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Wirtschaftswissenschaftliche Fakultät
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Some additional considerations regarding efficient allocations in a federation

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Für wertvolle Hinweise bedanke ich mich bei Wolfgang Wiegard.

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1 Introduction

An important result of recent publications in the field of regional economics is the insight that Nash-competing local authorities will—under certain conditions—achieve a Pareto efficient allocation. Provided that individuals are perfectly mobile, Myers (1990) and Krelove (1992) showed that local governments need a combination of two tax instruments to establish a socially efficient outcome in an economy with one private and one purely local public good. The result still holds, if the public good causes spillover effects into adjoining regions (Wellisch, 1993), or if households are only imperfectly mobile (Mansoorian and Myers, 1993).

The actual tax design is thereby of minor importance. More important is the point that one tax affects solely the inhabitants of the region itself, while the second tax instrument generates interregional transfers. This transfer which causes in a world of immobile households, the market failure commonly known as the 'beggar my neighbour' principle, acts here as an instrument to control migration. It internalizes the external effect, which a person exerts on the utility of others when he moves. No local government is able to exploit the situation, because the equal utility condition required for a migration equilibrium accomplishes that the benefit of an excessive tax export would be more than offset by the cost of induced migration.

Efficiency is usually examined by comparing the dezentralized market solution with the solution of the corresponding central planners problem. If both situations yield identical first order conditions (f.o.c.'s), the point is made. This, of course, reflects the standard procedure used either to prove efficiency, or to highlight the causes of inefficiency of a particular market structure. However, if one allows for a variable population size, the problem might become ill-behaved. Under the usual assumptions—i.e. a decreasing marginal product with respect to labor and identical treatment of all individuals living in one region—the production possibility frontier (with respect to population size) is a convex function and multiple as well as unstable migration equilibria may emerge. See for example Stiglitz (1978) for a graphical outline of various cases. This tendency is not necessarily eliminated by the additional introduction of interregional transfers. The set of allocations characterized by the necessary conditions of the planners problem might well be larger than the set of Pareto-efficient allocations. Even without any additional model refinements like spillover effects of local public goods, congestion effects etc., there may be several allocations compatible to a migration equilibrium. Thus there also exist several candidates for a Nash-equilibrium, each associated with a different utility level.

Since there is no conflict of interests in the objectives of the two local governments, obviously, as long as individuals have perfect information, the allocation yielding the highest utility level will be reached. If this assumption is weakened in that households can only observe the utility level attained by inhabitants of the other region (and vice versa), the realization of an efficient allocation is not guaranteed in any case, at least not in a single step.

The following sections illustrate this line of argumentation with a simple numerical example. After a short description of the underlying model, the f.o.c.'s of the planners problem will be computed and the resulting opportunity locus (in terms of utility levels) will be plotted graphically in the following section. The causes of the strange shape of this set will be examined more closely in section 3, while Section 4 deals with the consequences in relation to the decentralized market setting.

2 The planners solution

Since the model comprises only the basic setup for this type of problem, its description will be kept rather short. A more detailed discussion can be found, for example, in Wildasin (1980) or Myers (1990). There exist two regions, each endowed with a fixed amount of land $\overline{T_i}$ ($i = 1, 2$). The respective production functions $f_i(n_i, \overline{T_i})$ use labor and land as inputs. It is assumed that each person provides one unit of labor. Since labor cannot be exported—for example due to high transportation costs—n_i is also interpretable as the number of individuals living in region i. The good produced, x, is purely private, but it can be transformed without any additional costs into a local public good Z (i.e. $MRT_{xZ} = 1$). Furthermore the total number of identical individuals is fixed to N. Under the assumption that all persons living in one region are treated identically, the utility level $U_i = U(x_i, Z_i)$ of every person of region i depends upon the level of private consumption and upon the level of the local public good region i provides.

The central planners problem is therefore to allocate individuals and consumption levels optimally across these two regions. In formal notation:

$$\begin{aligned} \max_{x_i, Z_i, n_i} \; L =& U(x_1, Z_1) \\ &+ \lambda\left[U(x_2, Z_2) - \overline{U_2}\right] && \text{(1a)} \\ &+ \mu\left[f_1(n_1, T_1) + f_2(n_2, T_2) - n_1 x_1 - n_2 x_2 - Z_1 - Z_2\right] && \text{(1b)} \\ &+ \Psi[N - n_1 - n_2]. && \text{(1c)} \end{aligned}$$

The utility of any individual in region 1 is maximized under the restrictions that individuals living in region 2 reach a given utility level (1a), that total output equals total consumption (1b), and that all persons are located somewhere (1c).

