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Fabian Herweg und Daniel Müller: Overconfidence in the Market for Lemons

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Overconfidence in the Market for Lemons^{*}

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We extend Akerlof's (1970) "Market for Lemons" by assuming that some buyers are overconfident. Buyers in our model receive a noisy signal about the quality of the good that is at display for sale. Overconfident buyers do not update according to Bayes' rule but take the noisy signal at face value. The main finding is that the presence of overconfident buyers can stabilize the market outcome by preventing total adverse selection. This stabilization, however, comes at a cost: rational buyers are crowded out of the market.

JEL classification: D82; L15

Keywords: Adverse Selection; Market for Lemons; Overconfidence

1. Introduction

There are many markets for used goods that seem to work quite well. In particular nowadays in the age of the Internet, there exist many platforms where tremendous amounts of used goods of all kinds are traded, most prominently eBay. For instance, the trade volume of eBay Motors in the United States is approximately 36000 cars sold each month (Lewis, forthcoming). Markets for used goods, in particular used cars where stakes are relatively high, having a high volume of trade is puzzling from the perspective of standard economics. It seems reasonable to presume that the potential seller of a used commodity, say the used car, has a good idea of the quality of her car, whereas the potential buyer obtains only little information whether the car on display is a "lemon" or a "cream puff". Given this information structure, standard models of adverse selection predict that only low-quality goods are traded in equilibrium and that the volume of trade is low

^{*}We have benefited from comments made by Takeshi Murooka and Xiaoyu Xia. All errors are of course our own.

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(Akerlof, 1970). In the extreme, the asymmetric information problem can lead to a complete market breakdown. A seller who owns a cream puff prefers not to offer her car on the market, which reduces the average quality on the market and in turn the trade volume. Besides casual observations that markets for used goods function well, also empirical works investigating markets for used cars find only weak evidence for adverse selection (Bond, 1982; Genesove, 1993).

We provide one possible explanation for this puzzling gap between theoretical predictions and empirical observations in a fairly simple but standard framework of a market with asymmetric information: if the buyer is overconfident with positive probability, then there always exists an equilibrium in which high-quality goods are traded. Moreover, if it is sufficiently likely that the buyer is overconfident, then all pure-strategy equilibria are pooling equilibria in which all qualities are traded. The presence of overconfident buyers thus can stabilize the market and helps to prevent the total adverse selection outcome. This market stabilization, however, comes at a cost: rational buyers do not participate in the market anymore.

2. The Model

A seller (S) owns one unit of an indivisible good, the quality of which is $q \in \{q_L, q_H\}$ with $0 \leq q_L < q_H$.¹ Her reservation price for quality q is $r(q)$, with $r(q_L) = r_L$ and $r(q_H) = r_H$. The probability of S being endowed with high quality is $\alpha \in (0, 1)$. The buyer (B)'s valuation for quality q , $v(q)$, is normalized such that $v(q_L) = q_L$ and $v(q_H) = q_H$.

Assumption 1.

- (i) $q_H - r_H \geq q_L - r_L$
- (ii) $q_L - r_L > 0$
- (iii) $r_H > \bar{q} := \alpha q_H + (1 - \alpha)q_L$

Assumption 1 (i) and (ii) together guarantee that there are positive gains from trade for all quality levels, an assumption usually imposed in the literature.² Moreover, we are interested in markets in which there is an adverse selection problem in the sense that high-quality sellers value the item above the average quality, Assumption 1 (iii). If Assumption 1 (iii) is violated there does always exist a pooling equilibrium. It is important to note that Assumption 1 (iii) implies that B 's willingness to pay for a low-quality item is below the reservation price of a high-quality seller, $q_L < r_H$.

While B cannot directly observe quality, he receives a private signal $s \in \{L, H\}$, which is positively correlated with true quality, $\gamma = \Pr(s = H|q = q_H) = \Pr(s =$

¹This setup is common in the literature and used, for instance, by Ellingsen (1997).

²Assumption 1 (i) is made in order to simplify the exposition.

$L|q = q_L) \in (1/2, 1)$. We refer to $s = H$ and $s = L$ as the “good” and the “bad” signal, respectively.

There are two potential types of buyers, $\tau \in \{R, O\}$: with probability $1 - \lambda$, B is a rational Bayesian updater ($\tau = R$), who draws the correct informational inference from the signal he receives. With probability $\lambda \in [0, 1)$, B is overconfident ($\tau = O$) in his own ability to judge the quality of the item from his private signal in the sense that he believes the signal to be perfectly informative.

