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Pigou Meets Ramsey: Gender-Based Taxation with Non-Cooperative Couples

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Pigou Meets Ramsey: Gender-Based Taxation with Non-Cooperative Couples*

Abstract

This paper explores the implications of gender-based income taxation in a non-cooperative model of household behavior. In a first step, we show how gender-based taxes can act as Pigou taxes and correct the externality induced by a non-cooperative household equilibrium. We find that the first-best Pigou tax rules are solely determined by spouses’ relative marginal rates of substitution between the public household good and private consumption. Breaking down this general rule into the primitives of the model, the spouse with a comparative advantage in home production should be taxed at a higher rate. In a second step, we embed our non-cooperative framework in a standard second-best planning problem in which taxes serve a revenue-raising purpose. In this case, the optimal structure of differential taxation by gender is partly determined by a Ramsey-type inverse elasticity rule and partly by a Pigouvian tax element. We show that these two forces work in opposite directions in determining whether men or women should be taxed at a higher rate, and that either one could be dominant, depending on the revenue-raising position of the government. This result is robust to the introduction of two groups of households that differ in their mode of decision-making, which can be either cooperative or non-cooperative.


Keywords: Gender-based taxation; non-cooperative family decision-making.

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1. Introduction

Should husband and wife be taxed differently? This question lies at the heart of the debate on how to tax family income. As constitutional rulings usually require governments to treat men and women alike, the space of political decision-making is largely limited to the choice between individual taxation and joint taxation. The latter formally implies that marginal tax rates of husband and wife are always identical, regardless of individual shares in generating total family income. However, if framed in terms of primary and secondary earner, a system of joint taxation with a progressive tax rate schedule disincentivizes secondary earners because the tax on their earned income starts at the highest marginal tax rate of the primary income. With this interpretation in mind, it is quite conceivable to construct a politically acceptable income tax schedule prescribing lower marginal taxes on secondary earners, appropriately defined. While such a specification may not be challenged as discriminatory, it would effectively allow for lower tax rates on female earnings.

Public finance theory has long acknowledged the importance of gender by highlighting the differences in the labor supply behavior of men and women and their implications for optimal income taxation. A common theme in most contributions is Ramsey’s optimal taxation criterion whereby tax rates should be inversely proportional to the labor supply elasticity of the taxpayer. Since married women’s labor supply is more elastic than that of men or single women (Pencavel, 1986; Evers et al., 2008), gender-based taxation with lower rates for women than for men—as advocated by Alesina et. al (2011, henceforth AIK)— and Apps and Rees (2011a) is desirable on grounds of economic efficiency. If the policy choice set is restricted to individual versus joint taxation, the logic of Ramsey taxation yields a preference in favor of the former method (Boskin and Sheshinski, 1983; Apps and Rees, 1999a; Meier and Wrede, 2013). The literature further argues that individual incomes should not be taxed independently (Brett, 2007), but probably in a fashion where marginal tax rates of the secondary earner fall in the income of the primary earner (Kleven et al., 2009). Several authors focus on the relevance of household production as an alternative to labor supply in the market, which remains untaxed in both its production and its trade component (Apps and Rees, 1999b) and may exhibit public good characteristics on the household level. Moreover, household production may involve time inputs of both husband and wife. Hence, not only the amount of production may be a source of inefficiency, but also the structure of time inputs. In this respect, Piggott and Whalley (1996) stress that joint taxation has the advantage to induce a symmetric distortion. Kleven and Kreiner (2007) argue that optimal marginal taxation of secondary earners may not fall short of taxation of primary earners if taxation of input goods that can be used in household production is taken into account.

Though existing studies of household taxation differ in many respects, they share the common characteristic of assuming that couples are able to determine their time allocations in an efficient manner. Justifications of this assumption point to cooperative bargaining or relational contracts within households (AIK, 2011; Apps and Rees, 2011b). Yet, in line with the theoretical argument that transaction costs may prevent couples from reaching cooperative outcomes (Pollak, 1985), recent research casts doubt on the systematic recourse to this efficiency assumption. For example, econometric evidence from time allocation models which allow for both efficient and inefficient intrahousehold
behavior suggests that a sizeable proportion of couples behaves non-cooperatively (Del Boca and Flinn, 2012). Similarly, results from experiments that study family behavior in social dilemma games indicate that cooperation is not ubiquitous among maritally living couples (Cochard et al., 2009). We therefore find it important to examine the implications of gender-based taxation in a model that allows for inefficient family decision-making. As empirical observations indicate a huge variety in labor supply behavior and time allocation under various tax regimes (Apps and Rees, 2009), it also seems desirable to analyze gender-based taxation in a framework of heterogeneous modes of household behavior.

Our analysis proceeds in two steps. In the first part of the paper, we consider a simple non-cooperative framework of a family’s time allocation between market work and providing a home-produced public good. We postulate a linear tax schedule and study the extent to which men and women should be taxed differently. Since labor supply decisions are made non-cooperatively, couples fail to provide the optimal level of the family public good. In our basic setup, the main function of gender-based taxes is to push non-cooperative couples from some point inside the Pareto frontier to the Pareto frontier. Put differently, labor income taxation serves the purpose of counteracting the externalities created by non-cooperative behavior in Pigouvian fashion. We show that the slopes of the optimal gender-specific tax schedules are solely determined by spouses’ relative marginal rates of substitution between the public household good and the private good. In particular, gender-based taxation with higher taxes on men is optimal when the marginal rate of substitution of men is smaller than the marginal rate of substitution of women, and vice versa. Breaking down this general rule into the primitives of the model, we find that the optimal structure of differential taxation by gender depends on the deeper causes that sustain a gendered allocation of time. On the one hand, if women assume more household duties than men because they have higher valuations of home-produced public goods, then they should be taxed at a lower rate than men. However, if women perform more household production tasks than men because they have a comparative advantage in them, then gender-based taxation with higher marginal tax rates on women are optimal. These two results may combine to imply a higher optimal tax rate on female labor supply.

In the second part of the paper, we address the policy-relevance of our approach by recognizing that real income taxes are not Pigou taxes but serve a revenue-raising purpose. Thus, we embed our non-cooperative framework in a standard second-best planning problem in which a government has to raise taxes to finance some exogenous spending level and which allows for leisure as a third form of time use. We first solve the planning problem for a representative non-cooperative household, and then consider an environment in which two groups of households are distinct in their mode of decision-making which can either be cooperative or non-cooperative. It turns out that the optimal structure of differential taxation by gender is partly determined by a Ramsey-type inverse elasticity rule and partly by Pigouvian considerations. The way in which these two forces interact depends on the fiscal position of the government. On the one hand, if the government sets taxes according to a relatively low revenue requirement, then the Pigouvian element of labor income taxation dominates the Ramsey element. Thus, when women take on more home duties than men because they have a comparative advantage in them, the government faces an incentive to tax women more than men in order to strengthen the corrective impact of taxation. Interestingly, this case may arise even as the proportion
of cooperative households in the population becomes large. On the other hand, if the government sets taxes according to a relatively high revenue requirement, then Ramsey taxation considerations dominate Pigou considerations. Thus, when women take on more home duties than men because they have a comparative advantage in them, the planner’s solution involves higher marginal tax rates on men relative to women. Finally, there is a “knife-edge case” with a moderate tax revenue requirement in which Ramsey and Pigou forces offset one another so that identical marginal tax rates across members of the same household are optimal.

Our results stand in contrast to those prevailing in the efficiency-based household taxation literature: AIK study gender-based taxation in a model in which labor supply elasticities emerge endogenously from a cooperatively bargained allocation of goods and time in the family. Their main conclusion is that a system of selective taxation with lower marginal tax rates for women is superior to an ungendered tax code, independently of the deeper reasons that sustain gendered allocations of time. The main advantage of structuring tax schedules in favor of women in AIK’s setup is that it minimizes the distortionary cost of taxation. In parallel, it offers the benefit of endogenously balancing the allocation of work across genders. In our setup, gender-based taxation with higher marginal tax rates on women may be optimal because they counteract the externalities created by the non-cooperative labor supply behavior of couples.

From a policy perspective, our goal is not to argue that re-balancing the tax structure in favor of women is undesirable but to highlight a particular effect that strikes a cautionary note on thinking about its welfare consequences: efficiency models of the family and theories based on non-cooperative behavior may suggest distinct and sometimes mutually exclusive optimal taxation criteria. Moreover, once we acknowledge the possibility of heterogeneous modes of household behavior—with some couples behaving cooperatively and others acting non-cooperatively—Ramsey and Pigou forces will interact in determining the optimal tax treatment of couples. Our results indicate that these two forces work in opposite directions in determining whether men or women should be taxed at a higher rate, and that either one could be dominant, depending on the revenue-raising position of the government.

Apart from complementing the literature on the optimal taxation of couples, this study is also related to the literature on non-cooperative family decision-making. From an empirical viewpoint, the underlying motivation of our work stems from econometric estimates of Del Boca and Flinn (2012), which suggest roughly one-fourth of households behaves non-cooperatively. Relatedly, Jia (2005) empirically examines labor supply of retiring couples and concludes that more than one-half of households behaves according to a non-cooperative model of family decision-making. From a theoretical perspective, a close antecedent to our paper is Konrad and Lommerud (1995). They show that it is possible to influence non-cooperative household outcomes by lump-sum redistribution from one spouse to the other, and that such redistribution might lead to a Pareto-improvement. The non-cooperative approach has also been adopted by Anderberg (2007) analyzing the mix of government spending when family behavior is inefficient, and by Gugl (2009) who investigates the impact of tax regimes on inequality within the household. Finally, in our own work (Meier and Rainer, 2012a) we show that joint taxation may be Pareto-superior to individual taxation under a Stackelberg equilibrium modeling assumption with a household public good.
The remainder of this paper is organized as follows. Section 2 introduces our basic model and illustrates how gender-based taxes can push non-cooperative couples from some point inside the Pareto frontier to the Pareto frontier. Section 3 embeds a non-cooperative model of household behavior in a standard second-best tax problem with an exogenous revenue requirement. After discussing gender-based taxation for a representative non-cooperative household, it analyzes optimal income taxation of couples in a framework of heterogeneity in decision-making across households. Section 4 concludes and indicates possible directions for further research.