The above maximization problem leads to the following well known efficiency conditions: first, the Samuelson rule for the provision of public goods must be met in both regions,

$$n_i \frac{\partial U_i / \partial Z_i}{\partial U_i / \partial x_i} = 1, \tag{2a}$$

and second. the marginal net benefit of the population must be equalized across regions, i.e.

$$\frac{\partial f_1(n_1,T_1)}{\partial n_1} - x_1 = \frac{\partial f_2(n_2,T_2)}{\partial n_2} - x_2. \tag{2b}$$

To illustrate the resulting opportunity locus for this optimization, I will use standard textbook case functions with a rather simple numerical specification. Both are listed in Table 1[1]. Since labor is the only variable input of the production function, it is more convenient to write $f_i(n_i)$ for regional, and $f(n_1,n_2)$ for total output.

Table 1:
Functional forms and parameter values

	Production function		Utility function
Functional forms	$f_i(n_i) = A_i n_i^{\alpha_i} \overline{T}_i^{1-\alpha_i}$		$U_i(x_i,Z_i) = x_i^\beta Z_i^{1-\beta}$
Parameter values			$\beta = 0.5$
Case 1 (identical regions)			
	$A_{1,2} = 10$	$N = 100$	
	$\alpha_{1,2} = 0.3$	$\overline{T}_{1,2} = 100$	
Case 2 (differences in land size only)			
	$\overline{T}_1 = 93$	$\overline{T}_2 = 100$	

The system (2a),(2b) can be solved for the 'optimal' private consumption levels, if one starts off with the feasability constraint (1b) and incorporates all optimality conditions successively. The Samuelson rule for public good provision has the form

$$\gamma n_i x_i = Z_i, \qquad \text{where} \qquad \gamma = \frac{1-\beta}{\beta} \tag{3}$$

for the chosen type of the utility function. Substituting into equation (1b) yields,

$$\begin{aligned}\beta f(n_1,n_2) &= n_1x_1 + n_2x_2 \\ &= n_1(x_1 - x_2) + Nx_2.\end{aligned} \tag{4}$$

In the last step, the right hand side of the equation was extended by $\pm n_1x_2$. Efficiency condition (2b) is equivalent to $\Delta f' := f_1'(n_1) - f_2'(n_2) = x_1 - x_2$. Therefore x_2 is given by

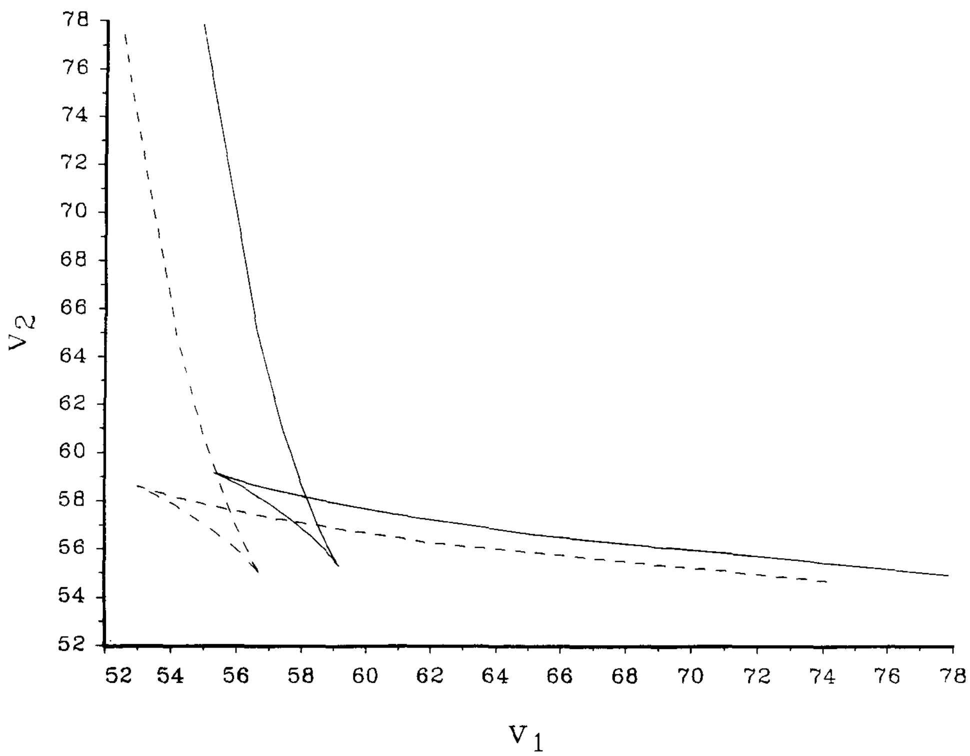
$$x_2 = \frac{\beta f(n_1,n_2)}{N} - \frac{n_1}{N}\Delta f' \tag{5}$$

[1]Case 2 serves mainly to show, how the shape of the opportunity locus is affected by a change of parameter values.

for any allocation of individuals across the two regions.

