

Chamberlain, Thomas

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# **SMALL PERTURBATION APPROACH TO A TRANSIENT INTER-REGIONAL ECONOMY ACCOUNTING FOR WAGES, PRICES, AND TRANSACTION COSTS**

**By**

**Thomas E. Chamberlain<sup>\*</sup>**

Given that the only certain fact is the intensity of pleasure felt at an instant of time, the only epistemologically sound approach is to take intensity as the primary concept. (Georgescu-Roegen, in his introduction to the English translation of Hermann Gossen's book ([1854] 1983, lxxxii)).

## **ABSTRACT**

*Meaningful and useful representation of inter-regional economics over time requires the explicit modelling of human activity in production, consumption, and rest—a representation that has been foreclosed in standard (neoclassical) economics due to a crucial misstep in utility theory during the marginal revolution of the 1870s. In this error, utility (satisfaction) was directly identified with consumables, rather than exclusively with the process-of-knowing attending all activity including consumption, resulting in the suppression of differential time in expectational planning. Differential time in a canonical methodology has been restored to economic theory in four earlier conference-papers by incorporating 20<sup>th</sup> century understanding of subjective uncertainty, intertemporal time preference, expectation theory, and psychosomatic cognitive-function into Gossanian utility theory. Application of this approach has yielded explicit modelling of liquidity preference, endogenous prices and interest rates, return on (labor) investment, and social psychology. Progress is extended in the present work by formulating endogenous labor-rates and commodity prices in an inter-regional market that evolves over a one-year intertemporal period. The method of small perturbations, well-established in applied physics, is introduced to help deal with complexity and nonlinearity in economic behavior. In an initial expository treatment, small differences in productive-capability among three geographical regions are assumed resulting in corresponding first-, second-, and third-order assessments of agent and regional behavior. Due to the simplifying assumption of mirror-image agents throughout the three-region economy, finite transactions in the labor and commodity markets are delayed to the third-order level, although first-order approximations to the wage-rates and commodity-prices are formulated. Transaction cost, estimated to comprise over 50% of all economic activity, is given an introductory formulation.*

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<sup>\*</sup> Chamberlain: Independent Researcher. Los Angeles, CA (USA) / Rev. 0.  
Phone: 1-310-547-5097, Fax (same), E-mail: tomchamb@ix.netcom.com

## I. INTRODUCTION

It is more than of passing interest that instantaneous-utility (i.e., empirically measurable pleasure (Rolls 1975)) must be the primary concept in understanding economics. In this regard, utility (satisfaction) is the time-integral of instantaneous-utility (see, e.g., Strotz 1956)—or, in a similar statement, if a finite intensity of cardinal instantaneous-utility is combined with (multiplied by) a finite interval of cardinal time, the result is a corresponding cardinal magnitude of utility. But what has happened here? Just as differential time in Newton's laws of motion is (partially) extinguished upon integration over time, so differential time in economics is similarly lost when instantaneous-utility is time-integrated—the necessary consequence when utility, not instantaneous-utility, is taken as the primary concept. Accordingly, in economic analyses and models where utility is the primary concept—as, of course, is generally the case in mainstream economics—time has been undermined.

Does it matter that time as a substantive parameter is absent from modern economic theory in its basic or "canonical" formulation (see Hausman 1992)? In answering this question, it may first be noted that time plays an even more profound role in economic behavior (and human behavior in general) than it does in the natural world. The reason is that the evolution of economic systems depends on two modes of time: (1) imaginary (or *expectational*) time; and (2) real (or clock) time (see also Arthur (1999)). More to the point, people naturally prepare expectational plans in imaginary time to guide their real-time activities; these plans then unfold with the progress of real time. Inevitably (and frequently) surprise, with its attending new knowledge, invalidates the operative expectational plan, to a finite degree however small, resulting in a revised plan. Briefly returning to the world of physics, everything is changing or evolving, on diverse time scales. Clearly, our understanding of the physical world, with its diverse time scales, would be insufficient to satisfactorily support modern society were this understanding to be fundamentally static or timeless. Since time plays an even greater (compounded) role in economics, analogy suggests that society would be better served by theory that substantively accounts for time in its basic formulation.

Previous articles by the present writer (1997, 1998a, 1998b, 1999) have helped to present the case for "time in economic theory," showing, first, how the temporal approach, employing the recently introduced *emotive equation*, provides new insight (e.g., the labor-capital relationship, interest, uncertainty, and liquidity preference), and, second, how this approach may be extended to the study and modelling of macroeconomic problems. The present article continues along this path by addressing an inter-regional economy—this being a simplified treatment, in accordance with the proven approach in science of systematically proceeding from the elementary to the complex in developing theory.

A primary purpose of the present work, in addition to further demonstrating the analytic and explanatory power of the Gossenian utility theory (as modified and enhanced), is to introduce the small perturbation approach for dealing with nonlinear problems of high complexity—an approach with proven success in diverse branches of applied physics (see, e.g., Van Dyke 1964). In a variation of this approach, one or more salient parameters in the problem are incrementally changed, or perturbed, leading to the resolution of the representative mathematics into first, second, third, etc. order terms and corresponding equations. (As usual, there are mathematical conditions or requirements that must be satisfied for the legitimate treatment.) Small inter-regional differences in productive capability comprise the perturbation in the considered problem.

A number of simplifying assumptions have been made in the analysis—besides the above-mentioned postulate that only expected productive capability is given small changes among the three regions. The most prominent is the assumption of mirror-image individuals throughout the three regions—i.e., their having identical process-of-knowing (P-O-N) instantaneous-utility functions, the same assessment of uncertainty (future is expectationally certain), the same weighting of expected instantaneous-utility throughout the single intertemporal year (no discounting), and the same expected constraints in number, form, and parameter values. Because of this assumption—ignoring for the moment, the perturbations in productive capability—all individuals conduct their near-subsistence lives (production of food and food-producing appliance, food consumption, and rest) in parallel, with no finite interaction. The formulation nevertheless retains explanatory power by permitting the derivation of market-determined wage-rates and commodity prices.

It is with the assumed small differences in productive capability from region to region that the market-economy “comes to life.” In particular, the participants expect to interact via the inter-regional labor and commodity markets to gradually work-out (over time) the induced expectational imbalances. The result of these assumptions is an approach wherein the first-order approximation represents all participants as following identical and noninteractive “steady-state” (equilibrium) activity-regimens; the second-order component represents all individuals as still noninteractive, but expecting to adjust their respective regimens, as appropriate; and the third-order component (discussion only) reflects full participation in the inter-regional labor and commodity markets.

## **II. REGIONALLY-INTERACTIVE COMMUNITY**

### **IIa OVERVIEW AND GENERAL FORMULATION**

Early in the development of aeronautical science, the challenge of providing initial predictions of flow fields about supersonic wings and bodies at angle of attack was successfully met through use of simplifying assumptions in the aerophysics and mathematics. For example, a considerable simplification was to postulate inviscid flow, thereby eliminating the boundary (shear) layers that originate on the surfaces and eventually separate forming regions of vorticity above and downstream of the configuration. Mathematical complexity was further reduced by assuming slender configurations at small angles of attack, thereby transforming nonlinear three-dimensional governing equations into linear two-dimensional counterparts that applied only to the cross-flow plane. This approach to a mathematically challenging science provided meaningful and useful insight to the aircraft designers of the time.

It is proposed that a similar approach applied to the canonical formulation of behavior (i.e., the emotive equation) will help advance our understanding of economics. Of course, the use of simplifying assumptions is not new to economics. In this regard, Marshall promoted the *ceterus paribus*—“other things remaining equal”—assumption in addressing a given economic sector exclusive of “cross-talk” with other sectors. However, the present approach differs by accounting for interaction throughout the economy—on the basis of a simplifying “small perturbation” of an elementary general formulation.

The "elementary general formulation" comprises a considerable reduction of complexity—for the present introductory or expository purpose. Instead of the complete modern economy with a seemingly infinite variety of commodities and occupations, the considered economy contains just two commodities and the corresponding two productive activities. Each individual has appliance and food production as the first two activities of the daily regimen, with food consumption and rest concluding the day. Three separate regions comprise the economic system, each with its own population and productive capability. In addition to producing and consuming his or her own food, each individual trades labor and commodities in both domestic and inter-regional markets. Wage-rates and commodity-prices, invariant across the three-region economy, are determined in units of a numeraire currency such that corresponding supply versus demand is cleared or balanced. Transaction costs affect both trade-amounts and prices. A single intertemporal year of 365 days is postulated with each individual having a rigorously periodic activity sequence, in modes and durations. (Multi-year formulations will accommodate differing activity durations from year-to-year. See the IAES paper.) All participants have perfect expectational foresight of the coming year (no uncertainty) with a uniform time preference, and all negotiate a mutual comprehensive plan to the end of the year, and no further.

The three-region economic system described above comprises the basis for the perturbation treatment to follow in Section IIb. This treatment consists of three orders of mathematical representation, each resolving new aspects of economic behavior at a deeper level, much as successively higher-powered microscopes reveal organisms previously unseen. At all three orders of approximation, however, a conceptual and mathematically helpful assumption is employed—every individual in all three regions is assumed to have identical personal attributes, i.e. they are assumed to be “mirror-images” of each other. Furthermore, they are assumed to have equivalent commodity and currency amounts at the beginning of the year. The only departure from a uniform distribution of personal attributes and circumstances across the three region economy is an imposed, or postulated, small difference in per capita food and/or appliance productivity from region to region. At the first-order level this difference is unseen, resulting in a three-region ensemble of behaviorally identical individuals, who lead parallel, effectively isolated lives but nevertheless determine (first-order) market wage-rates and commodity-prices. Because

of postulated equilibrium at the first-order level, commodity and currency amounts are invariant throughout the year. At the second-order level, the productivity perturbation has its effect on the activity regimens of participants as isolated agents, as they accordingly adjust productive and consumptive activity-durations in response to their expected advantage or disadvantage—i.e., their nonparticipation in the markets continues in the second-order formulation. Labor and commodity markets come to life at the third-order level, addressed verbally in the present work, as nonlinearities in the P-O-N instantaneous-utility functions and the commodity production/depletion functions have their finite effect on wage-rates and commodity-prices.

Before proceeding to the perturbation analysis and discussion, the general formulation is introduced.