2. Pigouvian Income Tax Rules for Non-Cooperative Couples

In this section, we show how gender-based taxes can act as Pigou taxes and correct the externality induced by a non-cooperative household equilibrium. To this end, we will consider a simple model of a couple’s time allocation between market work and providing a household public good. In the second part of the paper, we will introduce leisure as a third form of time use, but for now we will be able to get a lot of important insights from the simplest possible setup.

2.1. Setup

Consider a representative family consisting of two decision-makers, a (he) and b (she). Throughout we use the notation $i \in \{a, b\}$ to refer to either one of the two. Individual $i$’s preferences are represented by a strictly increasing and strictly quasi-concave utility function defined over a private good, $c_i$, and a home-produced public good, $q$. Examples of the public good comprise the upbringing and education of children, and care for the elderly. Formally, the utility functions of $a$ and $b$ are

$$U^i(c_i, q) \text{ for } i = a, b. \quad (1)$$

We denote by $U_k^i(c_i, q)$ the first-order partial derivative of $U^i$ with respect to its $k$-th argument ($k = c, q$). The second-order partial derivatives are represented by $U_{kl}^i(c_i, q)$ (or simply $U_k^i$), where $k, l = c, q$.

Each partner has a unit of active time endowment, which can be allocated between working in the outside labor market $(1 - \ell_i)$ and working at home ($\ell_i$), thereby contributing to the production of a household public good. The household production function $f$ depends on time inputs $\ell \equiv (\ell_a, \ell_b)$:

$$q = f(\ell) \quad (2)$$

For each $i = a, b$, we denote by $f_i(\ell)$ and $f_{ii}(\ell)$ the first-order and second-order partial derivative of $f$ with respect to $\ell_i$. We assume that $f$ is increasing and concave in its first two arguments: $f_i(\ell) > 0$ and $f_{ii}(\ell) < 0$.

Spouses may be differently productive in market work and home production. The productivity in the labor market is given by the gross market wage, $w_i$. The marginal productivity in household production is captured by $f_i(\ell)$. 

5
The consumption levels of $a$ and $b$ are

$$c_i = w_i(1 - \tau_i)(1 - \ell_i) + \vartheta_i \quad \text{for } i = a, b \quad (3)$$

where $\tau_i \geq 0$ is the marginal tax rate and $\vartheta_i$ is a lump-sum transfer.

The sequence of events is as follows. First, the governments sets labor taxes $\tau = (\tau_a, \tau_b)$ and determines the lump-sum transfers. Second, the spouses non-cooperatively decide on how to allocate their time between market work and home production. To characterize equilibrium time allocations, we will consider both a simultaneous-move and a sequential-move game between the spouses. Labor taxes $\tau = (\tau_a, \tau_b)$ will be set to correct the externality from non-cooperative behavior. Moreover, we consider the case of lump-sum redistributed tax proceeds $\vartheta = (\vartheta_a, \vartheta_b)$ whereby each individual $i$ receives a transfer $\vartheta_i$ that is equal to her labor income taxes.

$$\vartheta_i = \tau_i w_i(1 - \ell_i) \quad \text{for } i = a, b \quad (4)$$

Thus, all wage taxes paid are returned to family members as lump-sum benefits, implying a tax revenue requirement of zero. The specification of type-specific lump-sum transfers where taxes paid are returned in full allows for ruling out distributional goals of the government, ensuring that labor income taxes serve purely allocative purposes.

For convenience, we let

$$MRS^i_q = \frac{U^i_q(c_i, q)}{U^i_c(c_i, q)}$$

denote $i$’s marginal rate of substitution between the public and the private good.

### 2.2. First-Best Time Allocation

To derive the first-best benchmark, we now replicate a Pareto efficient allocation without labor income taxes and lump-sum redistributed tax proceeds. Thus, we maximize one partner’s utility subject to a given level of the other and the resource constraint. The Lagrangian reads:

$$\mathcal{L} = U^a(c_a, q) + \lambda \left[ U^b(c_b, q) - T \right] + \mu \left[ \sum_{i \in \{a, b\}} \left( w_i(1 - \ell_i) - c_i \right) \right]$$

where $\lambda$ and $\mu$ are Lagrange multipliers, and $w_i$ is the gross market wage. In any interior solution, the first-order conditions are

$$U^a_q(c_a, q) - \mu = 0, \quad (5)$$
$$\lambda U^b_c(c_b, q) - \mu = 0, \quad (6)$$
$$\left[ U^a_q(c_a, q) + \lambda U^b_q(c_b, q) \right] f_a(\ell) - \mu w_a = 0, \quad (7)$$
$$\left[ U^a_q(c_a, q) + \lambda U^b_q(c_b, q) \right] f_b(\ell) - \mu w_b = 0. \quad (8)$$

These conditions can be simplified to express the Samuelson rule, stating that the sum of the marginal rates of substitution between the public and the private good must be equal
to the marginal rate of transformation between these two goods.

\[ MRS_{qc}^a + MRS_{qc}^b = \frac{w_i}{f_i(\ell)} \quad \text{for } i = a, b \]  

(9)

Additionally, the (first-best) socially efficient allocation is also characterized by the marginal rates of transformation being equated across partners:

\[ \frac{w_a}{f_a(\ell)} = \frac{w_b}{f_b(\ell)}. \]  

(10)

This ensures efficiency in the production of the public good.

2.3. The First-Best Pigouvian Tax Rule

If there are no limits to cooperation, we would expect couples to achieve some first-best allocation as described above. We now follow the literature on non-cooperative family decision-making (see, e.g., Bergstrom, 1989; Lundberg and Pollak, 1993; Konrad and Lommerund, 1995; Chen and Woolley, 2001; Anderberg, 2007) in supposing that couples are not able to reach efficient outcomes. There are a number of ways of modeling non-cooperative family behavior. Our basic model focuses on an environment in which the individuals play a simultaneous move game and determine their time allocation independently. Throughout, we will focus on interior private provision equilibria in which neither partner fully specializes in market work.

The two partners simultaneously and non-cooperatively choose how to divide their time endowments between market work and home production. We analyze the resulting time allocations that constitute a Nash equilibrium. An interior provision equilibrium can be characterized as follows. Each partner consumes quantities \( q^* \) and \( c^*_i \) of the public and the private good, respectively. Moreover, \( q^* = f(\ell^*) \) and \( c^*_i = w_i(1 - \tau_i)(1 - \ell^*_i) + \vartheta_i \) satisfy

\[ \frac{MRS_{qc}^i}{1 - \tau_i} = \frac{w_i}{f_i(\ell)} \quad \text{for } i = a, b \]  

(11)

which depends on \( \tau = (\tau_a, \tau_b) \) and \( \vartheta = (\vartheta_a, \vartheta_b) \) as well as on the other model parameters. In equilibrium, each partner allocates her time between market work and home production such that her marginal rate of substitution between the public and the private good, multiplied by the tax wedge \( \frac{1}{1 - \tau_i} \) (i.e., the ratio of gross wage and net wage), equals the marginal rate of transformation between the two goods.

Compared to the first-best, non-cooperative behavior implies that there is no self-enforcing mechanism that induces the partners to internalize the impact of their choices on each other. As a consequence, each partner tends to supply an inefficiently high amount of time to the labor market, implying an inefficiently low provision of the household public good. In the presence of this inefficiency, wage taxes are no longer necessarily distortionary. Instead, they have a corrective element that may fully address the externality from non-cooperative behavior. We have:

**Proposition 1.** The couple can be induced to attain the first-best allocation of time between market work and home production by implementing a set of corrective (Pigouvian)
labor income taxes with lump-sum transfers to each individual that are equal to his or her labor income taxes. The first-best inducing labor income taxes \( \tau^* = (\tau_a^*, \tau_b^*) \) satisfy:

\[
\tau_a = \frac{MRS_{qc}^b}{MRS_{qc}^a + MRS_{qc}^b} \quad \text{and} \quad \tau_b = \frac{MRS_{qc}^a}{MRS_{qc}^a + MRS_{qc}^b}
\]  

(12)

**Proof.** In order to implement a socially efficient allocation, the partners’ equilibrium choices have to satisfy both the Samuelson condition [eq. 9] and the home production efficiency requirement [eq. (10)]. Both conditions are simultaneously fulfilled if and only if

\[
\frac{MRS_{qc}^a}{1 - \tau_a} = MRS_{qc}^a + MRS_{qc}^b = \frac{MRS_{qc}^b}{1 - \tau_b}.
\]

Inserting (12) into the individual’s first-order condition (11) shows that this condition will indeed hold.

