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Self-enforcing capital tax coordination

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Self-enforcing capital tax coordination*

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Abstract
Capital tax competition is known to result in inefficiently low tax rates and an undersupply of public goods. The provision of public goods and with it the welfare of all countries can be enhanced via tax coordination. Based on the standard Zodrow-Mieszkowski-Wilson tax-competition model this paper analyses the conditions under which tax coordination by a group of countries is self-enforcing. It is shown that there always exists a rather small stable tax coalition. For some subset of the parameter space the grand coalition may be stable as well, even if the total number of countries is large. The small stable coalition is not very effective in mitigating the inefficiency of the non-cooperative Nash equilibrium. The ineffectiveness is increasing in the total number of countries.

JEL classification: C72, H70, H73
Key words: tax coordination, tax competition, coalition, self-enforcing

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1 Introduction

During the past decades, international economic integration and capital mobility have increased significantly, and median statutory corporate tax rates decreased between 1980 and 2010 from about 45% to about 25% (Keen and Konrad 2012, Figure 1). The basic model of capital tax competition (Zodrow and Mieszkowski 1986, Wilson 1986) provides an explanation for the downward trend of capital taxes in response to increasing capital mobility. In that model, benevolent governments provide national public goods financed through capital taxes, and tax rates are inefficiently low in the resulting non-cooperative Nash equilibrium (= business as usual, BAU). The inefficiency arises, because all countries make strategic use of their tax rates. On the one hand, capital importing [exporting] countries gain by choosing high [low] tax rates. On the other hand, increasing [reducing] the tax rate erodes [expands] the tax base. That tax base effect changes the country’s tax revenues, ceteris paribus, and with them the provision of public goods. Since each country seeks to avoid the erosion of its tax base, capital tax competition becomes a ‘race toward low tax rates’ and thus results in an undersupply of public goods in all countries.

Since tax rates are too low in BAU, all countries can obviously gain, in principle, through suitable tax rate hikes.\(^1\) However, as there is no benevolent and omniscient central planner, increasing the average level of tax rates requires some international \textit{coordinated} action. Up to now, there have been no serious \textit{global} negotiations toward coordinating international capital taxes. On the smaller scale of European economic integration, capital tax coordination or harmonization has vividly and controversially been debated ever since the Treaties of Rome 1957. But to date, the EU failed to reach the unanimous approval required by the Treaty of the European Union. The Treaty also provides the option to bypass the unanimity mandate of all member states through so-called Enhanced Cooperation Agreements that allow a minimum of nine member states to cooperate without the other member states participating. Such agreements have already been successfully concluded in the realm of judicial cooperation, criminal matters, security policy and defense. However, no such agreement exists for capital taxation which suggests that the barriers to voluntary capital tax coordination are particularly high.

In the present paper we model policy coordination as coalition formation and envisage tax coalitions as subgroups of countries signing a tax agreement and committing to a co-

\(^1\)There is a tax rate higher than the average tax rate in BAU such that the maximum aggregate welfare (= social optimum) would be attained, if all countries would adopt that rate. However, to make all countries better off in the transition from BAU to the social optimum one may need suitable transfers, if countries are heterogeneous. See Bucovetsky and Smart (2006) and Hindriks et al. (2008).
operative tax policy that promotes the aggregate coalition welfare (in a way to be specified later). The crucial question is what the countries’ incentives are for policy coordination through coalition formation.

To our knowledge Burbidge et al. (1997) present the only systematic formal analysis of coalition formation in a model with capital tax competition and heterogeneous countries (but without public goods). In their model an equilibrium coalition structure is a demanding game theoretical concept in which each coalition takes all possible alternative coalition structures into account. They show that with two countries the unique equilibrium structure is the grand coalition, while with more than two countries other coalition structures may form. Owing to the inherent complexity of coalition structures, it is extremely difficult to obtain informative results. "Understanding how and which subgroups might form becomes a challenging question" (Keen and Konrad 2012, p. 37) which has not yet been answered satisfactorily. Our paper aims at improving the understanding of coalition formation.

An important intermediate step toward that goal is to compare the outcome of BAU with that of a given coalition structure. That is done by Konrad and Schjelderup (1999) in a model of identical countries and rather general functional forms. They find that a coalition of a subset of countries increases the welfare of all countries compared to BAU, if tax rates are strategic complements. Bucovetsky (2009) considers heterogeneous countries and shows in a parametric model that if two or more formerly independent coalitions set a common tax rate the average tax rate of all countries increases and also enhances the welfares of all countries outside the merged coalitions. The countries' welfare positions in given coalition structures will also be studied systematically in the present paper because they are an important precondition for the analysis of coalition formation. However, in contrast to Konrad and Schjelderup (1999) and Bucovetsky (2009) we then proceed to the analysis of coalition formation.

Altogether, the literature on tax coordination among a subset of countries is rather small (Keen and Konrad 2012, Section 3.3), and offers only limited insights due to the formidably complex analysis of coalition formation. To make progress, we will choose

\footnote{Vrijburg and de Mooij (2010) set up a three-country model and generalize the analysis of Konrad and Schjelderup (1999) by allowing heterogeneous countries and show that the key results in standard tax competition models regarding the impact of global tax harmonization do not carry over to partial tax harmonization in a subset of countries.}

\footnote{Bucovetsky (2009) also presents a sufficient condition under which the grand coalition Pareto-dominates all other coalition structures. We will refer to that interesting result in Section 4 below.}

\footnote{Cardarelli et al. (2002), Cantenaro and Vidal (2006) and Itaya et al. (2008) study tax coordination of two asymmetric countries in a repeated game setting.}
a highly simplified approach combining the symmetry assumption made by Konrad and Schjelderup (1999) with the parametric functional forms employed by Bucovetsky (2009). Although asymmetric countries certainly are an important feature in the real world, symmetry has been a prominent assumption in numerous tax competition studies for reasons of tractability and for the benefit of a clear focus on the problem at hand. Although symmetry is a very strong assumption, we find it acceptable in the present context because the symmetric BAU allocation is still inefficient due to the tax base effect combined with the institutional setup of public goods being entirely financed by capital tax revenues. Thus studying the welfare-enhancing potential of coalition formation remains a relevant issue even in a world of identical countries.

The analytical relief provided by the symmetry assumption is a drastic reduction in the set of relevant coalition structures. With symmetry, it suffices to consider a single coalition of alternative size along with the countries outside the coalition, called fringe countries. The coalition is assumed to set uniform tax rates for its members to maximize the aggregate coalition welfare while playing Nash against all fringe countries. The fringe countries play Nash against the coalition and against all fellow fringe countries, as in BAU. The question then is whether a coalition will form and, if so, how many members it will have. An obvious minimum requirement for the formation of a coalition is that it makes all members better off than in BAU. That will be shown to hold for any given coalition size. We also show that if a coalition exists, the fringe countries gain compared to BAU and compared to coalition countries. That may create an incentive for coalition countries to defect such that it is not clear whether an individual country is better off inside or outside the coalition. Since agreements among sovereign states cannot be credibly enforced, countries will choose to be members of a coalition - be it global or sub-global - if and only if it is in their own interest. Hence coordination agreements will be concluded, if and only if they are self-enforcing in the sense that no coalition member gains by leaving the coalition (internal stability) and no non-member gains by joining the coalition (external stability).

The objective of the present paper is to investigate the conditions under which stable tax coalitions exist and to characterize them with regard to the model parameters, in particularly regarding their size. We prove that compared to BAU in coalition-fringe equilibria all countries gain, that fringe countries gain more than coalition countries, and that all countries’ gains are strictly increasing in the coalition size. Then we specify in detail, how the equilibrium allocations (capital inputs, public goods, tax rates, welfare levels etc.)

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5 That concept of coalition stability was first employed in the theory of cartels (D’Aspremont et al. 1983). In the literature there are other and more sophisticated concepts of coalition formation. For an overview of these concepts we refer to Ray and Vohra (2013).
vary with the coalition size. We show, in particular, that the equilibrium welfare levels of both coalition and fringe countries increase progressively with the coalition size and that the welfare gain of fringe countries over coalition countries increases as well. The precise forms of these functional relations turn out to be crucial for coalition formation. We first establish analytically that stable grand coalitions exist for a non-empty parameter subspace in which the total number of countries may be large. That parameter subspace is characterized. Then we prove that for any values of the parameters there always exists a very small stable coalition (consisting of 3 to 6 countries out of \( n \) countries) while the grand coalition is a second stable coalition only under certain parameter constellations.

