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Taxation and the Sustainability of Collusion: Ad Valorem versus Specific Taxes

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Taxation and the Sustainability of Collusion: Ad Valorem versus Specific Taxes

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Taxation and the Sustainability of Collusion: Ad Valorem versus Specific Taxes

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Abstract

Assuming constant marginal cost, it is shown that a switch from specific to *ad valorem* taxation has no effect on the critical discount factor required to sustain collusion. This result is shown to hold for Cournot oligopoly as well as for Bertrand oligopoly when collusion is sustained with Nash-reversion strategies or optimal-punishment strategies. In a Cournot duopoly model with linear demand and quadratic costs, it is shown that the critical discount factor is lower with an *ad valorem* tax than with a specific tax. However, in contrast to Colombo and Labrecciosa (2013), it is shown that revenue is always higher with an *ad valorem* tax than with a specific tax.

JEL Classification: H21, H22, L13, L41, C72, C73

Keywords: Taxes, Imperfect Competition, Oligopoly, Cartel, Supergame.

1. Introduction

An *ad valorem* tax and specific tax that result in the same consumer price will yield the same tax revenue under perfect competition, but an *ad valorem* tax will yield higher tax revenue than a specific tax under monopoly.¹ The systematic comparison of *ad valorem* and specific taxes under oligopoly began with the article by Delipalla and Keen (1992). In a conjectural variation oligopoly model they demonstrate that an *ad valorem* tax is superior to a specific tax by considering tax reform that increases the *ad valorem* tax and reduces the specific tax in such a way that the first-order effect on revenue is zero (denoted as a *P*-shift). Skeath and Trandel (1994a) demonstrate that a specific tax can be replaced by a Pareto superior *ad valorem* tax under monopoly and under oligopoly if the tax rate is sufficiently high. The topic has also been addressed for tariffs in the international trade literature, see Kowalczyk and Skeath (1994), Skeath and Trandel (1994b). Assuming constant marginal cost, Anderson, de Palma, and Kreider (2001) demonstrate that an *ad valorem* tax will yield higher tax revenue than a specific tax that results in the same consumer price.

Recently, Colombo and Labrecciosa (2013) compared the sustainability of collusion with *ad valorem* and specific taxation under Cournot and Bertrand oligopoly using the *P*-shift employed by Delipalla and Keen (1992). They consider infinitely-repeated supergames where collusion is sustained by either Nash-reversion or optimal punishment strategies, and claim that a shift from specific to *ad valorem* taxation makes it easier for firms to sustain collusion. Consequently, in contrast to conventional wisdom, they claim to demonstrate that the specific tax may yield higher tax revenue than an *ad valorem* tax when collusion is sustainable with the *ad valorem* tax but is not sustainable with the specific tax. However, their analysis seems to be flawed in the way that they use the *P*-shift, as it would involve either different tax rates or different changes in tax rates in the various phases of the supergame. Also, their

¹ See Suits and Musgrave (1953) for a proof and details of the earlier literature on this topic going back to Wicksell in 1896.

demonstration of the superiority of a specific tax does not allow for the possibility of partial collusion when full collusion is not sustainable.

In this paper, the sustainability of collusion with *ad valorem* and specific taxes will be reconsidered using a different approach. The assumption of constant marginal cost will be used so that it is possible to find a specific tax that results in the same consumer price as an *ad valorem* tax in all phases of the supergame with general demand functions under Cournot and Bertrand oligopoly. It will be shown that the critical discount factor required to sustain collusion is the same with an *ad valorem* tax as with a specific tax that results in the same price. Using specific functional forms, linear demand and quadratic costs, it will be shown that it is easier to sustain collusion with an *ad valorem* tax than with a specific tax that results in the same price. However, in contrast to Colombo and Labrecciosa (2013), it is shown that an *ad valorem* tax always yields higher revenue than a specific tax when the possibility of partial collusion is considered.

2. Cournot Oligopoly

Consider an infinitely-repeated Cournot oligopoly where firms produce a homogeneous product, and the firms have identical and constant marginal cost. There are two or more firms, $n \geq 2$, in the industry. All firms have the same cost function: $c(q_i) = \kappa q_i$, where q_i is the output of the i th firm and its marginal cost is $c'(q_i) = \kappa > 0$, which is constant.² The inverse demand function is: $P = P(Q)$, where P is the consumer price and $Q = \sum_{j=1}^n q_j$ is the total output of the firms, and it is assumed to be downward sloping so $P'(Q) < 0$. Since this is a symmetric Cournot oligopoly, the existence of equilibrium is

² The assumption of constant marginal cost is consistent with the assumption of Colombo and Labrecciosa (2013) that the cost function is convex, and with the assumption of Delipalla and Keen (1992) that the Seade (1980) stability condition is satisfied. Constant marginal cost is a fairly standard assumption in oligopoly models, especially, in the analysis of collusion.

implied by the results of McManus (1964, 1962). To ensure the uniqueness (and stability) of the Cournot equilibrium, it will be assumed that $(n+1)P' + QP'' < 0$, see Kolstad and Mathiesen (1987). The government imposes either an *ad valorem* consumption tax: τ (expressed as a proportion of the producer price), or a specific (per unit) consumption tax: t at the beginning of the game (stage zero), where $\tau \geq 0$ and $t \geq 0$.³ The comparison of the effects of the two forms of taxation will be achieved by comparing an *ad valorem* tax with a specific tax that results in the same price in all phases of the game. It will turn out that for a given *ad valorem* tax, τ , the equivalent specific tax that results in the same price is: $t = \tau\kappa$. After the government sets the *ad valorem* or specific tax, the Cournot oligopoly stage game is played an infinite number of times by the firms with profits discounted by a discount factor: δ , where $0 < \delta < 1$.

When the other $n-1$ firms each produce: q_j , the profits of the i th firm with an *ad valorem* tax and with a specific tax are, respectively:

$$\begin{aligned}\pi_i^\tau &= \frac{P(q_i + (n-1)q_j)}{1 + \tau} q_i - \kappa q_i \\ \pi_i^t &= P(q_i + (n-1)q_j) q_i - \kappa q_i - t q_i\end{aligned}\tag{1}$$

In an infinitely-repeated game, the folk theorem implies that collusion can be sustained at the monopoly price if the discount factor is sufficiently high. As in Colombo and Labrecciosa (2013), the sustainability of collusion will be analysed for the case of Nash-reversion (grim trigger) strategies and for the case of optimal-punishment (stick and carrot) strategies.