The first-best inducing marginal tax rates have the striking feature that they sum-up to one. Moreover, they solely depend on *gender differences in marginal rates of substitution*. In particular, gender-based taxation with higher taxes on men is optimal when the marginal rate of substitution of men is smaller than the marginal rate of substitution of women, and *vice versa*:

\[
\tau_a^* \geq \frac{1}{2} \geq \tau_b^* \quad \text{if and only if} \quad MRS_{qc}^a \leq MRS_{qc}^b
\]

This result has a simple logic. It follows from the observation that the expression \( \frac{MRS_{qc}^i}{MRS_{qc}^a + MRS_{qc}^b} (i = a, b) \) captures the relative degree to which individual \( i \) “under-invests” into home production activities relative to the first-best. Indeed, the lower is partner \( i \)’s marginal rate of substitution relative to the sum of marginal rates of substitution, the more severe is his or her underinvestment relative to that of the other spouse. The optimal gender-specific tax rates fully eliminates the inefficiency arising from non-cooperative behavior by imposing a tax rate on each individual based on the partner’s relative degree of underinvestment. By requiring a higher marginal tax rate for the partner whose equilibrium choice more severely deviates from the socially efficient allocation, a *deviation from the first-best* principle is established. It should be noted that these Pigouvian taxes depend on the exact form of the first-order condition of the household equilibrium since these taxes are derived directly from them.

### 2.4. Gendered Equilibria and Gender-Based Taxation

So far we have discussed differential taxation by gender in terms of marginal rates of substitution. However, the marginal rates of substitution cannot be taken as exogenous primitives, but depend endogenously on marginal tax rates, lump-sum transfers, wages, home productivities, and preference parameters. In other words, our main results so far do not provide an explicit solutions for the first-best inducing marginal tax rates, but characterize \( \tau^* = (\tau_a^*, \tau_b^*) \) and \( \vartheta^* = (\vartheta_a^*, \vartheta_b^*) \) as functions of the primitives of the model. Our analysis now proceeds as follows. First, we unearth the reasons that sustain
a gendered allocation of time in our non-cooperative decision-making framework. Second, we ask how the optimal proportional tax rates on men relative to women depend on the reasons that sustain a gendered equilibrium.

We develop our results in a simpler framework where payoffs are additive. Let

\[ U^i(c_i, q) = (1 - \gamma_i)v(c_i) + \gamma_i z(q) \quad \text{for } i = a, b, \]

where \( \gamma_i \) is a preference parameter which measures the relative importance of the household public good. We assume that \( v(\cdot) \) and \( z(\cdot) \) are well-behaved increasing and concave functions. To keep the analysis tractable, we additionally assume that the partners’ time inputs into household production are “independent”. Thus, we let the composition of the utility function \( z(q) \) with the household production function \( q = f(\ell) \) be given by

\[ z(q) = z(f(\ell)) = \xi_a \mu(\ell_a) + \xi_b \mu(\ell_b), \]

where \( \mu(\cdot) \) is a well-behaved increasing and concave function. The parameters \( \xi_a \) and \( \xi_b \) capture the partners’ productivity in home production. As an example, combining a Cobb-Douglas production function \( q = (\ell_a)^{\xi_a}(\ell_b)^{\xi_b} \) with logarithmic utility, \( z(q) = \ln q \), would yield \( \mu(\ell_i) = \ln \ell_i \).

Under the above assumptions, the partners’ time inputs into home production are neither complements nor substitutes. Therefore, each partner has a strictly dominant time allocation strategy. Indeed, an interior provision equilibrium now fulfills

\[ \frac{MRS_{qc}^i}{1 - \tau_i} = \frac{w_i}{\xi_i \mu'(\ell_i)} \quad \text{with} \quad MRS_{qc}^i = \frac{\gamma_i z'(q)}{(1 - \gamma_i)v'(c_i)} \]

which only depends on \( \ell_i \). In general, the effect on \( i \)'s own wage on \( \ell_i \) is ambiguous due to conflicting income and substitution effects. Throughout the paper we will assume, however, that the substitution effect dominates so that individual \( i \)'s time allocated to market work increases in the own wage. Formally, we impose \( \varepsilon v'c \equiv cv''(c)/v'(c) \in (-1, 0] \) and \( \varepsilon z'q \equiv qz''(q)/z'(q) \in (-1, 0] \). The latter ensures that when household productivity increases, the rising marginal utility of the respective time input is not offset by the diminishing marginal utility due to a higher level of the household good at given behavior.

We now discuss the extent to which the equilibrium gives rise to a gendered allocation of time. We have

**Proposition 2.** Suppose that one of the three cases holds:

(a) \( \gamma_a < \gamma_b \) with \( w_a = w_b \) and \( \xi_a = \xi_b \).

(b) \( w_a > w_b \) with \( \xi_a = \xi_b \) and \( \gamma_a = \gamma_b \).

(c) \( \xi_a < \xi_b \) with \( w_a = w_b \) and \( \gamma_a = \gamma_b \).

Holding constant \((\tau_a, \tau_b)\) and \((\bar{\vartheta}_a, \bar{\vartheta}_b)\) at some arbitrary levels \( \bar{\tau} \) and \( \bar{\vartheta} \) respectively, men work more in the labor market than women and take less home duties than women \( (\ell^*_a < \ell^*_b) \).

**Proof.** See the Appendix. \( \square \)
The proposition describes three cases which sustain a gendered equilibrium. In the first case, women have, for exogenous reasons, a higher preference for the household public good than men ($\gamma_a < \gamma_b$). Econometric studies of family behavior which identify preference parameters suggest that the average weight placed on home-produced public goods is indeed greater for women than for men (Del Boca and Flinn, 2012). In the second case, men receive exogenously (e.g., due to gender discrimination) a higher wage than women in the labor market ($w_a > w_b$). In the third case, women are exogenously (e.g., due to biological differences) more productive than men in performing home duties ($\xi_a < \xi_b$). Under any of these specifications, a gendered allocation of time arises in which women assume more responsibilities for home production.

What are the implications of these three cases for the optimal proportional tax rates on men relative to women? First, suppose that a gendered allocation of time stems from gender differences in preferences. We have:

**Proposition 3.** If women value the household public good more than men ($\gamma_a < \gamma_b$ with $w_a = w_b$ and $\xi_a = \xi_b$) then gender-based taxation with higher marginal tax rates on men is optimal.

*Proof.* See the Appendix.

To illustrate one specific case, suppose the utility function of each individual is linear in consumption, i.e., let $v_i(c_i) = c_i$. In this case, the optimal marginal tax rates are given by

$$
\tau^*_a = \frac{\kappa_b}{\kappa_a + \kappa_b} \quad \text{and} \quad \tau^*_b = \frac{\kappa_a}{\kappa_a + \kappa_b},
$$

where $\kappa_i = \frac{\gamma_i}{1 - \gamma_i}$.

Each partner attaches a relative preference weight of $\kappa_i$ to the public good when choosing how to allocate his or her time between work and home production. The socially efficient allocation, however, would require him or her to attach a relative weight of $\kappa_a + \kappa_b$ to the public good. Thus, if men value the public good less than women, their time allocation choice will deviate more from the first-best than that of women. Gender-based taxation with higher marginal tax rates on men address the relative severity of the underprovision problem among them, while at the same time guaranteeing that women also are incentivized to choose the first-best.

Next, consider the other two cases in which women assume more home duties than men because they have a comparative advantage in them:

**Proposition 4.** If men receive a higher wage than women in the labor market ($w_a > w_b$ with $\xi_a = \xi_b$ and $\gamma_a = \gamma_b$), or if women are more productive than men in performing home duties ($\xi_a < \xi_b$ with $w_a = w_b$ and $\gamma_a = \gamma_b$), then gender-based taxation with higher marginal tax rates on women is optimal.

*Proof.* See the Appendix.

When women have a comparative advantage in home duties, then the marginal tax rate on women is higher at the optimum than that on men. The intuition behind this result is as follows. While even in our noncooperative setting there is a tendency towards specialization between partners in market work and the provision of household...
public goods, there is less specialization than in a cooperative model. Thus, if gender differences are assumed to originate from women’s comparative advantage in home duties, then women’s inputs into home production are more distorted than that of men from an allocative point of view. The optimal marginal tax rates ensure—by weighing relative input distortions—that the partners’ joint contributions to the household public good correspond to the first-best.

To conclude the discussion of the main results so far, we briefly summarize their implications. The main message here is as follows. When family members behave non-cooperatively, the optimal structure of differential taxation by gender depends on the deeper causes that sustain a gendered allocation of time. If men and women are almost identical in their market and home productivity but women value household public goods more than men, then women should be taxed at a lower rate than men. However, if men and women are almost identical in their preferences, while women assume more home duties than men because they have a comparative advantage in them, then gender-based taxation with higher marginal tax rates on women is optimal. Ultimately, the optimal gender-specific proportional tax rate on men relative to women depend in a non-trivial way on three sets of parameters – the partners’ valuations of household public goods and market and home productivities.

In an earlier version of this paper (Meier and Rainer, 2012b), we have examined the robustness of our results so far to some alternative model specifications. First, it can be shown that introducing altruistic “caring” preferences reduce the first-best implementing tax rates because the problem of underprovision of the family public good becomes less severe. Second, in a scenario where time allocation decisions are made sequentially and side payments between family members are feasible, the tax treatment of the primary earner changes by a term reflecting whether inputs in household production function are complements or substitutes. Third, when accounting for ex-ante career choices, optimal tax rates are lower to reduce disincentives in human capital accumulation. However, in all of these alternative model specifications, the structure of Pigouvian taxation by gender continues to depend crucially on spouses’ relative marginal rates of substitution between the public household good and private consumption.