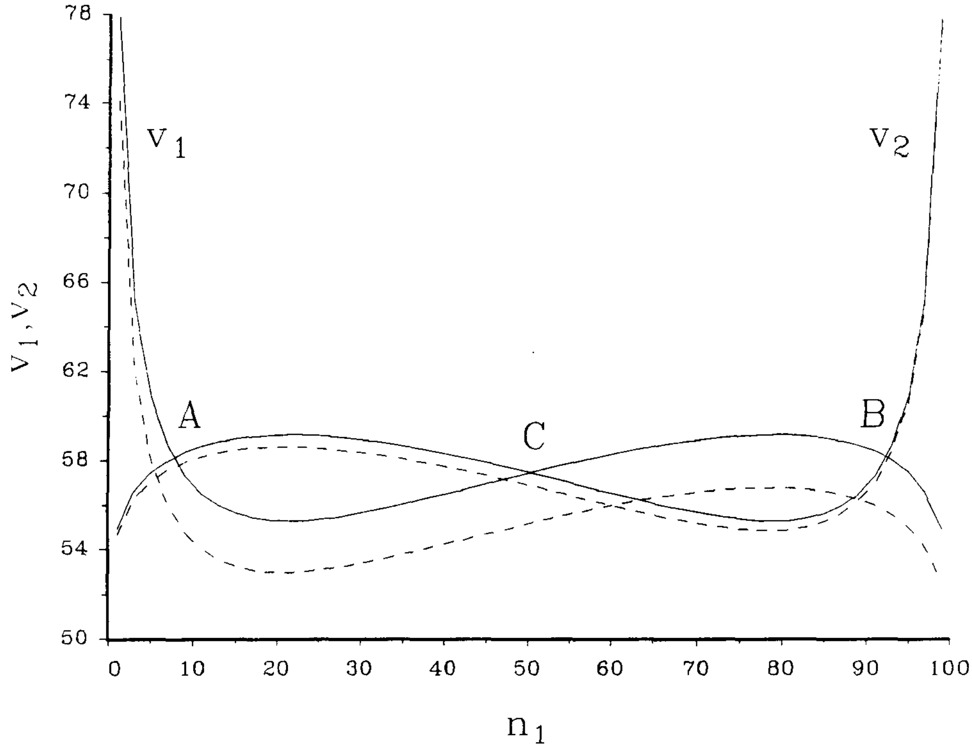
The remaining consumption levels are also quickly found. In short: $Z_2 = \gamma n_2 x_2$, $x_1 = (\beta f(n_1, n_2) + n_2 \Delta f')/N$, and $Z_1 = \gamma n_1 x_1$. Substituting these consumption levels back into the utility function yields the utility levels $v_i(n_1)$, which conform to the solution of the central planners problem. Since the total number of persons N is fixed, the v_i's depend only on the number of persons living in region 1. Finally, the opportunity locus can be obtained by plotting v_1 against v_2 for every value of n_1. The resulting curve is shown in Figure 1, where the solid line represents the identical regions case and the dashed line corresponds to case 2 parameter values. An uneven distribution of land size shifts the curve towards the lower left corner of the plot, but does not change its shape significantly.

Figure 1: Opportunity locus in the utility space



Obviously utility does not vary monotonously. Furthermore, since the curve intersects itself, some allocations are preferable to others. In fact, the diagram suggests that all allocations between this intersection point and the origin should be ruled out on efficiency grounds. However, one should be careful with this line of argumentation, as will be seen in the following section. More straightforward is the feature, that both curves contain three intersection points with the dotted 45°-line. So in both cases there exist three allocations, in which utility is equalized across regions. In the identical regions case, two points coincide because of symmetry, whereas in the case of differing land endowments all three utility levels associated with the intersection points are distinct[2].