³ Note that in Colombo and Labrecciosa (2013) and Delipalla and Keen (1992) the *ad valorem* tax is expressed as a proportion of the consumer price, but this does not alter the results of the analysis.

2.1 Nash-Reversion Strategies

Collusion can be sustained at the monopoly price in this infinitely repeated game by the threat of reversion to the Nash equilibrium as in Friedman (1971). The strategy of each firm, in the collusive phase, is to produce the joint profit-maximising output as long as all the other firms have done so in all the previous periods. If a firm deviates from the collusive phase then, in the deviation phase, it will maximise its profits while all the other firms produce the collusive output for one period. Following any deviation, from the next period onwards, in the punishment phase, all firms will produce the Cournot-Nash outputs. In the collusive phase, the firms maximise joint profits, $\Pi = \sum_{j=1}^n \pi_j$, so denoting the joint profit-maximising output of each firm as q_M^τ or q_M^t , the first-order conditions for joint-profit maximisation with an *ad valorem* tax and with a specific tax are, respectively:

$$\begin{aligned} \frac{\partial \Pi^\tau}{\partial q_i} &= \frac{P(nq_M^\tau) + nq_M^\tau P'(nq_M^\tau)}{1 + \tau} - \kappa = 0 \Rightarrow P(nq_M^\tau) + nq_M^\tau P'(nq_M^\tau) = \kappa(1 + \tau) \\ \frac{\partial \Pi^t}{\partial q_i} &= P(nq_M^t) + nq_M^t P'(nq_M^t) - \kappa - t = 0 \Rightarrow P(nq_M^t) + nq_M^t P'(nq_M^t) = \kappa + t \end{aligned} \quad (2)$$

Note that if $t = \tau\kappa$ then the right-hand-sides of the two equations are equal and therefore $q_M^\tau = q_M^t = q_M$ so the collusive output of the firms is the same with both taxes, and it follows that the collusive (monopoly) price is the same with both taxes, $P(nq_M) = P_M$. When $t = \tau\kappa$, the profits of each firm in the collusive phase of the game with an *ad valorem* tax and with a specific tax are, respectively:

$$\begin{aligned} \pi^\tau(q_M) &= \frac{P_M q_M}{1 + \tau} - \kappa q_M = \frac{1}{(1 + \tau)} [P_M q_M - (1 + \tau)\kappa q_M] \\ \pi^t(q_M) &= P_M q_M - \kappa q_M - t q_M = P_M q_M - (1 + \tau)\kappa q_M = (1 + \tau)\pi^\tau(q_M) \end{aligned} \quad (3)$$

When the *ad valorem* and specific tax both result in the same price and output, profits with the specific tax are $1+\tau$ times the profits with the *ad valorem* tax. A similar result is obtained by Anderson, de Palma, and Kreider (2001).

In the deviation phase, while the other $n-1$ firms each produce the collusive output q_M , the deviating i th firm produces the profit-maximising deviation output q_D^τ or q_D^t , given by the first-order conditions, which with an *ad valorem* tax and with a specific tax are, respectively:

$$\begin{aligned}\frac{\partial \pi_i^\tau}{\partial q_i} &= \frac{P(q_D^\tau + (n-1)q_M) + q_D^\tau P'(q_D^\tau + (n-1)q_M)}{1+\tau} - \kappa = 0 \\ \Rightarrow P(q_D^\tau + (n-1)q_M) + q_D^\tau P'(q_D^\tau + (n-1)q_M) &= \kappa(1+\tau) \\ \frac{\partial \pi_i^t}{\partial q_i} &= P(q_D^t + (n-1)q_M) + q_D^t P'(q_D^t + (n-1)q_M) - \kappa - t = 0 \\ \Rightarrow P(q_D^t + (n-1)q_M) + q_D^t P'(q_D^t + (n-1)q_M) &= \kappa + t\end{aligned}\tag{4}$$

Again, if $t = \tau\kappa$ then the right-hand sides of the two equations are equal so the outputs with the two taxes are the same, $q_D^\tau = q_D^t = q_D$, and it follows that the price in the deviation phase is the same with both taxes, $P(q_D + (n-1)q_M) = P_D$. When $t = \tau\kappa$, as in (3), the profits of the deviating firm in the deviation phase of the game with a specific tax are $1+\tau$ times the profits with the *ad valorem* tax $\pi_D^t(q_M) = (1+\tau)\pi_D^\tau(q_M)$.

In the punishment phase, all the firms produce the symmetric Cournot-Nash equilibrium outputs q_N^τ or q_N^t given by the first-order conditions, which with an *ad valorem* tax and with a specific tax are, respectively:

$$\begin{aligned}\frac{\partial \pi_i^\tau}{\partial q_i} &= \frac{P(nq_N^\tau) + q_N^\tau P'(nq_N^\tau)}{1 + \tau} - \kappa = 0 \Rightarrow P(nq_N^\tau) + q_N^\tau P'(nq_N^\tau) = \kappa(1 + \tau) \\ \frac{\partial \pi_i^t}{\partial q_i} &= P(nq_N^t) + q_N^t P'(nq_N^t) - \kappa - t = 0 \Rightarrow P(nq_N^t) + q_N^t P'(nq_N^t) = \kappa + t\end{aligned}\tag{5}$$

Again, if $t = \tau\kappa$ then the right-hand sides of the two equations are equal so the output is the same with the two taxes $q_N^\tau = q_N^t = q_N$, and it follows that the price in punishment phase is the same with both taxes, $P(nq_N) = P_N$. When $t = \tau\kappa$, as in (3), the profits of each firm in the punishment phase of the game (the Cournot-Nash equilibrium) with specific tax are $1 + \tau$ times the profits with the *ad valorem* tax: $\pi^t(q_N) = (1 + \tau)\pi^\tau(q_N)$.

