **Exchange Rate Uncertainty and Labour Market Adjustment under Fixed and Flexible Exchange Rates**

**Yu-Fu Chen  
Michael Funke**

The subject of this paper is assigned to the HWWA's research programme "International Financial Markets".

**HWWA DISCUSSION PAPER**

**Edited by the  
PRESIDENTIAL DEPARTMENT**

Hamburgisches Welt-Wirtschafts-Archiv (HWWA)  
Hamburg Institute of International Economics

Öffentlichkeitsarbeit  
Neuer Jungfernstieg 21 – 20347 Hamburg  
Telefon: 040/428 34 355  
Telefax: 040/428 34 451  
e-mail: [hwwa@hwwa.de](mailto:hwwa@hwwa.de)  
Internet: <http://www.hwwa.de/>

Yu-Fu Chen  
Department of Economic Studies  
University of Dundee  
Dundee DD 1 4HN, Great Britain  
E-mail: [y.f.chen@dundee.ac.uk](mailto:y.f.chen@dundee.ac.uk)

Michael Funke  
Department of Economics  
Hamburg University  
Von-Melle-Park 5  
20146 Hamburg, Germany  
E-mail: [funke@hermes1.econ.uni-hamburg.de](mailto:funke@hermes1.econ.uni-hamburg.de)

## CONTENTS

Abstract	VI
I. Introduction	1
II. Exchange Rate Uncertainty and Labour Demand	2
III. Numerical Solutions to the Model	12
IV. Summary and Discussion	20
Appendix	22
References	29

### List of Figures

Figure 1: The Impact of $\lambda$ Upon the Exchange Rates Thresholds and Hours Thresholds for $T = 0.1$ and $F = 0.6$	14
Figure 2: The Impact of $\lambda$ Upon the Exchange Rates Thresholds and Hours Thresholds for $T = 0.2$ and $F = 0.8$	14
Figure 3: The Impact of $\lambda$ Upon the Exchange Rates Thresholds and Hours Thresholds for $T = 0.3$ and $F = 0.9$	15
Figure 4: The Impact of $\lambda$ Upon the Exchange Rates Thresholds and Hours Thresholds for $H_S = 0.9$	17
Figure 5: The Impact of $\lambda$ Upon the Exchange Rates Thresholds and Hours Thresholds for $a = 1.2$ and $a = 1.8$	18
Figure 6: The Impact of the Speed of Mean Reversion ( $\lambda$ ) Upon the Exchange Rates Thresholds Thresholds in the Overtime and in the No Overtime Case	19

## **Abstract**

The standard literature on working time has modelled the decisions of firms in a deterministic framework in which firms can choose between employment and overtime (given mandated standard hours). Contrary to this approach, we follow the real options approach, which allows us to investigate the value to a firm of waiting to adjust labour when the firm's revenues in domestic currency are stochastic and adjustment costs are sunk. The simulations reject the null hypothesis that all exchange rate regimes obey common employment adjustment thresholds.

**Keywords:** Real Option Theory, Uncertainty, Working Time, Employment, Labour Demand, Exchange Rates

**JEL-Classification:** D81, E24, J23, F31

## I. Introduction

This paper presents theoretical results on the relationship between exchange rate uncertainty and labour demand across exchange rate regimes. The aim of our analysis is twofold. First, we would like to contribute to the long-running debate on employment. It is noteworthy that much of Europe has been struggling with high structural unemployment, low growth, rigid labour markets, excessive regulation and an overgenerous welfare state. The obvious and unflattering comparison has been with the United States whose economy proved itself more flexible and more resilient than Europe's. Secondly, we expect to reach some conclusion to the role of alternative exchange rate regimes. An important feature of the past two decades has been the increasing globalisation of production. This suggests heightened potential for employment and growth to be affected by exchange rate uncertainty. Both phenomena have therefore rightly absorbed the attention of economists and have spurred interest in labour market policy issues and discussion of the appropriate exchange rate regime has mushroomed. To date, however, exchange rate regimes and labour adjustment decisions have mainly been assessed in isolation, i.e. no attempt has been made to date to embed the above described exchange rate uncertainty across exchange rate regimes into a model dealing with labour demand decisions. In this paper, we will therefore analyse the implications of various exchange rate regimes for labour market policymaking. While it seems natural for the average person on the street to think that higher uncertainty means lower labour demand, results from theoretical models critically depend upon hypotheses made regarding the type of production technology, agents' preferences and the market regime. To those who are not familiar with the literature, learning that such disagreement exists may come at somewhat of a surprise.<sup>1</sup>

In this paper, we employ the recent real options literature on factor demand which highlights the potentially negative impact of uncertainty on those expenditures which are postponable and entail a fixed cost.<sup>2</sup> The general idea behind the theory of real

---

1 The results are ambiguous and depend – among other things – on what methodology is employed. For example, it is well-known that factor demand and uncertainty are positively correlated in a standard neoclassical model. This result depends upon the convexity of the firm's profit function with respect to the stochastic variable. A mean-preserving increase in uncertainty, by Jensen's inequality, determines a higher expected present value of future profits and, ceteris paribus, makes it convenient to expand employment. The positive relationship between factor demand and uncertainty is, however, at odds with what seems to occur in the real world, where media often report the concern of firms for the negative effects of uncertainty on project returns and, therefore, on their willingness to expand capacity.

2 A substantial number of papers have been developed within the real options approach. Amran and Kulatilaka (1999), Coy (1999) and Dixit and Pindyck (1994) provide comprehensive surveys. Dixit



options is that each investment operation can be assimilated, in its nature, to the purchase of the investor of a financial call option, where the investor pays a premium price in order to get the right to buy an asset for some time at a price predetermined, and eventually different from the spot market price of the asset. Taking into account this approach, the calculus of profitability of each labour adjustment decision cannot be done simply applying the net present value, but has rather to consider the following particular characteristics of a labour adjustment decision:

1. there is uncertainty over the future rewards from the employment decision;
2. there is some leeway about the timing of the employment decision;
3. the employment decision is partially or completely irreversible.

The first characteristic of labour adjustment decisions derives from the fact that firms have no perfect information. As a result, they form expectations and beliefs on the future behaviour of the driving economic variables, which cannot be predicted with certainty. The second characteristic is directly related to the uncertainty: for an individual firms the decision involved in labour adjustment is not just of *whether* to adjust labour, but *when*.<sup>3</sup> This of course entails an opportunity cost of waiting. Finally, the firm has to take into account the fact that labour adjustment costs are at least partially sunk. As a result, the weight of the uncertainty in the determination of the labour adjustment decisions is higher the higher is the sunk cost of the investment.

The remainder of this paper is structured as follows. Section II develops a theoretical model which describes how exchange rate uncertainty might influence labour adjustment decisions across exchange rate regimes. Section III discusses the calibration results. Finally, section IV touches on some of the policy implications of our findings and suggests avenues for future research.

## **II. Exchange Rate Uncertainty and Labour Demand**

Following in the footsteps of Bentolila and Bertola (1990) and Booth et al. (2002) we begin by developing a continuous-time model of labour demand where the exchange

---

(1989) discusses a single firm's export decision problem under uncertain exchange rates. His model on export-market penetration is something of a classic now. Focusing upon the employment impacts of exchange rate uncertainty, we take up an idea originally proposed by Dixit (1989, p. 624, fn. 3).

3 The assumption implies that firms are not pressed to undertake the employment adjustment in order to gain first mover advantages.

rate evolves in a stochastic manner. The novelty of this paper is that we model employment and hours adjustment under exchange rate uncertainty in different exchange rate regimes. To the best of our knowledge, this relationship has not yet been modelled.<sup>4</sup>

A firm is defined by its access to a particular production technology. In this paper the constant returns production function of the representative firm in terms of value added ( $Y$ ) is denoted by a general three factor *CES* production function which has the form

$$(1) \quad Y = \phi \left[ \theta_K K^{-\mu} + \theta_H g(H)^{-\mu} + (1 - \theta_K - \theta_H) N^{-\mu} \right]^{-1/\mu},$$

where  $-1 < \mu < \infty$  is the substitution parameter ( $\mu \neq 0$ ), and  $0 < \theta_K, \theta_H < 1$  are the distribution parameters,  $H$  is actual working hours,  $K$  is the given capital stock,  $N$  is the number of employees,  $x$  the fixed costs of employment, and  $\phi$  is the efficiency parameter. In (1) we treat employment ( $N$ ), which is the extensive margin by which labour input can be varied, as a separate input from hours ( $H$ ), which is the intensive margin. The elasticity of substitution is given by  $b = 1/(1+\mu)$ . In accordance with the concept of a short run production function, the capital stock  $K$  is taken as given at any point in time, giving rise to strict concavity of the production function.

The model is based on a standard partial equilibrium model of a small open economy. Since we are dealing with a small open economy, firms take goods prices set in world markets as given. We assume that the firm has some market power and faces an isoelastic demand function

$$(2) \quad p = E(p^* Y)^{(1-\psi)/\psi}, \quad \psi \geq 1,$$

where  $p^*$  denotes the constant foreign price,  $p^* Y$  is the firm's revenue in foreign currency,  $\psi$  is an elasticity parameter that takes its minimum value of 1 under perfect competition and  $E$  denotes the nominal exchange rate which is assumed to move unpredictably over time.<sup>5</sup> Given these assumptions, exchange rate uncertainty gets

---

4 The theoretical work that has been done on how exchange rate uncertainty influences labour adjustment is very scant. Belke and Göcke (1999) suppose a discrete binary stochastic change in the exchange rate which is either non-recurrent or only occurs twice. This implies that they take a much simpler technical approach which is unrealistic in large part because it is totally resolved after one (two) period(s).

