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(Un)anticipated Monetary Policy in a DSGE Model
with a Shadow Banking System

Institute for Monetary and Financial Stability
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(Un)anticipated monetary policy in a DSGE model with a shadow banking system

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Abstract

Motivated by the U.S. events of the 2000s, we address whether a too low for too long interest rate policy may generate a boom-bust cycle. We simulate anticipated and unanticipated monetary policies in state-of-the-art DSGE models and in a model with bond financing via a shadow banking system, in which the bond spread is calibrated for normal and optimistic times. Our results suggest that the U.S. boom-bust was caused by the combination of (i) too low for too long interest rates, (ii) excessive optimism and (iii) a failure of agents to anticipate the extent of the abnormally favorable conditions.

Keywords: DSGE model, shadow banking system, too low for too long, boom-bust

JEL codes: E32, E44, E52, G24
1 Introduction

Many observers have suggested that the Federal Reserve has kept short-term interest rates too low for too long in the early 2000s, inducing the credit boom that eventually led to the post-2007 financial and economic bust known as the Great Recession. For example, Taylor (2009) shows that following the 2001 recession, the federal funds rate (FFR) has been unusually below the level prescribed by the standard Taylor rule during 2002-2005, and that in the counterfactual scenario of a Taylor rule-driven policy there would not be a comparable boom and bust.

While persistently low interest rates encourage excessive leverage and risk taking, both apparent in the 2000s boom, monetary policy has surely not been the only cause of the boom and bust. A variety of authors (from academics – e.g. Borio, 2008 and Blanchard, 2009 – to policy-makers – e.g. Trichet, 2009, Bean et al., 2010 and Bernanke, 2010) have argued that the crisis resulted from the interaction of macro and micro factors. Macro factors include, besides the persistently loose policy, the global imbalances and savings glut, the financial globalization, and the hangover from the Great Moderation. Micro factors include the boom of securitization, the development of a vast shadow banking system with unregulated institutions, products and markets, the increased complexity of innovative financial products with the difficulty to measure and locate financial risk, and myopic and biased incentives within financial institutions (see e.g. de Larosiére et al., 2009 and de Larosiére, 2010).

As Semmler and Bernard (2012, p. 442-3) put it, in all boom-bust episodes “the boom period triggers overconfidence, overvaluation of assets, over-leveraging, and the underestimation of risk; then follows a triggering event and the market mood turns pessimistic”; in the U.S. during the 2000s, “The complex securities (…) have, jointly with the changes in the macroeconomic environment, actually accelerated not only the boom, but also the bust”.

We focus on monetary policy as one key element of the macro environment in the 2000s that may have helped triggering the boom and the bust, as its relative role is still an open issue – see e.g. the discussion in the 2010 Jackson Hole Symposium, especially between Bean et al. (2010) and Taylor (2010). In particular, the role of monetary policy in the 2000s boom-bust has not been addressed in the context of a dynamic stochastic general equilibrium (DSGE) model.

Against this background, this paper assesses the role of a too low for too long interest rate policy in the development of a boom and bust, by means of quantitative simulations of DSGE models. We start by

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1 See Borio (2008) and Brunnermeier (2009) for a chronology of the events relating to the subprime crises.
assessing whether such a policy may trigger a boom-bust in state-of-the-art DSGE models, namely two versions of the Christiano, Motto and Rostagno (2010) model (CMR hereafter): one version including a banking system in the spirit of Bernanke et al. (1999) (henceforth BGG) and thus the well-known financial accelerator, and another version without such a sector, that corresponds closely to the Smets and Wouters (2003) model. After demonstrating their difficulty to engender financial boom-busts, we suggest a DSGE model that extends the CMR model with a shadow banking system. Specifically, our model includes a bond market populated by investment banks that finance part of the entrepreneurial sector activity. We realistically calibrate the model for normal times and for times of over-optimism, on the basis of the cyclical sensitivity of the spread in bond finance over the long run and during the 2000s’ boom. When excessive optimism in financial markets is coupled with a too low for too long interest rate policy, the model generates a boom-bust in the price of capital, investment and output, associated with a strong rise in leverage alongside a fall in the bond finance spread. Our main contribution in this paper is, thus, suggesting a DSGE model that reconciles the state-of-the-art models with financial and economic boom-bust events.

To conduct our simulations, the concepts of too low and of too long must be precisely defined from the outset. We take too low to mean a constant policy interest rate below its steady-state level, not responding to economic conditions and, as such, deviating from the interest rate that would result from the monetary policy rule in force. Specifically, to approximate the events of the post-2001 boom, we simulate a FFR 100 basis points below its steady-state level: in fact, when the U.S. economy was exiting the 2001 recession, the FFR target was 2 percent and then it was further lowered to 1 percent from June 2003 onward (see figure 1). We take too long to mean that such policy interest rate path lasts for 6 quarters; the period is too lengthy in the sense that in its course, real activity and inflation start reacting, which, in normal times, would imply a reaction of monetary policy according to the rule in effect. In our simulations, the Taylor-type policy rule is only reactivated after the 6-quarters period of too low interest rate; then, the economy eventually returns to its steady state.\(^2\)

In the literature, there are two traditions for simulations such as ours, namely (a) an unanticipated policy scenario and (b) an anticipated policy scenario.

The unanticipated policy scenario corresponds to a simulation in the spirit of Sims and Zha (2006)

\(^2\)Throughout the paper all the policy simulations will consist of a 6-quarters period of constant policy rate below its steady-state level. The length of the simulation exercise is arbitrary, but in line with comparable experiments in the literature (see e.g. Laseen and Svensson, 2011). Furthermore, we have checked that plausible variations of the simulation period (ranging from 4 to 8 quarters) do not change qualitatively the results.
and Bernanke et al. (1997, 2004), in which the short-term interest rate is kept constant and away from its steady-state level for some periods by means of a sequence of unanticipated policy shocks. Economically, it mimics a situation in which agents do not know how long the monetary ease will last, and are continuously surprised during the ease and, also, at its end. The anticipated policy scenario corresponds to a simulation in the spirit of Laseen and Svensson (2011), in which the policy projection is a credibly pre-announced policy rate path. Economically, it represents situations where the policy-maker transparently announces that it plans to implement a particular policy rate path and this plan is believed and anticipated by the private sector.\footnote{Another sort of possible policy simulations are those of Reis (2009) and Milani and Treadwell (2012), who study the effects of news about future monetary policies.}

It could be argued that the unanticipated policy scenario is inappropriate for the specific case that we study. In fact, the U.S. monetary policy of the first half of the 2000s is seen by many as a case of an anticipated persistent monetary ease, as the Federal Open Market Committee (FOMC) held the FFR target consistently at 1 percent between June 2003 and June 2004, and stated on August, September, October and December 2003 that it believed policy accommodation could be maintained for a considerable period (see panel A of table 1). However, in its January and March 2004 meetings, the FOMC surprised the markets, changing the tone by delivering a new statement in which it mentioned the eventual removal of its policy accommodation. As figure 1 shows, short-term interest rates (and overall bond yields, not depicted) started increasing after March 2004, even though the FFR target has increased only in June 2004. The rise in yields thus anticipated the change in policy and happened because of an unanticipated change in the FOMC tone in January that became clear only in March.