Collusion can be sustained by Nash-reversion strategies if the discounted present value of profits in the collusive phase exceeds the discounted present value of profits from deviation for one period followed by Cournot-Nash equilibrium profits in the punishment phase:

$$\frac{1}{1 - \delta} \pi^z(q_M) \geq \pi_D^z(q_M) + \frac{\delta}{1 - \delta} \pi^z(q_N) \quad z = \tau, t\tag{6}$$

Collusion is sustainable if the discount factor is greater than the critical value defined when (6) holds with equality. Hence, the critical discount factors with an *ad valorem* tax and with a specific tax are, respectively:

$$\delta^\tau = \frac{\pi_D^\tau(q_M) - \pi^\tau(q_M)}{\pi_D^\tau(q_M) - \pi^\tau(q_N)} \quad \delta^t = \frac{\pi_D^t(q_M) - \pi^t(q_M)}{\pi_D^t(q_M) - \pi^t(q_N)}\tag{7}$$

When $t = \tau\kappa$, profits with the specific tax are $1 + \tau$ times the profits with the *ad valorem* tax in each phase of the game. Hence, the critical discount factor required to sustain collusion is the same with both taxes. This leads to the following proposition:

Proposition 1. *In the Cournot oligopoly supergame with collusion being supported by Nash reversion strategies the critical discount factor is the same with an *ad valorem* tax as with a specific tax that results in the same price in the collusive phase.*

If both taxes lead to the same price in the collusive phase then the critical discount factor is the same with both taxes.⁴ The intuition for this result is that although the two taxes have different effects on profits, they both have the same effect on the profits from collusion relative to the profits from deviation. Since relative profitability is unaffected by the form of taxation, the discount factor is the same with both taxes.

This result contradicts proposition one of Colombo and Labrecciosa (2013), where it is claimed that a shift from a specific to an *ad valorem* tax will lead to a strict reduction in the critical discount factor, but their analysis seems to be flawed. To derive their results, they use the *P*-shift developed by Delipalla and Keen (1992) in a static oligopoly model with identical firms where specific and *ad valorem* taxation are used simultaneously. They consider a marginal tax reform that shifts the balance from specific taxation towards *ad valorem* taxation such that: $-dt = Pd\tau / (1 + \tau)^2 > 0$, which implies that the (first-order) effect on tax revenue is zero.⁵ In proposition three, Delipalla and Keen (1992) show that this tax reform will lead to a strict reduction in profits, except in the polar case of joint profit-maximisation when profits are unaffected. When Colombo and Labrecciosa (2013) apply this result to analyse collusion in an infinitely repeated game, they do not seem to appreciate that the price will be different in all three phases of the game (P_M, P_D, P_N in, respectively, the collusive, deviation and punishment phases), and hence the necessary tax reforms (*P*-shifts) will be different in each phase if the result of Delipalla and Keen (1992) is to be valid. Therefore,

⁴ Note that if both taxes lead to the same price in the collusive phase then prices with the two taxes will also be the same in the deviation phase and in the punishment phase.

⁵ The formula here is different to the formula in Delipalla and Keen (1992) as the *ad valorem* tax is defined here as a proportion of the producer price rather than as a proportion of the consumer price.

either their analysis is invalid or they are considering different tax rates (or different changes in tax rates) in the various phases of the game. However, since the government sets the tax rates before the firms set their outputs, the government cannot set different taxes depending upon the phase of the game.⁶

2.2 Optimal Punishment Strategies

Collusion can be sustained at the monopoly price in this infinitely repeated game by the use of optimal symmetric punishments as in Abreu (1986), where the punishment lasts for one period and then the firms revert to collusion. The strategy of each firm, in the collusive phase, is to produce the collusive output provided there has been no deviation in the previous stage. Following a deviation, each firm will produce punishment output for one period, the punishment phase, and then revert to the collusive phase if all firms went along with the punishment. If a firm deviates from the punishment phase, then the punishment phase will continue for another period. In the collusive phase, the joint profit-maximising output of each firm will be the same with both taxes if $t = \kappa\tau$, and is given by (2). Similarly, the output of a firm when it deviates from the collusive phase will be the same with both taxes if $t = \tau\kappa$, and is given by (4).

In the punishment phase, suppose that each firm produces output q_p , which is assumed to be the same with both taxes if $t = \tau\kappa$. Later, it will be verified that this assumption is justified. If the output of each firm is q_p then the price is $P(nq_p) = P_p$, and the profits of each firm in the punishment phase with the specific tax are $1 + \tau$ times the profits with the *ad valorem* tax: $\pi^t(q_p) = (1 + \tau)\pi^\tau(q_p)$.

⁶ For example, if the government set a different tax rate in the deviation phase it would have to know that a firm was going to deviate when it set its tax, and setting a different tax in the deviation phase would inform all the firms that a firm was going to deviate, which would lead all firms to deviate from collusion.

If the i th firm deviates from the punishment phase, while the other $n-1$ firms each produce the punishment output q_P , then the first-order conditions for the profit-maximising deviation by the i th firm with an *ad valorem* tax and with a specific tax are, respectively:

$$\begin{aligned}
\frac{\partial \pi_i^\tau}{\partial q_i} &= \frac{P(q_{DP}^\tau + (n-1)q_P) + q_{DP}^\tau P'(q_{DP}^\tau + (n-1)q_P)}{1 + \tau} - \kappa = 0 \\
\Rightarrow P(q_{DP}^\tau + (n-1)q_P) + q_{DP}^\tau P'(q_{DP}^\tau + (n-1)q_P) &= \kappa(1 + \tau) \\
\frac{\partial \pi_i^t}{\partial q_i} &= P(q_{DP}^t + (n-1)q_P) + q_P^t P'(q_{DP}^t + (n-1)q_P) - \kappa - t = 0 \\
\Rightarrow P(q_{DP}^t + (n-1)q_P) + q_P^t P'(q_{DP}^t + (n-1)q_P) &= \kappa + t
\end{aligned} \tag{8}$$

Again, if $t = \tau\kappa$ then the right-hand sides of the two equations are equal so $q_{DP}^\tau = q_{DP}^t = q_{DP}$, and it follows that the price when a firm deviates from the punishment phase is the same with both taxes, $P(q_{DP} + (n-1)q_P) = P_{DP}$. When $t = \tau\kappa$, the profits of the firm deviating from the punishment phase of the game with the specific tax are $1 + \tau$ times the profits with the *ad valorem* tax: $\pi_D^t(q_P) = (1 + \tau)\pi_D^\tau(q_P)$.

