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**Working Paper**

## The term structure of interest rates in a small open economy DSGE model with Markov switching

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# FinMaP-Working Paper No. 22



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FINMAP –

**FINANCIAL DISTORTIONS AND MACROECONOMIC  
PERFORMANCE: EXPECTATIONS, CONSTRAINTS AND  
INTERACTION OF AGENTS**

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**TITLE**

**The Term Structure of Interest Rates in a  
Small Open Economy DSGE Model with  
Markov Switching**

**by: Roman Horváth and Aleš Maršál**

## ABSTRACT

We lay out a small open economy dynamic stochastic general equilibrium (DSGE) model with Markov switching to study the term structure of interest rates. We extend the previous models by opening up the economy and adding a foreign demand channel. As a result, we explain the term structure of Czech interest rates and that the open economy version of the model fits reasonably well the period after the adoption of inflation targeting, which was characterized by two regimes: 1) a disinflation regime and 2) a price stability regime.

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# 1 Introduction

The term structure of interest rates is the key source of information in macroeconomics and finance. The yield curve has been established as an essential tool in predicting the business cycle, and it is a fundamental input in asset pricing and debt management. However, many macroeconomic models have had difficulties in matching the macro and financial data. For this reason, the estimates of term structure are usually derived from latent factor financial models. This dichotomous modeling approach leads to several problems.

First, as emphasized by Rudebusch and Swanson (2008), the importance of the joint modeling of both macroeconomic and finance variables within a dynamic stochastic general equilibrium (DSGE) framework is often under-appreciated. Macroeconomics and the asset pricing theory are closely related. This fact is clearly formulated by Cochrane (2001), who notes that asset markets are the mechanism by which consumption and investment are allocated across time and states of nature in such a way that the marginal rates of substitution and transformation are equalized. Hordahl et al. (2007) argues that the inability of macro models to match asset prices can be justified to some extent since the expected future profitability of individual firms is unobservable and difficult to evaluate. Equity prices may therefore be thought to be subject to fluctuations that are disconnected from the real economy. However, this reasoning is not valid for bond prices. The term structure of interest rates incorporate expectations of future monetary policy decisions, which have been relatively predictable in the past two decades.

Second, financial models typically do not account for monetary policy and macroeconomic fundamentals, as stressed by Rudebusch and Wu (2008). The short-term interest rate is the basic building block of the yield curve, which is largely influenced by the monetary authority. Long-term interest rates represent the risk-adjusted expectations for the short-term interest rates; hence, the behavior of the central bank is an important source of information when determining the shape of the yield curve.

Third, many interesting questions in economics are related exactly to the interaction between macroeconomic variables and asset prices. For example, the recent problems of many countries in rolling over their government debt and excessive debt financing in general arise questions how dramatically the increase in the term premium is able to affect the economy. Our work contributes to the discussion related to modeling the term structure of interest rates within the DSGE framework. However, contrary to other authors, e.g., (Rudebusch and Swanson, 2008), (Hordahl et al., 2007), (Andreasen, 2008), and (Ferman, 2011), who concentrate entirely on a closed-economy framework, we focus on the open economy implications of the term structure of interest rates within the DSGE model. To our knowledge, this question has been neglected by the macro-finance literature. The idea that motivates our research question relies on the fact that the driving force that determines the shape of the term structure of interest rates is closely tied to consumption and inflation dynamics. Galí (2002) shows that the foreign demand channel may significantly alter the dynamics of consumption

and prices. Intuitively, consumption-smoothing households in a closed economy react to positive shock by decreasing the number of hours worked. However, in an open economy, households do not have to decrease the number of hours worked in order to smooth consumption because of the foreign demand channel. They can keep consumption constant, increase the number of hours worked and sell the extra production to the rest of the world. Nevertheless, eventually, the accumulated wealth leads to a rise in consumption.

The literature on the modeling of the term structure of interest rates within the DSGE framework differs mostly in the specification of utility. The form of the utility function is the most significant determinant of consumption dynamics. The models with CRRA preferences were shown to deliver a negative slope for term structure, as first argued by Backus, Gregory, and Zin (1989). This paper presents a new open economy model with the term structure of interest rates, including the foreign demand channel in the model with CRRA preferences.

The inability of standard DSGE models to fit the volatility of term structure data has been attributed to the fact that the term premium is time-varying (see, for example, Ravenna and Seppala (2006)). Two main approaches to introducing time-varying term premia into the DSGE model have been mentioned in the literature: a third-order approximation, e.g., Ravenna and Seppala (2006), (Rudebusch and Swanson, 2008), or regime switching in terms of the volatility of exogenous shocks, e.g., Amisano and Tristani (2010) and Amisano and Tristani (2011). However, Ferman (2011) argues that to obtain an unbiased estimation of the yield curve, regime switching in monetary policy rules must be included. This argument is based on the observation in the data that the yield curve is steeper on average for the mid-1980s than for the Great Inflation of the 1970s. This fact is puzzling because one would expect that higher macroeconomic uncertainty would be reflected in higher term premia; hence, the slope of the term structure of interest rates should intuitively be higher in the 1970s. To solve this puzzle, Ferman (2011) proposes a framework in which investors incorporate into their beliefs the possibility that the economy may switch across different regimes. The nominal short-term rate fluctuates around different means across regimes. In this case, investors will require an additional premium, which Ferman (2011) calls “level risk,” to compensate for the risk of regime switch. Conditional on the economy being in a regime with low interest rates, long-term bonds will lose value in the case of a shift to the regime with higher rates.

We argue that the term structure of interest rates in the Czech Republic is formed by regime switches. The Czech Republic originally adopted inflation targeting in 1998 as a disinflation strategy, and within a few years, the central bank was able to deliver price stability. The related empirical evidence shows the asymmetry in the monetary policy rule of the Czech central bank (see Horvath (2008) and Vasicek (2012)) as well as the overall decline in the natural rate of interest rate (Horvath, 2009). This motivates us to model the Czech economy using the Ferman (2011) methodology. The model is characterized by Markov chain switching in monetary policy and solved to the second order. In

addition, it is important to note that the Czech Republic is one of the most open economies in the world, so opening up our model to include foreign interactions and fluctuations in a systematic manner is a natural choice.

Introducing the foreign demand channel into the DSGE model has the following consequences: the model calibrated to fit Czech moments is capable of delivering a positive and sizable term premium and solve Backus, Gregory, and Zin (1989)'s puzzle without introducing habit formation at the cost of increased model volatility.

We use the two-country model developed by Bergin et al. (2007) to derive a small open economy model. The model is suitable because it offers a relatively rich model representation of the economy with money in the utility function, intermediate and final markets and habits in consumption; moreover, this model can be easily extended, for example, to account for currency substitution, e.g., (Colantoni, 2010). Although we simplify the Bergin et al. (2007) framework for our benchmark model, the model can again be easily extended to study the implications of particular model specifications of the open economy model on the term structure of interest rates.

The findings of Backus, Gregory, and Zin (1989) and den Haan (1995) that the DSGE models cannot generate the term premia of a magnitude that is comparable to what we can observe in actual data have triggered research in this area. Consequently, there have been several relatively successful attempts to fit macro and term structure data into a DSGE model. Hordahl et al. (2007) use the stochastic discount factor to model the term premium. They assume an expectations hypothesis, which implies that the term premium is constant over time. The success of their model in fitting macro and finance data relies on a relatively large number of exogenous shocks, a long memory and a high degree of interest rate smoothing. The nominal rigidities have an indirect effect; sticky prices imply monetary non-neutrality. The number of papers tries to match the data using third-order approximations, e.g., (Rudebusch and Swanson, 2008). This method allows for a variable-term premium. Nevertheless, Rudebusch and Swanson (2008) concludes that in order to match the financial data in the DSGE model, it is necessary to seriously distort the ability to fit other macroeconomic variables. Caprioli and Gnocchi (2009) uses a collocation method with Chebyshev polynomials to investigate the impact of the credibility of monetary policy on the term structure of interest rates. Andreasen (2008) addresses the fact that stationary shocks to the economy have only moderate effects on interest rates with medium and long maturities. Hence, they introduce non-stationary shocks. They argue that whereas a highly persistent stationary shock may also affect interest rates with longer maturities, this shock is likely to distort the dynamics of the macroeconomy, and this is not the case for permanent shocks.

This paper is organized as follows. Section 2 presents the macro part of the model, which consists of a small open economy DSGE model. The section 3 discusses the calibration of the benchmark model. Section 4 provides the solution method. The finance part is presented in section 5, where we outline the general characteristics of the data on the term structure of interest rates and derive the yield curve implied by the DSGE model. In section 6, we evaluate the

results of the model simulations compared to the data from the Czech economy. Section 7 concludes.

## 2 Macro part: Model

This section presents a DSGE model, which has three types of agents: *i*) households, *ii*) firms, and *iii*) monetary authority. The economy is assumed to be driven by two persistent shocks that come from foreign output and productivity. The small economy framework is derived as a limiting case of the two-country model similar to Bergin et al. (2007). The technique that we employ to solve for a small open economy model builds on the method developed by Obstfeld and Rogoff (1995) and used by Sutherland (2006) and De Paoli (2009). The specification of the model allows us to produce deviations from purchasing power parity that arise from the existence of home bias in consumption. Moreover, agents believe that the switching of central bankers implies a positive probability of regime change. The regime-switching part borrows heavily from Ferman (2011). The extension of the baseline model incorporates habit formation.

