Kohler, Stefan

Article
Incomplete information about social preferences explains equal division and delay in bargaining

Games

Provided in Cooperation with:
MDPI – Multidisciplinary Digital Publishing Institute, Basel

Suggested Citation: Kohler, Stefan (2012) : Incomplete information about social preferences explains equal division and delay in bargaining, Games, ISSN 2073-4336, MDPI, Basel, Vol. 3, Iss. 3, pp. 119-137, http://dx.doi.org/10.3390/g3030119

This Version is available at:
http://hdl.handle.net/10419/98510

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.
Incomplete Information about Social Preferences Explains Equal Division and Delay in Bargaining

Stefan Kohler

Institute for Social Medicine, Epidemiology and Health Economics, Charité University Medical Center, Luisenstrae 57, 10117 Berlin, Germany; E-Mail: stefan.kohler@charite.de

Received: 30 July 2012; in revised form: 23 August 2012 / Accepted: 27 August 2012 / Published: 13 September 2012

Abstract: Two deviations of alternating-offer bargaining behavior from economic theory are observed together, yet have been studied separately. Players who could secure themselves a large surplus share if bargainers were purely self-interested incompletely exploit their advantage. Delay in agreement occurs even if all experimentally controlled information is common knowledge. This paper rationalizes both regularities coherently by modeling heterogeneous social preferences, either self-interest or envy, of one bargaining party as private information in a three period game of bargaining and preference screening and signaling.

Keywords: alternating-offer bargaining; asymmetric information; envy; fairness; inequality aversion; uncertainty

1. Introduction

In bargaining situations, the price and terms of a transaction can be disputed by the seller and the buyer of a good or service before an eventual agreement. Thereby, each bargaining party has bargaining power that can shift, for instance, through credibly establishing a not directly observable quality like social preferences. In order to investigate how one of numerous individually rational and Pareto optimal divisions might be agreed, Ståhl [1] and Rubinstein [2] first proposed structural models of alternating-offer bargaining that postulate strategic interaction between two bargainers. The presumed bargaining outcome depends on both bargainers’ preferences and how bargainers capitalize on their relative impatience. For purely self-interested, similarly impatient bargainers and complete information, the equilibrium predicts
immediate agreement on a larger share for the bargainer that starts the bargaining. This prediction is in contrast with the practical experience of delay in agreement and divisions more equal than forecast; both of which are well-documented regularities also studied in experiments (e.g., Bolton, Ochs and Roth, Weg et al. [3–5]).

This paper shows how equal division or one to two periods of delay occur in the equilibrium of a three period alternating-offer bargaining model with social preferences if a heterogeneity in one bargainer’s preferences is private information. Previous works explained either delay by incomplete information or the bias toward the equal division by social preferences that include a form of interpersonal comparison. Building on the framework of Ståhl, I assume that the first mover, who starts the bargaining and is arbitrarily called the seller, is commonly known to be entirely self-interested whereas the second mover, named the buyer, can be either self-interested or perfectly envious. Ample evidence indicates that at least some people compare themselves to others, which can include aspects of envy (e.g., Camerer, Cooper and Kagel, Smith [6–8]). The perfectly envious buyer prefers disagreement to anything less than the equal division. This simplified distributional concern stylizes the response to absolute and relative payoffs that motivates some social preferences. Envy can also be interpreted as loss aversion with respect to a fixed reference point at the equal division. Furthermore, bargaining with perfect envy is strategically equivalent to bargaining with a credible reputation of not accepting less than equal division.

The optimal behavior in the perfect Bayesian equilibrium of a three period alternating-offer bargaining model is either an immediate equal division of the surplus, and its certain acceptance, or gradual reduction of an initially high demand over the course of the game. Latter behavior leads to an occasional acceptance of the high demand by a self-interested buyer or to up to two periods of delay before an agreement on the equal division else. One period of delay is caused by the seller’s desire to discover the buyer’s true type allowing the interpretation of delay as the result of a failed first mover gamble that the second mover is self-interested. Further delay results from the signaling game in which the envious buyer wants to credibly show his type after an initial high demand by the seller. Whether the equilibrium of the three period alternating-offer bargaining model supports pooling or separating the two buyer types depends on their frequency and the bargainers’ impatience to reach agreement. The game only has an equilibrium in pure strategies if the share of self-interested buyers or the bargainers’ impatience remains low enough. As the perfectly envious buyer achieves at least the equal division, his situation is strategically equivalent to bargaining with an outside option of half of the surplus.

The next section reviews related literature. Section 3 presents the model and Section 4 derives the optimal behavior under uncertainty about the buyer preferences. Section 5 discusses the bargaining outcomes, then Section 6 concludes.

2. Related Literature

Informational disparities between bargaining parties are understood as a fundamental reason for delay in agreement. Various models of alternating-offer bargaining (e.g., Chatterjee and Samuelson, Cramton, Grossman and Perry, Gul and Sonnenschein, Rubinstein [9–14]) and of one-sided offer bargaining (e.g., Cramton, Fudenberg et al., Sobel and Takahashi [15–17]) employ incomplete information about a characteristic of the bargainers to explain delay. Often the first mover is assumed to be uninformed (e.g., Ausubel and Deneckere, Grossman
Valuation of the bargaining good, outside options, the individual cost of bargaining or personal risk attitude, all may be private information and have been studied. Ausubel et al. [19] and Roth [20] provide overviews. In a bargaining setting with asymmetrically informed inequality averse parties, a fully efficient mechanism in the form of a double auction exists only if compassion is strong. Less compassionate parties do not trade in the double auction in the limit of infinitely strong envy [21]. Given one-sided incomplete information in two sequential bargaining experiments, the role of inferences was tested by Srivastava [22] with experimentees who completed questionnaires that measured perceptions of fairness and opponent’s level of competitiveness. Srivastava conjectured that uninformed bargainers infer their opponents’ competitiveness rather than assess and refine a probabilistic assessment of the private information based on the informed bargainer’s behavior. Studying uncertainty about the opponent’s social preferences in an alternating-offer bargaining model supplements these literatures and bridges the theoretical gap between well-understood consequences of incomplete information in bargaining and the research that suggest that some players care about distributional aspects.

Uncertainty about social preferences has already been studied in different contexts. Von Siemens [23] considers investment incentives in a hold-up problem, in which the returns to relationship-specific investments are allocated by bargaining amongst individuals with heterogeneous social preferences that are private information. The investments can signal preferences and, thereby, influence beliefs and bargaining behavior. In consequence, bargainers might choose not to signal unfavorable information. Von Siemens [24] studies employment contracts if workers ability and their social preferences are private information. The paper finds that it is “not possible to screen workers according to their social preferences within the firm” but that “the firm can exclude inequity averse workers with low ability by offering employment contracts that do not compensate any suffering from rent differences in the firm.”