5 The assumption that domestic firms produce only for foreign markets is for heuristic simplicity and not required for our results. The key requirement is for foreign demand fluctuations to be more strongly correlated with the exchange rate than is domestic demand for domestic products. For

translated into uncertainty about a firm's revenue in domestic currency and/or product demand. The production function and the demand function together give us an expression for the firm's revenues

$$(3) \quad \Pi = \phi E p^* \frac{1-\psi}{\psi} \left[ \theta_K K^{-\mu} + \theta_H g(H)^{-\mu} + (1 - \theta_K - \theta_H) N^{-\mu} \right]^{\frac{-1}{\mu\psi}} - [w(H) + x]N$$

where  $w(\cdot)$  is the wage, and  $x$  is the fixed costs of employment.<sup>6</sup> It is important to note that  $x$  (for example, work space for the workers) is interpreted as a flow in (3). The existence of fixed costs per worker  $x$  tends to make firms want higher hours in order to spread these costs over more hours of work. The risk-neutral firm chooses actual hours and employment to maximise its expected discounted value of profits. The firm encounters no firing and hiring costs and aims to maximise the expected discounted profit stream, taking as given the quantity of the quasi-fixed factor, capital:

$$(4) \quad V = \max_{N,H} \int_0^{\infty} \left[ \phi E p^* \frac{1-\psi}{\psi} \left[ \theta_K K^{-\mu} + \theta_H g(H)^{-\mu} + (1 - \theta_K - \theta_H) N^{-\mu} \right]^{\frac{-1}{\mu\psi}} - [w(H) + x]N \right] e^{-rs} ds$$

where  $r$  is the constant real interest rate. To introduce uncertain exchange rates across different exchange rate regimes in an analytical tractable manner, we assume that the exchange rate follows the mean-reverting Ornstein-Uhlenbeck process in continuous time

$$(5) \quad dE = \lambda(\bar{E} - E)Edt + \sigma Ed\varpi,$$

where  $\lambda$  is the speed of mean-reversion,  $\bar{E}$  is the equilibrium exchange rate level and  $\sigma$  is the variance parameter. The component  $\varpi$  in (5) is a Wiener process, and

---

purposes of exposition, we have also put all uncertainty into the revenues and none into the costs of labour. It should be obvious from the model that it is uncertainty about the profitability of the project which is crucial.

6 We take the wage as given although hiring and firing costs give incumbent workers some market power. The potential for incumbent workers to extract rents from hiring and firing costs was used by Lindbeck and Snower (1988) to explain hysteresis in unemployment. Additionally, van der Ploeg (1987) has shown that partial irreversibility may result in a time inconsistency because unions may have the irresistible urge to increase wages at a high rate in the future. In either case, firms would learn that the union may undertake actions different from those initially announced and try to anticipate its choices. The ideal would therefore be a fully dynamic model with an endogenous wage. Unfortunately, such models have thus far proved intractable. Hamermesh and Pfann (1996, p. 1270) comment that a departure from the assumption of a fixed wage leads to „substantial complications“.

$d\varpi = \varepsilon\sqrt{dt}$ , where  $\varepsilon \sim N(0,1)$  is a white noise stochastic process. Equation (5) indicates that exchange rates follow a geometric Brownian motion without drift if  $\lambda = 0$  and a mean-reverting process if  $\lambda > 0$ . Thus, the generality of the approach allows for an examination of employment responses across exchange rate regimes. For  $\lambda = 0$ , the stochastic process tends to wander far from its starting point and therefore represents freely floating exchange rates. For  $\lambda > 0$ , the process fluctuates randomly up and down in the short run, but in the medium to long run it tends back towards the equilibrium value (central parity). In economic terms, this means that equation (5) characterises the reduced form behaviour of an exchange rate in a (credible) target zone model.<sup>7</sup>

To emphasise that the results have nothing to do with risk aversion, it is assumed that firms are risk neutral.<sup>8</sup> By Itô's lemma we know that the market value of the firm expected at time zero satisfies the following condition

$$(6) \quad rV = \max_{N,H} \left\{ \phi E p^* \frac{1-\psi}{\psi} \left[ \theta_K K^{-\mu} + \theta_H g(H)^{-\mu} + (1 - \theta_K - \theta_H) N^{-\mu} \right]^{\frac{-1}{\mu\psi}} - [w(H) + x]N + \lambda(\bar{E} - E)EV_E + \frac{1}{2}\sigma^2 E^2 V_{EE} \right\}$$

where  $V_E$  and  $V_{EE}$  are the first and second derivatives of  $V$  with respect to  $E$ . The first term on the right-hand side is revenue,  $[w(H) + x]N$  is the employment-related bill, and the last two terms represent the changes caused by exchange rates fluctuations.

Since future revenues are stochastic, we cannot expect to find specific hiring or firing dates; therefore the objective is to find a rule stating under which conditions is optimal to *stop* or to *continue*. In the case of the employment decision, *stop* means to hire or fire, while *continue* means to postpone such a decision and reserve the option to adjust labour. In other words, the issue of finding an optimal stopping rule is analogous to optimally exercising an American call option in the case of hiring, and an American

---

7 For details on this, see Dixit and Pindyck (1984), pp. 74-79. The parameters  $\lambda$  and  $\sigma$  determine the degree of mean-reversion and therefore the width of the exchange rate band. We do not address the issue of uncertainty associated with potential currency realignments, i.e. we assume that monetary policy is credibly committed to avoiding realignments. The sceptical reader can regard the following as a normative exercise extendable in a straightforward way to cases involving stochastic realignments by mixing the Ornstein-Uhlenbeck process with a discrete Poisson-process [see Böhm and Funke (2002) for further details].

8 The assumption of risk-neutrality can be relaxed and risk-aversion can be assumed instead, for example by using the CAPM and calculating a risk-adjusted discount rate [see Harrison and Kreps (1979)].

put option in the case of firing. The problem is simplified by considering the optimal solution for  $H$  first. The first-order condition for  $H$  is given by:

$$(7) \phi \frac{\theta_H}{\psi} E p^* \frac{1-\psi}{\psi} g(H)^{-\mu-1} g'(H) \left[ \theta_K K^{-\mu} + \theta_H g(H)^{-\mu} + (1-\theta_K - \theta_H) N^{-\mu} \right]^{\frac{-1}{\mu\psi}-1} - w'(H)N = 0$$

After solving (7), the variable  $H$  become a function of  $E$  given the functions  $w(\cdot)$  and  $g(\cdot)$ . The firm can choose to adjust employment at time  $t_0$  but the time  $t_0$  at which the project is initiated is at the discretion of the firm. For the sake of simplicity, we first deal with identification of uncertainty effects in the special case where hiring and firing costs are zero. This special case turns out to be useful as a starting point and for comparisons. Then we turn to the more general case with positive hiring and firing costs. To find the optimal condition for employment adjustment *without* firing costs and hiring costs, we need to obtain the value of the marginal employed worker ( $v = V_N$ ). The derivative of (6) with respect to  $N$  is

$$(8) \quad rv = \Phi E - [w(H) + x] + \lambda(\bar{E} - E)EV_E + \frac{1}{2}\sigma^2 E^2 V_{EE}$$

where  $\Phi = \frac{1-\theta_K - \theta_H}{\psi} \phi p^* \frac{1-\psi}{\psi} N^{-\mu-1} \left[ \theta_K K^{-\mu} + \theta_H g(H)^{-\mu} + (1-\theta_K - \theta_H) N^{-\mu} \right]^{\frac{-1}{\mu\psi}-1}$  and

$v = V_N$  is the value of employing the marginal worker. The solution for  $v(E)$  consists of the particular solution (integral) and the complementary function. In Appendix A, we show that the particular integral may be expressed as the following approximate expected present value of the marginal employed workers  $v^P(E)$ :

$$(9) \quad v^P(E) = \Phi \bar{E} \int_0^{\infty} \frac{e^{-rs}}{1 + \left( \frac{\bar{E} - E}{E} \right) e^{-\bar{E}\lambda s}} ds - \frac{w(H) + x}{r},$$

which is the expected present value of the marginal employed worker. It can easily be demonstrated that in the limiting case  $\lambda \rightarrow 0$  ("free float"), the particular solution becomes

$$(10) \quad v^P(E) = \frac{\Phi E}{r} - \frac{w(H) + x}{r},$$

which is the case of the geometrical Brownian motion without drift. On the other hand, in the limiting case  $\lambda \rightarrow \infty$  (“monetary union”, “currency board system” or “dollarization”), the particular solution is given by the simple formulae

$$(11) \quad v^P(E) = \frac{\Phi \bar{E}}{r} - \frac{w(H) + x}{r}.$$

Equation (11) has an interesting and immediate interpretation: direct inspection verifies that a huge positive (negative) shock is required to make firms hire (fire) a marginal employee when the speed of mean-reversion is high. Considering only distinct two exchange rate regimes is obviously a simplification. Frankel (1999) identifies nine exchange rate regimes: currency union, currency board, truly fixed exchange rates, adjustable peg, basket peg, target zone or band, managed float or free float. It is, however, important to stress that most firms export to a large number of countries with different regimes. This implies that exporting firms cannot be assigned to a clearly identifiable group or legal regime and therefore intermediate regimes prevail *de facto*. In other words, in order to capture exchange rate uncertainty we have to rank firms on a continuous scale of exchange rate risk exposure rather than lumping them in arbitrarily defined regimes.<sup>9</sup>