Figure 2: Utility and population size



In figure 2 utility levels are plotted against region 1's population size. Solid curves again represent utility levels for case 1 parameter values, dashed curves case 2 values accordingly. The figure shows the same situation as figure 1, but it reveals a little more information about the nature of the potential migration equilibria. The non-monotonic behaviour of utility (with respect to increasing values of n_1) can also be seen more clearly. An algebraic representation is given in the appendix. In the identical regions case, points A, B, and C are candidates for a migration equilibrium. Point C, for example, would represent a migration equilibrium with no interregional transfers (autarky case). Since both regions are identical, individuals will be distributed equally across the regions and all efficiency conditions are satisfied. Starting from point C though, an increase in n_1 will also raise utility v_1. Thus equilibrium C is unstable—A and B, however, are not. In

[2]Multiple intersections with the equal utility line may also occur, if the Cobb-Douglas utility function is replaced by a CES (constant elasticity of substitution) function. With all other parameter values unchanged, an opportunity locus similar to that of figure 1 will emerge, if the elasticity of substitution in consumption is set to a value between 1.0 and 0.88. For smaller values the opportunity locus is a smooth curve and has only one intersection with the 45°-line. In the identical regions case, this intersection is always identical to the autarky migration equilibrium.

point A a net transfer flows from region 1 to region 2. Hence region 1 benefits from being 'less overpopulated' while region 2 benefits mainly from a larger provision of the local public good, which in turn reaches more people. Again, since the regions are identical, point B is simply the mirror image of A.

One question emerges immediately: what are the implications for the decentralized market? But before we turn to the Nash-players problem, I will in short illustrate how the shape of the opportunity locus is influenced by the allocation of goods on the one hand and the allocation of individuals on the other hand. This decomposition reveals that although combining a particular distribution of individuals (across the two regions) with the proper amount of transfers is necessary to achieve an efficient allocation, it may also be the cause for the existence of multiple potential migration equilibria.

3 Autarky and Efficiency

The central planners instruments to achieve an efficient allocation can also be interpreted in the following way: via n_i he controls migration and global output, via x_i, Z_i he controls interregional transfers and the intraregional allocation of goods. Without transfers—but still satisfying the Samuelson condition for an efficient allocation of private and public goods—all allocations along the dotted curve in figure 3 are attainable. The curve characterizes the opportunity locus for the autarky situation with respect to all possible distributions of population across the two regions.

For a given number of individuals n_i, the utility level $\tilde{v}_i(n_i)$ is determined by

$$\tilde{v}_i(n_i) = \left(\frac{\beta}{n_i}\right)^{\beta} (1-\beta)^{1-\beta} f_i(n_i), \tag{6}$$

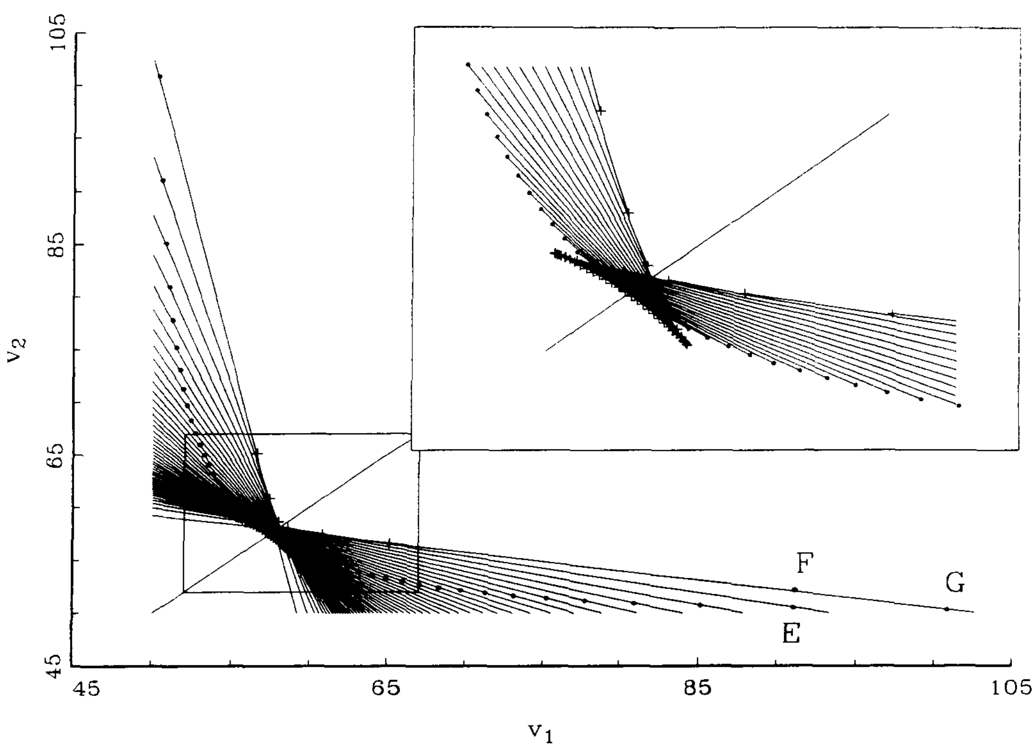
and the equilibrium point will be reached at $n_1 = n_2 = 50$. In this case, utility decreases monotonously with increasing n_i as long as $\beta > \alpha_i$, since