3. Pigou Meets Ramsey

In the first part of the paper, we have shown how gender-based taxes can push a non-cooperative household from some point inside the Pareto frontier to the Pareto frontier. However, real income taxes are not Pigou taxes but revenue-raising taxes. It might well be that these already distort choices away from taxed labor supply towards household production by far more than is sufficient to correct the externality induced by a non-cooperative household equilibrium. In this section, we will address this issue by considering an optimal income tax problem in which there is a given revenue constraint and which allows for leisure as a third form of time use. We first solve the optimal income tax problem for a representative non-cooperative household. We then consider an environment in which some proportion of households is able to implement an efficient allocation of time, while the remaining proportion behaves non-cooperatively.
3.1. Setup

We explore the implications of an optimal income tax problem in a version of the framework developed in Alesina et al. (2011). Thus, we adopt the following utility function:

\[ U_i = c_i + \gamma_i q - \frac{1}{1 + \phi} (n_i + \ell_i)^{1+\phi}, \quad (16) \]

where \( c_i \) is consumption of a private good, \( q_i \) is a home-produced public good, \( n_i \) is hours of market work, \( \ell_i \) is the amount of home duties, and \( \phi > 0 \) is the curvature of the disutility of working a total of \( n_i + \ell_i \) hours. As in the previous section, \( \gamma_i \) is a preference parameter measuring \( i \)'s valuation of the household public good. For simplicity of presentation, from now onwards, we will abstract away from gender differences in preferences by setting \( \gamma_i = \gamma \) for every partner \( i = a, b \). The household production function is given by:

\[ q = \frac{\xi_f(\ell_f)^{\alpha} + \xi_m(\ell_m)^{\alpha}}{\alpha} \quad (17) \]

where \( \xi_i \) captures \( i \)'s productivity in home production. We assume that there are decreasing returns to scale in home production and therefore let \( \alpha < 1 \). The consumption level of partner \( i \) is given by

\[ c_i = w_i(1 - \tau_i)n_i + \vartheta_i \quad (18) \]

3.2. Optimal Labor Taxation of Non-Cooperative Couples

Consider first a representative household in which two partners simultaneously and non-cooperatively choose their hours of market work and the amount of home duties, respectively. It is straightforward to show that, for a given tax system \( \tau_i \) and \( \vartheta_i \), the following time allocations constitute a dominant strategy Nash equilibrium. The amount of home duties of every partner \( i = a, b \) is given by

\[ \ell_i^* = \left[ \frac{\gamma_i \xi_i}{(1 - \tau_i)w_i} \right]^\frac{1}{\alpha - 1}, \quad (19) \]

while hours of market work are

\[ n_i^* = \left[ (1 - \tau_i)w_i \right]^\frac{1}{\phi} - \ell_i^*. \quad (20) \]

It is readily verified that household production time input \( \ell_i^* \) increases with \( i \)'s marginal tax rate \( (\tau_i) \), while market work \( n_i^* \) decreases with it. By contrast, an increase in \( i \)'s wage rate \( (w_i) \) decreases \( \ell_i^* \) but increases \( n_i^* \). Finally, a higher valuation of the household public good \( (\gamma_i) \) and a higher productivity in home production \( (\xi_i) \) is associated with a higher level of \( \ell_i^* \) and a lower level of \( n_i^* \). Thus, Proposition 2 from the previous section carries over to the framework considered in this section. For future reference, we denote by \( \Delta_i^* = \ell_i^*/n_i^* \) the ratio of home duties over market work for every partner \( i = a, b \). It is
also useful to note that the wage elasticity of labor supply can be written as:

\[ \varepsilon_i^* = \frac{\partial n_i^*}{\partial w_i} \left( \frac{w_i n_i^*}{w_i} \right) = (1 - \alpha)(1 + \Delta_i^*) + \phi \Delta_i^* \frac{\partial n_i^*}{\partial w_i} \]

The key feature of the wage elasticity of labor supply is that it increases in the ratio of home duties over market work, \( \Delta_i^* \).

Now suppose that a social planner chooses gender-specific linear tax schedules to maximize

\[ \max_{\tau_a, \tau_b, \vartheta_a, \vartheta_b} W = \omega V(\hat{U}_a) + (1 - \omega)V(\hat{U}_b), \]

where \( \hat{U}_i \) is the indirect utility of partner \( i \), and \( V \) is a strictly increasing and strictly concave function. The maximization problem is subject to the government budget constraint

\[ \tau_a w_a n_a^* + \tau_b w_b n_b^* - \vartheta_a - \vartheta_b \geq R, \]

where \( R \) denotes the revenue requirement. We exclude lump-sum taxes on both partners, \( \vartheta_a < 0 \) and \( \vartheta_b < 0 \), from our analysis. The first-order conditions of the planner’s problem are given by:

\[ \tau_a \geq 0 : -\omega V'(\hat{U}_a)w_a n_a^* + (1 - \omega)V'(\hat{U}_b)\gamma \xi_a (\ell_a^*)^{\alpha - 1} \frac{\partial \ell_a^*}{\partial \tau_a} \leq -\lambda \left[ w_a n_a^* + w_a \tau_a \frac{\partial n_a^*}{\partial \tau_a} \right] \]

\[ \tau_b \geq 0 : \omega V'(\hat{U}_a)\gamma \xi_b (\ell_b^*)^{\alpha - 1} \frac{\partial \ell_b^*}{\partial \tau_b} - (1 - \omega)V'(\hat{U}_b)w_b n_b^* \leq -\lambda \left[ w_b n_b^* + w_b \tau_b \frac{\partial n_b^*}{\partial \tau_b} \right] \]

\[ \vartheta_a \geq 0 : \omega V'(\hat{U}_a) \leq \lambda \]

\[ \vartheta_b \geq 0 : (1 - \omega)V'(\hat{U}_b) \leq \lambda \]

\[ \lambda \geq 0 : \tau_a w_a n_a + \tau_b w_b n_b - \vartheta_a - \vartheta_b \geq R, \]

where \( \lambda \) is the multiplier attached to the government budget constraint. Note that an increase in \( \tau_a \) has three effects (similar points hold for an increase in \( \tau_b \)). First, it has a negative effect on \( a \)'s equilibrium utility by decreasing his net labor income. Second, it has a positive effect on \( b \)'s equilibrium utility by increasing \( a \)'s contribution to the household public good. Third, it changes tax revenues by increasing the tax collected from \( a \) (for a given \( n_a^* \)) and by decreasing his hours of market work.

We now characterize the solution of the planning problem. As a key distinction between our non-cooperative approach and the cooperative setting considered by Alesina et al. (2011), we note that positive lump-sum transfers to both spouses \( (\vartheta_a > 0 \text{ and } \vartheta_b > 0) \) may be optimal in our non-cooperative setting. In this case of an interior solution, we obtain [from eqs. (25) and (26)] an equalization of social marginal utilities to the multiplier \( \lambda \):

\[ \omega V'(\hat{U}_a) = (1 - \omega)V'(\hat{U}_b) = \lambda \]

With this equality, it is straightforward to establish:

**Proposition 5.** At an interior solution with positive lump-sum transfers to both spouses...
\(\vartheta^*_a > 0 \text{ and } \vartheta^*_b > 0\), the optimal tax rates \(\tau^*_a\) and \(\tau^*_b\) are implicitly characterized by

\[
\tau^*_a = \frac{\Delta^*_a}{\Delta^*_a + (1 - \alpha) \epsilon^*_a} \quad \text{and} \quad \tau^*_b = \frac{\Delta^*_b}{\Delta^*_b + (1 - \alpha) \epsilon^*_b},
\]  

(30)

respectively.

**Proof.** See the Appendix.

On the one hand, the optimal tax rate on each spouse decreases with his or her labor supply elasticity, \(\epsilon^*_i\). This property reflects Ramsey’s “inverse elasticity rule”, which—when considered in isolation—calls for a lower marginal tax rate on the spouse with a higher ratio of home duties over market work [see eq. (21)]. However, opposite to this inverse elasticity element, the optimal tax rate on each spouse also increases directly with his or her ratio of home duties over market work, \(\Delta^*_i\). This reflects the Pigouvian tax element, which works to partially correct the externality induced by non-cooperative household behavior. To gain some insight into the relative strength of the Ramsey versus the Pigou tax element, we combine eqs. (21) and (30) to obtain the following characterization of the optimal tax rates:

\[
\tau^*_a = \frac{\phi \Delta^*_a}{2\phi \Delta^*_a + (1 - \alpha)(1 + \Delta^*_a)} \quad \text{and} \quad \tau^*_b = \frac{\phi \Delta^*_b}{2\phi \Delta^*_b + (1 - \alpha)(1 + \Delta^*_b)},
\]  

(31)

It is now straightforward to check that

\[
\tau^*_a \gtrless \tau^*_b \quad \text{if and only if} \quad \Delta^*_a \gtrless \Delta^*_b.
\]

Thus, gender-based taxation with higher taxes on men is optimal when the ratio of home duties over market work for men is larger than the ratio of home duties over market work for women. This suggests that Pigou considerations dominate Ramsey considerations in the optimal income tax characterizations. This conjecture is confirmed when we recognize that \(\Delta^*_a\) and \(\Delta^*_b\) cannot be taken as exogenous primitives, but depend endogenously on tax rates, wages, and home productivities:

**Proposition 6.** Consider an interior solution of the planning problem with positive lump-sum transfers to both spouses. When women perform more home duties than men because they have a comparative advantage in them (\(w_a > w_b\) with \(\xi_a = \xi_b\) or \(\xi_a < \xi_b\) with \(w_a = w_b\)), then the marginal tax rate on women is higher at the optimum than that on men: \(\tau^*_a < \tau^*_b\).