### General Formulation.

In producing the general formulation, the starting point is the canonical emotive equation and corresponding expected constraints:

$$(1a) \quad E_k^i = \sum_{w=1, \infty} [f_{kw}^i \int_0^\infty \lambda_{kw}^i(.,.,.,.,t) P_{kw}^i(.,.,.,.,t) dt] \quad ; \quad \textbf{Emotive Equation}$$

$$(1b) \quad \Phi_{kw}^{ic} = 0, \quad c(w) = 1, \infty. \quad ; \quad \textbf{Expected Constraints}$$

The emotive equation and constraints comprise a comprehensive representation of individual  $i$ 's expectational plan  $k$ , from the real-time datum to the intertemporal horizon (set at infinity as a mathematical convenience). Included in the constraints are the supply versus demand “continuity equations.”<sup>1</sup>

In the emotive equation, the product of expected instantaneous-utility  $P_{kw}^i$  with differential imaginary-time  $dt$ —yielding differential utility  $P_{kw}^i dt$ —is mapped by the emotive mapping function  $\lambda_{kw}^i$  into the real-time datum as a corresponding differential anticipatory-pleasure  $dE_k^i$ . The result,  $\lambda_{kw}^i P_{kw}^i dt$ , is integrated over the intertemporal period from  $0 \rightarrow \infty$ , diminished by the expectational occurrence probability  $f_{kw}^i$ , and summed over all worldlines  $w$  to obtain the expectational plan anticipatory pleasure  $E_k^i$ . Each individual, in his or her purposeful planning,

maximizes  $E_k^i$  ( $dE_k^i = 0$ ;  $d^2E_k^i < 0$ ) subject to the expected constraints for each of the contending plans, choosing the plan that provides the maximum  $E_k^i$ .

Advances in neuropsychology are substantiating or confirming the emotive equation—e.g., measurable instantaneous-utility (Rolls 1975); feeling as a crucial adjunct to practical thought (Damasio 1994; Bechara, et. al. 1997); and measurable intention (Snyder 1997). A derivation of the emotive equation has been provided in the AAAS paper (1998a), with a critical comparison vis-à-vis standard or mainstream mathematical economic theory given in the WEAI paper (1997).<sup>2</sup>

As mathematically developed in the IAES paper (1999), the emotive equation for a daily four-activity regimen (appliance-production, food production, food consumption, and rest) over a 3-year intertemporal period is expressed:

**EMOTIVE EQUATION (For Each Member of the Community):**

$$(2) \quad E^i = \quad \underline{\lambda 1}^i u^i(L_{[1\backslash 4]11}^i, L_{[1\backslash 4]12}^i, C_{[1\backslash 4]1}^i, R_{[1\backslash 4]1}^i) \\ + \underline{\lambda 2}^i \{ f_{[1\backslash 2]}^i u^i(L_{[1\backslash 2]21}^i, L_{[1\backslash 2]22}^i, C_{[1\backslash 2]2}^i, R_{[1\backslash 2]2}^i) \\ + f_{[3\backslash 4]}^i u^i(\dots, \dots, C_{[3\backslash 4]2}^i, R_{[3\backslash 4]2}^i) \} \\ + \underline{\lambda 3}^i \{ f_{[1\backslash 1]}^i u^i(L_{[1\backslash 1]31}^i, L_{[1\backslash 1]32}^i, C_{[1\backslash 1]3}^i, R_{[1\backslash 1]3}^i) \\ + f_{[2\backslash 2]}^i u^i(\dots, \dots, C_{[2\backslash 2]3}^i, R_{[2\backslash 2]3}^i) \\ + f_{[3\backslash 3]}^i u^i(L_{[3\backslash 3]31}^i, L_{[3\backslash 3]32}^i, C_{[3\backslash 3]3}^i, R_{[3\backslash 3]3}^i) \\ + f_{[4\backslash 4]}^i u^i(\dots, \dots, C_{[4\backslash 4]3}^i, R_{[4\backslash 4]3}^i) \}.$$

where  $\underline{\lambda 1}^i$ ,  $\underline{\lambda 2}^i$ , and  $\underline{\lambda 3}^i$  are the postulated constant emotive discount (mapping) coefficients for years 1, 2, and 3, respectively;  $u^i$  is the total annual-utility expected to be experienced in year  $y$  by the individual, for the corresponding (rigorously periodic) daily-activity durations  $L_{[r\backslash s]y1}^i$ ,  $L_{[r\backslash s]y2}^i$ ,  $C_{[r\backslash s]y}^i$ , and  $R_{[r\backslash s]y}^i$ ; and  $f_{[r\backslash s]}^i$  represents the expectational occurrence probability for worldlines  $r$  through  $s$  combined. (Only one plan is now addressed, and the designation  $k$  is dropped.) In the present application we are concerned with the single-year intertemporal period with zero expectational uncertainty, and only the first line of (2) is accordingly retained. Additionally, a new labor-activity  $\Re L_{13}^i$  representing *transaction cost* is introduced—the activity accounting for the spending of (scarce) time in acquiring market information and in buying/selling commodities (the

former assumed fixed and the latter variable). Furthermore, recognizing differing economic performance from region to region, the designation  $\Re$  is introduced as a (preceding) superscript. ( $\Re$  will acquire integer values 1, 2, and 3, for the three separate regions.) In consonance with  $\Re$ , the individual designation  $i$  is converted to  $i(\Re)$ . With these changes (2) becomes, dropping the worldline designation [1\1] as superfluous,

$$(3) \quad \Re E^i = \underline{\lambda} 1^i u^i(\Re L_{11}^i, \Re L_{12}^i, \Re L_{13}^i, \Re C_1^i, \Re R_1^i)$$

where  $\Re L_{13}^i$ , to repeat, accounts for transaction cost—a major component of economic life.<sup>3</sup>

The corresponding constraint relations are obtained as a contraction and modification of (15a-d) of the IAES paper. The first step is to express these equations in a simplified nomenclature consistent with the single-year intertemporal period:

### **EXPECTATIONAL CONSTRAINTS (For Each Member of the Community):**

#### **Single Worldline - Year (Appliance and Food Production)**

$$(4a) \quad \Phi^{\text{indivTime}}_1 = \tau - (L_{11}^i + L_{12}^i + C_1^i + R_1^i) = 0 \quad \text{Time}$$

$$(4b) \quad \Phi^{\text{indivAppl}}_1 = Q^{iA}_0 + \int_{0\mathfrak{I}} F^{iA}_1 dt^* + \int_{0\mathfrak{I}} D^{iA}_1 dt^* + N_{d/y} \Delta Q^{iA}_1 - Q^{iA}_1 = 0 \quad \begin{array}{l} \text{Appliance} \\ \text{(Capital)} \end{array}$$

$$(4c) \quad \Phi^{\text{indivFood}}_1 = Q^{iF}_0 + \int_{0\mathfrak{I}} F^{iF}_1 dt^* + \int_{0\mathfrak{I}} D^{iF}_1 dt^* + N_{d/y} \Delta Q^{iF}_1 - Q^{iF}_1 = 0 \quad \text{Food}$$

$$(4d) \quad \Phi^{\text{indivBudg}}_1 = \langle \underline{P} \Delta Q \rangle^{iA}_1 + \Delta Q^{iF}_1 = 0 \quad \text{Budget}$$

where food, for the present, is retained as the numeraire. In accordance with our general formulation, the above is modified accounting for (1) currency as the numeraire, (2) productive labor both hired and supplied in market transactions, (3) transaction cost, and (4) (three) geographic regions:

$$(5a) \quad \Re \Phi^{\text{indivTime}}_1 = \tau - (\Re L_{11}^i + \Re L_{12}^i + \Re L_{13}^i + \Re C_1^i + \Re R_1^i) = 0 \quad \text{Time}$$

$$(5b) \quad \Re \Phi^{\text{indivT(1)}}_1 = \Re L_{11}^i - (\Re L_{11}^i + \Re \delta L_{11}^i) = 0 \quad \begin{array}{l} \text{Time (1)} \\ \backslash \quad \quad \quad \backslash \text{ wage labor} \\ \quad \quad \quad \backslash \text{ household labor} \end{array}$$

\ aggregate production-labor

$$(5c) \quad \Re \Phi^{\text{indiv} \overline{\text{I}(2)}}_1 = \Re L_{12}^i - (\Re L_{12}^i + \Re \delta L_{12}^i) = 0$$

Time (2)

$$(5d) \quad \Re \Phi^{\text{indiv} \overline{\text{I}(3)}}_1 = \Re L_{13}^i - C_{L3} + [\alpha_A \Re \Delta Q_{11}^{iA} + \alpha_F \Re \Delta Q_{11}^{iF}] = 0$$

\ market-related labor

Time (3)

$$(5e) \quad \Re \Phi^{\text{indiv} \overline{\text{Appl}}}_1 = \Re Q_0^{iA} + \int_{0\mathbb{I}} \Re F_{11}^{iA} dt^* + \int_{0\mathbb{I}} \Re D_{11}^{iA} dt^* + N_{d/y} \Re \Delta Q_{11}^{iA} - \Re Q_{11}^{iA} = 0$$

Appliance Production \ / Appliance depletion

Appliance  
(Capital)

$$(5f) \quad \Re \Phi^{\text{indiv} \overline{\text{Food}}}_1 = \Re Q_0^{iF} + \int_{0\mathbb{I}} \Re F_{11}^{iF} dt^* + \int_{0\mathbb{I}} \Re D_{11}^{iF} dt^* + N_{d/y} \Re \Delta Q_{11}^{iF} - \Re Q_{11}^{iF} = 0$$

Food Production \ / Food depletion

Food

$$(5g) \quad \Re \Phi^{\text{indiv} \overline{\text{Exchng}}}_1 = \Re \langle \underline{P} \Delta Q \rangle_{11}^{iA} + \Re \langle \underline{P} \Delta Q \rangle_{11}^{iF} + \Re \underline{w}_{11}^i \Re \delta L_{11}^i + \Re \underline{w}_{12}^i \Re \delta L_{12}^i + \Re \Delta Q_{11}^{iM} = 0$$

Exchange

$$(5h) \quad \Re \Phi^{\text{indiv} \overline{\text{Money}}}_1 = \Re Q_0^{iM} + N_{d/y} \Re \Delta Q_{11}^{iM} - \Re Q_{11}^{iM} = 0$$

Money

The budget equation (4d) is replaced by two constraints—exchange (5g) and money (5h). All expected exchanges by the individual during the day are recognized in (5g) and result in a “per day” net change of the currency-in-hand  $\Re \Delta Q_{11}^{iM}$ , this, in turn, entering (5h) to give the currency-in-hand at year-end  $\Re Q_{11}^{iM}$ . Another major change consists of the hire-for-wages terms,  $\Re \underline{w}_{11}^i \Re \delta L_{11}^i$  and  $\Re \underline{w}_{12}^i \Re \delta L_{12}^i$ , where  $\Re \underline{w}_{11}^i$  and  $\Re \underline{w}_{12}^i$  are the market wages per unit time of supplied labor  $\delta L_{11}^i$  and  $\delta L_{12}^i$ , respectively. Note that  $\Re \delta L_{11}^i$  and  $\Re \delta L_{12}^i$  may be positive or negative, depending on whether the individual hires the labor (positive) or supplies the labor (negative). The remaining terms,  $\Re \langle \underline{P} \Delta Q \rangle_{11}^{iF}$  and  $\Re \langle \underline{P} \Delta Q \rangle_{11}^{iA}$ , represent the total cost (per day) in the purchase ( $\Delta Q$  positive) or sale ( $\Delta Q$  negative) of food and appliance, respectively.