The coalition formation in our tax competition game is reminiscent to coalition formation in two other literature strands: cartel formation in oligopolies and international environmental agreements (IEAs). In the cartel formation game Donsimoni et al. (1986) point out that the stable cartel is either unique or there exist two stable cartels. One of the two stable cartels comprises all firms and the other is small or large depending on the firms’ costs, consumers’ marginal willingness-to-pay etc. The literature on self-enforcing IEAs finds that the stable coalition is unique and small consisting of less than four countries if the coalition plays Nash (Finus 2001, Hoel 1992) or if the coalition acts as Stackelberg leader (Barrett 1994, Rubio and Ulph 2006, Diamantoudi and Sartzetakis 2006).

The paper is organized as follows. Section 2 introduces the model and briefly analyzes the business-as-usual scenario and the social optimum which serve as benchmarks throughout the paper. Section 3 characterizes the coalition-fringe equilibria for given coalitions of alternative size and prepares for the stability analysis in Section 4. Section 5 concludes.

### 2 The model

Consider an economy with \( n \geq 2 \) identical countries. Each country \( i \) hosts a representative firm that employs \( k_i \) units of capital to produce good \( X \) according to the production function

\[
X(k_i) = ak_i - \frac{b}{2}k_i^2 \quad \text{for} \quad 0 \leq k_i \leq \frac{b}{a}.
\]  

The output of good \( X \) can be used either as a private consumption good or as a local public good provided to its residents by each country’s government. The utility of the representative consumer of country \( i \) is

\[
u_i = x_i + (1 + \varepsilon)g_i,
\]
where \( x_i \) and \( g_i \) denote consumption of private and public goods, respectively, and where \( \varepsilon \geq 0 \) is a parameter reflecting the preference intensity for the local public good. We choose the restrictive functional forms (1) and (2) for reasons of tractability.\(^6\)

Capital and good \( X \) are traded on perfectly competitive world markets. We denote by \( r \) the rate of return on capital and normalize to one the price of good \( X \). The capital market equilibrium condition is

\[
\sum_j k_j = n\bar{k},
\]

where \( \bar{k} \) is the fixed capital endowment of the consumer of each country.

The firms’ demand for capital is straightforward. Taking the rate of return on capital \( r \) and country \( i \)’s capital tax rate \( t_i \) as given, the producer of country \( i \) maximizes profits

\[
\Pi_i = X(k_i) - (r + t_i)k_i.
\]

The resulting first-order condition is

\[
a - bk_i = r + t_i. \tag{4}
\]

Equations (3) and (4) determine the equilibrium capital allocation \( \{k_i\}_{i=1}^n \) and the rate of return on capital \( r \) as a function of tax rates as follows:

\[
r = a - \bar{k}b - \frac{\sum_j t_j}{n}, \quad k_i = \bar{k} - \frac{t_i}{b} + \frac{\sum_j t_j}{nb}. \tag{5}
\]

Differentiation of (5) yields

\[
\frac{\partial r}{\partial t_i} = -\frac{1}{n} < 0, \quad \frac{\partial k_i}{\partial t_i} = -\frac{(n - 1)}{nb} < 0. \tag{6}
\]

As expected, the unilateral increase in some country’s capital tax rate reduces investment in that country via a decline in the rate of return on capital. The government of each country uses the tax revenues for the provision of a local public good. Under a balanced-budget constraint the supply of the public good is

\[
g_i = t_i k_i. \tag{7}
\]

Implicit in (7) is the assumption that the output \( X \) can be transformed one-to-one into the public good. As the consumer of country \( i \) owns her country’s firm, her budget constraint is

\[
x_i = \underbrace{X(k_i) - (r + t_i)k_i}_{=\Pi_i} + r\bar{k}. \tag{8}
\]

\(^6\)The dilemma between the quest for general assumptions and informative outcomes is particularly well known in the tax-competition literature. Production functions of type (1) are employed by Bucovetsky (1991), Grazzini and van Ypersele (2003) and by Devereux, Lockwood and Redoano (2008). A linear utility function of type (2) is assumed by Keen and Lahiri (1998). In fact, various contributions to the capital tax literature use the functional forms (1) and (2), e.g. Peralta and van Ypersele (2005), Bucovetsky (2009), Kempf and Rota-Graziosi (2010) and Ogawa (2012).
where \( \Pi_i \) is the profit income and \( r \bar{k} \) is the capital income. Inserting (7) and (8) in (2) yields the welfare function

\[
W^i(t_1, \ldots, t_n) := r \bar{k} + (a - r + \varepsilon t_i)k_i - \frac{b}{2}k_i^2 \tag{9}
\]
with \( r \) and \( k_i \) as specified in (5).

**Business as usual.** For the later use as a benchmark, we briefly characterize the non-cooperative Nash equilibrium which we refer to as business as usual (BAU). The government of country \( i \) chooses the tax rate which maximizes \( W^i(t_1, \ldots, t_n) \) for given tax rates \( (t_1, \ldots, t_{i-1}, t_{i+1}, \ldots, t_n) \). Consideration of (5), differentiation of (9) with respect to \( t_i \) and accounting for (4) yield the first-order condition

\[
W^i_{t_i} = (1 + \varepsilon)t_i \frac{\partial k_i}{\partial t_i} + (\bar{k} - k_i) \frac{\partial r}{\partial t_i} + \varepsilon k_i = 0. \tag{10}
\]

Combined with (5) and (6), equation (10) determines country \( i \)'s best-reply function

\[
t_i = \tilde{R} \left( \sum_{j \neq i} t_j \right) = \frac{n^2 \varepsilon \bar{k} b}{(n - 1)(n + 2n \varepsilon + 1)} + \frac{(n \varepsilon + 1)}{(n - 1)(n + 2n \varepsilon + 1)} \sum_{j \neq i} t_j. \tag{11}
\]

The reaction function is upward sloping and characterizes the countries’ tax rates as strategic complements. Since \( \frac{dt_i}{\sum_{j \neq i} t_j} \in [0, 1] \) and constant, there is a unique Nash equilibrium, say \( (t_{1o}, \ldots, t_{no}) \). With all countries being alike, the Nash equilibrium is characterized by \( t_{1o} = t_{2o} = \ldots = t_{no} \equiv t_o \) and hence also by \( k_{1o} = k_{2o} = \ldots = k_{no} = \bar{k} \). Due to the symmetry assumption equation (10) simplifies to \( W^i_{t_i} = (1 + \varepsilon)t_i \frac{\partial k_i}{\partial t_i} + \varepsilon k_i = 0 \) and making use of (5) and (6) we obtain \( t_o = \frac{n \varepsilon \bar{k} b}{(n - 1)(1 + \varepsilon)} \). We insert \( t_o \) in (5) and determine the equilibrium rate of return to capital as \( r_o = a - \bar{k} b - \frac{n \varepsilon \bar{k} b}{(n - 1)(1 + \varepsilon)} \). It is reasonable to restrict the focus on positive rates \( r_o \) and therefore we assume

\[
\frac{a}{b \bar{k}} > 1 + \frac{n \varepsilon}{(n - 1)(1 + \varepsilon)}. \tag{12}
\]

**Social optimum.** With \( \varepsilon > 0 \) and free choice between consumption of the private and public good, consumers would maximize utility by turning the entire gross income into the public good. However, owing to the assumed institutional arrangement the public good can only be publicly provided and must be financed via capital tax revenues. That places an upper bound on feasible tax rates, because the rate of return to capital is required to be non-negative, \( r \geq 0 \). When this condition is combined with (5) and symmetry, the social optimum is characterized by \( \hat{t} = a - b \bar{k} \) for all \( i \) and \( \hat{r} = 0 \). To ensure that in the social
optimum capital is fully employed we constrain the parameter space in the sequel by
\[
\frac{a}{bk} > 1 - \frac{\varepsilon}{1 + 2\varepsilon}.
\] (13)

Since \(k_i = \bar{k}\) in BAU and in the social optimum, and the output \(ak - \frac{b}{2}k^2\) is the same in BAU and in the social optimum, \(r_o > 0\) and \(\hat{r} = 0\) imply \(t_o \bar{k} = g_o < \hat{t} \bar{k} = \hat{g}\) and \(x_o = ak - \frac{b}{2}k^2 - t_o \bar{k} > \hat{x} = ak - \frac{b}{2}k^2 - \hat{t} \bar{k}\). From \(\hat{g} > g_o\) and \(\varepsilon > 0\) follows, in turn, that the countries’ welfare is higher in the social optimum than in BAU (\(\hat{w} > w_o\)).