Overall, we argue that the U.S. monetary policy in the early 2000s may not be considered clearly pre-announced and entirely anticipated by economic agents. First, both the beginning and the end of the monetary ease are hard to locate precisely from the FOMC minutes, given the evolution of decisions and statements showed in panel A of table 1. Second, many agents and markets actually changed their behavior before the actual end of the 2003-2004 monetary loosening, seemingly surprised by the announcement of a change in the monetary policy stance. Hence, we consider the monetary policy of the early 2000s in the U.S. as an intermediate case between anticipated and unanticipated policy: during part of the monetary ease most agents knew that policy would be loose for a prolonged period, but it is very hard to argue that most agents knew for sure the future policy path from its outset.\footnote{A different case could be made regarding the 2011-12 monetary ease, as, in our view, the information in panel B suggests that the announcements of future monetary policy by the FOMC in 2011 and 2012 conform more closely to an anticipated loosening of monetary policy. In fact, in August, September, November and December 2011, the FOMC}
All considered, throughout the whole paper we implement both the unanticipated and the anticipated policy simulations, for all models considered, explicitly comparing their results. A first advantage of our approach is that considering both simulations may allow for a better approximation to the real world events of the 2000s. A second advantage is that simulating both policies allows for assessing whether the anticipation of the policy path influences its dynamic impacts; and, as a corollary, allows for drawing lessons regarding the communication of future monetary actions.

The key conclusions of the paper may be summarized as follows. First, in line with the literature, anticipated periods of *too low for too long* interest rates generate a very much larger and quicker boom in real activity and asset price than similar policies that are unanticipated. Second, the state-of-the-art DSGE models fail to predict a sizeable boom and bust following a *too low for too long* monetary policy, even when extended with financial frictions. Third, our DSGE model with a shadow banking system predicts a substantial boom and bust following a *too low for too long* policy, when adequately calibrated with a cyclical sensitivity of the spread in bond finance in line with the 2002-2007 data. Our framework therefore highlights the central role that spreads may have on the transmission of monetary policy, as recently suggested by Gilchrist and Zakrajsek (2012b).

The remaining of the paper is structured as follows. In section 2 we simulate anticipated and unanticipated policies of *too low for too long* interest rates in two versions of the CMR (2010) model, after briefly discussing their main features relevant for the issue addressed in this paper. In section 3 we present our model, which extends the CMR model adding a shadow banking system as a second financial sector in addition to the standard Bernanke et al. (1999) banking system; we calibrate the model for normal times and for periods of excessive optimism such as those seen in the first part of the 2000s; and we simulate policies of *too low for too long* interest rates and discuss the results. Finally, we present some concluding remarks in section 4.5

2 State-of-the-art DSGE models

In this section we assess whether a *too low for too long* interest rate policy generates a boom and a bust in state-of-the-art DSGE models. In subsection 2.1 we briefly describe the models we focus on. Then stated that it foresaw that economic conditions were likely to warrant exceptionally low levels for the federal funds rate at least through mid-2013 and in January, March and April 2012 extended that period through late 2014.

5 Appendix A presents the complete model, while technical details are relegated to appendix B.
in subsection 2.2 we explain the two alternative policy simulations, we show the impulse responses and
discuss the results.

2.1 The models

The DSGE models that are currently the benchmark macroeconomic models resulted from the fusion
of the Real Business Cycle models of the 1980s with the New Keynesian sticky-price models of the
early 1990s. Some current versions still feature frictionless financial markets and a passive role for
financial intermediaries, thus being utterly unsuitable for the analysis of financial booms and busts.
This is the case of DSGE models currently used for monetary policy analysis at the main central banks – e.g. the SIGMA model at the FED (Erceg et al., 2006), the Smets and Wouters model at the ECB
(Smets and Wouters, 2003) and the Bank of England's Quarterly Model (Harrison et al., 2005).

Following the seminal paper of BGG, who include a banking sector that amplifies the effects of monetary
shocks via a mechanism termed financial accelerator, the structure and role of the financial sector in
DSGE models has been developed along several lines. Iacoviello (2005) extended the BGG model by
introducing collateral constraints for firms, as in Kiyotaki and Moore (1997). CMR (2003, 2008 and
2010) and Goodfriend and McCallum (2007) considered a perfectly competitive banking sector that
offers agents a variety of financial assets with different returns, while Kobayashi (2008) and Gerali et al.
(2010) considered imperfect competition in the banking sector so as to model the setting of interest
rates by banks. Curdia and Woodford (2010) allowed for a time-varying spread between deposits and
lending rates. Finally, a number of papers (see, for instance, Van den Heuvel, 2008, de Walque et al.,
2010, Meh and Moran, 2010 and Gertler and Karadi, 2011) studied the role of bank capital in the
transmission of macroeconomic shocks.

Most importantly for our purposes, we consider and take the Financial Accelerator Model described
in CMR (2010) as the state-of-the-art DSGE model. It essentially corresponds to the models in Smets
and Wouters (2003) and Christiano et al. (2005) enlarged with the financial accelerator mechanism
developed by BGG.

Given the widespread awareness of the CMR model, we skip any analytical or detailed presentation,
except for the retail banking system that forms its financial core. Very briefly, the model is composed
of households, final- and intermediate-good firms, capital producers, entrepreneurs, banks, and gov-
ernment. Households consume, save and supply labor services monopolistically. They employ their
savings in time deposits offered by retail banks. On the production side, monopolistically competitive intermediate-good firms use labor (supplied by households) and capital (rented from entrepreneurs) to produce a continuum of differentiated intermediate goods. Perfectly competitive final-good firms buy intermediate goods and produce the final output, which is then converted into consumption, investment and government goods. Capital producers combine investment goods with undepreciated capital purchased from entrepreneurs to produce new capital, which is then sold back to entrepreneurs. Capital services are supplied by entrepreneurs, who own the stock of physical capital and choose how intensively to use it. Entrepreneurs purchase capital using their own resources as well as external finance, namely bank loans. Government expenditures represent a constant fraction of final output and are financed by lump-sum taxes imposed to households, with the government budget systematically balanced. The Central Bank sets the nominal interest rate according to a Taylor-type interest rate rule.

Banks finance the part of the entrepreneurs' purchases of physical capital that cannot be financed with their net worth. Entrepreneurial loans are risky because the returns on their investments are subject to idiosyncratic shocks, which, if unfavorable and large enough, may lead to bankruptcy and thus incapacity to pay the required interest and re-pay their debt. Financial frictions arise because the idiosyncratic shock is observed by the entrepreneurs at no cost, while the bank has to incur in a monitoring cost to observe the shock. To deal with the problem of asymmetry in information, entrepreneurs and bank sign a debt contract, according to which the entrepreneur commits to pay back the loan principal and a non-default interest rate, unless he declares default, in which case the bank verifies the residual value of the entrepreneur's assets and takes in all of the entrepreneur's net worth, net of monitoring costs.

The retail bank funds its lending activity by issuing time deposits held by households. Although individual entrepreneurs are risky, the bank itself is not: by lending to a large number of entrepreneurs, the bank can diversify the idiosyncratic risk and thus can guarantee a safe return on households' deposits. Nevertheless, financial frictions – reflecting the costly state verification problem between entrepreneurs and the bank – imply that bank hedges against credit risk by charging a premium over the rate at which it can borrow from households.