Two prominent notions of overconfidence are *overprecision* and *overestimation*. While overprecision refers to people believing their estimates to be more accurate than they actually are, overestimation alludes to people overestimating their own abilities.³ Our model captures both these biases: if the buyer wrongly believes his noisy signal to be perfectly accurate, then he is prone to overestimation; if he wrongly believes to be an expert who can perfectly determine the quality of the item on display by inspecting it, then he suffers from the overestimation.

Both S and B are risk neutral and maximize expected profits and expected utility, respectively. While S 's profit from selling at price p is $\pi = p - r$, B 's utility from purchasing quality q at price p is $u = q - p$. If no trade takes place, S 's profit and B 's utility are zero.

The sequence of events is as follows. (i) Nature draws $q \in \{q_L, q_H\}$. (ii) S observes q and sets a take-it-or-leave-it price $p \in \mathbb{R}$. A pure strategy for S is a pair (p_L, p_H) , where p_L (p_H) denotes the price for a low-quality (high-quality) item. (iii) Nature draws $\tau \in \{R, O\}$ and $s \in \{L, H\}$. (iv) Upon privately observing p and s , B decides whether to buy the item. A pure strategy for a buyer of type τ is a function $b^\tau : \mathbb{R} \times \{L, H\} \rightarrow \{0, 1\}$, where $b^\tau(p, s) = 1$ means “buy” and $b^\tau(p, s) = 0$ means “don't buy” at price p and for signal s . (v) Both parties receive their respective payoffs, $\pi = (p - r)b^\tau(p, s)$ and $u = (q - p)b^\tau(p, s)$.⁴

We augment the usual notion of (weak) perfect Bayesian equilibrium in pure strategies in order to allow for an overconfident buyer.⁵ Let $\mu_s^\tau(p)$ denote the belief held by buyer type $\tau \in \{R, O\}$ about high quality being offered when observing price p and receiving signal s . Let $\mu^\tau = \{(\mu_L^\tau(p), \mu_H^\tau(p)) \in [0, 1]^2 | p \in \mathbb{R}\}$.

Definition 1. A market equilibrium is a vector $(\mu^R, \mu^O, (p_L, p_H), b^R(p, s), b^O(p, s))$ that satisfies the following conditions:

(B1) If $p_L \neq p_H$, then $\mu_L^R(p_L) = \mu_H^R(p_L) = 0$ and $\mu_L^R(p_H) = \mu_H^R(p_H) = 1$.

(B2) If $p_L = p_H = \bar{p}$, then

$$\mu_H^R(\bar{p}) = \frac{\gamma\alpha}{\gamma\alpha + (1 - \gamma)(1 - \alpha)} \quad \text{and} \quad \mu_L^R(\bar{p}) = \frac{(1 - \gamma)\alpha}{(1 - \gamma)\alpha + \gamma(1 - \alpha)}. \quad (1)$$

³See Alpert and Raiffa (1982) or Soll and Klayman (2004) on overprecision, and Clayton (2005) or Buehler, Griffin, and Ross (1994) on overestimation.

⁴The analysis would remain unchanged if all draws by nature occurred in step (i).

⁵Focusing on pure-strategy equilibria often allows us to obtain a unique equilibrium outcome without applying an equilibrium refinement.

(B3) For all $p \in \mathbb{R}$ and $s \in \{L, H\}$, $\mu_L^O(p) = 0$ and $\mu_H^O(p) = 1$.

(SR) At each information set, the concerned player's strategy is a best response to the other player's strategy under the belief induced by μ at that information set.

(B1) and (B2) specify a rational buyer's beliefs on the equilibrium path: if S charges different prices for different qualities, an incompletely informed rational buyer should—irrespective of the signal he receives—believe the item to be of low quality if he observes price p_L and of high quality if he observes price p_H ; if S charges the same price for both high-quality and low-quality items, then a rational buyer's beliefs are determined by Bayes' rule. (B3) specifies that an overconfident buyer takes the signal at face value irrespective of the price. Last, (SR) requires sequential rationality.

Finally, let ρ_L and ρ_H denote the trading probabilities in equilibrium for low-quality and high-quality goods, respectively.