Algebraically, the firm’s option value of hiring in the future and its option value of firing once the worker is employed are measured by the general (complementary) function

$$(12) \quad rv = \lambda(\bar{E} - E)EV_E + \frac{1}{2}\sigma^2 E^2 V_{EE}$$

It is known that the solutions for the hiring and firing options have the general form (see Appendix B and Dixit and Pindyck (1994), p. 162)

$$(13) \quad v_r^G(E) = A_1 E^{\beta_1} \Gamma(E; \beta_1),$$

and

---

9 Even the adoption of the Euro may not reduce the exchange rate uncertainty substantially. Nominal exchange rate volatility is completely eliminated within the EMU, but some amount of real exchange rate volatility will remain, due to differences in inflation rates and productivity. Moreover, if the Euro develops persistent exchange rate uncertainties *vis-à-vis* the other major currencies such as the US Dollar or the Japanese Yen, then the adoption of the single currency in itself will not remove the nominal exchange rate exposure.

$$(14) v_F^G(E) = A_2 E^{\beta_2} \Gamma(E; \beta_2),$$

where  $\Gamma(\cdot)$  denotes the confluent hypergeometric function and  $\beta_1$  and  $\beta_2$  are the positive and negative roots of the characteristic equation (B5) in Appendix B, respectively. To satisfy the boundary conditions  $v_T^G(0)=0$  and  $v_F^G(\infty)=0$ , we use the positive solution for  $v_T^G(E)$  and the negative solution for  $v_F^G(E)$ . For  $\lambda = 0$ , equations (13) and (14) simplify to

$$(15) v_T^G(E) = A_1 E^{\beta_1}$$

and

$$(16) v_F^G(E) = A_2 E^{\beta_2}$$

when the speed of mean-reversion approaches zero (“free float”).

In the scenario hypothesised so far, we have not considered hiring and firing costs. Now we explore the complications that arise when we add asymmetric fixed marginal *hiring* ( $T$ ) and *firing* ( $F$ ) costs to the model with both  $T$  and  $F$  being payable by the firm. Hiring and firing costs are a crucial feature of the labour market. In the model, firing a worker costs a constant amount  $F$  per worker in discounted present value terms. These costs of firing include the firing procedure itself (mandatory consultation with the work councils, notification of dismissals to a public agency and special protection for certain groups), as well as any termination payments or litigation. Hiring a worker costs  $T$  which includes advertising and selection, outlays on firm-specific training, and lost output while a new worker learns about the firm’s operations. Firm-specific training is probably the largest component of these hiring costs. Finally, observe that, in line with the existing empirical evidence, we are assuming that the firm incurs no costs of adjusting hours worked per worker. Emerson (1988) and Nicoletti et al. (1999) give detailed overviews on employment-protection legislation and other restrictions on hiring and firing in the industrialised countries.

The existence of hiring and firing costs implies that labour adjustment is partially irreversible. The firm will therefore consider the option value of maintaining her current position against the alternative of hiring or firing. The value of the marginal, employed worker is equal to the sum of  $v^P$  and  $v_F^G$  in the continuation region. In order to derive

the two thresholds for hiring and firing, we compare the value of the worker to the direct and indirect costs of hiring (firing) the workers. The definitions of the hiring and firing barriers,  $E_T$  and  $E_F$ , are given by the value-matching and smooth-pasting conditions below. It is straightforward to show that according to the value-matching conditions the firm would find it optimal to exercise its option to hire or fire the marginal worker once  $E$  hits one of the following two barriers:

$$(17) \quad \Phi \bar{E} \int_0^{\infty} \frac{e^{-rs}}{1 + \left( \frac{\bar{E} - E_T}{E_T} \right) e^{-\bar{E}\lambda s}} ds - \frac{w(H) + x}{r} + A_2 E_T^{\beta_2} \Gamma(E_T; \beta_2) = T + A_1 E_T^{\beta_1} \Gamma(E_T; \beta_1)$$

$$(18) \quad - \left[ \Phi \bar{E} \int_0^{\infty} \frac{e^{-rs}}{1 + \left( \frac{\bar{E} - E_F}{E_F} \right) e^{-\bar{E}\lambda s}} ds - \frac{w(H) + x}{r} \right] + A_1 E_F^{\beta_1} \Gamma(E_T; \beta_1) = F + A_2 E_F^{\beta_2} \Gamma(E_T; \beta_2)$$

When these conditions are violated, then the firm would adjust labour at suboptimal times. The left-hand sides of (17) and (18) show the marginal benefit from hiring/firing a worker and the right-hand sides the corresponding marginal costs. The marginal benefit of hiring a worker is equal to the sum of the present discounted value of his productivity net of wages and the value of the option to fire him. The firm's ability to fire raises the benefit from employing a worker. The marginal cost of hiring is the sum of the direct hiring costs and the sacrificed option to hire him in the future. By hiring a worker today, the opportunity to do so in the future – when conditions may be more favourable – is sacrificed. Similarly, by firing a worker, the opportunity to do so in the future – when demand conditions may be even more adverse – is sacrificed, and the opportunity to hire him again is gained. The value of the two options depends on expectations about changes in demand. The option to hire is valuable if firms expect demand to increase in the future, while the option to fire is the more important if they expect it to fall. The smooth-pasting conditions ensure that hiring (firing) is not optimal either before nor after the hiring (firing) threshold is reached. In technical terms, this means:

$$(19) \quad \Phi \bar{E} \int_0^{\infty} \frac{\bar{E} \cdot e^{-(r+\bar{E}\lambda)s}}{\left[ E_T + (\bar{E} - E_T) e^{-\bar{E}\lambda s} \right]^2} ds = A_1 \left[ \beta_1 E_T^{\beta_1-1} \Gamma(E_T; \beta_1) + E_T^{\beta_1} \Gamma'(E_T; \beta_1) \right] \\ - A_2 \left[ \beta_2 E_T^{\beta_2-1} \Gamma(E_T; \beta_2) + E_T^{\beta_2} \Gamma'(E_T; \beta_2) \right]$$



and

$$(20) \quad -\Phi \bar{E} \int_0^{\infty} \frac{\bar{E} \cdot e^{-(r+\bar{E}\lambda)s}}{\left[ E_F + (\bar{E} - E_F) e^{-\bar{E}\lambda s} \right]^2} ds = -A_1 \left[ \beta_1 E_F^{\beta_1-1} \Gamma(E_F; \beta_1) + E_F^{\beta_1} \Gamma'(E_F; \beta_1) \right] \\ + A_2 \left[ \beta_2 E_F^{\beta_2-1} \Gamma(E_F; \beta_2) + E_F^{\beta_2} \Gamma'(E_F; \beta_2) \right]$$

Equations (17) - (20) form a non-linear system of equations with four unknown parameters,  $E_T$ ,  $E_F$ ,  $A_1$ , and  $A_2$ , and can be solved for numerically once the solutions for  $\beta_1$  and  $\beta_2$  are obtained from (B5) in Appendix B and optimal values for  $H$  are found for the the values of  $E_T$  and  $E_F$  via

$$(21) \quad \phi \frac{\theta_H}{\psi} E_T p^* \frac{1-\psi}{\psi} g(H)^{-\mu-1} g'(H) \left[ \theta_K K^{-\mu} + \theta_H g(H)^{-\mu} + (1-\theta_K - \theta_H) N^{-\mu} \right]^{\frac{-1}{\mu\psi-1}} - w'(H)N = 0$$

and

$$(22) \quad \phi \frac{\theta_H}{\psi} E_F p^* \frac{1-\psi}{\psi} g(H)^{-\mu-1} g'(H) \left[ \theta_K K^{-\mu} + \theta_H g(H)^{-\mu} + (1-\theta_K - \theta_H) N^{-\mu} \right]^{\frac{-1}{\mu\psi-1}} - w'(H)N = 0.$$

When the speed of mean-reversion  $\lambda$  is zero (“free float”), then (17) - (20) simplify to:

$$(23) \quad \frac{\Phi E_T}{r} - \frac{w(H)+x}{r} + A_2 E_T^{\beta_2} = T + A_1 E_T^{\beta_1},$$

$$(24) \quad - \left[ \frac{\Phi E_F}{r} - \frac{w(H)+x}{r} \right] + A_1 E_F^{\beta_1} = F + A_2 E_F^{\beta_2},$$

$$(25) \quad \frac{\Phi}{r} - A_2 \beta_2 E_T^{\beta_2-1} = A_1 \beta_1 E_T^{\beta_1-1}$$

$$(26) \quad - \frac{\Phi}{r} + A_1 \beta_1 E_F^{\beta_1-1} = A_2 \beta_2 E_F^{\beta_2-1}$$