As in Abreu (1986), for the punishment to be credible, the gain from deviating in the punishment phase in any period is less than the present discounted value of the loss in the next period:

$$\delta[\pi^\tau(q_M) - \pi^\tau(q_P)] \geq \pi^\tau(q_P) - \pi^\tau(q_P) \quad z = \tau, t \tag{9}$$

The optimal punishment output is the largest output that solves (9) when it holds with equality. Since profits with the specific tax are $1 + \tau$ times the profits with the *ad valorem* tax when $t = \tau\kappa$, any solution for an *ad valorem* tax is also a solution for a specific tax. Therefore, as assumed above, the optimal punishment output is the same with both taxes:

$$q_P^\tau = q_P^t = q_P.$$

It is also necessary that the firms find it profitable to continue with the supergame following any deviation from the collusive phase. The participation constraint of the firms requires that the discounted future profits from collusion must exceed any losses in the punishment phase: $\pi(q_P) + \delta\pi(q_M)/1 - \delta \geq 0$. The participation constraint of the firms is assumed to be satisfied.⁷

For collusion to be sustainable, the gain from deviating in the collusive phase in any period is less than the present discounted value of the loss in the next period:

$$\delta[\pi^z(q_M) - \pi^z(q_P)] \geq \pi_D^z(q_M) - \pi^z(q_M) \quad z = \tau, t \quad (10)$$

Collusion is sustainable if the discount factor is greater than the critical value defined when (10) holds with equality. Hence, the critical discount factors with an *ad valorem* tax and with a specific tax are, respectively:

$$\delta_P^\tau = \frac{\pi_D^\tau(q_M) - \pi^\tau(q_M)}{\pi^\tau(q_M) - \pi^\tau(q_P)} \quad \delta_P^t = \frac{\pi_D^t(q_M) - \pi^t(q_M)}{\pi^t(q_M) - \pi^t(q_P)} \quad (11)$$

When $t = \tau\kappa$, profits with the specific tax are $1 + \tau$ times the profits with the *ad valorem* tax in each phase of the game so the critical discount factor required to sustain collusion is the same with both taxes. This leads to the following proposition:

Proposition 2. *In the Cournot oligopoly supergame with collusion being supported by optimal symmetric punishment strategies the critical discount factor is the same with an ad valorem tax as with a specific tax that results in the same price in the collusive phase.*

The result with optimal punishment strategies is the same as with Nash-reversion strategies, and the intuition is also the same. This result contradicts proposition two of Colombo and Labrecciosa (2013), where it is claimed that a shift from a specific to an *ad*

⁷ With linear demand and differentiated products, Lambertini and Sasaki (1999) show that the participation constraint will only bind at the critical discount factor in the case of Bertrand oligopoly and perfect substitutes.

valorem tax will lead to a strict reduction in the critical discount factor. In this case there are two flaws in their analysis. First, rather than characterising the optimal punishment strategies in terms of the credibility condition (9) and the sustainability condition (10), they use the participation constraint of the firm, which they assume binds, but which Lambertini and Sasaki (1999) show only binds in extreme cases, although they only consider linear demand functions. Second, as with proposition one, the use of the P -shifts when prices differ in the various phases of the game is not valid.

3. Bertrand Oligopoly

Now consider the case of Bertrand oligopoly with differentiated products rather than Cournot oligopoly with homogeneous products. The demand functions facing the firms are symmetric and the demand facing the i th firm is: $q_i = D_i(P_i, \mathbf{P})$ where $\mathbf{P}$ is the vector of prices set by the $n-1$ other firms. The rest of the model is the same as in section two so it is assumed that the Bertrand-Nash equilibrium exists and is unique. The profits of the i th firm with an *ad valorem* tax and with a specific tax are, respectively:

$$\begin{aligned}\pi_i^\tau &= \left(\frac{P_i}{1+\tau} - \kappa \right) D(P_i, \mathbf{P}) \\ \pi_i^t &= (P_i - \kappa - t) D(P_i, \mathbf{P})\end{aligned}\tag{12}$$

As in the previous section, the sustainability of collusion will be analysed for the case of Nash-reversion (grim trigger) strategies and for the case of optimal-punishment (stick and carrot) strategies.

3.1 Nash-Reversion Strategies

In the collusive phase, the firms set prices to maximise joint profits, $\Pi = \sum_{j=1}^n \pi_j$, so denoting the joint profit-maximising (symmetric) price as P_M^τ or P_M^t , the first-order

conditions for joint profit maximisation with an *ad valorem* tax and with a specific tax are, respectively:

$$\begin{aligned}
\frac{\partial \Pi^\tau}{\partial P_i} &= \left(\frac{P_M^\tau}{1+\tau} - \kappa \right) \left(\frac{\partial D_i(P_M^\tau, \mathbf{P}_M^\tau)}{\partial p_i} + \sum_{j \neq i} \frac{\partial D_j(P_M^\tau, \mathbf{P}_M^\tau)}{\partial p_i} \right) + \frac{D_i(P_M^\tau, \mathbf{P}_M^\tau)}{1+\tau} = 0 \\
\Rightarrow (P_M^\tau - \kappa - \kappa\tau) &\left(\frac{\partial D_i(P_M^\tau, \mathbf{P}_M^\tau)}{\partial p_i} + \sum_{j \neq i} \frac{\partial D_j(P_M^\tau, \mathbf{P}_M^\tau)}{\partial p_i} \right) + D_i(P_M^\tau, \mathbf{P}_M^\tau) = 0 \quad (13) \\
\frac{\partial \Pi^t}{\partial P_i} &= (P_M^t - \kappa - t) \left(\frac{\partial D_i(P_M^t, \mathbf{P}_M^t)}{\partial p_i} + \sum_{j \neq i} \frac{\partial D_j(P_M^t, \mathbf{P}_M^t)}{\partial p_i} \right) + D_i(P_M^t, \mathbf{P}_M^t) = 0
\end{aligned}$$

Note that if $t = \tau\kappa$ then the solutions to both equations are the same: $P_M^\tau = P_M^t = P_M$ and $D_i(P_M, \mathbf{P}_M) = q_M$. Hence, when $t = \tau\kappa$, profits with the specific tax are $1 + \tau$ times the profits with the *ad valorem* tax as in the previous section: $\pi^t(P_M) = (1 + \tau)\pi^\tau(P_M)$.