$$\tilde{v}_i'(n_i) = \frac{\partial \tilde{v}_i}{\partial n_i} = \frac{\alpha_i - \beta}{n_i} \left(\frac{\beta}{n_i}\right)^{\beta} (1-\beta)^{1-\beta} f_i(n_i). \tag{7}$$

To derive the above equation $f_i' = (\alpha_i f_i(n_i))/n_i$ was used. If the productivity of labor is relatively low and the preference for private consumption is high, both regions tend to be "overpopulated" and the resulting migration equilibrium will be stable. Otherwise, starting from the equilibrium point, any person could increase his/her utility by moving to the other region and a corner solution with all individuals in either of the two regions will emerge. An equivalent stability condition for the central planners problem of section 1 would require that the equalized marginal net benefit must be negativ, i.e.

$$f_i'(n_i) - x_i = (\alpha_1 - \beta)\frac{f_1(n_1)}{N} + (\alpha_2 - \beta)\frac{f_2(n_2)}{N}, \tag{8}$$

Figure 3: Autarky and transfers (identical regions)



which leads to the same condition for the parameter values.

Now, starting from a particular point on the dotted curve, the attainable utility levels $\tilde{v}_i$ can be varied along a straight line through interregional transfers (whereby the intraregional allocation of goods is implicitly adjusted to meet the Samuelson condition). The slope of any of these transfer lines is given by

$$\frac{dv_2}{dv_1} = -\left(\frac{n_1}{n_2}\right)^{\beta}, \tag{9}$$

which is shown in the appendix. The slope is thus constant and only depends upon the relative population sizes of the two regions.

The envelope of all transfer lines then generates the opportunity locus shown in figure 1 (solid line). This can be seen more clearly in the magnified area in the upper right corner of the diagram; plus signs hereby represent $(v_1; v_2)$ combinations of the planners solution.

There are two points worth mentioning. First, only the combination of transfers and migration makes the problem become ill-behaved. Neither instrument on its own would lead to ambiguity. Second, the central planners optimization problem is not simply a formal representation of the Pareto principle—as one might think at first. The latter point can best illustrated with a concrete example: Allocation E in

Figure 3 is not a solution of the planners problem. As the figure suggests allocation F will be preferred, because it yields the same utility level for inhabitants of region 1 and a higher utility level for inhabitants of region 2—compared to E. However, starting from E one person has to be reallocated from the better off region 1 to the worse off region 2 (move to point G), accompanied by compensating transfers from region 1 to region 2 (move along the transfer line from G to F). This procedure disregards that the reallocated person will not be fully compensated for the utility change he underwent. He will be left worse off. Thus, on the basis of the Pareto principle alone, which only requires that nobody can be made better off without making someone else worse off, allocation E cannot be ruled out as inefficient.

4 Decentralized Solution

To answer the remaining question, namely whether a Nash-equilibrium is still feasable and efficient, we set up the local Nash-players problem as follows. Each local government has three instruments: a head tax, the amount of the local public good Z_i and the possibility to pay direct gross transfers S_{ij} to the other region (j). Only two instruments can be set independently. The third—say the tax rate—is then determined via the budget equation. So the objective function for the local government is to maximize utility for residents by setting gross transfers and local public good provision to the optimal level, while taking the other regions choice of Z_j and S_{ji} as given. Formally we get for the assumed form of the utility function:

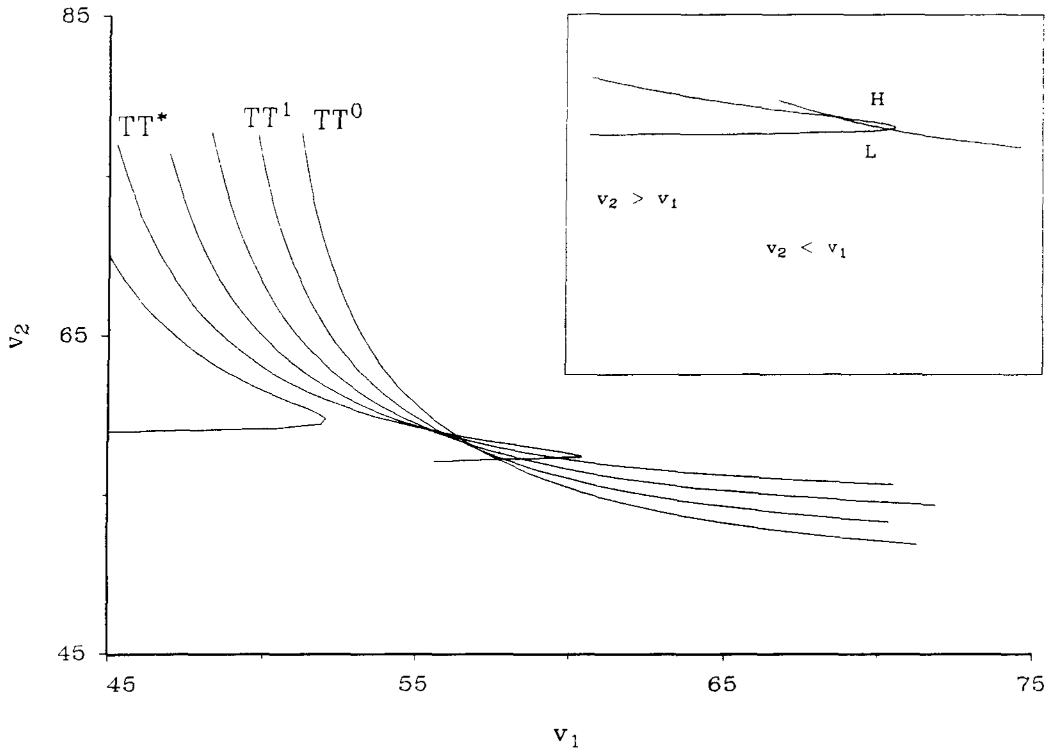
$$\begin{aligned} \max_{Z_i, S_{ij}} U_i &= \left(\frac{f_i(n_i) + S^e_{j,i} - S_{i,j} - Z_i}{n_i} \right)^{\beta} Z_i^{1-\beta} \\ \text{s.t.} \quad U_i &= U_j, \end{aligned} \tag{10}$$

where the maximization constraint generates the implicit migration response to variations of Z_i and S_{ij}.

Unfortunately it is very difficult to plot the Nash-reaction-curve directly from the first order conditions of (10), due to the functional forms used here. Therefore, we use a stepwise illustration of the local authorities options instead. Suppose region 1 expects a certain gross transfer $\overline{S^e_{21}}$ from region 2. Region 1 would then choose the S_{12}, Z_1 combination which yields the highest utility level under the equal utility condition. To illustrate this we assume—for the moment—that both regions vary Z_i with respect to n_i so that the Samuelson condition is implicitly satisfied. Utility for the inhabitants of region 1 can then be expressed as $v_1(n_1; S_{12}, \overline{S^e_{21}})$, where gross transfers enter the function parametrically. So the utility combinations $[v_1, v_2]$ can be plotted for any particular pair of transfers over the whole range of population distributions. The TT^0-curve in Figure 4, for example, corresponds to the pair $[S^0_{12}, \overline{S^e_{21}}]$ and its intersection with the 45°-line identifies region 1's payoff

$\tilde{v}_1(=\tilde{v}_2)$ for this particular answer to $\overline{S^e_{21}}$. As can be seen from figure 4 S^1_{12} (resp. TT^1) yields a higher payoff than S^0_{12}, etc.

Figure 4: Utility-combinations for given gross transfers



Interestingly enough, not only the location but also the shape of the TT-curves changes for increasing values of S_{12}. Note that the curve TT*, which produces the highest intersection with the equal-utility-line, also contains a second intersection point at a lower utility level. The situation does not change dramatically, if we give up the simplifying assumption of an implicitly adjusted value of Z_i and set the local public good provision throughout to Z^*_i, i.e. the value which conforms to the highest possible $v_1(=v_2)$ level. This can be seen in the upper right window of Figure 4. Here, the allocation underlying the higher intersection point, H, is the same as allocation A in Figure 2. L, on the other hand, is inefficient and has no counterpart on the opportunity locus shown in Figure 1.