3. The Analysis

The following observations hold irrespective of B 's type: first, sequential rationality requires $b^\tau(p, s) = 1$ for $p \leq q_L$ and $b^\tau(p, s) = 0$ for $p > q_H$. Thus, in any equilibrium low quality must be traded with strictly positive probability at a price of at least q_L and at most q_H —otherwise S could profitably deviate to $p = q_L$. Likewise, high quality will never be priced strictly below r_H because S could profitably deviate to $p > q_H$. Hence, there are three potential kinds of equilibria: (i) pooling equilibrium, i.e., $p_L = p_H = \bar{p} \in [r_H, q_H]$, (ii) separating equilibrium with trade of high-quality items, i.e., $p_L = q_L$ and $p_H \in [r_H, q_H]$ and (iii) separating equilibrium without trade of high-quality items (total adverse selection), i.e., $p_L = q_L$ and $p_H \geq q_H$.

3.1. No Overconfidence and Total Adverse Selection

If the signal is sufficiently imprecise, then no pooling equilibrium exists. Intuitively, with $\bar{q} < r_H$, B purchases at the pooling price $\bar{p} \geq r_H$ only if he receives positive information about the quality on display, i.e., $s = H$. If the signal is highly uninformative, however, then B 's willingness to pay after receiving $s = H$ is below r_H and thus no pooling equilibrium exists.

Lemma 1. *Suppose that Assumption 1 holds. For $\lambda = 0$ there does not exist a pure-strategy pooling equilibrium if*

$$\gamma < \tilde{\gamma} := \frac{(r_H - q_L)(1 - \alpha)}{(q_H - r_H)\alpha + (r_H - q_L)(1 - \alpha)} \in \left(\frac{1}{2}, 1\right). \quad (2)$$

Moreover, for a sufficiently imprecise signal there does not exist a separating equilibrium with trade of high-quality items. Intuitively, a pure-strategy separating equilibrium is most likely to exist if B purchases the high-quality item only after receiving $s = H$, which requires that $p_H = q_H$. However, even if B follows this strategy, then a low-quality seller can profitably mimic a high-quality seller if B receiving a good signal is sufficiently likely to occur also in the case of low quality.

Lemma 2. *Suppose that Assumption 1 holds. For $\lambda = 0$ there does not exist a pure-strategy separating equilibrium with $\rho_H > 0$ if*

$$\gamma < \hat{\gamma} := \frac{q_H - q_L}{q_H - r_L} \in \left(\frac{1}{2}, 1\right). \quad (3)$$

In what follows, we restrict attention to a rather signal noisy.

Assumption 2. $\gamma < \min\{\hat{\gamma}, \tilde{\gamma}\}$.

We can always support a separating equilibrium without trade of high quality—this is most obvious for pessimistic out-of-equilibrium beliefs, i.e., $\mu_s^R(p) = 0$ for all $p < q_H$ and $s \in \{L, H\}$. Thus, for a fairly noisy signal, the market exhibits the well-known adverse selection problem (Akerlof, 1970).

Proposition 1. *Suppose Assumptions 1 and 2 hold. For $\lambda = 0$, in all pure-strategy equilibria it holds that $p_L = q_L$, $p_H \geq q_H$, $\rho_L = 1$ and $\rho_H = 0$.*

3.2. Overconfidence and Market Stabilization

The first observation is that total adverse selection is not a feasible equilibrium outcome if there are overconfident buyers in the market. A seller of high quality can always ask for a price higher than her reservation value— $p_H \in (r_H, q_H]$ —that allows her to sell with positive probability because she sells at least to an overconfident buyer who receives a good signal. Even if there are only few overconfident buyers and even if the signal is fairly noisy, this strategy leads to strictly positive profits in expectations. Thus, if there exists a pure-strategy equilibrium, then both qualities are traded in this equilibrium. In the following, we characterize this equilibrium and establish conditions for its existence.

