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Visitations and Transfers in Non Intact Households

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CENTER DISCUSSION PAPER NO.807

VISITATIONS AND TRANSFERS IN NON INTACT HOUSEHOLDS

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Visitations and Transfers in Non Intact Households

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Abstract

Recent research reveals a negative impact of divorce on children's welfare as a consequence of the reduction in monetary and time contributions by the non-custodian parent. When the custody arrangement is sole custody, the variables that link the absent parent to the child are visitations and child support transfers. We explain visitations and child support transfers using a behavioral model of competitive equilibrium in which both variables are the results of competitive allocations realized in a decentralized non-cooperative manner.

In our framework the mother has control over visitations and the father has control over child support. Estimates of the model are used to simulate the effects of alternative endowment levels on the proportion of time spent with the noncustodial parent and the ex-post parental income distribution. Our results show that a more equal allocation of time with the child, though beneficial to the children, may have a negative effect for the mother's welfare, increasing the income gap between ex-spouses.

Key Words: Divorce, Visitations, Child support transfers

Introduction

One of the negative consequences of the increasing number of divorces is the reduced income for the custodial parents and the children living with her/him. Despite several attempts to improve the system of child support transfers in the United States, only about half of the women entitled to child support payments receive the full amount they are due, and one fourth of households receive no payment at all.

The economic decline in income is not the only way in which children are affected by parents' separation or divorce. Marital dissolution can alter the amount of time spent with the children by the non-custodial parent (generally the father)¹. A reduction in time spent by the non residential parents with their children can determine children outcomes much beyond the effect attributable to income contribution, affecting negatively human capital accumulation, educational attainment, as well as starting wages and job performance later on.

There is an important relationship between time spent with the non-custodian parents and money transfers: parents that maintain close contact with their children after divorce have more time to enjoy with them and more ways to monitor the effect of their transfers on child well-being.

The following table is a cross tabulation of transfers by visitations (low versus high for each). Note that 80 percent of parents with high visitation rates have high transfers, while only 36 percent of parents with low transfers have high visitation rates.

Table 1

Transfers by Visitation Rate

Transfers	Visitations		Total
	High	Low	
High	275	67	342
Low	130	229	359
Total	405	296	701

¹Research focusing on the time spent with father after divorce shows that the proportion of children seeing their father once a week is quite small, while the proportion of children who have lost contact with their father is about forty per cent (Furstenberg et al. 1987)

In this paper we focus on the link between visitations and child support transfers. As Maccobby and Mnookin (1992) remark, the links between visitation and child support payments are reinforced by social values. A father who does not support his children “may in popular perception no longer be entitled to maintain a relationship with his minor children if the custodian mother objects”. In the same way, a mother who purposely denies her ex-husband any relationship with the children “may be viewed as no longer entitled to his support”. The amount of time the children spend with each parent is the outcome of the exchange between mother and father after divorce, where the mother has control on the time of the child and the father uses the income available to acquire time with their children.

While some empirical findings and anecdotal evidence show that more time with the father after divorce would be beneficial for the children in several dimensions, no literature to our knowledge has tried to investigate the effect of alternative time arrangements on the distribution of resources between the parents. We use the results of our model to simulate the effect of a visitation arrangement similar to joint custody. The results indicate that an allocation of time equivalent to joint physical custody would leave the mother economically worse off (relatively to a full custody arrangement) increasing the initial unequal distribution of resources between the ex-spouses. The increase in child expenditures however partly compensates for the loss of income. We discuss various interpretations and policy issues.

The paper is organized as follows. In Section 1 we discuss some aspects of the recent literature. Section 2 presents the theoretical model in which parents derive utility on the time they spend with the children. Section 3 describes the empirical implementation of the model. Section 4 presents the data. Section 5 contains the solutions of the model. We solve the model for the equilibrium parameters and use them to simulate the effect of an endowment of equal time on income distribution between the two parents. Section 6 extends the model to include the case in which the parents derive utility from direct expenditures on the children. Section 7 contains some conclusions and research directions.

1 Recent Literature

While there is an extensive literature analyzing the transfers to the children by the non-custodial parents after divorce, only a limited number of studies

have analyzed the amount of time children spend with their parents after divorce. It has been shown that a reduction in time spent with one parent can affect children's lives along many dimensions including their human capital accumulation and labor market performance (Haveman and Wolfe, 1995, Beller, 1993).

The results of the few empirical studies on the relationship between non-custodial contributions to children and their involvement in their life after divorce are quite mixed. A positive relationship has been found in cross-sectional studies showing that money and time are complements. Paying child support may change the father-child relationship by increasing the mothers' willingness to let them see each other. Peters et al (1993) show that the continued involvement between fathers and children can result in self-enforcing parental visitation and payment arrangement similar to those in intact families.

According to Wallerstein and Huntington (1983), visitations and the post-divorce relationship, among other factors, were strongly correlated with child support payments. In their study, the duration of each visit was found to be a more accurate measure of the father's interest in the child than the frequency of contact. Fustenberg et. al. (1983) also found a positive relationship between compliance with child support award and contact between the father and the child, proposing that the existence of child support rather than the amount seemed to be related to the maintenance of the relationship between the father and his child.

Other research has found a negative relationship implying that child support payment may increase the conflict between the two parents if the child support enforcement is hostile. When forced to pay child support fathers see their children less frequently and live further from them (Mac Lanahan et. al., 1997). Anderson (1993) used data from the National Survey of Children to estimate child support transfers and visitations in a simultaneous equation framework and found that child support transfers and visitation allowance were used strategically. The mother uses her power as custodian to decide on the time the child spends with the father to guarantee the child support payments, whereas the father uses his control over child support transfers to elicit some behavior from the mother with respect to visitations. Veum (1993) also analyzes visitations and child support contribution as joint decisions using instrumental variables. His analysis, based on the National Longitudinal Survey of Youth, reports no relationship between child support and visitations. These different results point to the need of further research

on this issue.

Using a game-theoretic approach, Weiss and Willis (1985, 1993) provide an interpretation of the low compliance with child support that has to do with the time spent with the children. The father's non compliance with the child support order is caused by his lack of monitoring power over the allocation of the transfer by the mother. In their view, a father who wants to support his child, but not his ex-spouse, would tend to pay less than the amount he would like to spend on his child, because he cannot guarantee that the money will be spent on the child. This implies that more time with the children may facilitate the non-custodian's control over the allocation of resources, allowing to monitor the outcomes of their expenses. Del Boca and Flinn (1991, 1995) have provided a theoretic framework in which the behaviors of divorced parents regarding the sharing of living expenses of their children are the result of optimal decision making. Following their lines, in this paper we analyze behaviorally the exchange between visitations and child support payments that the evidence has shown to happen in real life.

Visitation arrangements and child support transfers are crucial issues in the negotiation following the divorce of a couple with children. A mother may object to the children visiting the father regularly, if she does not receive child support transfers from her ex husband. Similarly, a non-custodian father, who is denied visitations to his children, may be more unwilling to pay child support than a father who is allowed to spend time with them (Maccobby and Mnookin, 1992).

