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Transport Cost Sharing and Spatial
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Transport Cost Sharing and Spatial Competition

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September, 2003; Revised February 2004

Abstract

We consider a linear city model where both firms and consumers have to incur transport costs. Following a standard Hotelling (1929) type framework we analyze a duopoly where firms facing a continuum of consumers choose locations and prices, with the transportation rate being linear in distance. From a theoretical point of view such a model is interesting since mill pricing and uniform delivery pricing arise as special cases. Given the complex nature of the profit function for the two-stage transport cost sharing game, we invoke simplifying assumptions and solve for two different games. We provide a complete characterization for the equilibrium of the location game between the duopolists by removing the price choice from the strategy space. We then find that if the two firms are constrained to locate at the same spot, the resulting price competition leads to a mixed strategy equilibrium with discriminatory rationing. In equilibrium both firms always have positive expected profits. Finally, we derive a pure strategy equilibrium for the two-stage game. Results are then compared with the mill pricing and uniform delivery pricing models.

Keywords: Spatial Competition, Cost Sharing

JEL Classification: R1, L13, D43, C72

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1 Introduction

The spatial competition literature in the Hotelling tradition has two main strands. One concerns itself with models of *mill pricing* in which firms choose location and prices, while the spatially dispersed consumers pay the cost of travelling to the firm to buy the product. The other strand of the literature assumes that firms absorb the transport cost of shipping the item to the consumers and is called *uniform delivery pricing* since all consumers pay the same price.¹ In this paper we analyze a model of a linear city that

¹A third concept, less frequently encountered is that of *spatial price discrimination* (Hoover, (1937)). For an insightful exposition of this issue see Anderson, de Palma and

incorporates features of both mill pricing and uniform delivery pricing. We assume that firms charge the same price to all consumers but have a cost of delivering to all those who purchase from them as in the models of uniform delivery pricing. Buyers on the other hand pay the price and also incur a transport cost which, for instance, reflects the delivery time associated with the good. This delivery time increases with the consumer's distance from the firm and is a source of disutility. It captures the opportunity cost of being able to consume sooner than later.² The consumers' share of transport cost can be interpreted broadly to include time, effort and other transaction costs, besides from the costs of travel. This feature is common to models of mill pricing. Thus our model is a hybrid of the standard mill price and uniform delivery price models.

Buyers and sellers in the real world are dispersed over geographical space. It has been argued that the dispersed nature of market activities can be a source of market power for firms. Each firm has only a few rivals in its immediate neighborhood. Similarly, consumers who are at a considerable distance from a firm will not buy from that firm since they have to pay very high transport costs. The relative location of the firms with respect to the consumers is a crucial determinant of the degree of competition. Consequently, once one recognizes the importance of space, it is obvious that competition in the real world occurs only among a few and is best analyzed in a strategic game setting.

The economic relevance of location games does not stem exclusively from their initial geographical set-up. The idea can be extended to competition among firms selling differentiated products where each firm's product is viewed as a point in the characteristic space. This product differentiation aspect of location theory dates back to Hotelling's (1929) seminal work. He recognized that while location was a source of market power in itself, it could also be a proxy for other characteristics of the product. The following quote serves to illustrate this point quite well: "*... distance, as we have used it for illustration, is only a figurative term for a great congeries of qualities. Instead of sellers of an identical commodity separated geographically we might consider two competing cider merchants side by side, one selling a sweeter liquid than the other.*"

Aside from the purely theoretical aspects of the model, one encounters many examples of this sort in the real world. Retailers bear the cost of

Thisse (1989).

²One need look no further than the wide array of shipping options provided to consumers by FedEx, UPS and the United States Postal Service to be convinced of the value of consuming earlier.

bringing the commodity over to the shopping center, while the buyers must drive there to actually inspect and purchase the items. Buying furniture usually involves a trip to the furniture store and selecting the desired items, and the furniture store usually delivers the items to the consumer location free of charge. An almost perfect example is the Chicago based video rental firm *Facets Cinémathèque* or *Facets Video Rent By Mail*. Members can order videos of their choice by mail. The firm pays shipping and handling one way, while the consumer incurs the mailing expenses involved in returning the video. The labor market also has similar features. The commute time to work has to be borne by the employees. Hence, one consideration for firms in choosing to locate in the suburbs is the desire to avoid traffic congestion thereby making the job attractive to workers. The large numbers of hi-tech firms located in sub-urban Washington D.C. provide ample testimony to this fact.

A similar phenomenon can also be observed in certain types of differentiated product markets. In particular, it is quite common in some segments of the software industry. Often each firm produces its own standard product and then customizes it to suit the needs of individual buyers, while buyers have to learn the intricacies of the software. For example, software for Supply Chain management in the food industry differs from that designed for the apparel industry (which is equivalent to choosing location) and the software firm has to tailor the package to suit the needs of individual clients in each of those industries. The cost of learning new software or customizing it to suit the individual client's needs can be treated as transport cost in our framework. Also, firms often attempt to reduce their buyers' learning costs by providing training classes, on site implementation and customer service. This is definitely true for the Electronic Resource Planning [ERP] segment of the industry where firms like *PeopleSoft* and *SAP* constantly provide training to their clients and have created a new professional class called information technology consultants.

For the purpose of modelling these issues one might imagine that there is a total cost for moving a commodity from the store to the consumer's location. We then assume that the total pecuniary burden of shipping a commodity from the firm to a consumer is shared by both buyers and sellers. So consumers in our model pay an exogenously set proportion of the transport cost while firms pay the remainder. For most of the examples listed above, assuming an exogenously given transport cost sharing rule is reasonable since the consumers have their own transport cost, while firms have to incur transport costs which are specific to them. Notice that when the consumers' share of costs goes to zero we have the uniform delivery

price model and when they bear the entire cost we obtain the mill pricing formulation.

In the subsequent sections we develop a model to analyze the two stage game. We first analyze for a pure location game with exogenous prices and its counter-part where firms locate at the same spot (thereby removing location choice from the strategy space) and compete in prices. The most interesting finding for the location game is that when transport costs are shared, a unique and symmetric equilibrium in locations will exist. We find that as the consumers' share of the transport cost decreases, both firms move further away to the center to reduce their transport cost bill. On the other hand when the exogenously given price in the location game is raised, firms move towards the center to minimize transport costs. We also find that when the two firms are constrained to locate at the same spot, the resulting price competition leads to a mixed strategy equilibrium with *discriminatory rationing*. Both firms always have positive expected profits in equilibrium. Using the insights from these two games, we then model the two stage location price game. These allow us to simplify the otherwise complex and intricate expression obtained for the profit function of each firm. We identify conditions under which the two stage game has a pure strategy equilibrium. We also demonstrate that Hotelling's claim about increasing transport costs leading to greater profits is incomplete – properly stated, it must account for consumer reservation prices.

The remainder of the paper is organized as follows. The next section provides a brief overview of the related literature. Section 3 provides the basic model setup. In Section 4 we solve for the location equilibria, assuming fixed prices. Section 5 analyzes a particular price game where both firms are located at the same spot. The following section solves for the location-price equilibrium of a two stage game where the two firms first choose their locations and then compete in prices. We look for subgame perfect equilibria in this game. Section 7 contains concluding remarks.

2 Review of Literature

Given the plethora of work both on models of mill pricing and uniform delivery pricing, an exhaustive survey of all aspects of the literature would be a considerable digression. We limit the scope of our review only to those results which are pertinent to the model under consideration. Graitson (1982) is an early survey of the literature. A more up-to date and comprehensive survey can be found in Anderson, de Palma and Thisse (1992). The litera-

ture on mill price is more abundant and we will start by discussing those.

The mill price models trace their heritage from the original Hotelling (1929) model.³ Typically in these models firms choose locations and then prices and consumers incur the transportation cost. Hotelling ((1929), pg. 53) claimed that under mill pricing the two firms in the market would “...*crowd together as closely as possible.*” , while he noted the possibility of Bertrand competition for the extreme concentration case only.⁴ Fifty years later d’Aspremont, Gabszewicz, and Thisse (1979) (henceforth DGT) revisited the model and formally characterized the flawed nature of Hotelling’s solution.⁵ They found that the price equilibrium found by Hotelling holds only if the two firms are sufficiently far apart. If the two firms were located close to each other, undercutting the opponent is profitable. Higher profits destroy the pure strategy equilibrium in prices. Consequently, finding the location equilibrium for the two stage game is also jeopardized. Of course as pointed out by Hotelling, when the two firms are exogenously located at the same spot, the game reduces to pure Bertrand competition. What he missed though (and was pointed out by DGT), was that price undercutting or Bertrand competition would arise ‘earlier’ – long before the firms ‘arrived’ at the same location. The tendency to undercut which allows the successful firm to capture the entire market would arise as soon as the firms are sufficiently close. DGT suggest one way out of the nonexistence problem: assume quadratic transport costs. The game now exhibits a ‘centrifugal’ location tendency rather than central location tendency. The firms would like to locate outside the linear city, and hence in equilibrium the two firms charge the same price and locate at the endpoints of the line segment.