As shown by Bernanke et al. (1999), the first order condition of the contracting problem yields the following relationship linking the expected return on capital \( R_{t+1}^{k,HR} \) relative to the risk-free interest...
rate \( R_{t+1} \) and the entrepreneurs’ leverage ratio \( \frac{Q_{k,t} R_{t+1}^{HR}}{N_{t+1}^{HR}} \):

\[
E_t \left( \frac{1 + P_{t+1}^{k,HR}}{1 + R_{t+1}^e} \right) = \Psi \left( \frac{Q_{k,t} R_{t+1}^{HR}}{N_{t+1}^{HR}} \right),
\]

where \( Q_{k,t}, R_{t+1}^{HR} \) and \( N_{t+1}^{HR} \) denote, respectively, the price of capital, the entrepreneur’s stock of capital and the entrepreneur’s net worth and the function \( \Psi \) is such that \( \Psi' > 0 \) for \( N_{t+1}^{HR} < Q_{k,t} R_{t+1}^{HR} \). The ratio \( \frac{E_t (1 + P_{t+1}^{k,HR})}{1 + R_{t+1}^e} \), which Bernanke et al. (1999) interpreted as the external finance premium faced by the entrepreneur, depends positively on the entrepreneur’s leverage ratio. All else equal, higher leverage means higher exposure, implying a higher probability of default and thus a higher credit risk, which leads the bank to require a higher return on lending.

In the CMR model, the cost of borrowing fluctuates endogenously with the cycle due to two general equilibrium mechanisms.

The first, and quantitatively more important, is the BGG financial accelerator effect, whereby changes in the asset price change the value of the collateral that the entrepreneur can pledge and, thus, the contractual loan rate. A positive shock to the asset price, resulting for instance from a monetary easing, increases the entrepreneur’s net worth and decreases the external finance premium, which in turn stimulates the demand for investment. The increase in net worth also reduces the expected default probability and allows the entrepreneur to take on more debt and to further expand investment. Hence, an accelerator effect arises, as the boom in investment raises the asset price, further pushing up the entrepreneur’s net worth and, thus, investment.

The second mechanism, quantitatively less important and absent in BGG, is a Fisher (1933) debt deflation effects that arise from the fact that debt contracts are set in nominal terms. Since the return received by households on their deposits is nominally non-state contingent, while loans to entrepreneurs are state-contingent, unexpected movements in the price level change the ex-post real burden of entrepreneurial debt and, hence, the entrepreneur’s net worth. For example, after an unexpected increase in inflation, the real resources transferred from the entrepreneur to households fall and consequently the entrepreneur’s net worth increases.\(^6\)

The financial accelerator and Fisher effect reinforce each other in the case of shocks that move inflation and output in the same direction (e.g. monetary policy shocks), whereas they counterbalance each other in the case of shocks that move inflation and output in opposite directions.\(^6\)

\(^6\) Iacoviello (2005) and Gerali et al. (2010) also consider this effect.
other in the case of shocks that move inflation and output in opposite directions (e.g. technology shocks). Evidently, a persistently loose monetary policy should have amplified effects in the CMR model, compared with a similar model without the banking sector.

In what follows, we simulate the *too low for too long* interest rate policies in the version of the CMR that includes the BGG financial accelerator (henceforth CMR-FA) and in a version without the retail banking system, which is very close to the model of Smets and Wouters (2003) (termed CMR-Simple). The purpose is to isolate the effect of the retail banking system in the transmission of the persistently loose monetary policy, as well as to assess whether it depends on the ability of agents to anticipate the policy.

### 2.2 Policy simulations

We now conduct two simulations for each model, namely the unanticipated policy scenario and the anticipated policy scenario. In both, the policy (risk-free) nominal interest rate is held constant below its steady-state level by 100 basis points for 6 quarters; afterward, the nominal interest rate set by the central bank re-starts being determined by the following Taylor-type policy rule, which includes interest rate smoothing and responses of the policy rate to deviations of expected inflation ($E_t \pi_{t+1}$) and current output ($Y_t$) from their steady states:

\[
R_t^e = \bar{\rho} R_{t-1}^e + (1 - \bar{\rho}) \left[ R^e + \alpha_\pi (E_t \pi_{t+1} - \bar{\pi}) + \alpha_y (Y_t - \bar{Y}) \right] + \varepsilon_{t}^{MP},
\]

where $R^e$, $\pi$ and $Y$ are the steady-state values of $R_t^e$, $\pi_t$ and $Y_t$, respectively, $\alpha_\pi$ and $\alpha_y$ are the weights assigned to expected inflation and output, $\bar{\rho}$ captures interest rate smoothing and $\varepsilon_{t}^{MP}$ is a monetary policy shock. Based on CMR, we calibrate $\bar{\rho}$ to 0.88 and $\alpha_\pi$ and $\alpha_y$ to 1.82 and 0.11, respectively (we maintain this policy rule throughout the whole paper).

The unanticipated policy simulation mimics a situation in which agents do not know how long the monetary ease will last, and are continuously surprised during the whole policy path. Technically, the policy interest rate is held constant by means of a sequence of shocks $\varepsilon_{t}^{MP}$ determined residually at each of the 6 periods of the simulation. These are unanticipated by economic agents, who then

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7 Compared to the original CMR model, both versions also exclude long-run growth, the fixed cost in the production function and distortional taxes on capital, labor income and household consumption. While not changing the model's dynamic responses to monetary policy shocks, these simplifications reduce its complexity. The Dynare codes for replicating all the results in this paper are available from the authors upon request.
change their inter-temporally optimizing decisions each period accordingly. The anticipated policy simulation mimics a situation in which the policy-maker announces a specific policy rate path and it is entirely believed by the private sector. Technically, the policy path is written as a function of the initial state of the economy, which includes a set of anticipated shocks to the policy rule consistent with the announced path, i.e. the set \( \{ \varepsilon_{MP}^t \}_{t=1}^6 \) is perfectly known to agents at \( t = 0 \). As agents know the shocks and the policy rate path from the outset, including the return of monetary policy to the prevailing rule after the ease, they adjust their optimizing decisions immediately after the credible announcement.

Figure 2 plots the impulse response functions of the most important variables – nominal interest rate, output, investment, inflation, price of capital (the model’s indicator for the price of assets in the financial market) and, for the CMR-FA version, the interest rate on loans and the entrepreneurs’ leverage and loans.

A first conclusion that stands out is that anticipated policies (panel A) generate a much larger and immediate response of macroeconomic and financial variables than unanticipated policies (panel B). This is in line with results elsewhere in the literature (see Laseen and Svensson, 2011, fig. 3), and it was expected: when policy is anticipated, agents change immediately their decisions knowing that the nominal interest rate will be below its steady-state level and unresponsive to the state of the economy for 6 quarters. Output, investment and inflation react with a hump-shaped pattern in both simulation scenarios, even though peaking sooner and at a very much higher level when policy is anticipated.

While the behavior of net worth is similar, the price of capital behaves quite differently in the two simulations: when the too low for too long interest rate policy is unanticipated, it jumps slightly on impact and then is virtually constant until the end of the monetary ease, then re-converging to its steady-state level; when policy is anticipated, the price of capital jumps markedly on impact (by a factor of 5 compared to the alternative scenario) and then falls rapidly and goes through a small bust, before converging to the steady state. Clearly, no scenario predicts a gradual build up of the price of capital as apparently was the case during the 2000s boom.

A second conclusion is that the BGG’s financial accelerator has a rather limited amplification effect in the CMR model. The impulse response functions of output, inflation and price of capital in the CMR-FA and the CMR-Simple model are quite similar, irrespectively of the (un)anticipation of the monetary policy path. When policy is anticipated, in the model with the banking system there is a larger jump on impact of the price of capital, while when policy is unanticipated not only the initial
jump is slightly larger but there is also some build up in the price of capital until the end of the monetary ease – but the differences are not striking. In both policy scenarios, investment reacts with a hump-shaped pattern, and it peaks at a rather higher level in the CMR-FA model – here, the difference is more noteworthy.