First, we show the existence of a pooling equilibrium in which high-quality items are traded with strictly positive probability even when the signal is very uninformative, i.e., Assumption 2 holds. In any pooling equilibrium with trade of high quality we must have $\bar{p} \in [r_H, q_H]$. Upon observing price $\bar{p}$, beliefs of a rationally updating buyer are determined according to (B2), whereas beliefs of an overconfident buyer are given by (B3). In consequence, upon observing price $\bar{p}$ and signal $s \in \{H, L\}$, a rational buyer purchases the item if

$$\bar{p} \leq \bar{q}(s) \equiv \mu_s(\bar{p})q_H + (1 - \mu_s(\bar{p}))q_L. \quad (4)$$

Thus, for $\gamma < \hat{\gamma}$, rational buyers do not purchase at price $\bar{p}$ because $\bar{q}(L) < \bar{q}(H) < r_H$. An overconfident buyer, on the other hand, purchases if $\bar{p} \leq q_s$, and therefore buys the item at price $\bar{p}$ if he receives $s = H$. Hence, irrespective of her item's quality, for any price $\bar{p} \in [r_H, q_H)$ the seller has an incentive to deviate to the highest price at which trade possibly takes place, such that the only price consistent with a pooling equilibrium is $\bar{p} = q_H$. To establish existence of a pooling equilibrium with price $\bar{p} = q_H$, consider pessimistic out-of-equilibrium beliefs for a rational buyer, i.e., $\mu_s(p) = 0$ for all $p \neq \bar{p}$ and $s \in \{L, H\}$. Since the seller of a low-quality item can always sell for sure at a price q_L , $\bar{p} = q_H$ can only be part of a pooling equilibrium if $q_L - r_L \leq \lambda(1 - \gamma)(q_H - r_L)$, or equivalently, if

$$\lambda \geq \frac{q_L - r_L}{(1 - \gamma)(q_H - r_L)} =: \bar{\lambda}. \quad (5)$$

Note that $\gamma < \hat{\gamma}$ implies $\bar{\lambda} < 1$. Since a high-quality seller obviously has no reason to deviate, there exists a pooling equilibrium even for a fairly noisy signal as long as the share of overconfident buyers is sufficiently large.

Proposition 2. *Suppose that Assumptions 1 and 2 hold. Then, for $\lambda \in (0, 1)$ there does not exist a pure-strategy separating equilibrium. If $\lambda \geq \bar{\lambda}$, then there exists a pure-strategy equilibrium with a pooling price $p_L = p_H = q_H$ and trading probabilities $\rho_L = \lambda(1 - \gamma)$ and $\rho_H = \lambda\gamma$.*

From the above proposition, the next result—for the case where there are almost no gains from trade for low-quality items—follows immediately.

Corollary 1. *Suppose that Assumptions 1 and 2 hold and that $q_L \rightarrow r_L$. Then, for $\lambda \in (0, 1)$ there exists a pure-strategy equilibrium and any pure-strategy equilibrium is a pooling equilibrium.*

According to Corollary 1, if the low-quality item is basically useless, then an arbitrary small proportion of overconfident buyers in the market is sufficient to avoid the undesirable total adverse selection outcome.

3.3. Equilibrium in Mixed Strategies

One question is immediately at hand from Proposition 2: what happens for $\lambda < \bar{\lambda}$, i.e., if neither a pure-strategy pooling equilibrium nor a pure-strategy separating equilibrium exists. In this case, there exists a mixed-strategy separating equilibrium with trade of high-quality items. The equilibrium prices are $p_L = q_L$ and $p_H = q_H$. In one equilibrium, with pessimistic out-of-equilibrium beliefs, the buyer plays the mixed strategy: $b(p_H, H) = \sigma \in (0, 1)$, $b(p, s) = 1$ for $p \leq q_L$, and $b(p, s) = 0$ in the remaining cases. If the mixing probability σ is sufficiently small, then the low-quality seller has no incentives to mimic the high-quality seller. For pessimistic out-of-equilibrium beliefs the mixing probability needs to satisfy

$$\sigma \leq \bar{\lambda} - \lambda =: \bar{\sigma}. \quad (6)$$

In this equilibrium, rational buyers as well as overconfident buyers and both types of sellers are active. A mixed-strategy separating equilibrium with trade of high-quality items also exists in the case without overconfident buyers ($\lambda = 0$). In this mixed-strategy equilibrium, the maximum trading probability for high quality is

$$\rho_H(\lambda) = [\bar{\lambda} - \lambda(\bar{\lambda} - \lambda)]\gamma. \quad (7)$$

With $\rho(\lambda) < \rho(0)$ for $\lambda \in (0, \bar{\lambda})$, in this mixed-strategy equilibrium the presence of overconfident buyers reduces the probability that high quality is traded which in turn depletes market efficiency.