**Proof.** See the Appendix.\(^1\)

So far, we have shown that a benevolent government should tax non-cooperative couples by weighing the social loss from distortionary labor income taxation against the social gain from corrective labor income taxation. In the interior solution of the planning problem, with positive lump-sum transfers to both spouses, the corrective element dominates

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\(^1\)If we relax the simplifying assumption of identical preferences across spouses, it can be shown that gender-based taxation with higher marginal tax rates on men is optimal when women value the household public good more than men *ceteris paribus* (\(\gamma_a < \gamma_b\) with \(w_a = w_b\) and \(\xi_a = \xi_b\)).
the distortionary element. As in the previous section, a comparative advantage in home production relative to market work then implies a higher marginal tax rate.

We now discuss a second possible solution of the planning problem, namely the case of redistributive transfers across spouses (as in Alesina et al., 2011). Thus, we let \( \vartheta_a = \vartheta \) and \( \vartheta_b = -\vartheta \) without restricting the sign of \( \vartheta \). It is also useful to set \( \tau_b = \tau \) and \( \tau_a = \tau + \sigma \), where \( \sigma \in (-\tau, 1 - \tau) \) represents the gender difference in marginal tax rates. With this modified setup, the social planner solves

\[
\max_{\tau, \sigma, \vartheta} W = \omega V(\hat{U}^a) + (1 - \omega) V(\hat{U}^b) \quad \text{s.t.} \quad (\tau + \sigma)w_a n_a^* + \tau w_b n_b^* = R \tag{32}
\]

At an interior solution of the planning problem with redistributive transfers across spouses, the optimal tax parameters \((\tau^*, \sigma^*)\) simultaneously solve

\[
\frac{1 - \alpha - \Delta^*_a}{1 - \alpha - \Delta^*_b} = \frac{1 - \epsilon^*_a \left(\frac{\tau + \sigma}{1 - \tau}\right)}{1 - \epsilon^*_b \left(\frac{\tau}{1 - \tau}\right)} \quad \text{and} \quad (\tau + \sigma)w_a n_a^* + \tau w_b n_b^* = R.
\]

We now have:

**Proposition 7.** Suppose that \((\tau^*, \sigma^*)\) induce a gendered allocation of time in which women take on more home duties than men. There exists a critical value for the optimal undifferentiated marginal tax rate \(\tau^*\), given by

\[
\hat{\tau} = \frac{\phi}{2(\phi + 1) - \alpha},
\]

such that the optimal gender difference in marginal tax rates, \(\sigma^*\), has the following properties:

(a) If \(\tau^* < \hat{\tau}\), then gender-based taxation with lower marginal tax rates on men is optimal: \(\sigma^* < 0\).

(b) If \(\tau^* > \hat{\tau}\), then gender-based taxation with higher marginal tax rates on men is optimal: \(\sigma^* > 0\).

**Proof.** See the Appendix.

Table 1 illustrates this result using a numerical example. The proposition highlights two possible cases. First, when the revenue requirement of the government is relatively low, then the optimal undifferentiated tax rate \(\tau^*\) will fall short of the threshold \(\hat{\tau}\). In this case, the Pigouvian element of labor income taxation dominates the distortionary element. Thus, when women take on more home duties than men, the government faces an incentive to tax women higher than men in order to strengthen the corrective impact of taxation, and so the planner’s solution involves \(\sigma^* < 0\). Second, when the revenue requirement of the government is relatively high, then the optimal undifferentiated tax rate \(\tau^*\) will exceed the threshold \(\hat{\tau}\). In this case, Ramsey taxation considerations dominate Pigou considerations. Thus, when women take on more home duties than men, the planner’s solution now involves higher marginal tax rates on men relative to women, \(\sigma^* > 0\). The

\[\text{See the Appendix for a derivation of these conditions.}\]
Table 1: A Numerical Example

<table>
<thead>
<tr>
<th></th>
<th>Low Revenue Requirement</th>
<th>High Revenue Requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( R = 4 )</td>
<td>( R = 5 )</td>
</tr>
<tr>
<td>( \hat{\tau} )</td>
<td>0.286</td>
<td>0.286</td>
</tr>
<tr>
<td>( \tau^* )</td>
<td>0.227</td>
<td>0.308</td>
</tr>
<tr>
<td>( \sigma^* )</td>
<td>-0.020</td>
<td>+0.008</td>
</tr>
<tr>
<td>( n^*_a )</td>
<td>3.071</td>
<td>2.599</td>
</tr>
<tr>
<td>( n^*_b )</td>
<td>2.133</td>
<td>1.843</td>
</tr>
<tr>
<td>( \ell^*_a )</td>
<td>0.099</td>
<td>0.134</td>
</tr>
<tr>
<td>( \ell^*_b )</td>
<td>0.186</td>
<td>0.232</td>
</tr>
</tbody>
</table>

Notes: The numerical example assumes that \( w_a = 4, w_b = 3, \alpha = 0.5, \phi = 1, \gamma = 1, \xi_a = 1, \xi_b = 1. \)

intuition is as follows. If the government needs to generate large tax revenues, then the high income taxes necessary to do so distort choices away from taxed labor supply towards untaxed household production by more than is sufficient to correct the externality induced by non-cooperative household behavior. As consequence, the government faces the incentive to tax women less than men in order to minimize the distortionary costs of taxation according to the inverse elasticity rule.

3.3. Heterogenous Modes of Household Behavior

We now consider an optimal income tax problem in which some proportion of households \((\beta)\) displays an efficient allocation of time, while the remaining proportion \((1 - \beta)\) behaves non-cooperatively. We derive the effects of the presence of the latter on the optimal tax structure, given that it is not possible to differentiate between the two household types in the taxes they have to pay.\(^3\) To do so, we continue to work with the utility and home production specification in eqs. (16) and (17). To describe an efficient allocation of time, we impose an exogenous intra-household rule for time allocation and consumption choices. The rule is as follows:

(a) Net of tax household income is shared between \(a\) and \(b\) according to \(\{(\mu_a, \mu_b) \in (0, 1)^2| \mu_a + \mu_b = 1\}\), such that consumption for every partner \(i = a, b\) is

\[
c_i = \mu_i [w_a(1 - \tau_a)n_a + w_b(1 - \tau_b)n_b + \vartheta_a + \vartheta_b],
\]

and each spouse \(i\) bears his or her own cost of working a total of \(n_i + \ell_i\) hours, which is given by \((n_i + \ell_i)^{1+\phi}/(1 + \phi)\).

(b) With this sharing rule in place, cooperative couples choose \((n_a, \ell_a, n_b, \ell_b)\) to maxi-
mize joint utility net of taxes,
\[ c_a + c_b + 2\gamma q - \frac{1}{1+\phi}(n_a + \ell B)^{1+\phi} - \frac{1}{1+\phi}(n_b + \ell B)^{1+\phi}. \]

For couples with an efficient allocation of time, the amount of home duties is given by
\[ \ell^e_i = \left[ \frac{2\gamma \xi_i}{(1-\tau_i)w_i} \right]^{\frac{1}{1-\alpha}}, \tag{33} \]
while hours of market work are
\[ n^e_i = \left[ (1-\tau_i)w_i \right]^{\frac{1}{\phi}} - \ell^e_i. \tag{34} \]

Let us now assume that a social planner chooses gender-specific linear tax schedules to maximize a linearly additive welfare function given by:
\[ \max_{\tau_a, \tau_b, \vartheta_a, \vartheta_b} W = \beta (\tilde{U}_a + \tilde{U}_b) + (1-\beta) (\hat{U}_a + \hat{U}_b), \tag{35} \]
where \( \tilde{U}_i \) and \( \hat{U}_i \) denote the indirect utility of spouse \( i \) in an efficient and a non-cooperative household, respectively. The case of a utilitarian social welfare function without diminishing social marginal utilities is interesting and important because it allows us to focus on the efficiency aspects of labor income taxation with heterogeneous modes of household behavior. If the social welfare function exhibited diminishing social marginal utilities [as in eq. (22)], redistribution between efficient and non-cooperative households would become important in the determination of the optimal marginal tax rates—an issue we do not address here. One implication of maximizing the linearly additive welfare function in eq. (36) is that only the sum of lump-sum transfer, \( \vartheta_a + \vartheta_b \), but not gender-specific transfers can be determined as part of the optimum. The maximization problem is subject to the government’s budget constraint
\[ \beta(\tau_a n^e_a + \tau_b n^e_b) + (1-\beta)(\tau_a n^*_a + \tau_b n^*_b) - \vartheta_a - \vartheta_b \geq R. \tag{36} \]

The following proposition describes the interior solution of the planning problem.

**Proposition 8.** At an interior solution with positive lump-sum transfers to each household \( (\vartheta^*_a + \vartheta^*_b > 0) \), the optimal tax rate \( \tau^e_i \) of spouse \( i \) \( (i = a, b) \) is implicitly characterized by
\[ \tau^e_i = \frac{(1-\beta)\Delta^*_i}{(1-\beta)\Delta^*_i + (1-\alpha)(1-\beta)\epsilon^*_i + \beta K^*_i}, \tag{37} \]
where \( \kappa^*_i = \frac{n^*_i}{\ell^*_i} \) is the ratio of efficient market work over non-cooperative market work.