In the deviation phase, while the other $n-1$ firms each set the collusive price P_M , the deviating firm sets the profit-maximising deviation price P_D^τ or P_D^t given by the first-order conditions, which with an *ad valorem* tax and with a specific tax are, respectively:

$$\begin{aligned}
\frac{\partial \pi_i^\tau}{\partial P_i} &= \left(\frac{P_D^\tau}{1+\tau} - \kappa \right) \frac{\partial D_i(P_D^\tau, \mathbf{P}_M)}{\partial p_i} + \frac{D_i(P_D^\tau, \mathbf{P}_M)}{1+\tau} = 0 \\
\Rightarrow (P_D^\tau - \kappa - \kappa\tau) &\frac{\partial D_i(P_D^\tau, \mathbf{P}_M)}{\partial p_i} + D_i(P_D^\tau, \mathbf{P}_M) = 0 \quad (14) \\
\frac{\partial \pi_i^t}{\partial P_i} &= (P_D^t - \kappa - t) \frac{\partial D_i(P_D^t, \mathbf{P}_M)}{\partial p_i} + D_i(P_D^t, \mathbf{P}_M) = 0
\end{aligned}$$

Again, if $t = \tau\kappa$ then the solutions to the two equations are the same: $P_D^\tau = P_D^t = P_D$ and $D(P_D, \mathbf{P}_M) = q_D$. When $t = \tau\kappa$, profits with the specific tax are $1 + \tau$ times the profits with the *ad valorem* tax as in the previous section: $\pi_D^t(P_M) = (1 + \tau)\pi_D^\tau(P_M)$.

In the punishment phase, all the firms set the symmetric Bertrand-Nash equilibrium prices, P_N^τ or P_N^t , defined by the first-order conditions, which with an *ad valorem* tax and with a specific tax are, respectively:

$$\begin{aligned}\frac{\partial \pi_i^\tau}{\partial P_i} &= \left(\frac{P_N^\tau}{1+\tau} - \kappa \right) \frac{\partial D_i(P_N^\tau, \mathbf{P}_N^\tau)}{\partial p_i} + \frac{D_i(P_N^\tau, \mathbf{P}_N^\tau)}{1+\tau} = 0 \\ \Rightarrow \quad (P_N^\tau - \kappa - \kappa\tau) \frac{\partial D_i(P_N^\tau, \mathbf{P}_N^\tau)}{\partial p_i} + D_i(P_N^\tau, \mathbf{P}_N^\tau) &= 0 \\ \frac{\partial \pi_i^t}{\partial P_i} &= (P_N^t - \kappa - t) \frac{\partial D_i(P_N^t, \mathbf{P}_N^t)}{\partial p_i} + D_i(P_N^t, \mathbf{P}_N^t) = 0\end{aligned}\tag{15}$$

Again, if $t = \tau\kappa$ then the solutions to both equations are the same so $P_N^\tau = P_N^t = P_N$ and $D_i(P_N, \mathbf{P}_N) = q_N$. When $t = \tau\kappa$, profits with the specific tax are $1+\tau$ times the profits with the *ad valorem* tax as in the previous section: $\pi^t(P_N) = (1+\tau)\pi^\tau(P_N)$.

Now consider the sustainability of collusion using Nash-reversion strategies. Analogously to (7), the critical discount factors with an *ad valorem* tax and with a specific tax are, respectively:

$$\delta^\tau = \frac{\pi_D^\tau(P_M) - \pi^\tau(P_M)}{\pi_D^\tau(P_M) - \pi^\tau(P_N)} \quad \delta^t = \frac{\pi_D^t(P_M) - \pi^t(P_M)}{\pi_D^t(P_M) - \pi^t(P_N)}\tag{16}$$

When $t = \tau\kappa$, profits with the specific tax are $1+\tau$ times the profits with the *ad valorem* tax in each phase of the game so the critical discount factor is the same with the *ad valorem* tax as with the specific tax. This leads to the following proposition:

Proposition 3. *In the Bertrand oligopoly supergame with collusion being supported by Nash reversion strategies the critical discount factor is the same with an ad valorem tax as with a specific tax that results in the same prices in the collusive phase.*

The result under Bertrand oligopoly is the same as under Cournot oligopoly, and the intuition is also the same. This result contradicts proposition four of Colombo and Labrecciosa (2013), where it is claimed that a shift from a specific to an *ad valorem* tax will lead to a strict reduction in the critical discount factor. The flaw in the analysis, as with proposition one, is that the use of the P -shifts when prices vary in the different phases of the game is not valid.

3.2 Optimal-Punishment Strategies

In the collusive phase, the joint profit-maximising price set by each firm will be the same with both taxes if $t = \kappa\tau$, and is given by (13). Similarly, the price set by a firm when it deviates from the collusive phase will be the same with both taxes if $t = \tau\kappa$, and is given by (14). In the punishment phase, suppose that all the firms set price P_p and output is $q_p = D(P_p, \mathbf{P}_p)$ then the profits of each firm in the punishment phase with a specific tax are $1 + \tau$ times profits with an *ad valorem* tax: $\pi^t(P_p) = (1 + \tau)\pi^\tau(P_p)$.

If the i th firm deviates from the punishment phase, while the other $n - 1$ firms each set price P_p , then the first-order conditions for the profit maximising deviation with an *ad valorem* tax and with a specific tax are, respectively:

$$\begin{aligned} \frac{\partial \pi_i^\tau}{\partial P_i} &= \left(\frac{P_{DP}^\tau}{1 + \tau} - \kappa \right) \frac{\partial D_i(P_{DP}^\tau, \mathbf{P}_p)}{\partial p_i} + \frac{D_i(P_{DP}^\tau, \mathbf{P}_p)}{1 + \tau} = 0 \\ \Rightarrow \quad (P_{DP}^\tau - \kappa - \kappa\tau) \frac{\partial D_i(P_{DP}^\tau, \mathbf{P}_p)}{\partial p_i} + D_i(P_{DP}^\tau, \mathbf{P}_p) &= 0 \\ \frac{\partial \pi_i^t}{\partial P_i} &= (P_{DP}^t - \kappa - t) \frac{\partial D_i(P_{DP}^t, \mathbf{P}_p)}{\partial p_i} + D_i(P_{DP}^t, \mathbf{P}_p) = 0 \end{aligned} \tag{17}$$

Again, if $t = \tau\kappa$ then the solutions to the two equations are the same: $P_{DP}^\tau = P_{DP}^t = P_{DP}$ and $D(P_{DP}, \mathbf{P}_P) = q_{DP}$. When $t = \tau\kappa$, the profits of the deviating firm with a specific tax are $1 + \tau$ times the profits with an *ad valorem* tax: $\pi_D^t(P_P) = (1 + \tau)\pi_D^\tau(P_P)$.