Turning to the remaining constraint equations, the time constraint (5a) exhibits the additional activity duration  $\Re L_{13}^i$ , representing transaction cost (in terms of activity time) attending market-related activity. As seen in (5d),  $L_{13}^i$  has an “overhead” burden  $C_{L3}$  irrespective

of the level of purchases and sales. ( $C_{L3}$  represents, hypothetically, time spent reading about the market or visiting the market in familiarization sessions.). To  $C_{L3}$  is added the transaction expression  $[\alpha_A \Re \Delta Q_1^{iA} + \alpha_F \Re \Delta Q_1^{iF}]$ , where  $\alpha_A$  and  $\alpha_F$  are positive constants. Each of the two components is positive for the consumer of marketed commodities and negative for the producer.<sup>4</sup> This means that the consumer enjoys a benefit at the producer's expense—perhaps free transportation to and from the market.

The appliance constraint (5e) is functionally equivalent to its counterpart in the IAES paper,<sup>5</sup> except that the operative labor term in  $F_1^{iA}$  is now  $\Re L_{11}^i$  (along with  $\Re L_{12}^i$ ) rather than  $L_{11}^i$ , i.e.,

$$\Re F_1^{iA}(\Re Q_0^{iA}, N_{d/y}, \Re \Delta Q_1^{iA}, \Re L_{11}^i, \Re L_{12}^i, t^*)$$

The reason is that the individual now participates in the labor market, either hiring or supplying appliance-production labor  $\Re \delta L_{11}^i$  (and food production labor  $\Re \delta L_{12}^i$ ) as represented in (5b) and (5g), so  $\Re L_{11}^i$  cannot represent the actual labor entering the production function. More to the point,  $\Re L_{11}^i$  represents the labor entering appliance production, where this labor may be exclusively provided by the individual ( $\Re \delta L_{11}^i$  is negative—i.e. he or she supplies rather than hires appliance-producing labor) or partially provided by others ( $\Re \delta L_{11}^i$  is positive—i.e., labor is hired to work with the individual). Similar considerations apply to appliance depletion  $\Re D_1^{iA}$  and food production  $\Re F_1^{iF}$ . (As will be seen, food depletion  $\Re D_1^{iF}$  in the food constraint is due only to consumption.)

### **EXPECTATIONAL CONTINUITY EQUATIONS (Aggregates Over All members of the Community):**

To the emotive equation and constraints must be added the expectational “continuity” equations (as has been noted, formally part of the constraints), requiring that labor and commodities neither originate nor disappear in market transactions—conditions that serve to determine the wage-rates and commodity prices:

$$(6a) \quad \sum_{\Re=1,3} \left[ \sum_{i(\Re)=1,n} \Re \delta L_{11}^i \right] = 0 \quad \begin{array}{l} \text{Labor:} \\ \text{Appliance Production} \end{array}$$

$$(6b) \quad \sum_{\mathfrak{R}=1,3} \left[ \sum_{i(\mathfrak{R})=1,n} \mathfrak{R} \delta L_{12}^i \right] = 0 \quad \text{Labor: Food Production.}$$

$$(6c) \quad \sum_{\mathfrak{R}=1,3} \left[ \sum_{i(\mathfrak{R})=1,n} \mathfrak{R} \Delta Q_1^{iA} \right] = 0 \quad \text{Appliance}$$

$$(6d) \quad \sum_{\mathfrak{R}=1,3} \left[ \sum_{i(\mathfrak{R})=1,n} \mathfrak{R} \Delta Q_1^{iF} \right] = 0 \quad \text{Food}$$

$$(6e) \quad \sum_{\mathfrak{R}=1,3} \left[ \sum_{i(\mathfrak{R})=1,n} \mathfrak{R} \Delta Q_1^{iM} \right] = 0 \quad \text{Money}$$

As usual, one of the equations (discretionary) does not enter into the solution process, due to implicit representation in the constraints (see Intriligator 1971).

The general formulation is completed with the postulate that the inter-regional community rigorously and comprehensively negotiates all human activity, and the results therefrom, prior to the initial instant of the given intertemporal year. As a consequence, each individual's expectationally-certain plan for the coming year is in rigorous accord with the plans of all others. In the absence of surprise, this mutually negotiated plan provides an exact prediction of all economic behavior for the year.<sup>6</sup>

Each agent is unique at this point, having self-specific P-O-N instantaneous-utility functions, self-specific quantities of commodities on hand at the start of the year, self-specific food and appliance productivities, etc. This will change in the section to follow. In particular, all participants in the economy will be postulated identical (mirror-images) in their personal attributes and circumstances, except for expected small differences in productive capability between the three regions. In the first-order formulation these differences in productivity are not evident resulting expectedly inactive markets due to symmetry. It is later in the section that uniqueness is (partially) restored—with the expected differences in appliance and food productive-capacity among the three regions producing second-order adjustments in the expected activity regimens. Nonlinear third-order adjustments, verbally addressed in the present work, reflect the desire of the participants to trade labor and commodities in the markets.

## **IIb SMALL PERTURBATION APPROACH**

Marshall was clearly correct in observing that “The element of time is the centre of the chief difficulty of almost every economic problem (1890).” It is also true that standard economic theory is fundamentally timeless. From these statements it could be concluded that the best is yet to come in the discipline’s investigation of economic behavior. A difficulty, however, is the complexity of economics—owing to the dual nature of time (imaginary and real) in the science, and the great number of parameters.

Guidance on how to proceed in the face of complexity may be obtained from the natural sciences. In this regard, it cannot be accepted that difficulty in understanding and formulating (human) behavior is singular in comparison to the difficulties faced in the natural sciences—physics, say. While it is true that time has a dual character in human behavior, versus the unitary (real) nature in physics, time in relativity and quantum mechanics may be judged sensibly transcendent—e.g., in both disciplines, the location of a particle in space-time is indefinite. Furthermore, the level of mathematics in physics exceeds that envisioned for economics. General relativity, for example, is approached with tensor calculus and Riemann geometry of curved space-time, while behavioral theory may not require mathematical acumen much beyond the method of Lagrange multipliers of advanced calculus. Faced with their own conceptual and analytic challenges, basic and applied mathematical physicists have frequently simplified the governing equations, particularly in the earliest development. Mathematical economists may use similar techniques in obtaining initial solutions to the emotive equation and associated constraints.

In the present paper, an analytical method similar to that employed in mathematical physics is introduced. The approach retains the salient aspects of economics (e.g., endogenous preferences and market prices) while modelling economic change over the intertemporal year. As noted earlier, the method consists of three steps: [1] a “first-order” formulation, wherein all agents, expectedly identical in their attributes and circumstances, plan parallel, noninteractive lives; [2] a “second-order” formulation, wherein all agents, still expecting to lead noninteractive lives, adjust their planned activity-regimens to accommodate the expected shift in productive capability; and [3] a third-order formulation, where nonlinearities in instantaneous-utility and other functions effect adjustments in expected wage-rates and commodity prices such that inter-

regional trading is planned. (Only the first and second-order steps are formulated in the present work.) The perturbation method is described. In the next section, followed by mathematical development of the first- and second-order formulations.

### Perturbation Method

The formal procedure is to replace each of the dependent parameters in (3), (5a-h) and (6a-e) with the sum of its first-, second-, and third-order components. Inasmuch as the third-order approximation is not mathematically formulated, only the first- and second-order terms are explicitly recognized below.

(7)

|                                                                                        |                                                         |
|----------------------------------------------------------------------------------------|---------------------------------------------------------|
| $\Re L_{11}^i = L_{11}^{[1]i} + \Re L_{11}^{[2]i}$                                     | : Labor duration per day in appliance production. $\Re$ |
| $L_{12}^i = L_{12}^{[1]i} + \Re L_{12}^{[2]i}$                                         | : Labor-duration per day in food production.            |
| $\Re L_{13}^i = L_{13}^{[1]i} + \Re L_{13}^{[2]i}$                                     | : Labor duration per day in market-related activity.    |
| $\Re L_{11}^{i'} = L_{11}^{[1]i'} + \Re L_{11}^{[2]i'}$                                | : Labor entering the appliance production function.     |
| $\Re L_{12}^{i'} = L_{12}^{[1]i'} + \Re L_{12}^{[2]i'}$                                | : Labor entering the food production function.          |
| $\Re \delta L_{11}^i = \delta L_{11}^{[1]i} + \Re \delta L_{11}^{[2]i}$                | : Market-labor/day entering appliance production.       |
| $\Re \delta L_{12}^i = \delta L_{12}^{[1]i} + \Re \delta L_{12}^{[2]i}$                | : Market-labor/day entering food production.            |
| $\Re C_1^i = C_1^{[1]i} + \Re C_1^{[2]i}$                                              | : Food consumption duration per day.                    |
| $\Re R_1^i = R_1^{[1]i} + \Re R_1^{[2]i}$                                              | : Rest duration per day.                                |
| $\Re Q_1^{iA} = Q_1^{[1]iA} + \Re Q_1^{[2]iA}$                                         | : Appliance quantity at end-of-year.                    |
| $\Re \Delta Q_1^{iA} = \Delta Q_1^{[1]iA} + \Re \Delta Q_1^{[2]iA}$                    | : Appliance transfer to/from the market per day.        |
| $\Re Q_1^{iF} = Q_1^{[1]iF} + \Re Q_1^{[2]iF}$                                         | : Food quantity at end-of-year.                         |
| $\Re \Delta Q_1^{iF} = \Delta Q_1^{[1]iF} + \Re \Delta Q_1^{[2]iF}$                    | : Food transfer to/from the market per day.             |
| $\Re Q_1^{iM} = Q_1^{[1]iM} + \Re Q_1^{[2]iM}$                                         | : Currency quantity at end-of-year .                    |
| $\Re \Delta Q_1^{iM} = \Delta Q_1^{[1]iM} + \Re \Delta Q_1^{[2]iM}$                    | : Currency transfer to/from the market per day.         |
| $\Re \underline{W}_{11}^i = \underline{W}_{11}^{[1]i} + \Re \underline{W}_{11}^{[2]i}$ | : Wage-rate for appliance production.                   |
| $\Re \underline{W}_{12}^i = \underline{W}_{12}^{[1]i} + \Re \underline{W}_{12}^{[2]i}$ | : Wage-rate for food production.                        |
| $\Re \underline{P}_1^{iA} = \underline{P}_1^{[1]iA} + \Re \underline{P}_1^{[2]iA}$     | : Market price per unit appliance.                      |
| $\Re \underline{P}_1^{iF} = \underline{P}_1^{[1]iF} + \Re \underline{P}_1^{[2]iF}$     | : Market price per unit food.                           |