**Proof.** See the Appendix. \( \square \)

The proposition shows that optimal taxes in this realistic scenario of heterogeneity in decision-making depends both on Pigouvian and on Ramsey-type considerations. Compared with Proposition 5, inverse elasticity considerations now enter the formula not only
through the gender-specific labor supply elasticity in non-cooperative households ($\varepsilon^e_i$),
but also through the gender-specific labor supply elasticity in efficient households ($\varepsilon^n_i$).
The formula nicely illustrates that the relative shares of efficient and non-cooperative household have the expected impact on the weights of Pigouvian and Ramsey-type elements. In particular, the inverse elasticity tax element strictly increases with the share of efficient households in the population. However, it is also interesting to note that the extent to which optimal tax rates are driven by inverse elasticity considerations decreases with the degree of inefficiency in non-cooperative households, which is large when the ratio of efficient market work over non-cooperative market work ($\kappa_i^e/\kappa_i^n$) is small. Thus, the larger the externality induced by non-cooperative behavior, the less relative weight a social planner should put on Ramsey versus Pigou-type tax considerations. Interestingly, at an interior solution with positive lump-sum transfers, Pigou considerations continue to dominate Ramsey considerations in the optimal income tax characterization. To see this, observe that:

$$\tau^e_a \geq \tau^e_b \quad \text{if and only if} \quad \Delta_a^* \geq \Delta_b^*.$$  

Despite the presence of households with an efficient allocation of time, spouses with a larger ratio of home duties over market work should optimally face a larger marginal tax rate. As long as the equilibrium remains interior and is characterized by positive lump-sum transfers, this results holds for any $\beta \in (0, 1)$. Thus, imposing a higher marginal tax rate on spouses who take on more home duties may be efficient even as the share of cooperative households in the population becomes large.

However, an interior solution with positive lump-sum transfers to each household is not obtained for all parameter values. It is a possible outcome when the revenue requirement of the government is low, but may become infeasible once the revenue requirement of the government exceeds a certain threshold. In the latter scenario, it is interesting to consider (i) the case of purely redistributive transfers across spouses ($\vartheta_a = \vartheta$ and $\vartheta_b = -\vartheta$), and (ii) the case of no lump-sum transfers ($\vartheta_a + \vartheta_b = 0$). In both cases, the following two conditions simultaneously characterize the optimal tax treatment of the family:

$$\frac{(1-\alpha)[1-\beta(1-\kappa^n_a^e)] - \Delta_a^*}{(1-\alpha)[1-\beta(1-\kappa^n_b^e)] - \Delta_b^*} = \frac{1-\beta(1-\kappa^n_a^e) - \left(\frac{\tau^i}{1-\tau}\right)[(1-\beta)e^*_a + \beta\kappa^n_a^e\varepsilon^n_a]}{1-\beta(1-\kappa^n_b^e) - \left(\frac{\tau^i}{1-\tau}\right)[(1-\beta)e^*_b + \beta\kappa^n_b^e\varepsilon^n_b]}$$

where $\tau_b = \tau$ and $\tau_a = \tau + \sigma$, i.e., $\sigma \in (-\tau, 1-\tau)$ again captures the gender difference in the marginal tax rates. Letting $(\tau^e, \sigma^e)$ denote the optimal tax parameters, we have:

**Proposition 9.** Suppose that $(\tau^e, \sigma^e)$ induce a gendered allocation of time in which women take on more home duties than men. There exists a critical value for the optimal

\[\frac{\epsilon^n_a}{\pi^a} \geq \frac{1}{\pi^a}(1-\beta)[(1-\alpha)(n^*_a + \ell^*_a) + \phi^*_a] + \frac{1}{\pi^a}\beta[(1-\alpha)(n^*_b + \ell^*_b) + \phi^*_b] - \left(\frac{\tau^i}{1-\tau}\right)[(1-\beta)e^*_a + \beta\kappa^n_a^e\varepsilon^n_a] + \frac{1}{\pi^a}\beta[(1-\alpha)(n^*_b + \ell^*_b) + \phi^*_b].\]

The claim that $\tau^e_a \geq \tau^e_b$ if and only if $\Delta_a^* \geq \Delta_b^*$ follows immediately after noting that $n^*_a + \ell^*_a = n^*_i + \ell^*_i$ and $\ell^*_a \ell^*_b = \ell^*_b \ell^*_a$. 

undifferentiated marginal tax rate $\tau^{e,*}$, given by
\[
\tilde{\tau} = \frac{\phi}{1 + \phi + \Gamma(\beta)(1 + \phi - \alpha)} \quad \text{with} \quad \Gamma'(\beta) > 0,
\]
such that the optimal gender difference in marginal tax rates, $\sigma^*$, has the following properties:

(a) If $\tau^{e,*} < \tilde{\tau}$, then gender-based taxation with lower marginal tax rates on men is optimal: $\sigma^{e,*} < 0$.

(b) If $\tau^{e,*} > \tilde{\tau}$, then gender-based taxation with higher marginal tax rates on men is optimal: $\sigma^{e,*} > 0$.

Proof. See the Appendix.

The model with heterogeneous modes of household behavior has two possible solutions corresponding to the two cases described in Proposition 7: one in which women should optimally face a higher marginal tax rate than men [part (a)], and one in which women should be taxed at lower rate than men [part (b)]. Thus, depending on the revenue requirement of the government, the corrective element of labor income taxation may still dominate the inverse elasticity element ($R$ low so that $\tau^{e,*} < \tilde{\tau}$) or be dominated by it ($R$ high so that $\tau^{e,*} > \tilde{\tau}$). However, since the tax threshold $\tilde{\tau}$ now strictly decreases with $\beta$, Proposition 9 adds the additional insight that the parameter range under which inverse elasticity considerations dominate Pigou considerations expands with the share of efficient households in the population.

4. Concluding Remarks

Economists have recently started to examine models of household behavior in which couples endogenously sort into efficient and inefficient time allocation regimes. Econometric estimates suggest that a model of inefficient family decision-making adequately captures the behavior of a substantial share of households. Motivated by this finding, this paper has explored the implications of gender-based taxation using a non-cooperative approach to household behavior. Our analysis allows for a rich set of specifications, giving rise to some clear-cut principles. In a basic model with only two uses of time—i.e., market work and home production—gender-based taxes can push non-cooperative couples from some point inside the Pareto frontier to the Pareto frontier. In this case, empirical regularities suggest lower marginal tax rates on female earnings due to women’s higher valuation of household public goods. However, this can be more than offset when women display a comparative advantage in home production.

Extending the basic model to a standard optimal tax framework with leisure as an alternative use of time, a tax revenue requirement, and heterogeneous modes of household behavior shows that gender-based tax rules are determined by a mix of Pigou and Ramsey-type considerations. These two forces work in opposite directions, and either one could be dominant, depending on the fiscal stance of the government. While a high tax revenue requirement pushes the balance in favor of Ramsey-type considerations and thus implies a lower marginal tax rate on female earnings, the opposite outcome cannot a priori be ruled
out when the government sets taxes according to a low revenue requirement. Moreover, while a high share of couples acting cooperatively implies that a social planner should put more relative weight on Ramsey versus Pigou-type tax considerations, it is not, by itself, sufficient to ensure that inverse elasticity considerations are the dominant force, i.e., that women should be taxed at a lower rate than men. Finally, there is a "knife-edge case" with a moderate tax revenue requirement in which Ramsey and Pigou forces offset one another so that identical marginal tax rates across members of the same household are optimal. If policy choices were restricted to individual versus joint taxation, this case suggests a preference in favor of the latter method for a non-empty set of exogenous parameter values.

One important caveat is that we have not modelled the reason for why some households behave cooperatively while others act in a non-cooperative manner. In a model in which the mode of household behavior is determined endogenously, certain tax policies could actually change the values of the two alternative behavioral regimes and get couples to switch from one regime to another. This is issue has so far not been addressed in the literature, and might be an interesting avenue for future research, especially when complemented with structural estimation methods.

References


Appendix

Proof of Proposition 2. Given our assumptions, the first-order conditions for an interior provision equilibrium, $\ell_i^*$, can be written as:

$$-(1 - \gamma_i)v'(c_i)w_i(1 - \tau_i) + \gamma_i\xi_i\mu'(\ell_i) = 0 \tag{38}$$

where $c_i = w_i(1 - \tau_i)(1 - \ell_i) + \partial_i$. Clearly, if the spouses are identical in every respect ($\gamma_a = \gamma_b$, $w_a = w_b$, $\xi_a = \xi_b$), they will make identical time allocation choices. The proposition then describes three cases which sustain a gendered allocation of time: (a) $\gamma_a < \gamma_b$ with $w_a = w_b$ and $\xi_a = \xi_b$; (b) $w_a > w_b$ with $\xi_a = \xi_b$ and $\gamma_a = \gamma_b$; and (c) $\xi_a < \xi_b$ with $w_a = w_b$ and $\gamma_a = \gamma_b$. These three cases follow immediately from noting that:

$$\frac{\partial \ell_i}{\partial \gamma_i} = -\frac{w_i(1 - \tau_i)v'(c_i) + \xi_i\mu'(\ell_i)}{\chi_i} > 0 \tag{39}$$
$$\frac{\partial \ell_i}{\partial w_i} = \frac{(1 - \gamma_i)(1 - \tau_i)[v'(c_i) + w_i(1 - \tau_i)(1 - \ell_i)v''(c_i)]}{\chi_i} < 0 \tag{40}$$
$$\frac{\partial \ell_i}{\partial \xi_i} = -\frac{\gamma_i\mu'(\ell_i)}{\chi_i} > 0. \tag{41}$$

where $\chi_i \equiv (1 - \gamma_i)w_i^2(1 - \tau_i)^2v''(c_i) + \xi_i\gamma_i\mu''(\ell_i) < 0$ and (40) can be signed due to our assumption $\epsilon_{v\ell} > -1$.