A third conclusion, related with the previous, is that including a BGG banking system in the state-of-the-art DSGE model is not sufficient for a period of too low for too long interest rates to induce a dynamics of the price of capital and investment – let alone output and inflation – akin to the one seen in the boom and bust of the 2000s.

A fourth and key conclusion is that, irrespectively of the ability of agents to anticipate the policy path, the CMR-FA model predicts that a too low for too long interest rate policy induces a fall in the leverage of entrepreneurs during the period of booming investment, output and prices (of final good and of capital). This is clearly at odds with the huge overall increase in leverage that occurred in the U.S. during the first half of the 2000s. In fact, it is at odds with the stylized facts of all boom episodes, namely overconfidence, overvaluation of assets, over-leveraging, and the underestimation of risk (Semmler and Bernard, 2012). We thus conclude that the state-of-the-art DSGE model has important difficulties to scrutinize the boom and bust events such as the one in the U.S. during the 2000s. Hence the motivation for our model proposed and analyzed in the next section.

3 A DSGE model with a shadow banking system

In this section we suggest a DSGE model that realistically features over-optimism and over-leveraging in the course of the boom. The section has three subsections. In the first, we present the new part of the model, namely the shadow banking system that parallels the retail banking system; special attention is given to the modeling of normal times as opposed to times of optimism associated with booms. In the second, we present the calibration, again focusing on the working of the shadow banking system in normal and optimistic times. Finally, we present the simulation of anticipated and unanticipated monetary policies of too low for too long interest rates and discuss the results.

8 Descriptions of the U.S. boom-bust of the 2000s typically emphasize the excessive risk taking and leverage of financial institutions, rather than of non-financial economic agents (as are the entrepreneurs in these DSGE models). Stylized as the model might be with this respect, one should not forget that the financial intermediaries’ leverage is a sign of the leverage took on by the ultimate economic agents, firms and households who buy goods and services.
3.1 The model

Figure 3 sketches the structure of the model. Agents and flows depicted in solid lines correspond to the CMR-FA model discussed in section 2, while dashed lines represent the innovations of our model. In short, we augment the CMR-FA model with a shadow banking system, which may be thought of as representing the intermediation of funds between households and part of the entrepreneurial sector through securitized finance.\(^9\) Our model’s financial system is thus composed of two different financial sectors – loan and bond market – with different financial intermediaries – retail and investment banks – that intermediate financial flows (granting loans and underwriting bonds) between households (lenders) and two groups of entrepreneurs (borrowers).

The setting up of the shadow banking system is paralleled by the division of the entrepreneurial sector into two groups, each having access to one of the sources of external funding. Usually, in the corporate finance literature (see, among many others, Diamond, 1991, Chemmanur and Fulghieri, 1994, Holmstrom and Tirole, 1997, Berlin and Loeyes, 1988, Bolton and Freixas, 2000, 2006 and Hale, 2007), firms are segmented by risk classes in their choice of funding, with safer firms having access to bond financing and riskier firms resorting to bank loans. Accordingly, we assume that riskier entrepreneurs (henceforth HR, to denote high risk) obtain financing via retail bank loans, while safer entrepreneurs (henceforth LR – low risk) issue bonds resorting to investment banks.\(^{10}\) The entrepreneurs that resort to retail bank financing have higher risk because, as seen in section 2, they may default due to a low realization of the productivity shock. We model the remaining set of entrepreneurs as safer assuming that they always have enough net worth to repay their debt, \textit{i.e.} they never default. Accordingly, in equilibrium, safer entrepreneurs obtain finance at a lower interest rate than riskier entrepreneurs.

The shadow banking system is populated by a continuum of monopolistic competitive investment banks, who set the coupon rate on bonds seeking to maximize profits. These are then related to the

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\(^9\) The expression “shadow banking system” has been suggested originally by Paul McCulley of PIMCO at the 2007 Jackson Hole conference, where he defined it as “the whole alphabet soup of levered up non-bank investment conduits, vehicles, and structures” (McCulley, 2007, pag. 2). The shadow banking system in our model is very stylized and does not come anywhere near the variety and complexity of products, institutions and markets that exist in reality (see Pozsar et al., 2010 for a comprehensive and up-to-date description of the shadow banking system). For example, during the 2000s’ boom in the U.S., a large part of assets intermediated by the shadow banking system were related to the housing market, involving mortgage-backed securities, while our model does not feature a housing market. The modeling of the shadow banking system stems from the macroeconomic nature of our model: adding into a DSGE model a too complex shadow banking system would be highly demanding and hardly cost-effective given that we want to focus on aggregate phenomena.

\(^{10}\) Typically, a firm going public hires an investment bank to sell its securities. The investment bank (the underwriter) acts as an intermediary between the issuing firm and the ultimate investors. The most common type of underwriting arrangement is the firm commitment underwriting, according to which the underwriter buys the entire stock of bonds from the firm and resells it to investors at a higher price (\textit{i.e.}, at a lower interest rate). This spread represents the investment bank’s profits. See Ellis et al. (2000) for an in-depth analysis of the underwriting process.
stockholders, i.e. to the households. The assumption of monopolistic competition in the bond market is consistent with the empirical evidence about the U.S. market of bond underwriting. For example, Fang (2005) shows that the largest five investment banks underwrite more than 60% of all deals, and the largest fifteen banks account for roughly 95% of all deals. The decision of the bond coupon rate is taken by the investment bank manager, whom we call henceforth the underwriter. In our framework, the coupon interest rate of the bond is set as a markup over the risk-free interest rate. We defer to the end of this subsection and, most especially, to the next subsection, details on the spread in bond finance. For now, we make three brief notes about the bond finance spread. First, its behavior will be crucial for the dynamics of the model. Second, the data clearly reveal that it is time-varying and counter-cyclical (see figure 4). Third, the data also tell that in times of over-optimism, the elasticity of the spread to the business cycle is larger than in normal times.

In what follows we only describe the shadow banking system and the agents involved in this new block of the economy. The rest of the model is standard in the literature (corresponds to the CMR-FA model) and is set out in appendix A.

We assume that riskier entrepreneurs are a fraction $\eta$ of the total population of entrepreneurs, so that the share of safer entrepreneurs is $1 - \eta$. The superscripts "LR" and "LR,l" refer to variables associated with the safer entrepreneurs.

**Safer entrepreneur’s profit maximization problem**

At the beginning of period $t$, the representative $l$-th LR entrepreneur provides capital services to intermediate-good firms. Capital services, $K^{LR,l}_t$, are related to the entrepreneur's stock of physical capital, $\bar{K}^{LR,l}_t$, according to $K^{LR,l}_t = u^{LR,l}_t \bar{K}^{LR,l}_t$, where $u^{LR,l}_t$ denotes the level of capital utilization.