As in Abreu (1986), for the punishment to be credible, the gain from deviating in the punishment phase in any period is less than the present discounted value of the loss in the next period:

$$\delta[\pi(P_M) - \pi(P_P)] \geq \pi_D(P_P) - \pi(P_P) \quad (18)$$

The optimal punishment price is the lowest price that solves (18) when it holds with equality. Since profits with the specific tax are $1 + \tau$ times the profits with the *ad valorem* tax when $t = \tau\kappa$, any solution for an *ad valorem* tax is also a solution for a specific tax. Therefore, as assumed above, the optimal punishment price is the same with both taxes: $P_P^\tau = P_P^t = P_P$.

For collusion to be sustainable, the gain from deviating in the collusive phase in any period is less than the present discounted value of the loss in the next period:

$$\delta[\pi(P_M) - \pi(P_P)] \geq \pi_D(P_M) - \pi(P_M) \quad (19)$$

Collusion is sustainable if the discount factor is greater than the critical value defined when (19) holds with equality. Hence, the critical discount factors with an *ad valorem* tax and with a specific tax are, respectively:

$$\delta_P^\tau = \frac{\pi_D^\tau(P_M) - \pi^\tau(P_M)}{\pi^\tau(P_M) - \pi^\tau(P_P)} \quad \delta_P^t = \frac{\pi_D^t(P_M) - \pi^t(P_M)}{\pi^t(P_M) - \pi^t(P_P)} \quad (20)$$

When $t = \tau\kappa$, profits with the specific tax are $1 + \tau$ times the profits with the *ad valorem* tax in each phase of the game so the critical discount factor required to sustain collusion is the same with both taxes. This leads to the following proposition:

Proposition 4. *In the Bertrand oligopoly supergame with collusion being supported by optimal symmetric punishment strategies the critical discount factor is the same with an *ad valorem* tax as with a specific tax that results in the same prices in the collusive phase.*

This result contradicts proposition five of Colombo and Labrecciosa (2013), where it is claimed that a shift from a specific to an *ad valorem* tax will lead to a strict reduction in the critical discount factor.

4. Cournot Duopoly with Linear Demand and Quadratic Costs

In the previous two sections, the assumption of constant marginal cost allowed clear-cut results to be obtained with general demand functions. This section will consider the case of increasing marginal cost, but this will require the use of specific functional forms so that explicit solutions can be obtained for outputs and profits. Also, this section will consider the possibility of partial collusion when full collusion is not possible.

Consider an infinitely-repeated Cournot duopoly, $n = 2$, where firms produce a homogeneous product, and the firms have identical quadratic cost functions. The i th firm has the cost function: $c(q_i) = \kappa q_i + \theta q_i^2 / 2$, where $\kappa \geq 0$ and $\theta \geq 0$, and hence its marginal cost is $c'(q_i) = \kappa + \theta q_i \geq 0$, which is increasing in output if $\theta > 0$ and constant if $\theta = 0$. The inverse demand function is linear: $P(Q) = \alpha - \beta(q_1 + q_2)$, where $\alpha > (1 + \tau)(\kappa + t) \geq 0$ and $\beta > 0$. It is useful to define the variable $\mu \equiv \theta / \beta \geq 0$, which is the slope of a firm's marginal cost curve relative to the slope of the demand function, and is equal to zero in the case of constant marginal cost. Also, to simplify the expressions later in the paper, it is useful to define the

following terms: $A \equiv \alpha - (1 + \tau)(\kappa + t) > 0$, $B_I \equiv I + \mu(1 + \tau) > 0$, where $I = 1, 2, \dots, 6$, $D_1 \equiv (3 + \mu)^2 - \delta(5 + 2\mu) > 0$ and $D_2 \equiv (3 + \mu)^2 - 4(2 + \mu)\delta > 0$. Note that only A is a function of the specific tax, t , while D_1 and D_2 do not depend upon either the *ad valorem* or the specific tax. With these demand and cost functions, the profits of the- i th firm when its competitor produces output q_j , given the *ad valorem* and specific taxes, are:

$$\pi_i = \frac{P(q_i + q_j)}{1 + \tau} q_i - c(q_i) - tq_i = \frac{\alpha - \beta(q_i + q_j)}{1 + \tau} q_i - \kappa q_i - \theta \frac{q_i^2}{2} - tq_i \quad (21)$$

It is straightforward to solve for the joint profit-maximising output and profits of each firm as functions of the two taxes:

$$q_M = \frac{A}{\beta B_4}, \quad \pi(q_M) = \frac{A^2}{2\beta(1 + \tau)B_4} \quad (22)$$

The solutions for an *ad valorem* tax are obtained by setting the specific tax equal to zero, $t = 0$, and are denoted by a superscript τ . Similarly, the solutions for a specific tax are obtained by setting the *ad valorem* tax equal to zero, $\tau = 0$, and are denoted by a superscript t . For example, the joint profit-maximising output is $q_M^\tau = (\alpha - \kappa - \tau\kappa)/\beta B_4$ with an *ad valorem* tax and is $q_M^t = (\alpha - \kappa - t)/\beta(4 + \mu)$ with a specific tax.

4.1 Nash-Reversion Strategies

When the discount factor is less than the critical value, it is not possible to sustain collusion at the joint profit-maximising price, but partial collusion at a lower price may still be possible using Nash-reversion strategies. To find the maximum level of collusion that can be sustained for a given discount factor, let q_C be the collusive output. Then, the profits of the firms from colluding are:

$$\pi(q_C) = \frac{q_C(2A - \beta B_4 q_C)}{2(1 + \tau)} \quad (23)$$

If the other firm produces output q_C then the profit-maximising output and profits for a firm that deviates are:

$$q_D(q_C) = \frac{A - \beta q_C}{\beta B_2}, \quad \pi_D(q_C) = \frac{(A - \beta q_C)^2}{2\beta(1 + \tau)B_2} \quad (24)$$

Following a deviation by either firm, in the punishment phase, both firms will produce the Cournot-Nash output forever thereafter. It is straightforward to show that the Cournot-Nash equilibrium output and profits are:

$$q_N = \frac{A}{\beta B_3}, \quad \pi(q_N) = \frac{A^2 B_2}{2\beta(1 + \tau)B_3^2} \quad (25)$$