In order to calculate the thresholds for hiring ( $E_T$ ) and firing ( $E_F$ ) a marginal worker, we have to select a functional form for  $g(H)$  and  $w(H)$ . Following Hart (1987) and Santamäki (1984) we model labour services as a piece-wise continuous and nonlinear function of mandated standard hours  $H_s$  and actual hours worked  $H$ :

$$(27) \quad g(H) = \begin{cases} H_s^\gamma (H / H_s)^\delta & H > H_s \\ H^\gamma & H \leq H_s \end{cases}$$

where  $H_s$  is standard contract hours. Standard contract hours are exogenously given to the firm which determines actual hours as a control variable besides employment.<sup>10</sup> Following the literature we assume that  $0 < \delta < 1$  so that  $g(H)$  is strictly concave and the problem of the firm is well defined. An exogenous reduction of  $H_s$  may increase or decrease  $g(H)$  and – depending on the overtime wage premium – increase or decrease employment and labor services. Following Hart (1987) we introduce a piece-wise linear wage equation

$$(28) \quad w(H) = \begin{cases} w_s H_s + a w_s (H - H_s) & H > H_s \\ w_s H & H \leq H_s \end{cases}$$

according to which firms pay a constant premium  $a > 1$  on overtime hours  $(H - H_s)$ .<sup>11</sup> In other words,  $a$  is the legally determined multiple of the standard wage  $w_s$  paid for regular hours. Equations (3) and (28) imply that we allow for quasi-fixed labour costs and wage schedules that are increasing in hours worked. The corresponding thresholds for hours ( $H_T$  and  $H_F$ , respectively) can be calculated in a similar way. It should be evident that the hiring and firing policy of the optimising firm is discontinuous. In some periods the optimal strategy of the firm will be to adjust hours of work. In other circumstances, the firm will fire or hire. More specifically, employment inaction will always be chosen when deviations of the expected marginal product of labour from the optimal level do not justify the costs of employment adjustment. In such situations, the firm prefers to adjust the actual hours of work, i.e. overtime work provides "flexibility at the margin". Adjustments to the workforce (hirings or firings) will only be observed when deviations in the expected marginal revenue product of labour from the optimal level are large enough to compensate for the hiring and firing costs. In other words, hiring and firing costs generate a corridor of inaction within which firms do not change their workforce. This region is identified by the upper and lower control barriers. The cyclical implications of this no action corridor are clear; firing costs increase

---

10  $H_s$  is best interpreted as standard hours either set by the government or, more realistically, determined in the bargaining between firms and unions beyond which an overtime premium must be paid. We think that this is a rich and relevant specification, because we believe that firms indeed use the whole set of available labour adjustment mechanisms in order to make optimal choices.

11 The overtime hours wages typically exceed the wages of standard hours although empirical evidence shows that this is not always the case, and sometimes there is even no compensation for overtime work. See, for instance, Trejo (1993) and Pannenberg and Wagner (2001).

employment in troughs and reduce employment in peaks. But it is unclear what the effects are on the average employment level. Bentolila and Bertola (1990) find in their model that firing costs actually increase average employment since the effect that they prevent firings dominates the effect from lower hiring.<sup>12</sup> Their paper described necessarily abstracted from some important issues, including hours of work. Our paper accordingly asks how endogenous hours of work change the impact of regulations protecting jobs upon labour market adjustment. In other words, hiring (firing) and lengthening (shortening) the workweek are alternative strategies to increase (decrease) labour inputs.

To determine the optimal labour demand policy of the firm one therefore needs to identify this no-action region, this involves calculating the optimal upper and lower control barriers as functions of the parameters of the model. There are no closed-form solutions to the model, but we can explore further the differences across exchange rate regimes by use of numerical simulations of the model.

### III. Numerical Solutions to the Model

To have a feel on the quantitative importance of the various factors discussed above, we present some numerical examples. For this reason, the models are calibrated in order to match characteristics of the German economy. The unit time length corresponds to one year. Our base parameters are  $\sigma = 0.12$ ,  $K = 1$ ,  $H_S = 1$ ,  $x = 0.45$ ,  $T = 0.1$ ,  $F = 0.6$ ,  $\Psi = 1.5$ ,  $r = 0.08$ ,  $\mu = 0.4825$ ,  $\alpha = 0.3$ ,  $\theta_K = 0.333$ ,  $\theta_H = 0.333$ ,  $w_S = 1$ ,  $a = 1.5$ ,  $\delta = \gamma = 0.9$ ,  $N = 1$ ,  $\bar{E} = 1$  and  $\phi$  is normalised so that for  $w = w_S$  and  $H = H_S$  we have  $E = \bar{E}$ . Where possible, parameter values are drawn from empirical labour studies. The firing and hiring parameters are consistent with those in Bentolila and Bertola (1990) for Germany. Their estimated firing costs for Germany are in the range  $0.562 \leq F \leq 0.750$  and their hiring cost estimate (excluding on-the-job-training) for Germany is 0.066 of the average annual wage. Our baseline specification ( $T = 0.10$ ) is also broadly consistent with the recruiting and training cost of two months in Mortenson and Pissarides' (1999) calibration. They suggest that this number is consistent with survey results reported in

---

12 While similar in spirit to our research, Bentolila and Bertola (1990) have considered working time as one of the determinants of the reduced-form revenue function, i.e. the function that returns the cash flow as a function of employment heads only, after all flexible decisions are taken – like, use of material, ... and of overtime. Risager and Sorensen (1997) have suggested that the effect on employment is sensitive to the demand elasticity under endogenous capital formation.

Hamermesh (1993).<sup>13</sup> The substitution elasticity  $b = 1/(1+\mu) = 0.7$  has been taken from Pissarides (1998). Hart (1984) documents that the share of quasi-fixed costs in labour costs is non-negligible. In line with Hart and Kawasaki (1988), we set  $x = 0.45$ . The overtime wage  $a = 1.5$  is consistent with most German bargaining agreements and therefore a reasonable approximation to reality [see, for example, Bauer and Zimmermann (1999) and Trejo (1993)]. As a guide to the calibration of  $\lambda$  and  $\sigma$ , special attention is paid in Appendix D to the standard deviation of the Ornstein-Uhlenbeck process. In the following simulations, we always assume  $\sigma = 0.12$  and a wide range of parameters for  $\lambda$ . Analysing labour adjustment for various  $\lambda$ 's implies that we don't narrow the discussion to a unidimensional spectrum of fixity-versus-flexibility. Finally, the price elasticity of demand parameter is set at  $\Psi = 1.50$  as in Bovenberg et al. (1998). The determination of some parameters, however, requires the use of judgement, i.e. they reflect a back-of-the-envelope calculation.

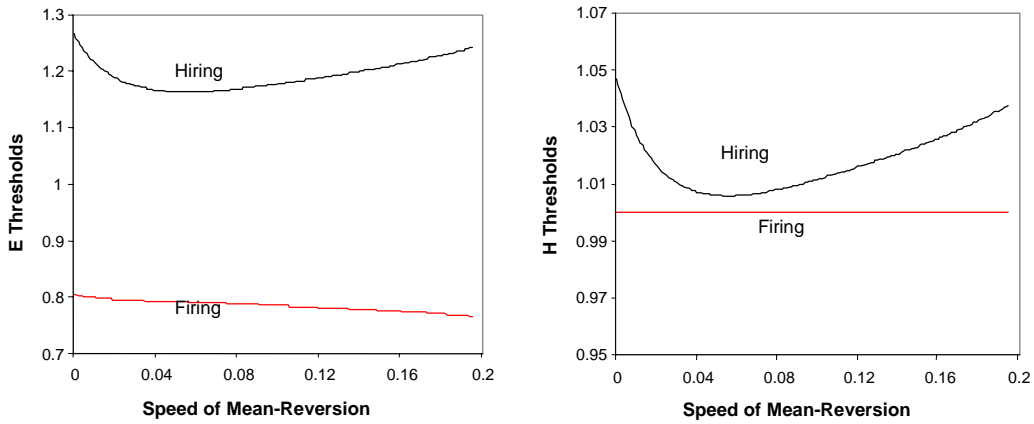
To illustrate the properties of the real option model and to get some qualitative ideas of the impact exercised by the parameters of the model, we give here some numerical calculations of the thresholds.<sup>14</sup> We call these thresholds a „band of hysteresis“. First, we consider a policy which changes hiring and firing (layoff) costs. Despite the fact that liberalisation of labour markets has ranked highly in European policy debates, few effective changes to the stringent nature of the employment constraints facing European firms appear to have been implemented over the last decade. Moreover, in a number of European countries the general trend towards greater employment protection would actually appear to have continued. The results for alternative hiring and firing costs are given in Figure 1 - 3 below.

---

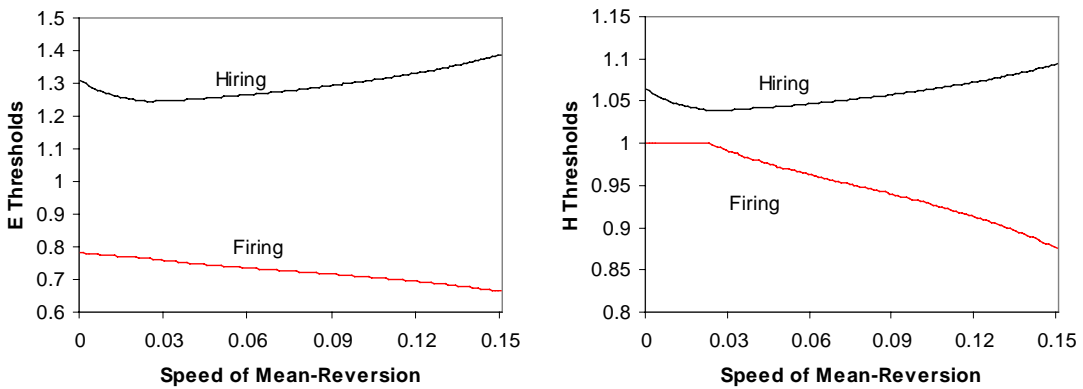
13 There exists surprisingly little empirical evidence on the size and form of hiring and firing costs. The main difficulty is that the data is not collected by government statistical agencies. Hamermesh and Pfann (1996) summarise the existing survey evidence. Although they lament the lack of data, their conclusions (p. 1268) are that the external costs alone of adjusting labour demand are large, with various studies suggesting they amount to as much of one year of payroll cost for the average worker.