When choosing the capital utilization rate, the entrepreneur takes into account the increasing and convex utilization cost function $a\left(u^{LR,l}_t\right)$, that denotes the cost, in units of final goods, of setting the utilization rate to $u^{LR,l}_t$.\(^{11}\)

Then, at the end of period $t$, the entrepreneur sells the undepreciated capital to capital producers at price $Q_{k,t}$, pays the nominal coupon rate ($R^{coupon}_t$) on bonds issued and purchases new capital from

\[^{11}\text{The functional form that we use is } a\left(u^{LR,l}_t\right) = \frac{r^{k,LR}}{a''(1)} \left[ \exp^{\sigma^{LR}_{u} \left(u^{LR,l}_{t-1}\right)} - 1 \right], \text{ where } r^{k,LR} \text{ is the steady-state value of the rental rate of capital, } a(1) = 0, a''(1) > 0 \text{ and } \sigma^{LR}_{u} = a''(1)/a'(1) \text{ is a parameter that controls the degree of convexity of costs.}\]
capital producers at price $Q_{k, t}$. The capital acquisition is financed partly by his net worth, $N_{t+1}^{LR}$, and partly by issuing new bonds. The amount of bonds issued, $BI_{t+1}^{LR}$, is given by:

$$BI_{t+1}^{LR} = Q_{k, t} R_{t+1}^{LR} - N_{t+1}^{LR}. \tag{1}$$

The entrepreneur’s time-$t$ profits, $\Pi_t^{LR}$, are given by:

$$\Pi_t^{LR} = \left[ u_t^{LR} r_t^{k, LR} - a \left( u_t^{LR} \right) \right] R_t^{LR} P_t + (1 - \delta) Q_{k, t} R_{t+1}^{LR}$$

$$- Q_{k, t} R_{t+1}^{LR} - R_{t+1}^{coupon} \left( Q_{k, t+1} R_{t+1}^{LR} - N_{t+1}^{LR} \right),$$

where $r_t^{k, LR}$ denotes the real rental rate, $P_t$ the price of the final good and $\delta$ the depreciation rate.

In period $t$ the entrepreneur chooses the capital utilization rate and the desired capital to use in period $t + 1$ so as to maximize $\Pi_t^{LR}$. taking as given the coupon rate to be paid on the bonds issued. The first order conditions with respect to $u_t^{LR}$ and $K_{t+1}^{LR}$ are, respectively:

$$r_t^{k, LR} = a' \left( u_t^{LR} \right) \tag{2}$$

$$Q_{k, t} = \beta E_t \left\{ u_{t+1}^{LR} r_{t+1}^{k, LR} - a \left( u_{t+1}^{LR} \right) \right\} P_{t+1} + (1 - \delta) Q_{k, t+1} R_{t+1}^{LR} - R_{t+1}^{coupon} Q_{k, t} \tag{3}.$$
we assume a constant probability of death: in each period, the entrepreneur exits the economy with probability $1 - \gamma^{LR}$. In that case, he rebates his equity to households in a lump-sum way:

$$\text{transfer to households} = (1 - \gamma^{LR}) V_t^{LR,l}.$$ 

To keep the entrepreneurs’ population constant, a new entrepreneur is born with probability $1 - \gamma^{LR}$. The total entrepreneur’s net worth $N_{t+1}^{LR,l}$ combines total equity and a transfer, $W_t^{e,LR,l}$, received from households, which corresponds to the initial net worth necessary for the entrepreneur’s activity to start. The law of motion for the entrepreneur’s net worth is:

$$N_{t+1}^{LR,l} = \gamma^{LR} V_t^{LR,l} + W_t^{e,LR,l}.$$

**Safer entrepreneur’s financing cost minimization problem**

There is a continuum of investment banks, indexed by $z \in [0, 1]$, and each investment bank $z$ has some market power when conducting its intermediation services. An entrepreneur seeking an amount of borrowing for period $t + 1$ equal to $B_{t+1}^{LR,l}$, defined by (1), would allocate his borrowing among different investment banks, $B_{t+1}^{LR,l}(z)$, so as to minimize the total repayment due. At the end of period $t$, the entrepreneur chooses how much to borrow from bank $z$ by solving the following problem:

$$\min_{B_{t+1}^{LR,l}(z)} \int_0^1 \left[ 1 + R^{\text{coupon}}_{t+1} (z) \right] B_{t+1}^{LR,l} (z) \, dz$$

subject to a Dixit-Stiglitz aggregator

$$B_{t+1}^{LR,l} = \left\{ \int_0^1 \left[ B_{t+1}^{LR,l} (z) \right]^{\epsilon^{\text{coupon}}_{t+1} - 1} \, dz \right\}^{\epsilon^{\text{coupon}}_{t+1} - 1},$$

where $R^{\text{coupon}}_{t+1} (z)$ is the interest rate charged by the $z$-th bank and $\epsilon^{\text{coupon}}_{t+1} > 1$ is the time-varying interest rate elasticity of the demand for funds. The first order condition yields the following entrepreneur’s demand for funds:

$$B_{t+1}^{LR,l} (z) = \left( \frac{1 + R^{\text{coupon}}_{t+1} (z)}{1 + R^{\text{coupon}}_{t+1}} \right)^{-\epsilon^{\text{coupon}}_{t+1}} B_{t+1}^{LR,l}.$$
where \( R_{t+1}^{\text{coupon}} \) is the nominal average coupon rate prevailing in the market at time \( t + 1 \), defined as:

\[
1 + R_{t+1}^{\text{coupon}} = \left\{ \int_0^1 \left[ 1 + R_{t+1}^{\text{coupon}} (z) \right]^{1 - \epsilon_{t+1}}^{1 - \epsilon_{t+1}} \, dz \right\}^{1 - \epsilon_{t+1}}.
\]

As expected, the funds demand curve has a negative slope: when the interest rate that the \( z \)-th bank sets increases relatively to the average rate, the entrepreneur wishes to borrow less funds from that particular bank.

**Investment banks**

The investment banks are owned by households. To keep the analysis as simple as possible, we follow the recent DSGE banking literature and assume perfect competition in the market for households’ deposits in these banks (see e.g. Kobayashi, 2008). We rule out the entry and exit of investment banks. The investment bank therefore maximizes its profits, taking as given the return to pay to the households. In appendix A we show that the required return on bonds by households is equal to the risk-free rate, i.e. the central bank nominal interest rate.

At the end of period \( t \), the \( z \)-th investment bank thus solves the following profit maximization problem:

\[
\max_{R_{t+1}^{\text{coupon}}} \Pi_{t+1}^I (z) = \left\{ [1 + R_{t+1}^{\text{coupon}} (z)] B_{t+1}^{LR,l} (z) - [1 + R_{t+1}^{e}] B_{t+1}^{LR,l} (z) \right\}
\]

subject to

\[
B_{t+1}^{LR,l} (z) = \left( \frac{1 + R_{t+1}^{\text{coupon}} (z)}{1 + R_{t+1}^{e}} \right)^{-\epsilon_{t+1}} B_{t+1}^{LR,l}.
\]

Deriving the first-order condition, imposing a symmetric equilibrium and rearranging yields

\[
1 + R_{t+1}^{\text{coupon}} = \frac{\epsilon_{t+1}}{\epsilon_{t+1} + 1} (1 + R_{t+1}^{e}),
\]

that is, the coupon rate is a time-varying markup, \( \frac{\epsilon_{t+1}}{\epsilon_{t+1} + 1} \), over the policy interest rate. The profits of the investment banking sector in period \( t + 1 \) are given by \( \Pi_{t+1}^I = (R_{t+1}^{\text{coupon}} - R_{t+1}^{e}) (1 - \eta) B_{t+1}^{LR,l} \) and are rebated to households.