In alternating-offer bargaining, social preferences have been studied in a complete information context. De Bruyn and Bolton [25], Goeree and Holt [26], and Bolton [3] study the influence of distributional concerns and self-interest on finite horizon alternating-offer bargaining behavior. The first group of authors estimates quadratic inequality aversion preferences, introduced by Bolton and Ockenfels [27], for a representative agent based on preexisting data from different bargaining games. They conclude that self-interest and distributional concern “offer better predictions than traditional preference models that ignore fairness considerations.” Goeree and Holt [26] conducted an alternating-offer bargaining experiment in which, independently of the bargaining outcome, role-specific commonly known payments are made to the participants. In their experiment, average offers are biased toward the equal division of the overall income. They argue that a distributional concern is necessary to explain the observed behavior and estimate significant linear inequality aversion preferences as introduced by Fehr and Schmidt [28] based on the experimental data. How guilt or envy affects either party’s bargaining power and, thus, the outcome of open-ended alternating-offer bargaining is shown by Kohler [29,30]. Compassionate bargainers reach agreement in the first period. If guilt is strong, they divide the bargaining surplus equally. If guilt is weak, the bargaining outcome is tilted away from the Rubinstein [2] division towards a more unequal split. Also envious bargainers reach agreement in the first period. Bargaining shares increase in the strength of own envy. Envy can cause divisions more unequal than predicted by Rubinstein, but, if equally patient bargainers exhibit a similar degree of envy, then the bargaining outcome is tilted
away from the Rubinstein division towards the equal split. Similarly, envy’s more generic form—loss aversion—may be a disadvantage in bargaining [31]. When comparisons with others cause guilt and envy, the underlying preferences are often called inequality or inequity aversion. Inequality aversion can also increase the asymmetry of the payoff division in bargaining games like legislative bargaining games, in which unanimity is not required [32]. None of these papers consider the influence of incomplete information about social preferences on bargaining.

Bolton’s [3] paper on fairness preferences in bargaining is the most closely related work. He collected data in a large scale bargaining experiment and postulated a formal model, in which bargainers receive utility from two sources. One source is the amount of money earned from agreement while the other is a relative comparison of money earnings incorporated into the utility function as an index. Utility includes this fairness index and is defined such that it combines self-interest with an additional utility gain from obtaining a division closer to the equal division for the bargainer who receives the smaller share. If a player can achieve an advantageous share, then, *ceteris paribus*, an earlier period agreement is preferred to a later agreement. This comparative model is consistent with five enumerated bargaining regularities. First, offers deviate from the narrow self-interest equilibrium in the direction of the equal division. Second, opening offers are frequently rejected. Third, disadvantageous counteroffers frequently come after rejected first period offers. Fourth, there is a consistent proposer advantage. Fifth, discounting affects the outcome. The comparative model predicts all but delay of agreement under complete information. Bolton emphasized that the “comparative model assumes that subjects have complete information about one another’s utility function. In reality, however, they do not. [...] The marginal rate of substitution between absolute and relative money most likely varies by individual, making utility functions private information.” Evaluating the observed experimental behavior, Bolton developed an intuition how subjects handle the supposable preference uncertainty: “Proposers must search. [...] Searching proceeds roughly as follows. Based on a subject’s prior, he makes an offer. If it is rejected, he makes a more generous offer in the next period.” The following analysis of the optimal behavior under that kind of incomplete information about heterogeneous preferences formally elicits the screening and signaling of preferences within bargaining at the cost of occasional delay of agreement in equilibrium conjectured by Bolton.

### 3. Model

Within three periods, two players, a seller and a buyer of a good, have to reach agreement on the division of a trade surplus, which is normalized to one, by naming a price $p_t \in [0, 1]$. The price $p_t$ defines the share of the surplus that the seller receives and $1 - p_t$ is the residual share of the surplus received by the buyer in period $t \in \{1, 2, 3\}$. The surplus results from a potential buyer’s valuation of an object that exceeds the seller’s reservation price. It depreciates with a common discount factor $0 \leq \delta \leq 1$ after any disagreement.

Before the bargaining starts, nature moves by drawing the buyer preferences. By asking a price in periods 1 and 3, the seller proposes a sharing rule for the surplus that the buyer can accept or reject. The game ends once the buyer accepts a price. If the buyer rejects the seller’s price $p_1$ in period 1, then the buyer can propose a new sharing rule $p_2$ in period 2 that the seller can accept or reject. If the seller accepts the buyer’s counteroffer, the bargaining ends. If the buyer rejects the seller’s price in period 3, then the surplus shrinks to zero and the game ends.
The self-interested seller $s$ derives utility from his absolute payoff $u_s(p_t) = p_t$. The buyer $b \in \{c, e\}$ can be one of two types. With probability $q \in (0, 1)$ he is also self-interested (or competitive) and has utility $u_c(p_t) = 1 - p_t$. With probability $1 - q$ he cares about absolute and relative payoffs such that his utility is affected by a loss factor of infinity, interpreted as envy, when receiving less than half of the surplus:

$$u_e(p_t) = \begin{cases} 1 - p_t & \text{if } p_t \leq 0.5 \\ -\infty & \text{if } p_t > 0.5 \end{cases}$$

The buyer type is private information, but the seller knows the distribution of the preferences in the population and, at any time, holds a belief $\mu^t$ that the other bargaining party is self-interested. The utility of the envious buyer represents an asymmetric limiting case of inequality aversion, put forward by Fehr and Schmidt or Bolton and Ockenfels [27,28], and extensions thereof [33,34] or loss aversion, introduced by Kahneman and Tversky [35], with respect to an equal split. The model of a perfectly envious buyer and a self-interested seller is included in Bolton’s [3] comparative model. Its implied inflexible demands have been studied before in the context of bargaining and reputation (e.g., Abreu and Gul, Myerson [36,37]). By simplifying some distributional social preferences, the model links different literatures and gives a behavioral reinterpretation the previous ad hoc assumption of inflexible demands.

A game with a maximum length of three periods is the shortest game to study different length of delay and in which the seller, who is the first mover, can make one counteroffer. If players share a common discount factor, stronger bargaining power is with the proposing player that also collects at least half of the surplus in open-ended alternating-offer bargaining.