14 The numerical computation of the trigger points is less complex than one would think. The numerical boundary value problem is solved with the method of Newton-Raphson for nonlinear systems. For a description of the algorithm used to compute the numerical simulations, see Press et al. (2002).

**Figure 1: The Impact of  $\lambda$  Upon the Exchange Rates Thresholds and Hours Thresholds for  $T = 0.1$  and  $F = 0.6$**



**Figure 2: The Impact of  $\lambda$  Upon the Exchange Rates Thresholds and Hours Thresholds for  $T = 0.2$  and  $F = 0.8$**



**Figure 3: The Impact of  $\lambda$  Upon the Exchange Rates Thresholds and Hours Thresholds**  
**for  $T = 0.3$  and  $F = 0.9$**

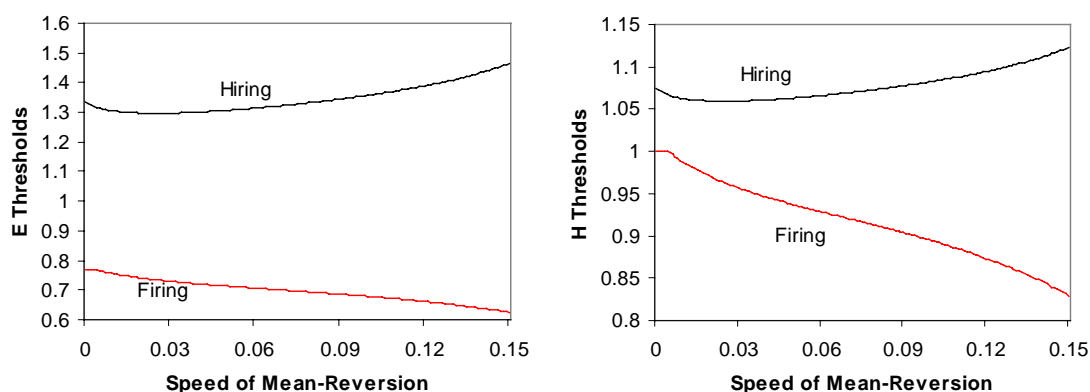


Figure 1 – 3 indicates that the band through which exchange rates can fluctuate without inducing employment changes is wider the larger are hiring and/or firing costs, i.e. hiring and firing costs act to the detriment of employment adjustment. This implies that Europe with more regulation of the employment relationship and higher firing costs will have a wider band.<sup>15</sup> The simulation results therefore support the notion that if the US and Europe face the same exchange rate shock, the US economy will be pushed more into the region where firms hire, and so we would expect the job creation record of the US to be better.<sup>16</sup> Another interesting feature is that  $H$  is smaller than  $H_S = 1$  for very large firing costs. In other words, when firing costs are very high ( $F = 0.8$  or  $F = 0.9$ ), short-time work (a partial layout) turns out to be attractive for firms.

The simulations, however, produce another surprising twist. The results in Figure 1 – 3 indicate that the hiring threshold is *U-shaped* in the speed of mean reversion ( $\lambda$ ), i.e. hiring costs do not exert a monotonic influence on the threshold. How can this quite paradoxical result be explained? The intuition for this result is the following: a higher  $\lambda$  induces a less cautious hiring behaviour by the firms because exchange rate uncertainty

15 Europe here means the continental European countries which generally have highly regulated labour markets. Finer differences between countries within Europe are ignored although not all of Europe has fallen behind. Several smaller countries, such as Denmark and the Netherlands have prospered mightily. In any event, it should be evident that an evaluation of European labour markets which is relevant for policymakers cannot proceed far without a rigorous assessment of labour adjustment costs.

16 Although some structural reforms have been made in several European countries, attempts at thoroughly reforming labour markets have generally been half-hearted. Therefore, one has to take into careful consideration the „all or nothing“ warning issued by Coe and Snower (1997). They argue that piecemeal labour market reforms may have had so little success because they disregarded the complementarities between a broad range of policies and institutions. Hence, what is needed is a fundamental labour market reform which is both broad and deep.

and therefore the option value of waiting is declining.<sup>17</sup> In other words, the no action area is declining and labour adjustment decisions are more likely to be undertaken rather than postponed. On the other hand, a higher  $\lambda$  implies that exchange rate shocks are less persistent, driving down the incentive to adjust labour. This leads to an opposite tendency: labour adjustment is less profitable because shocks are more transitory. So two opposing influences play and the remaining question is which is the predominant effect? By looking at Figure 1 - 3, we see that the U-shaped curve balances these two forces.<sup>18</sup> The important lesson here is that the inactivity band is widened substantially for overly tight and free-float regimes. A possible policy implication is that countries inside EMU should adopt policies leading to flexible labour markets.

Second, we consider a policy which restricts the standard hours of all employees. In countries where unemployment (about 4 million in Germany) is seen as a national emergency, cutting working hours is becoming a popular solution. Germany's most powerful trade union, *IG Metall*, is campaigning to cut the work week from 35 hours to 32 hours. How many jobs will be created by such a policy? Figure 4 plots the employment thresholds as a function of the speed of mean reversion ( $\lambda$ ) for the baseline parameters and  $H_S = 0.9$ . The comparison of the numerical results in Figure 1 and Figure 4 reveals that a reduction of  $H_S$  to 0.9 leads to a widening of the no-action-area. Again the effects are intuitively obvious. A cut in  $H_S$  is qualitatively the same as an increase in fixed costs per worker,  $x$ . For given output, the marginal cost of an employee (the so-called extensive margin) rises but the marginal cost of an overtime hour (the intensive margin) remains constant, and so the firm substitutes away from employment towards hours. These results provide a warning about some potential, perhaps unforeseen, effects of a mandatory hours restriction. A reduction in standard hours not only leads to a decline in employment, but also results in an increase of overtime. In other words, the 32-hour week looks like a Trojan horse.<sup>19</sup>

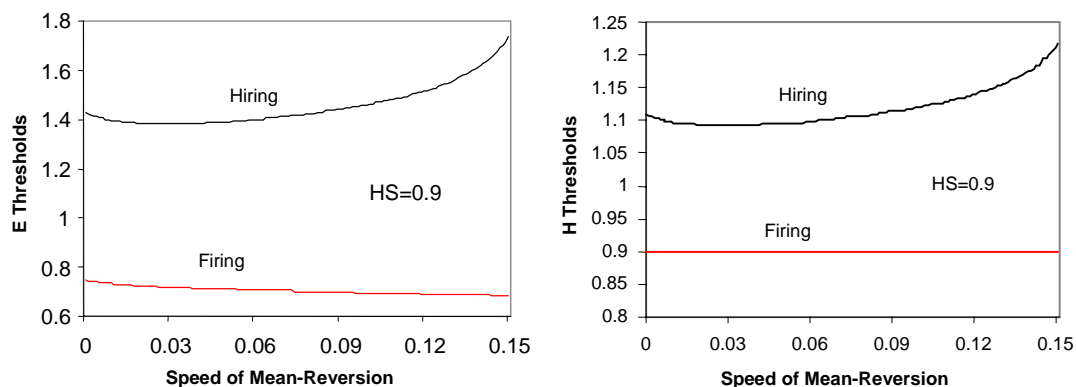
---

17 In other words, for  $\lambda \rightarrow \infty$ , the put option becomes valueless.

18 This feature bears similarity to Rose (2000) who has recently shown that irreversible investment is a hump-shaped function of the interest rate. It is also interesting to compare this feature with the well-known hump-shaped relationship between the degree of bargaining coordination and the real wage level of Calmfors and Driffill (1988) saying that labour markets work best in those countries with either very decentralised or very centralised wage formation systems.

19 In the wake of German unification, a modified labour adjustment mechanism was implemented in some Germany tariff agreements which allows more flexible working time arrangements at the firm level. Even though their main provision is that periods of overtime work will be compensated by reduced hours over a longer time period, tariff agreements may also allow for working time reductions, with an aim towards avoiding lay-offs, or for reductions of overtime in favour of full-time employment and more flexible labour hours. Some tariff agreements also include a „hardship“ clause which permits working time to be reduced at the company level, with corresponding cuts in wages. The German experience, however, shows that the impact of this flexibility mechanism is not

**Figure 4: The Impact of  $\lambda$  Upon the Exchange Rates Thresholds and Hours Thresholds**  
**for  $H_S = 0.9$**



In Figure 5 we plot the firm's optimal thresholds within which the firm optimally maintains the status quo for different overtime premiums ( $a = 1.2$  and  $a = 1.8$ , respectively) and our baseline parameters. The results are very appealing. They indicate that a (an) decrease (increase) of the overtime premium leads to a widening (shrinking) of the no-action-area („band of hysteresis“). For  $a = 1.80$  firms do not ask workers to work overtime when demand conditions are relatively buoyant because it is not profitable any more (the region of inaction degenerates to a straight line). The reason why the width of the band depends upon  $a$  is again fairly straightforward. When the overtime premium increases (decreases), the option value of hiring a new worker increases (decreases), the hiring cutoff decreases (increases), and firms hire more (less).<sup>20</sup> Another interesting feature of the simulation results which emerges from Figure 4 is that  $a$  has an asymmetric impact on the optimal employment and hours trigger points. More specifically, when the firms are confronted with lower overtime premiums, the hiring thresholds show a much more pronounced increase. Another feature is that higher mean reversion again increases the sluggishness of employment adjustment. Thus, failure to control for mean reversion could result in misleading inference about the employment thresholds.