In these expressions, the superscripts [1] and [2] identify, respectively, the first and second-order terms, and  $\mathfrak{R}$  identifies the geographical region— $\mathfrak{R}=1$ : marginally diminished ability to produce both appliance and food;  $\mathfrak{R}=2$ : marginally increased ability to produce appliance; and  $\mathfrak{R}=3$ : marginally increased ability to produce food. Because the participants across the three-region economy are identical in all respects in the first-order formulation, the designation  $\mathfrak{R}$  is superfluous and accordingly omitted—although it should be recognized that different regional populations  $n(\mathfrak{R})$  are assumed (i.e.,  $i=1, n(\mathfrak{R})$ ). At the second-order level, all terms are (linearly) proportional to the incremental (increase or decrease) of the individual's productive capability. However, as an insight provided by the perturbation approach, the wage and price components are zero at the second-order level, acquiring their first finite adjustment at the third-order level.

The production and dissipation functions for both appliance and food are similarly resolved into their first- and second-order components. In this regard, the production functions are the determining factors in the perturbation analysis. More to the point, the second and higher-order terms in the three regions acquire magnitudes that are driven or effected by imposed inter-regional differences in these functions. In imposing the inter-regional differences, the basic production-functions  $F^{iA}_1$  and  $F^{iF}_1$  (each having the same form for all three regions) are multiplied by corresponding factors  $[1 + \mathfrak{R}\epsilon^{[2]iA}]$  and  $[1 + \mathfrak{R}\epsilon^{[2]iF}]$  to obtain the region-dependent functions

$$\begin{aligned}
 (8) \quad \mathfrak{R}F^{iA}_1 &= (1 + \mathfrak{R}\epsilon^{[2]iA}) F^{iA}_1(\mathfrak{R}Q^{iA}_0, N_{dy}, \mathfrak{R}\Delta Q^{iA}_1, \mathfrak{R}L^{iA}_{11}', \mathfrak{R}L^{iA}_{12}', t^*) \\
 &= (1 + \mathfrak{R}\epsilon^{[2]iA}) (F^{[1]iA}_1 + \mathfrak{R}F^{[2]iA}_1 + \dots) \\
 &= F^{[1]iA}_1 + \mathfrak{R}\epsilon^{[2]iA} F^{[1]iA}_1 + \mathfrak{R}F^{[2]iA}_1 + \dots \\
 &\quad \downarrow \text{1st order} \quad \quad \quad \downarrow \text{2nd order} \quad \quad \quad \downarrow \text{3rd order} \quad \quad \quad \downarrow \text{4th order} \quad \quad \quad \downarrow \text{5th order} \\
 &= F^{[1]iA}_1 + \mathfrak{R}\epsilon^{[2]iA} F^{[1]iA}_1 + [\partial F^{[1]iA}_1 / \partial L^{[1]i}_{11}'] \mathfrak{R}L^{[2]i}_{11}', \\
 &\quad + [\partial F^{[1]iA}_1 / \partial L^{[1]i}_{12}'] \mathfrak{R}L^{[2]i}_{12}', \\
 &\quad + [\partial F^{[1]iA}_1 / \partial \Delta Q^{[1]iA}_1] \mathfrak{R}\Delta Q^{[2]iA}_1 \\
 &\quad + (\text{higher-order terms})
 \end{aligned}$$

and

$$(9) \quad \mathfrak{R}F^{iF}_1 = (1 + \mathfrak{R}\epsilon^{[2]iF}) F^{iF}_1(\mathfrak{R}Q^{iA}_0, N_{dy}, \mathfrak{R}\Delta Q^{iA}_1, \mathfrak{R}L^{iA}_{11}', \mathfrak{R}L^{iA}_{12}', t^*)$$

$$\begin{aligned}
 &= (1 + \Re \epsilon^{[2]iF}) (F^{[1]iF}_1 + \Re F^{[2]iF}_1 + \dots) \\
 &= F^{[1]iF}_1 + \Re \epsilon^{[2]iF} F^{[1]iF}_1 + \Re F^{[2]iF}_1 + \dots \\
 &\quad \downarrow \text{1st order} \quad \quad \quad \downarrow \text{2nd order} \quad \quad \quad \downarrow \text{2nd order} \quad \quad \quad \downarrow \text{2nd order} \\
 &= F^{[1]iF}_1 + \Re \epsilon^{[2]iF} F^{[1]iF}_1 + [\partial F^{[1]iF}_1 / \partial L^{[1]i}_{11}] \Re L^{[2]i}_{11}, \\
 &\quad \quad \quad + [\partial F^{[1]iF}_1 / \partial L^{[1]i}_{12}] \Re L^{[2]i}_{12}, \\
 &\quad \quad \quad + [\partial F^{[1]iF}_1 / \partial \Delta Q^{[1]iA}_1] \Re \Delta Q^{[2]iA}_1 \\
 &\quad \quad \quad + (\text{higher-order terms})
 \end{aligned}$$

where, consistent with the earlier discussion, the perturbation parameters  $\Re \epsilon^{[2]iA}$  and  $\Re \epsilon^{[2]iF}$  are defined

(10)

$$\begin{aligned}
 \text{Region 1:} \quad &^1 \epsilon^{[2]iA} = -\pi; \quad ^1 \epsilon^{[2]iF} = -\pi : \text{Appliance and food production} \\
 &\quad \quad \quad \text{disadvantaged;} \\
 \text{Region 2:} \quad &^2 \epsilon^{[2]iA} = \pi; \quad ^2 \epsilon^{[2]iF} = 0 : \text{Appliance production advantaged;} \\
 \text{Region 3:} \quad &^3 \epsilon^{[2]iA} = 0; \quad ^3 \epsilon^{[2]iF} = \pi : \text{Food production advantaged;}
 \end{aligned}$$

with  $\pi \ll 1$ . The appliance and food depletion functions are similarly defined, but without perturbation coefficients:

(11)

$$\begin{aligned}
 \Re D^{iA}_1 &= D^{iA}_1(\Re Q^{iA}_0, N_{d/y}, \Re \Delta Q^{iA}_1, \Re L^{i}_{11}, \Re L^{i}_{12}, t^*) \\
 &= D^{[1]iA}_1 + \Re D^{[2]iA}_1 + \dots \\
 &\quad \downarrow \text{1st order} \quad \quad \quad \downarrow \text{2nd order} \quad \quad \quad \downarrow \text{2nd order} \quad \quad \quad \downarrow \text{2nd order} \\
 &= D^{[1]iA}_1 + [\partial D^{[1]iA}_1 / \partial L^{[1]i}_{11}] \Re L^{[2]i}_{11}, \\
 &\quad \quad \quad + [\partial D^{[1]iA}_1 / \partial L^{[1]i}_{12}] \Re L^{[2]i}_{12}, \\
 &\quad \quad \quad + [\partial D^{[1]iA}_1 / \partial \Delta Q^{[1]iA}_1] \Re \Delta Q^{[2]iA}_1 \\
 &\quad \quad \quad + (\text{higher-order terms})
 \end{aligned}$$

and

(12)

$$\begin{aligned}
 \Re D^{iF}_1 &= D^{iF}_1(N_{d/y}, \Re C^i_1, t^*) \\
 &= D^{[1]iF}_1 + \Re D^{[2]iF}_1 + \dots \\
 &\quad \downarrow \text{1st order} \quad \quad \quad \downarrow \text{2nd order} \quad \quad \quad \downarrow \text{2nd order} \\
 &= D^{[1]iF}_1 + [\partial D^{[1]iF}_1 / \partial C^{[1]i}_1] \Re C^{[2]i}_1 \\
 &\quad \quad \quad + (\text{higher-order terms}).
 \end{aligned}$$

Note that the food depletion function is simplified for the present purposes, accounting only for consumption (i.e., neglecting spoilage).

Upon substituting equations (7) and (8-12) into (3), (5a-h), and (6a-e), the first and second-order terms are readily separated resulting in the corresponding sets of first- and second-order formulations. These formulations are presented below, followed by discussion in Section III.

### **First-Order Formulation.**

Inserting the perturbation relations (7) into the general formulation and retaining the first-order terms yields a formulation that is functionally equivalent to (3), (5), and (6), except for the first-order designation—i.e., superscript [1]. However, as noted earlier, the intent is to employ assumptions which greatly simplify the mathematics. In particular, it is postulated that all individuals are “mirror images” of each other in all respects, except for the small expected inter-regional perturbations in productive capability and their (expected) effects. In accordance with this assumption, the participants lead parallel (subsistence) lives at the first-order level without finite trading but nevertheless, in their infinitesimal interactions in the market, determine the *endogenous* wage-rates and commodity prices. It is additionally postulated that equilibrium exists (to first-order). The solution to the first-order formulation provides, in turn, the foundation for the higher-order solutions, accounting for disequilibrium and the (expected) growth/decline of assets and income over time.