3 Tax competition between a given tax coalition and the fringe countries

In this section we convert the non-cooperative \(n\)-country game into a game with two groups of countries. Specifically, we lump together the first \(m\) countries, \(2 \leq m < n\), in one group, denoted group \(C = \{1, 2, \ldots, m\}\) (with \(C\) for coalition), and collect the remaining countries in another group, denoted group \(F = \{m + 1, \ldots, n\}\) (with \(F\) for fringe). Throughout the present section the number of countries, \(m\), in group \(C\) is exogenous, and we assume that all \(m\) countries have committed themselves to cooperative tax policy, while all other countries in group \(F\) continue abstaining from cooperation. Each fringe country plays Nash against all other fringe countries and against group \(C\), but the coalition now acts as a single player. Its payoff is the coalition members’ aggregate welfare and it plays Nash against all fringe countries. The coalition takes into account that \(t_i = t_c\) for all \(i \in C\) is a necessary condition for reaching its goal and hence maximizes over \(t_c\),

\[
\sum_{j \in C} W_j \left( t_c, \sum_{j \in F} t_j, m \right) = m \left[ r_c \bar{k} + \left( a - r_c + \varepsilon t_c \right) k_c - \frac{b}{2} k_c^2 \right].
\] (14)

where \(r_c := a - b \bar{k} - \Theta_c\), \(k_c := \bar{k} - \frac{t_c}{b} + \Theta_c\) and \(\Theta_c := \frac{mt_c + \sum_{i \in F} t_i}{n}\). The pertaining first-order condition is

\[(1 + \varepsilon) t_c \frac{\partial k_c}{\partial t_c} + (\bar{k} - k_c) \frac{\partial r_c}{\partial t_c} + \varepsilon k_c = 0\]

or equivalently

\[-m(t_c - \Theta_c) - (n - m)(1 + \varepsilon)t_c + \varepsilon(nb \bar{k} - nt_c + n\Theta_c) = 0.\] (15)

---

7 The condition (13) is an analogue to Assumption 1 in Bucovetsky (2009). Observe that (13) is sufficient for (12).
Denote by \( t_i = t_f \) the tax rate of an individual fringe country \( i \) and rewrite the first-order condition (10) as

\[
(1 + \varepsilon) t_f \frac{\partial k_f}{\partial t_f} + (\bar{k} - k_f) \frac{\partial r_f}{\partial t_f} + \varepsilon k_f = 0,
\]

where \( r_f := a - b\bar{k} - \Theta_f \), \( k_f := \bar{k} - \frac{t_f}{b} + \frac{\Theta_f}{b} \) and \( \Theta_f := \frac{t_f + \sum_{j \neq f} t_j}{n} \). Accounting for \( r_f \), \( k_f \) and \( \Theta_f \) turns (16) into

\[
-(t_f - \Theta_f) - (n - 1)(1 + \varepsilon)t_f + \varepsilon(nb\bar{k} - nt_f + n\Theta_f) = 0.
\]

Since all fringe countries are alike, a necessary condition for a symmetric equilibrium is \( t_i = t_f \) for all \( i \in F \) and thus

\[
\Theta_c = \Theta_f = \frac{mt_c + (n - m)t_f}{n}; \quad \Theta.
\]

A coalition-fringe equilibrium is a \( n \)-tuple \((t^*_c, \ldots, t^*_c, t^*_f, \ldots, t^*_f)\) of tax rates satisfying (15), (17) and (18).

To characterize the allocation in the coalition-fringe equilibrium and to compare it with the BAU scenario we will mark by the subscript "o" all BAU variables and with an asterix all variables in the coalition-fringe equilibrium. We denote by \( W^v(t^*_c, t^*_f, m) \) the welfare level of country \( v = c, f \) in the coalition-fringe equilibrium with a coalition of size \( m \) and prove in the Appendix A

**Proposition 1.** The coalition-fringe equilibrium with a coalition of size \( m \in \{2, \ldots, n-1\} \) compares with the non-cooperative Nash equilibrium (BAU) as follows:

(i) \( W^f(t^*_c, t^*_f, m) > W^c(t^*_c, t^*_f, m) > w_o \),

(ii) \( t^*_c > t^*_f > t_o \).

The good news of Proposition 1(i) is that coalition members are better off than in BAU which may constitute an incentive for countries to form a tax coalition. Even more remarkable is the finding that in the presence of the tax coalition the benefits of fringe countries exceed those of coalition countries. That indicates an incentive to defect on the part of coalition countries. However, the analysis of the relation between these incentives and coalition stability will be postponed to the next section. Here we stick to the preliminary assumption of an exogenously given coalition size \( m \) and proceed with explaining the asymmetry in outcome for fringe and coalition countries. The first of two effects generating that asymmetry is the tax base effect
which creates the well-known (positive) fiscal externality. In formal terms:

\[ \frac{\partial k_c}{\partial t_c} = -\frac{n - m}{nb}, \quad \frac{\partial k_f}{\partial t_f} = -\frac{n - 1}{nb}, \quad \text{and hence } \left| \frac{\partial k_f}{\partial t_f} \right| > \left| \frac{\partial k_c}{\partial t_c} \right|. \quad (19) \]

Since the tax base erosion of an increase in \( t_f \) is more pronounced than that of increasing \( t_c \), the tax rate of fringe countries is smaller than that of coalition countries \((t_f^* < t_c^*)\). As a consequence, more capital is employed in fringe than in coalition countries \((k_f^* > k_c^*)\) which implies, in turn, that the coalition countries export and the fringe countries import capital \((k_f^* > \bar{k} > k_c^*)\).\(^8\)

The second differential effect is the impact of the countries’ tax policies on the rate of return on capital which represents the terms of trade in our model. We find

\[ \frac{\partial r_c}{\partial t_c} = -\frac{m}{n}, \quad \frac{\partial r_f}{\partial t_f} = -\frac{1}{n}, \quad \text{and hence } \left| \frac{\partial r_f}{\partial t_f} \right| < \left| \frac{\partial r_c}{\partial t_c} \right|. \quad (20) \]

Since coalition countries export capital (in the relevant neighborhood of equilibrium), an increase in the tax rate \( t_c \) reduces their export income, ceteris paribus, while fringe countries enjoy lower import bills. The relatively small tax base effect encourages the coalition to raise its tax rate and the relatively large adverse terms-of-trade effect discourages tax increases. The opposite holds for fringe countries.

The comparison of the allocations in BAU and the coalition-fringe equilibrium is straightforward. As already observed, the coalition countries export and fringe countries import capital while in BAU (net) trade flows are zero. Since the tax rates of all countries are higher than in BAU (Proposition 1(ii)), the equilibrium return on capital is lower than in BAU \((r^* < r_o)\). Also, the producer per-unit cost of capital in coalition countries \((r^* + t_c^*)\) is higher than in fringe countries \((r^* + t_f^*)\) and the latter must be lower than in BAU \((r_o + t_o)\).\(^9\)

Correspondingly, output in coalition [fringe] countries is lower [higher] than in BAU. With respect to the gross national product \( X(k_v) + (\bar{k} - k_v)r \) of country \( v = c, f \), we find in the Appendix B that

\[ X(k_f^*) + (\bar{k} - k_f^*)r^* > X(k_o) + (\bar{k} - k_o)r_o > X(k_c^*) + (\bar{k} - k_c^*)r^*. \quad (21) \]

Since the coalition countries’ gross national product is smaller than in BAU, a necessary condition for \( W^c(t_c^*, t_f^*, m) > w_o \) is that the amount of the public good provided in coalition countries is larger than in BAU \((g_c^* = t_c^*k_c^* > g_o = t_o\bar{k})\). Essentially, the coalition succeeds in raising its members’ welfare above BAU through its strategy of substantially increasing the tax rate. Although that strategy erodes the tax base \((k_c \downarrow)\) and diminishes gross national

\(^8\)The asymmetric taxes and capital flows generate production inefficiencies.