The lowest collusive output that can be sustained using Nash-reversion trigger strategies for any given discount factor can be obtained by solving:

$$\delta = \frac{\pi_D(q_C) - \pi(q_C)}{\pi_D(q_C) - \pi(q_N)} \quad (26)$$

Using (23), (24) and (25) to solve (26) for the collusive output as a function of the discount factor yields:

$$q_C(\delta) = \frac{A(B_3 - \delta(B_2 + B_3))}{\beta B_3(B_3^2 - \delta)} \quad (27)$$

Full collusion at the joint profit-maximising price can be sustained if the discount factor is greater than some critical value, and this critical value can be obtained by solving $q_C(\delta) = q_M$, which yields the critical discount factor: $\delta_N = B_3^2 / (17 + 2\mu(1 + \tau)B_6)$. Note that with constant marginal cost, $\mu = 0$, the critical discount factor is: $\delta_N = 9/17$, which does not

depend upon the tax rates. With increasing marginal cost, setting $t=0$ yields the critical discount factor with an *ad valorem* tax and setting $\tau=0$ yields the critical discount factor with a specific tax, respectively:

$$\delta_N^\tau = \frac{B_3^2}{17 + 2\mu(1+\tau)B_6}, \quad \delta_N^t = \frac{(3+\mu)^2}{17 + 2\mu(6+\mu)} \geq \delta_N^\tau \quad (28)$$

The critical discount factor with the specific tax does not depend upon the tax rate whereas the critical discount with the *ad valorem* tax is decreasing in the tax rate, $\partial \delta_N^\tau / \partial \tau < 0$, and they are equal when there are no taxes, $t = \tau = 0$. Therefore, there is a range of values for the discount factor, $\delta \in [\delta_N^\tau, \delta_N^t]$, where full collusion can be sustained with an *ad valorem* tax, but cannot be sustained with a specific tax. In this range of values for the discount factor, Colombo and Labrecciosa (2013) claim that tax revenue may be higher with a specific tax than that with an *ad valorem* tax that yields the same price. However, they assume that if full collusion is not possible then the result will be the Cournot-Nash equilibrium even though partial collusion can still be sustained with a specific tax. If instead one allows for the possibility of partial collusion with a specific tax then for $\delta \in [\delta_N^\tau, \delta_N^t]$ there will be full collusion with an *ad valorem* tax but partial collusion with a specific tax. The specific tax can be set so that the price will be the same as with the *ad valorem* tax so $q_M^\tau = q_C^t(\delta)$. Since both taxes lead to the same price and output, tax revenue will be higher with an *ad valorem* tax than with a specific tax if the difference in revenue per unit: $\Delta R_N = \tau P_M^\tau / (1+\tau) - t$ is positive. When $\delta \in [0, \delta_N^\tau]$ there will be partial collusion with both taxes and if the specific tax is set so that $q_C^\tau(\delta) = q_C^t(\delta)$ then the difference in revenue per unit is: $\Delta R_N = \tau P_C^\tau / (1+\tau) - t$. When $\delta \in [\delta_N^t, 1]$ there will be full collusion with both taxes and if the specific tax is set so that $q_M^\tau = q_M^t$ then the difference in revenue per unit is:

$\Delta R_N = \tau P_M^\tau / (1 + \tau) - t$. Allowing for all three possibilities, it can be shown that the difference in revenue per unit is:

$$\Delta R_N = \begin{cases} \frac{\tau(\alpha - (1 + \tau)\kappa)E}{(1 + \tau)B_3(B_3^2 - \delta)D_1} > 0 & 0 \leq \delta \leq \delta_N^\tau \\ \frac{(\alpha - (1 + \tau)\kappa)F}{(1 + \tau)B_4D_1} > 0 & \delta_N^\tau < \delta \leq \delta_N^t \\ \frac{2\tau(\alpha - (1 + \tau)\kappa)}{(1 + \tau)B_4} > 0 & \delta_N^t < \delta \leq 1 \end{cases} \quad (29)$$

$$\begin{aligned} E &= (3 + \mu)^2 B_3^2 - (35 + 14(2 + \tau)\mu + 6(1 + \tau)\mu^2)\delta^2 \\ &\quad + (2(1 + \mu)^2(3 + \mu)^2 + 4\mu(1 + \mu)(2 + \mu)(3 + \mu)\tau + \mu^2(7 + 8\mu + 2\mu^2)\tau^2)\delta \\ F &= (17 + 7\mu + (12 + 8\mu)\tau + 2(1 + \tau)\mu^2)\delta - (3 + \mu)^2(1 - \tau) \end{aligned}$$

The only terms where the sign is not immediately clear are E and F , but these terms can be signed quite easily. Since the term E is a concave quadratic in the discount factor, which is positive when $\delta = 0$ and when $\delta = \delta_N^\tau$, it will be positive for $\delta \in [0, \delta_N^\tau]$. Since the term F is positive when $\delta = \delta_N^\tau$ and it is increasing in the discount factor, it will be positive for $\delta \in [\delta_N^\tau, \delta_N^t]$. Therefore, an *ad valorem* tax yields a higher revenue than a specific tax that results in the same price in the collusive phase. This leads to the following proposition:

Proposition 5. *In the Cournot duopoly supergame with linear demand and quadratic costs where collusion is supported by Nash-reversion trigger strategies, tax revenue is higher with an ad valorem tax than with a specific tax that results in the same price in the collusive phase.*

Allowing partial collusion when the discount factor is lower than the critical value, restores the conventional wisdom that an *ad valorem* tax yields higher revenue than a specific tax that results in the same price.