Formally, the utility change with respect to population size is given by

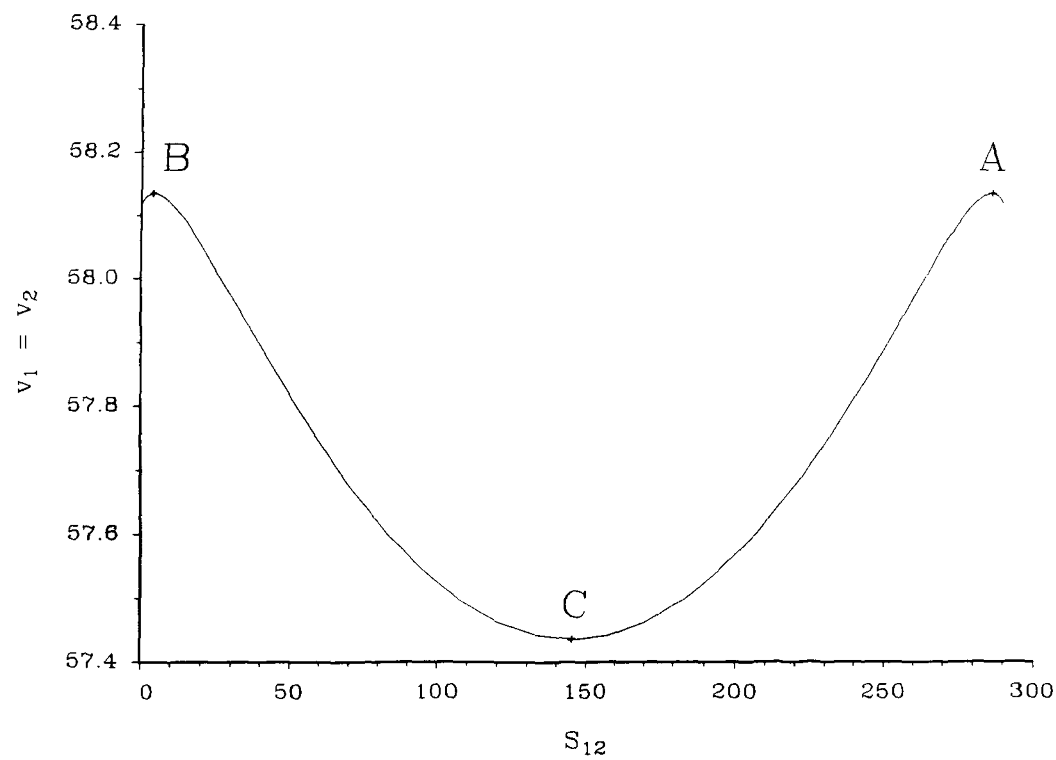
$$\frac{\partial v_1}{\partial n_1} = \frac{\dfrac{\alpha_1 f(n_1)}{n_1} - \dfrac{f(n_1) - S^*_{12} + \overline{S^e_{21}} - Z^*_1}{n_1}}{f(n_1) - S^*_{12} + \overline{S^e_{21}} - Z^*_1}\beta v_1 < 0 \qquad \text{if} \quad f'(n_1) - x_1 < 0. \quad (11)$$

Thus transfers affect the condition for the optimal regional population size, and there is some critical value for n_1, below which utility increases with an increase in population. Condition (11) also shows that the migration equilibrium corresponding to L is unstable.

Whether allocation H will be reached depends upon migration assumptions. Obviously, in the case of perfect information everybody knows that H yields the highest utility level. So S_{12}^*, Z_1^* is region 1's best answer to $\overline{S_{21}^e}$. If, however, only the utility level attained by the inhabitants of the other region is observable, then there exist situations in which a one shot game is not feasible. Imagine, for example, that migration was restricted initially. Then the initial population distribution may be such that the direct choice of S_{12}^*, Z_1^* would lead to a complete depopulation of region 1, since S_{12}^*, Z_1^* may lead to a utility combination (v_1, v_2) to the left of L on the TT*-curve.

Both local authorities are aware of this situation and thus S_{12}^*, Z_1^* will not be chosen. Without further assumptions it is not possible to determine which levels of S_{12} and Z_1 will be choosen instead. It is, of course, rather simple to overcome this problem, for example by introducing a peacemeal policy. But this in turn requires some kind of coordination of the local governments.