There are also some other approaches to deal with the non-existence problem. One of the more ingenious ones by de Palma *et al.* (1986) shows the existence of Nash equilibrium in pure strategies by introducing sufficiently heterogenous products. A different solution has been provided by Kats (1995) where the linear city was replaced by a one dimensional bounded space without boundary, i.e., a circle. In this case a pure strategy equilib-

³Note however that Ferreira and Thisse (1996) provide evidence of the fact Launhardt had already proposed such a model of a spatial duopoly in 1885.

⁴In the industrial organization literature this result is also referred to as the *principle of minimum differentiation*. The term was coined by Boulding (1955) who used it among other things to explain the existence of similarities between Methodists, Quakers and Baptists.

⁵As noted in Osborne and Pitchik (1987), Vickrey (1964) had already identified the problem with Hotelling’s analysis.

rium does exist – any pair of locations where the firms are at least a quarter away from each other and $p_1^* = \frac{1}{2} = p_2^*$ constitutes a pure strategy equilibrium. Another approach is to characterize the mixed strategy equilibrium. This line of research stems from the two Dasgupta and Maskin (1986) papers on games with discontinuous payoffs guaranteeing the existence of an equilibrium in Hotelling type models. Osborne and Pitchik (1987) undertake the task of actually identifying the equilibrium mixed strategy price distribution functions of Hotelling’s original model. They identify a support for mixed strategies in prices when the firms locate close to each other. They find a unique pure strategy equilibrium in locations, in which the firms are located at about 0.27 from the respective endpoints. For those locations, the equilibrium support for prices consists of two distinct line segments. As an intuitive explanation Osborne and Pitchik suggest a parallel with the phenomenon of ‘sales’. It is worth emphasizing that during this analysis they encounter highly nonlinear equations and resort to computational methods to come up with approximate numbers.

For the uniform delivery pricing models, where each firm quotes a single delivery price to all its customers, the non-existence problem is even more severe. It arises because the rationing of some consumers by one firm allows its rival to service this segment of the market at a high price. This gives the first firm an incentive to undercut, thereby destroying the equilibrium (see Beckmann and Thisse (1986)).⁶ A comprehensive analysis of the circular space scenario can be found in Kats and Thisse (1993). After showing the nonexistence of a pure strategy equilibrium in prices, they invoke Dasgupta and Maskin (1986) and characterize the mixed strategy equilibrium in prices. The location equilibrium in the first stage of the game is in pure strategies. The second part of their paper is devoted to the endogenous choice of the pricing policy by the firms. For the monopoly case, uniform delivery pricing is the optimal policy, partly because it allows the monopolist to extract all the surplus from the consumers. In the duopoly case, the consumer’s reservation price r is the crucial parameter. For low $r < \frac{5}{8}$, both firms choosing uniform delivery pricing is the unique equilibrium of the pricing policy game. For higher r , the competitive region (the overlapping market area) for the two firms becomes larger, intensifying price competition between the firms, making mill pricing quite attractive. Hence both price policies can be sustained as equilibria for the duopoly with mill pricing re-

⁶For more on uniform delivery pricing models also see Greenhut and Greenhut (1975) and de Palma, Portes and Thisse (1987).

sulting in higher profits. Another solution to the nonexistence problem also using heterogenous products can be found in de Palma, Labbé and Thisse (1986). The interested reader may also refer to Anderson, de Palma and Thisse (1989) for an excellent comparison of the above two pricing policies, as well as spatial price discrimination using a heterogeneous product formulation.

3 The Model

Consider a linear city of length l with a continuum of consumers distributed uniformly on this line. Each consumer derives a surplus from consumption (gross of price and transportation costs) denoted by V . In keeping with the terminology used in the spatial competition literature we will refer to this as the consumer's reservation price. Consumers are assumed to have unit demands when their reservation value exceeds the price plus the transport cost they incur. Otherwise, they do not purchase the commodity. The transportation rate t is assumed to be linear in distance. Consumers pay a proportion α and firms pay a proportion $(1 - \alpha)$ of the transport cost. Consequently, a consumer who travels a distance of d pays αtd as transport cost and the firm pays the remaining $(1 - \alpha)td$ of the cost. For notational convenience we set $(1 - \alpha)t = s$ and $\alpha t = t - s$. Due to the sharing of transport costs by consumers and firms, consumers face horizontal product differentiation and firms engage in some price discrimination in our model. There are two firms in the market called A and B . The firms are located at respective distances a and b from the ends of the line ($a + b \leq l$, $a \geq 0$, $b \geq 0$), and charge prices of p_1 and p_2 respectively. In order to focus on the transport cost issue, we assume that there are zero marginal costs. Utility maximizing consumers buy from the firm that quotes the smallest effective price (mill price plus their share of the transport cost). The location of the indifferent customer is denoted by $z = \frac{p_2 - p_1}{2(t-s)} + \frac{1}{2}(l - b + a)$. Firms in the model first choose a location and then quote a price. Based on the price and the transport cost consumers make their purchase decision. *Figure 1* (all figures have been attached at the end) represents the most general situation, i.e., the two stage location-price game and provides a graphical depiction of the notation developed here.

We are now in a position to obtain the profit function of the two stage game. Note that the expression below is derived for firm A . We require that $a \leq l - b$, or that firm A is located to the left of firm B . For the firm on the

right, a symmetric expression applies with only relevant change in notation. Set

$$\begin{aligned}
\Delta &= \max\{0, \min\{a, \max\{0, l - b - \frac{p_2}{s}\}\} - \max\{0, a - \frac{V - p_1}{t - s}, a - \frac{p_1}{s}\}\}, \\
\Phi &= \min\{\max\{a, l - b - \frac{p_2}{s}\}, a + \frac{V - p_1}{t - s}\} - a, \\
\Gamma &= \max\{0, \min\{l, a + \frac{p_1}{s}, a + \frac{V - p_1}{t - s}\} - \max\{a, (b + \frac{p_2}{s})\}\}, \\
H &= \min\left[\left\{\frac{p_2 - p_1}{2(t - s)} + \frac{1}{2}(l - b + a) - a\right\}, \frac{p_1}{s}, \frac{V - p_1}{t - s}\right], \\
K &= a - \max\{0, a - \frac{V - p_1}{t - s}, a - \frac{p_1}{s}\}, \\
M &= \min\{\frac{p_1}{s}, \frac{V - p_1}{t - s}, l - a\}, \\
P &= (t - s)(l - b - a).
\end{aligned}$$

Then the general expression for the profit function is as follows:

$$\Pi_1(p_1, p_2, a, b) = \begin{cases} (i) \quad \Delta \cdot \{p_1 - s \cdot \max[0, a - (b - \frac{p_2}{s})]\} \\ \quad - \frac{s}{2}\Delta^2 + \Phi \cdot p_1 - \frac{s}{2}\Phi^2 + \Gamma \cdot p_2 - \frac{s}{2}\Gamma^2 & \text{if } p_1 > p_2 + P; \\ (ii) \quad H \cdot p_1 - \frac{s}{2}H^2 + K \cdot p_1 - \frac{s}{2}K^2 & \text{if } |p_1 - p_2| \leq P; \\ (iii) \quad M \cdot p_1 - \frac{s}{2}M^2 + K \cdot p_1 - \frac{s}{2}K^2 & \text{if } p_1 < p_2 - P. \end{cases}$$

Notice that the expression depends on the relationship between the price difference $p_1 - p_2$ and $P = (t - s)(l - b - a)$, the cost to a consumer to go the extra way from a to b . Further, observe that if the firm serves an adjacent market area (an interval immediately to its left or to its right) of length N , then the revenue from these customers is $N \cdot p_1$ and the cost of serving them is $\frac{s}{2} \cdot N^2$. It remains to determine that Δ, Φ, \dots are the correct market sizes. We will explain the first component of the profit function in detail and provide brief discussions of the last two cases.

(i) $p_1 > p_2 + (t - s)(l - b - a)$. This case occurs when firm A is being undercut by firm B . Here Δ is the size of the market area to the left of its location and Φ is the size of the market area to the right of its location. We now discuss each part in some detail.