4. Perfect Bayesian Equilibrium

The model is solved for a pure strategy equilibrium. Due to the incomplete information, the solution concept applied is a perfect Bayesian equilibrium (PBE). The PBE is one of two types depending on whether the self-interested seller can distinguish a self-interested from an envious buyer. In a separating equilibrium, the seller can infer the buyer’s private information completely. In a pooling equilibrium, the seller learns nothing about the buyer’s preferences. Whether there is a pooling or separating equilibrium is determined by the probability $q$ of bargaining with a self-interested buyer, which is implied by the frequency of this preference in the population, and the players’ common discount factor $\delta$ (Figure 1).

Proposition 1. Let $(p_t, t)$ denote the bargaining outcome if a player accepts $p_t$ in period $t$:

(P) If $\delta \leq 0.5$ and $q \leq \frac{1 - \delta^2}{2 - 2\delta}$, if $0.5 < \delta \leq \frac{2}{3}$ and $q \leq \frac{1 - \delta^2}{2 - \delta(2 - \delta)}$ or if $q \leq 0.5$, then there is a pooling equilibrium. Its outcome is $(0.5, 1)$.

(S1) If $\delta \leq 0.5$ and $q > \frac{1 - \delta^2}{2 - 2\delta}$, then there is a separating equilibrium. Its outcome is $((1 - \delta(1 - \delta), 1)$ if the buyer is self-interested. Its outcome is $(\delta, 2)$ if the buyer is envious.

(S2) If $0.5 < \delta \leq \frac{2}{3}$ and $q > \frac{1 - \delta^2}{2 - \delta(2 - \delta)}$, then there is a separating equilibrium. Its outcome is $(1 - \delta (1 - \delta), 1)$ if the buyer is self-interested. Its outcome is $(0.5, 3)$ if the buyer is envious.

Proof Follows from lemmas 1–3 in the appendix.
**Proposition 2.** If $\delta > \frac{2}{3}$ and $0.5 < q < 1$, then no PBE in pure strategies exists.

**Proof.** See appendix.

**Proposition 3.** The PBE in pure strategies is unique if $\delta \leq \frac{2}{3}$ or $q \leq 0.5$, and if the seller’s belief remains unchanged, i.e., $\mu^t = \mu^{t-1}$, whenever Bayesian updating is not possible.

**Proof.** Solving the game by backward induction in lemmas 1–3 given the equilibrium beliefs rules out other equilibria in pure strategies. □

**Figure 1.** Perfect Bayesian equilibrium for $q \in (0, 1)$.

Notes: $P$ pooling equilibrium with no delay. $S_1$ separating equilibrium with one period delay if buyer is envious. $S_2$ separating equilibrium with two periods delay if buyer is envious. $\emptyset$ no pure strategy equilibrium.

The strategies and seller beliefs supporting the equilibrium are formally reported in the appendix. Figures 2(a)–2(c) also summarize them, grouped by parameter values, in the game tree of the bargaining problem. The game trees start with the seller decision in period 1, in which he offers to a self-interested buyer with probability $q$ or to an envious one with probability $1 - q$. The seller’s respective belief to bargain with a self-interested buyer is denoted in square brackets at his decision nodes. Players’ utility values (rather than payoffs) are denoted in parentheses. The information sets and utilities of minor interest are suppressed. The game is truncated after period 2, but relevant period 3 continuation games are represented below the dotted line. Optimal behavior and the equilibrium outcomes are emphasized. Red lines indicate the paths to the pooling and separating equilibrium outcomes, which are marked with a dotted and dashed border, respectively. Blue lines indicate continuation games that are only reached by mistake (black lines) in equilibrium. The behavior after a division not represented in the game tree equals the behavior illustrated in the continuation game of the incrementally higher seller/lower buyer reservation price depicted in the corresponding period.
Figure 2. (a) Pooling equilibrium. (b) Pooling or separating equilibrium with one period of delay. (c) Pooling or separating equilibrium with two periods of delay.

Notes: Seller $s$. Self-interested buyer $c$. Envious buyer $e$. Division $\pi^L := 1 - \delta(1 - 0.5\delta)$. Division $\pi^H := 1 - \delta(1 - \delta)$. Equilibrium strategies and outcome are emphasized. Red lines indicate the path to the pooling equilibrium outcome marked with a dotted border. Blue lines indicate continuation games reached by mistake (black lines) in equilibrium. Utility is denoted in parentheses. Belief is denoted in squared brackets. Information sets are not drawn.
Notes: Seller \( s \). Self-interested buyer \( c \). Envious buyer \( e \). Division \( \pi^L := 1 − \delta(1 − 0.5\delta) \). Division \( \pi^H := 1 − \delta(1 − \delta) \). Equilibrium strategies and outcomes are emphasized. Red lines indicate the paths to the pooling and separating equilibrium outcomes marked with a dotted and dashed border, respectively. Blue lines indicate continuation games reached by mistake (black lines) in equilibrium. Utility is denoted in parentheses. Belief is denoted in squared brackets. Information sets are not drawn.
Figure 2. Cont.

Notes: Seller $s$. Self-interested buyer $c$. Envious buyer $e$. Division $\pi^L := 1 - \delta(1 - 0.5\delta)$. Division $\pi^H := 1 - \delta(1 - \delta)$. Equilibrium strategies and outcomes are emphasized. Red lines indicate the paths to the pooling and separating equilibrium outcomes marked with a dotted and dashed border, respectively. Blue lines indicate continuation games reached by mistake (black lines) in equilibrium. Dashed lines indicate omitted utility values. Utility is denoted in parentheses. Belief is denoted in squared brackets. Information sets are not drawn.
Figure 2(a) corresponds to $q \leq 0.5$, the lower rectangle in Figure 1, in which the equilibrium outcome is pooling the buyer types through an immediate proposal of the equal division that is instantly accepted by both buyer types. Figure 2(b) corresponds to $q > 0.5$ and $\delta \leq 0.5$, the upper left rectangle in Figure 1, in which, depending on the exact parameter values, the equilibrium outcome is either pooling or separating buyer types by risking one period of delay. Figure 2(c) corresponds to $q > 0.5$ and $0.5 < \delta \leq \frac{2}{3}$, the upper middle rectangle in Figure 1, in which, depending on the exact parameter values, the equilibrium outcome is either pooling or separating buyer types by risking two periods of delay. The dashed paths in the period 2 continuation games indicate omitted utility values.