On the basis of the adopted perturbation approach with the equilibrium and mirror-image assumptions, the first-order formulation consists of (3), (5a-h), and (6a-e) transcribed as is, except for a number of terms that cancel (due to equilibrium—signified by double underlines) or are infinitesimal (due to the mirror-image assumption—thick underline):

### **EMOTIVE EQUATION (First-Order):**

$$(13) \quad E^{[1]i} = \underline{\lambda} \underline{1}^i u^i (L^{[1]i}_{11}, L^{[1]i}_{12}, L^{[1]i}_{13}, C^{[1]i}_1, R^{[1]i}_1)$$

### **EXPECTATIONAL CONSTRAINTS (First-Order):**

$$(14a) \quad \Phi^{[1]indivTime}_1 = \tau - (L^{[1]i}_{11} + L^{[1]i}_{12} + L^{[1]i}_{13} + C^{[1]i}_1 + R^{[1]i}_1) = 0$$

Time

$$(14b) \quad \Phi^{[1]indivT(1)}_1 = L^{[1]i}_{11} - (L^{[1]i}_{11} + \delta L^{[1]i}_{11}) = 0$$

Time (1)

$\backslash$  wage labor  
 $\backslash$  household labor  
 $\backslash$  aggregate production-labor

$$(14c) \quad \Phi^{[1]indivT(2)}_1 = L^{[1]i}_{12} - (L^{[1]i}_{12} + \delta L^{[1]i}_{12}) = 0$$

Time (2)

$$(14d) \quad \Phi^{[1]indivT(3)}_1 = L_3^{[1]i} - C_{L3} + [\alpha_A \Delta Q^{[1]iA}_1 + \alpha_F \Delta Q^{[1]iF}_1] = 0$$

Time (3)

$\backslash$  market-related labor

$$(14e) \quad \Phi^{[1]indivAppl}_1 = \underbrace{Q^{[1]iA}_0}_{\text{Appliance Production}} + \int_{0\pm} F^{[1]iA}_1 dt^* + \int_{0\pm} D^{[1]iA}_1 dt^* \underbrace{-}_{\text{Appliance depletion}} + N_{d/y} \Delta Q^{[1]iA}_1 - \underbrace{Q^{[1]iA}_1}_{\text{Appliance (Capital)}} = 0$$

Appliance (Capital)

$$(14f) \quad \Phi^{[1]indivFood}_1 = \underbrace{Q^{[1]iF}_0}_{\text{Food Production}} + \int_{0\pm} F^{[1]iF}_1 dt^* + \int_{0\pm} D^{[1]iF}_1 dt^* \underbrace{-}_{\text{Food depletion}} + N_{d/y} \Delta Q^{[1]iF}_1 - \underbrace{Q^{[1]iF}_1}_{\text{Food}} = 0$$

Food

$$(14g) \quad \Phi^{[1]indivExchng}_1 = \langle P \Delta Q \rangle^{[1]iA}_1 + \langle P \Delta Q \rangle^{[1]iF}_1 + \underline{w}^{[1]i}_{11} \delta L^{[1]i}_{11} + \underline{w}^{[1]i}_{12} \delta L^{[1]i}_{12} + \Delta Q^{[1]iM}_1 = 0$$

Exchange

$$(14h) \quad \Phi^{[1]indivMoney}_1 = \underbrace{Q^{[1]iM}_0}_{\text{Money}} + N_{d/y} \Delta Q^{[1]iM}_1 - \underbrace{Q^{[1]iM}_1}_{\text{Money}} = 0$$

Money

The equilibrium and mirror-image assumptions are seen to significantly reduce the number of unknowns and attending mathematical complexity.

### EXPECTATIONAL CONTINUITY EQUATIONS (First-Order):

Added to the first-order emotive and constraint equations are the first-order continuity expressions (as earlier noted, formally part of the expectational constraints):

$$(15a) \quad \sum_{\Re=1,3} \left[ \sum_{i=1,n(\Re)} \delta L^{[1]i}_{11} \right] = 0 \quad \text{Labor: Appliance Production}$$

$$(15b) \quad \sum_{\mathfrak{R}=1,3} \left[ \sum_{i=1,n(\mathfrak{R})} \delta L_{12}^{[1]i} \right] = 0 \quad \text{Labor: Food Production.}$$

$$(15c) \quad \sum_{\mathfrak{R}=1,3} \left[ \sum_{i=1,n(\mathfrak{R})} \Delta Q_{1}^{[1]iA} \right] = 0 \quad \text{Appliance}$$

$$(15d) \quad \sum_{\mathfrak{R}=1,3} \left[ \sum_{i=1,n(\mathfrak{R})} \Delta Q_{1}^{[1]iF} \right] = 0 \quad \text{Food}$$

$$(15e) \quad \sum_{\mathfrak{R}=1,3} \left[ \sum_{i=1,n(\mathfrak{R})} \Delta Q_{1}^{[1]iM} \right] = 0 \quad \text{Money}$$

Here it might be argued that the equations are superfluous inasmuch as the mirror-image postulate ensures null transaction-magnitudes. However, in the Lagrangian method, the transaction-magnitudes are finite in the “search” for solution.

Equations (13), (14a-h) and (15a-e) may be solved using the method of Lagrange multipliers for a properly defined problem, resulting in a (first-order, equilibrium) timeline of activity and commodity production/consumption throughout the year. A solution is not necessary for the present expository purposes. However, the methodology does permit the derivation of expressions for salient parameters—e.g., (endogenous) wage-rates and commodity prices. Some attention will be given to this capability later in the paper.

### **Second-Order Formulation.**

While the preceding first-order formulation has an equilibrium or stationary-state character (by postulate), the second-order formulation to follow represents change over time. In this regard, the individual’s (expected) activity-regimen over the intertemporal year does not change—due to the assumed periodicity of diurnal activity. (Unchanging periodicity is adopted as a simplification for the present purposes—see the IAES paper (1999) for a more general treatment.) What does change, however, are the quantities of food and appliance on hand throughout the year—a consequence of each individual’s separate or isolated productive activity, there being no market transactions until the third-order approximation is reached.

In obtaining the second-order formulation, the initial step is to recognize the separate first- and second-order terms. The expanded form of the emotive equation (3) may be written

$$\begin{aligned}
 (16) \quad \Re E^i &= E^{[1]i} + \Re E^{[2]i} + \dots \\
 &= \underline{\lambda 1}^i [u^i(L^{[1]i}_{11}, L^{[1]i}_{12}, L^{[1]i}_{13}, C^{[1]i}_1, R^{[1]i}_1) \\
 &\quad + (\partial u^i / \partial L^{[1]i}_{11}) \Re L^{[2]i}_{11} : \text{Appliance production labor} \\
 &\quad + (\partial u^i / \partial L^{[1]i}_{12}) \Re L^{[2]i}_{12} : \text{Food production labor} \\
 &\quad + (\partial u^i / \partial L^{[1]i}_{13}) \Re L^{[2]i}_{13} : \text{Market-related labor} \\
 &\quad + (\partial u^i / \partial C^{[1]i}_1) \Re C^{[2]i}_1 : \text{Food consumption} \\
 &\quad + (\partial u^i / \partial R^{[1]i}_1) \Re R^{[2]i}_1 : \text{Rest} \\
 &\quad + \text{higher-order terms}
 \end{aligned}$$

where the first-order expression [1] is recognized from the preceding development. The second-order terms [2] consist of linear departures from the first-order solution.

### **EMOTIVE EQUATION (Second-Order):**

From (16) it is seen that the second-order emotive equation is

$$\begin{aligned}
 (17) \quad \Re E^{[2]i} &= \underline{\lambda 1}^i [(\partial u^i / \partial L^{[1]i}_{11}) \Re L^{[2]i}_{11} \\
 &\quad + (\partial u^i / \partial L^{[1]i}_{12}) \Re L^{[2]i}_{12} \\
 &\quad + (\partial u^i / \partial L^{[1]i}_{13}) \Re L^{[2]i}_{13} \\
 &\quad + (\partial u^i / \partial C^{[1]i}_1) \Re C^{[2]i}_1 \\
 &\quad + (\partial u^i / \partial R^{[1]i}_1) \Re R^{[2]i}_1]
 \end{aligned}$$

where, as before,  $\Re$  identifies the corresponding region.

Turning to the constraint relations, the second-order terms in each equation are readily distinguished except for the (nonlinear) product terms in the exchange equation. Writing this equation (i.e., (5g)) in its expanded form yields

$$\begin{aligned}
 (5g') \quad \Re \Phi^{\text{indivExchng}}_1 &= \Re \langle \underline{P} \Delta Q \rangle^{iA}_1 + \Re \langle \underline{P} \Delta Q \rangle^{iF}_1 \\
 &\quad + \Re \underline{w}^i_{11} \Re \delta L^i_{11} + \Re \underline{w}^i_{12} \Re \delta L^i_{12} \\
 &\quad + \Re \Delta Q^{iM}_1 = 0
 \end{aligned}$$

Each of the first-order exchange terms (e.g.,  $\Delta Q^{[1]jA}_1$  and  $\delta L^{[1]j}_{12}$ ) is zero due to the mirror-image assumption. Furthermore, the second-order products (e.g.,  $\Re \underline{P}^{[2]jF}_1 \Re \Delta Q^{[2]jF}_1$ ) are higher (third) order. Equation (5g') accordingly becomes

for the second-order expression. Combining this result with the remaining equations (5a-f and h), similarly contracted, yields

$$\begin{aligned}
 (18e) \quad \Re \Phi^{[2] \text{indiv} \text{Appl}}_1 &= \underbrace{\text{Appliance Production}}_{\text{Appliance Depletion}} \\
 &\int_{0 \leq t} \left\{ \Re \epsilon^{[2] \text{IA}} F^{[1] \text{IA}}_1 \right. \\
 &+ [\partial F^{[1] \text{IA}}_1 / \partial L^{[1] \text{i}}_{11}] \Re L^{[2] \text{i}}_{11}, \\
 &+ [\partial F^{[1] \text{IA}}_1 / \partial L^{[1] \text{i}}_{12}] \Re L^{[2] \text{i}}_{12}, \\
 &\left. + [\partial F^{[1] \text{IA}}_1 / \partial \Delta Q^{[1] \text{IA}}_1] \Re \Delta Q^{[2] \text{IA}}_1 \right\} dt^*
 \end{aligned}$$