### 2.1 Households

The economy is populated by a continuum of representative, infinitely long-living households that sum up to one. The representative households seek to maximize the following intertemporal sum of utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma_1}}{1-\sigma_1} - \omega \frac{N_t^{1+\sigma_2}}{1+\sigma_2} \right\} \quad (1)$$

where  $\beta \in (0, 1)$  is the subjective discount factor of a future stream of utilities.  $C$  is the aggregate consumption.

The representative households face the following budget constraint

$$P_t C_t + E_t Q_{t,t+1} B_{t+1} \leq B_t + D + W_t N_t + T_t \quad (2)$$

where  $Q_{t,t+1}$  is the one-period-ahead stochastic discount factor at time  $t$ . Agents have access to a complete array of state-contingent claims; thus,  $B_{t+1}$  can be understood as a single financial asset that pays a risk-free rate of return (one year risk-free bond).  $D$  is the share of the aggregate profits. Firms are assumed to be owned by households; therefore, profits serve as a resource for households.  $T_t$  are lump-sum government transfers. All variables are expressed in units of domestic currency.

#### 2.1.1 Preferences

The small open economy (SOE) representation induces the independence of the rest of the world from the domestic policy, and therefore, we can abstract from the strategic interaction between SOE and rest of the world (ROW).

Consumption  $C$  is represented by a Dixit-Stiglitz aggregator of home and foreign consumption.

$$C_t = \left[ \gamma^{\frac{1}{\rho}} (C_{H,t})^{\frac{\rho-1}{\rho}} + (1-\gamma)^{\frac{1}{\rho}} (C_{F,t})^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \quad (3)$$

where  $\rho > 0$  is the elasticity of substitution between home and foreign goods and  $C_H$  and  $C_F$  refer to the aggregate of home-produced and foreign-produced final goods. The parameter  $\gamma$  represents home consumers' preference of domestic and foreign goods. As in De Paoli (2009), the preference parameter is a function of the relative size of the foreign economy,  $1-n$ , and of the degree of openness,  $\lambda$ ; more specifically  $(1-\gamma) = (1-n)\lambda$ .

$$C_{H,t} = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\phi}} \int_0^n C_{H,t}(j)^{\frac{\phi-1}{\phi}} dj \right]^{\frac{\phi}{\phi-1}}, \quad C_{F,t} = \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\phi}} \int_n^1 C_{F,t}(l)^{\frac{\phi-1}{\phi}} dl \right]^{\frac{\phi}{\phi-1}} \quad (4)$$

where  $\phi$  is the elasticity of substitution between particular goods.

$P_t$  is the overall price index of the final good,  $P_{H,t}$  depicts the price index of home goods, and  $P_{F,t}$  denotes the foreign goods denominated in home currency.

$$P_t = \left\{ \gamma [P_{H,t}]^{1-\rho} + (1-\gamma) [P_{F,t}]^{1-\rho} \right\}^{\frac{1}{1-\rho}} \quad (5)$$

$$P_{H,t} = \left[ \left( \frac{1}{n} \right) \int_0^n [P_{H,t}(j)]^{1-\phi} dj \right]^{\frac{1}{1-\phi}}, \quad P_{F,t} = \left[ \left( \frac{1}{1-n} \right) \int_n^1 [P_{F,t}(l)]^{1-\phi} dl \right]^{\frac{1}{1-\phi}} \quad (6)$$

Next, a firm must solve the optimal composition of the basket of home and foreign goods. The cost minimization implies that

$$C_{H,t}(j) = \frac{1}{n} \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\phi} C_{H,t}, \quad C_{F,t}(l) = \frac{1}{1-n} \left( \frac{P_{F,t}(l)}{P_{F,t}} \right)^{-\phi} C_{F,t} \quad (7)$$

After choosing the optimal intrabasket demand, firms choose inputs to maximize their profit.

$$C_{H,t} = \gamma \left( \frac{P_{H,t}}{P_t} \right)^{-\rho} C_t, \quad C_{F,t} = (1-\gamma) \left( \frac{P_{F,t}}{P_t} \right)^{-\rho} C_t \quad (8)$$

The variables representing the (ROW) relative to the Czech Republic are denoted with an asterisk. The foreign economy has to solve the same problem as the SOE; therefore, the optimal behavior is analogical. However, the size of SOE relative to ROW approaches zero; thus, similarly to De Paoli (2009),  $\gamma^* = n\lambda$ , as  $n \rightarrow 0$  the rest of the world's version of the equation 5 implies that  $P_t^* = P_{F,t}^*$  and  $\pi_t^* = \pi_{F,t}^*$



### 2.1.2 Total demand for a generic good $j$ and $l$

Using consumer demands and market clearing conditions for goods  $j$  and  $l$ , we can derive the total demand for a generic good  $j$  produced in SOE and the demand for a good  $l$  produced in ROW. The real exchange rate is defined as  $RS_t = \frac{\varepsilon_t P_t^*}{P_t}$ .

$$Y_t(j) = nC_{H,t}(j) + (1-n)C_{H,t}^*(j) \quad (9)$$

$$Y_t(l) = nC_{F,t}(l) + (1-n)C_{F,t}^*(l) \quad (10)$$

Using the demand for  $C_{H,t}$  and  $C_{H,t}^*$  and some algebraic operations, then applying the definition of  $\gamma$  and  $\gamma^*$  and taking the limit for  $n \rightarrow 0$  as in De Paoli (2009), we can see in equations 11 and 12 that external changes in demand affect the small open economy, but not *vice versa*. In addition, fluctuations in the exchange rate do not influence the ROW's demand. Thus, the demand of the rest of the world is exogenous for the small open economy.

$$Y_t(j) = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\phi} \left\{ \left( \frac{P_{H,t}}{P_t} \right)^{-\rho} \left[ (1-\lambda)C_t + \lambda \left( \frac{1}{RS} \right)^{-\rho} C_t^* \right] \right\} \quad (11)$$

$$Y_t(l) = \left( \frac{P_{F,t}(l)}{P_{F,t}^*} \right)^{-\phi} \left\{ \left( \frac{P_{F,t}^*}{P_t^*} \right)^{-\rho} C_t^* \right\} \quad (12)$$

## 2.2 Pass-through and Deviations from PPP

We assume that there are no trade barriers and no market segmentation, and thus, the law of one price holds in equilibrium. Formally,

$$P_{F,t}(l) = \varepsilon_t P_{F,t}^*(l) \quad P_{H,t}(j) = \varepsilon_t P_{H,t}^*(j) \quad (13)$$

$$P_{F,t} = \varepsilon_t P_{F,t}^* \quad P_{H,t} = \varepsilon_t P_{H,t}^* \quad (14)$$

where  $\varepsilon_t$  is the nominal exchange rate. However, we also allow for the deviations from purchasing power parity  $P_t \neq \varepsilon_t P_t^*$ .

To track the sources of deviation from the aggregate PPP in this framework, it is useful to rewrite the real exchange rate<sup>1</sup>

$$\begin{aligned} RS_t &= \frac{\varepsilon_t P_t^*}{P_t} \\ &= \frac{\varepsilon_t P_t^* S_t}{g(S_t) P_{F,t}} \\ &= \Upsilon_{F,t} \frac{S_t}{g(S_t)} \end{aligned} \quad (15)$$

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<sup>1</sup>This can also be found in Monacelli (2005) for the log-linearized system.

where  $g(S_t)$  is defined in equation 21; because  $P_{F,t} = \varepsilon_t P_t^*$ , we know that  $\Upsilon_{F,t} = 1$  for all  $t$ . Thus, the distortion of PPP comes from the heterogeneity of consumption baskets between the small open economy and the rest of the world.

### 2.3 Goods sector

Goods are imperfect substitutes, and a continuum of firms that hire labor operate in the market. A firm has control over its price; nevertheless, it has to face a quadratic adjustment cost when changing the price.

The production function is given by

$$Y_t(j) = A_t N_t(j) \quad (16)$$

The total costs of the firm are:

$$TC = W_t N_t \quad (17)$$

Using the production function, we can write

$$TC = \frac{W_t Y_t}{A_t} \quad (18)$$

$\frac{\partial TC}{\partial Y_t(j)} :$

$$MC_t = \frac{W_t}{A_t} \quad (19)$$

All firms face the same marginal costs; therefore,  $MC_t = MC_t(j)$

#### 2.3.1 Phillips Curve

Solving the profit maximization problem gives the Phillip Curve.

$$\begin{aligned} P_{H,t} = \frac{\phi}{(\phi-1)} & \left( (1-\tau_p)MC_t + P_{H,t} \frac{\varphi_p}{2} \left[ \frac{\pi_{H,t}}{\bar{\pi}} - 1 \right]^2 \right) + P_{H,t} \frac{\varphi_p}{(\phi-1)} \left[ 1 - \frac{\pi_{H,t}}{\bar{\pi}} \right] \frac{\pi_{H,t}}{\bar{\pi}} \\ & + P_{H,t} \frac{\varphi_p}{(\phi-1)} E_t \left[ \frac{R_t}{R_{t+1}} \left[ \frac{\pi_{H,t+1}}{\bar{\pi}} - 1 \right] \frac{\pi_{H,t+1}^2}{\bar{\pi}} \right] \frac{Y_{t+1}}{Y_t} \end{aligned}$$

$\frac{\phi}{(\phi-1)}$  in *Phillips Curve* embodies the constant price markup that comes from the monopolistic competition on the market. The firm can choose a price that is higher than the marginal cost. As  $\phi \rightarrow \infty$  and  $\varphi_p = 0$ , we are approaching the competitive output market, where  $P_{H,t} = MC_t$ . The steady state inflation is given by  $\bar{\pi}$ . In the presence of the Rotemberg quadratic adjustment cost, Rotemberg (1982), price setting deviates from the monopolistic competition without price stickiness. Marginal cost is augmented with price adjustment costs on the unit of output. The second term in the previous equation depicts the fact that firms are unwilling to make significant price changes because it is

costly; for example, firms are concerned about their reputations. The second term represents the marginal adjustment cost of the unit of output (note that the term is actually negative). The last term represents the forward-looking part of price setting. If the firm expects large price changes in the future, it will tend to change the prices more today. Thus, a firm operating in monopolistic competition will set a higher price to be hedged against future price changes.