In the models developed in this paper, we find that child support transfers depend only on the father's income, and not on the income of the mother. Visitations depend on mother's and father's incomes, supporting the idea of a negotiation process whereby the father's income is exchanged against visitations allowed by the mother. Visitation arrangements on one hand guarantee non resident parents an endowment of time with their children. On the other hand, they ensure the mothers income transfers to compensate them for the reduction in welfare after divorce. The results show that a reduction of time under the mother's control implies a reduction in the income transfers from the father and therefore a loss in the mother's consumption levels.

2 Theoretical Model

In this section we explain visitations and child support payments and child expenditures using a behavioral model of competitive equilibrium, in which the variables are the result of competitive allocations realized in a decentralized non-cooperative manner. While the importance of institutional constraints on parents' decisions after divorce is well known (Del Boca and Flinn, 1995, Del Boca and Ribero, 1998), we will not consider these aspects here. In our framework the parents value the child in two ways: 1) they care about the welfare of the child and 2) they enjoy spending time with the child. While during marriage time with the child is a public good, after separation it becomes a private good.

In this model, parents are divorced and have had one child from the marriage. Each one of them has preferences defined over the amount of a representative consumption good c_j and the amount of time they spend with their child h_j , that can be represented by a Von Neumann-Morgenstern utility function $u_j(c_j, h_j)$, $j \in \{m, f\}$, where the subindex m stand for mother and the subindex f stand for father.

Both parents have access to two independent sources of income, y_m and y_f . At this point, these incomes are assumed to be independent of the time that each parent spends with the child. In case of the mother, this assumption is more problematic given the possible negative association between labor income and child care. We will address this issue in future developments of our model. Without loss of generality the total time the child can spend with the parents is normalized to one, so that $h_m + h_f = 1$. A critical assumption (that will be relaxed later) is that the mother has the sole physical custody of the child, because that guarantees that her initial endowment of "time with the child" is equal to one. Both parents act simultaneously and the equilibrium is determined in one period.

The behavior of the parents is decentralized and non-cooperative. Each of them derives their own demand from utility maximization subject only to their budget constraint, without knowledge of the demands or concern for the tastes of the other parent.² In this setting, prices are a signal of scarcity and parents interact with the market rather than with each other as in the bargaining models analyzed by Weiss and Willis (1985) and Del Boca

²Tastes and preferences of the divorced parents may be interdependent to some extent, for assortative mating argument given the two parents have been married for some time.

and Flinn (1994, 1995). However, by virtue of the first welfare theorem and without the existence of public goods, this competitive equilibrium is also Pareto optimal. The competitive allocation is therefore consistent with the maximization of utility of each parent subject to hold the utility of the other parent constant.

Normalizing the price of the consumption good to one, let p represent the monetary amount that the father is prepared to pay in order to stay one unit of time with the child. The budget constraint for each parent guarantees that the monetary value of the consumption vector cannot exceed the value of the initial endowment vector. Therefore, if the vector of prices of consumption good and time with the child is given by $\vec{p} = (1, p)$, the vector of mother's consumption is given by $\vec{x}_m = (c_m, h_m)$ and her initial endowment is given by $\vec{w}_m = (y_m, 1)$,³ then the budget constraint for the mother can be expressed as $\vec{p} \cdot \vec{x}_m \leq \vec{p} \cdot \vec{w}_m$. Given her total endowment of income and time with the child, the mother chooses a level of consumption of the private good and time to spend with the child, by solving the problem:

$$\begin{aligned} & \max u_m(c_m, h_m) & (1) \\ \text{s.t. } & c_m + ph_m \leq y_m + p. \end{aligned}$$

The solution of this problem is given by the mother's demands $c_m(p, y_m)$ and $h_m(p, y_m)$.

On the other hand, the father's initial endowment is given only by his monetary income $\vec{w}_f = (y_f, 0)$.⁴ Denoting his vector of consumption by $\vec{x}_f = (c_f, h_f)$, his budget constraint can be expressed as $\vec{p} \cdot \vec{x}_f \leq \vec{p} \cdot \vec{w}_f$. He will allocate his income between the consumption good and time with his child according to the solution of the problem:

$$\begin{aligned} & \max u_f(c_f, h_f) & (2) \\ \text{s.t. } & c_f + ph_f \leq y_f. \end{aligned}$$

The solution of this problem is given by the father's demands $c_f(p, y_f)$ and $h_f(p, y_f)$.

The equilibrium of the "market" is given when the sum of the demands for each good is equated to the aggregate supply, that is the sum of endowments

³Later, when we drop the mother's sole custody assumption, the mother's initial endowment will be given by $\vec{w}_m = (y_m, \varepsilon)$, where $0 < \varepsilon < 1$.

⁴Later, when we drop the mother's sole custody assumption, the father's initial endowment will be given by $\vec{w}_f = (y_f, 1 - \varepsilon)$, where $0 < \varepsilon < 1$.

of each good. Therefore, the clearing equations are:

$$\begin{aligned} c_m(p, y_m) + c_f(p, y_f) &= y_m + y_f \\ h_m(p, y_m) + h_f(p, y_f) &= 1. \end{aligned} \tag{3}$$

Solving for p in the equilibrium equations, we get the solution for the equilibrium price p and the equilibrium allocations c_m, h_m, c_f and h_f . These variables must satisfy the equilibrium conditions given by:

$$\begin{aligned} MRS_m(c_m, h_m) &= MRS_f(c_f, h_f), \\ c_m + c_f &= y_m + y_f, \\ h_m + h_f &= 1, \\ c_m + ph_m &= y_m + p \\ MRS_m(c_m, h_m) &= p, \end{aligned} \tag{4}$$

where MRS_j are j 's marginal rates of substitution between consumption and time with the child, $j \in \{m, f\}$.

The first equation means that the equilibrium allocation has to be Pareto optimal. The next two guarantee that it must be feasible. The fourth equation means that the allocation has to be in the budget constraint of the parents, and the last one that the price line is tangent to the indifference curves of both parents in equilibrium. The existence and uniqueness of the equilibrium is guaranteed by the assumptions made on the utility functions.⁵

The child support transfer is the amount of money that the father pays to the mother, which in this model represents the cost of the time to be spent with the child. Therefore, denoting the child support transfer by t , we have:

$$t(y_m, y_f) = p(y_m, y_f)h_f(y_m, y_f). \tag{5}$$

Define the total visitation time obtained by the father to be equal to h_f , that is, the time the father spends with the child. Thus, denoting the visitations by v , we have:

$$v(y_m, y_f) = h_f(y_m, y_f). \tag{6}$$

⁵Technically, for existence of the equilibrium, it is assumed that the utility functions of both parents are such that the "better than" sets are closed, and strictly convex, satisfy local non-satiation and the tangency between the two indifference curves does not occur in the corners. For uniqueness of the equilibrium, it is assumed that the consumption good and the time with the child are gross substitutes, i.e. that an increase in the price of one of the goods implies an increase in the excess demand of the other one.

The following figure shows how the exchange between visitations and transfers takes place. It is an Edgeworth box with the consumption good on the horizontal axis and the time to spend with the child on the vertical one. By assumption, the *vertical* side of the box has length one and the length of the horizontal side is the sum of the incomes of the father and the mother. The *bottom* left corner is the mother's origin and the top right corner is the father's origin. The initial endowment is represented by the point \mathbf{w} that correspond to the allocation $\vec{w}_m = (y_m, 1)$, from the mother's origin and to point $\vec{w}_f = (y_f, 0)$, from the father's origin.