\(^9\)The ranking \( r^* + t_c^* > r_o + t_o > r^* + t_f^* \) follows from \( k_f^* > k_o > k_c^* \), \( X_k = r + t \) and \( X_k > 0 \).
The product \( (X(k_c) + (\bar{k} - k_c)r \downarrow)\), the direct tax rate effect \((t_c \uparrow)\) overcompensates the tax base effect and thus increases the coalition countries' public good provision and welfare in spite of shrinking gross national product. Note that the public-good provision in fringe countries also exceeds the BAU level \((g^*_f = t^*_f k^*_f > g_o\) because \(t^*_f > t_o\) and \(k^*_f > \bar{k}\)), but the sign of \(g^*_c - g^*_f\) is unclear.

Proposition 1(i) gives rise to an interesting side result by reinterpreting our model as follows. So far we have looked at the model (and will continue to do so later) as consisting of \(n\) small identical countries that may or may not be members of a tax coalition. Alternatively, we may consider the same model as representing a world economy of \(n - m + 1\) countries. As before, \(n - m\) of these countries are supposed to be small and identical, but we now interpret the coalition as a single large country with the capital endowment \(m\bar{k}\), with \(m\) representative consumers, and with \(m\) firms each of which produces good \(X\) by means of the production function (1). \(W_f(t^*_c, t^*_f, m) > W_c(t^*_c, t^*_f, m)\) from Proposition 1(i) then means that the per capita utility of consumers in each small country is higher than in the large country. That implication is in line with the result known from the literature on asymmetric tax competition (Bucovetsky 1991, Bucovetsky and Wilson 1991 and Wilson 1991).

Another comment is in order concerning the relation between our Proposition 1 and the analysis of Konrad and Schjelderup (1999). As we do in the present section, these authors study capital tax competition with a given tax coalition. They also assume identical countries, but the functional forms they employ for the production and utility functions are much more general than our functions (1) and (2). They find (Proposition 2, p. 166) that in the coalition-fringe equilibrium countries enjoy higher welfare\(^{10}\) than in the non-cooperative equilibrium (BAU) if three conditions (p. 163) are satisfied which relate to the existence of a unique equilibrium and to the properties of the reaction functions but are not proved to be implied by the basic assumptions of the model. In our model with its much more restrictive functional forms the three conditions imposed by Konrad and Schjelderup are satisfied such that our Proposition 1 proves that Proposition 2 in Konrad and Schjelderup (1999) is non-empty. Our more restrictive assumptions pay off in terms of more specific information regarding the characterization, and comparison with BAU, of the coalition-fringe equilibrium. Note finally, that in our paper the characterization of coalition-fringe equilibria with coalitions of given size is only an intermediate step for tackling the issue of coalition stability which would be intractable without massive simplifications such as the functions (1) and (2).

\(^{10}\)In their Proposition 1 (p. 162) Konrad and Schjelderup (1999) find in an exercise with marginal changes of tax rates that compared to BAU all countries may be better off in the presence of a tax coalition. However, they do not establish our result of Proposition 1(i) that fringe countries benefit more than coalition countries.
Note that in coalition-fringe equilibrium all economic variables are uniquely determined by the coalition size \( m \). It is therefore natural to ask how the equilibrium values of the model variables vary with the (exogenous) coalition size. To address that issue we make explicit the dependence of equilibrium values on \( m \) and prove in the Appendix C Proposition 2.

Denote by \( t^*_v = T^v(m) \) and \( W^v(m) := W^v[T^c(m), T^f(m), m] \) for \( v = c, f \) the tax rates and welfare levels of coalition and fringe countries in the coalition-fringe equilibrium with a coalition of size\(^\text{11} \) \( m \in [1, n] \in \mathbb{R}_+ \).

(i) The welfare levels of a coalition countries, \( W^c(m) \), and of fringe countries, \( W^f(m) \), are strictly increasing in \( m \).

(ii) The tax rates of coalition countries, \( T^c(m) \), and of fringe countries, \( T^f(m) \), are strictly increasing in \( m \).

As expected, Proposition 2(i) confirms that larger coalitions are more effective than smaller ones in mitigating the inefficiencies of capital tax competition. More specifically, the economy-wide welfare gap between the coalition-fringe equilibrium and the social optimum shrinks monotonically with growing coalition size and becomes zero in the grand coalition. The intuition of Proposition 2(ii) is also clear. With increasing \( m \) the adverse tax base effect (19) of rising \( t_c \) weakens. The larger coalition therefore finds it advantageous to increase the provision of the public good, although the adverse terms-of-trade effect worsens due to (20). Interestingly, the tax base effect as well as the terms-of-trade effect of fringe countries are unaffected by variations of the coalition size.

Unfortunately, the equilibrium functions of all variables other than tax rates and welfare levels depend on \( m \) in such a complex way that the analytical specification of their dependence on the coalition size proved to be impossible. To make progress we resort to a numerical example. We choose the parameter values \( a = 10, b = 1, \bar{k} = 0.01, \varepsilon = 0.5 \) and \( n = 100 \) and refer to the resultant parameter constellation as Example 1. The Figures 1, 2 and 3 illustrate how the equilibrium values\(^\text{12} \) of Example 1, \( T^v(m), K^v(m), G^v(m) \) and \( W^v(m) \) for \( v = c, f \), vary over their domain \([1, n]\).

\(^{11}\)For analytical convenience we take the interval \([1, n]\) to be the domain of these functions, keeping in mind in our later conclusions that the domain of real-world coalitions is the set of integers \( \{1, \ldots, n\} \).

\(^{12}\)For \( v = c, f \) we write \( k^*_v = K^v(m) \) and \( g^*_v = G^v(m) \).
Figure 1: Welfare of coalition and fringe countries (Example 1)

In the Figures 1a, 1b and 2a the outcome of Example 1 is obviously in line with the Propositions 1 and 2. These figures add to the propositions the information that the positive differences $W^f(m) - W^c(m)$ and $T^c(m) - T^f(m)$ are strictly increasing in $m$. That is, the fringe countries’ ‘free ride’ on the coalition’s tax coordination is the more profitable, the larger the tax coalition. The slope of the $K^c$ curve in Figure 2b is negative, because the difference $T^c(m) - T^f(m)$ is positive and rising in $m$. The decline of $K^c(m)$ in $m$ is slowed down by the weakening tax base effect. In contrast, the $K^f$ curve is progressively increasing in $m$ although the fiscal externality generated by the coalition becomes weaker. The reason for that is the continuous reduction in the rate of return on capital which occurs because each remaining fringe country must employ more capital when more and more coalition countries reduce their capital demand. The falling rate of return on capital improves the fringe countries’ terms of trade and thus induces massive ‘capital leakage’.

\[13\] The observation that the welfare gap $W^f(m) - W^c(m)$ increases in $m$ holds not only in Example 1 but also for any values of the parameters. This can be seen by differentiation of (A7) in the Appendix A with respect to $m$. 

13
Since $G_f(m)$ is equal to $T_f(m) \cdot K_f(m)$ by definition, the progressively increasing $G_f(m)$ curve in Figure 3 is easily explained by the curvature of the $T_f$ curve and the $K_f$ curve in Figure 2. In Figure 3, $G_c(m)$ is also progressively increasing in $m$ because the positive tax rate effect, $T_c(m) \cdot K_c(m)$, overcompensates the negative tax base effect, $T_c(m) \cdot K_c(m)$. Interestingly, the difference $G_c(m) - G_f(m)$ is positive for all $m$ up to about $m = 92$ where the sign switches.

We have calculated many more examples by modifying the parameter values of Example 1 in various ways and always found that, in qualitative terms, the equilibrium values depend on the coalition size as illustrated in the Figures 1, 2 and 3. This is why we do not find it useful to present additional examples.