5. Discussion

For a low probability of bargaining with a self-interested buyer $q \leq 0.5$ the equilibrium strategies and belief support only a pooling outcome. Due to the low chance to bargain with a self-interested buyer, the expected utility of asking a price higher than half is never more than half. Thus, the seller prefers immediate agreement on the equal surplus division with all buyer types. In contrast, as it becomes more likely for the seller to bargain with a self-interested buyer, the equilibrium strategies and beliefs support either a pooling equilibrium or a separating outcome. If the proportion of self-interested buyers $q > 0.5$ is sufficiently large in relation to $\delta < \frac{2}{3}$, the seller maximizes his expected utility by offering such that only a self-interested buyer will accept in period 1. In such a separating strategy, delay results from disagreement and continuation of the bargaining with an envious buyer or an imitating self-interested type. If discounting $\delta \leq 0.5$ is low, imitation of envy is too costly for the self-interested buyer. He accepts the higher price $1 - \delta(1 - \delta)$ in the screening period 1. The envious buyer additionally signals his type by rejecting an unequal $p_1$. His period 2 counteroffer $\delta$ will be accepted by the seller. Thereby the envious buyer’s counteroffer is less than half of the pie but the seller prefers the low price to the equal division in the final period due to the low patience. If discounting $\delta \in (0.5; \frac{2}{3}]$ is moderate, it costs more time to exclude imitation by a self-interested type and agreement on equal division with an envious buyer follows in period 3 after he signaled his type by rejecting an unequal $p_1$ and offering an unacceptable price $p_2$ in period 2.

The preference between the pooling and separating outcomes depends on the seller’s patience in relation to the probability of bargaining with a self-interested buyer once the probability $q > 0.5$ is high. If $\delta \leq \frac{2}{3}$ and $q > 0.6$, then both separating outcome are preferred to the pooling outcome. Whether separating the buyer types in equilibrium requires one or two periods of delay depends merely on the discount factor $\delta$. As the perfectly envious buyer suffers infinitely from accepting any disadvantageous division, he never imitates self-interested behavior. In contrast, a self-interested buyer gains utility through being offered the equal division if he would pass undetected as an envious type. Therefore, the more patient the players become, the more time must pass between the disadvantageous opening price and the equal division that the seller would like to offer to the different buyers in order to separate buyer types.

If players are more patient, i.e., $\delta > \frac{2}{3}$, and the buyer is likely to be self-interested, i.e., $q > 0.5$, then a delay of two periods before dividing equally is no longer sufficient to deter the self-interested buyer from imitating the envious type. Yet, the seller does not maximize utility by offering the equal division to all buyer types for a sufficiently high chance to bargain with self-interested buyer. In an infinite horizon game, the incentive of a self-interested buyer to imitate envy could be eliminated by the seller and the
envious buyer through delaying their agreement on the equal division beyond two periods. In the finite horizon game, the scope for delay is limited through the length of the game. Nevertheless, the players can reduce the incentive of the self-interested buyer to imitate the envious type in addition to a physical delay by playing mixed strategies that exclude, at least sometimes, an agreement between the envious buyer and the seller in period 3. Such equilibria are not considered in the paper, but I established that for sufficiently high \( q \) and high \( \delta \) no pure strategy equilibria exist.

The model above postulates an uninformed self-interested seller and assumes perfect envy as the unknown social preferences of some buyers. The dynamics observed in this model, however, applies to a more general context. Suppose a different model of social preferences implied that some buyers accept also other divisions than the equal split while their behavior remains different from a self-interested buyer. Intuitively, when social preferences increase the attractiveness of the outside option to a buyer, then his obtainable share increases and the incentive of a self-interested buyer to imitate social preferences increases too. Therefore, in cases of lower envy, the seller has a lower incentive to separate the buyer types and a self-interested buyer has a reduced interest in imitating. That is, the seller’s incentive increases whereas a self-interested buyer’s incentive decreases the likelihood of observing the pooling equilibrium. For two bargainers with social preferences, Kohler [29,30] studied the impact of commonly known envy or guilt on the bargaining outcome in a model of open-ended alternating-offer bargaining. Even if both bargainers have similar social preferences, the bargaining outcome can differ from the division agreed by self-interested bargainers. The depicted deviations from the bargaining outcome of self-interested bargainers may cause imitating, screening and signaling when there is uncertainty about the opponent’s preferences like in the example of one-sided perfect envy.

6. Conclusion

This paper modeled heterogeneity of social preferences as the source of one-sided incomplete information in an alternating-offer bargaining problem. Assuming that a buyer who moves second is either perfectly envious (that is he rejects any offer less than the equal division of the trade surplus) or self-interested, equal as well as unequal division and occasional occurrence of delay have been rationalized in a perfect Bayesian equilibrium. In the studied three period game, the informed buyer may be screened by the uninformed seller in the first period, in which case the seller demands the larger share of the surplus accepted only by a self-interested buyer. An envious buyer is willing to forgo immediate agreement in order to signal his type. As the cost of learning the buyer type is one or more periods of delay if bargaining is with an envious buyer, the seller may not be interested in screening or receiving a signal. Therefore, two categories of optimal seller behavior exist depending on the players’ patience and the seller’s believed likeliness to encounter a buyer with the respective preferences. Confronted with a low probability to realize a large share against a self-interested buyer, the seller backs off from learning the buyer preferences and instantly offers the equal division, which is always accepted immediately in this pooling equilibrium. As the probability to bargain with a self-interested buyer increases, the risk-neutral expected utility maximizing seller no longer prefers immediate acceptance of all buyer types. If the probability of bargaining with a self-interested buyer is high (or the seller believes it is high), then the seller initially demands a large share and first period rejection by an envious buyer is observed. Through the delay of agreement the envious buyer signals his type. With relatively patient bargainers, a second
rejection may be observed as the envious buyer must make an unacceptable counteroffer, after rejecting the seller’s opening offer, to deter the self-interested buyer from worthwhile imitating and to thereby signal his type. Unsettled bargaining goes on and ends with the equal division at latest in period 3 although in both cases the buyer has revealed his envious type by his period 2 offer. If discounting and the probability to bargain with a self-interested buyer are high, the finite game is too short to send a credible signal about the uncertain preferences and no pure strategy equilibrium exists. In addition to divisions more equal than forecast by Rubinstein [2] and delay, the model reproduces further regularities of bargaining behavior not explicitly discussed in the paper: The bargaining outcome, inter alia, depends on the bargainers’ impatience. The payoff distribution on the equilibrium paths uniquely favors the proposing player who demands at least half of the surplus. In a separating equilibrium the seller eventually accepts disadvantageous counteroffers by an envious buyer or asks for less than the equal division himself after he learned the buyer type. As the highly stylized model of social preferences allows for alternative interpretations, relaxing the assumption of perfect envy (or its alternative interpretations) in different directions may help to either further discriminate or to unify competing explanations of bargaining behavior in future research.

Acknowledgments

I thank Karl Schlag, Pascal Courty, Simon Gächter, Jordi Brandts, Ulrich Berger and the anonymous reviewers for helpful and constructive comments. I gratefully acknowledge the financial support provided by the European University Institute and the German Academic Exchange Service at an early stage of this research.