## 2.4 Terms of trade

$$S_t = \frac{P_{F,t}}{P_{H,t}} \quad (20)$$

We can rewrite the price indexes using the definition for the terms of trade in the equation 20

$$\frac{P_t}{P_{H,t}} = \{\gamma + (1 - \gamma)[S_t]^{1-\rho}\}^{\frac{1}{1-\rho}} \equiv g(S_t) \quad (21)$$

$$\frac{P_t}{P_{F,t}} = \{\gamma[S_t]^{\rho-1} + (1 - \gamma)\}^{\frac{1}{1-\rho}} \equiv \frac{g(S_t)}{S_t} \quad (22)$$

$$\begin{aligned} \frac{\frac{P_t}{P_{H,t}}}{\frac{P_{t-1}}{P_{H,t-1}}} &= \frac{\{\gamma + (1 - \gamma)[S_t]^{1-\rho}\}^{\frac{1}{1-\rho}}}{\{\gamma + (1 - \gamma)[S_{t-1}]^{1-\rho}\}^{\frac{1}{1-\rho}}} \\ \pi_t^{1-\rho} &= \frac{\{\gamma + (1 - \gamma)[S_t]^{1-\rho}\}^{\frac{1}{1-\rho}}}{\{\gamma + (1 - \gamma)[S_{t-1}]^{1-\rho}\}^{\frac{1}{1-\rho}}} \pi_{H,t-1} \end{aligned} \quad (23)$$

## 2.5 Financial Markets

It has been shown for example in Cochrane (2001), De Paoli (2009) or Uribe (2009) that in complete markets, the contingent claim-price ratio is the same for all investors. Thus, in domestically and internationally complete markets with perfect capital mobility, the expected nominal return from the complete portfolio of state-contingent claims (a risk-free bond paying one in every state of the world) is equal to the expected domestic-currency return from foreign bonds  $E_t Q_{t,t+1} = E_t(Q_{t,t+1}^* \frac{\varepsilon_{t+1}}{\varepsilon_t})$ .

To determine the relationship between the real exchange rate and marginal utilities of consumption, we use the first-order condition with respect to bond holdings for the ROW.  $\mu$  is the marginal rate of consumption substitution.

$$\beta \left( \frac{\mu_{t+1}^*}{\mu_t^*} \right) \left( \frac{P_t^*}{P_{t+1}^*} \right) \left( \frac{\varepsilon_t}{\varepsilon_{t+1}} \right) = Q_{t,t+1} \quad (24)$$

Next, we use the first-order condition with respect to bond holdings for SOE together with the definition of the real exchange rate  $RS_t \equiv \frac{\varepsilon_t P_t^*}{P_t}$ ; it follows that

$$\left(\frac{C_t^*}{C_{t+1}^*}\right)^{\sigma_1} \left(\frac{P_t^*}{P_{t+1}^*}\right) \left(\frac{\varepsilon_t}{\varepsilon_{t+1}}\right) = \left(\frac{C_t}{C_{t+1}}\right)_1^\sigma \left(\frac{P_t}{P_{t+1}}\right) \quad (25)$$

This expression holds at all dates and under all contingencies. The assumption of complete financial markets implies that arbitrage will force the marginal utility of the consumption of the residents from the ROW economy to be proportional to the marginal utility of domestic residents multiplied by the real exchange rate.

$$C_t = \vartheta C_t^* R S_t^{\frac{1}{\sigma_1}} \quad (26)$$

$\vartheta$  is a constant consisting of the initial conditions. Since countries are perfectly symmetric, one can assume that at time zero, they start from the same initial conditions.

### 2.5.1 Uncovered Interest Rate Parity

The equilibrium price of the risk-less bond denominated in a foreign currency is given as in Galí and Monacelli (2005) by  $\varepsilon_t(R_t^*)^{-1} = E_t\{Q_{t,t+1}\varepsilon_{t+1}\}$ . Combining this with the domestic pricing equation  $R_t^{-1} = E_t\{Q_{t,t+1}\}$ , one can obtain a version of the uncovered interest parity condition:

$$E_t\{Q_{t,t+1}[R_t - R_t^*(\varepsilon_{t+1}/\varepsilon_t)]\} = 0 \quad (27)$$

Furthermore, because all prices are expressed in terms of trade, we need to substitute for the nominal exchange rate in the equation 27. Using the law of one price and the equation 20, the uncovered interest rate parity (UIP) takes the following form:

$$R_t = R_t^* \triangle S_t \frac{\pi_{t+1,H}}{\pi_{t+1}^*} \quad (28)$$

## 2.6 General Equilibrium

The equilibrium requires that all markets clear and all households and all firms behave identically. In particular, the equilibrium is characterized by the following system of stochastic differential equations.

### 2.6.1 Goods market equilibrium

The goods market clearing condition from the equation 9 and aggregate demand for a generic good  $j$  give aggregate demand<sup>2</sup>

$$Y_t = \left(\frac{1}{g(S_t)}\right)^{-\rho} \left[ (1-\lambda)C_t + \lambda \left(\frac{1}{RS}\right)^{-\rho} C_t^* \right] \quad (29)$$

Using international risk sharing (see the equation 26), we can write

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<sup>2</sup>Insert the equation 11 into the equation  $Y_t = \left[\left(\frac{1}{n}\right)^{\frac{1}{\phi}} \int_0^n Y_t(j)^{\frac{\phi-1}{\phi}} dj\right]^{\frac{\phi}{\phi-1}}$ .

$$Y_t = g(S_t)^\rho C_t^* \left[ (1 - \lambda) + \lambda R S_t^{\rho - \frac{1}{\sigma_1}} \right] \quad (30)$$

Next, if we use a Euler equation, we would be able to derive a dynamic IS equation. This is analytically tractable, however, only in a log-linearized form.

Aggregating the equation 12 over  $l$ , we can see that the SOE can treat  $C_t^*$  as exogenous.

$$Y_t^* = C_t^* \quad (31)$$

### 2.6.2 Aggregate Demand and Supply

In equilibrium, aggregate supply must be equal to the consumption and resources spent on adjusting prices.

$$g(S_t)Y_t = g(S_t)C_t + \frac{\varphi_p}{2} \left[ \frac{\pi_{H,t}}{\bar{\pi}} - 1 \right]^2 Y_t \quad (32)$$

Production function:

$$Y_t = A_t N_t \quad (33)$$

### 2.6.3 Labor market equilibrium

The real wage is defined  $\frac{W_t}{P_t} = w_t$ .

$$\omega N_t^{\sigma_2} = C_t^{-\sigma_1} w_t \quad (34)$$

$\omega$  is the scaling parameter equal to  $\bar{C}^{-\sigma_1}$ .

### 2.6.4 Monetary Policy

The central bank sets the short-term nominal interest rate based on the following rule:

$$r_t = \bar{r} + \Phi_{\pi(s_t)} \hat{\pi}_t + \Phi_{y(s_t)} \hat{y}_t \quad (35)$$

where  $\hat{\pi}_t$  and  $\hat{y}_t$  are log approximations of deviations from a deterministic steady state. This formulation of Taylor rule follows Ferman (2011) and introduces regime dependence into the model. The policy reaction coefficients  $\Phi_{\pi(s_t)}$  and  $\Phi_{y(s_t)}$  are given by the realization of regime  $s_t \in 1, 2$ . This means that for instance, in the situation when  $\Phi_{\pi(1)} > \Phi_{\pi(2)}$  and the world is in the state one, monetary authority is more active in fighting the deviation of inflation from its steady state relative to the second state.

### 2.6.5 Phillips Curve

First, we derive the relationship between domestic PPI and CPI inflation.

$$\pi_t = \frac{g(S_t)}{g(S_{t-1})} \pi_{H,t} \quad (36)$$

We thus derive

$$\begin{aligned} & \left( 1 - \varphi_p \left[ \frac{\pi_{H,t}}{\bar{\pi}} - 1 \right] \frac{\pi_{H,t}}{\bar{\pi}} - \phi + (1 - \tau_p) \phi m c_t + \phi \frac{\varphi_p}{2} \left[ \frac{\pi_{H,t}}{\bar{\pi}} - 1 \right]^2 \right) (Y_t) \\ & + E_t \left\{ \frac{R_t}{R_{t+1}} \varphi_p \left[ \frac{\pi_{H,t+1}}{\bar{\pi}} - 1 \right] \frac{\pi_{H,t+1}^2}{\bar{\pi}} \right\} (Y_{t+1}) = 0 \end{aligned} \quad (37)$$

We can rewrite marginal costs as follows:  $\bar{m}c_t \frac{P_t}{P_{H,t}} = m c_t g(S_t) = \frac{MC_t}{P_{H,t}}$ . Further, from cost minimization, we know that  $MC_t = W_t$

$$\begin{aligned} \frac{MC_t}{P_H} &= \frac{W_t}{P_t} \frac{P_t}{P_H} \\ m c_t &= w \times g(S_t) \end{aligned} \quad (38)$$

Marginal cost can be decomposed to

$$\begin{aligned} m c_t &= \omega N_t^{\sigma_2} C^{\sigma_1} g(S_t) \\ &= \omega Y_t^{\sigma_2} (Y^*)^{\sigma_1} S_t \end{aligned} \quad (39)$$

This is a convincing way to show that marginal costs are growing with positive foreign output shocks, increases in home output and a worsening of the terms of trade.