---

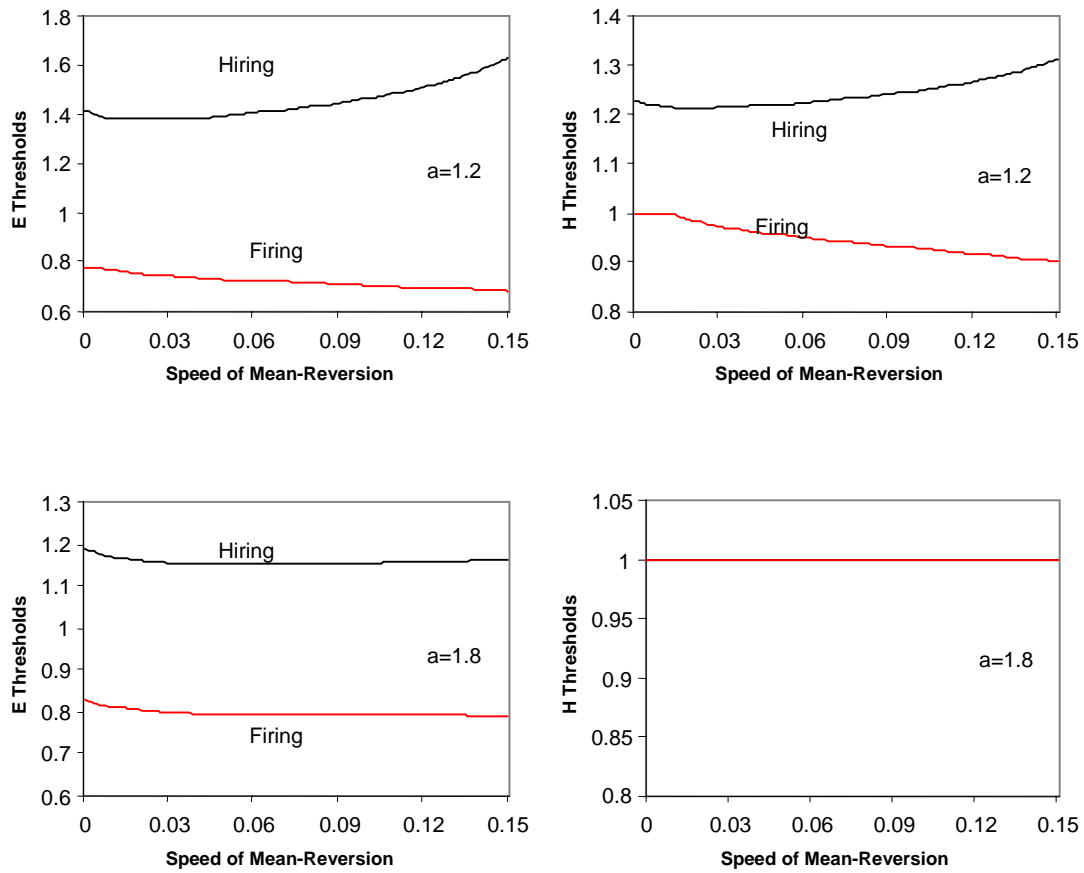
significant because exercising the clause is conditional on employment guarantees and requires the approval of both the union and the employers' association.

20 These numerical results imply that various recent flexible working deals between German firms and unions will have detrimental effects upon employment. These new „productivity deals“ typically require workers to forgo overtime pay and some bonuses, and to work extra hours when there is a big order-book, in return for paid time off when things are slack. Their basic pay is guaranteed at all times.



**Figure 5: The Impact of  $\lambda$  Upon the Exchange Rates Thresholds and Hours Thresholds**

**for  $a = 1.2$  and  $a = 1.8$**



Finally, we end this section considering the case of fixed hours of work in order to compare our results with those reported in Bentolila and Bertola (1990). The corresponding value-matching and smooth-pasting conditions for the „Working Time As Usual“ policy option are given in Appendix C. The numerically calculated optimal trigger points are given in Figure 6 below.

**Figure 6: The Impact of the Speed of Mean Reversion ( $\lambda$ ) Upon the Exchange Rates Thresholds Thresholds in the Overtime and in the No Overtime Case**



The graph examines the direction and the magnitude of the bias which may arise through failure to control for endogenous variations in hours worked. The basic interpretation is that demand fluctuates considerably more in our analysis than in the Bentolila and Bertola (1990) framework without warranting changes in employment because because firms can ask workers to work overtime when demand conditions are relatively buoyant. The results therefore indicate that particular care must be taken to ensure that misspecification of hours worked does not yield spurious estimates of the employment effects of firing and hiring costs and worksharing arrangements. Another qualitative feature is that the speed of mean reversion affects firms' behaviour much more in case of endogenous hours of work than in the „working-time-as-usual“ case. In particular, firms are more cautious hiring or firing employees when exchange rate uncertainty is low, i.e. the diminished volatility of the exchange rate comes at some cost.<sup>21</sup> Another interpretation is that EMU may strengthen the incentives for labour market reforms.

21 This result has the following awkward implication. A major economic goal of the EMU is to strengthen the Single European Market and to increase employment and investment. Expectations of reaching the goal are notably higher among policymakers than among economists. The former see the costs of currency exchange and exchange rate uncertainty as formidable barriers to employment and investment, while the latter have been unable in the past to find theoretical and empirical support for such a view. Our simulation results in Figure 6 do not provide much support for the hypothesis that the elimination of exchange rate volatility will significantly increase employment.

## IV. Summary and Discussion

The proper assessment of costs and benefits of alternative exchange rate regimes has been a hotly debated issue in recent years. The bulk of the literature has concentrated either on the tradeoff between monetary independence and credibility, or on the insulation properties of each arrangement in the face of various macroeconomic shocks. The research strategy of this paper was to develop a model that integrates labour demand decisions and exchange rate variability using analytical techniques developed in what is now loosely called the real options literature. We extend the standard labour demand model to include a detailed specification of the role of labour input in the production process and of the costs associated with it. In particular, we distinguish between employment, standard hours and overtime hours. The basic result is that the choice of regime has a bearing on employment adjustment. In particular, we have shown that a credible peg may imply a smaller region of inaction and may therefore be more conducive to employment. This latter finding is particularly important as it demonstrates that any unidimensional view of fixity-versus-flexibility is incomplete, and „intermediate regimes“ may have advantages with respect to employment.<sup>22</sup> Another novel policy implication is that due to the *U*-shaped behaviour of the hiring threshold flexible labour markets appear most necessary in truly fixed exchange rate regimes.<sup>23</sup>

There are obviously limitations to this work which make it difficult to know how strong conclusions to draw from this study. In our model the opportunity to adjust employment is assigned to one firm only. Extending the analysis to more than one firm would be reasonable. But if such an opportunity is available to any of several firms, then waiting may no longer be feasible. There can be strategic situations with more firms, where making the first move may have a commitment value. In practice, strategic considerations may call for early labour adjustment at the same time that information aspects suggest waiting: the optimal choice would have to balance the two. Another limitation of our

---

22 It is interesting to note that despite the growing support for the idea that „corner solutions“ – free float and currency union/dollarization – are the only long run sustainable options for emerging markets if not for all economies, still diversified „intermediate regimes“ like bands and crawling peg systems retain their advocates [notably Williamson (2000)]. Masson (2001) has demonstrated that the bipolar view is to a certain degree a fallacy.

23 It must be emphasised, that although the model is built around hiring and firing costs, the employment effects are not the result of changes in hiring and firing costs alone. Empirically, hiring and firing costs have not increased over the last decade in most OECD countries. They may even have fallen. In the present model, the labour demand changes are coming from the interaction of shocks and labour market institutions. See Blanchard and Wolfers (2000) for evidence on the role of shocks and institutions in the rise of European unemployment.

approach is that we have ignored the ability of, and incentives for, firms to diversify their capital stock internationally in times of exchange rate volatility.<sup>24</sup> The agenda for future research should consider realistic extensions of the model in these directions, especially in view of the important implications such work has for economic policy.