Consider $\{p_1 - s \cdot \max[0, a - (b - \frac{p_2}{s})]\}$ first. This is part of the expression for revenue. As noted above, if the firm were to serve an adjacent market

area, the revenue would be p_1 times the base, Δ . However, it is possible that the low price firm, by undercutting, is serving part of the market to the left of firm A 's location. This would correspond to the case when in the expression $\max[0, a - (b - \frac{p_2}{s})]$ the greater number is $a - (b - \frac{p_2}{s})$. Here the height of the rectangle representing revenues would be less than p_1 by the amount $s \cdot \max[0, a - (b - \frac{p_2}{s})]$.

Next consider Δ in more detail. The part $\max\{0, l - b - \frac{p_2}{s}\}$ within Δ represents situations where depending on parameters firm B 's left hand side bound could be to the left of a . The expression $\min\{a, \max\{0, l - b - \frac{p_2}{s}, l - b - \frac{V-p_2}{t-s}\}\}$ allows for the fact that the effective right hand side boundary will be determined by the minimum of a and the left hand side bound of firm B 's market segment as determined by the preceding expression. Now consider $\max\{0, a - \frac{V-p_1}{t-s}, a - \frac{p_1}{s}\}$. This depicts the possibility that firm A may not serve to all the customers to the left of a . It depends on whether the firm A 's constraint through price and transport cost is binding or the consumer's reservation price constraint goes into effect first. The difference Δ is the total market area available to firm A to the left of a .

We now move on to Φ . The term $\max\{a, l - b - \frac{p_2}{s}\}$ inside Φ represents the possible right-hand side bounds on firm A 's market area depending on the price and transport cost that firm B has to incur. Intuitively, by undercutting, firm B captures firm A 's territory and the parameters determine the limits of firm A 's market area. The $l - b - \frac{p_2}{s}$ expression here represents the case when the firm B 's transport cost determines the limits of its territory. Taking the maximum of this expression and a ensures that we confine ourselves to considering market segments to the right of firm A 's location. The next term $a + \frac{V-p_1}{t-s}$ represents the case when given firm A 's price, customers' decision about whether to buy from this firm determines its market area. Since both numbers, i.e., $\max\{a, l - b - \frac{p_2}{s}\}$ and $a + \frac{V-p_1}{t-s}$ represent possible bounds on the right hand side for firm A , the smallest of them is the effective bound. This accounts for the "min" in the expression. Finally, since we are considering only the right hand side market area, we need to subtract a from this expression.

The last two terms represent the possible case of 'leapfrogging' a far away market: Γ is a possible market area to the right of the opponent's territory. It occurs when the opponent loses market share from the right, thus making it feasible for the left-hand side firm to serve that chunk of the market by sufficiently raising its price. The $\min\{l, a + \frac{p_1}{s}, a + \frac{V-p_1}{t-s}\}$ part of Γ represents the possible right-hand side boundary of that market segment. It could be either l (the right-hand side boundary of our linear city), or a

bound arising either because the seller is unwilling to sell or the buyers do not wish to purchase on the far right hand side area of the linear city. The term $\max\{a, (b + \frac{p_2}{s})\}$ represents the possible left-hand side boundary for that market segment. The maximum of zero and the above has to be taken, to account for the possibility that the right hand side border of firm B 's market area is to the right of $\min\{l, a + \frac{p_1}{s}, a + \frac{V-p_1}{t-s}\}$.

(ii) $|p_1 - p_2| \leq (t-s)(l-b-a)$. In this case no firm is able to undercut its rival. The first two terms here signify profits from the right hand side and the last two terms signify profits from the left hand side. H is the minimum of the three following possibilities: either the line segment between a and location of the indifferent consumer, or $\frac{p_1}{s}$ which is total the market the firm A would like to serve (on its right hand side), or only the line segment representing the locations of those consumers (located to the right of a) who would like to buy from firm A . Since this is the no undercutting case, the (left-hand side) extent of the market area to which firm B is willing to sell plays no role here. The expression K just depicts the firm A 's captive market on the left hand side.

(iii) $p_1 < p_2 - (t-s)(l-b-a)$. This case occurs when firm A undercuts firm B . The first two terms are profits from the market area of size M on the right and the last two are profits from the market area of size K on the left. Consider M . Firm A can sell to the market segment it wishes to, given by $\frac{p_1}{s}$, unless of course some of those customers themselves do not want to purchase from it which is given by $(\frac{V-p_1}{t-s})$. Finally, it allows for the fact that firm A sells to the entire line segment, from its own location a up to the right hand side boundary of the linear space, l . The last two terms of this part of the profit function represent the market area of the firm to the left of its location.

The general expression for the profit function given below indicates a host of possibilities from which one may surmise that multiple equilibria can exist in our setting. Clearly, it will not be possible to analyze the model without making some simplifying assumptions. Any equilibrium outcome of the model will be determined by the interplay of the consumer's reservation value and the firm's choice of location and prices. Since our model combines elements from both the mill pricing and the uniform delivery pricing models, absence of sales can occur for two reasons. *First*, consumers may not wish to purchase the product at the price offered by the firm because of low reservation utilities. *Secondly*, for certain location-price pairs, it is also

possible that a firm may not want to sell to some consumers who are willing to buy from it. Keeping these in mind we analyze two different games to gain some insight into the two-stage game. We first study a pure location game. Here V does not play any role and we are able to focus on the interaction between price and location choice. We then look at the situation where the firms are located at the same spot. In this case choice location does not play a role and allows us to concentrate on the interaction between V and the prices set by the firm. Finally, in the two stage game all three parameters are allowed to vary.

4 The Location Game

In this section we assume that price is exogenously given, as in a regulator's world. This can also happen if prices have been chosen earlier in the distribution channel by manufacturers or wholesalers, and retailers are subject to resale-price maintenance. In this section we assume that the price set by the regulator is low enough to ensure that consumers can buy from either firm.⁷ In order to look for equilibria we define three price ranges based the firms' ability to recover the transport cost of shipping to consumers and identify the equilibrium for each case. Our first proposition concerns one of these ranges.

Proposition 1. *If $\frac{1}{4}sl \leq p \leq sl$, there exists a unique equilibrium in locations. The equilibrium locations are symmetric and are given by*

$$a^* = b^* = \frac{l}{6} + \frac{p}{3s}$$

Equilibrium profits for each seller are identical and given by

$$\Pi_i(a^*, b^*) = \frac{1}{72s} [40pls - 8p^2 - 5(sl)^2].$$

Proof: See Appendix.

In contrast to the original Hotelling model, here transport cost considerations in maximizing profits prevent the firms from locating at the center in all instances. Next, in equilibrium the indifferent consumer is always located at $\frac{l}{2}$ irrespective of the location of the two firms. Further, $a^* \in [\frac{l}{4}, \frac{l}{2}]$ with

⁷We analyze the role of reservation prices in subsequent sections of the paper.

the firm never locating to the left of $\frac{l}{4}$ to ensure that cost minimization. The corresponding profits lie in the range $[\frac{2}{8}pl, \frac{3}{8}pl]$ with profits increasing as the firm A moves to the right. When $p \in [\frac{1}{4}sl, sl]$ as assumed in the proposition, the optimal location varies inversely with s and directly with p . Comparative statics results suggest that firms in our model also have a central location tendency. Clearly, $\frac{da^*}{dp} > 0$, suggesting that both firms want to locate closer to the center as the exogenously given price grows. As the regulator raises the price, each firm can sell to a larger segment of the market and in order to minimize costs moves towards the centre. Since this is true for both firms, equilibrium behavior ensures that the position of z remains unchanged. Finally, using the fact that $s = (1 - \alpha)t$, it is also possible to show that $\frac{da^*}{d\alpha} > 0$. Thus as the consumers bear a greater proportion of the transport cost, we get an outcome closer to Hotelling's mill pricing model. This is intuitive since the closer the situation is to Hotelling's case, the stronger is the central location tendency.

Equilibrium location for the remaining two cases cannot be obtained using standard first order conditions and is discussed next.