### 2.6.6 Euler Equation

$$1 = \beta E_t R_t \left( \frac{C_t}{C_{t+1}} \right)^{\sigma_1} \frac{P_t}{P_{t+1}} \quad (40)$$

A stationary rational expectation equilibrium is a set of stationary stochastic processes  $\{S_t, C_t, Y_t, N_t, \pi_t, \pi_{H,t}, R_t, w_t, S_t, g(S_t)\}_0^\infty$  and exogenous processes  $\{Y_t^*, A_t\}_0^\infty$ .

## 3 Calibration

The model is calibrated using data for the Czech Republic obtained from the Czech Statistical Office, the Czech National Bank and World Bank. Furthermore, we follow Ravenna and Natalucci (2008), Vasicek and Musil (2006) and (Colantoni, 2010) in choosing the parameter values.

### 3.1 Preferences

The quarterly discount factor  $\beta$  is fixed at 0.99, which means that households have a high degree of patience with respect to their future consumption, and it implies a real interest rate of 4 percent in the steady state. The elasticity of intertemporal substitution is calibrated at values somewhat higher than is implied by the microeconomic evidence. The utility consumption curvature is

set to  $\sigma_1 = 9.5$ , which means that the elasticity of intertemporal substitution is 0.105. The intertemporal elasticity of substitution can be interpreted as a willingness of households to agree with deviation from their current consumption path. In other words, with higher elasticity, households smooth consumption more over time, and they are willing to give up a larger amount of consumption today to consume a little more in the future. The elasticity of the labor supply is chosen to be 2 in the baseline calibration, implying  $\sigma_2 = 0.5$ . The increase of the real wage by 1% brings a 2 percentage point increase in the labor supply, which indicates that the labor supply is elastic.

### 3.2 Technology

The degree of monopolistic competition,  $\phi = 11$ , brings a markup of 10%. To set the elasticity between imported goods and domestic goods, we follow Ravenna and Natalucci (2008) and set the value to 0.5. The exact rate is difficult to compute, but in general, the elasticity has increased in the Czech Republic recently with the development of the economy. The degree of openness,  $\lambda$ , is assumed to be 0.75, implying a 75% import share of the GDP and determining the parameter  $\gamma$  (share of domestic goods in the consumption basket) to be 0.25. The degree of openness is calibrated based on the time series of imports to GDP share data for the Czech Republic. We set the price adjustment costs to the standard value,  $\varphi_p = 50$ , as in Bergin et al. (2007), implying that 95 percent of the price has adjusted four periods after a shock.

### 3.3 Shocks

There are only two shocks in the benchmark model: one from the world economy and the other one from productivity. Both shocks are characterized by a degree of persistence. The foreign output inertia is set close to Vasicek and Musil (2006)'s estimate of  $\rho_y = 0.8$ . The standard deviation of the foreign output shock is calibrated at 2.5% in order to generate enough uncertainty in the model to match the size of the term premium. The variance of innovations for technology is set at  $\sigma_a^2 = 0.005^2$ .

### 3.4 Monetary Policy

The behavior of the central bank is characterized by the Taylor rule. A weight connected to inflation is set in such a way that the ratio between inflation and output is approximately 20. In a more passive monetary policy regime, the central bank prefers to stabilize inflation three times more than output. In particular,  $\Phi_{\pi(s_t=1)} = 13.5$  and  $\Phi_{\pi(s_t=1)} = 1.5$ , which is the lowest value for which the model equilibrium does not explode. The weight on output stabilization is  $\Phi_{\pi(s_t=s)} = 0.5$ .

### 3.5 Transition matrix

To calibrate the transition probabilities, we borrow from Ferman (2011), who estimated  $p_{11} = 0.993$  and  $p_{11} = 0.967$ , because the Czech time series are relatively short and the estimation of the transition matrix can be surrounded by some margins of uncertainty. In addition, for the sake of the comparison of the effects of the foreign demand channel, the values suggested by Ferman (2011) are sufficient.

## 4 Solution method

To solve the model, we rely on the perturbation methods applied to the second-order approximation of the nonlinear relationships that link all endogenous variables to the predetermined variables. The point around which the approximation is computed is the non-stochastic steady state. The second-order approximation is necessary since the first-order approximation of the model eliminates the term “premium” entirely and the covariance term from equation 48 is zero. This property is known as certainty equivalence in linearized models<sup>3</sup> when agents in equilibrium behave as if they were risk-neutral. The model is a highly nonlinear system of equation without a closed-form solution; therefore, it has to be solved numerically. We use Matlab, particularly the Dynare package, to find the model solution, see Julliard (2010). The Dynare solution algorithm is based on the perturbation techniques suggested by Schmitt-Grohe and Uribe (2004). Nevertheless, the presented model contains a discrete state variable, and the Schmitt-Grohe and Uribe (2004) method is therefore not directly applicable. We follow Ferman (2011) and define an extended system of equations in which the dependence of the control variables on regimes is made explicit.

Similar to Ferman (2011), we write the Markov chain in a vector representation,  $\zeta_{t+1} = \mathbf{P}\zeta_t + \nu_t$ , where  $\mathbf{P}$  is the transition matrix,  $\nu_t$  is the vector of innovations characterized by a zero mean and heteroskedastic variance, and  $\zeta_t$  is the set of realizations. Furthermore, the continuously differentiable exogenous variables are represented by  $n_x \times 1$  vector  $\mathbf{x}_t$ , and  $\mathbf{y}_t$  represents the  $n_y \times 1$  vector of state variables.

$$E_t[f(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{x}_t, \zeta_t)] = 0$$

$$\mathbf{x}_{t+1} = (I - \mathbf{\Lambda})\bar{\mathbf{x}} + \mathbf{\Lambda}\mathbf{x}_t + \Sigma\sigma\epsilon_{t+1}$$

$$\zeta_{t+1} = \mathbf{P}\zeta_t + \nu_t \tag{41}$$

where  $\sigma$  is the perturbation parameter, as in Schmitt-Grohe and Uribe (2004),  $\mathbf{\Lambda} = \text{diag}(\rho_y, \rho_A)$  are the coefficients of persistence,  $\bar{\mathbf{x}} = (\bar{A}, \bar{Y}^*)'$  are the steady-state exogenous variables and  $\nu_t = (\nu_t^A, \nu_t^{Y^*})'$  is the vector of innovations.

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<sup>3</sup>Alternatively, one can use the log-linear/log-normal approach, see Emiris (2006).



As mentioned above, to solve the rational expectation model with regime switching, Schmitt-Grohe and Uribe (2004)'s method cannot be used. The solution to a first-order approximation has been proposed by, e.g., Davig and Leeper (2007). The certainty equivalence feature of the term structure, however, requires that the model is solved at least to the second order. Ferman (2011) suggests to reformulate the problem and make the dependence of control variables on the state of the world explicit. The variables for expectations conditional on  $\Omega_t$  are parametrized in the following way:

$$E[y_{t+1}|\Omega_t^{-s}, s_t = s] = p_{s1}E[y_{t+1(1)}|\Omega_t^{-s}] + p_{s2}E[y_{t+1(2)}|\Omega_t^{-s}] \quad (42)$$

Inserting the equation 42 into the equation 41, one can re-write the system of equations as:

$$F(\mathbf{y}_{t+1(1)}, \mathbf{y}_{t+1(2)}, \mathbf{y}_{t(1)}, \mathbf{y}_{t(2)}, \mathbf{x}_t) = E_t \begin{bmatrix} f_1(\mathbf{y}_{t+1(1)}, \mathbf{y}_{t+1(2)}, \mathbf{y}_{t(1)}, \mathbf{y}_{t(2)}, \mathbf{x}_t) | \Omega_t^{-s} \\ f_2(\mathbf{y}_{t+1(1)}, \mathbf{y}_{t+1(2)}, \mathbf{y}_{t(1)}, \mathbf{y}_{t(2)}, \mathbf{x}_t) | \Omega_t^{-s} \end{bmatrix} = 0$$

$$\mathbf{x}_{t+1} = (I - \Lambda)\bar{\mathbf{x}} + \Lambda\mathbf{x}_t + \Sigma\sigma\epsilon_{t+1}$$

$$\zeta_{t+1} = \mathbf{P}\zeta_t + \nu_t \quad (43)$$

The function  $F(\bullet)$  contains variables from each possible realization of  $\zeta_t$ . Intuitively, the above-stated model equilibrium indicates that the expectations of the variable are the weighted averages of the values in each possible realization. The weights are the probabilities of switching from the transition matrix. Being in each regime implies a chance of ending up in a different regime in the future; thus, agents have to consider different states of the world when forming their beliefs. For this reason, the solution for each regime fundamentally depends on the behavior of the economy in the alternative regime. Agents in the model, in their decision process, consider that the central bank's policy may switch. This fact leads to a different equilibrium compared with a model with a single policy regime. One can see this possibility in the equation 43, where the functions  $f_1$  and  $f_2$  depend on variables belonging to both regimes. Comparing the equation 43 to the equation 41, one can see that the equation 43 has only differentiable predetermined variables. Hence, we can employ the perturbation techniques suggested by Schmitt-Grohe and Uribe (2004). The second-order approximate solution to the vector of control variables conditional on the state of the world  $s_t$  around the zero deterministic steady state is given by