Figure 5: Equal utility levels for variable gross transfers



Finally, we plot the utility levels attainable in the stable intersection points against the related gross transfer levels S_{12} (Figure 5). Here, $\overline{S_{21}^e}$ has been set to a value slightly higher than the optimal net transfer S_{21}^*, which is implied by allocation B of Figure 2. This has been done merely for didactical reasons, because the range of reasonable values for S_{21} is maximized in this way[3]. Utility increases at first with increasing S_{12} until the optimal net transfer from region 2 to region 1 is reached (allocation B in figure 2). Afterwards utility decreases up to point C, where both gross transfers offset each other (autarky point) and then utility rises again until the optimal net transfers flow in the opposite direction (allocation A).

Both regions face this reaction curve with respect to gross transfers. Therefore, if the two local maxima are not identical—as in the case of different land endowments, for example—both local authorities head for the global maximum. Since both local authorities have perfect information about each others reaction curve, the region which should receive a net transfer will rationally expect the other region to do so and will choose a zero gross transfer (and vice versa). Only in cases as shown in Figure 5 is it not quite clear, which allocation will be reached. Obviously, simultaneous actions will cause coordination problems. However, these problems could also be omitted if, for example, the two players move succesively. There is no strategic advantage in having the first move because both objective functions are linked together by the equal utility condition.

5 Concluding remarks

It has been demonstrated how 'efficient' transfers affect the attainable utility levels in the two region case. Even if one uses standard textbook case functions the set of allocations characterized by the first order conditions of the central planners maximization problem might be larger than the set of efficient allocations. In particular multiple migration equilibria exist, all of which are interior solutions. Clearly, a decentralized market structure will pick the best alternative, if all acteurs are perfectly informed. However, if individuals can only observe the utility level obtained in the other region there exist situations in which an efficient Nash-equilibrium cannot be accomplished without further assumtions.

Appendix

Marginal utility change for increasing n_l $(\mathrm{d}v_l/\mathrm{d}n_l)$: with Samuelsonian provision of the local public good, utility can be expressed as

$$v_1 = x_1^{\beta}(\gamma n_1 x_1)^{1-\beta} = x_1\,(\gamma n_1)^{1-\beta}. \tag{12}$$

Partial derivation with respect to n_1 leads to

$$\frac{\partial v_1}{\partial n_1} = \left[\frac{\partial x_1}{\partial n_1} + \frac{x_1(1-\beta)}{n_1}\right](\gamma n_1)^{1-\beta}. \tag{13}$$

[3]If region 1 expects a lower transfer $\hat{S}^e_{21}$ from region 2 then the curve will be shifted horizontally to the left exactly by the difference between S^e_{21} and $\hat{S}^e_{21}$.

The first term inside the brackets of (13) is negative, since

$$\frac{\partial x_1}{\partial n_1} = \frac{1}{N}\Big(n_2(f_1'' + f_2'') - (1-\beta)(f_1' - f_2')\Big)$$
$$= \frac{1}{N}\Big(\underbrace{(\alpha_2 - \beta)\, f_2'}_{<0} - \underbrace{\big[(1-\alpha_1)\frac{n_2}{n_1} + (1-\beta)\big] f_1'}_{>0}\Big) \; < \; 0, \tag{14}$$

where $f_i'' = (\alpha_i - 1)f_i'/n_i$ has been used. The second term of (13), however, is positive, so utility is decreasing for very low and for very high levels of n_1, as figure 2 shows.

Slope of transfer line: For a given population size n_i $(i = 1, 2)$ and a given amount A_i of goods consumed in region i, the utility level v_i—with Samuelson condition satisfied—is given by

$$v_i = \frac{\beta A_i}{n_i}(\gamma n_i)^{1-\beta}. \tag{15}$$

A change in the level of A_i leads therefore to a change in utility of

$$\frac{\mathrm{d}v_i}{\mathrm{d}A_i} = \frac{\beta}{n_i}(\gamma n_i)^{1-\beta}. \tag{16}$$

For $\mathrm{d}A_2 = -\mathrm{d}A_1$ the slope of any transfer line comes to

$$\left.\frac{\mathrm{d}v_2}{\mathrm{d}v_1}\right|_{\mathrm{d}A_2 = -\mathrm{d}A_1} = -\frac{\frac{\beta}{n_2}(\gamma n_2)^{1-\beta}}{\frac{\beta}{n_1}(\gamma n_1)^{1-\beta}} = -\left(\frac{n_1}{n_2}\right)^{\beta}. \tag{17}$$

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