The contract curve is the set of allocations in which the marginal rates of substitution are equal for the father and the mother, represented in the graph by the curve \mathbf{cc} . The equilibrium allocation is represented by the point \mathbf{e} , a point on the contract curve. The line that crosses \mathbf{w} and \mathbf{p} is the price line, and has slope $-p^{-1}$. The horizontal distance between \mathbf{w} and \mathbf{r} is the child's support transfer $t(y_m, y_f)$, and the vertical distance is the visitations time: $v(y_m, y_f)$.

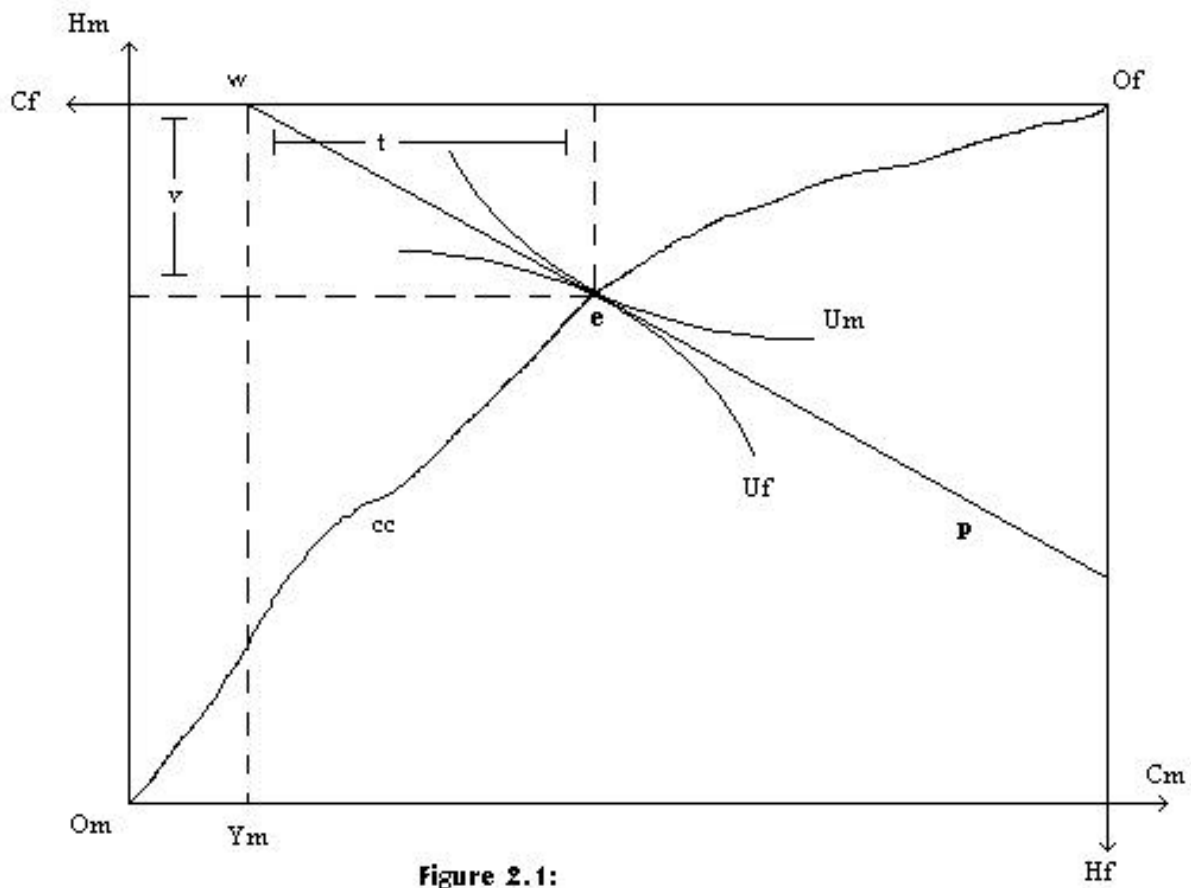


Figure 2.1:

The comparative statics predictions of this model are summarized in the following lemma.

Lemma 1: Under the assumption that:

$$\left(\frac{du_m^2(c_m, h_m)}{dc_m dh_m} - p \frac{du_m^2(c_m, h_m)}{dc_m^2}\right) h_f - \frac{du_m(c_m, h_m)}{dc_m} \leq 0, \quad (7)$$

the equilibrium allocations change with respect to income variations in a way described by the following derivatives:

$$\begin{aligned} dc_m(y_m, y_f)/dy_m &\geq 0, & dc_m(y_m, y_f)/dy_f &\geq 0, & dc_f(y_m, y_f)/dy_m &\leq 0, \\ dc_f(y_m, y_f)/dy_f &\geq 0, & dh_m(y_m, y_f)/dy_m &\geq 0, & dh_m(y_m, y_f)/dy_f &\leq 0, \\ dh_f(y_m, y_f)/dy_m &\leq 0, & dh_f(y_m, y_f)/dy_f &\geq 0, & & \\ dp(y_m, y_f)/dy_f &\geq 0, & dp(y_m, y_f)/dy_m &\leq 0. & & \end{aligned} \quad (8)$$

These relationships are proved by taking total derivatives in the equations that define the equilibrium and following standard comparative statics methods. See Appendix 1 for details.

The signs of the derivatives are consistent with the intuition of the exchange that takes place between the parents. According to the model, the higher the income of the father, the less time the mother spends with the child because fathers with higher income have more capacity to buy time with the child as well as their own consumption good. Similarly, higher income fathers spend more time with their children. Also, the mothers who have more income spend more time with the child, leaving less time for the father to spend with the child.

Some of the signs of the derivatives are ambiguous. For example, it is uncertain how a change in the income of the mother will affect the consumption allocation of the father or the equilibrium price. On one hand, it may be that the increase in y_m increases her demand for time with the child. This may affect the equilibrium price making more expensive for the father to buy time with the child. Then he can either increase his consumption of the other good c_f , or he may pay the higher price for time with the child decreasing his consumption of the good.

The derivatives for the variables of interest are derived from the derivatives in 8 and the definitions of t and v given in 5 and 6. They can be summarized as:

$$\begin{aligned} dt(y_m, y_f)/dy_f &\geq 0, & dt(y_m, y_f)/dy_m &\leq 0, \\ dv(y_m, y_f)/dy_f &\geq 0, & dv(y_m, y_f)/dy_m &\leq 0. \end{aligned}$$

The model implies that fathers with higher incomes transfer more and visit more, that the mother's income has an ambiguous effect on child support transfers and that mothers with higher incomes allow fewer visitations. These comparative statics results are helpful in understanding the changes in transfers and visitations that may occur in different situations. For example, if the mother gets remarried, this may imply an increase in her income, since she is now with a partner that may support her. In the context of the model, this would imply an increase in her consumption of the good and of time with the child, with a consequent decrease in the father's visitation time. A joint physical custody agreement that we consider below may be considered a reallocation of time endowment with consequence on child support transfers and consequently on the mothers total consumption levels. Mandatory child support orders such as the ones implemented in some states like Wisconsin, might be considered like an "ad hoc" decrease in the income of the father, with possible detrimental consequences on the visitations time for the fathers.