---

24 For a model which allows for the diversification of exchange rate risk see Kogut and Kulatilaka (1994).

## Appendix A: The Derivation of Equation (9)

Equation (9) is derived as follows. The particular solution for equation (8) in the text can be written as the following particular integral:

$$(A1) \quad v^P(E) = \int_0^{\infty} [\Phi E - (w(H) + z)] e^{-rs} ds$$

Equation (5) is the stochastic Verhulst equation

$$(A2) \quad dX_t = (\lambda X_t - X_t^2) dt + \sigma X_t dW_t .$$

From Kloeden and Platen (1992), p. 125 we get

$$(A3) \quad X_t = \frac{X_0 \exp((\lambda - 0.5\sigma^2)t + \sigma W_t)}{1 + X_0 \int_0^t \exp((\lambda - 0.5\sigma^2)s + \sigma W_s) ds} .$$

Equation (5) is given as  $dE = \lambda(\bar{E} - E)Edt + \sigma Ed\varpi = (\lambda\bar{E}E - \lambda E^2)dt + \sigma Ed\varpi$ . Let us define a new variable  $X$  as  $X = \lambda E$ . Then, we have

$$(A4) \quad \frac{dX}{\lambda} = \left( \bar{E}X - \frac{X^2}{\lambda} \right) dt + \sigma \frac{X}{\lambda} d\varpi$$

and

$$(A5) \quad dX = (\lambda\bar{E}X - X^2)dt + \sigma X d\varpi .$$

Substituting into (A3) yields

$$(A6) \quad X_s = \frac{X_0 \exp[(\lambda\bar{E} - \sigma^2/2)s + \sigma\varpi]}{1 + X_0 \int_0^s \exp[(\lambda\bar{E} - \sigma^2/2)t + \sigma\varpi] dt}$$

From the definition of  $X$  we get

$$(A7) \quad \lambda E_s = \frac{\lambda E_0 \exp[(\lambda\bar{E} - \sigma^2/2)s + \sigma\varpi]}{1 + \lambda E_0 \int_0^s \exp[(\lambda\bar{E} - \sigma^2/2)t + \sigma\varpi] dt}$$

and similarly

$$(A8) \quad E_s = \frac{E_0 \exp[(\lambda \bar{E} - \sigma^2 / 2)s + \sigma \omega]}{1 + \lambda E_0 \int_0^s \exp[(\lambda \bar{E} - \sigma^2 / 2)t + \sigma \omega] dt}.$$

For  $\sigma$  approaching zero, we get

$$(A9) \quad E_s = \frac{E_0 e^{\lambda \bar{E} s}}{1 + \lambda E_0 \int_0^s e^{\lambda \bar{E} t} dt} = \frac{E_0 e^{\lambda \bar{E} s}}{1 + \frac{E_0}{\bar{E}} (e^{\lambda \bar{E} s} - 1)} = \frac{\bar{E}}{1 + \left( \frac{\bar{E} - E_0}{E_0} \right) e^{-\bar{E} \lambda s}}$$

Substituting (A9) into (A1) with  $E = E_0$ , finally leads to

$$(A10) \quad v^p(E) = \Phi \bar{E} \int_0^\infty \frac{e^{-rs}}{1 + \left( \frac{\bar{E} - E}{E} \right) e^{-\bar{E} \lambda s}} ds - \frac{w(H) + x}{r}$$

It has been shown in Booth et al. (2002) that this approximation yields accurate values for the particular solution.

### Appendix B: The Derivation of Equation (13) and (14)

Equations (13) and (14) can be derived as follows. Suppose the general solution to equation (12) has the following functional form

$$(B1) \quad v = AE^\beta h(E)$$

where A is a constant, still to be determined. This yields

$$(B2) \quad \lambda(\bar{E} - E)E v_E = \beta \lambda(\bar{E} - E)AE^\beta h(E) + \lambda(\bar{E} - E)EAE^\beta h'(E)$$

and

$$(B3) \quad \frac{1}{2} \sigma^2 E^2 v_{EE} = \frac{1}{2} \sigma^2 E^2 \beta(\beta - 1)AE^\beta h(E) + \sigma^2 E \beta AE^\beta h'(E) + \frac{1}{2} \sigma^2 E^2 AE^\beta h''(E).$$

Equations (B1) through (B3) can be manipulated to obtain

$$(B4) \quad E^\beta h(E) \left[ \frac{1}{2} \sigma^2 \beta(\beta-1) + \lambda \bar{E} \beta - r \right] \\ + E^{\beta+1} \left[ \frac{1}{2} \sigma^2 E h''(E) + (\sigma^2 \beta + \lambda(\bar{E} - E)) h'(E) - \beta \lambda h(E) \right] = 0$$

Equation (B4) must hold for any value of  $v$ . Therefore the terms in the two square brackets must be equal zero.

$$(B5) \quad \frac{1}{2} \sigma^2 \beta(\beta-1) + \lambda \bar{E} \beta - r = 0$$

$$(B6) \quad \frac{1}{2} \sigma^2 E h''(E) + (\sigma^2 \beta + \lambda(\bar{E} - E)) h'(E) - \beta \lambda h(E) = 0$$

By making the substitution  $x = 2\lambda E / 2\sigma^2$ , we can transform equation (B6) into a standard form. Let  $h(E) = f(x)$ . Then we get

$$(B7) \quad x f''(x) + (b - x) f'(x) - \beta f(x) = 0$$

where  $b = 2\beta + 2\lambda \bar{E} / \sigma^2$ . Equation (B7) is the well-known Kummer equation [see Dixit and Pindyck (1994), p. 163]. Its solution is the confluent hypergeometric function  $\Gamma(x; \beta, b)$ , which has the following series representation:

$$(B8) \quad \Gamma(x; \beta, b) = 1 + \frac{\beta}{b} x + \frac{\beta(\beta+1)}{b(b+1)} \frac{x^2}{2!} + \frac{\beta(\beta+1)(\beta+2)}{b(b+1)(b+2)} \frac{x^3}{3!} + \dots$$

Thus, (B1) becomes

$$(B9) \quad v^G(E) = A_1 E^{\beta_1} \Gamma(E; \beta_1) + A_2 E^{\beta_2} \Gamma(E; \beta_2)$$

where  $\Gamma(E; \beta_1) \equiv \Gamma\left(\frac{2\lambda}{\sigma^2} E; \beta_1, b_1\right)$  and  $\Gamma(E; \beta_2) \equiv \Gamma\left(\frac{2\lambda}{\sigma^2} E; \beta_2, b_2\right)$ .

### Appendix C: „The Working Time As Usual“ Case

The case of no-working time flexibility implies that there will be no working time optimisation for hiring and firing and therefore  $g(H)$  and  $w(H)$  are given by

$$(C1) \quad g(H) = H_S^\gamma$$

and

$$(C2) \quad w(H) = w_S H_S.$$

It is straightforward to rewrite the corresponding value-matching conditions (17) and (18) in the following form:

$$(C3) \quad \Phi \bar{E} \int_0^\infty \frac{e^{-rs}}{1 + \left( \frac{\bar{E} - E_T}{E_T} \right) e^{-\bar{E}\lambda s}} ds - \frac{w_S H_S + x}{r} + A_2 E_T^{\beta_2} \Gamma(E_T; \beta_2) = T + A_1 E_T^{\beta_1} \Gamma(E_T; \beta_1)$$

and

$$(C4) \quad - \left[ \Phi \bar{E} \int_0^\infty \frac{e^{-rs}}{1 + \left( \frac{\bar{E} - E_F}{E_F} \right) e^{-\bar{E}\lambda s}} ds - \frac{w_S H_S + x}{r} \right] + A_1 E_F^{\beta_1} \Gamma(E_T; \beta_1) = F + A_2 E_F^{\beta_2} \Gamma(E_T; \beta_2)$$

where

$$(C5) \quad \Phi = \frac{1 - \theta_K - \theta_N}{\psi} \phi p^{*\frac{1-\psi}{\psi}} N^{-\mu-1} \left[ \theta_K K^{-\mu} + \theta_H H_S^{-\mu\gamma} + (1 - \theta_K - \theta_H) N^{-\mu} \right]^{-1}.$$

The smooth-pasting conditions are identical to (19) and (20) except that the parameter  $\Phi$  follows (C5). Therefore all qualitative results obtained above remain valid.



## Appendix D: The Steady-State Standard Deviation of the Ornstein-Uhlenbeck Process

For our mean-reverting process (5) in the text, we obtain the corresponding Kolmogorov forward equation in steady state, i.e. when time approaches infinity

$$(D1) \quad \frac{d[\lambda(\bar{E} - E)E\bar{p}(E)]}{dE} - \frac{1}{2} \frac{d^2[\sigma^2 E^2 \bar{p}(E)]}{dE^2} = 0,$$

where  $\bar{p}$  denotes the steady state density function (see, Dixit and Pindyck (1994), p. 88). Integrate the differential equation over  $E \geq 0$  and use the asymptotic behaviour of  $\bar{p}$  to get the following first order equation:

$$(D2) \quad \lambda(\bar{E} - E)E\bar{p}(E) - \frac{\sigma^2}{2} \frac{d[E^2 \bar{p}(E)]}{dE} = 0.$$

By expanding and rearranging, we get

$$(D3) \quad \lambda(\bar{E} - E)E\bar{p} - \frac{\sigma^2}{2} \left( 2E\bar{p} + E^2 \frac{d\bar{p}}{dE} \right) = 0$$

$$(D4) \quad \lambda(\bar{E} - E)\bar{p} - \frac{\sigma^2}{2} \left( 2\bar{p} + E \frac{d\bar{p}}{dE} \right) = 0$$

$$(D5) \quad \frac{2}{\sigma^2} \lambda(\bar{E} - E)\bar{p} - 2\bar{p} = E \frac{d\bar{p}}{dE}$$

$$(D6) \quad 2 \left( \frac{\lambda}{\sigma^2} (\bar{E} - E) - 1 \right) \bar{p} = E \frac{d\bar{p}}{dE}$$

$$(D7) \quad 2 \frac{\left( \frac{\lambda}{\sigma^2} (\bar{E} - E) - 1 \right)}{E} dE = \frac{d\bar{p}}{\bar{p}}.$$