Remark 1. Case (i) $p < \frac{1}{4}sl$: Prices are so low in this range that the firms are unable to serve the entire market. At best each firm can only sell to a market size of $\frac{l}{2}$ (see *Figure 2*). To see this we equate $a^* + \frac{p}{s} = (l - b^*) - \frac{p}{s}$ and solve for p . This equation enables us to find the price at which the market areas of the two firms are adjacent without overlapping which occurs at $p = \frac{1}{4}sl$. For prices below this the firms can have isolated markets. In fact the firms will locate such that their market segments do not overlap while maximizing the market area served. They choose locations so as (1) not to have overlapping market areas ($\frac{p}{s} < \frac{l-b-a}{2}$), and (2) to ensure that $(p - as)$ is nonpositive. Thus, it is possible to have a whole range of locations as equilibria in this instance. ■

Remark 2. Case (iii) $p > sl$: Here each firm can cover the entire market by itself. Both firms locate at $\frac{l}{2}$. From the previous proposition we have already seen that no seller wants to choose a location to the left of their original location with increases in price. By symmetry of the profit functions rightward movements are ruled out, as this amounts to relabelling the firms and therefore both firms choose $\frac{l}{2}$. ■

Thus, with a slight modification of the location game we find that Boulding's ubiquitous principle of minimum differentiation is no longer so pervasive. The implications of this for the regulator are also fairly obvious. If the regulator decides to lower prices after the firms have chosen locations,

each firm's location will be sub-optimal. Hence, in order to maximize profits both firms will deliver to a smaller market than prior to the price reduction. Similarly, if prices are raised locations will still be sub-optimal, but fewer customers will be left out. We next solve for a price game where both firms are at the same location by decree.

5 A Special Price Game

In this section we analyze a price game when both players are constrained to be at the same location while retaining all other assumptions of the previous section. Admittedly, this is an extreme assumption, but it is also the analogue of the problem in the previous section where the firms faced exogenously given prices and were allowed to compete only in locations.⁸ Given that transport cost is shared between the firms and consumers some interesting possibilities can arise in this game. First, for low V some consumers will not purchase the product at the price offered by the firm. Another possibility is the existence of prices at which a firm does not sell to some willing consumers. Then given a sufficiently high V , the other firm can alter (raise) its price and sell to the excluded section of the market. Thus, the two firms may sell the *same product* to different segments of the markets at different prices. Clearly, rationing of certain consumers is a distinct possibility in this model. Furthermore, this rationing will be of a discriminatory nature, as each additional consumer will pay a higher effective price based on the distance from the seller's location.

Since both firms are located at the same place, there is no horizontal product differentiation – we have a case of pure price competition. As shown by DGT, in the Hotelling model there exists a pure strategy equilibrium for such a price subgame where prices are equal to the marginal cost, i.e., zero. Given positive transport costs for firms however, the standard Hotelling result no longer holds. We find that there is no pure strategy equilibrium in the price subgame. Instead, we show the existence of a mixed strategy equilibrium exists where prices always exceed the marginal cost. This is similar to the results in models of Bertrand-Edgeworth competition (see for example Allen and Hellwig (1986), Dasgupta and Maskin (1986), and Kats and Thisse (1993) in the context of spatial models.

We will first establish a result about the upper (p^u) and lower (p^l) bounds on prices. Without loss of generality, consider a realization of the mixed

⁸One might imagine a situation where fastidious city planners will only let firms set up shop at a particular location!

strategy where firm A charges a low price and firm B sets a high price.⁹ We obtain p^u by computing the monopoly price that takes V into account. Consequently, there are two possible upper bounds on price. When the reservation price is below a certain threshold (say \widehat{V}), at the price upper bound denoted by p_r^u some consumers in the market will not wish to purchase from firm B .¹⁰ When $V \geq \widehat{V}$, the upper bound is given by the highest price at which firm B can sell to consumers located furthest from it, and is denoted by p_a^u ($> p_r^u$) since the firm sells to all residual consumers.

Lemma 1. *The support of any equilibrium in the (same location) price game is a strict subset of $[0, V]$.*

Proof: See Appendix.

The next proposition shows the precise support of this mixed strategy equilibrium when the firms locate at the center. We then argue that this can be generalized to asymmetric location choices of the firms.

Proposition 2. *For $a = b = \frac{l}{2}$, the price game has no pure strategy equilibrium. A mixed strategy equilibrium does exist for this price game. For $V < \widehat{V}$, the support is given by $\left[sV \left(\frac{t - \sqrt{2ts - s^2}}{(t-s)^2} \right), \frac{sV}{\sqrt{2ts - s^2}} \right]$ and for $V \geq \widehat{V}$, the support is given by $\left[s(l-a) \left(1 + \frac{s(l-a)}{2(V - (l-a)(t-s))} \right), V - (l-a)(t-s) \right]$.*

Proof: See Appendix.

We now argue that the same result is also true for any other location of the two firms. The problem becomes asymmetric in this case and the critical value of V on the right hand side segment of the firm's location can differ from the critical value on the left hand side. This alters the profits of the high price firm and consequently the value of the bounds. Given that the problem is computationally intensive and qualitatively no different from the one shown above, we just provide the rationale for the argument without explicitly computing the bounds.

Remark 3. *For $a + b = l$, the price game has no pure strategy equilibrium. A mixed strategy equilibrium however, does exist in this price game.*

⁹Although we often refer to firm A as the low price firm and firm B as the high price firm, here we have in mind only a particular realization of the mixed strategy.

¹⁰The precise value of the threshold reservation price is not relevant to the argument here. Exact computations are shown for the next result.

To deal with the case of any location, we will consider situations where $a < \frac{l}{2}$ and $a + b = l$, using the rationale suggested above. While the method for computing the upper and lower bound remains the same, three distinct possibilities can arise in this situation. Two of these situations have already been described above (see also *Figure 3*). The third possibility arises in the asymmetric case because there is an intermediate value of the reservation price at which some consumers to the right of a will not be able to purchase at the monopoly price. This case will also yield different values of p^h and p^l . Thus we will have three different inequalities which have to be solved using a technique similar to the one used for $a = \frac{l}{2}$. The main difference with the previous case therefore stems from the fact that the computation of monopoly profits changes. This affects the high price firm's best response and consequently the lower bound without altering the logic of the calculation. Since Theorem 5 of Dasgupta and Maskin (1986) holds in this case as well, the mixed strategy equilibrium exists. ■

It is worth pointing out that while the upper bound can be the monopoly price, the lower bound differs from zero and from s as well. Since s may be thought of as the marginal cost to the firm of (delivering) an additional unit, this is different from the usual lower bound of the support of the mixed strategy in rationing models. These differences arise because the rationing mechanism in our model can be described as *discriminatory rationing*. Consumers not served by the low price firm are served by the high price firm, but each additional consumer pays a higher effective price which is proportional to the distance from the firm's location. It is precisely this reason which also prevents the price from going down to zero due to price undercutting, as it becomes worthwhile for one firm to sell to the market segment that is left out instead of lowering prices further.

The mixed strategy equilibrium in our formulation has another attractive feature. Equilibrium profits in the DGT model are always zero when $a + b = l$ and involves the play of a pure strategy with both firms choosing zero prices. In general, a mixed strategy equilibrium is often considered unattractive as players are indifferent between all the pure strategies involved. Moreover, it does not give any reason to select among these different strategies (see Osborne and Rubinstein, (1994) for more on interpretations and criticisms of mixed strategies, including points on which even the authors of the book disagree). The redeeming feature of the equilibrium mixed strategy in our model is the fact that expected profits are always positive, whereas in the DGT framework they are always zero. In fact, profits are positive for any realization of the mixed strategies since p^l is always positive in our model.

The insights from these two special games and their implications for the two-stage game are discussed in the concluding section.

6 The Location–Price Game

In this section we will look for pure strategy equilibria of the two stage game where firms first choose their locations and then set prices. *Figure 1* is a representative situation for this case. The subgame perfect equilibrium obtained here is the central result of this paper and ties together all the elements of the last two sections. Given the general setting of this section multiple equilibria cannot be ruled out. However we focus on identifying a pure strategy equilibrium.¹¹ This is the most frequently sought after equilibrium in the spatial competition literature, and is perhaps the most interesting one as well. Besides being an interior equilibrium we find that it also has other intuitive properties. We compare our results with those of DGT and the uniform delivery price models.

The strategy for constructing the proof is as follows. We identify three conditions to simplify the profit function for the two stage game and give it the relevant shape, *i.e.*, one that enables us to find a pure strategy equilibrium – the only one that is “*strictly interior in all respects*” for the two stage game. Then we show that an interior pure strategy equilibrium cannot exist unless these conditions are satisfied, thereby justifying these conditions. Thus while the constructed equilibrium might seem to require endogenous conditions, we show that if these conditions do not hold an equilibrium cannot exist. These conditions essentially allow us to focus on a specific part of range of the profit function to construct the desired equilibrium.