3 Empirical Implementation

Consider the specification of the model with Cobb-Douglas utility functions for both parents:

$$u_j(c_j, h_j) = \delta_j \log(c_j) + (1 - \delta_j) \log(h_j) \quad (9)$$

$$\delta_j \in (0, 1), \quad j \in \{m, f\}, \quad c_j > 0, \quad h_j > 0.$$

Given her total endowment of income and time with the child, the mother chooses a level of own consumption of the private good and time to spend with the child, solving problem 1. Her demands are:

$$c_m(\delta_m, p, y_m) = \delta_m(p + y_m), \quad (10)$$

$$h_m(\delta_m, p, y_m) = (1 - \delta_m) \frac{p + y_m}{p}.$$

The father allocates his income between consumption good and time with his child, according to the solution of problem 2. His demands are:

$$c_f(\delta_f, p, y_f) = \delta_f y_f, \quad (11)$$

$$h_f(\delta_f, p, y_f) = (1 - \delta_f) \frac{y_f}{p}.$$

The equilibrium price and allocations are given by:

$$\begin{aligned}
p(\delta_m, \delta_f, y_m, y_f) &= \frac{(1 - \delta_m)y_m + (1 - \delta_f)y_f}{\delta_m}, \\
c_m(\delta_f, y_f, y_m) &= (1 - \delta_f)y_f + y_m, \\
h_m(\delta_m, \delta_f, y_m, y_f) &= \frac{(1 - \delta_m)[y_m + (1 - \delta_f)y_f]}{(1 - \delta_m)y_m + (1 - \delta_f)y_f}, \\
c_f(\delta_f, y_f) &= \delta_f y_f, \\
h_f(\delta_m, \delta_f, y_m, y_f) &= \frac{\delta_m(1 - \delta_f)y_f}{(1 - \delta_m)y_m + (1 - \delta_f)y_f}.
\end{aligned} \tag{12}$$

This utility function satisfies assumption 7. Under this specification, changes in the income of the mother do not alter the consumption allocation of the father. The child support transfer t is given by:

$$t(\delta_f, y_f) = ph_f(\delta_m, \delta_f, y_m, y_f) = (1 - \delta_f)y_f, \tag{13}$$

and the visitations time v is given by:

$$v(\delta_m, \delta_f, y_m, y_f) = h_f(\delta_m, \delta_f, y_m, y_f) = \frac{\delta_m(1 - \delta_f)y_f}{(1 - \delta_m)y_m + (1 - \delta_f)y_f}. \tag{14}$$

The child support transfer depends solely on the father's income. The visitations depend on the income of the father as well as on the income of the mother.

In cases in which it is assumed that the mother is *not* endowed with all of the child's time, but only with a portion ε of the time ($\varepsilon < 1$), the equilibrium allocations are given by:

$$\begin{aligned}
p(\delta_m, \delta_f, y_m, y_f, \varepsilon) &= \frac{(1 - \delta_f)y_f + (1 - \delta_m)y_m}{\delta_m\varepsilon + \delta_f(1 - \varepsilon)}, \\
c_m(\delta_m, \delta_f, y_m, y_f, \varepsilon) &= \frac{\delta_m[(\varepsilon + \delta_f(1 - \varepsilon))y_m + (1 - \delta_f)\varepsilon y_f]}{\delta_m\varepsilon + \delta_f(1 - \varepsilon)}, \\
h_m(\delta_m, \delta_f, y_m, y_f, \varepsilon) &= \frac{(1 - \delta_m)[(\varepsilon + \delta_f(1 - \varepsilon))y_m + (1 - \delta_f)\varepsilon y_f]}{(1 - \delta_m)y_m + (1 - \delta_f)y_f}, \\
c_f(\delta_m, \delta_f, y_m, y_f, \varepsilon) &= \frac{\delta_f[(1 - \delta_m)(1 - \varepsilon)y_m + (1 - \varepsilon + \delta_m\varepsilon)y_f]}{\delta_m\varepsilon + \delta_f(1 - \varepsilon)}, \\
h_f(\delta_m, \delta_f, y_m, y_f, \varepsilon) &= (1 - \delta_f)(1 - \varepsilon) + \frac{[\delta_f^2(\varepsilon - 1) - \delta_f(\delta_m\varepsilon + \varepsilon - 1) + \delta_m\varepsilon]y_f}{(1 - \delta_m)y_m + (1 - \delta_f)y_f}.
\end{aligned} \tag{15}$$

In this case the child support transfer is computed considering that the father pays only for the time with the child additional to his endowment $(1 - \varepsilon)$. Similarly, the visitations time is defined as the time that the father spends with the child apart from his endowment $(1 - \varepsilon)$. The child support transfer and visitations are given by:

$$\begin{aligned}
t(\delta_m, \delta_f, y_m, y_f, \varepsilon) &= p[h_f - (1 - \varepsilon)] & (16) \\
&= \frac{\delta_m(1 - \delta_f)\varepsilon y_f - y_m\delta_f(1 - \varepsilon)(1 - \delta_m)}{\delta_m\varepsilon + \delta_f(1 - \varepsilon)} \\
v(y_f, \delta_f, \varepsilon) &= h_f - (1 - \varepsilon) \\
&= (-\delta_f)(1 - \varepsilon) + \frac{[\delta_f^2(\varepsilon - 1) - \delta_f(\delta_m\varepsilon + \varepsilon - 1) + \delta_m\varepsilon]y_f}{(1 - \delta_m)y_m + (1 - \delta_f)y_f}.
\end{aligned}$$

4 Data issues.

The data for this study are from the NLS of the high school class of 1972 - 5th follow up. This wave of the survey was taken in 1986. Out of the 12,841 respondents of the survey, we selected those who have been legally married and divorced or separated at least once, have had one child from that marriage and the physical custody of the child was assigned to the mother. The important issue of non response bias is analyzed in Ribero (1994). We also select cases with positive non-custodian incomes and child support transfers, because the model does not allow for corner solutions. The respondents in the sample know the marital status of their ex-spouses at the time of the survey and have provided information regarding visitations behavior as well as child support and expenditures besides child support. The selection process for the data set is described in Appendix 2.

Table 2 reports descriptive statistics of the variables. The proportion of remarried fathers is higher than the proportion of remarried mothers, and the income of fathers is higher than the income of mothers. Child support transfers are on average \$2,321 a year. Visitations are on average 46 days per year.⁶ The construction of the visitations variable is reported in Appendix 3. The data set also contains information on child expenditures of the non-custodian parent after child support. The construction of this variable is described in Appendix 3.

⁶This is equivalent to 12.6% of 365 days a year.

Table 3 reports an ordinary least squares analysis of visitations, child support transfers and the child expenditures of the fathers to offer a first look at the data. Visitations are positively influenced by parental incomes, while child support transfers depend only on the father's income. Father remarriage has a negative effect on the time spent with children as well as on child support transfers. His income is reduced by the responsibilities for the new family, and, as a consequence, he will pay less transfers and visit the child less. On the other hand, the remarriage of the mother may instead imply a higher level of her income, with the implication of less time allowed to the father. The duration of marriage has a positive effect on visitations, while it is not significant. The longer the marriage before separation, the higher the probability that parents act more cooperatively as well as the father and the children had time to build stronger ties. The empirical results are quite coherent with the predictions and interpretations obtained by the model described in Section 2.