$$\begin{aligned}
 & + \int_{0T} \left\{ \left[ \partial D^{[1]iA}_1 / \partial L^{[1]i}_{11} \right] \Re L^{[2]i}_{11} \right. \\
 & + \left[ \partial D^{[1]iA}_1 / \partial L^{[1]i}_{12} \right] \Re L^{[2]i}_{12} \\
 & + \left. \left[ \partial D^{[1]iA}_1 / \partial \Delta Q^{[1]iA}_1 \right] \Re \Delta Q^{[2]iA}_1 \right\} dt^* \\
 & + N_{d/y} \Re \Delta Q^{[2]iA}_1 - \Re Q^{[2]iA}_1 = 0 \\
 & \text{Appliance}
 \end{aligned}$$

$$\begin{aligned}
 (18f) \quad \Re \Phi^{[2]i \text{divFood}}_1 &= \\
 & \quad \text{Food Production} \\
 & \int_{0T} \left\{ \Re \varepsilon^{[2]iF}_1 F^{[1]iF}_1 \right. \\
 & + \left[ \partial F^{[1]iF}_1 / \partial L^{[1]i}_{11} \right] \Re L^{[2]i}_{11} \\
 & + \left[ \partial F^{[1]iF}_1 / \partial L^{[1]i}_{12} \right] \Re L^{[2]i}_{12} \\
 & + \left. \left[ \partial F^{[1]iF}_1 / \partial \Delta Q^{[1]iA}_1 \right] \Re \Delta Q^{[2]iA}_1 \right\} dt^* \\
 & \quad \text{Food Depletion} \\
 & + \int_{0T} \left\{ \left[ \partial D^{[1]iF}_1 / \partial C^{[1]i}_1 \right] \Re C^{[2]i}_1 \right\} dt^* \\
 & + N_{d/y} \Re \Delta Q^{[2]iF}_1 - \Re Q^{[2]iF}_1 = 0 \\
 & \text{Food}
 \end{aligned}$$

$$\begin{aligned}
 (18g) \quad \Re \Phi^{[2]i \text{divExchng}}_1 &= \underline{P}^{[1]iA}_1 \Re \Delta Q^{[2]iA}_1 + \underline{P}^{[1]iF}_1 \Re \Delta Q^{[2]iF}_1 \\
 & + \underline{W}^{[1]i}_{11} \Re \delta L^{[2]i}_{11} + \underline{W}^{[1]i}_{12} \Re \delta L^{[2]i}_{12} \\
 & + \Re \Delta Q^{[2]iM}_1 = 0 \\
 & \text{Exchange}
 \end{aligned}$$

$$\begin{aligned}
 (18h) \quad \Re \Phi^{[2]i \text{divMoney}}_1 &= N_{d/y} \Re \Delta Q^{[2]iM}_1 - \Re Q^{[2]iM}_1 = 0 \\
 & \text{Money}
 \end{aligned}$$

As before, the market-exchange continuity equations are added (below), ensuring that the net amounts of labor exchanges and commodity exchanges in the market are zero.

### EXPECTATIONAL CONTINUITY EQUATIONS (Second-Order):

$$\begin{aligned}
 (19a) \quad \sum_{\Re=1,3} \left[ \sum_{i=1,n(\Re)} \Re \delta L^{[2]i}_{11} \right] &= 0 \quad \text{Labor:} \\
 & \text{Appliance Production}
 \end{aligned}$$

$$\begin{aligned}
 (19b) \quad \sum_{\Re=1,3} \left[ \sum_{i=1,n(\Re)} \Re \delta L^{[2]i}_{12} \right] &= 0 \quad \text{Labor:} \\
 & \text{Food Production.}
 \end{aligned}$$

$$\begin{aligned}
 (19c) \quad \sum_{\Re=1,3} \left[ \sum_{i=1,n(\Re)} \Re \Delta Q^{[2]iA}_1 \right] &= 0 \quad \text{Appliance}
 \end{aligned}$$

$$(19d) \quad \sum_{\mathfrak{R}=1,3} \left[ \sum_{i=1,n(\mathfrak{R})} \mathfrak{R} \Delta Q^{[2]iF}_1 \right] = 0 \quad \text{Food}$$

$$(19e) \quad \sum_{\mathfrak{R}=1,3} \left[ \sum_{i=1,n(\mathfrak{R})} \mathfrak{R} \Delta Q^{[2]iM}_1 \right] = 0 \quad \text{Money}$$

Similar to the first-order formulation, a solution may be obtained to the foregoing emotive equation, constraint equations, and continuity equations using the method of Lagrange multipliers.<sup>7</sup>

As a brief commentary on the preceding developments, contrary to the first-order formulation which has been assigned the stationary or equilibrium character, the second-order formulation recognizes the changing conditions affecting the three regional populations over the intertemporal year (assuming, of course, that the economy unfolds according to plan, without surprises). However, these changing conditions are due exclusively to the efforts and activities of the individuals (effectively) in isolation from each other—i.e., finite market transaction between the three regions retains its null character (also, wage-rates and commodity prices are unchanged). The labor and commodity markets become active at the non-linear third-order approximation, as the agents of the productivity-advantaged regions recognize the profits to be gained by hiring labor and trading commodities at the revised market-clearing prices.

### III. DISCUSSION

It is interesting to speculate on what the state of fluid mechanics would be if the science had decided that steady-state (equilibrium) theory was sufficient or appropriate, as did economics in the twentieth century. In fact, fluid mechanics did have a static equilibrium theory about the time of the marginal revolution in economics in the late nineteenth century—so-called *potential theory*. For the mathematics to work, the air flow had to be “irrotational”—e.g., no attached or separated shear layers—in addition to steady-state. The solutions looked intuitively satisfying but were deficient in crucial respects. For example, real air-flows about airplanes have shear (boundary) layers on all external surfaces, without which flight would be impractical if not impossible. The boundary layers are crucial because wings need “circulation” to generate lift, and

boundary-layer separation at specified locations initiates and maintains the circulation. Our current worldwide air-transport industry simply would not have developed had airflow theorists chosen to adhere to the equilibrium models of early theory—the necessary theoretical tools would not have been available. And, of course, theoretical fluid mechanics is important in numerous other applications—e.g., design of gas turbine engines, hydraulic power generators, oil pipelines, air conditioners, etc. It is doubtful whether many of the great technological advances of the twentieth century could have occurred on the basis of a cursory (equilibrium) theory of fluid mechanics.

Could there be a concern here for economics as a science? Is it possible to substantively understand, and mathematically represent, the many facets of economics—e.g., capital function, return on investment, liquidity preference, interest rates, unemployment, inflation, financial crises, inter-regional and intra-regional growth, etc.—without a theory that takes explicit account of time in the canonical formulations? In the late twentieth century, with a state of economic science that many consider seriously inadequate (see the 1<sup>st</sup> quarter 1998 issue of the *Journal of Economic Literature*) and in a state of crisis (Bell and Kristol 1981), the affirmative answer cannot be credited.

The emotive equation, combining the contributions of prominent psychologists and economists over the past 140+ years, provides the basis for advancing economic theory where standard theory has faltered. The present work adds to the previous four conference papers in demonstrating how properly accounting for time yields substantive analytic power and a deeper insight into behavior. An equilibrium first-order formulation developed earlier in this paper provided the basis for the second-order transient formulation that followed. While all individuals comprising the three-region economic system were “mirror-image” identical in all attributes and circumstances in the first-order formulation, in the second-order formulation the uniformity was altered by small differences in appliance and food productive-capacity among the regions. These alterations in expected productive-capacity induced corresponding expectational adjustments in the activity regimens of all participants as independent or isolated agents—i.e., the labor and commodity markets remained inactive. It is in the third-order (nonlinear) perturbation that the interactive evolution of the regional economic system emerges.

In the discussion to follow, the first-order formulation is first addressed resulting in expressions for the endogenous wage-rates and commodity prices. The second-order formulation is then reviewed, followed by some perspectives regarding the third-order treatment.

### **First-Order Considerations (Wage-Rates and Commodity-Prices)**

By assuming “mirror-image” individuals, the three regional sets of representative equations are greatly simplified in the number of unknowns. The formulation nevertheless retains its explanatory character: In addition to modelling the regimen of the (effectively) isolated individual (reflecting the model in the IAES paper), endogenous wage-rates and commodity prices can be expressed in terms of expected P-O-N instantaneous-utility subject to expected constraints.<sup>8</sup>

**Endogenous Wage-Rates.** The concern is not in demonstrating how we may compute wage-rates in the actual economy but in demonstrating endogeneity—i.e., how wages (and prices) are (ultimately) controlled from within the system, contrary to standard theory with its basically exogenous prices.<sup>9</sup>

In obtaining the wage-rate expression by means of the method of Lagrange multipliers, it is first recognized that each of the first-order constraint equations (14a-h) is in product with a corresponding multiplier in forming the composite emotive equation (see (22) of the IAES paper). Three subsidiary relations are necessary in determining the expression for the wage-rate  $\underline{w}^{[1]i}_{11}$  in the production of appliance. The first of these relations is for the individual’s personal labor in the production of appliance  $L^{[1]i}_{11}$ , and the second for rest  $R^{[1]i}_1$ ,

$$(20) \quad L^{[1]i}_{11} : \quad \underline{\lambda}^i_1 (\partial u^i / \partial L^{[1]i}_{11}) \quad - \quad l^{[1]iT}_1 \quad - \quad l^{[1]iT(1)}_1 = 0;$$

$$(21) \quad R^{[1]i}_{[1\backslash 4]1} : \quad \underline{\lambda}^i_1 (\partial u^i / \partial R^{[1]i}_1) \quad - \quad l^{[1]iT}_1 \quad = 0$$

Eliminating  $l^{[1]iT}_1$  between the two equations gives

$$(23) \quad \underline{\lambda}^i_1 \{ (\partial u^i / \partial L^{[1]i}_{11}) - (\partial u^i / \partial R^{[1]i}_1) \} - l^{[1]iT(1)}_1 = 0$$

and replacing the total-utility partial derivatives with their marginal P-O-N pleasure (instantaneous-utility) equivalents (see the IAES paper) yields

$$(24) \quad N_{d/y} \lambda 1^i \{pL1^i(L^{[1]i}_{11}) - pR^i(R^{[1]i}_1)\} - l^{[1]iT(1)}_1 = 0;$$

The third relation is for the wage-labor  $\delta L^{[1]i}_{11}$ ,

$$(25) \quad \delta L^{[1]i}_{11} : -l^{[1]iT(1)}_1 + l^{[1]iE}_1 w^{[1]i}_{11} = 0;$$

And the fourth is for the corresponding payment  $\Delta Q^{[1]iM}_1$  (obtained from (14g & h))

$$(26) \quad \Delta Q^{[1]iM}_1 : l^{[1]iE}_1 + N_{d/y} l^{[1]iM}_1 = 0.$$

Solving (24, 25, and 26) for the appliance-production labor-rate yields

$$(27) \quad \underline{w}^{[1]i}_{11} = l^{[1]iT(1)}_1 / l^{[1]iE}_1$$

$$= \frac{\lambda 1^i \{pL1^i(L^{[1]i}_{11}) - pR^i(R^{[1]i}_1)\}}{l^{[1]iM}_1}$$

where the numerator is the net marginal value (anticipatory pleasure) per unit time of appliance-production labor and the denominator is the marginal value of the monetary unit. The equal-value exchange yields the wage-rate in units of currency per unit of time worked.