The spread in bond finance, i.e. the spread between the bond coupon rate and risk-free nominal interest rate is

\[
\text{spread}_{t+1} = R_{t+1}^{\text{coupon}} - R_{t+1}^{e} = \frac{1}{\epsilon_{t+1} + 1} (1 + R_{t+1}^{e}) \cdot
\]

16
Equation (5) is crucial for the dynamic behavior of the model. If the elasticity of the demand for funds in the bond market was constant, the model would predict that the spread in bond finance would depend only on the policy interest rate. Yet, it is well-known that spreads co-move with the business cycle overall, from spreads between interest rates on short term commercial paper and Treasury bills (see e.g. Friedman and Kuttner, 1998) to spreads between long corporate bond yields and long Treasury yields (see e.g. Gertler and Lown, 1999). We do not attempt to provide micro-foundations for the counter-cyclical behavior of the spread in bond finance, as their multiple and complex determinants would be very hard to pin down in a DSGE model – thus deferring that task to more stylized models of finance. Rather, we adopt an empirical approach and calibrate the counter-cyclical behavior of the spread in bond finance according to U.S. data, as further detailed in the next subsection. We firstly specify a linear relation between the elasticity of demand for funds in the bond market and the cyclical state of the economy, which rules the baseline behavior of the spread in bond finance. In addition, motivated by the above referred fact that boom periods trigger overconfidence, overvaluation of assets, over-leveraging, and the underestimation of risk (see e.g. Semmler and Bernard, 2012) we distinguish between normal times and periods of optimism (associated with financial booms). To do so, we adopt a two-tier approach to model the counter-cyclical behavior of the spread in bond finance.

First, we specify the following baseline relation between the elasticity of the demand for funds in bond finance and the output gap:

\[ \varepsilon_{t+1}^{\text{normal}} = \bar{\varepsilon} + \alpha_1 \left( Y_t - \bar{Y} \right) , \]  

so that the interest rate on bonds in normal times, \( R_{t+1}^{\text{coupon,normal}} \), is given by

\[ 1 + R_{t+1}^{\text{coupon,normal}} = \frac{\varepsilon_{t+1}^{\text{normal}}}{\varepsilon_{t+1}^{\text{normal}} - 1} (1 + R_{t+1}^c) \]  

and the spread during normal times is

\[ \text{spread}_{t+1}^{\text{normal}} = \frac{1}{\varepsilon_{t+1}^{\text{normal}} - 1} \left( 1 + R_{t+1}^c \right) . \]

As further explained below, calibrating equation (6) with very long historical time series for the U.S., allows us to deem equations (6)-(8) as driving the cyclical sensitivity of the spread in normal times.

The second tier of our modeling strategy for the behavior of the spread in bond finance consists of proposing two additional equations that are inactive in normal times, but are activated in times of
over-optimism. The associated under-estimation of risk and excessive leverage inexorably co-exist with abnormally low spreads in securitized finance, as has been the case in the U.S. 2000s’ boom (see e.g. Semmler and Bernard, 2012). A multiplicity of factors may have triggered the over-confident behavior and it is beyond the scope of this paper to suggest thorough micro-foundations for over-confidence. Rather, we distillate those multiple factors in what we call optimism: whenever the underwriter becomes optimistic, he underwrites bonds at a lower that the normal coupon rate.

We model this second tier with two additional equations.

The first states that the underwriter becomes optimistic whenever the entrepreneurs are able to pledge a higher (than the steady-state level) value as collateral. To see the potential of this assumption, note that this could happen, for example, because the market price of collateral is increasing. We thus model underwriter’s optimism, \( \chi_t \), as a positive function of the entrepreneur’s net worth. Moreover, to take into account the fact that human beliefs are very persistent (Kurz and Motolese, 2011), we furthermore model optimism as an \( AR(1) \) process with high persistence. Accordingly, the law of motion for optimism is given by

\[
\chi_t = \rho \chi_{t-1} + (1 - \rho \chi) \left[ \bar{\chi} + \alpha_2 \left( N_{LR,l}^{t+1} - N_{LR,l}^{\text{steady}} \right) \right],
\]

where \( \bar{\chi}, \bar{\chi} = 0 \), is the steady-state level of optimism, \( \rho \) captures the degree of persistence in optimism and \( \alpha_2 > 0 \) the sensitivity of optimism with respect to the deviation of the entrepreneur’s net worth from its steady-state value (\( N_{LR,l}^{\text{steady}} \)).

The second states that whenever the underwriter is optimistic, the elasticity of the demand for funds in the bond market deviates positively from its normal times level:

\[
\varepsilon_{t+1}^{\text{optimistic}} = \varepsilon_{t+1}^{\text{normal}} (1 + \chi(t)).
\]

The optimistic elasticity results in a lower coupon rate, which may be seen substituting (10) into (4), yielding the following expression

\[
1 + R_{t+1}^{\text{coupon, optimistic}} = \frac{\varepsilon_{t+1}^{\text{normal}} (1 + \chi_t)}{\varepsilon_{t+1}^{\text{normal}} (1 + \chi_t) - 1} \left( 1 + R_r^{t+1} \right),
\]

where \( R_{t+1}^{\text{coupon, optimistic}} \) is the coupon rate that the optimistic underwriter sets on the bonds issued. Comparing (11) and (7), it is clear that the optimistic underwriter underwrites bonds at a lower than...
the normal interest rate.

To summarize: in the steady state, when optimism and the output gap are null, the spread in bond finance is given by a constant elasticity of the demand and depends only on the central bank interest rate; when the economy records cyclical fluctuations but times are normal as regards the financial markets and economic mood, the spread moves counter-cyclically, as the elasticity moves with the output gap according to equation (6); when some set of factors induces over-optimism, perceived as increases in the net worth of the entrepreneurial sector, we allow the elasticity of demand for funds in the bond market to change with optimism (in addition to changing with the cyclical state of the economy) and therefore the discount on bond finance increases further than would happen merely because of the business cycle.

3.2 Calibration

The model is calibrated for the U.S. economy assuming the quarter as the time unit. We focus on the calibration of the parameters associated with the shadow banking system and the fraction of entrepreneurs that employ bond finance – the innovative part of our model. The values of the remaining parameters are calibrated in line with the established New Keynesian literature and are shown in table 2. Tables 3 and 4 report the steady-state implications of the model and their empirical counterparts. These tables show how the model successfully reproduces most of the salient features of the U.S. economy, namely key macroeconomic and leverage ratios, interest rates and, importantly, its financial market structure.

The share of entrepreneurs that resort to the retail banking system for finance, $\eta$, is set to 0.278, which (given the bond and loan amounts in the steady state) allows us to exactly match the ratio of bank to bond finance in the U.S. economy (as reported by De Fiore and Uhlig, 2011).\footnote{The values of the parameters related with this part of the entrepreneurial sector are primarily chosen to match the cost of external finance, i.e. the contractual, no-default interest rate on entrepreneurial debt ($Z_t$ resulting from A.6). Setting the fraction of realized payoffs lost in bankruptcy, $\mu$, to 0.15 and the standard deviation of the entrepreneurial idiosyncratic productivity shock, $\sigma$, to 0.55 yields $Z = 6.81\%$ / year. This is in line with the data and is realistically above the steady-state risk-free interest rate, which is 5.16%, given that we set the discount factor $\beta$ to 0.9875. Moreover, it reasonably guarantees that, in equilibrium, bond financing is cheaper than bank financing. To match the observed leverage ratio, we set the survival rate $\gamma^{HR}$ to 0.97.}

Equations (6) and (8) show that the steady-state bond spread depends on the interest rate elasticity $\varepsilon$. Chen et al. (2007) report an average annual yield spread of AAA bonds of 84 basis points. Accordingly, we set $\varepsilon$ to 510, so that the annual bond spread is around 84 basis points in the steady state, and
the steady-state coupon rate paid by the safer entrepreneurs is 5.99% per year. To match the average leverage ratio in the data, we set the survival probability $\gamma^{LR}$ to 0.96.