Table 2
Means and standard deviations
(Sample with positive child support transfers)

Variables	Mean	Std. Dev.
<i>% Mother remarriage</i>	57.9%	-
<i>%Father remarriage</i>	62.2%	-
<i>Mother's income</i>	7,155	6,102
<i>Father's income</i>	16,822	9,354
<i>%Mother with some college</i>	27%	-
<i>%Father with some college</i>	35%	-
<i>Duration of marriage</i>	5.02	2.81
<i>Duration of divorce</i>	6.26	3.84
<i>Child support transfers</i>	2,321	2,028
<i>Visitations (days per year)</i>	46	39.5
<i>Index of child expenditures</i>	0.375	0.268
<i>Number of cases</i>	233	

Table 3

OLS estimates of visitations, child support transfers and father's expenditures on the child

Standard errors in parentheses

Variables	Visitations	Transfers	Index of child expenditures
<i>Constant</i>	33.716	145.406	1.200
<i>Father's income</i>	.490 (.149)	.221 (.110)	.026 (.007)
<i>Mother's income</i>	.459 (.245)	.136 (.181)	.013 (.011)
<i>Father remarriage</i>	-1.489 (3.148)	20.487 (22.050)	.102 (.149)
<i>Mother remarriage</i>	-2.387 (3.098)	-25.975 (22.051)	-.073 (.147)
<i>Duration of marriage</i>	-.179 (.765)	.847 (5.319)	.031 (.035)
<i>Duration of divorce</i>	-1.038 (.591)	-8.500 (4.130)	-.023 (.027)
<i>F-test</i>	6.15	4.53	6.02
<i>Number of cases</i>	635	595	701

5 Solutions

The empirical implementation of the model analyzed in Section 3 can be used to solve for the equilibrium levels of visitation and transfers conditional on the model parameters and on the distribution of endowments. In this case, the variables which fully capture the endowments are y_m , y_f , and ε , where ε is the proportion of time with the child which is given to the mother as an endowment. That is, if the mother is assumed to be entitled to all of the child's time, then $\varepsilon = 1$. We will also consider the case that the father is entitled, or endowed, with one day each week of time with the child, then $\varepsilon = 0.86$.⁷ Given the values of δ_m and δ_f and the endowment ε , we can solve for t and v from equations 13 and 14. Alternatively, given values of t and v and the endowment ε , these equations can be inverted to solve for the (implied) preference parameters of the parents. This procedure assumes that the preferences are heterogeneous in the population of divorced parents. The advantage of this technique is that no assumptions regarding the joint distribution or constancy of parental preferences are required. The main drawback is the fact that no provision is made for measurement errors or other types of data unreliability.

Equations 13 and 14 implicitly assume that $\varepsilon = 1$, that is, that the mother is endowed with all of the child's time. In this case, inverting the model as described above gives the following expressions for the parental preference parameters:

$$\begin{aligned}\delta_m(\varepsilon = 1) &= \frac{v(t + y_m)}{t + vy_m}, \\ \delta_f(\varepsilon = 1) &= 1 - \frac{t}{y_f}.\end{aligned}\tag{17}$$

In the cases that $\varepsilon < 1$, the solution of equations 16 for the preferences parameters gives:

$$\begin{aligned}\delta_m(\varepsilon) &= \frac{v(t + y_m)}{t\varepsilon + vy_m}, \\ \delta_f(\varepsilon) &= \frac{v(y_f - t)}{t(1 - \varepsilon) + vy_f}.\end{aligned}\tag{18}$$

⁷This number results from calculating 52 days per year and dividing by 365.

In the numerical exercises reported below, we have obtained values for the parental preference parameters under the alternative assumptions that $\varepsilon = 1$ or $\varepsilon = 0.86$.

The values (means and standard deviations) of the parameters obtained from these equations are in Table 4. The father's preferences parameter δ_f is 0.85 and the mother's preferences parameter δ_m is 0.31, implying that the mothers are less selfish than the fathers. The values of the mother's preference parameter is very different from the father's preference parameter (smaller difference between the two parameters was found in Del Boca and Flinn, 1995). This result depends crucially on the assumption of full time endowment to the mother. Under the assumption that the father has an initial endowment of one day a week ($\varepsilon = 0.86$), the obtained values are less different from each other, the mother preference parameter is 0.34 and the father's preference parameter is 0.64.

We are now interested in simulating the outcomes in terms of mother's available income and father's visitations under $\varepsilon = 0.5$, an initial endowment that we can claim gives both parents the same rights in terms of time to spend with the children (such as a joint custody agreement).

Table 5 reports the results of this exercise using the preference parameters obtained under the alternative assumptions that $\varepsilon = 1$ or $\varepsilon = 0.86$, compared with the observed values. These results show that under an agreement in which both parents are entitled to the same amount of time, visitations are 34 percent of the total time and 47 percent respectively (using the preference parameters obtained under the two alternative assumptions). The income available to the mother (the sum of her income and the father's transfer) decreases when the time with the child is distributed equally among both parents under both sets of parameters. This means that in that case the mother cannot "sell" all the time to the father given that a portion of time is endowed to him. In both cases her available income is lower than the observed one. Using the values of the preference parameters obtained under $\varepsilon = 0.86$, the father is less selfish and he sells less time with the child to the mother.

6 Extensions

The model can be extended by assuming that the parents derive utility not only from their own consumption and the time spent with the child, but

also from the amount of consumption good allocated to the child. Let k_f be the private expenditures the father makes on the child. The price of the consumption good for the child is assumed to be one, the same as the price of the parents consumption. The utility functions of the parents are assumed to be Cobb-Douglas of the form:

$$\begin{aligned}
u_m(c_m, h_m) &= \delta_m \log(c_m) + (1 - \delta_m) \log(h_m) \\
u_f(c_f, h_f, k_f) &= \delta_{1f} \log(c_f) + \delta_{2f} \log(h_f) + \delta_{3f} \log(k_f) \\
\sum_{i=1}^3 \delta_{if} &= 1, \quad \delta_m \in (0, 1), \delta_{if} \in (0, 1), \quad i \in \{1, 2, 3\}, \quad c_j > 0, \quad h_j > 0.
\end{aligned} \tag{19}$$

We assume that the mother's expenses on the child's consumption are equivalent to her own consumption expenses, given that she lives with the child and it is difficult to distinguish between her consumption and the child's consumption. Then c_m in this case denotes the consumption of the mother and the child together. This assumption is necessary because of data constraints, given that the data does not contain information about expenditures of the custodian parent in the child. Given her total endowment of income and time with the child, the mother chooses a level of own consumption of the private good and time to spend with the child, solving problem 1. Her demands for c and h are the same as in 10.

The father allocates his income between consumption good, time with his child and private expenditures on the child, according to the solution of a problem similar to 2 but adapted for his new utility function and budget constraint. See Appendix 4 for details of this model. His new demands are:

$$\begin{aligned}
c_f(p, y_f) &= \delta_{1f} y_f, \\
h_f(p, y_f) &= \delta_{2f} \frac{y_f}{p}, \\
k_f(p, y_f) &= \delta_{3f} y_f.
\end{aligned} \tag{20}$$

Solving for the equilibrium allocations and based on the definitions of t , v and k_f , we have the following:

$$\begin{aligned}
t(y_f) &= p h_f(\delta_m, \delta_f, y_m, y_f) = \delta_{2f} y_f, \\
v(y_m, y_f) &= h_f(\delta_m, \delta_f, y_m, y_f) = \frac{\delta_m \delta_{2f} y_f}{(1 - \delta_m) y_m + \delta_{2f} y_f}, \\
k_f(y_f) &= \delta_{3f} y_f.
\end{aligned} \tag{21}$$

Given the observed values of t, v and k , we can invert the model to solve for the “implied” preference parameters of the parents: $\delta_m, \delta_{1f}, \delta_{2f}$ and δ_{3f} . Once again, we are assuming that the preferences are heterogeneous in the population of divorced parents.