### 4 Self-enforcing tax coordination

In the preceding Section 3 a tax coalition of given size existed by presupposition, and our focus was on characterizing the coalition-fringe equilibrium and its dependence on the exogenous coalition size $m$. Now we turn to the issue of coalition stability. Since supranational authorities for the effective enforcement of international tax coordination agreements are not available, such agreements will not be concluded unless they are self-enforcing in the sense that no coalition country has an incentive to defect (internal stability) and no fringe country has an incentive to join the tax agreement (external stability).\footnote{This notion of self-enforcement or stability was originally introduced by D’Aspremont et al. (1983) in the context of cartel formation. It was then applied in various different areas, in particular in the study of stability of international environmental agreements initiated by Barrett (1994).} In formal language, an international tax coalition with $m \in \{2, \ldots, n\}$ member countries is said to be self-enforcing...
or stable if it satisfies the internal stability condition

\[ W_c(m) \geq W_f(m - 1) \]

and the external stability condition

\[ W_f(m) \geq W_c(m + 1). \]

First we will examine a polar case, the stability of the grand coalition,\(^{15}\) for which it suffices to check the internal stability condition \( \hat{w} \geq W_f(n - 1). \) For \( n = 2 \) the grand coalition is trivially stable, because defection from the grand coalition then amounts to moving from the social optimum back to BAU. The crucial question is whether the grand coalition is also stable for more than two countries. The answer is given by the equivalence

\[ \hat{w} \geq W_f(n - 1) \iff \frac{a}{bk} \geq F(\varepsilon, n) \]

which we establish in the Appendix D, where we also characterize the function \( F \) as follows:\(^{16}\)

\[ F(0, n) = 1 \quad \text{for all } n \geq 2; \]
\[ F_\varepsilon(\varepsilon, n) > 0 \quad \text{for all } \varepsilon > 0 \text{ and } n \geq 2; \quad \lim_{n \to \infty} F(\varepsilon, n) = \infty; \]
\[ F_\varepsilon(\varepsilon, n) > 0 \quad \text{for all } \varepsilon \geq 0 \text{ and } n \geq 2; \quad \lim_{\varepsilon \to \infty} F(\varepsilon, n) =: \bar{F}(n) > 0 \text{ and } \bar{F}_n > 0. \]

Owing to (13) and the conditions (25) - (27) there are feasible parameter constellations for which (24) is or is not satisfied. To be more specific, for our Example 1 we calculate \( \frac{a}{bk} = 1000 > F(0.5, 1000) = 418.23 \) which implies that the grand coalition is stable in Example 1. If we replace \( a = 10 \) by \( a = 1 \) in Example 1 keeping all other parameters unchanged we calculate \( \frac{a}{bk} = 100 > F(0.5, 100) = 418.23 \) so that the grand coalition is not stable anymore in that slightly modified example.

Proposition 3 summarizes our results on the stability of the grand coalition in a more systematic way and spells out further information gained by exploiting the properties (25) - (27) of the function \( F \).

**Proposition 3.**

\(^{15}\)Burbidge et al. (1997) and Bucovetsky (2009) also analyze the stability of the grand coalition in their models of heterogeneous countries. Burbidge et al. (1997) show that the grand coalition among all countries is realized as a unique equilibrium if the number of countries is only two, but this is not the case if there are more than three countries. Bucovetsky (2009) points out that the grand coalition is stable if the population share of the smallest country exceeds some threshold.

\(^{16}\)The function \( \bar{F}(n) \) in (27) is further specified in the Appendix D.
(i) The grand coalition is stable, if and only if \((a, b, \varepsilon, \bar{k}, n) \in M^n\), where 
\[ M^n := \{(a, b, \varepsilon, \bar{k}, n) \mid \varepsilon \geq 0, n \geq 0, (13) \text{ and } (24) \text{ are satisfied}\}. \]
The set \(M^n\) is non-empty and a proper subset of the set of feasible economies. \(M^n\) is the larger, the larger the expression \(\frac{a}{\bar{k}}\).

(ii) For all \((a, b, \varepsilon, \bar{k}, n) \in M^n\) satisfying \(\varepsilon > 0\) and \(\frac{a}{\bar{k}} > F(\varepsilon, n)\) there exists \(\hat{n} > n\) such that \((a, b, \varepsilon, \bar{k}, \hat{n}) \left\{ \begin{array}{l} \in M^n \\ \not\in M^n \end{array} \right\} \iff \hat{n} \left\{ \begin{array}{l} \leq \\ > \end{array} \right\} n.

(iii) If \((a, b, \varepsilon, \bar{k}, n) \in M^n\) and \(n \leq F^{-1}\left(\frac{a}{\bar{k}}\right)\), then \((a, b, \varepsilon, \bar{k}, n) \in M^n\) for all \(\varepsilon \geq 0\).

(iv) For all \((a, b, \varepsilon, \bar{k}, n) \in M^n\) satisfying \(n > F^{-1}\left(\frac{a}{\bar{k}}\right)\) there exists \(\varepsilon > \varepsilon\) such that 
\[(a, b, \varepsilon, \bar{k}, n) \left\{ \begin{array}{l} \in M^n \\ \not\in M^n \end{array} \right\} \iff \varepsilon \left\{ \begin{array}{l} \leq \\ > \end{array} \right\} \varepsilon.

An important and surprising message of Proposition 3 is that the grand coalition may be stable even in a world economy with a large number of countries.\(^{17}\) According to Proposition 3 the stability of the grand coalition depends on certain parameter constellations. In particular, it identifies \(n\) and \(\varepsilon\) as key parameters for stability. Given the economy’s technology and capital endowment as represented by the parameters \(a, b\) and \(\bar{k}\), stability of the grand coalition cannot be secured unless the total number of countries is sufficiently small\(^{18}\) and/or the preference weight of the public good is small. The maximum stability-preserving value of \(n\) is the larger, the larger \(\frac{a}{\bar{k}}\) and the smaller \(\varepsilon\). Large values of \(\varepsilon\) may also destabilize the grand coalition but only if combined with sufficiently large values of \(n\).

In order to understand the role the total number of countries plays in Proposition 3(ii) for the stability of the grand coalition recall that according to Proposition 1 \(\mathcal{W}^f(n-1) > \mathcal{W}^c(n-1)\) is satisfied for all \(n \geq 2\). Also, \(\mathcal{W}^c(n-1) < \hat{w}\) for all \(n \geq 2\), because total welfare cannot exceed \(nw\) for \(m < n\). But \(\mathcal{W}^c(n-1)\) is increasing in \(n\) and approaches \(\hat{w}\) from below. We combine the information \(\mathcal{W}^f(n-1) > \mathcal{W}^c(n-1)\) and \(\mathcal{W}^c(n-1) \to \hat{w}\) for \(n \to \infty\) to conclude that there exists \(\hat{n}\) such that \(\mathcal{W}^f(n-1) > \hat{w}\) for all \(n > \hat{n}\). The intuition for Proposition 3(iii) is less obvious. According to that proposition the stability of the grand coalition depends on the combined values of \(\varepsilon\) and \(n\). If there is a stable grand

\(^{17}\)In the literature on self-enforcing international environmental agreements the grand coalition is never stable for \(n > 2\), to our knowledge. The decisive difference between that environmental issue and tax competition is that in the former case we deal with free riding on the provision of a global public good while the latter case is about the provision of local (national) public goods.

\(^{18}\)Bucovetsky (2009, Proposition 8) establishes a result comparable to Proposition 3(ii) in a model of heterogeneous countries. Proposition 3 is more specific than his result because it fully characterizes the parameter sub-space for which the grand coalition is stable.
coalition with a sufficiently small total number of countries, \( n < \bar{F}^{-1}\left(\frac{a}{b}\right) \), then variations in \( \varepsilon \) preserve stability (Proposition 3(iiiia)). However, if there is a stable grand coalition in a world economy with \( n \) countries and if \( n > \bar{F}^{-1}\left(\frac{a}{b}\right) \), then reductions in \( \varepsilon \) preserve stability, but increases in \( \varepsilon \) eventually destabilize the grand coalition (Proposition 3(iiiib)). It is the more likely that an increase in the preference weight \( \varepsilon \) for the public good destroys the stability of the grand coalition, the larger is the total number of countries \( n \).