Appendix

Definition. A perfect Bayesian equilibrium (PBE) is a strategy pair \((\hat{s}_s, \hat{s}_b)\) and a set of beliefs of the seller that the buyer is self-interested satisfying the following requirements:

(E1) The seller’s equilibrium strategy \(\hat{s}_s\) maximizes the seller’s discounted expected utility given the buyer’s equilibrium strategy \(\hat{s}_b\).

(E2) For each buyer type \(b \in \{c, e\}\), the buyer’s equilibrium strategy \(\hat{s}_b\) maximizes the buyer’s discounted utility given the seller’s equilibrium strategy \(\hat{s}_s\).

(E3) The seller belief \(\hat{\mu}_t\) in period \(t\) is formed by Bayes’ theorem from the equilibrium strategies at each information set.

Notation \(u^t_i(s_j | \tilde{s}^t_j)\) indicates the utility \(u^t_i\) of player \(i\) in period \(t\) if player \(j\) adopts his strategy \(s_j = \{s^t_1, s^t_2, s^t_3\}\), in which the \(t\)-th element is substituted by \(\tilde{s}^t_j\) given the other player’s equilibrium strategy \(\hat{s}_i\). As \(\partial u^t_b / \partial p_t \leq 0\) and \(\partial u^t_b / \partial p_t > 0\), it is sufficient to look at the maximal/minimal prices at which behavior of the buyer types/seller changes. \(\pi^H := 1 - \delta(1 - \delta)\) denotes the division that corresponds to the backwards induction outcome of a three period bargaining game with a self-interested buyer. \(\pi^L := 1 - \delta(1 - 0.5\delta)\) denotes the division a self-interested buyer just accepts in period 1 if he could obtain the share of an envious buyer in the continuation game.
Proof of Proposition 1

The proof of proposition 1 follows from three lemmas. Lemma 1 establishes the PBE for \( q \leq 0.5 \) (Figure 2(a)), lemma 2 for \( q > 0.5 \) and \( \delta \leq 0.5 \) (Figure 2(b)) and lemma 3 for \( q > 0.5 \) and \( \delta \in (0.5, \frac{2}{3}] \) (Figure 2(c)). In each lemma, I show for each player and type that there is no profitable deviation from the equilibrium strategy, emphasized in the figures, given the beliefs at the decision nodes and the equilibrium strategy of the other player. Afterwards, I prove that the assumed beliefs follow from Bayes’ theorem given the equilibrium strategies.

**Lemma 1.** If \( q \leq 0.5 \), then there is a pooling equilibrium with outcome \((0.5,1)\). The equilibrium is supported by the strategy pair \((\hat{s}_s, \hat{s}_b)\) and beliefs \(\hat{\mu}^t\) given in Table 1.

<table>
<thead>
<tr>
<th>The seller’s equilibrium strategy (\hat{s}_s) and belief (\hat{\mu}^t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(t)</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The buyer’s equilibrium strategy (\hat{s}_b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(t)</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

**Proof** I show that the strategy pair \((\hat{s}_s, \hat{s}_b)\) and belief \(\hat{\mu}^t\) satisfy the PBE requirements:

(E1) In period 3, both buyer types maximize utility by accepting the equal division. Given \(\hat{\mu}^3 \in \{0, q\}\) and \(q \leq 0.5\), the seller maximizes expected utility with \(\hat{s}_s^3 = 0.5\) because \(u^3_s(s_0 \backslash 0.5) = 0.5\) is weakly larger than \(E u^3_s(\pi_s \backslash 1) = \hat{\mu}^3\). In period 2, the utility of \(s^2_c \in \{Y, N\}\) is \(u^2_c = p_2\) and \(u^2_c = 0.5\delta\), respectively. Hence, \(Y \succeq_s N\) if \(p_2 \geq 0.5\delta\) as implied by \(\hat{s}_s^2\). In period 1, the utility of \(s^1_c \in \{0.5, \pi^L, 1\}\) is maximized by \(\hat{s}_s^1 = 0.5\) as \(u^1_c(s_0 \backslash 0.5) = 0.5\), \(E u^1_s(\pi_s \backslash 1) = \hat{\mu} \pi^L + 0.5\delta^2(1 - \hat{\mu}) \leq 0.5\) if \(\hat{\mu} \leq 0.5\) and \(E u^1_s(\pi_s \backslash 1) = 0.5\delta^2 \leq 0.5\).

(E2) In period 3, the self-interested buyer’s best response \(\hat{s}_c^3\) is to accept any price as, otherwise, the bargaining ends in disagreement with zero payoff. In period 2, the utility of \(s^2_c \in \{0, 0.5\delta\}\) is maximized by \(\hat{s}_c^2 = 0.5\delta\) as \(u^2_c(s_0 \backslash 0.5) = 1 - 0.5\delta\) is larger than \(u^2_c(s_0 \backslash 0) = 0.5\delta\). In period 1, the utility of \(s^1_c \in \{Y, N\}\) is \(u^1_c(s_0 \backslash Y) = 1 - p_1\) and \(u^1_c(s_0 \backslash N) = \delta(1 - 0.5\delta)\), respectively. Hence, \(Y \succeq_c N\) if \(p_1 \leq \pi^L\) as implied by \(\hat{s}_c^1\). In period 3, the envious buyer’s best response \(\hat{s}_c^3\) is to accept any price equal or less than half and to reject otherwise as, afterwards, the bargaining ends with zero payoff to both. In period 2, \(\hat{s}_c^2\) equals \(\hat{s}_c^2\) because payoffs are larger than half and similar for both buyer types in this subgame. In period 1, the utility of \(s^1_c \in \{Y, N\}\) is \(u^1_c(s_0 \backslash Y) = 1 - p_1\) if
\( p_1 \leq 0.5, \text{ else } -\infty, \) and \( u^1(s_e\backslash N) = \delta(1 - 0.5\delta) \), respectively. Hence, \( Y \geq e \) if \( p_1 \leq 0.5 \) as implied by \( \hat{s}^3 \).