The equations for t, v and k above implicitly assume that $\varepsilon = 1$, that is, that the mother is endowed with all of the child’s time. In this case, inverting equations 21 gives the following expressions for the parental preference parameters:

$$\begin{aligned}\delta_m(\varepsilon = 1) &= \frac{v(t + y_m)}{t + vy_m}, \\ \delta_{1f}(\varepsilon = 1) &= 1 - \frac{t}{y_f} - \frac{k_f}{y_f}, \\ \delta_{2f}(\varepsilon = 1) &= \frac{t}{y_f}, \\ \delta_{3f}(\varepsilon = 1) &= \frac{k_f}{y_f}.\end{aligned}\tag{22}$$

In the cases in which $\varepsilon < 1$, the child support transfer, the visitations (apart from the endowment) and the amount of expenditures of the father on the child other than child support are given by:

$$\begin{aligned}t(\delta_m, \delta_f, y_m, y_f, \varepsilon) &= p[hf - (1 - \varepsilon)] \\ &= \frac{(\delta_{2f} - 1)(1 - \delta_m)(1 - \varepsilon)y_m + \delta_m \delta_{2f} \varepsilon y_m}{\delta_m \varepsilon + (1 - \delta_{2f})(1 - \varepsilon)}, \\ v(y_f, \delta_f, \varepsilon) &= hf - (1 - \varepsilon) \\ &= (1 - \delta_{2f})(1 - \varepsilon) + \frac{[(1 - \delta_{2f})(1 - \varepsilon) + \delta_m \varepsilon] \delta_{2f} y_f}{(1 - \delta_m)y_m + \delta_{2f} y_f}, \\ k_f(y_f, y_m, \varepsilon) &= \frac{\delta_{3f}[(1 - \delta_m)(1 - \varepsilon)y_m + (1 - \varepsilon + \delta_m \varepsilon)y_f]}{\delta_m \varepsilon + (1 - \delta_{2f})(1 - \varepsilon)}.\end{aligned}\tag{23}$$

In particular for this implementation it is possible to sign the derivative of k_f with respect to the amount of endowment that is initially given to the mother (ε). The derivative of k_f with respect to ε is negative, i.e. that the father that is given more time with the child spends more in other items different than child support. A similar result is reported in Del Boca & Ribero (1997).

Combining these equations it is possible to solve for the parameters in terms of the observables, which gives:

$$\begin{aligned}
\delta_m(\varepsilon, y_m, t, v) &= \frac{v(t + y_m)}{vy_m + \varepsilon t}, \\
\delta_{1f}(\varepsilon, y_f, t, v, k_f) &= \frac{v(y_f - k_f - t)}{y_f v + (1 - \varepsilon)t}, \\
\delta_{2f}(\varepsilon, y_f, t, v) &= \frac{t(v + 1 - \varepsilon)}{t(1 - \varepsilon) + y_f v}, \\
\delta_{3f}(\varepsilon, y_f, t, v, k_f) &= 1 - \delta_{1f} - \delta_{2f}.
\end{aligned} \tag{24}$$

The numerical exercises reported in Table 6 show the values for the parental preference parameters under the alternative assumptions that $\varepsilon = 1$ or $\varepsilon = 0.86$. In the exercises reported below the father's endowments are arbitrarily added to the observed visitations from the data.

We are interested in simulating the outcomes in terms of mother's available income and father's visitations under $\varepsilon = 0.5$, an initial endowment similar to joint custody. Table 7 reports the results of this exercise using the preference parameters obtained under the assumptions that $\varepsilon = 1$ and that $\varepsilon = 0.86$ alternatively, compared with the observed values. The results indicate that an endowment of equal time for both parents, though beneficial for the children, leaves the mother worse off by increasing the income disparity between the two parents. However, the increase in the father's voluntary transfers to the child could partially compensate the mother's reduction in total consumption (by reducing her expenditures on the child).

Some of the expenditures considered here concern health care, clothes and gifts for the child and are related to the father's involvement with the child's everyday life. Other expenditures instead may not substitute mother's expenditures since they are related more closely with the child's life with the father. When the child spends a considerable amount of time with the non residential father (like in joint custody arrangements) this implies having a room in his home as well as a set of clothes and items necessary to meet the child's needs. These are expenditures that *both* parents have to make in order to allow the child to live with them.

Therefore visitations have a crucial role in determining the post-divorce distribution of incomes between parents. On one hand they guarantee the father a positive amount of time with the child. On the other hand they guar-

antee the mother the ability to continue to share her ex-husband's income. These results confirm our previous finding (Del Boca and Ribero, 1998) that households with joint custody arrangements have higher child expenditures than households with sole custody arrangements. A custody arrangement that allows both parents to share responsibility and time with the child has a positive effect on mandated as well as voluntary transfers to the child.

Table 4
Values of mother's and father's preferences parameters

Basic model

	Mean	Standard deviation
$\delta_m(\varepsilon = 1)$.311	.249
$\delta_f(\varepsilon = 1)$.847	.092
$\delta_m(\varepsilon = 0.86)$.340	.261
$\delta_f(\varepsilon = 0.86)$.640	.194

Table 5
Mother's consumption and father's time with child

Outcomes under $\varepsilon = 0.5$

	Observed	Using $\hat{\delta}(\varepsilon = 1)$	Using $\hat{\delta}(\varepsilon = 0.86)$
c_m	9,476 (6,640)	4,559 (5,054)	5,904 (5,346)
v	.126 (.108)	.345 (.304)	.473 (.084)

Table 6
Values of mother's and father's preferences parameters

Extended model

	Mean	Standard deviation
$\delta_m(\varepsilon = 1)$.311	.249
$\delta_{1f}(\varepsilon = 1)$.765	.135
$\delta_{2f}(\varepsilon = 1)$.152	.092
$\delta_{3f}(\varepsilon = 1)$.083	.079
$\delta_m(\varepsilon = 0.86)$.340	.261
$\delta_{1f}(\varepsilon = 0.86)$.586	.193
$\delta_{2f}(\varepsilon = 0.86)$.359	.194
$\delta_{3f}(\varepsilon = 0.86)$.060	.058

Table 7
Mother's consumption, father's time with child and father's expenditures on child
 Outcomes under $\varepsilon = 0.5$

	Observed	Using $\hat{\delta}(\varepsilon = 1)$	Using $\hat{\delta}(\varepsilon = 0.86)$
c_m	9,476 (6,640)	4,569 (5054)	5,904 (5,346)
v	0.126 (0.108)	.345 (0.161)	.473 (0.084)
k	1,125 (805)	1,627 (1,403)	1,478 (1,171)

7 Conclusions

In this paper, we have provided a neoclassical interpretation of the exchange between divorced parents when they make decisions on child support transfers and visitations. We explain visitations and child support using a behavioral model of competitive equilibrium in a framework in which the mother has control over visitations and the father has control over child support. We analyze and compare two models: 1) parents derive utility only from their own consumption and the time spent with the child, 2) parents derive utility also from the amount of consumption good allocated to the child. We use estimates of the models to simulate the effects of alternative endowment levels on the proportion of time spent with the noncustodial parent and the ex-post parental income distribution as well as on child expenditures.