For future reference, it is also useful to have:

$$\frac{\partial \ell_i}{\partial \tau_i} = -\frac{(1 - \gamma_i)w_i[v'(c_i) + w_i(1 - \tau_i)(1 - \ell_i)v''(c_i)]}{\chi_i} > 0 \tag{42}$$
$$\frac{\partial \ell_i}{\partial \ell_i} = \frac{(1 - \gamma_i)w_i(1 - \tau_i)v''(c_i)}{\chi_i} > 0 \tag{43}$$

Proof of Propositions 3 and 4. Expressing marginal rates of substitution in terms of primitives, the set of labor income taxes implementing the first best $\tau^* = (\tau^*_a, \tau^*_b)$, together with lump-sum redistributed tax proceeds $\vartheta^* = (\vartheta^*_a, \vartheta^*_b)$, are implicitly characterized by:

$$\tau^*_a + \tau^*_b = 1 \tag{44}$$
$$\frac{(1 - \tau^*_a)\gamma_b}{(1 - \gamma_b)v'(c^*_b)} = \frac{(1 - \tau^*_b)\gamma_a}{(1 - \gamma_a)v'(c^*_a)} \tag{45}$$
$$\vartheta^*_a = \tau^*_a w_a(1 - \ell^*_a) \tag{46}$$
$$\vartheta^*_b = \tau^*_b w_b(1 - \ell^*_b) \tag{47}$$

where $c^*_i = w_i(1 - \tau^*_i)(1 - \ell^*_i) + \vartheta^*_i$ with $\ell^*_i = \ell^*_i(\tau^*_i, \vartheta^*_i, w_i, \gamma_i, \xi_i)$. Clearly, $\tau^*_i = \tau^*_i(\gamma_a, \gamma_b, w_a, w_b, \xi_a, \xi_b)$ and $\vartheta^*_i = \vartheta^*_i(\gamma_a, \gamma_b, w_a, w_b, \xi_a, \xi_b)$. Now recall that, if the spouses are identical in every respect (i.e., $\gamma_a = \gamma_b$, $w_a = w_b$, $\xi_a = \xi_b$), they will make identical time allocation choices. In this case, it follows immediately from (44) and (45) that $\tau^*_a = \tau^*_b = \frac{1}{2}$. To establish Propositions 3 and 4, it is therefore sufficient to show that: (a) $\tau^*_a$ is strictly decreasing in $\gamma_b$, while $\tau^*_a$ is strictly increasing in $\gamma_b$; (b) $\tau^*_a$ is strictly decreasing in $w_b$, while $\tau^*_a$ is strictly increasing in $w_b$; and (c) $\tau^*_b$ is strictly increasing in $\xi_b$, while $\tau^*_a$ is strictly decreasing in $\xi_b$ [see Figure 2].
we obtain:

\[ \frac{\partial \tau^*_i}{\partial \gamma} = \frac{\gamma \mu'(\ell^*_i)}{\mu''(\ell^*_i)} > 0 \]

The first-order conditions determining \( \tau^*_i \) and \( \chi^*_i \) where Propositions 3 and 4 now follow immediately.

Proof of Proposition 6. We substitute eq. (29) into eq. (24) and eq. (25), and obtain a condition \( F_i \) determining the optimal \( \tau^*_i \) for every partner \( i = a, b \):

\[ F_i \equiv \gamma \chi_i(\ell^*_i)^{\alpha-1} \frac{\partial F_i}{\partial \tau_i} + w_i \tau^*_i \frac{\partial n_i}{\partial \tau_i} = 0 \]  

(48)

The first-order conditions determining \( \ell^*_i \) imply that \( \gamma \chi_i(\ell^*_i)^{\alpha-1} = (1 - \tau_i) w_i \). It is straightforward to verify that \( \frac{\partial F_i}{\partial \tau_i} = \frac{\ell^*_i}{(1 - \tau_i)(1 - \alpha)} \). Finally, we note that \( w_i \tau^*_i \frac{\partial n_i}{\partial \tau_i} = -w_i n_i \epsilon_i^* \left( \frac{\tau_i}{1 - \tau_i} \right) \), where \( \epsilon_i^* \) is the wage elasticity of labor supply [see eq. 21]. Thus, eq. (48) can be rewritten as:

\[ F_i \equiv \frac{\ell^*_i}{1 - \alpha} - n_i \epsilon_i^* \left( \frac{\tau_i}{1 - \tau_i} \right) = 0 \]  

(49)

The proposition follows immediately after setting \( \ell^*_i = n_i \Delta_i^* \) in eq. (49) and solving it for \( \tau_i \).

Proof of Proposition 6. We prove this result by showing that \( \tau^*_i \) (\( i = a, b \)) is a strictly
decreasing function of $w_i$ and a strictly increasing function of $\xi_i$. Since the derivative of $F_i$ with respect to $\tau_i$ must be negative at the optimum, it is sufficient to show that

\[
\frac{\partial F_i}{\partial w_i} < 0 \quad \text{and} \quad \frac{\partial F_i}{\partial \xi_i} > 0
\]

Using eq. (21), we may rewrite eq. (49) as:

\[
F_i \equiv \phi(1 - \tau_i)\ell_i^* - \tau_i \left[(1 - \alpha)(n_i^* + \ell_i^*) + \phi\ell_i^*\right], \tag{50}
\]

where $\ell_i^*$ and $n_i^*$ are given by eqs. (19) and (20), respectively. Differentiating this expression with respect to $w_i$ we obtain:

\[
\frac{\partial F_i}{\partial w_i} = -\left[\frac{\phi\ell_i^*(1 - 2\tau_i)}{(1 - \alpha)w_i} + \frac{\tau_i(1 - \alpha)(n_i^* + \ell_i^*)}{\phi w_i}\right] < 0 \tag{51}
\]

This expression is strictly negative since the optimal $\tau_i^*$ is strictly smaller than $\frac{1}{2}$ [see eq. (31)]. Differentiating eq. (50) with respect to $\xi_i$, we obtain:

\[
\frac{\partial F_i}{\partial \xi_i} = \frac{\phi\ell_i^*(1 - 2\tau_i)}{(1 - \alpha)\xi_i} > 0 \tag{52}
\]

This expression is strictly positive since the optimal $\tau_i^*$ is strictly smaller than $\frac{1}{2}$ [see eq. (31)]. The proposition now follows immediately since the comparative statics in eqs. (51) and (52) hold for every partner $i = a, b$.

**Proof of Proposition 7.** The first-order conditions of the planner’s problem are given by:

\[
\sigma : \quad -\omega V'(U^a)w_an_a^* + (1 - \omega)V'(U^b)\gamma\xi_a(\ell_a^*)^{\alpha - 1} \frac{\partial \ell_a^*}{\partial \sigma} = -\lambda \left[w_an_a^* + w_a(\tau + \sigma)\frac{\partial n_a^*}{\partial \sigma}\right] \tag{53}
\]

\[
\tau : \quad -\omega V'(U^a) \left[w_an_a^* - \gamma\xi_b(\ell_b^*)^{\alpha - 1} \frac{\partial \ell_b^*}{\partial \tau}\right] - (1 - \omega)V'(U^b) \left[w_bn_b^* - \gamma\xi_a(\ell_a^*)^{\alpha - 1} \frac{\partial \ell_a^*}{\partial \tau}\right] = -\lambda \left[w_bn_b^* + w_b(\tau + \sigma)\frac{\partial n_b^*}{\partial \tau}\right] + w_a(\tau + \sigma)\frac{\partial n_a^*}{\partial \tau} \tag{54}
\]

\[
\vartheta : \quad (1 - \omega)V'(U^a) = (1 - \omega)V'(U^b) \tag{55}
\]

\[
\lambda : \quad \tau_a w_a n_a + \tau_b w_b n_b = R, \tag{56}
\]

where $\lambda$ is the multiplier attached to the government budget constraint. The first-order conditions determining $\ell_i^*$ imply that $\gamma\xi_i(\ell_i^*)^{\alpha - 1} = (1 - \tau_i)w_i$. It is straightforward to verify that $\frac{\partial \ell_i^*}{\partial \eta_i} = \frac{\ell_i^*}{(1 - \eta_i)(1 - \alpha)}$. Finally, we note that $w_i\tau_i \frac{\partial n_i^*}{\partial \eta_i} = -w_i n_i \epsilon_i^* \left(\frac{\tau_i}{1 - \tau_i}\right)$, where $\epsilon_i^*$ is the wage elasticity of labor supply [see eq. 21]. After combining eqs. (53) and (54) and using eq. (55) to simplify the resulting expression, the first-order conditions determining $\sigma^*$ and $\tau^*$ reduce to

\[
\frac{1 - \alpha - \Delta_a^*}{1 - \alpha - \Delta_b^*} = \frac{1 - \epsilon_a^* \left(\frac{\tau + \sigma}{1 - \tau - \sigma}\right)}{1 - \epsilon_b^* \left(\frac{\tau}{1 - \tau}\right)} \tag{57}
\]
and
\[(\tau + \sigma)w_a n_a^* + \tau w_b n_b^* = R\]  
(58)

We now solve eq. (57) for \(\sigma\) and obtain
\[\sigma = \frac{(1 - \tau) \left[ 1 - \frac{\tau - \tau^*}{1 - \tau} \right] - \chi}{1 + \epsilon_a^* - \chi}\]  
where \(\chi = \left[ \frac{1 - \frac{\tau - \tau^*}{1 - \tau}}{1 - \frac{\Delta_a}{1 - \alpha}} \right] \left( 1 - \frac{\Delta_a}{1 - \alpha} \right)\)  
(59)