When the economy is not in the steady state, the spread in bond finance exhibits a counter-cyclical dynamics. Our two-tier approach to modeling the behavior of the bond spread requires calibrating (i) parameter $\alpha_1$ in equation (6), to govern the behavior of the spread in normal times, and (ii) parameter $\alpha_2$ in equation (9), to add a specific behavior of the spread in times of over-optimism.

We calibrate the parameter $\alpha_1$ using very long historical time series for the U.S., so that the equation may actually be thought of as driving the cyclical sensitivity of the bond spread in normal times.

Data are from the U.S. for 1953 through 2011, with a quarterly periodicity. Regarding the spread we follow a recent literature that focuses on yields of corporate bonds with long average residual maturity vis-a-vis yields of government bonds with comparable maturity (see Gilchrist et al., 2009 and Gilchrist and Zakrajsek, 2012a). Specifically, we use the difference between (quarterly averages of) the Moody’s Seasoned Baa Corporate Bond yields and the 10-Year Treasury constant maturity yields. To measure the business cycle we consider the official output gap, i.e. the difference between real output and the real potential output estimated by the Congressional Budget Office. Figure 4 shows the long time series of bond spread and output gap, as well as the official dating of U.S. recessions by the NBER.

On average, in the 10 recessions identified by the NBER during this period, each percentage point of decrease in the output gap has been associated with an increase in the bond spread of 0.21 percentage points. Consistently, an OLS regression of the spread on a constant and the output gap yields an estimate of $-0.2$ for the coefficient associated to the gap. In a VAR model of inflation, output gap, spread and the federal funds rate, a monetary policy shock generates a counter-cyclical dynamics of the spread which, at the height of the impacts, corresponds to a rise in the spread of 0.26 percentage points per each percentage point of decrease in the output gap. Overall, we find it reasonable to calibrate the model so that in its baseline specification – when optimism is absent – a monetary policy shock causes the output gap and the spread to move in opposite directions with each percentage point of decrease in the output gap.

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13 See the business cycle dating at http://www.nber.org/cycles/cyclesmain.html. For each recession, we compute the difference between the bond spread (gap) in the last quarter of the recession and its value in the quarter prior to the beginning of the recession. The behavior of spreads during expansions is essentially symmetric.

14 All data have been downloaded from FRED on March 2012 [http://research.stlouisfed.org/fred2/]. The series references are: BAA and GS10 [corporate and treasury yields]; GDP1C1 and GDPPO1T [real and potential output]; FEDFUNDS [effective federal funds rate]; and PCECTPI [personal consumption expenditures price index]. The monthly yield and fed funds data have been converted to a quarterly periodicity with simple 3-month averages. Inflation is the year-on-year growth rate of the quarterly PCECTPI. The reported dynamic elasticities have been computed from the impulse response functions of a VAR(3) model estimated over 1954-Q3-2011-Q4, in which the peak response of the output gap occurs at quarter 4 and the peak response of the spread occurs at quarter 6.
deviation of real output from its steady-state level associated with 0.22 percentage points of deviation of the corporate bond coupon rate from the risk-free interest rate.

Simulating our model, it turns out that for $\alpha_1 = 30000$ an expansionary monetary policy shock generates a hump-shaped response of the output gap and (in the opposite direction) of the bond spread, with peak and trough respectively at quarters 3 and 4 and an implied output gap-elasticity of the spread of $-0.22$ at both quarters. We thus assume $\alpha_1 = 30000$ for the calibration of the behavior of the spread in bond finance in normal times.

We then calibrate the parameter $\alpha_2$ considering only data for the 2002-2007 period, as the equations driven by this parameter (9-11) are meant to capture times of over-optimism. As figure 4 shows, in the 2000s the spread has been abnormally counter-cyclical: in fact, in the expansion identified by the NBER during this period (2002:Q1-2007:Q3), each percentage point of increase in the output gap has been associated with a decrease in the bond spread of 0.95 percentage points. For 2002:Q1-2008:Q2, a period that encompasses the boom and does not include the financial crisis, an OLS regression of the spread on a constant and the output gap yields an estimate of $-0.7$ for the coefficient associated to the gap.

We thus simulate the model in search for the value for $\alpha_2$ that mimics such degree of sensitivity. It turns out that for $\alpha_2 = 24$ an expansionary monetary policy shock generates a hump-shaped response of the output gap and a symmetric response of the bond spread, with peak and trough respectively at quarters 3 and 5 and an implied output gap-elasticity of the spread of $-0.72$ at both quarters.

So, adopting this calibration, in the next subsection we simulate the effects of a *too low for too long* interest rate policy, anticipated and unanticipated, to assess whether the model is able to generate a boom-bust event similar to the one of the 2000s in the U.S. Given the structure of the model, we can easily shut down equation (9) – setting $\alpha_2 = 0$ – and thus simulate the model for normal times; and, alternatively, setting $\alpha_2 = 24$ we can assess the role of the optimistic shadow banking system in the transmission of the *too low for too long* monetary policy.

### 3.3 Policy simulations

Figures 5 and 6 plot the impulse response functions of selected variables of our model to monetary policies of *too low for too long* interest rates, when the policy path is anticipated and unanticipated, respectively. The layout of the figures is identical. First, crossed lines correspond to responses of
our model in normal times, while circled lines correspond to impulse responses in times of optimism. Second, both figures have 3 panels: the top panel relates to aggregate variables, the middle one to variables relative to the safer entrepreneurs, and the bottom panel to variables relative to the riskier entrepreneurs.

A first broad conclusion from the comparison of the two figures is that anticipated policies generate a much larger and immediate response of the aggregate macroeconomic and most financial variables than unanticipated policies. This was expected and is entirely in line with what we have found for the models scrutinized in section 2.

A second general conclusion is that the effects of the too low for too long interest rate policy on macroeconomic variables—output, investment, inflation and price of capital—are very similar to those of the CMR-FA model, when our model with a shadow banking system is calibrated for normal times. This happens both for unanticipated and anticipated policies. Hence, when the spread in bond finance features a counter-cyclical behavior in line with the U.S. long-run average, our model does not improve on the CMR-FA model as regards predicting a dynamics of the price of capital and investment—let alone output and inflation—akin to the one seen in the boom and bust of the 2000s.

However, even in times of normal behavior of the spread in bond finance, our model adds relevant and reasonable results to the CMR-FA model. This is a third key conclusion. To see that, recall that the CMR-FA model predicts that a too low for too long interest rate policy leads to a fall in leverage and especially when policy is unanticipated—a fall in loans (after an initial surge caused by the fact that the value of capital reacts more rapidly than the entrepreneurial net worth). In our model, the response of the riskier entrepreneurs’ leverage is quite similar to the one predicted by the CMR-FA model (this is expected, since this part of the entrepreneurial sector closely follows the CMR-FA model). However, the safer entrepreneurs react differently: after a below-the-steady-state level for some quarters (caused by a stronger reaction of investment and the price of capital, compared to the reaction of their net worth), their leverage becomes higher than the steady-state level, and the amount of bonds reacts accordingly. Most importantly, the results from our model with a shadow banking system, even in its normal times version (crossed lines), are substantially new in the sense that the safer entrepreneurs are not merely crowding out finance from the riskier entrepreneurs. That is especially clear in figure 5, when the policy is anticipated: in fact, the total amount of finance in the economy (the sum of retail bank loans and investment banks bonds), after the initial increase on impact, is never below its steady-state level. In figure 6, when the policy path is unanticipated, the response of total finance
is qualitatively similar to the one predicted by the CMR-FA model, but both the duration and the magnitude of the transitional period of below-the-steady-state total credit is quantitatively smaller than in the CMR-FA model.