While empirical findings have indicated that more time with the father after divorce would be beneficial for the children in several dimensions, no literature to our knowledge has tried to investigate the effect on the distribution of resources between the parents. The results of our analysis show that a more equal share of time with the children leaves the mother economically worse off. However, the solution of our second model indicates that a more equal distribution of time increases child expenditures, partially compensating for the mother's loss of welfare.

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Appendix 1

Comparative Static Analysis for the Model of Exchange between Child Support and Visitations

In this appendix we explain the process to derive the conclusions of Lemma 1. Rewriting the equilibrium conditions from the model, we have:

$$\frac{u_2^m}{u_1^m} = \frac{u_2^f}{u_1^f} \quad (25)$$

$$c_m + c_f = y_m + y_f \quad (26)$$

$$c_f + ph_f = y_f \quad (27)$$

$$h_m + h_f = 1 \quad (28)$$

$$\frac{u_2^m}{u_1^m} = p. \quad (29)$$

Differentiating totally equation 25 we get:

$$Ndc_m + Adh_m + Cdc_f + Ddh_f = 0, \quad (30)$$

where:

$$\begin{aligned} N &= (u_1^f u_{12}^m - u_2^f u_{11}^m) \\ A &= (u_1^f u_{22}^m - u_2^f u_{21}^m) \\ C &= (u_2^m u_{11}^f - u_1^m u_{12}^f) \\ D &= (u_2^m u_{21}^f - u_1^m u_{22}^f). \end{aligned}$$

Differentiating totally equations 26, 27, 28 and 29 we get:

$$dc_m + dc_f = dy_m + dy_f \quad (31)$$

$$dc_f + pdh_f + h_f dp = dy_f \quad (32)$$

$$dh_m = -dh_f \quad (33)$$

$$(u_{12}^m - pu_{11}^m)dc_m + (u_{22}^m - pu_{21}^m)dh_m - u_1^m dp = 0 \quad (34)$$

Replacing equation 33 in equations 30, 31, 32 and 34 we get:

$$\begin{bmatrix} N & A - D & C & 0 \\ 1 & 0 & 1 & 0 \\ 0 & -p & 1 & h_f \\ R & B & 0 & -u_1^m \end{bmatrix} \begin{bmatrix} dc_m \\ dh_m \\ dc_f \\ dp \end{bmatrix} = \begin{bmatrix} 0 \\ dy_m + dy_f \\ dy_f \\ 0 \end{bmatrix}, \quad (35)$$

where:

$$\begin{aligned} R &= u_{12}^m - pu_{11}^m, \\ B &= (u_{22}^m - pu_{21}^m). \end{aligned}$$

The signs of the derivatives that are consistent with the assumption of concave utility functions are as follows:

$$\begin{aligned} u_j^i &\geq 0; \\ u_{jj}^i &\leq 0, \\ u_{11}^i u_{22}^i - (u_{12}^i)^2 &\geq 0 \end{aligned}$$

for all $i \in \{m, f\}$ and all $j \in \{1, 2\}$.

We have that:

$$\det \begin{bmatrix} N & A - D & C & 0 \\ 1 & 0 & 1 & 0 \\ 0 & -p & 1 & h_f \\ R & B & 0 & -u_1^m \end{bmatrix} = (N - C)(Bh_f - u_1^m) - (A - D)(Rh_f - u_1^m).$$

Assuming that $u_{jk}^i \geq 0$ for all $j, k \in \{m, f\}$ and all $i \in \{m, f\}$, we have the following signs for the variables:

$$\begin{aligned} N &\geq 0, & A &\leq 0, & C &\leq 0, \\ D &\geq 0, & R &\geq 0, & B &\leq 0, \end{aligned}$$

and therefore:

$$\begin{aligned} N - C &\geq 0, \\ Bh_f - u_1^m &\leq 0, \\ A - D &\leq 0, \end{aligned}$$

but the sign of $Rh_f - u_1^m$ is not determinate. Therefore we state assumption 1:

$$Rh_f - u_1^m \leq 0.$$

If this assumption holds, then:

$$\delta = \det \begin{bmatrix} N & A - D & C & 0 \\ 1 & 0 & 1 & 0 \\ 0 & -p & 1 & h_f \\ R & B & 0 & -u_1^m \end{bmatrix} \leq 0.$$

We will also assume that this determinant is different than zero, in order for the system to have solution. Under this assumption, taking the equations system 35 and assuming that $dy_f = 0$, we get:

$$\begin{bmatrix} N & A - D & C & 0 \\ 1 & 0 & 1 & 0 \\ 0 & -p & 1 & h_f \\ R & B & 0 & -u_1^m \end{bmatrix} \begin{bmatrix} dc_m/dy_m \\ dh_m/dy_m \\ dc_f/dy_m \\ dp/dy_m \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}.$$

Using Cramer's rule to get the derivatives with respect to y_m , we have that:

$$\begin{aligned} dc_m/dy_m &= [(A - D)u_m^1 + C(pu_m^1 - Bh_f)]/\delta \geq 0, \\ dh_m/dy_m &= [-Nu_m^1 + RCh_f]/\delta \geq 0, \\ dc_f/dy_m &= [-N[pu_m^1 - Bh_f] - (A - D)Rh_f]/\delta, \\ dp/dy_m &= [-NB + R(A - D) + pRC]/\delta. \end{aligned}$$

The latter two derivatives can not be signed without further assumptions. Taking the equations system 35 and assuming that $dy_m = 0$, we get:

$$\begin{bmatrix} N & A - D & C & 0 \\ 1 & 0 & 1 & 0 \\ 0 & -p & 1 & h_f \\ R & B & 0 & -u_1^m \end{bmatrix} \begin{bmatrix} dc_m/dy_f \\ dh_m/dy_f \\ dc_f/dy_f \\ dp/dy_f \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}.$$

Using Cramer's rule you get the derivatives with respect to y_f :

$$\begin{aligned} dc_m/dy_f &= [C(pu_m^1 - Bh_f)]/\delta \geq 0, \\ dh_m/dy_f &= [(-u_m^1 + Rh_f)C]/\delta \leq 0, \\ dc_f/dy_f &= [-N[pu_m^1 - Bh_f] - (A - D)(Rh_f - u_m^1)]/\delta \geq 0, \\ dp/dy_f &= [(-B + Rp)C]/\delta \geq 0. \end{aligned}$$

Appendix 2

Data selection

The Table included in this appendix describes the process of selection to generate the data set used in the paper. The universe of analysis contained the 701 divorced couples with one child provided by the survey. The additional selection criteria are specified in bold at the left column.