Condition 1: *Buyer Participation condition.* This condition consists of two inequalities and is assumed by most models of spatial competition (see for example DGT or Graitson’s (1982) survey).

$$\begin{aligned} (i) \quad p_1 + \max\{(t-s)(l-a), (t-s)a\} &< V \\ (ii) \quad p_2 + \max\{(t-s)(l-b), (t-s)b\} &< V \end{aligned}$$

These inequalities require that the reservation price be so high that consumers are not prevented from buying from either firm. The absence of this condition will lead to the type of outcomes associated with *Case (i)* of the location model described in Section 4.

¹¹Following Dasgupta and Maskin (1986), it can be shown that the profit function satisfies conditions for the existence of a mixed strategy equilibrium and is available from the authors on request.

Condition 2: *Seller Participation condition.* All of the four inequalities given below must be satisfied for no firm to lose any market share.

$$\begin{aligned} (i) \quad p_1 - as &\geq 0 \text{ and } p_1 - (z - a)s \geq 0 \text{ and,} \\ (ii) \quad p_2 - (l - b)s &\geq 0 \text{ and } p_2 - (l - b - z)s \geq 0, \end{aligned}$$

where $z = \frac{p_2 - p_1 + (t-s)(l-b+a)}{2(t-s)}$ denotes the location of the indifferent consumer. It ensures that the total cost imposed on each firm by the market segment both to its left and right hand side can be represented by the area of a triangle. Essentially it says that the prices are so high that after accounting for costs, the firm is willing to sell to all consumers who wish to purchase from it, in its captive market on one side and up to the indifferent consumer on the other side. Note that this condition has consequences for both location and price choices and affects the possibility of ‘leapfrogging’ by firms.

Condition 3: *Market Capture condition.*

$$|p_1 - p_2| < (t - s)(l - b - a)$$

This is the situation that frequently appears in mill pricing models. It implies that no single firm can set its price to sell to the entire market by completely undercutting its rival.¹²

Next, assuming that Conditions (1) – (3) hold, we have surprisingly well behaved profit functions for both firms given by

$$\begin{aligned} \Pi_1(p_1, p_2) &= p_1 z - \frac{s}{2}(2a^2 + z^2 - 2az) \quad \text{and} \\ \Pi_2(p_1, p_2) &= p_2(l - z) - \frac{s}{2}(b^2 + (l - b)^2 + z^2 - 2(l - b)z). \end{aligned}$$

This is easy to verify since Condition 3 eliminates a large portion of the profit function and the other two conditions give us the desired expression. Region 3 of (*Figure 5*) depicts a profit function of this sort. We now show that a (interior) pure strategy equilibrium in prices and locations does not exist when either Condition 2 or Condition 3 is not satisfied.

¹²Although we call it the *Market Capture* condition, this is somewhat of a misnomer. When the firm’s share of transport costs is high, the firm that undercuts may not choose to sell to all the customers who wish to purchase from it. Condition 2 prevents the occurrence of this situation.

Lemma 2. *A pure strategy equilibrium of the price-location game does not exist when the Seller Participation condition does not hold.*

Proof: See Appendix.

Lemma 3. *A pure strategy equilibrium of price-location game does not exist when the Market Capture condition is violated.*

Proof: See Appendix.

Note that in the DGT framework Condition 3 is enough to guarantee that the profit functions are well-behaved. However, in our framework this condition does not suffice, as the firm that has undercut its rival may not wish to sell to all the willing consumers. The other firm then will be able to get the residual consumers. By raising the price firm A is able to reach the boundary where the customers wish to purchase from it. Note that the profit function of firm A will have a kink here. Consequently, the profit function will not be well behaved, which renders it impossible to solve the two stage game in the standard fashion. However, Conditions (1) – (3) together whose violation ensures that a pure strategy equilibrium will not exist enables us to circumvent those sorts of problems for the current analysis.

Define

$$a^* = b^* = \frac{1}{8}lt \frac{4t - 3s}{s(3t - 2s)} \quad \text{and} \quad p_1^* = p_2^* = \frac{1}{8}l \frac{20t^2 + 8s^2 - 25st}{3t - 2s}$$

At these prices and location profits are given by

$$\Pi_1^* = \Pi_2^* = \frac{l^2}{64s} \frac{-96s^4 + 416s^3t - 609s^2t^2 + 312st^3 - 16t^4}{(3t - 2s)^2}.$$

Proposition 3. *Suppose Conditions (1) – (3) are satisfied. Then for $a + b < l$, the tuple (a^*, b^*, p_1^*, p_2^*) as defined above is the unique symmetric equilibrium in pure strategies of the location-price game.*

Proof: See Appendix.

We are now in a position to compare our results with those of the mill pricing and uniform delivery pricing models. As demonstrated by DGT, the

Hotelling model is inherently unstable. Optimal prices require that firms be far apart but $\partial\Pi_1(p_1^*, p_2^*)/\partial a$ and $\partial\Pi_2(p_1^*, p_2^*)/\partial b$ are positive and both firms have a tendency to move to the center. Thus the Market Capture condition is violated and the second stage equilibrium does not exist. We employ a similar reasoning to investigate the existence of our pure strategy equilibrium. Let $\Theta = (\Pi_i^* - \Pi_i') > 0$ where Π_i' denotes profits from capturing the whole market. We find that $\Theta > 0$ when $s > 0.5034t$ suggesting that firms must share more than half of the total transport cost to ensure that equilibrium profits are higher than undercutting profits. This is equivalent to the DGT condition requiring firms to be sufficiently far enough. In other words a low s facilitates capturing the market. It becomes easier for the two firms to locate closer to each other which facilitates undercutting.

Note that $\Theta = 0$ at $s = 1$, which coincides with the uniform delivery pricing case. When $s = 1$, optimal profits are the same as those from undercutting, simply because optimal prices are the same as undercutting prices: $p_1' = p_1^* - (l - b^* - a^*)(t - s)$, and at $s = t$, clearly $p_1' = p_1^*$. This can also be explained intuitively. As $s \rightarrow 1$, $p^* \rightarrow \frac{3}{8}lt$ and $a^* = b^* \rightarrow \frac{1}{8}l$. Therefore, the right hand side bound of the desirable market area for firm A given by $a^* + \frac{p^*}{s}$ can be written as $\frac{1}{8}l + \frac{3}{8t}lt = \frac{l}{2}$. Obviously, the desirable market areas for two firms in this case do not overlap. They are contiguous meeting at the center of the city. Also, as mentioned earlier, the optimizing price here is the same as undercutting price. In this situation, each firm has an incentive to *increase its price*, provided the buyer participation constraint is satisfied. By doing so the firm loses only an infinitesimal amount in profits as a result of market loss to the competitor. On the other hand it gains a larger amount in profits due to the higher prices. This is the usual explanation for the non-existence of price equilibria in uniform delivery price models (see de Palma, Labbé and Thisse, 1986). Hence, for the case of $s = 1$ or in the uniform delivery price model, there is no pure strategy equilibrium in prices. Thus, while a mill pricing type situation does not arise unless the firm share of transport cost is greater than half the total transport cost, the uniform delivery pricing type situation arises only when firms pay the entire cost.

Next we find that $\partial\Theta/\partial s$ is a concave function with the maximum at $s \simeq 0.7$. The concavity of this function can be explained by the fact that it affects both prices and locations differently and the value of Θ clearly depends on which of these two dominates the other. Our model suggests that differential profits are maximized when both firms and consumers incur a part of the transport cost. This echoes the findings of Kats and Thisse (1993) on the issue endogenous pricing policy choice. They find that for high

reservation values (as in our model) both mill pricing and uniform delivery pricing can be sustained here as an equilibrium. It is also worth mentioning that a special case of Proposition 3 arises when sellers are constrained to choosing only symmetric locations around the center. In this case we find that $a = b = \frac{l}{4}$, and optimal prices are given by $p = l(t - \frac{3}{4}s)$.

An intriguing feature of optimal profits seems to be the fact that profits rise with an increasing t .¹³ This however is fairly intuitive. As the transport technology becomes more expensive, it becomes easier for the firms to engage in monopolistic behavior. The consumers' reservation price must also increase in order to enable the consumers to purchase the commodity at the higher effective price. Since the equilibrium here assumes that Conditions (1) – (3) must hold, it leads to higher profits for the two firms. On the other hand, if V is fixed, then the profits shrink at t increases. This is easily verifiable if we use the intuition from the problem where firms are located at the same place and compete only in prices. Recall that the optimal profit functions there are given by $\Pi_1 = \Pi_2 = s \left(\frac{V}{(t-s)^2} \right)^2 \left(t - \sqrt{2ts - s^2} \right)^2$, where V is the fixed reservation price. Thus profits tend to zero as t increases.