Next we wish to answer the crucial question whether there exist stable sub-global coalitions. Unfortunately, the functions \( W^c \) and \( W^f \) are too complex to allow deriving informative analytical results. We therefore resort to examining the stability conditions (22) and (23) for the numerical Example 1 introduced in the previous Section 3.1.

![Figure 4: Coalition stability in Example 1](image)

Figure 4 depicts the graphs of the functions \( W^c(m) - W^f(m - 1) \) and \( W^f(m) - W^c(m + 1) \) for Example 1. There is one and only one interval of coalition sizes in which both functions take on non-negative values (thus satisfying (22) as well as (23)), and this interval contains one and only one integer, namely \( m^* = 4 \). The good news is that there exists a unique stable coalition of size greater than one. The bad news is that the stable coalition is very small in relation to the total number of countries which is \( n = 100 \) in Example 1.

It is obvious from the stability conditions (22) and (23) that the stability of coalitions depends on the properties of the functions \( W^c \) and \( W^f \). In the left panel of Figure 1 we see that \( W^f(m) - W^c(m) \), the vertical difference between the welfare curves \( W^f \) and \( W^c \), is zero for \( m = 1 \) and positive for all \( m > 1 \). That difference can be interpreted as the free-rider advantage of fringe countries over coalition countries. In our Example 1 that free-rider advantage grows with the coalition size suggesting that the incentive to leave the coalition increases and the incentive to join declines with the coalition size. However, the defining
criterion for coalition stability is the horizontal rather than the vertical distance between
the welfare curves $W^I$ and $W^C$. To formalize that observation, we introduce the function

$$H : [1, n] \rightarrow \mathbb{R}_+, \text{ where } h = H(m), \text{ if and only if } W^I(m - h) = W^C(m).$$

$H(m)$ measures the horizontal distance between the welfare curves $W^I$ and $W^C$ at the level $W^C(m)$ above the horizontal $m$-axis. For Example 1 Figure 5 shows that the function $H$ is strictly increasing in $m$ for $m < 80$ and strictly decreasing in $m$ for $81 < m < 99$.\textsuperscript{19} and that there is a unique value $\tilde{m} \in [2,99[$ satisfying $H(\tilde{m}) = 1$. The stability conditions (22) and (23) imply that if for some $\{3,4,\ldots,99\}$ $H(m) < 1$ [$H(m) > 1$] and $H(\tilde{m}) < 1$ [$H(\tilde{m}) > 1$] for all $\tilde{m}$ in a sufficiently large neighborhood of $m$, then the coalition of size $m$ is externally [internally] unstable. The stability conditions also imply that since $H_m(m) > 0$ holds in a sufficiently large neighborhood of $\tilde{m}$, there exists a stable coalition of size $m^*$ satisfying $m^* \in [\tilde{m} - 1, \tilde{m}]$. As Figure 5 shows, $\tilde{m} \in ]4,5[$ and therefore $m^* = 4$. The coalitions of size $\{3,4,\ldots,99\}$ are externally [internally] unstable for all $m < m^*$ [$m > m^*$]. Hence the coalition of size $m^* = 4$ is the only stable coalition in the set of coalitions of size $\{3,4,\ldots,99\}$.

We know already that the grand coalition is stable in Example 1. Since the function $H$ is discontinuous between $m = 99$ and $m = 100$, the stability of the grand coalition manifests itself in Figure 5b in the point $A$ which is located above $m = 100$ and below the $H(\tilde{m}) = 1$ line. In other words, we have $H(100) < 1$ and hence $W^I(99) < \hat{w}$, so that no member of the grand coalition has an incentive to defect. Taking the magnitude of $H(m)$ (with $H(m) > 1$) as an indicator for the strength of the coalition countries’ incentive to defect, we observe in Figure 5b that these incentives decline for very large coalitions ($m > 81$). Whether we get $H(100) < 1$ or $H(100) > 1$ appears to depend on how strongly these incentives diminish.

In order to examine the robustness of the outcome of Example 1, we take the parameter values of that example as a point of departure and examine continuous variations of the parameters $\alpha, \beta, \varepsilon$ and $\tilde{k}$, one at a time. It can be shown that the function $H$ is invariant with respect to changes in $\alpha, \beta$, and $\tilde{k}$. To indicate the dependence of $H(m)$ on $\varepsilon$, we write $H(m;\varepsilon)$ and also define $\tilde{m} = \tilde{M}(\varepsilon)$ by the equation $H[\tilde{M}(\varepsilon);\varepsilon] = 1$. Figure 6 plots the graph of the function $H$ for the values $\varepsilon = 0.01$, $\varepsilon = 0.05$ and $\varepsilon = 10$ with the obvious result that the larger $\varepsilon$ the larger is $\tilde{m}$ and the larger is the stable coalition size $m^*$. Figure

\textsuperscript{19}In addition, the function $H$ is discontinuous between $m = 99$ and $m = 100$. But we need not care about that discontinuity because it is only the integers that count.
Figure 5 illustrates the graph of the function $\tilde{m}$ and confirms for continuous variations of $\varepsilon$ that $\tilde{m}$ is strictly increasing in $\varepsilon$. For very small $\varepsilon$, $\tilde{M}(\varepsilon)$ increases sharply, but then levels off satisfying $\tilde{M}(\varepsilon) < 7$ for all $\varepsilon \geq 0$. Table 1 demonstrates that this pattern is not specific to $n = 100$; it is hardly affected by variations in the total number of countries, $n$. 

Figure 6: Function $H$ for parametric variations in $\varepsilon$ ($n = 100$)

Figure 7: Function $\tilde{M}$ for variations in $\varepsilon$ ($n = 100$)
We summarize our findings in

**Proposition 4.** Suppose that (13) is satisfied and that \( n \in \{10, 11, \ldots, 200\} \). Then

(i) there exists a stable coalition of size \( m^* \in \{3, 4, \ldots, 7\} \);

(ii) the size of the stable coalition is invariant with respect to changes in \( a, b \) and \( \bar{k} \);

(iii) the size of the stable coalition is the larger the larger \( \varepsilon \).

According to Proposition 4(i) the number of countries in the stable coalition is always small (between 3 and 7), but in relative terms, the differences are large. The members of the stable coalition as a share of all countries is at most \( \frac{m^*}{n} = 0.06 \) for \( n = 100 \), but at most \( \frac{m^*}{n} = 0.7 \) for \( n = 10 \). These remarkable differences give rise to the question, how the effectiveness of the stable coalition varies with the total number of countries. Effectiveness is meant to be the capacity of the stable coalition to make the countries better off than in BAU or the degree to which the stable coalition reduces the inefficiency prevailing in BAU. As a measure of relative effectiveness we introduce the indicator

\[
RW := \frac{[m^*W^c(m^*) + (n - m^*)W^f(m^*)] - nw_o}{nw - nw_o} \cdot 100\%.
\]  

(28)

In the denominator of (28) we have the world welfare gap between the social optimum and BAU and the numerator denotes the difference between world welfare in the equilibrium with the stable coalition \( m^* \) (= term in square brackets) and BAU. Inserting in (28) the pertaining numbers for Example 1 yields \( RW = 0.000041 \). Hence in Example 1 the capacity of the stable coalition to reduce the BAU inefficiency is negligibly small.

<table>
<thead>
<tr>
<th>( n )</th>
<th>10</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M(0) )</td>
<td>3.45</td>
<td>3.08</td>
<td>3.04</td>
<td>3.03</td>
<td>3.02</td>
</tr>
<tr>
<td>( \lim_{\varepsilon \to \infty} M(\varepsilon) )</td>
<td>7.28</td>
<td>6.82</td>
<td>6.90</td>
<td>6.93</td>
<td>6.94</td>
</tr>
</tbody>
</table>

Table 1: \( \tilde{M} \) for variations in \( n \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>10</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n^* )</td>
<td>4.06</td>
<td>0.06</td>
<td>0.02</td>
<td>0.015</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 2: RW for variations in \( n \)

It is necessary, of course, to further investigate the range of relative effectiveness. Variations in the indicator \( RW \) can be expected, of course, because the relative size \( \frac{n^*}{n} \) of
the stable coalition varies with $n$ and $m^*$ varies with $\varepsilon$ for any given $n$. To get an estimate of the range of the indicator $\text{RW}$, when we account for variations in $n$, we have computed many more numerical examples with different parameter values. Table 2 shows the upper bounds of $\text{RW}$ that are attained in these examples for alternative values of $n$. The clear message of Table 2 is that, ceteris paribus, the relative effectiveness of the (respective) stable coalition is strictly decreasing in the total number of countries. Taking into account that the maximum value $\text{RW} = 18\%$ for $n = 10$ is fairly small and that the total number of countries in the real world is about $n = 200$ Table 2 suggests that the formation of a sub-global stable coalition is hardly worth the effort.