Following a similar procedure gives the food-production labor-rate

$$(28) \quad \underline{w}^{[1]i}_{12} = l^{[1]iT(2)}_1 / l^{[1]iE}_1$$

$$= \frac{\lambda 1^i \{pL2^i(L^{[1]i}_{12}) - pR^i(R^{[1]i}_1)\}}{l^{[1]iM}_1}$$

The significance of (27) and (28) is not, to reiterate, with the prospect of computing wage rates for given P-O-N instantaneous-utility functions, etc., but in demonstrating their endogenous character, as part of an overall methodological coherence. Beyond this is the additional consideration that advances in semi-empirical formulation—i.e., econometrics—is properly dependent on a substantive and coherent mathematical theory of economic behavior.

**Endogenous Commodity-Prices.** Commodity-price has been mathematically addressed in earlier papers (see 1998b and 1999), albeit with food as the numeraire rather than currency. The monetary numeraire is accommodated in the present work. Following a procedure similar to that used in obtaining the labor-rate results in

$$(29) \quad \underline{P}^{[1]iA}_1 = [1/l^{[1]iM}_1] \left\{ l^{[1]iT(3)}_1 \alpha_A + l^{[1]iA}_1 \left[ \int_{0\tau} [\partial F^{[1]iA}_1 / \partial \Delta Q^{[1]iA}_1] dt^* + \int_{0\tau} [\partial D^{[1]iA}_1 / \partial \Delta Q^{[1]iA}_1] dt^* + N_{d/y} \right] + l^{[1]iF}_1 \int_{0\tau} [\partial F^{[1]iF}_1 / \partial \Delta Q^{[1]iA}_1] dt^* \right\}$$

for the market price of appliance and

$$(30) \quad \underline{P}^{[1]iF}_1 = [1/l^{[1]iM}_1] \left\{ l^{[1]iT(3)}_1 \alpha_F + l^{[1]iF}_1 N_{d/y} \right\}$$

for the market price of food. (The simpler form of  $\underline{P}^{[1]iF}_1$  versus  $\underline{P}^{[1]iA}_1$  is due to food not being a factor in production, or depletion—except for consumption.)

Equations (29) and (30) are similar to the wage-rate expressions in having the currency Lagrange multiplier in the denominator, this of course introducing the monetary unit into the expressions. Introduction of the commodity units into the corresponding expressions is more complicated due to the integrals over the intertemporal year  $\tau$ . Here something similar to the chain rule of calculus applies. Consider (29), and the term

$$l^{[1]iF}_1 \int_{0\tau} [\partial F^{[1]iF}_1 / \partial \Delta Q^{[1]iA}_1] dt^*.$$

$F^{[1]iF}_1$  has the dimensions [GOOD (food)/TIME],  $\Delta Q^{[1]iA}_1$  the dimension [GOOD (appliance)], and  $t^*$  the dimension [TIME], yielding the dimension set [GOOD (food)/GOOD (appliance)].  $l^{[1]iF}_1$  has the dimension set [PLEASURE/GOOD (food)], and its product with the integral's dimension set gives [PLEASURE/GOOD (appliance)]. Returning to (29), the term  $l^{[1]iT(3)}_1 \alpha_A$  has of course the same dimensions as the foregoing—i.e.,  $l^{[1]iT(3)}_1 \sim [\text{PLEASURE/TIME}]$  in product with  $\alpha_A \sim [\text{TIME/GOOD (appliance)}]$  yields [PLEASURE/GOOD (appliance)]. The remaining terms (in the bold brackets) are similarly addressed. Dividing the dimension set in the braces of (29) by the dimension set for the monetary multiplier [PLEASURE/GOOD (money)] yields the dimensions of the appliance price  $\underline{P}^{[1]iA}_1$

$$\begin{aligned} \underline{p}^{[1]iA}_1 &\sim [\text{PLEASURE/GOOD (appliance)}] / [\text{PLEASURE/GOOD (money)}] \\ &\sim [\text{GOOD (money)/GOOD (appliance)}] \end{aligned}$$

with the conclusion:

$$(31) \quad \underline{p}^{[1]iA}_1 = \frac{\text{Number of Monetary Units}}{\text{Unit of Appliance}}.$$

On a similar basis, the food-price definition is obtained:

$$(32) \quad \underline{p}^{[1]iF}_1 = \frac{\text{Number of Monetary Units}}{\text{Unit of Food}}.$$

Equations (31) and (32) demonstrate, once again, how market exchange ratios are directed or guided by wants or preferences internal (endogenous) to the economic system, rather than on the basis of exogenously imposed preferences (commodity utilities). In this regard, commodities acquire (marginal) specific utility (or, more fundamentally, marginal specific anticipatory pleasure) in the mathematics, but on the basis of imputation rather than spurious direct assignment. As a related consideration, market performance/behavior in general—e.g., supply/demand, wage-rates, commodity prices, interest rates—are plan-dependent. In this regard, even if no one in the economy changed their operative plan over time the market could change, e.g. as demographics evolve to a more heavily weighted retirement population. Furthermore, plan-change due to surprise—e.g., due to a natural disaster or international financial-panic—affects prices and wages. It could be considered obvious that this character of real economic behavior is beyond the reach of fundamentally timeless standard (neoclassical) economics with its short-circuited utility theory.

### **Second-Order Considerations (Individuals Independently Adjust To Production Perturbations)**

While transient or time-dependent economic behavior is represented in the second-order approximation as the agents revise their plans in response to expected changes in productive

capability, this representation is limited in that there is no interaction in the labor and commodity markets. There are two reasons for the noninteractive markets: (1) the mirror-image postulate ensures null labor and commodity transfers thereby extinguishing the terms  $\Re \underline{P}^{[2]iA}_1 \Delta Q^{[1]iA}_1$ ,  $\Re P^{[2]iF}_1 \Delta Q^{[1]iF}_1$ ,  $\Re w^{[2]i}_{11} \delta L^{[1]i}_{11}$ , and  $\Re w^{[2]i}_{12} \delta L^{[1]i}_{12}$ , and their promotion of finite wage-rates and commodity prices; and (2) the second-order approximation rejects, or suppresses, the sources of nonlinearity in the instantaneous-utility and production/depletion functions, with the same effect of defeating finite prices. In the absence of wage-rate and commodity-price changes there is no basis or motivation (e.g., profit) for finite market transactions.

Despite the failure of the (second-order) market to provide “jobs” and “customers,” each participant in the inter-regional economy seeks to improve his or her well-being in the effectively isolated condition by increasing productive activity if disadvantaged or decreasing the same if advantaged. This is accomplished by the individual’s expectational calculus: Upon recognizing that the productive capability of food in region 3 is increased, for example, he or she would expect an imbalance in P-O-N utility at the activity margins. More to the point, for an increased food-consumption duration but with the productive-activity durations the same (i.e., rest is partially eclipsed), the individual would see that the net marginal utility of food consumption ( $\{pC^i(^3C^{[2]i}_1) - pR^i(^3R^{[2]i}_1)\} d[^3C^{[2]i}_1]$ ) has a smaller absolute magnitude than the net marginal utility of food production ( $\{pL^i(^3L^{[2]i}_{11}) - pR^i(^3R^{[2]i}_1)\} d[^3L^{[2]i}_{11}]$ ). The resolution is to curtail productive activity to restore the balance between the net marginal utility of consumption and the net marginal dis-utility of production. Of course, in the interactive economy with functioning markets the individual can, additionally, trade in labor and goods to help achieve the balance. This capability is contained in the third-order approximation.

### **Third-Order Considerations<sup>10</sup>** **(Inter-Regional Adjustment Through Market Transactions)**

An interesting feature of the present perturbation analysis is the emergence of active labor and commodity markets at the third-order level. Inspection of the mathematics suggests that even if the mirror-image assumption were not employed thereby permitting finite market transactions at the first-order level, the second-order price components would still be zero with corresponding

zero components of the labor and commodity transactions. This conclusion follows from the emotive equation in its linearized second-order form. To be specific, application of the method of Lagrange multipliers produces multiplier relationships—and values—which are identical to those of the first-order approximation. Accordingly, the second-order wage-rate and commodity-price components must be zero for the second-order exchange constraint (5g') to have the same functional form as its first-order counterpart (14g), thereby being compatible with the “shared” multipliers.

Full inter-regional market participation of the agents then emerges at the third-order level, as nonlinearities in the P-O-N instantaneous-utility functions and production/consumption functions effect finite (third-order) wage-rate and commodity-price components. The production-advantaged agents in regions 2 and 3 accordingly may realize a profit by hiring labor at a favorable wage-rate as a means of achieving expectational balance in accordance with their intertemporal productivity-advantage. Region 1 agents, disadvantaged in productive-capability, may supply their labor at the market rate to similarly achieve expectational balance. Finally, the wages in monetary terms are used to purchase food and appliance thereby closing the commercial cycle. Transaction, or institutional, costs have their effect in increasing prices and diminishing trade.