7 Conclusion

This paper analyzes a model where both firms and consumers have transportation costs. In the standard mill pricing model the pure strategy equilibrium breaks down since firms have an incentive to move to the center and this makes it easier for the rival to undercut. The firm that undercuts successfully gains the entire market. In our model while choosing locations the firms also have to ensure that they minimize their of transport cost bill. So, while there is a central location tendency, in our model there is also a countervailing force. The firm that undercuts its rival may not be able to sell to the entire market. Similarly, note that in the uniform delivery price models one firm may charge a high price and sell to customers who its rival may be unwilling to service. In our model the 'rationed' consumers may be not be willing to buy from a high price firm since the price inclusive of transport costs may exceed their reservation price. However, by imposing

¹³Hotelling's remark in this context is especially interesting. "*These particular merchants would do well, instead of organizing improvement clubs and booster associations to better the roads, to make transportation as difficult as possible. Still better would be their situation if they could obtain a protective tariff to hinder the transportation of their commodity between them.*" (pg. 51) Thus Hotelling's intuition seems to be incomplete as the effect of the reservation price of the consumers is not taken into account.

a set of simple requirements and invoking two lemmas, we are able to rule out such possibilities in the location-price game. Under certain parameter values on the transport cost share, we find a unique symmetric pure strategy equilibrium where firms do not locate at the city center. This result differs from the earlier results on location games and is clearly a consequence of requiring both types of players to incur transport costs.

We also solve two other games to develop some insights for the two-stage game. In the first of these we consider parametric prices, thereby restricting firms to choosing a location only. We find that when the share of transport costs borne by the consumers increase firms move closer to the center. An interesting feature of the model is that there is a symmetric location equilibrium where the firms chose their location keeping their transport cost in mind. It means that the firms would never locate at the end points. In fact there is a threshold location ($\frac{l}{4}$) which the firms will never cross. In the second game firms are assumed to locate at the same spot. Thus location choice is no longer an element of the strategy space. Here we find the existence of a mixed strategy equilibrium whose support is identified in the paper. This is useful since the existence problem is resolved indicating that the two stage game would at least have a mixed strategy equilibrium. This game also revealed that when the firms locate too close to each other there cannot be an equilibrium in pure strategies, i.e., the incentive to undercut is too strong. The importance of the reservation price was also demonstrated by the this game. Another interesting feature of this price game is the possibility of discriminatory rationing. These key insights were valuable for finding the pure strategy equilibrium of the two-stage game.

Finally, the paper also raises another interesting question – the issue of endogenizing the transport cost sharing decision. We believe that this will provide an alternative approach to modeling the choice between uniform delivery pricing or mill pricing for firms.

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Appendix:

Proof of Proposition 1: The location of the consumer who is indifferent between buying from A and buying from B simplifies to $z = \frac{l-b+a}{2}$. Then we can write the profit function as $\Pi_1(a, b) = pz - \frac{(1-\alpha)t}{2}\{a^2 + (z-a)^2\}$, and $\Pi_2(a, b) = p(l-z) - \frac{(1-\alpha)t}{2}\{b^2 + (l-b-z)^2\}$. In each of these profit functions the first term denotes revenues and the second term is the share of the transport cost. Geometrically, total costs are triangles whose areas the firms try to minimize. The proof consists of taking the derivative of each firm's profit function with respect to its location and solving the following system of two equations obtained from the first order conditions.

$$a = \frac{4\frac{1}{2}p + \frac{1}{4}ls - \frac{1}{4}sb}{5s} \quad \text{and} \quad b = \frac{4\frac{1}{2}p + \frac{1}{4}ls - \frac{1}{4}sa}{5s}$$

We also verify that the second order conditions are satisfied. Substituting the optimal locations in the profit functions yields the equilibrium profits. Furthermore, we can check that firm A does not wish to locate to the right of firm B . Given b^* , we know that $l - b^*$ is less than $\frac{l}{2}$ which denotes the location of the indifferent consumer in the above equilibrium. Consequently, in the above equilibrium firm A has half the market. If firm A locates to the right of firm B then the new indifferent consumer will lie in the interval $l - b^*$ and firm A 's market area will be strictly less than half the market area $\frac{l}{2}$. Hence, firm A will not gain by selecting a location to the right of firm B . By symmetry, firm B will never locate to the left of firm A . So, (a^*, b^*) constitutes an equilibrium. ■

Proof of Lemma 1: We demonstrate that the interval $[p^l, p^u] \subset [0, V]$ for each of the two possible cases. We show that when firm A raises its price starting from zero, firm B will not raise its price beyond p^u . Similarly, firm A will not reduce its price below a lower bound p^l . *Case (I):* For $V < \widehat{V}$, p_r^u is the candidate upper bound. By definition, we know that p_r^u dominates all prices in the interval $(p^u, V]$. When firm A raises its price beyond zero, firm B loses customers from the center of its market. The optimal response for firm B is to either lower its price and sell to the consumers previously left out, or to undercut firm A . Thus prices do not exceed p_r^u . *Case (II):* For $V \geq \widehat{V}$, the candidate upper bound is p_a^u . When firm A raises its prices, firm B can continue to sell to the residual market at p_a^u , or undercut firm A . Thus in either case there is an upper bound on prices. Now consider the

existence of the associated lower bounds for the two possible cases. *Case (III)*: Let $V < \widehat{V}$. Since firm A 's profits are increasing in its price, it will prefer to raise its price. The lower bound on prices can be obtained by equating $\Pi_2(p_1, p_2'(p_1))$ (where $p_2'(p_1)$ is firm B 's best response) with firm A 's profit when it raises price. Prices in the range $[0, p_r^l)$ are dominated by p_r^l and for prices above p_r^l , the rival firm will have an incentive to undercut. *Case (IV)*: For $V \geq \widehat{V}$, the candidate lower bound is p_a^l . A similar argument establishes the lower bound on prices for this case. Thus prices will not fall below p^l . This allows us to conclude that prices above p^u are dominated by it. Also, choosing p^l yields higher profits than prices below it. So there cannot be a pure strategy equilibrium outside this interval. Hence there cannot be one in mixed strategies either as it would involve the play of dominated strategies. ■

Proof of Proposition 2: From Lemma 1 we know that any equilibrium must lie in the interval $[p^l, p^u]$. Assuming $a = \frac{1}{2}$ we will now compute the critical V and the two associated intervals. The threshold V is found by taking the derivative of firm B 's profit function at the price when the furthest consumer is indifferent between buying and not buying from firm B and occurs at $p = V - (t - s)\frac{1}{2}$. For $V < \frac{1}{2}(2t - s) = \widehat{V}$, the upper bound is the monopoly solution. When firm B behaves as a monopolist on the residual demand (area) left by firm A , its profits are $\Pi_2 = \frac{1}{2}\left(\frac{V-p_2}{t-s} - \frac{p_1}{s}\right)(2p_2 - s\left(\frac{V-p_2}{t-s} + \frac{p_1}{s}\right))$. Hence its best response is $p_2'(p_1) = \frac{stV - p_1(t-s)^2}{(2t-s)s}$. Using this we compute profits of firm B in terms of p_1 which are given by $\Pi_2(p_1, p_2'(p_1)) = \frac{1}{2}\frac{(sV - p_1 t)^2}{(2t-s)s^2}$. Clearly $\frac{\partial \Pi_2(p_1, p_2'(p_1))}{\partial p_1} < 0$. For the lower bound on prices we equate the profits of the two firms using the fact that $\Pi_1(p_1, p_2) = \frac{p_1^2}{2s}$. Since firm A is the low price firm, its profit expression does not contain a p_2 term. Equating the two profit expressions gives us $p_1 = sV \left(\frac{t \pm \sqrt{2ts - s^2}}{(t-s)^2}\right)$. The root with the positive discriminant yields a negative p_2 and hence is eliminated. The optimal value of p_1 which is the lower bound is then given by

$$p_r^l = sV \left(\frac{t - \sqrt{2ts - s^2}}{(t-s)^2}\right) \quad \text{and} \quad p_r^u = \frac{sV}{\sqrt{2ts - s^2}}$$

This is intuitive: a high p_1 implies that the low-price firm is selling to a large section of consumers leaving out very little for the other firm. To find the upper bound we use the root with the negative discriminant and it can be checked that $p_r^u > p_r^l$. Also, using these prices we find that profits are

$$\Pi_1 = \Pi_2 = s \left(\frac{V}{(t-s)^2} \right)^2 \left(t - \sqrt{2ts - s^2} \right)^2.$$