(E3) In period 1, the seller’s unconditional belief is the frequency of the self-interested buyer in the population. Both buyer types accept an price \( p_1 \leq 0.5 \) and reject \( p_1 \in (\pi^L, 1] \). In period 2, after rejection of \( p_1 \notin (0.5, \pi^L] \) both types make counteroffer \( p_2 = 0.5\delta \) and the seller receives no information, i.e., \( \hat{\mu}^2 = q \). If \( p_1 \in (0.5, \pi^L] \), then only the self-interested buyer accepts and, hence, \( \hat{\mu}^2 = 0 \). In period 3, the seller moves between information sets and receives no information. \( \square \)

Lemma 2. If \( q \in (0.5, \frac{1 - 2\delta^2}{2 - 2\delta}] \) and \( \delta \leq 0.5 \), then there is a pooling equilibrium with outcome \((0.5, 1)\). If \( q > \frac{1 - 2\delta^2}{2 - 2\delta} \) and \( \delta \leq 0.5 \), then there is a separating equilibrium. Its outcome is \((\pi^H, 1)\) if bargaining is with a self-interested buyer. Its outcome is \((\delta, 2)\) if bargaining is with an envious buyer. The equilibrium is supported by the strategy pair \((\hat{s}_s, \hat{s}_b)\) and beliefs \( \hat{\mu}^t \) given in Table 2.

Table 2. Strategies and beliefs if \( q \in (0.5, \frac{1 - 2\delta^2}{2 - 2\delta}] \) and \( \delta \leq 0.5 \).

<table>
<thead>
<tr>
<th>The seller’s equilibrium strategy ( \hat{s}_s ) and belief ( \hat{\mu}^t )</th>
<th>( t )</th>
<th>( \hat{s}_s^t )</th>
<th>( \hat{\mu}^t )</th>
<th>History</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5 if ( q \in (0.5, \frac{1 - 2\delta^2}{2 - 2\delta}] ), else ( \pi^H )</td>
<td>( q )</td>
<td>( q )</td>
<td>( p_1 \notin (0.5, \pi^H] )</td>
</tr>
<tr>
<td>2</td>
<td>( Y ) if ( p_2 \geq \delta ), else ( N )</td>
<td>( q )</td>
<td>( q )</td>
<td>( p_1 \in (\pi^L, \pi^H] )</td>
</tr>
<tr>
<td>( Y ) if ( p_2 \geq \delta ), else ( N )</td>
<td>0 if ( p_2 = \delta ), else ( q )</td>
<td>( p_1 \in (0.5, \pi^L] )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Y ) if ( p_2 \geq 0.5\delta ), else ( N )</td>
<td>0 if ( p_2 = 0.5\delta ), else ( q )</td>
<td>( p_1 \in (0.5, \pi^L] )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>( \mu^2 )</td>
<td>( \mu^2 )</td>
<td>( p_1 \in (\pi^L, \pi^H] )</td>
</tr>
<tr>
<td>0.5 if ( p_2 = \delta ), else 1</td>
<td>( \mu^2 )</td>
<td>( \mu^2 )</td>
<td>( p_1 \in (\pi^L, \pi^H] )</td>
<td></td>
</tr>
<tr>
<td>0.5 if ( p_2 = 0.5\delta ), else 1</td>
<td>( \mu^2 )</td>
<td>( \mu^2 )</td>
<td>( p_1 \in (0.5, \pi^L] )</td>
<td></td>
</tr>
</tbody>
</table>

The buyer’s equilibrium strategy \( \hat{s}_b \)

<table>
<thead>
<tr>
<th>( t )</th>
<th>( \hat{s}_b^t )</th>
<th>( \hat{s}_c^t )</th>
<th>History</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( Y ) if ( p_1 \leq \pi^H ), else ( N )</td>
<td>( Y ) if ( p_1 \leq 0.5 ), else ( N )</td>
<td>( \hat{s}_c^t )</td>
</tr>
<tr>
<td>2</td>
<td>( \delta )</td>
<td>( \delta )</td>
<td>( \hat{s}_c^t )</td>
</tr>
<tr>
<td>( 0.5\delta )</td>
<td>( 0.5\delta )</td>
<td>( \hat{s}_c^t )</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>( Y ) if ( p_3 \leq 0.5 ), else ( N )</td>
<td>( \hat{s}_c^t )</td>
<td></td>
</tr>
<tr>
<td>Any</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Proof I show that the strategy pair \((\hat{s}_s, \hat{s}_b)\) and belief \( \hat{\mu}^t \) satisfy the PBE requirements:

(E1) In period 3, the action of one or both buyer types changes if \( s^3_s \in \{0.5, 1\} \). Given \( \hat{\mu}^3 \in \{0, q\} \) and \( q > 0.5 \), the seller maximizes expected utility \( Eu^s(s_b|\hat{\mu}^t) \) with \( \hat{s}^3_s \) because \( u^3(s_e\backslash 0.5) = 0.5 \) is smaller than \( Eu^3(s_e\backslash 1) = q \) if \( \hat{\mu}^t = q \), i.e., for histories other than \( p_1 \in (0.5, \pi^L] \) and \( p_2 = 0.5\delta \) or \( p_1 \in (\pi^L, \pi^H] \) and \( p_2 = \delta \). In period 2, the utility of \( s^2_s \in \{Y, N\} \) is \( u^2_s = p_2 \) and \( Eu^2_s = 0.5\delta \) or \( Eu^2_s = \hat{\mu}\delta \), respectively. Hence, \( Y \geq s \) if \( p_1 \in (0.5, \pi^L] \) and \( p_2 > 0.5\delta \) or \( p_1 \in (\pi^L, \pi^H] \) and \( p_2 = \delta \) as implied by \( \hat{s}^2_s \). In period 1, the utility of \( s^1_s \in \{0.5, \pi^L, \pi^H\} \) is maximized by \( \hat{s}^1_s = 0.5 \) if \( q \leq \frac{1 - 2\delta^2}{2 - 2\delta} \) and by \( \hat{s}^1_s = \pi^H \) otherwise as, for \( \hat{\mu}^1 = q \) and \( q > 0.5 \), the expected utility \( Eu^1(s_e\backslash \pi^L) = \hat{\mu}\pi^L + 0.5\delta^2(1 - \hat{\mu}) \leq 0.5 \) is weakly smaller than \( u^1(s_e(0.5) = 0.5 \) and \( Eu^1(s_e\backslash \pi^H) = q\pi^H + (1 - q)\delta^2 \). The comparison of latter implies the threshold.
In period 3, the self-interested buyer’s best response $\hat{s}_b^3$ is to accept any price as, otherwise, the bargaining ends in disagreement with zero payoff. In period 2, the utility of $s_c^2 \in \{0, 0.5\delta, \delta\}$ is maximized by $\hat{s}_c^2 = 0.5\delta$ if $p_1 \in (0.5, \pi^L]$ and by $\hat{s}_c^2 = \delta$ otherwise as, depending on the history, these prices are the discounted period 3 agreements just accepted by the seller who rejects $s_c^2 = 0$ giving the buyer utility $u^2_c(s_c \mid 0) = 0$. In period 1, the utility of $s_c^1 \in \{Y, N\}$ is $u^1_c(s_c \mid Y) = 1 - p_1$ and $u^1_c(s_c \mid N) = \delta(1 - 0.5\delta)$, respectively. Hence, $Y \succeq_e N$ if $p_1 \leq \pi^L$ as implied by $\hat{s}_c^1$. In period 3, the envious buyer’s best response $\hat{s}_c^3$ is to accept any price equal or less than half and to reject otherwise as, afterwards, the bargaining ends with zero payoff to both. In period 2, $\hat{s}_c^2$ equals $\hat{s}_c^2$ because payoffs are larger than half and similar for both buyer types in this subgame. In period 1, the utility of $s_c^1 \in \{Y, N\}$ is $u^1_c(s_c \mid Y) = 1 - p_1$ if $p_1 \leq 0.5$, else $-\infty$, and $u^1_c(s_c \mid N) = \delta(1 - 0.5\delta)$, respectively. Hence, $Y \succeq_e N$ if $p_1 \leq 0.5$ as implied by $\hat{s}_c^1$.