$$y_{t(s)} \cong \bar{g} + \bar{g}_x^s(\mathbf{x}_t - \bar{\mathbf{x}}) + \frac{1}{2}E_t \begin{bmatrix} (\mathbf{x}_t - \bar{\mathbf{x}})' \bar{g}_{xx[1]}^s (\mathbf{x}_t - \bar{\mathbf{x}}) \\ \dots \\ (\mathbf{x}_t - \bar{\mathbf{x}})' \bar{g}_{xx[n_y]}^s (\mathbf{x}_t - \bar{\mathbf{x}}) \end{bmatrix} + \frac{1}{2} \bar{g}_{\sigma\sigma}^s \sigma^2$$

where as in Schmitt-Grohe and Uribe (2004),  $\bar{g}_x^s$  is the  $n_y \times n_y$  matrix of the first derivatives of  $g^s(\bullet)$  with respect to the state variables evaluated at the

deterministic steady state.  $\bar{g}_{xx[k]}^s$  are the derivatives to the second order. The vector  $n_y \times 1$  of second-order derivatives with respect to scalar  $\sigma$  is represented by  $\bar{g}_{\sigma\sigma}^s$ .  $\sigma$  stands for uncertainty in the model and is responsible for a non-zero term premium. Up to the second order,  $\sigma$  is constant; therefore, conditional on the monetary policy regime, the implied term premium is also constant. To obtain the volatile term premium, the model must be approximated at least to the third order or, as suggested by Ferman (2011), exposed to the possibility of a regime switch.

It is convenient to calculate the model in percentage deviations from the steady state. Hence, we approximate the solutions of the natural logs of the variables rather than variables in levels. It is easy to show that when we rewrite the first-order conditions by using exponentials of the logs of variables, linearizing the model will deliver the percentage deviations of the variables from the steady state.

The model solution is represented by policy functions of the form:

$$\begin{bmatrix} \mathbf{y}_{t(1)} \\ \mathbf{y}_{t(2)} \end{bmatrix} = \begin{bmatrix} g^1(\mathbf{x}_t, \sigma) \\ g^2(\mathbf{x}_t, \sigma) \end{bmatrix} \quad (44)$$

$$\mathbf{x}_{t+1} = (I - \Lambda)\bar{\mathbf{x}} + \Lambda\mathbf{x}_t + \Sigma\sigma\epsilon_{t+1}$$

$$\zeta_{t+1} = \mathbf{P}\zeta_t + \nu_t \quad (45)$$

Once we compute an approximate solution to the model, we compare the model and the data using macro and finance-simulated moments and data for the Czech Republic.

## 5 The finance part

In this section, we borrow from Hordahl et al. (2007) to briefly summarize some basic stylized facts on the term structure of interest rates for a closed economy (the case of the U.S.), and we show the fundamental characteristics of the data in a small open economy represented by the Czech Republic. We discuss the properties of Czech term structure. In addition, we provide data and qualitative reasoning that support the integration of Markov switching in the model. In the second part, we derive the yield curve implied by the DSGE model outlined in the 2 section.

### 5.1 Finance-related data

Table 1 summarizes some of the most well-known stylized facts for the U.S. First, the yield curve is, on average, upward-sloping. The mean of the 10-year, zero-coupon annualized quarterly rate exceeds the mean of the one-quarter annualized yield by 116 basis points over the period 1961Q2 - 2007Q2. On the other hand, volatilities have tended over the analyzed period to be slightly

| Maturity | 3m   | 6m   | 1Y   | 3Y   | 5Y   | 10Y  |
|----------|------|------|------|------|------|------|
| mean     | 5.88 | 5.96 | 6.12 | 6.52 | 6.72 | 7.04 |
| Std.Dev. | 2.88 | 2.84 | 2.8  | 2.64 | 2.56 | 2.4  |

Table 1: **Summary statistics for U.S. Yield Curve.** Quarterly U.S. data - 1961Q2 - 2007Q2, annualized in percentages. Source: Hordahl et al. (2007)

downward-sloping. The volatility of a 10-year zero-coupon bond reached 2.4%, which is only 0.4 percentage points higher than corresponding volatility for one-year zero-coupon rate.

The moments describing the Czech yield curve in Table 2 are calculated from the dataset of zero-coupon swap rates provided by Bloomberg. The figures in table 2 suggest that the characteristics of the Czech yield curve are largely similar to those of the U.S. yield curve. The curve is on average upward-sloping, and its volatility is decreasing. Compared to the U.S. yield curve, the Czech term structure of interest rates is parallelly shifted downwards. However, the yield curve is steeper in the case of the Czech Republic. The average 10-year slope of the U.S. term structure is approximately 1.16 percentage points, whereas in the case of the Czech Republic, the corresponding slope is approximately 1.7 percentage points. The U.S. term structure of interest rates is about twice as volatile as its Czech counterpart. The volatility in the 10-year maturity range drops at a much lower rate in the case of the Czech Republic, the volatility of the U.S. curve drops of 20%, while the volatility of the Czech yield curve is near constant and decreases only by 3% between the 1- and 10-year yields.

The implications of these key stylized facts for our study come primary from the differences in the empirical evidence from the closed and open economy. The open economy model has to be able to replicate significantly higher slopes than the closed economy model. This is a challenging task, as it is known that DSGE models have difficulties in generating a sufficient size of the slope.

In summary, we conclude that if the theoretical model moments generate a mean of the annualized 10-year yield at 4.5% and 3% for the 3-month rate, implying a slope of 1.7 percentage points and theoretical standard deviations at 1.3 for the 10-year rate and 1.4 for the 3-month rate, the model fits the actual data very well.

The second part of this subsection is focused on evidence of Markov switching within a single monetary policy regime. Among others, Ferman (2011) and Davig and Leeper (2007) emphasize the link between macroeconomic volatility and monetary policy regimes and note the implications of the possibility of policy change in agents' beliefs. Ferman (2011) argues that not using Markov switching in the model necessarily leads to a biased estimate of the term premium. The U.S. nominal yield curve was steeper for the mid-1980s than for the 1970s. However, the macroeconomic uncertainty measured by the volatility of consumption growth as a proxy for the stochastic discount factor was higher at the time of the Great Inflation in the 1970s. Ferman (2011) refers to this contra-

|                 | Zero-coupon bond swap rate |      |      |      |      |      |
|-----------------|----------------------------|------|------|------|------|------|
| <b>Maturity</b> | 3M                         | 1Y   | 3Y   | 5Y   | 10Y  | 20Y  |
| Mean            | 2.94                       | 3.12 | 3.66 | 4.01 | 4.51 | 4.81 |
| Std.Dev.        | 1.39                       | 1.42 | 1.41 | 1.42 | 1.34 | 1.19 |
| Slope           | 0                          | 0.18 | 0.73 | 1.07 | 1.57 | 1.9  |

Table 2: **The Term Structure of Interest Rates for the Czech Republic.** Source: Bloomberg, means and standard deviations are calculated from quarterly data from 1Q 2000 till 4Q 2011 from risk-free zero-coupon bonds

intuitive observation as the 'Slope Volatility Puzzle.' One would expect that at a time of high uncertainty, bond holders will want to be compensated for the risk with a higher term premium. By estimating the MS-VAR, Ferman (2011) demonstrates that the term premium was actually higher in the 1970s than in Great Moderation starting in the mid-1980s. This is so because agents update their beliefs about the way monetary policy is conducted. Assuming a state with high macroeconomic volatility and moderate size of slope, bond holders assign a certain probability to the likelihood of shift in the conduct of monetary policy to a low-inflation environment. A policy regime that fights inflation more actively implies that the price of high-yield bonds from the time of the Great Inflation will rise in times of low inflation. This opportunity for potential revenue in the future implies that agents are willing to hold bonds with longer maturity for a smaller yield. Ferman (2011) uses U.S. data to measure slope and macro-volatility to demonstrate that splitting the dataset into sub-samples based on the FED chairman confirms the argument that monetary policy shifts have an impact on the slope of the term structure of interest rates. We follow Ferman (2011) and use the same analysis for Czech data. Note, however, that the purpose of our argumentation is to qualitatively justify the application of Markov switching in the model. We do not aim to provide a quantitative analysis that estimates the exact effects of regime switching. An estimation of the transition matrix and implied term structure decomposition is left for further research.