Let $V \geq \frac{l}{2}(2t - s) = \widehat{V}$. A similar argument establishes the lower bound on prices for this case. The only difference is that the monopoly profit of firm B is now different. So, $p_a^u = V - (l - a)(t - s)$ and $p_a^l = s(l - a) \left(1 + \frac{s(l - a)}{2(V - (l - a)(t - s))} \right)$. Now let us consider what happens when prices are in the range $[p^l, p^u]$. Suppose a firm is charging the price p^u . Then by charging a price $p^u - \varepsilon$ (where $\varepsilon > 0$, and small) its rival can undercut the firm completely. This phenomenon of successive undercutting will occur for any price above p^l . Once prices reach p^l , one of the firms is better off selling to the remaining consumers at a price of p^h instead of undercutting its rival further. However, the firm charging p^l would now prefer to undercut the high price firm. Hence there are no pure strategy equilibria. Finally, it can be shown that the two stage location-price game satisfies *Theorem 5* of Dasgupta and Maskin (1986, pg. 14). Hence it is satisfied for this price subgame. Hence we assert that a mixed strategy equilibrium identified here exists. ■

Proof of Lemma 2: This is a non-existence result that proves the necessity of the Seller Participation condition. *Case (i)* $p_1 - as < 0$. Here firm A has market loss on its left hand side. Also assume that $\alpha > 1 - \alpha$. Then it is easy to check that by charging a price $p_1 + \varepsilon$ ($\varepsilon > 0$), firm A can increase its profits. Suppose $\alpha \leq 1 - \alpha$ as shown in *Figure 4*. Then there exists a price pair (p_1, p_2) which is an equilibrium in pure strategies. However, it is not robust to the firm's location choice decision. It is easy to check that firm A can always do better by moving to its left. Since we require conditions for an equilibrium in the two stage game, any candidate equilibrium must survive the next stage of subgame perfection – the choice of optimal locations. Note that the tendency to move to the left for firm A is present irrespective of the relationship between α and $1 - \alpha$. Hence it is not possible for a pure strategy equilibrium to exist in this case. *Case (ii)* $p_1 - (z - a)s < 0$. Firm A now has market loss from the right hand side. One can check that by charging a price $p_1 + \varepsilon$, firm A is better off. It gains market share and sells at a higher price. *Case (iii)* $p_1 - as < 0$, and $p_1 - (z - a)s < 0$. Here firm A is losing market areas on both sides. By raising its price firm A will increase market areas on both sides, and sell to all customers at the higher price eventually leading to one of the two situations described above. The second part of Condition 2 consists of symmetric conditions for firm B . ■

Proof of Lemma 3: Suppose that (p_1, p_2) is an equilibrium but $|p_1 - p_2| > (t - s)(l - b - a)$. Then there are two possibilities. First let $p_2 - as \geq 0$. Here the high price firm has no market area and will lower its price down to (at least) the delivered price of the opponent at its own location, ensuring a positive profit. Hence this case cannot be sustained in equilibrium. Next let $p_2 - as < 0$. Here the high price firm has some residual market area. If this firm does not alter its location then a mixed strategy equilibrium (as in Section 5) is the only possibility. Alternatively, the firm can move inwards to make greater profits. Thus there is no pure strategy equilibrium. Finally, let $p_2 - p_1 = (t - s)(l - b - a)$ where the modulus has been ignored for the sake of simplicity. If $p_1 = 0$, then firm A can gain by charging a price less than $p_2 + (t - s)(l - b - a)$. If $p_1 > 0$, given the general form of our profit function firm A can sell to buyers up to the location of firm B . So firm A can reduce its price slightly and increase its market share making larger profits. Alternatively if at $p_1 > 0$ and firm A was not selling to all the customers up to firm B 's location, it can increase profits by raising prices. Hence for price equilibrium we need that $|p_1 - p_2| < (t - s)(l - b - a)$. ■

Proof of Proposition 3: We know from Lemma 2 and 3 that if any of the conditions are not satisfied then a pure strategy equilibrium does not exist. So assuming Conditions (1) – (3) are satisfied, we have well behaved profit functions in this range given by $\Pi_1 = p_1z - \frac{s}{2}(2a^2 + z^2 - 2az)$ and $\Pi_2 = p_2(l - z) - \frac{s}{2}(b^2 + (l - b)^2 + z^2 - 2(l - b)z)$. Assuming fixed locations we compute the optimal prices in terms of locations. These are given by

$$p_1 = \frac{1}{2} \frac{4s^2a + 2ls^2 + stb - 7sat - 7slt + 2at^2 + 6lt^2 - 2t^2b}{3t - 2s}$$

$$p_2 = \frac{1}{2} \frac{4s^2b + 2ls^2 + sat - 7stb + 2t^2b - 7slt + 6lt^2 - 2at^2}{3t - 2s}$$

Substituting these in the profit function we obtain the optimal locations stated in the proposition. Note that in order to satisfy $a^* \leq \frac{l}{2}$ we need $s > 0.32195t$. These are substituted back into the price equations to obtain the equilibrium prices. Recall using optimal locations and prices profits are given by $\Pi_1^* = \Pi_2^* = \frac{l^2 - 96s^4 + 416s^3t - 609s^2t^2 + 312st^3 - 16t^4}{64s(3t - 2s)^2}$. While it is quite cumbersome due to the higher order polynomials involved in the profit expression, we verify that profits are almost always positive through graphical analysis. The expression for optimal profits is positive for all $s > 0.0575t$. Hence (a^*, b^*, p_1^*, p_2^*) is the unique equilibrium. ■