4. Conclusion

On the one hand, Akerlof (1970)’s seminal contribution spawned a large literature investigating markets with asymmetric information. On the other hand, overconfident agents have been introduced into various economic settings, e.g. Grubb (2009), Englmaier (2010, 2011) or Sandroni and Squintani (2007). Nevertheless, to the best of our knowledge, this is the first paper that incorporates overconfident buyers into the classic market for “lemons”. We find that the presence of overconfident buyers increases the trading volume of high quality and thus prevents total adverse selection. Since buyers’ overconfidence can lead to an improvement of market efficiency, the often found suggestion that buyers should be educated to make better judgments in uncertain environments deserves more thoughtful consideration. Our findings complement the observation in Eyster and Rabin (2005) that the presence of “behavioral actors” may improve market outcomes.

References

- AKERLOF, G. A. (1970): “The market for “lemons”: quality uncertainty and the market mechanism,” *Quarterly Journal of Economics*, 84, 488–500.
- ALPERT, M., AND H. RAIFFA (1982): “A Progress Report on the Training of Probability Assessors,” in *Judgment Under Uncertainty: Heuristics and Biases*, ed. by D. Kahneman, and A. Tversky, pp. 294–305, Cambridge. Cambridge University Press.
- BOND, E. W. (1982): “A Direct Test of the “Lemons” Model: The Market for Used Pickup Trucks,” *American Economic Review*, 72, 836–840.
- BUEHLER, R., D. GRIFFIN, AND M. ROSS (1994): “Exploring the Planning Fallacy: Why People Underestimate Their Task Completion Times,” *Journal of Personality and Social Psychology*, 67, 366–381.

- CLAYSON, D. E. (2005): “Performance Overconfidence: Metacognitive Effects on Misplaced Student Expectations,” *Journal of Marketing Education*, 27, 122–129.
- ELLINGSEN, T. (1997): “Price Signals Quality: The Case of Perfectly Inelastic Demand,” *International Journal of Industrial Organization*, 19, 1347–1361.
- ENGLMAIER, F. (2010): “Managerial Optimism and Investment Choice,” *Managerial and Decision Economics*, 31, 286–291.
- (2011): “Commitment in R&D Tournaments via Strategic Delegation to Overoptimistic Managers,” *Managerial and Decision Economics*, 32, 63–69.
- EYSTER, E., AND M. RABIN (2005): “Cursed Equilibrium,” *Econometrica*, 73, 1623–1672.
- GENESOVE, D. (1993): “Adverse Selection in the Wholesale Used Car Market,” *Journal of Political Economy*, 101, 644–665.
- GRUBB, M. D. (2009): “Selling to Overconfident Consumers,” *American Economic Review*, 99, 1770–1807.
- LEWIS, G. (forthcoming): “Asymmetric Information, Adverse Selection, and Online Disclosure: The Case of Ebay Motors,” *American Economic Review*.
- SANDRONI, A., AND F. SQUINTANI (2007): “Overconfidence, Insurance, and Paternalism,” *American Economic Review*, 97, 1994–2004.
- SOLL, J. B., AND J. KLAYMAN (2004): “Overconfidence in Interval Estimates,” *Journal of Experimental Psychology. Learning, Memory, and Cognition*, 30, 299–314.

A. Proofs of Propositions and Lemmas (not for publication)

Proof of Lemma 1. Suppose, in contradiction, that the price $\bar{p}$ constitutes a pooling equilibrium. We know that we must have $\bar{p} \in [r_H, q_H]$. Given price $\bar{p}$, a rational buyer’s belief about the item being of high quality after receiving signal $s \in \{L, H\}$ is given by $\mu_s(\bar{p})$ according to (B2) and he will buy the item if and only if

$$\bar{p} \leq \mu_s(\bar{p})q_H + (1 - \mu_s(\bar{p}))q_L =: \bar{q}(s). \quad (8)$$

Since $\bar{q}(L) < \bar{q} < r_H$, for a pooling equilibrium with trade of high-quality items to exist, we must have $\bar{p} \in [r_H, \bar{q}(H)]$. Put differently, if a pooling equilibrium exists then the buyer will purchase the item only after receiving the good signal

$s = H$. Note that $\lim_{\gamma \searrow 1/2} \bar{q}(H) = \bar{q}$, $\lim_{\gamma \nearrow 1} \bar{q}(H) = q_H$, and $d\bar{q}(H)/d\gamma > 0$. Hence, by the intermediate-value theorem, there exists a unique value $\tilde{\gamma}$ such that $\bar{q}(H) = r_H$.