In period 1, the seller’s unconditional belief is the frequency of the self-interest buyer in the population. Both buyer types accept a price $p_1 \leq 0.5$ and reject $p_1 \in (\pi^H, 1]$. In period 2, after rejection of $p_1 \notin (0.5, \pi^H]$ both types make the same counteroffer $p_2 \in \{0.5\delta, \delta\}$ and the seller receives no information, i.e., $\hat{\mu}^2 = q$. If $p_1 \in (0.5, \pi^H]$, then only the self-interested buyer accepts $p_1 \in (0.5, \pi^H]$. Hence, $\hat{\mu}^2 = 0$ if his acceptance is followed by rational action. In period 3, the seller moves between information sets and receives no information.

Lemma 3. If $q \in (0.5, \frac{1-\delta^2}{2-\delta(2-\delta)})$ and $\delta \in (0.5, \frac{2}{3}]$, then there is a pooling equilibrium with outcome $(0.5, 1)$. If $q > \frac{1-2\delta^2}{2-\delta(2+\delta)}$ and $\delta \in (0.5, \frac{2}{3}]$, then there is a separating equilibrium. Its outcome is $(\pi^H, 1)$ if bargaining is with a self-interested buyer. Its outcome is $(0.5, 3)$ if bargaining is with an envious buyer. The equilibrium is supported by the strategy pair $(\hat{s}_a, \hat{s}_b)$ and beliefs $\hat{\mu}^t$ given in Table 3.

<table>
<thead>
<tr>
<th>$t$</th>
<th>$\hat{s}_a^t$</th>
<th>$\hat{\mu}^t$</th>
<th>History</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5 if $q \in (0.5, \frac{1-\delta^2}{2-\delta(2-\delta)})$, else $\pi^H$</td>
<td>$q$</td>
<td>$\pi^H$</td>
</tr>
<tr>
<td>2</td>
<td>$Y$ if $p_2 \geq \delta$, else $N$</td>
<td>0 if $p_2 = 0$, 1 if $p_2 = \delta$, else $q$</td>
<td>$p_1 \notin (0.5, \pi^L]$</td>
</tr>
<tr>
<td></td>
<td>$Y$ if $p_2 \geq 0.5\delta$, else $N$</td>
<td>0 if $p_2 = 0.5\delta$, else $q$</td>
<td>$p_1 \in (0.5, \pi^L]$</td>
</tr>
<tr>
<td>3</td>
<td>0.5 if $p_2 = 0$, else 1</td>
<td>$\mu^2$</td>
<td>$p_1 \notin (0.5, \pi^L]$</td>
</tr>
<tr>
<td></td>
<td>0.5 if $p_2 = 0.5\delta$, else 1</td>
<td>$\mu^2$</td>
<td>$p_1 \in (0.5, \pi^L]$</td>
</tr>
</tbody>
</table>
Table 3. Cont.

The buyer’s equilibrium strategy \( \hat{s}_b \)

<table>
<thead>
<tr>
<th>( t )</th>
<th>( s^1_t )</th>
<th>( s^2_t )</th>
<th>History</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( Y ) if ( p_1 \leq \pi_H ), else ( N )</td>
<td>( Y ) if ( p_1 \leq 0.5 ), else ( N )</td>
<td>–</td>
</tr>
<tr>
<td>2</td>
<td>( \delta )</td>
<td>0</td>
<td>( p_1 \notin (0.5, \pi_L] )</td>
</tr>
<tr>
<td>0.5( \delta )</td>
<td>0.5( \delta )</td>
<td>( p_1 \in (0.5, \pi_L] )</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>( Y ) if ( p_3 \leq 0.5 ), else ( N )</td>
<td>Any</td>
<td></td>
</tr>
</tbody>
</table>

**Proof** I show that the strategy pair \((\hat{s}_s, \hat{s}_b)\) and belief \( \hat{p}^t \) satisfy the PBE requirements:

- (E1) In period 3, the action of one or both buyer types changes if \( s^3_s \in \{0.5, 1\} \). Given \( \hat{p}^3 \in \{0, q, 1\} \) and \( q > 0.5 \), the seller maximizes expected utility with \( \hat{s}^3_b \) because \( u^3_s(s_0 \setminus 0.5) = 0.5 \) is smaller than \( Eu^2_s(s_1 \setminus 1) = \hat{\pi}^3 \in \{q, 1\} \), i.e., for histories other than \( p_1 \notin (0.5, \pi_L] \) and \( p_2 = 0 \) or \( p_1 \in (0.5, \pi_L] \) and \( p_2 = 0.5\( \delta \). In period 2, the utility of \( s^2_s \in \{Y, N\} \) is \( u^2_s = p_2 \) and \( Eu^2_s = 0.5\( \delta \) or \( Eu^2_s = \hat{\mu}\( \delta \), respectively. Hence, \( Y \geq s \) if \( p_1 \in (0.5, \pi_L] \) and \( p_2 \geq 0.5\( \delta \) or \( p_1 \notin (0.5, \pi_L] \) and \( p_2 = \delta \) as implied by \( \hat{s}^2_b \). In period 1, the utility of \( s^1_s \in \{0.5, \pi_L, \pi_H\} \) is maximized by \( \hat{s}^1_s = 0.5 \) if \( q \leq \frac{1-\delta^2}{2-\pi(2-\delta)} \) and by \( \hat{s}^1_s = \pi_H \) otherwise as, for \( \hat{\mu}^1 = q \) and \( q > 0.5 \), the expected utility \( Eu^1_s(s_0 \setminus \pi_L) = \hat{\mu}\pi_L + 0.5\( \delta \) \left(1 - \hat{\mu}\right) \leq 0.5 \) is weakly smaller than \( u^1_s(s_0 \setminus 0.5) = 0.5 \) and \( Eu^1_s(s_0 \setminus \pi_H) = q\pi_H + 0.5\( \delta \) \left(1 - q\right) \). The comparison of latter implies the threshold.