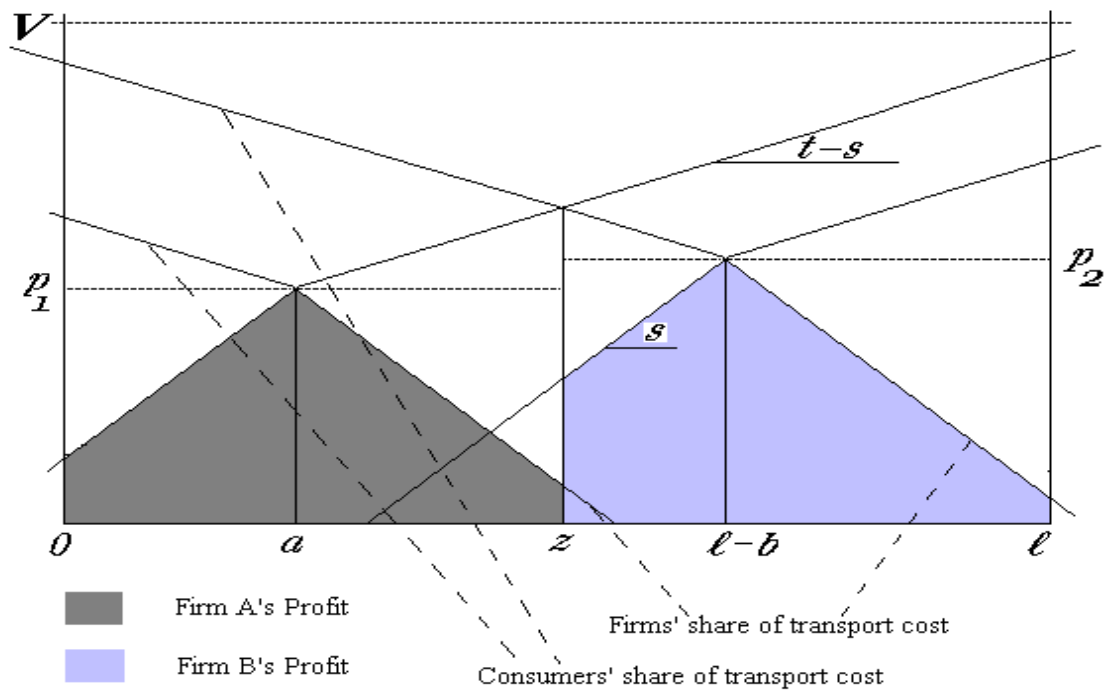


Figure 1: The Location-Price Game

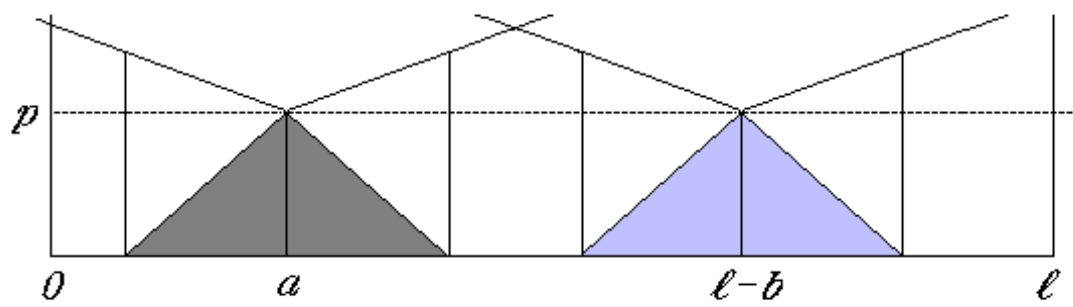


Figure 2: The Low Prices Case

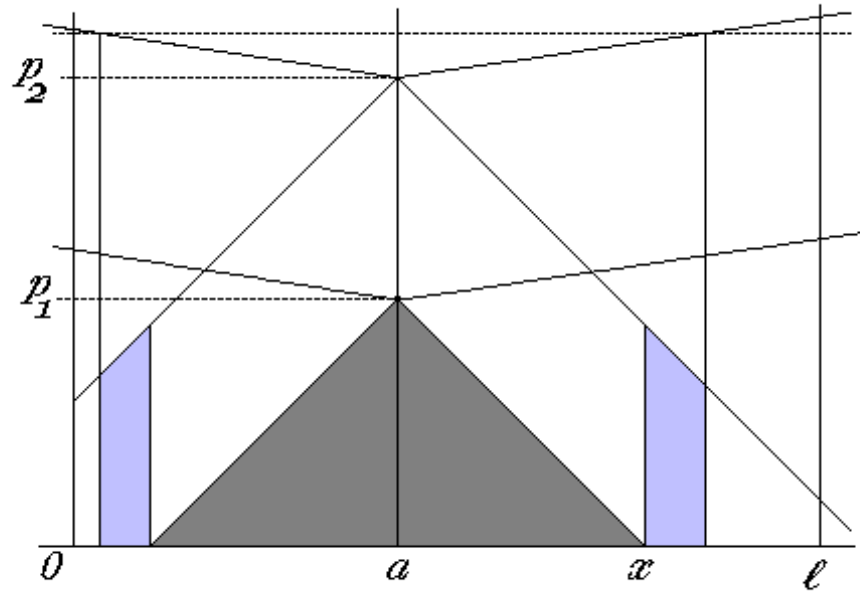


Figure 3: Mixed Strategy Equilibrium for $a+b=l$

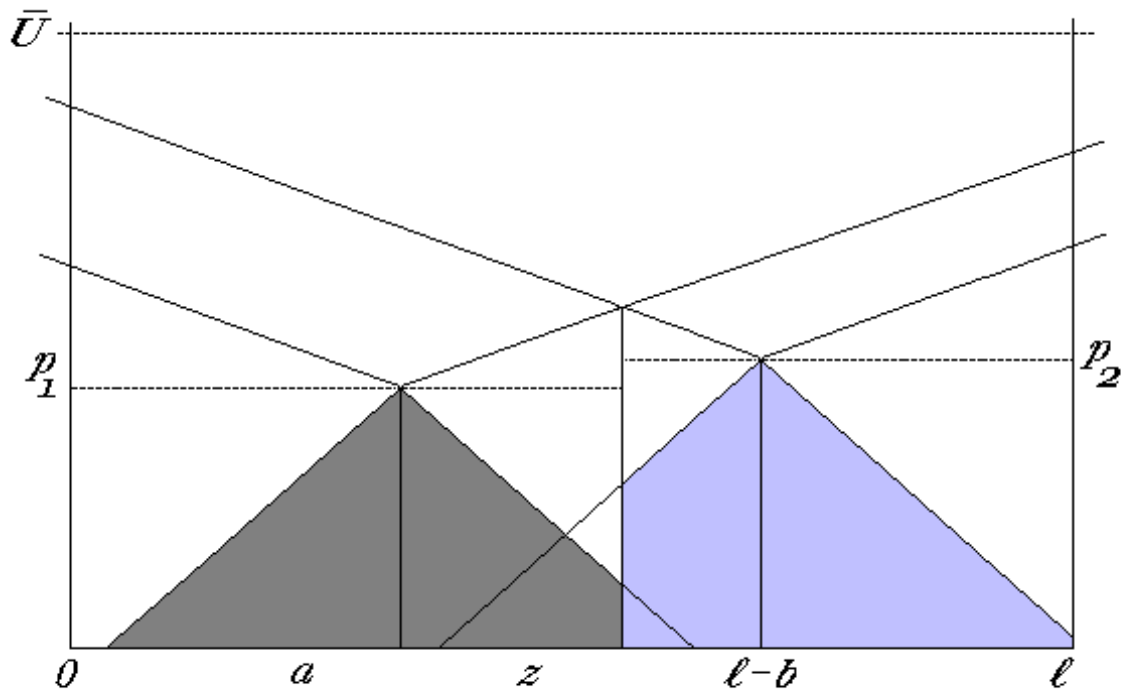


Figure 4: Seller Participation Condition violated for Firm A

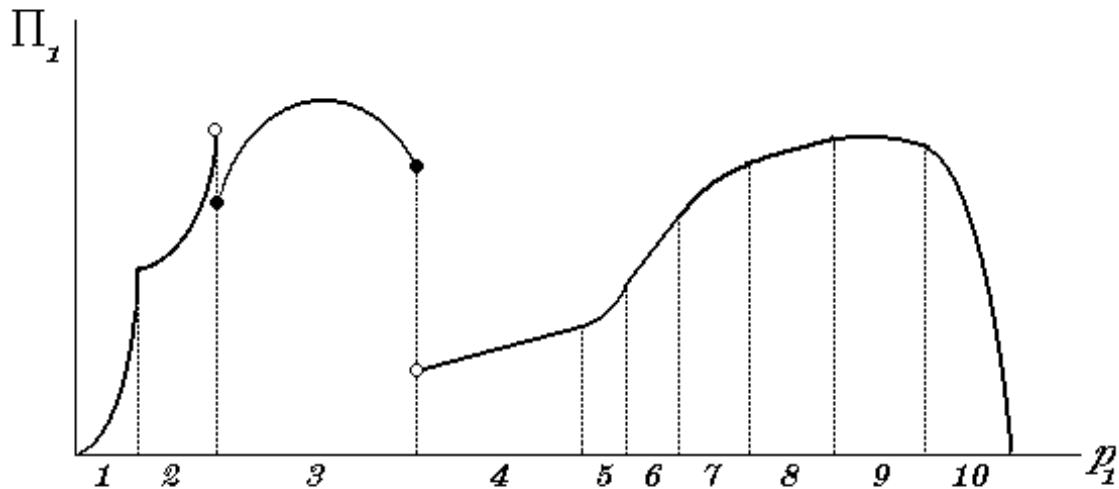


Figure 5: Profit Function for the Location-Price Game

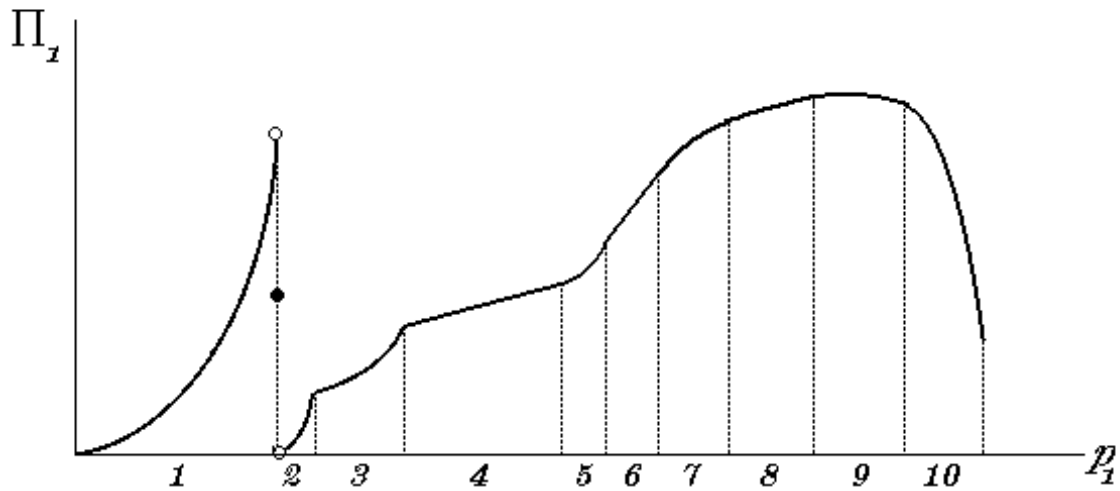


Figure 6: Profit Function for the Special Price Game