Criteria	Dropped	Kept
Custody	Joint custody [n=66]	Sole custody [n=635]
	Sole custody father [n=41]	Sole custody mother [n=594]
Visitations	Visitations denied [n=50]	Visitations allowed [n=525]
	Visitations missing [n=19]	
Father's income y_f	$y_f = 0$ [n=103]	$y_f \neq 0$ [n=422]
	$y_f < 12 * t$ [n=4]	$y_f \geq 12 * t$ [n=418]
Mother's income y_m	y_m missing [n=27]	y_m not missing [n=391]
Child support transfer t	$t = 0$ [n=122]	$t > 0$ [n=269]
	t missing [n=36]	
		Final sample size [n=233]

Appendix 3

Construction of the visitations variable and the index of expenditures of the non-custodian father on the child different than child support

In this appendix we explain the way in which the visitations variable and the index representing the expenditures of the non-custodian father on the child different than child support were built. The information for these two variables comes from the National Longitudinal Survey of High School Class of 1972 - 5th Follow Up in the categoric formats described below.

Question regarding visitations:

“What was the visitations agreement?”

- a) Sees the child once a week
- b) Sees child twice a month
- c) Sees child once a month
- d) Sees child on vacations
- e) Sees child at no specific times
- f) No visitations allowed

This information is used to build a variable representing a proxy for the number of times in a year in which the non-custodian parent visits the child. It is assumed that each visit lasts at least 2 days.⁸ The visitation variable v is defined as the proportion of days in the year in which a visit took place to match the theory we construct in the next section. The “continuous” visitations variable is built as follows:

⁸This is also confirmed by data from the Stanford Child Custody Project that provided detailed information on visitation arrangements (Peters et. al., 1993).

Construction of the visitations variable

Answers	Number of days	v	Frequency
Sees child once a week 104 0.285 83
Sees child twice a month 52 0.142 24
Sees child once a month 26 0.071 3
Sees child on vacations 13 0.036 60
Sees child at no specific times 8 0.022 63
No visitations allowed 0 0 53

Question regarding expenditures of the father on the child different than child support :

Other than child support payments that your first spouse may make, how regularly does your first spouse do the following?

- a) pay for clothes for the child?
- b) pay for gifts for the child?
- c) take the child on vacation?
- d) pay for routine dental care?
- e) carry medical insurance for the child?
- f) pay child's medical bills?

Each question above had an answer in the format shown in the second column below:

Frequency	Code	Value
very regularly 1 1
regularity scale 2 2 0.5
regularity scale 3 3 0.33
regularity scale 4 4 0.25
never 5 0

In order to use this information we used a transformation of the answers that assigns to the answers a number between zero and one in the way indicated in the last column above. For each father, the corresponding values

in the last column for the six questions were added, and the result was divided by six. The final number constitutes an index between zero and one that indicates the degree of frequency with which the father makes “other expenditures different than child support”. To build the variable k_f used in the model estimation, this index was multiplied \$3,000, assuming that this is the amount spent per year by a non-custodian father who spends “very regularly” in all the items. Even though the value of \$3,000 as a maximum seems rather large, less than 8% of non-custodian parents have “very regularly” in all items. With this assumption 50% of parents would be spending less than \$660 per year in “other expenditures different than child support.”

Appendix 4

Derivation of the model with father's private expenditures on the child

In this appendix the model is developed for the case in which the private consumption of the child for the non-custodian father is considered as an argument of the non-custodian father's utility function. The assumptions of this model are the same as those for the theoretical model from Section 2. Let k_f be the private consumption of the child. The utility function of the father is assumed to be of the form $u_f(c_f, h_f, k_f)$ and the one of the mother has the form $u_m(c_m, h_m)$.

The problem of the mother and the derived demands are the same as in 1 and 10. The problem for the father is to:

$$\begin{aligned} \max u_f(c_f, h_f, k_f) \\ \text{s.t. } c_f + ph_f + k_f \leq y_f. \end{aligned} \quad (36)$$

The equilibrium conditions are:

$$\begin{aligned} c_m(p, y_m) + c_f(p, y_f) + k_f(p, y_f) &= y_m + y_f \\ h_m(p, y_m) + h_f(p, y_f) &= 1. \end{aligned} \quad (37)$$

The solution for the equilibrium price and allocations results from equations:

$$\begin{aligned} MRS_m(c_m, h_m) &= MRS_f(c_f, h_f) = p \\ MRS_m(c_m, h_m) &= MRS_f(k_f, h_f) \\ MRS_f(c_f, k_f) &= 1 \\ c_m + c_f + k_f &= y_m + y_f, \\ h_m + h_f &= 1, \\ c_m + ph_m &= p + y_m. \end{aligned} \quad (38)$$

Lemma 2:

Under assumption 7, the comparative static results of Lemma 1 hold for the model of private consumption of the child. In addition we have:

$$dk_f(y_m, y_f)/dy_f \geq 0, \quad dk_f(y_m, y_f)/dy_m \leq 0.$$

Under the assumption that the utility functions of the father and mother are Cobb-Douglas of the form described in 19, the equilibrium price and allocations are given by:

$$\begin{aligned} p(y_m, y_f) &= \frac{(1 - \delta_m)y_m + \delta_{2f}y_f}{\delta_m}, \\ c_m(y_f, y_m) &= \delta_{2f}y_f + y_m, \\ h_m(y_m, y_f) &= \frac{(1 - \delta_m)[y_m + \delta_{2f}y_f]}{(1 - \delta_m)y_m + \delta_{2f}y_f}, \\ c_f(y_f) &= \delta_{1f}y_f, \\ h_f(y_m, y_f) &= \frac{\delta_m\delta_{2f}y_f}{(1 - \delta_m)y_m + \delta_{2f}y_f}, \\ k_f(y_f) &= \delta_{3f}y_f. \end{aligned} \tag{39}$$

When the model is solved for the case in which $\varepsilon < 1$, the equilibrium price and allocations are given by:

$$\begin{aligned} p(y_m, y_f, \varepsilon) &= \frac{\delta_{2f}y_f + (1 - \delta_m)y_m}{\delta_m\varepsilon + (1 - \delta_{2f})(1 - \varepsilon)}, \\ c_m(y_f, y_m, \varepsilon) &= \frac{\delta_m[(1 - \delta_{2f}(1 - \varepsilon))y_m + \delta_{2f}\varepsilon y_f]}{\delta_m\varepsilon + (1 - \delta_{2f})(1 - \varepsilon)}, \\ h_m(y_m, y_f, \varepsilon) &= 1 - \delta_{2f}(1 - \varepsilon) - \frac{[(1 - \delta_{2f})(1 - \varepsilon) + \delta_m\varepsilon]\delta_{2f}y_f}{(1 - \delta_m)y_m + \delta_{2f}y_f}, \\ c_f(y_f, y_m, \varepsilon) &= \frac{\delta_{1f}[(1 - \delta_m)(1 - \varepsilon)y_m + (1 - \varepsilon + \delta_m\varepsilon)y_f]}{\delta_m\varepsilon + (1 - \delta_{2f})(1 - \varepsilon)}, \\ h_f(y_m, y_f, \varepsilon) &= \frac{\delta_{2f}[(1 - \delta_m)(1 - \varepsilon)y_m + (1 - \varepsilon + \delta_m\varepsilon)y_f]}{(1 - \delta_m)y_m + \delta_{2f}y_f}, \\ k_f(y_f, y_m, \varepsilon) &= \frac{\delta_{3f}[(1 - \delta_m)(1 - \varepsilon)y_m + (1 - \varepsilon + \delta_m\varepsilon)y_f]}{\delta_m\varepsilon + (1 - \delta_{2f})(1 - \varepsilon)}. \end{aligned} \tag{40}$$