- (E2) In period 3, the self-interested buyer’s best response \( \hat{s}^3_b \) is to accept any price as, otherwise, the bargaining ends in disagreement with zero payoff. In period 2, for \( \delta \leq \frac{q}{2} \), the utility of \( s^2_s \in \{0, 0.5\( \delta \), \( \delta \} \) is maximized by \( \hat{s}^2_s = 0.5\( \delta \) if \( p_1 \in (0.5, \pi_L] \) and by \( \hat{s}^2_s = \delta \) otherwise as, depending on the history, these prices are the discounted period 3 agreements just accepted by the seller who rejects \( s^2_s = 0 \) giving the buyer utility \( u^2_s(s_0 \setminus 0) = 0.5\( \delta \). In period 1, the utility of \( s^1_s \in \{Y, N\} \) is \( u^1_s(s_0 \setminus Y) = 1 - p_1 \) and \( u^1_s(s_0 \setminus N) = \delta(1-\delta) \), respectively. Hence, \( Y \geq s \) if \( p_1 \leq \pi_H \) as implied by \( \hat{s}^1_s \). In period 3, the envious buyer’s best response \( \hat{s}^3_b \) is to accept any price equal or less than half and to reject otherwise as, afterwards, the bargaining ends with zero payoff to both. In period 2, the envious buyer utility of asking an acceptable price \( p_2 \) may be lower than the utility he gets from a rejection if he can thereby signal his type. The utility of \( s^2_s \in \{0, 0.5\( \delta \), \( \delta \} \) is maximized by \( \hat{s}^2_s = 0.5\( \delta \) if \( p_1 \in (0.5, \pi_L] \) and by \( \hat{s}^2_s = 0 \) otherwise because, depending on the history, \( u^2_s(s_0 \setminus 0.5\( \delta \)) = 1 - 0.5\( \delta \) is larger than \( u^2_s(s_0 \setminus 0) = 0 \) if \( p_1 \in (0.5, \pi_L] \) but \( u^2_s(s_0 \setminus \delta) = 1 - \delta \) is smaller than \( u^2_s(s_0 \setminus 0) = 0.5\( \delta \) if \( p_1 \notin (0.5, \pi_L] \). In period 1, the utility of \( s^1_s \in \{Y, N\} \) is \( u^1_s(s_0 \setminus Y) = 1 - p_1 \) if \( p_1 \leq 0.5 \), else \( -\infty \), and \( u^1_s(s_0 \setminus N) = \delta(1-0.5\( \delta \)) \), respectively. Hence, \( Y \geq s \) if \( p_1 \leq 0.5 \) as implied by \( \hat{s}^1_s \).

- (E3) In period 1, the seller’s unconditional belief is the frequency of the self-interest buyer in the population. Both buyer types accept a price \( p_1 \leq 0.5 \) and reject \( p_1 \in (\pi_H, 1] \). In period 2, after a \( p_1 \in (0.5, \pi_L] \) both types would make the same counteroffer \( p_2 = 0.5\( \delta \) but the seller updates his belief on the equilibrium path to \( \hat{\mu}\( \delta \) = 0 \) from the self-interested buyer’s acceptance of this price. If \( p_1 \notin (0.5, \pi_L] \), each types may make a different counteroffer \( p_2 \in \{0, \delta \} \) and the seller updates his belief to \( \hat{\mu}\( \delta \) = 0 \) after \( p_2 = 0 \), made only by the envious buyer, and to \( \hat{\mu}\( \delta \) = 1 \) after \( p_2 = \delta \),
made only by the self-interested buyer. In period 3, the seller moves between information sets and receives no information. □

Proof of Proposition 2

I show that the best pure strategy of the seller is dominated by a mixed strategy for \( q > 0.5 \) and \( \delta > \frac{2}{3} \).

For \( \delta > \frac{2}{3} \), the self-interested buyer will reject \( p_1 = \pi^H \) if he can realize either outcome \((0.5\delta, 2)\) or outcome \((0.5, 3)\) in a later period pooling equilibrium. An optimal strategy of the seller \( \hat{s}_s \), however, must include a starting price that is at least accepted by the self-interested buyer. Thus, the seller’s best pure strategy candidate either accepts \( p_1 = 0.5 \) in a pooling strategy, or it asks \( p_1 = \pi^L \) in a separating strategy, or it asks \( p_1 = \pi^H \), accepts \( p_2 \geq \delta \) and demands \( p_3 = 1 \) in another separating strategy. Comparing the seller utility of the two separating strategies, \( q\pi^L + 0.5(1 - q)\delta^2 < q\pi^H \) implies that immediate agreement on \( \pi^H \) at the cost of excluding the envious buyer from trade is preferred. Yet, the seller can further gain utility by choosing a mixed separating strategy \( \tilde{s}_s \) that divides equally in period 3 with probability \( l \) such that the envious buyer accepts sometimes but the self-interested buyer continues to weakly prefer immediate acceptance. That is, \( E\tilde{u}_s^1(s_s \backslash \tilde{s}_s^3) = q\pi^H + 0.5l(1 - q)\delta^2 > q\pi^H \) with probability \( l \) such that \( u_1^1(s_s \backslash \pi^H, s_c \backslash Y) = \pi^H \) is just equal to \( u_1^1(s_s \backslash \pi^H, s_c \backslash N) = 0.5l\delta^2 \). In period 1, the seller utility is maximized by the equal division if \( q < \frac{1 - l\delta^2}{2\pi^H - l\delta^2} \) and by the mixed separating strategy \( s_s \backslash \tilde{s}_s^3 \) otherwise. The comparison of \( u_1^1(s_s \backslash 0.5) = 0.5 \) and \( E\tilde{u}_s^1(s_s \backslash \tilde{s}_s^3) \) implies the threshold. The preference for a pooling strategy is bounded by \( \lim_{l \to 0} \frac{1 - l\delta^2}{2\pi^H - l\delta^2} = \frac{1}{2\pi^H} \) (dashed red curve in Figure 1). As sequential rationality requires the mixed strategy in the separating continuation game even if the pooling outcome is preferred, no PBE exists in pure strategies for \( \delta > \frac{2}{3} \) and \( q > 0.5 \). □

References


© 2012 by the author; licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution license (http://creativecommons.org/licenses/by/3.0/.)