Nevertheless, our ambition is to apply our newly developed modelling framework to the data from one of emerging market economies in which monetary policy is likely to be characterized by Markov switching during our sample period. We motivate our empirical exercise by previous research that examines the non-linearity in monetary policy rules in the Czech Republic (see Horvath (2008) and Vasicek (2012)). For example, Horvath (2008) finds that the Czech central bank at the early stage of inflation-targeting regime was far more concerned when the inflation forecast was heading above the inflation target than below the target. This finding is not surprising because the Czech central bank implemented the inflation-targeting measure as a disinflation strategy in 1998. This asymmetry in the monetary policy rule dissipated over time once price stability represented by inflation targets of 2 or 3 percent were achieved. We

|                          | Kysilka<br>12:97-98:7 | Tosovsky<br>98:8 - 00:11 | Tuma I<br>00:12-05:2 | Tuma II<br>05:3-10:06 | Singer<br>10:7-11:6 |
|--------------------------|-----------------------|--------------------------|----------------------|-----------------------|---------------------|
| $y_{10Y} - y_{1Q}$       | -4.63                 | 1.79                     | 1.86                 | -0.38                 | 1.7                 |
| Std. ( $\pi$ )           | 1.23                  | 1.37                     | 1.56                 | 2.1                   | 0.25                |
| Std. ( $\Delta c$ )      | 1.48                  | 1.012                    | 1.009                | 1.234                 | 0.478               |
| corr ( $\Delta c, \pi$ ) | -0.92                 | 0.18                     | 0.078                | 0.46                  | 0.96                |

Table 3: **The Slope - Volatility Puzzle: Different Governors**, the mean slope is calculated from annualized quarterly zero swap rates, for the Kysilka period, daily data for rates are used, and macro variables are defined in the next section.

label these two periods as 1) the disinflation regime and 2) the price stability regime.

For the U.S. data, we can see that more passive monetary policy regimes characterized by higher consumption growth and inflation variability imply a lower slope of the term structure of interest rates. We examine whether this feature is also present in the Czech data. We present two tables; see Table 3 and Table 4. Both tables show the consumption and inflation volatility with the slope of the term structure. The first one splits the sample period according to governance changes. The second one splits the sample according to changes in the inflation target.

Table 3 shows that the first governor Tuma’s term is characterized by lower macroeconomic uncertainty (measured by the *ex-post* standard deviation of consumption growth and inflation) accompanied by a steeper yield curve. The economy in the second period is more volatile, and the yield curve is downward-sloping on average. Looking at the second-order approximation of the term structure of interest rates, the slope is explained not only by the macroeconomic volatility but also by the correlation between the stochastic discount factor and inflation. This relationship is approximated by the last line of Table 3. The stochastic discount factor is an inverse to consumption growth; thus, with a higher positive correlation, the term premium decreases. For this reason, we cannot exclude the possibility that the negative slope in Tuma’s second term is given by the fact that when inflation rises, it is complemented by higher consumption growth. Nevertheless, the purpose of our discussion is to demonstrate that a more active (inflation-fighting) monetary policy regime in Tuma’s first term implies a higher slope of the term structure of interest rates than the more passive monetary policy regime in Tuma’s second term. This observation corresponds to the U.S. data and demonstrates the slope-volatility puzzle in the Czech term structure of interest rates.

In addition, we further support our argument that the aggressiveness of monetary policy has a direct effect on the term structure of interest rates by examining Table 3, which splits the sample period according to the level of the inflation target. The idea behind this analysis is similar to that of the previous

|                          | Core infl<br>3.5 – 5.5%<br>(00-01) | Target range<br>2 – 4%<br>(01 - 05) | Point in range<br>$3 \pm 1\%$<br>(05 - 09) | Point in range<br>$2 \pm 1\%$<br>(09 - 11) |
|--------------------------|------------------------------------|-------------------------------------|--------------------------------------------|--------------------------------------------|
| $y_{10Y} - y_{1Q}$       | 1.81                               | 1.86                                | 1.2                                        | 1.73                                       |
| Std. ( $\pi$ )           | 0.2                                | 1.62                                | 2.09                                       | 0.80                                       |
| Std. ( $\Delta c$ )      | 0.73                               | 0.95                                | 1.26                                       | 1.21                                       |
| corr ( $\Delta c, \pi$ ) | 0.14                               | -0.32                               | 2.31                                       | 0.85                                       |

Table 4: **The Slope - Volatility Puzzle: Different Inflation Targets**; the mean slope is calculated from the annualized quarterly zero swap rates.

case. By changing the inflation target, the weight put on inflation and output stabilization changes; consequently, the slope of the term structure of interest rates spreads or shrinks. We concentrate our argumentation again on the two periods that cover most of the first 10 years of the 21st century. Between the years 2001 and 2005, the Czech central bank set the inflation target to a decreasing range over time. From 2005 to 2009, the central bank turned to targeting the point in the announced range.

Table 4 displays the 10-year slope of the zero-coupon swap rate, the standard deviation of inflation and consumption growth, and in the last row, the contemporaneous correlation between consumption growth and inflation is presented. Because the studied periods largely overlap with the appointments for the governor of the central bank from the previous example, it is not surprising that we can find an analogous pattern in the data. Time intervals at which the target was set as a decreasing range starting at 3-5 % and decreasing to 2-4 % are characterized by lower macroeconomic volatility and a higher slope of the term structure of interest rates compared to the subsequent period. In addition, a negative contemporaneous correlation between inflation and consumption growth further supports the result of higher slope in this period. Between 2005 - 2009, the central bank pursued a policy with the target fixed at a point  $3 \pm 1$  %. We can see from Table 4 that the standard deviation of inflation and consumption growth increased relative to the previous period and that the slope of the yield curve declines.

In summary, the Czech data show some evidence that monetary policy conduct is related to the slope of the term structure of interest rates. In addition to the findings of the previous literature about non-linearity in the monetary policy rule (Horvath (2008) and Vasicek (2012)), this motivates us to model the Czech monetary policy using a Markov chain transition matrix in the monetary policy rule.

## 5.2 Term structure of interest rates

The assumption of complete markets and no arbitrage in the DSGE model implies that we can price all financial assets in the economy. Once we specify a

time-series process for a one-period discount factor  $Q_{t,t+i}$ , we can determine the price of any bond by chaining the discount factors  $P_t^{(i)} = E_t\{Q_{t,t+1}, Q_{t+1,t+2} \dots Q_{t+i-1,t+i}\}$ , see Cochrane (2001). We solve the discount factors forward to get particular maturities. Hence, the price of a zero-coupon bond for a CRRA utility that pays 1 dollar on the maturity date  $i$  is

$$P_t^{(i)} = E_t \left[ \beta^i \left( \frac{C_t}{C_{t+i}} \right)^{\sigma_1} \prod_{j=1}^i \frac{1}{\pi_{t+j}} \right] \quad (46)$$

where the price of a default-free one-period zero-coupon bond that pays one dollar at maturity  $P_t^{(1)} \equiv R_t^{-1}$ ,  $R_t$  is the gross interest rate and  $P_t^{(1)} \equiv 1$  (*i.e.*, the time  $t$  price of one dollar delivered at time  $t$  is one dollar). The equation 46 shows that the price of the bonds is defined by the behavior of consumption and inflation.

One can rewrite the nominal default-free bond with maturity  $i$  as follows:

$$P_t^{(i)} = E_t\{Q_{t,t+1}P_{t+1}^{(i-1)}\} \quad (47)$$

Next, using the definition of covariance:

$$P_t^{(i)} = P_t^{(i-1)} E_t P_{t+i-1}^{(1)} + Cov_t\{Q_{t,t+i-1} P_{t+i-1}^{(1)}\} \quad (48)$$

The equation 48 says that the price of the risk-free bond is equal to the expected price of a one-period bond at time  $t+i-1$  discounted by the discount factor for the period  $i-1$ . However, note that although the bond is default-free, it is still risky in the sense that its price can covary with the households' marginal utility of consumption. In this case, households perceive the nominal zero-coupon bond as being very risky because it loses its value exactly when households value consumption the most. Another way of thinking about the covariance term is through a precautionary savings motive. As we elaborate below, if the bond price and consumption fall at the same time, consumption-smoothing households wish to save some of their consumption for an unfortunate time when the economy is hit by a shock and the price of bonds fall with consumption. However, this is not possible in the equilibrium; thus, price of bonds must increase to distract the demand. We can see that the covariance term is the approximation for the risk premium.<sup>4</sup>

We follow the term structure literature and denote  $ym_t^{(i)} = \log(P_t^{(i)})$ . The logarithm of price has a convenient interpretation. If the price of a one-year zero-coupon bond is 0.98, the log price is  $\ln(0.98) = -0.0202$ , which means that the bonds sells at 2 percent discount. Furthermore, we define the nominal

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<sup>4</sup>A possible extension is to add a preference shock to capture the fact that households perceive risk differently in time, for example, in a recession, the foreign output shock is typically more painful, and households would demand a higher compensation for holding the bond.

interest rates as yields to maturity.

$$P^{(i)} = \frac{1}{[Y^{(i)}]^{(i)}}$$

$$ytm_t^{(i)} = -\frac{1}{i} \log(P_t^{(i)}) \quad (49)$$

The equation 49 is the annualized yield to maturity of the bond with  $i$  quarters to maturity.

In general, as is common in finance theory, the yield to maturity of a default-free bond with maturity  $i$  is given by the conditional expectations of the series of one-period yields between time  $t$  and the bond's maturity plus the nominal term premium, which represents the risks of holding a bond. Formally,  $r_{i,t} = \frac{1}{i} \sum_{j=0}^{i-1} E_t[i_{t+j}] + NTP_{i,t}$ . By substructing the one-period yield to maturity from the previous equation, we can define the  $i$ -period yield curve slope:

$$r_{i,t} - r_t = \left( \frac{1}{i} \sum_{j=0}^{i-1} E_t[i_{t+j}] - i_t \right) + NTP_{i,t} \quad (50)$$

Ferman (2011) calls the term in parentheses the “expectations hypothesis component of the slope.” In the standard models without a Markov-switching component, this term is equal to zero because of the covariance-stationary character of the model, and the slope of the yield curve is determined using solely the nominal term premium.