## 5 Concluding remarks

In the theoretical literature on capital tax competition the inefficiency of non-cooperative national tax policies is well-understood, and it is also clear that, in principle, tax coordination has a potential to alleviate or even mitigate the distortions caused by fiscal externalities. The few existing studies addressing the issue of international capital tax coordination offer limited information only due to the formidable analytical complexity of coalition formation. To make progress we consider a model with identical countries and parametric functional forms as is not uncommon in the tax competition literature. With these simplifications we are able to fully determine the conditions under which the grand coalition is stable. With reference to Burbridge et al. (1997), Vrijburg and de Mooij (2010) suggest that "[only] when countries are sufficiently similar, will the grand coalition of all countries arise in equilibrium". We show that symmetry is not sufficient for the formation of the grand coalition and according to Bucovetsky (2009) it is not necessary either. An increasing total number of countries makes the stability of the grand coalition less likely. However, our example of a stable grand coalition of 100 countries supports the view that a global tax agreement may not be out of reach.

Since even under favorable conditions the grand coalition may fail to be stable, a crucial question is whether there exist sub-global stable coalitions. While the extant literature offers no pertaining results, our answer is in the affirmative. Our analysis shows that a rather small stable coalition (consisting e.g. of 3 to 6 countries in a model world of 100 countries) always exists. That small coalition is either the unique stable coalition or it coexists with a stable grand coalition. The finding that there is a small stable coalition may be considered good news, but the bad news is that the small stable coalition is ineffective in the sense that all countries gain very little compared to BAU; the fringe countries gain a little bit more than
the coalition countries.

It must be (re)emphasized that our results cannot be taken as a straightforward guide to practical policy. The real world is far more complex than our model, notably regarding the heterogeneity of countries. Asymmetries complicate the analysis of tax coordination drastically. Our model of identical countries provides limited but informative insights in the potential of coalition formation for welfare gains.

References


**Appendix**

A: Proof of Proposition 1.

Solving (15) and (17) with respect to $t_c$ and $t_f$ yields

$$
t^*_c = \frac{\{m(n + 1) - m^2 + n[1 + 2\varepsilon] - 1 - \varepsilon]\varepsilon \bar{k}b}{(n - m)(1 + \varepsilon)(1 + \varepsilon)[m(1 + \varepsilon) + (n - 1)(1 + 2\varepsilon)]} > 0, \quad (A1)
$$

$$
t^*_f = \frac{[n^2(1 + 2\varepsilon) - m^2 - m(n\varepsilon - 1)]\varepsilon \bar{k}b}{(n - m)(1 + \varepsilon)[m(1 + \varepsilon) + (n - 1)(1 + 2\varepsilon)]} > 0. \quad (A2)
$$

From (A1), (A2) and \( t_o = \frac{n\varepsilon \bar{k}b}{(n - 1)(1 + \varepsilon)} \) we get

$$
t^*_c - t^*_f = \frac{(m - 1)n\varepsilon \bar{k}b}{(n - m)[m(1 + \varepsilon) + (n - 1)(1 + 2\varepsilon)]} > 0, \quad (A3)
$$

$$
t^*_c - t_o = \frac{(m - 1)[m(\varepsilon + 1) + (n - 1)n(1 + \varepsilon)]\varepsilon \bar{k}b}{(n - 1)(n - m)(1 + \varepsilon)[m(1 + \varepsilon) + (n - 1)(1 + 2\varepsilon)]} > 0, \quad (A4)
$$

Subtracting \( W^c(t_c, t_f, m) \) from \( W^f(t_c, t_f, m) \) yields

$$
W^f(t_c, t_f, m) - W^c(t_c, t_f, m) = \frac{(t_c - t_f)[2m\varepsilon(t_c - t_f) + n(t_c + t_f + 2t_c\varepsilon - 2b\varepsilon \bar{k})]}{2bn}. \quad (A6)
$$

Next, we insert \( t^*_c \) and \( t^*_f \) from (A1) and (A2) in (A6) to obtain after some rearrangement of terms

$$
W^f(t^*_c, t^*_f, m) - W^c(t^*_c, t^*_f, m) = \frac{(m - 1)n\varepsilon \bar{k}b^2 [(m + 1)n(1 + 2\varepsilon) - 2m\varepsilon]}{2(n - m)^2 [m(1 + \varepsilon) + (n - 1)(1 + 2\varepsilon)]^2} > 0. \quad (A7)
$$
Substracting $W^c(t_o, t_o, m) = \left(a - \frac{b_k}{2} + t_o \varepsilon \right) \bar{k}$ from $W^c(t_c, t_f, m)$ yields

$$W^c(t_c, t_f, m) - W^c(t_o, t_o, m) = \frac{m^2 (t_c - t_f)^2 + 2mn (t_c - t_f) (t_f + \varepsilon t_c) + n^2 \left[ t_f^2 - t_c^2 (1 + 2\varepsilon) - 2bt_c \varepsilon \bar{k} + 2t_c \varepsilon (t_f + \bar{k}) \right]}{2bn^2}. \quad (A8)$$

Next, we insert $t^*_c$ and $t^*_f$ from (A1) and (A2) and $t_o = \frac{nekb}{(n-1)(1+\varepsilon)}$ in (A8) to obtain after some rearrangement of terms

$$W^c(t^*_c, t^*_f, m) - W^c(t_o, t_o, m) = \frac{m^2 (n+1) + (n-1)(nm + m - n) + [m^2(3n+1) + (n-1)(5mn - 3n + 3n^2)] \varepsilon}{2(n-m)(n-1)(1+\varepsilon) [m(1+\varepsilon) + (n-1)(1+2\varepsilon)]^2} + \frac{[2m^2n + (n-1)(6mn - 2n)] \varepsilon^2}{2(n-m)(n-1)(1+\varepsilon) [m(1+\varepsilon) + (n-1)(1+2\varepsilon)]^2} > 0. \quad (A9)$$

(A7) and (A9) establishes $W^f(t^*_c, t^*_f, m) > W^c(t^*_c, t^*_f, m) > W^c(t_o, t_o, m) \equiv w_o$. \quad ■

**B: Proof of (21).**

Making use of (A1), (A2), (1), (5) and $k_o = \bar{k}$ in $X(k) + (\bar{k} - k)r$ we get after rearrangement of terms

$$X(k^*_c) + (\bar{k} - k^*_o)r^* - \left[ X(k_o) + (\bar{k} - k_o)r_o \right] = \frac{[2n^2 - m^2 + m + (4n^2 + m^2 - m - 2mn)\varepsilon] b(m - 1) \varepsilon^2 \bar{k}^2}{2(n-m)^2(1+\varepsilon) [m(1+\varepsilon) + (n-1)(1+2\varepsilon)]^2} > 0, \quad (B1)$$

$$X(k^*_c) + (\bar{k} - k^*_o)r^* - \left[ X(k_o) + (\bar{k} - k_o)r_o \right] = \frac{[m^2 + n - 2m^2 - m(n+1) + (n + m + nm - 4n^2 - m^2)\varepsilon] b(m - 1) \varepsilon^2 \bar{k}^2}{2(n-m)(1+\varepsilon) [m(1+\varepsilon) + (n-1)(1+2\varepsilon)]^2} < 0. \quad (B2)$$