It is straightforward to verify that \(1 + \epsilon_a^* - \chi\) is strictly positive. Thus,
\[\sigma \geq 0\]  
if and only if \(1 - \frac{\tau - \tau^*}{1 - \tau} \geq 0\)  
(60)

Substituting into this condition \(\chi\) and then the respective expressions for \(\epsilon_a^*\) and \(\epsilon_b^*\), it can be simplified to
\[\sigma \geq 0\]  
if and only if \(\frac{1}{\phi(\Delta_b^* - \Delta_a^*)} [\tau(2 - \alpha) + \phi(2\tau - 1)] \geq 0\)  
(61)

The claim follows immediately from this condition. \(\square\)

Proof of Proposition 8. The first-order conditions for an interior solution of the planner’s problem are given by:

\[\tau_a : (1 - \beta) \gamma \xi_a (\ell_a^*)^{\alpha - 1} \frac{\partial \ell_a^*}{\partial \tau_a} - w_a [(1 - \beta) n_a^* + \beta n_a^*] = -\lambda \left[ (1 - \beta) \left( w_a n_a^* + w_a \tau_a \frac{\partial n_a}{\partial \tau_a} \right) + \beta \left( w_a n_a^* + w_a \tau_a \frac{\partial n_a}{\partial \tau_a} \right) \right]\]  
(62)

\[\tau_a : (1 - \beta) \gamma \xi_b (\ell_b^*)^{\alpha - 1} \frac{\partial \ell_b^*}{\partial \tau_b} - w_a [(1 - \beta) n_b^* + \beta n_b^*] = -\lambda \left[ (1 - \beta) \left( w_a n_b^* + w_a \tau_a \frac{\partial n_a}{\partial \tau_a} \right) + \beta \left( w_a n_b^* + w_a \tau_a \frac{\partial n_a}{\partial \tau_a} \right) \right]\]  
(63)

\[\vartheta_a : 1 = \lambda\]  
(64)

\[\vartheta_b : 1 = \lambda\]  
(65)

\[\lambda : \tau_a w_a n_a + \tau_b w_b n_b - \vartheta_a - \vartheta_b = R,\]  
(66)

The first-order conditions determining \(\ell_i^*\) imply that \(\gamma \xi_i (\ell_i^*)^{\alpha - 1} = (1 - \tau_i) w_i\). It is straightforward to verify that \(\frac{\partial \ell_i^*}{\partial \tau_i} = \frac{\xi_i}{(1 - \tau_i)(1 - \alpha)}\). Finally, we note that \(w_i \tau_i^* \frac{\partial n_i}{\partial \tau_i} = -w_i n_i^* \epsilon_i^* \left( \frac{\tau_i}{1 - \tau_i} \right)\) and \(w_i \tau_i^* \frac{\partial n_i}{\partial \tau_i} = -w_i n_i^* \epsilon_i^* \left( \frac{\tau_i}{1 - \tau_i} \right)\), where \(\epsilon_i^*\) and \(\epsilon_i^*\) capture wage elasticities of labor supply of spouse \(i\) in a non-cooperative and efficient household, respectively. Thus, with \(\lambda = 1\), eqs. (62) and (63) can be rewritten as:

\[\frac{(1 - \beta) \Delta_i^*}{1 - \alpha} = \frac{(1 - \beta) \epsilon_i^* \tau_i}{1 - \tau_i} + \frac{\beta \kappa_i^* \epsilon_i^* \tau_i}{1 - \tau_i} \]  
i = a, b  
(67)
where \( \kappa_i^{e^*_a} = \frac{w^{e^*_a}}{n_i^{e^*_a}} \). The claim follows immediately after solving eq. (67) for \( \tau_i \).

**Proof of Proposition 9.** We consider the case of no lump-sum transfers (\( \vartheta_a + \vartheta_b = 0 \)). The proof of the case of redistributive transfers across spouses is mathematically identical.

With no lump-sum transfers, the first-order conditions determining \((\tau, \sigma, \lambda)\) are given by:

\[
\tau_a : (1 - \beta)\gamma \xi_a (\ell^*_a)^{\alpha - 1} \frac{\partial \ell^*_a}{\partial \tau_a} - w_a [(1 - \beta)n^*_a + \beta n^*_a] =
\]

\[
- \lambda \left[ (1 - \beta) \left( w_a n^*_a + w_a \tau_a \frac{\partial n^*_a}{\partial \tau_a} \right) + \beta \left( w_a n^*_a + w_a \tau_a \frac{\partial n^*_a}{\partial \tau_a} \right) \right]
\]

(68)

\[
\tau_b : (1 - \beta)\gamma \xi_b (\ell^*_b)^{\alpha - 1} \frac{\partial \ell^*_b}{\partial \tau_b} - w_b [(1 - \beta)n^*_b + \beta n^*_b] =
\]

\[
- \lambda \left[ (1 - \beta) \left( w_b n^*_b + w_b \tau_b \frac{\partial n^*_b}{\partial \tau_b} \right) + \beta \left( w_b n^*_b + w_b \tau_b \frac{\partial n^*_b}{\partial \tau_b} \right) \right]
\]

(69)

\[
\lambda : \tau_a w_a n_a + \tau_b w_b n_b = R,
\]

(70)

As before, note that the first-order conditions determining \( \ell^*_i \) imply that \( \gamma \xi_i (\ell^*_i)^{\alpha - 1} = (1 - \tau_i)w_i \). Recall that \( \frac{\partial \ell^*_i}{\partial \tau_i} = \frac{\ell^*_i}{(1 - \tau_i)(1 - \alpha)} \), \( w_i \tau_i \frac{\partial n^*_i}{\partial \tau_i} = -w_i n^*_i \epsilon^*_i \left( \frac{\tau_i}{1 - \tau_i} \right) \) and \( w_i \tau_i \frac{\partial n^*_i}{\partial \tau_i} = w_i n^*_i \epsilon^*_i \left( \frac{\tau_i}{1 - \tau_i} \right) \), where \( \epsilon^*_i \) and \( \epsilon^*_i \) capture wage elasticities of labor supply of spouse \( i \) in a non-cooperative and efficient household, respectively. Thus, eqs. (68) and (69) can be combined to yield the condition:

\[
\frac{(1 - \alpha)[1 - \beta(1 - \kappa^{e^*_a})] - \Delta^*_a}{(1 - \alpha)[1 - \beta(1 - \kappa^{e^*_b})] - \Delta^*_b} = \frac{1 - \beta(1 - \kappa^{e^*_a}) - (\frac{\tau + \sigma}{1 - \tau - \sigma}) \left[ (1 - \beta)\epsilon^*_a + \beta \kappa^{e^*_a} \epsilon^*_a \right]}{1 - \beta(1 - \kappa^{e^*_b}) - (\frac{\tau}{1 - \tau - \sigma}) \left[ (1 - \beta)\epsilon^*_b + \beta \kappa^{e^*_b} \epsilon^*_b \right]}
\]

(71)

We now solve eq. (71) for \( \sigma \) and obtain

\[
\sigma = \frac{(1 - \tau) \left[ 1 - \beta(1 - \kappa^{e^*_a}) - (\frac{\tau}{1 - \tau}) \left[ (1 - \beta)\epsilon^*_a + \beta \kappa^{e^*_a} \epsilon^*_a \right] - \chi \right]}{1 - \beta(1 - \kappa^{e^*_a}) + [(1 - \beta)\epsilon^*_a + \beta \kappa^{e^*_a} \epsilon^*_a] - \chi}
\]

(72)

where

\[
\chi = \frac{1 - \beta(1 - \kappa^{e^*_a}) - (\frac{\tau}{1 - \tau}) \left[ (1 - \beta)\epsilon^*_a + \beta \kappa^{e^*_a} \epsilon^*_a \right]}{(1 - \alpha)[1 - \beta(1 - \kappa^{e^*_b})] - \Delta^*_b}
\]

(73)

It is straightforward to verify that the denominator in eq. (72) is strictly positive. Thus, \( \sigma \geq 0 \) if and only if

\[
\frac{1 - \beta(1 - \kappa^{e^*_a}) - (\frac{\tau}{1 - \tau}) \left[ (1 - \beta)\epsilon^*_a + \beta \kappa^{e^*_a} \epsilon^*_a \right]}{(1 - \alpha)[1 - \beta(1 - \kappa^{e^*_a})] - \Delta^*_a} \leq \frac{1 - \beta(1 - \kappa^{e^*_b}) - (\frac{\tau}{1 - \tau}) \left[ (1 - \beta)\epsilon^*_b + \beta \kappa^{e^*_b} \epsilon^*_b \right]}{(1 - \alpha)[1 - \beta(1 - \kappa^{e^*_b})] - \Delta^*_b}
\]

(74)
Using the definitions for \((\Delta_i^*, \epsilon_i^*, \epsilon_i^e, \kappa_i^e^*)\), as well as the fact that \(n_i^* + \ell_i^* = n_i^e + \ell_i^e\) and \(\ell_i^e = 2\tau^{i\alpha_0} \ell_i^*\), we obtain

\[
\sigma \geq 0 \quad \text{if and only if} \quad (\Delta_b^* - \Delta_a^*) \left[ \tau[1 + \Gamma(\beta)(1 - \alpha)] + \phi[\Gamma(\beta) + 1 - 1] \right] \geq 0 \quad (75)
\]

where \(\Gamma(\beta) = 1 + \beta \left(2^{\frac{1}{1-\alpha_0}} - 1\right) > 0\). The claim now follows immediately. \(\square\)
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