To better understand the term premium, it is useful to derive the second-order approximation of the yield to maturity around the log-steady state.

$$\widehat{ytm}_t^{(i)} = \frac{1}{i} \left\{ \begin{array}{l} \sigma_1 E_t[\Delta^{(i)} \hat{\mathbf{c}}_{t+i}] + \sum_{n=1}^i E_t[\hat{\pi}_{t+n}] - \frac{1}{2} \sigma_1^2 Var_t [\Delta^{(i)} \hat{\mathbf{c}}_{t+i}] \\ - \frac{1}{2} Var_t [\Delta^{(i)} \hat{\pi}_{t+i}] - \sigma_1 Cov_t \left[ \sum_{n=1}^i \hat{\pi}_{t+n}, \Delta^{(i)} \hat{\mathbf{c}}_{t+i} \right] \end{array} \right\} \quad (51)$$

Equation 51 illustrates that risk-averse agents make precautionary savings if there is uncertainty about future consumption. The higher propensity to save decreases the yield to maturity. The first term says that the high level of expected consumption growth drives the yield to maturity up through the income effect. Consumption-smoothing households want to consume a part of their future consumption in the current period. The increased demand for borrowing projects into higher yields. The next term in equation 51 means that a high level of expected inflation pushes the yield to maturity up. This is because households care about real variables and demand compensation for the lower purchasing power of savings. The next two terms reflect the uncertainty that is faced by households. A higher variance of consumption growth encourages agents to make precautionary savings for bad days. Because all households want to hedge their exposure to low consumption growth, they want to buy bonds. This drives bond prices up and yields down accordingly. Inflation uncertainty lowers the yield to maturity because an unexpected price increase implies lower



consumption; thus, utility-maximizing households buy bonds to protect themselves against the drop in consumption. Buying more bonds is, however, not feasible in equilibrium; therefore, the bond price must go up to distract additional demand. The last term of equation 51 shows that in the economy with high inflation and low consumption growth, the households require higher compensation for holding the bond because bonds lose their value exactly when households need resources the most.

Furthermore, we present the second-order approximation of the slope of the term structure of interest rates around the log-steady state. This exercise provides insight on the factors that determine the term premium, helps to identify the parameters that are important for the calibration of the model and provides the intuition to determine the main factors that affect the term premium.

$$\begin{aligned}
E[\widehat{ytm}_t^{(r)}] - \hat{\mathbf{i}}_t &= -\frac{1}{2}\sigma_1^2 \left( \frac{E[Var_t(\Delta^{(i)}\hat{\mathbf{c}}_{t+i})]}{i} - E[Var_t(\Delta\hat{\mathbf{c}}_{t+1})] \right) \\
&- \frac{1}{2} \left( \frac{E[Var_t(\sum_{n=1}^i \hat{\pi}_{t+n})]}{i} - E[Var_t(\hat{\pi}_{t+1})] \right) \\
&- \sigma_1 \frac{E[Cov_t(\sum_{n=1}^i \hat{\pi}_{t+n}, \Delta^{(i)}\hat{\mathbf{c}}_{t+i})]}{i} + \sigma_1 E[Cov_t(\hat{\pi}_{t+1}, \Delta\hat{\mathbf{c}}_{t+1})]
\end{aligned} \tag{52}$$

The first bracket in equation 52 is the real term premium and represents the so-called Backus, Gregory, and Zin (1989) puzzle. In the data, the first-order autocorrelation of consumption growth is positive, so it is obvious that aggregate consumption varies more over a 10-year period than a 3-month period. Intuitively, the uncertainty about consumption growth is larger at more distant periods. When bad times come, they typically last longer than one period, and households tend to buy bonds with longer maturities to hedge against these bad times. Therefore, the variance in the consumption growth over a longer period is higher than the variance in one period of consumption growth. This implies that the term in the first bracket of equation 52 is positive. For this reason, the yield curve should have on average negative slope, which is typically not supported by data. As a result, it appears that the model is not able to generate a positive slope of the yield curve together with a positive serial correlation.

When constructing the model with the term structure of interest rates, one must be able to break the relationship between the serial correlation of consumption growth and the sign of the slope of the yield curve in order to generate a positive slope of the term structure of interest rates. Another option is to make the other components of equation 52 sufficiently large to compensate for the negative effect of the real term premium. However, many authors, such as Piazzesi and Schneider (2007) and Buraschi and Jiltsov (2005), argue that in U.S. data, the real term premia are quantitatively less important and are close to zero. Therefore, it may be enough for the model with the term structure of interest rates to minimize the effect of real term premia provided that the other

terms of equation 52 are positive.

To obtain a positive auto-correlation of consumption growth and a lower variance of consumption growth over a longer period at the same time, the impulse response function of consumption to an exogenous shock must be hump-shaped. A hump-shaped impulse response of consumption implies strong persistence in consumption growth and simultaneously mitigates the precautionary savings effect. A growing impulse response function of consumption followed by a smooth return to zero indicates a positive autocorrelation of consumption growth and means that the effect of shock dies out so that the variance of consumption growth is lower over a longer time span than in a short time period. In particular, habits are known to produce hump-shaped impulse responses of consumption. To sum up the discussion of real term premia, one needs to make the consumption impulse response function that are implied by CRRA preferences hump-shaped in order to generate a positive slope of term structure and a positive auto-correlation of consumption growth.

The next two brackets in equation 52 refer to inflation risk premia. The first one represents the convexity term and is not as quantitatively important. What matters more is the last component. The level term of inflation risk premium says that if the price of bonds is high when the consumption growth is low, bond holders will demand higher yields for holding such bonds. If this holds over a longer period rather than a shorter one, the term structure of interest rates will have a positive slope. The last term in equation 52 is the one-period-ahead inflation co-variability risk and is expected to be as low as the short period. For this reason, covariance over the long period will dominate. Negative covariance will increase the slope of the term structure, and positive covariance will decrease it. Note that the parameter  $\sigma_1$  is the scaling element of the above-described effects.

To account for the probability of Markov switching in agents' beliefs, we follow Ferman (2011). Ferman (2011) shows that by allowing investors to explicitly incorporate the possibility of regime shifts into their beliefs, the expectation hypothesis element of the slope of the yield curve changes substantially. The method applied by Ferman (2011) relies on two assumptions: *i*) there are two possible states  $s_t \in \{1, 2\}$ , and the regime-switching probabilities are constant given by the exogenous  $2 \times 2$  matrix. The matrix elements are  $Pr(s_{t+1} = j | s_t = k) \equiv p_{k,j}$  for  $k, j \in 1, 2$ . The matrix  $\mathbf{P}$  is known to all agents, and agents observe the realization of the world,  $s_t$ , at the beginning of the period. *ii*) The Markov-switching process for the short rate is covariance-stationary in each regime  $E[r_t | S_t] = E[r_t | s_t]$ , where  $S_t \equiv \{s_0, s_1, \dots, s_t\}$  is the history of regimes realized up to period  $t - 1$ .

Ferman (2011) demonstrates the method on the example of bonds with two-period maturity,  $i = 2$ .  $\Omega_t$  represents the complete information set available to investors, and the slope of the term structure is then

$$E[r_{2,t} - r_t | s_t = s] = \frac{E[r_t | s_t = s] + E[E_t[r_{t+1}] | s_t = s]}{2} - E[r_t | s_t = s] + E[NTP_{2,t} | s_t = s] \quad (53)$$

Using the law of iterated expectations and the transition matrix  $\mathbf{P}$  to parametrize the expectations, we can write

$$E[E_t[r_{t+1}]/s_t = s] = p_{s1}E_t[r_{t+1}/s_t = s, s_{t+1} = 1] + p_{s2}E_t[r_{t+1}/s_t = s, s_{t+1} = 2] \quad (54)$$

Combining the last two equations, we can derive

$$E[r_{2,t} - r_t | s_t = s] = \frac{p_{s1}E_t[r_{t+1}|s_{t+1} = 1] + p_{s2}E_t[r_{t+1}|s_{t+1} = 2] - E[r_t | s_t = s]}{2} + E[NT P_{2,t} | s_t = s] \quad (55)$$

In the model where agents assign positive probability to the belief that the monetary policy regime may change, the mean slope conditional on regime  $s$  is not equal to the mean term premium in that particular regime. Ferman (2011) refers to the term in parentheses as the *level premium*. The expectation hypothesis component does not cancel out the expectations and accounts for the risk that the average level of the one-period yield to maturity will change due to a regime change.

Ferman (2011) further shows that the equation 55 can be generalized for maturity  $i > 2$  as follows:

$$E[r_{i,t} - r_t | s_t = 1] = \left( \frac{i-1}{i} - \frac{1}{i} \sum_{l=1}^{i-1} [\mathbf{P}^l]_{11} \right) D_s E[r_t | s_t] + E[NT P_t^i | s_t = 1] \quad (56)$$

$$E[r_{i,t} - r_t | s_t = 2] = \left( \frac{i-1}{i} - \frac{1}{i} \sum_{l=1}^{i-1} [\mathbf{P}^l]_{22} \right) D_s E[r_t | s_t] + E[NT P_t^i | s_t = 2] \quad (57)$$

where  $D_s E[r_t | s_t] \equiv E[r_t | s_t = 2] - E[r_t | s_t = 1]$ .