#### IV. CONCLUSION

Faced with complex nonlinear problems, scientists and engineers in applied physics have frequently turned to perturbation analysis to achieve meaningful results. A primary purpose of the present work has been to demonstrate how this approach may be employed in economics. Toward this end in an expository analysis, a three-region economic system comprised of an arbitrary number of participants with near-subsistence activity regimens who trade labor and commodities in inter-regional markets was defined. Productive capability of food and appliance was, respectively, either increased, decreased, or left unchanged in the three regions. In responding to these expected conditions, all agents cooperated in detailed and expectationally-certain planning of their personal and interactive behavior over a one-year intertemporal period. The economy-

wide condition of “mirror-image” individuals in their personal attributes was postulated as a simplifying assumption, as was rigorously periodic “equilibrium” for the first-order formulation. Although wage-rates and commodity-prices receive functional (endogenous) expressions in the first-order development, it is not until the third-order that these market parameters are adjusted thereby encouraging all agents to expect to trade labor and commodities in balancing their intertemporal-calculus. While all agents continue to produce food for personal consumption at the third-order level, beneficiaries in the two regions advantaged by the productive-capability perturbation hire those of the remaining disadvantaged region on a “part-time” basis, thereby receiving a monetary profit for liquidity or for the purchase of food and/or appliance. Transaction cost is recognized in the intertemporal planning—in terms of the effect of this cost on commodity prices (increased) and level of trade (decreased). Food, food-producing appliance, and currency are redistributed at the third-order level as the agents in the three regions coordinate activities to maximize their personal welfare.

Part of the significance of the present study rests with the substantive accommodation of time across the inter-regional economy. Of course, it is not *real* time that is represented but imaginary or *expectational* time, this constituting the basis on which the individual connects his or her expected experience or action at one intertemporal-time with the expectation at an earlier or later intertemporal-time. During conscious experience, except for (usually) brief distraction following surprise, the operative expectational plan guides the individual’s activities through real time.

Time is, of course, more than an incidental that can be ignored to advantage in modelling, as is typically assumed in standard theory. When correctly represented, it permits insight into the relationships between cognition and the material world, this world including other conscious agents. While the substantive understanding of economic behavior—including capital function, capital-labor interaction, liquidity preference, and growth/decline—has not yielded to the fundamentally timeless theoretical approaches of standard theory, such understanding has been achieved by the present explicit representation of expected human activity (see (1997), (1998a), (1998b), and (1999)). Further evidence of this success is reflected in the present paper in the

mathematical formulation of endogenous wages-rates/commodity-prices, transaction cost, and inter-regional trade.

## NOTES

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<sup>1</sup> These equations are represented as “adjunct” or separate (expectational) constraints in the formulations, to help clarify the presentation and discussion.

<sup>2</sup> Copies of conference papers may be obtained from the writer.

<sup>3</sup> Transaction costs have been estimated to comprise 50% of all economic activity (see Chueng 1998).

<sup>4</sup> Preservation of the mirror-image character of the mathematics was a consideration in the formulation of transaction cost.

<sup>5</sup> The usual function conditionals have been dropped as either unnecessary or understood.

<sup>6</sup> Of course in the real world with its frequent surprises, exact prediction of concerted activity over extended periods is improbable. (See the IAES article for a preliminary treatment of uncertainty in planning.)

<sup>7</sup> Inasmuch as the second order formulation is a linear perturbation about the first-order basis, it follows (by inspection) that the first-order Lagrange multipliers are retained. Revised multiplier values arise in the nonlinear third order approximation.

<sup>8</sup> Wage rates and commodity prices are frequently set by institutions, and this feature of the real world can be represented in the present methodology.

<sup>9</sup> The salient consideration is that wage-rates and commodity-prices are endogenous in actual or real economic systems, whereas in standard theory they are basically exogenous.

<sup>10</sup> Pending a substantive mathematical treatment to be developed at a later date, the verbal assessments of the third-order approximation can be considered heuristic.

## NOMENCLATURE

|                                |                                                                                                                                                                |
|--------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $b$                            | Expected rate of food consumption per unit consumption time (dimension [GOOD/TIME]).                                                                           |
| $C$                            | Expected daily duration of food consumption (dimension [TIME]).                                                                                                |
| $D^A$                          | Expected appliance depletion function (dimensions [GOOD (appliance)/TIME]).                                                                                    |
| $D^F$                          | Expected food depletion function (dimensions [GOOD (food)/TIME]).                                                                                              |
| $E_k^i$                        | Pleasure experienced by individual $i$ in anticipation of expectational plan $k$ (dimension [PLEASURE]).                                                       |
| $F^A$                          | Expected appliance production function (dimensions [GOOD (appliance)/TIME]).                                                                                   |
| $F^F$                          | Expected food production function (dimensions [GOOD (food)/TIME]).                                                                                             |
| $f$                            | Expected worldline occurrence probability.                                                                                                                     |
| $I^T$                          | Lagrange multiplier for the primary time constraint (dimensions [PLEASURE/TIME]).                                                                              |
| $I^{T(1)}, I^{T(2)}, I^{T(3)}$ | Lagrange multipliers for the secondary time constraints (dimensions [PLEASURE/TIME]).                                                                          |
| $I^A, I^F$                     | Lagrange multipliers for the appliance and food constraints (dimensions [PLEASURE/GOOD]).                                                                      |
| $I^E, I^M$                     | Lagrange multipliers for the exchange and currency (money) constraints (dimensions [PLEASURE/GOOD (money)]).                                                   |
| $L_1$                          | Expected daily duration of labor activity in the production of appliance (dimension [TIME]).                                                                   |
| $L_1'$                         | Aggregate labor duration entering the appliance production function (i.e., personal labor $L_1$ plus wage-labor adjustment $\delta L_1$ ). (Dimension [TIME].) |
| $\delta L_1$                   | Wage-labor duration (purchased or worked) in the production of appliance. (Dimension [TIME].)                                                                  |
| $L_2$                          | Expected daily duration of labor activity in the production of food (dimension [TIME]).                                                                        |
| $L_2'$                         | Aggregate labor duration entering the food production function (i.e., personal labor $L_2$ plus wage-labor adjustment $\delta L_2$ ). (Dimension [TIME].)      |
| $\delta L_2$                   | Wage-labor duration (purchased or worked) in the production of food (dimension [TIME]).                                                                        |
| $N_{d/y}$                      | Number of days per year.                                                                                                                                       |
| $n(\mathfrak{R})$              | Number of participants, or agents, in each of the three regions $\mathfrak{R}$ .                                                                               |
| $P$                            | Expected process-of-knowing pleasure (or pain): datum is start of plan (dimension [PLEASURE]).                                                                 |
| $pL1$                          | Expected process-of-knowing instantaneous-utility in the production of food-producing appliance: datum is the start of activity (dimension [PLEASURE]).        |
| $pL2$                          | Expected process-of-knowing instantaneous-utility in the production of food-producing appliance: datum is the start of activity (dimension [PLEASURE]).        |
| $pC$                           | Expected process-of-knowing instantaneous-utility in the consumption of food: datum is the start of activity (dimension [PLEASURE]).                           |
| $pR$                           | Expected process-of-knowing instantaneous-utility of rest: datum is the start of activity (dimension [PLEASURE]).                                              |
| $\underline{P}^A$              | Expected price of appliance (in units of numeraire money per unit of appliance).                                                                               |
| $\underline{P}^F$              | Expected price of food (in units of numeraire money per unit of food).                                                                                         |
| $\underline{Q}^A$              | Expected quantity of appliance (dimension [GOOD (appliance)]).                                                                                                 |
| $\underline{Q}^F$              | Expected quantity of food (dimension [GOOD (appliance)]).                                                                                                      |

|              |                                                                                                              |
|--------------|--------------------------------------------------------------------------------------------------------------|
| $\Delta Q^A$ | Expected transferred amount of appliance.                                                                    |
| $\Delta Q^F$ | Expected transferred amount of food.                                                                         |
| $R$          | Expected daily duration of rest (dimension [TIME]).                                                          |
| $t$          | Expectational (intertemporal) time, from start of plan (dimension [TIME]).                                   |
| $t^*$        | Expectational (intertemporal) time, from start of year (dimension [TIME]).                                   |
| $u$          | Expected worldline-year utility (dimensions [PLEASURExTIME]).                                                |
| $w_1$        | Expected market-rate for appliance-production labor (in units of numeraire currency per unit-time of labor). |
| $w_2$        | Expected market-rate for food-production labor (in units of numeraire currency per unit-time of labor).      |

### Greek Symbols:

|                                                                       |                                                                                                                                                                                                            |
|-----------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $\alpha_A, \alpha_F$                                                  | Expected transactional (or institutional) cost coefficients.                                                                                                                                               |
| $\Re^{\{2\}jZ}$                                                       | Expected positive or negative second-order perturbation factor ( $ \varepsilon  < 1$ , dimension [TIME <sup>-1</sup> ]) applied to production function $Z$ (appliance or food) in region $\Re$ (see (10)). |
| $\lambda$                                                             | Expectational emotive mapping function (and expected pleasure/pain discount factor). (Dimension [TIME <sup>-1</sup> ].)                                                                                    |
| $\underline{\lambda}$                                                 | Expectational emotive mapping function, invariant within the worldline-year (dimension [TIME <sup>-1</sup> ]).                                                                                             |
| $\underline{\lambda}_1, \underline{\lambda}_2, \underline{\lambda}_3$ | Expectational emotive mapping functions, invariant within years 1, 2, and 3, respectively.                                                                                                                 |
| $\Phi$                                                                | Expectational constraint function.                                                                                                                                                                         |
| $\tau$                                                                | Expected length of day (dimension [TIME]).                                                                                                                                                                 |
| $\underline{\tau}$                                                    | Expected length of year (dimension [TIME]).                                                                                                                                                                |

### Superscripts:

|               |                                                                                   |
|---------------|-----------------------------------------------------------------------------------|
| $A$           | Appliance.                                                                        |
| $c$           | Expectational constraint (along worldline-year $wy$ of expectational plan $k$ ).  |
| $E$           | Exchange.                                                                         |
| $F$           | Food.                                                                             |
| $i$           | Individual $i$ . (Upper case for individual leading isolated [Crusoe] existence). |
| $M$           | Money (numeraire)                                                                 |
| $m$           | Virtual constraint (along worldline-day $wy$ of expectational plan $k$ ).         |
| $\Re$         | Geographical region 1, 2, or 3.                                                   |
| $T$           | Time.                                                                             |
| $y$           | Year                                                                              |
| $Z$           | Appliance $A$ , food $F$ , or currency (money) $M$ .                              |
| $[1],[2]$     | First and second-order, respectively.                                             |
| $(1),(2),(3)$ | Time constraint 1, 2, or 3.                                                       |

### Subscripts:

|                   |                                                                                 |
|-------------------|---------------------------------------------------------------------------------|
| $k$               | Expectational plan.                                                             |
| $[r \setminus s]$ | Identical worldline-days $r$ through $s$ combined (i.e., uncertainties summed). |
| $w$               | Worldline.                                                                      |
| $0$               | Initial quantity of commodity at beginning of year 1.                           |

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