**C: Proof of Proposition 2.**

Differentiation of $t^*_c = T^c(m)$ and $t^*_f = T^f(m)$ from (A1) and (A2) with respect to $m$ yields

$$\frac{dT^c(m)}{dm} = T^c_m = \frac{[n + \varepsilon(2n - 1)] [m^2 + n(n-1) + n\varepsilon(m + 2m - 2)] b\varepsilon \bar{k}}{(n-m)^2(1+\varepsilon) [m(1+\varepsilon) + (n-1)(1+2\varepsilon)]^2} > 0, \quad (C1)$$

$$\frac{dT^f(m)}{dm} = T^f_m = \frac{(n\varepsilon + 1) [(2m-1)n + (2n(2m - 1) - m^2)\varepsilon] b\varepsilon \bar{k}}{(n-m)^2(1+\varepsilon) [m(1+\varepsilon) + (n-1)(1+2\varepsilon)]^2} > 0. \quad (C2)$$
Define the welfare functions

\[ W^c(t_c, t_f, m) := X \left( \frac{k - t_c}{b} + \frac{\Theta}{b} \right) - (a - \bar{k}b - \Theta + t_c) \cdot \left( \frac{k - t_c}{b} + \frac{\Theta}{b} \right) + (a - \bar{k}b - \Theta) \bar{k} \]
\[ + (1 + \varepsilon) t_c \left( \frac{k - t_c}{b} + \frac{\Theta}{b} \right) \]
\[ = \frac{m^2 (t_c - t_f)^2 + 2mn(t_c - t_f)(t_f + \varepsilon t_c)}{2bn^2} \]
\[ + \frac{n^2 [t_f^2 - t_f^2(1 + 2\varepsilon) + (2\alpha - b\bar{k}) + 2t_c\varepsilon(t_f + b\bar{k})]}{2bn^2} \] (C3)

\[ W^f(t_c, t_f, m) := X \left( \frac{k - t_f}{b} + \frac{\Theta}{b} \right) - (a - \bar{k}b - \Theta + t_f) \cdot \left( \frac{k - t_f}{b} + \frac{\Theta}{b} \right) + (a - \bar{k}b - \Theta) \bar{k} \]
\[ + (1 + \varepsilon) t_c \left( \frac{k - t_f}{b} + \frac{\Theta}{b} \right) \]
\[ = \frac{m^2 (t_c - t_f)^2 + 2mn(t_c - t_f)t_f(1 + \varepsilon) + bn^2 \bar{k}(2\alpha + 2t_f\varepsilon - b\bar{k})}{2bn^2} \] (C4)

Total differentiation of \( W^c(m) := W^c(T^c(m), T^f(m), m) \) and \( W^f(m) := W^f(T^c(m), T^f(m), m) \) yields

\[ \frac{dW^c(m)}{dm} = W^c_{t_c} \cdot T^c_m + W^c_{t_f} \cdot T^f_m + W^c_m, \] (C5)
\[ \frac{dW^f(m)}{dm} = W^f_{t_c} \cdot T^c_m + W^f_{t_f} \cdot T^f_m + W^f_m, \] (C6)

where

\[ W^c_{t_f} = \frac{(n - m) [m(t_c - t_f) + n(t_f + t_c\varepsilon)]}{bn^2}, \] (C7)
\[ W^c_m = \frac{(t_c - t_f) [m(t_c - t_f) + n(t_f + t_c\varepsilon)]}{bn^2}, \] (C8)
\[ W^f_{t_c} = \frac{m [m(t_c - t_f) + nt_f(1 + \varepsilon)]}{bn^2}, \] (C9)
\[ W^f_{t_f} = \frac{m^2 (t_c - t_f) + mn(t_c - 2t_f)(1 + \varepsilon) + bn^2 \bar{k}}{bn^2}, \] (C10)
\[ W^f_m = \frac{(t_c - t_f) [m(t_c - t_f) + nt_f(1 + \varepsilon)]}{bn^2}. \] (C11)

Finally, we insert \( t^*_c = T^c(m) \) and \( t^*_f = T^f(m) \) from (A1) and (A2) in (C10) to obtain after rearrangement of terms

\[ W^f_{t_f} = \frac{(n - m - 1) [n + \varepsilon(2n - m)] \bar{k}}{(n - m) [m(1 + \varepsilon) + (n - 1)(1 + 2\varepsilon)]} > 0. \] (C12)

Observe that \( t^*_c > t^*_f > 0 \) implies \( W^c_{t_f} > 0 \) in (C7), \( W^c_m > 0 \) in (C8), \( W^f_{t_c} > 0 \) in (C9) and \( W^f_m > 0 \) in (C11). Using this information together with \( T^c_m > 0, T^f_m > 0 \) from (C1), (C2) and \( W^f_{t_f} > 0 \) from (C12) in (C5) and (C6) establishes \( \frac{dW^c(m)}{dm} > 0 \) and \( \frac{dW^f(m)}{dm} > 0. \)
D: Proof of (24)-(27).

Making use of \( \hat{w} = (1 + \varepsilon)a\bar{k} - b\bar{k}^2 \left( \frac{1}{2} + \varepsilon \right) \) and inserting \( t_c^* \) and \( t_f^* \) from (A1) and (A2) in \( W^f(t_c, t_f, m) \) yields after rearrangement of terms

\[
W^f(n - 1) - \hat{w} = a\bar{k} \\
\frac{[n^3\varepsilon(1 + 3\varepsilon + 2\varepsilon^2) + n^2\varepsilon(1 + 3\varepsilon + 4\varepsilon^2) + 4n(2 + 9\varepsilon + 13\varepsilon^2 + 5\varepsilon^3) - 2(4 + 18\varepsilon + 25\varepsilon^2 + 9\varepsilon^3)] b\bar{k}^2}{2(n - 1)(1 + \varepsilon)(2 + 3\varepsilon)^2}
\]

(D1)

Setting \( W^f(n - 1) - \hat{w} = 0 \) and solving for \( \frac{a}{bk} \) yields after rearrangement of terms

\[
\frac{a}{bk} = \frac{8(n - 1) + [n^2(n + 1) + 36(n - 1)] \varepsilon + [3n^2(n + 1) + 52n - 50] \varepsilon^2}{2(n - 1)(1 + \varepsilon)(2 + 3\varepsilon)^2} \\
+ \frac{[2n^2(n + 2) + 20n - 18] \varepsilon^3}{2(n - 1)(1 + \varepsilon)(2 + 3\varepsilon)^2} =: F(\varepsilon, n).
\]

(D2)

Then it is straightforward to show that

\[ \hat{w} - W^f(n - 1) \gtrless 0 \iff \frac{a}{bk} \gtrless F(\varepsilon, n). \]

(D4)

Differentiation of \( F(\varepsilon, n) \) with respect to \( \varepsilon \) and \( n \) yields

\[
F_\varepsilon = (1 + \varepsilon)^2(2 + 5\varepsilon)n^3 + (2n^2 + 8n - 8) + (9n^2 + 28n - 20)\varepsilon \\
\frac{2(n - 1)(1 + \varepsilon)^2(2 + 3\varepsilon)^3}{(n - 1)(1 + \varepsilon)^2(2 + 3\varepsilon)^3} > 0,
\]

(D5)

\[
F_n = \frac{(n^3 - n^2 - n) + [3(n^2 - n - 1)] \varepsilon + [2n^3 - n^2 - 4n] \varepsilon^2}{(n - 1)^2(1 + \varepsilon)(2 + 3\varepsilon)^2} > 0.
\]

(D6)

In addition, the function \( F \) has the properties

\[ F(n, 0) = 1, \quad \lim_{n \to \infty} F(\varepsilon, n) = \infty, \]

(D7)

\[ \lim_{\varepsilon \to \infty} F(\varepsilon, n) = \frac{n^3 + 2n^2 + 10n - 9}{9(n - 1)} =: \bar{F}(n) > 0, \]

(D8)

and it holds \( \bar{F}_n(n) = \frac{2n^3 - n^2 - 4n}{9(n - 1)^2} > 0 \) for \( n \geq 2 \).

E: Proof of Proposition 4.

The proof of Proposition 4 (i) follows from Table 1, Figure 8 which illustrates the function \( \tilde{M}(\varepsilon, n) \) for \( n \in [10, 200] \) and \( \varepsilon \in [0, 2] \), and \( \tilde{M}_e > 0 \).
Figure 8: The function $\tilde{M}(\varepsilon, n)$