Finally, note that $\tilde{\gamma} > 1/2$, or equivalently

$$(1 - \alpha)(r_H - q_L) > \alpha(q_H - r_H), \quad (9)$$

by Assumption 1 (iii), which concludes the proof. $\square$

Proof of Lemma 2. First, according to (B1), in any separating equilibrium with equilibrium prices $p_L \neq p_H$ we have $\mu_s(p_L) = 0$ and $\mu_s(p_H) = 1$ for $s \in \{L, H\}$. This implies that $p_L = q_L$: if $p_L > q_L$, then the buyer would not buy and the seller of a low-quality item could profitably deviate to a price $p \in (r_L, q_L]$, thereby selling for sure; if $p_L < q_L$, then the seller of a low-quality item could profitably deviate to $p = q_L$, thereby still selling for sure at a higher price. Next, whenever a high-quality seller sells with strictly positive probability in equilibrium, i.e., $\rho_H > 0$, we must have $p_H \in [r_H, q_H]$. Note that, since $\mu_s(p_H) = 1$, sequential rationality requires that the buyer always buys for $p_H \in [r_H, q_H]$. This, however cannot be part of any equilibrium because a low-quality seller could profitably deviate to price p_H . Therefore, in any separating equilibrium in which the seller of a high-quality item sells with strictly positive probability, we have $\rho_H < 1$ and thus $p_H = q_H$ (because B has to be indifferent between buying and not buying at price p_H). With our focus on pure strategies, for $\rho_H < 1$ to be feasible, the buyer has to condition his purchasing decision at price p_H on the realization of the signal he receives, i.e., $b(p_H, s) = 1$ for $s \in \mathcal{B} \subset \{L, H\}$ and $b(p_H, s) = 0$ for $s \in \{L, H\} \setminus \mathcal{B}$. With $\Pr(H|q_L) < \Pr(L|q_L)$, the seller of a low-quality item will definitely deviate to $p_H = q_H$ if $\Pr(H|q_L)(q_H - r_L) > (q_L - r_L)$, or equivalently, if $\gamma < \tilde{\gamma}$. Noting that $\tilde{\gamma} > 1/2$ if and only if $q_H - q_L > q_L - r_L$, which holds by Assumption 1, completes the proof. $\square$

Proof of Proposition 1. We show that a separating equilibrium without trade of high-quality items always exist. The proposition follows from this observation together with Lemmas 1 and 2.

In an adverse selection equilibrium, we must have $p_L = q_L$, $p_H \geq q_H$, and $\mu_s(p_L) = 0$ and $\mu_s(p_H) = 1$ for $s \in \{L, H\}$. With buyer behavior for prices $p \leq q_L$ and $p > q_H$ not depending on the buyer's beliefs, in order to support the equilibrium under consideration, we have to specify out-of-equilibrium beliefs for prices $p \in (q_L, q_H]$ such that neither type of seller has an incentive to deviate. One set of beliefs doing the job is given by $\mu_s(p) \leq (p - q_L)/(q_H - q_L)$ for $p \in (q_L, q_H]$ and $s \in \{L, H\}$. Since $\mu_s(p)q_H + (1 - \mu_s(p))q_L - p \leq 0$ it is sequentially rational for the buyer never to buy at prices $p \in (q_L, q_H]$, which in turn implies that no type of seller has an incentive to deviate to such prices.⁶ $\square$

⁶Here, it is assumed that the buyer does not purchase the item if he is indifferent.

Proof of Proposition 2. It remains to show that there does not exist a pure-strategy separating equilibrium with trade of high-quality items. This kind of equilibrium is most likely to exist if the Bayesian buyer has pessimistic out-of-equilibrium beliefs and purchases at p_H only after receiving $s = H$. This strategy is sequentially rational only for $p_H = q_H$. Moreover, we know that $p_L = q_L$. If the Bayesian buyer uses this strategy, then he behaves (on the equilibrium path) exactly as an overconfident buyer. Thus, the incentives to deviate to $p = q_H$ for a seller who possesses a good of low-quality are independent of the probability with which she faces an overconfident buyer. Hence, it follows immediately from Lemma 2 that a low-quality seller prefers to deviate if $\gamma < \hat{\gamma}$. Thus, there does not exist a pure-strategy separating equilibrium if Assumption 2 holds. $\square$