4.2 Optimal Punishment Strategies

Partial collusion can also be sustained using optimal punishment strategies as in Abreu (1986). The profits in the collusive phase when each firm produces output q_C are given by (23), and the profits if a firm deviates from the collusive phase are given by (24). Similarly, in the punishment phase, the profits when each firm produces output q_P are:

$$\pi(q_P) = \frac{q_P(2A - \beta B_A q_P)}{2(1 + \tau)} \quad (30)$$

The output and profits when a firm deviates from the punishment phase and the other firms produce the collusive output q_P are:

$$q_D(q_P) = \frac{A - \beta q_P}{\beta B_2}, \quad \pi_D(q_P) = \frac{(A - \beta q_P)^2}{2\beta(1 + \tau)B_2} \quad (31)$$

With partial collusion, the outputs in the collusive and the punishment phases are obtained by solving the credibility and sustainability conditions for a given discount factor, as in Abreu (1986):

$$\begin{aligned} \pi_D(q_P) - \pi(q_P) &= \delta(\pi(q_C) - \pi(q_P)) \\ \pi_D(q_C) - \pi(q_C) &= \delta(\pi(q_C) - \pi(q_P)) \end{aligned} \quad (32)$$

Solving for the outputs in the collusive and punishment phases as functions of the discount factor, and ignoring the trivial solution where both outputs are equal to the Cournot-Nash equilibrium outputs, $q_C = q_P = q_N$, yields:

$$q_C(\delta) = \frac{A}{\beta B_3} [B_3^2 - 4\delta B_2], \quad q_P(\delta) = \frac{A}{\beta B_3} [B_3^2 + 4\delta B_2] \quad (33)$$

Note that the two outputs are linear in the discount factor. Full collusion at the joint profit-maximising price can be sustained if the discount factor is greater than some critical

value, and this critical value can be obtained by solving $q_C(\delta) = q_M$, which yields the critical discount factor: $\delta_p = B_3^2 / 4B_2B_4$. Note that with constant marginal cost, $\mu = 0$, the critical discount factor is: $\delta_p = 9/32$, which does not depend upon the tax rates. With increasing marginal cost, setting $t = 0$ yields the critical discount factor with an *ad valorem* tax and setting $\tau = 0$ yields the critical discount factor with a specific tax, respectively:

$$\delta_p^\tau = \frac{B_3^2}{4B_2B_4}, \quad \delta_p^t = \frac{(3 + \mu)^2}{4(2 + \mu)(4 + \mu)} \geq \delta_p^\tau \quad (34)$$

The critical discount factor with the specific tax does not depend upon the tax rate whereas the critical discount with the *ad valorem* tax is decreasing in the tax rate, $\partial \delta_p^\tau / \partial \tau < 0$, and they are equal when there are no taxes, $t = \tau = 0$. Therefore, there is a range of values for the discount factor, $\delta \in [\delta_p^\tau, \delta_p^t]$, where full collusion can be sustained with an *ad valorem* tax, but cannot be sustained with a specific tax. However, partial collusion can be sustained with a specific tax.

As in the case of Nash-reversion strategies, the specific tax can be set so that the price in the collusive phase is the same as with the *ad valorem* tax. Since both taxes lead to the same output, and again allowing for all three possibilities, the difference in revenue per unit with the *ad valorem* and specific tax can be shown to be:

$$\Delta R_p = \begin{cases} \frac{\tau(\alpha - (1+\tau)\kappa)G}{(1+\tau)B_3^3 D_2} > 0 & 0 \leq \delta \leq \delta_p^r \\ \frac{(\alpha - (1+\tau)\kappa)H}{(1+\tau)B_4 D_2} > 0 & \delta_p^r < \delta \leq \delta_p^t \\ \frac{2\tau(\alpha - (1+\tau)\kappa)}{(1+\tau)B_4} > 0 & \delta_p^t < \delta \leq 1 \end{cases} \quad (35)$$

$$G = (3 + \mu)^2 B_3^2 - 32(2 + \mu)B_3^2 \delta^2$$

$$+ 4 \left((3 + \mu)^2 (2 + 2\mu + \mu^2) + \mu(3 + \mu)(5 + 6\mu + 2\mu^2)\tau + \mu^2 (2 + \mu)^2 \tau^2 \right) \delta$$

$$H = 4(2 + \mu)(B_4 + 2\tau)\delta - (3 + \mu)^2 (1 - \tau)$$

The only terms where the sign is not immediately clear are G and H , but these terms can be signed quite easily. Since the term G is a concave quadratic, which is positive when $\delta = 0$ and when $\delta = \delta_p^r$, it will be positive for $\delta \in [0, \delta_p^r]$. Since the term H is positive when $\delta = \delta_p^r$ and it is increasing in δ , it will be positive for $\delta \in [\delta_p^r, \delta_p^t]$. Therefore, an *ad valorem* tax yields a higher revenue than a specific tax that results in the same price in the collusive phase. This leads to the following proposition:

Proposition 6. *In the Cournot duopoly supergame with linear demand and quadratic costs where collusion is supported by optimal-punishment strategies, tax revenue is higher with an ad valorem tax than with a specific tax that results in the same price in the collusive phase.*

Allowing partial collusion when the discount factor is lower than the critical value, restores the conventional wisdom that an *ad valorem* tax yields higher revenue than a specific tax that results in the same price.

5. Conclusions

The analysis has compared the effects of *ad valorem* and specific taxes that result in the same price on the sustainability of collusion in infinitely repeated oligopoly models.

Assuming constant marginal cost, it was shown that a switch from specific to *ad valorem* taxation has no effect on the critical discount factor required to sustain collusion. This result was shown to hold for Cournot oligopoly, with homogeneous products and general demand functions, as well as for Bertrand oligopoly, with differentiated products and general demand functions, when collusion was sustained with Nash-reversion strategies or optimal-punishment strategies. The intuition for these results is that, although both taxes have different effects on profits, they have the same effect on relative profits because profits with an *ad valorem* tax are always proportional to profits with a specific tax. These results contradicted the results of Colombo and Labrecciosa (2013) who claimed that a shift from specific to *ad valorem* taxation leads to a strict reduction in the critical discount factor. However, their analysis is flawed as it uses P -shifts in all the phases of the game even though prices will be different in each phase of the game.

Also, in a Cournot duopoly model with linear demand and quadratic costs, it was shown that the critical discount factor was lower with an *ad valorem* tax than with a specific tax when marginal cost was increasing. In this case, there is a range of values for the discount factor where full collusion is possible with an *ad valorem* tax, but is not possible with a specific tax. In this region, in contrast to conventional wisdom, Colombo and Labrecciosa (2013) argue that revenue may be higher with a specific tax than with an *ad valorem* tax, but they assume that the outcome will be Nash (Cournot or Bertrand) equilibrium if the discount factor is lower than the critical value. However, partial collusion is still possible when the discount factor is lower than the critical value. Allowing for the possibility of partial collusion, it was shown that revenue is always higher with an *ad valorem* tax than with a specific tax thereby restoring conventional wisdom.

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