Equation (D7) has the following general solutions after integration

$$(D8) \quad \bar{p} = CE^{2 \left( \frac{\lambda \bar{E}}{\sigma^2} - 1 \right)} e^{-\frac{2\lambda}{\sigma^2} E},$$

where  $C$  is constant and is determined by probability:

$$(D9) \quad \int_0^{\infty} \bar{p} dE = C \int_0^{\infty} E^{2\left(\frac{\lambda\bar{E}}{\sigma^2}-1\right)} e^{-\frac{2\lambda}{\sigma^2}E} dE = 1$$

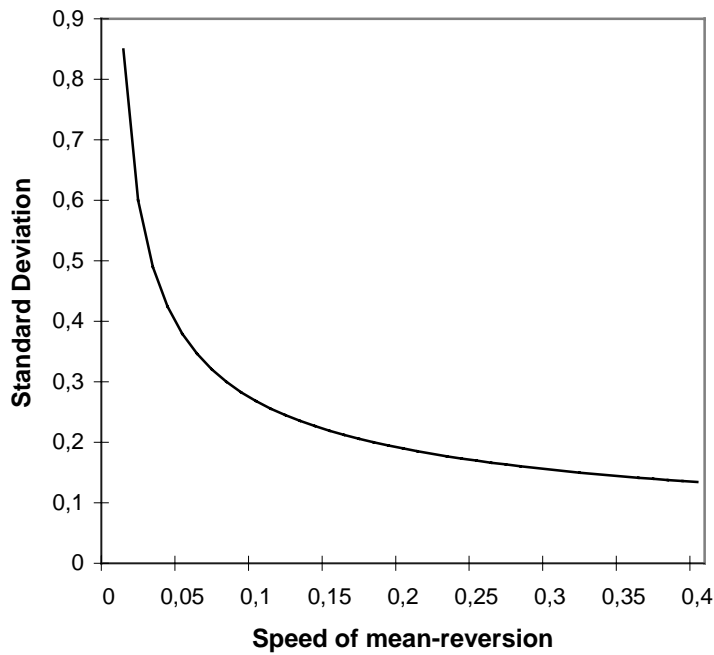
Thus,

$$(D10) \quad C = \frac{1}{\int_0^{\infty} E^{2\left(\frac{\lambda\bar{E}}{\sigma^2}-1\right)} e^{-\frac{2\lambda}{\sigma^2}E} dE}$$

The variance of the process at steady state is then given by

$$(D11) \quad \text{Var}[E - \bar{E}] = \frac{\int_0^{\infty} (E - \bar{E})^2 E^{2\left(\frac{\lambda\bar{E}}{\sigma^2}-1\right)} e^{-\frac{2\lambda}{\sigma^2}E} dE}{\int_0^{\infty} E^{2\left(\frac{\lambda\bar{E}}{\sigma^2}-1\right)} e^{-\frac{2\lambda}{\sigma^2}E} dE}.$$

Using our benchmark data ( $\bar{E} = 1$  and  $\sigma = 0.12$ ), we have



Speed s.d.	
0.010	0.8491
0.020	0.6000
0.030	0.4899
0.040	0.4243
0.050	0.3795
0.060	0.3464
0.070	0.3207
0.080	0.3000
0.090	0.2828
0.100	0.2683
0.110	0.2558

The variability for  $0.02 < \lambda \leq 0.20$  is similar in magnitude to the variability associated with real effective exchange rates in most G7-countries since 1980. Therefore one might argue that this range is relevant for any firm that is considering to start production devoted to exports.

## References

- Amran, M. and N. Kulatilaka (1999) *Real Options: Managing Strategic Investment in an Uncertain World*, Boston (Harvard Business School Press).
- Bauer, T. and K. Zimmermann (1999) "Overtime Work and Overtime Compensation in Germany", *Scottish Journal of Political Economy* 46, 419-436.
- Belke, A. and M. Göcke (1999) "A Simple Model of Hysteresis in Employment Under Exchange Rate Uncertainty", *Scottish Journal of Political Economy* 46, 260-286.
- Bentolila, S. and G. Bertola (1990) "Firing Costs and Labor Demand: How Bad is Euroclerosis?", *Review of Economic Studies* 57, 381-402.
- Blanchard, O. and J. Wolfers (2000) „The Role of Shocks and Institutions in the Rise of European Unemployment: The Aggregate Evidence“, *Economic Journal* 110, C1-C33.
- Böhm, H. and M. Funke (2001) "Does the Nominal Exchange Rate Regime Matter for Investment?", *CESifo Working Paper # 578*, Munich.
- Booth, A., Chen, Y.-F. and G. Zoega (2002) "Hiring and Firing: A Tale of Two Thresholds", *Journal of Labor Economics* 20, 217-248.
- Bovenberg, A.L., J.J. Graafland and R.A. de Mooij (1998) *Tax Reform and the Dutch Labor Market: An Applied General Equilibrium Approach*, CPB Netherlands Bureau for Economic Policy Analysis, Research Memorandum 143, The Hague.
- Calmfors, L. and J. Driffill (1988) „Bargaining Structure, Corporatism and Macroeconomic Performance“, *Economic Policy*, No. 3, 13-61.
- Coe, D. and D.J. Snower (1997) „Policy Complementarities: The Case for Fundamental Labour Market Reform“, *IMF Staff Papers* 44, 1-35.
- Coy, R. (1999) "Exploiting Uncertainty: The Real-Options Revolution in Decision-Making", *Business Week*, June 7, 118-124.
- Dixit, A. (1989) „Entry and Exit Under Uncertainty“, *Journal of Political Economy* 97, 620-638.
- Dixit, A.K. and R.S. Pindyck (1994) *Investment Under Uncertainty*, Princeton (Princeton University Press).
- Emerson, M. (1988) "Regulation or Deregulation of the Labour Market: Policy Regimes for the Recruitment and Dismissal of employees in the Industrialized Countries", *European Economic Review* 32, 775-817.
- Frankel, J. (1999) „No Single Currency Regime is Right for all Countries or at all Times“, *NBER Working Paper 7338*, Cambridge (Mass.).

- Hamermesh, D.S. (1993) *Labor Demand*, Princeton (Princeton University Press).
- Hamermesh, D. and G.A. Pfann (1996) „Adjustment Costs in Factor Demand“, *Journal of Economic Literature* 34, 1264-1292.
- Harrison, J. and D. Kreps (1979) “Martingales and Arbitrage in Multiperiod Securities Markets”, *Journal of Economic Theory* 20, 381-408.
- Hart, R.A. (1984) *The Economics of Non-Wage Labour Costs*, London & Boston (Allen & Unwin).
- Hart, R.A. (1987) *Working Time and Employment*, London & Boston (Allen & Unwin).
- Hart, R.A. and S. Kawasaki (1988) "Payroll Taxes and Labour Demand", in: *Research in Labour Economics. An Annual Compilation of Research*, Amsterdam & Greenwich (JAI Press), 257-285.
- Kloeden, Peter E., and Platen, Eckhard. *Numerical Solution of Stochastic Differential Equations*. Berlin: Springer-Verlag, 1992.
- Kogut, B. and N. Kulatilaka (1994) “Operating Flexibility, Global Manufacturing, and the Option Value of a Multinational Network”, *Management Science* 40, 5-28.
- Lindbeck, A. and D. Snower (1988) *The Insider-Outsider Theory of employment and Unemployment*, Cambridge/Mass. (MIT Press).
- Masson, P. (2001) „Exchange Rate Regime Transitions“, *Journal of Development Economics* 64, 571-586.
- Mortenson, D.T. and C.A. Pissarides (1999) "New Developments in Models of Search in the Labor Market", in: Ashenfelter, O.C. and D. Card (eds.) *Handbook of Labor Economics Vol. 3B*, 2567-2627 Amsterdam (Elsevier Science).
- Nicoletti, G., Scarpetta, S. and O. Boyland (1999) „Summary Indicators of Product Market Regulation with an Extension to Employment Protection Legislation“, *OECD Economics Department Working Paper # 226*, Paris.
- Pissarides, C. (1998) "The Impact of Employment Tax Cuts on Unemployment and Wages: The Role of Unemployment Benefits and Tax Structure", *European Economic Review* 42, 155-183.
- Ploeg, van der F. (1987) „Trade Unions, Investment, and Employment: A Non-Cooperative Approach“, *European Economic Review* 31, 1465-1492.
- Press, W.H., Teukolsky, S.A., Vetterling, W.T. and B.P. Flannery (2002) *Numerical Recipes in C++: The Art of Scientific Computing*, 2<sup>nd</sup> edition, Cambridge (Cambridge University Press).

Risager, O. and J.R. Sorensen (1997) "On the Effects of Firing Costs When Investment is Endogenous: An Extension of a Model by Bertola", *European Economic Review* 41, 1343-1353.

Rose, C. (2000) "The *I-r* Hump: Irreversible Investment Under Uncertainty", *Oxford Economic Papers* 52, 626-636.

Santamäki, T. (1984) "Employment and Hours Decision, and the Willingness to Work Overtime Hours", *Helsinki School of Economics Working Paper F-86*, Helsinki.

Trejo, S.J. (1993) "Overtime Pay, Overtime Hours, and Labor Unions", *Journal of Labor Economics* 11, 253-278.

Williamson, J. (2000) *Exchange Rate Regimes for Emerging Markets: Reviving the Intermediate Option*, Washington (Institute for International Economics).