The level risk increases with maturity because over the longer period, the probability of regime change is higher. With a higher transition probability ( $p_{12}$ ), the level premium increases as the likelihood of switching the regime rises. As the probability of remaining in the same regime approaches one, the level premiums becomes close to zero.

## 6 Comparing the Model with the Data

In this section, we present the results based on a calibrated version of our benchmark model along with the corresponding empirical moments for the quarterly Czech data from 4Q 1998 to 3Q 2011. The goal is to study implications for the open economy; therefore, for the sake of comparison, the results for the closed-economy version of the benchmark model are presented as well.

The second part of this section discusses the effects of monetary policy changes on the slope of the term structure of interest rates in an open-economy

framework. We argue that although the model fits some of the macro data relatively well, to generate the substantial size of the nominal term premium, the uncertainty in the model must be boosted to the extent that it compromises the moments of some macro variables. We show that in an open economy, the level premium differs even qualitatively from the closed-economy framework. This effect is puzzling, and we leave it for further research.

## 6.1 Macro Data

Table 5 compares the predictions from our closed and open economy models with the empirical moments. The moments are calculated from the quarterly data and reported in percentage points. The letter  $C$  denotes Czech households' consumption expenditures in year-2000 prices.  $N$  stands for the total hours worked, as reported by the Czech statistical office. Due to the data availability, the moments for hours worked are calculated from yearly data between 1995 - 2010.  $\Delta C$  is the annualized consumption growth at a quarterly frequency. Note that the time series are seasonally adjusted. The cyclical component in each variable is separated by a Hodrick-Prescott filter with  $\lambda = 1600$ . The standard deviations of the variables are computed as the logarithmic deviations of the cyclical component from the Hodrick-Prescott trend. The mean of the slopes comes from the annualized zero-coupon swap rate at the end of the quarter, as published by Bloomberg. The real interest rate is approximated by the quarterly nominal interest rate minus *ex-post* inflation. Inflation,  $\pi$ , is the annualized rate of the quarter to the quarter change in the consumer price index from the Czech central bank. The means and standard deviations of interest rates and inflation are computed using the original series rather than for filtered deviations from the trend.

Table 5 illustrates that the model fits the Czech data relatively well. The results also provide a rationale to model the economy as open because the volatility of consumption, inflation and hours worked is closer to the predictions of the open economy model rather than those of the closed economy. Closing the model delivers lower standard deviations because the contribution of sizable foreign output shocks is removed. To match the steepness of the slope of the term structure of interest rates, the households are modeled as substantially risk-averse. The low elasticity of substitution means that households are reluctant to postpone their consumption and agree with a higher deviation from their current consumption path. This fact contributes to the slightly higher volatility of consumption growth than we can see in the Czech data.

## 6.2 Finance data

We focus in this sub-section is on matching the features of the term structure described in section 5.

The slope of the term structure of interest rates consists of two components: the level risk and the nominal term premium. The level risk reflects the risk that

| <i>Variable</i><br>$s_t = 1$  | <b>Open economy</b><br>theoretical | <b>Closed economy</b><br>theoretical | <b>Czech data</b><br>4Q1998-3Q2011 |
|-------------------------------|------------------------------------|--------------------------------------|------------------------------------|
| <i>A. Macro Moments</i>       |                                    |                                      |                                    |
| $sd(C)$                       | 1.16                               | 0.91                                 | 1.13                               |
| $sd(\pi)$                     | 2.08                               | 1.77                                 | 2.29                               |
| $sd(N)$                       | 1.53                               | 1.32                                 | 1.69                               |
| $sd(\Delta C)$                | 4.78                               | 3.93                                 | 4.21                               |
| <i>B. Yield Curve Moments</i> |                                    |                                      |                                    |
| $mean[i_{10Y} - i_1]$         | 1.32                               | 2.11                                 | 1.57                               |
| =                             |                                    |                                      |                                    |
| $mean[NTP_{10Y}]$             | 1.22                               | 0.92                                 |                                    |
| +                             |                                    |                                      |                                    |
| $mean[LevelRisk_{10Y}]$       | -0.1                               | 1.19                                 |                                    |
| $mean[i_1]$                   | 2.13                               | 3.30                                 | 2.98                               |
| $sd[i_1]$                     | 24.8                               | 21.29                                | 1.39                               |
| $sd(i_{10Y} - i_1)$           | 23.9                               | 20.4                                 | 1.78                               |

Table 5: **Comparison of Model Predictions to Actual Data.** Empirical and theoretical moments are calculated as quarterly values expressed in percentages. Time series were seasonally adjusted. The cyclical component in each variable is separated by a Hodrick-Prescott filter, and the data sample for Czech Republic is 1998 Q4 - 2011 Q3.

|                              | MS-DSGE Model |                 | Actual Data  |                 |
|------------------------------|---------------|-----------------|--------------|-----------------|
| <b>Regime</b>                | Disinflation  | Price stability | Disinflation | Price stability |
| (1) Macro volatility:        |               |                 |              |                 |
| $\text{std}(\pi s_t)$        | 2.08          | 13.65           | 1.52         | 2.1             |
| $\text{std}(\Delta c_t s_t)$ | 4.78          | 2.94            | 4.04         | 4.96            |
| (2) Slope decomposition:     |               |                 |              |                 |
| $E(\text{slope}_{10Y})$      | 1.32          | 0.82            | 1.84         | -0.38           |
| $E(NTP_{10Y})$               | 1.42          | 0.36            |              |                 |
| $Level\ Risk_{10Y}$          | -0.10         | 0.46            |              |                 |

Table 6: **Term Structure Simulations and Monetary Policy Regime.** The price stability regime corresponds to governor Tuma's second term. The disinflation regime corresponds to the period before his second term. Source: Own calculations based on the model simulation.

conditional on being in the regime with low short-term interest rates, a long-term bond will lose value in the case of shifting to higher short-term interest rates (Ferman, 2011). The nominal term premium compensates mainly for the inflation and consumption growth co-variability.

The expectation hypothesis does not hold in the model because agents incorporate the beliefs about the possible changes in monetary policy conduct into their decision process. In other words, the term premium is not constant because of the level component of the term premium and its switching character (low vs. high short-term rates).

Table 6 compares the simulated model with the actual Czech data depending on whether the monetary policy operates in a disinflation regime or price stability regime. The first regime corresponds to the period of high short-term interest rates and a higher slope of the yield curve. The standard deviation of inflation is marginally higher in the disinflation regime than we can observe in the actual data. Suppose that the economy is exposed to strong foreign output shocks. The effect of shock from the foreign demand channel is dampened by the aggressive monetary policy reaction in the first regime. In the second regime, when the central bank puts less weight on inflation stabilization, the foreign output shock causes higher inflation volatility. Our results are in line with this logic, but the model overshoots the inflation volatility for the price stability regime quite substantially.

The volatility of consumption growth is higher in the price stability regime but only with the actual data. This feature is not replicated in our open economy model. Intuitively, the volatility of consumption growth should be higher in the economy with a lower degree of inflation stabilization. Our supposition is that the lower weight on inflation in Taylor rule implies a relatively higher weight assigned to output and consumption stabilization.

The steepness of the yield curve is matched for the disinflation regime. In line with Ferman (2011), we find that the level risk is negative in a disinflation regime characterized by high short-term interest rates. The nominal term premium is lower in the price stability regime. However, the slope of the term structure of interest rates is higher in the disinflation period than in the price stability regime. The nominal term premium is lower in the price stability regime.

## 7 Concluding Remarks

Our ambition is to contribute an assessment of what Ferman (2011) labels the slope-volatility puzzle. This puzzle is based on the observation that the U.S. yield curve was steeper in the 1980s, i.e., a period characterized by low inflation and a stabler macroeconomic environment, than in the more turbulent 1970s. This is puzzling because one would expect that higher macroeconomic uncertainty would be reflected in higher term premia, and hence, the slope of the term structure of interest rates should be higher in the 1970s according to this intuitive reasoning. To solve the puzzle, Ferman (2011) introduces Markov switching into the model, and the nominal short-term interest rate fluctuates

around the different means across the regimes. Investors require an additional premium, which Ferman (2011) calls “level risk,” to be compensated for the risk of regime switching. Conditional on the economy being in a regime with low interest rates, long-term bonds will lose value in case of a shift to the regime with higher rates and *vice versa*.

For this reason, we develop a dynamic stochastic general equilibrium model with Markov switching to study the term structure of interest rates. Our model extends the previous macro-finance models such as that by Ferman (2011) by opening the economy and adding the foreign demand channel. Using this newly developed model, we compare its predictions with actual data. We use Czech data for our empirical exercise because these data fulfill two conditions. First, the Czech Republic is one of the most open countries in the world, and therefore, a small open economy setting is a natural choice. Second, the Czech Republic originally introduced inflation targeting in 1998 as a disinflation strategy. Czech inflation indeed has fallen over several years, which is consistent with the notion of price stability. Therefore, we observe two regimes in monetary policy conduct: one characterized by disinflation attempts with an aggressive stance towards inflation and the other characterized by price stability. This creates an ideal setting to examine our open economy model with Markov switching.

We find that our model fits the macroeconomic data relatively well and that the open economy model matches the actual data better than the closed economy models do. Fitting the features of finance data is inherently more difficult. Our model shows that the slope of the term structure is steeper in the regime characterized by more aggressive inflation stabilization. This is in line with what we observe in the Czech data. However, we fail to reproduce some features of the closed economy model similar to Ferman (2011). We provide some explanations of why the results differ in this respect but believe that more research is needed to explain the slope-volatility puzzle.



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