At this point, we conclude that enhancing the state-of-the-art DSGE model with financial frictions associated to bond finance via a shadow banking system, while yielding more reasonable results, indicates that the policy of too low for too long interest rates has not been the only cause for the boom and bust of the U.S. in the 2000s. This was hardly unexpected, in view of the analyses mentioned at the outset of the paper stating that the 2000s’ boom and bust resulted from the interaction of macro and micro factors. While the too low for too long monetary policy has been one crucial macro factor, it should be thought of as a trigger for the boom (and for the bust, when interrupted), that operated only because there were further necessary conditions present at the microeconomic level. Also motivated by this reason, we have set a version of the model for times of optimism in the shadow banking system, calibrated in line with the degree of over-optimism observed in the U.S. during the boom. We now turn to the analysis of that version (circled lines).

Let us consider first the simulation of an anticipated policy path (figure 5). The responses of the variables associated with the riskier entrepreneurs are not visibly different from those simulated with the CMR-FA model (the only exception is that the amount of loans jumps much more initially and then falls much more and is below the steady state for a very long period). As regards the safer entrepreneurs, optimism leads to an immediate and much larger fall in the bond coupon interest rate and in the bond finance spread. While the price of capital jumps, investment reacts with some lag – due to adjustment costs – which, coupled with the jump in net worth, starts by decreasing leverage. But soon (a year after the announcement and beginning of the expansionary policy), as investment responds and the stock of capital increases rapidly and markedly, leverage rises above the steady-state level. The hump-shaped response of leverage puts it markedly above the steady-state level for a very long period, peaking at around 6 years after the beginning of policy. The amount of bonds underwritten reacts similarly to leverage and capital, with its hump at around 6 years more than 7 percent above its steady-state level. In the aggregate, while not impacting strikingly on output and inflation, optimism induces a significantly larger hump-shaped reaction of investment, a vastly bigger

\[ \text{\footnotesize 15} \]

Although not central for our purposes, a nice result of our model is that during the crisis – the period of sharp fall in asset prices and net worth – while bank loans are below their steady-state levels, the amount of bonds issued is above its long-run level (both when policy is anticipated and unanticipated). This is, to the best of our knowledge, the closer a DSGE model has come to the evidence reported by Adrian et al. (2012), who found that during crises bank lending to firms declines but bond financing increases.
increase in total credit (explained by the increase of bonds) and a somewhat higher initial jump of the price of capital. The subsequent dynamics of the price of capital is similar to the one seen in normal times, with a rapid decay to the steady-state level, but optimism induces levels slightly higher during the transition period. Overall, if the policy path is entirely anticipated by economic agents, our model with optimism in the shadow banking system creates the flavor of a boom and bust in some key variables, but does not produce the gradual build of the price of capital that is typically associated with booms and the period of asset prices below long-run equilibrium that is typically associated with busts.

The behavior of the model is quite different when agents do not anticipate the policy rate path (figure 6), especially regarding the safer entrepreneurs and some aggregate variables. As regards the riskier entrepreneurs, the only differences to the responses when policy is anticipated are that loans and leverage fall by a larger amount. Regarding the safer entrepreneurs, both the bond coupon interest rate and the spread in bond finance fall more gradually but end up lower at the end of the monetary ease (compared to when policy is anticipated) — in fact, much lower in the case of the spread. The initial fall in leverage is far smaller than when policy is anticipated; then, leverage starts increasing and becomes above the steady-state level quite faster; the peak of its hump occurs sooner, at 20 quarters, although at a slightly smaller level than when policy is anticipated. The amount of bonds underwritten reacts similarly to leverage and capital, increasing quite rapidly during the 2 years after the beginning of the policy of too low interest rates. The behavior of total credit is only slightly different, as it increases steadily for more than 7 years, not exhibiting the faltering in the second and third year after the beginning of the monetary ease that occurs when policy is anticipated. In the aggregate, again, optimism does not impact visibly on output and inflation, and while it induces a significantly larger hump-shaped reaction of investment, the magnitude of the rise in investment turns out to be inferior to the one found when policy is anticipated. Yet, the major difference happens in the dynamics of net worth and the price of capital: both jump on impact and keep on increasing until the end of the period of too low for too long interest rates — which does not happen without optimism; then, they both fall at a strong pace and, from quarter 20 onwards, they are persistently below their steady-state levels. Hence, if the policy path is not anticipated, our model with a shadow banking system and optimism is able to engender the gradual build of the price of capital that is typically associated with booms and the period of prices below long-run equilibrium that is typically associated with busts.

Our analysis therefore suggests that the boom and bust in the U.S. during the 2000s has been caused
by the combination of three factors – a policy of *too low for too long* interest rates, a mood of excessive optimism and a failure of agents to anticipate the duration of the abnormally favorable macro conditions (possibly due to a failure of the FED to clearly communicate its intentions).

4 Concluding remarks

In this paper we have assessed the role of a monetary policy of *too low for too long* interest rates as a trigger for a financial and economic boom, and then, when interrupted, as a trigger for a financial and economic bust.

The events of the 2000s in the U.S. were the key motivation for our analysis. Reviewing the FOMC decisions and statements after the 2001 recession, we argue that economic agents could hardly have anticipated the length and amplitude of the monetary ease. We thus conduct the analysis under both the more theoretically appealing scenario of an anticipated *too low for too long* policy path as well as the more pragmatic scenario of an unanticipated *too low for too long* policy path.

The study has been performed in the context of DSGE models and the method consisted of simulating anticipated and unanticipated policy paths of a constant policy interest rate 100 basis points below its steady-state level for 6 quarters, followed by an automatic shift of monetary policy to the Taylor-type policy rule.

We started by showing that state-of-the-art DSGE models, even when including financial frictions, fail to predict a sizeable boom and bust after such policies. Moreover, they generate a fall in leverage during the period of booming investment, output and prices, which is clearly at odds with the data.

We then suggested a DSGE model with a second segment in the financial system, populated by investment banks that underwrite bonds issued by entrepreneurs with a low level of risk. We call this sector the shadow banking system, as we deem it as representing the variety of financial firms that has

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16 We have submitted the model to a number of sensitivity analyses, from which we emphasize two. The first relates to the persistence of optimism. In line with the literature on the persistence of beliefs, we have run all simulations of our model with \( \rho_{\chi} = 0.7 \) instead of 0.9. The turning points of the impulse response functions [for the safer entrepreneurs] of leverage, bonds issued, capital and bond spread turn out to occur sooner, but otherwise the results are qualitatively similar. The second relates to our assumption that the capital input \( K_{t} \) in intermediate-good production is a composite of two entrepreneur-specific capital services: \( K_{t} = \left[ \eta \left( u_{t}^{HR,HR} K_{t}^{HR,HR} \right)^{\rho} + \left( 1 - \eta \right) \left( u_{t}^{LR,LR} K_{t}^{LR,LR} \right)^{\rho} \right]^{\frac{1}{\rho}} \), where \( \rho \) denotes the degree of substitutability between the two entrepreneur-specific capital services. In all simulations we have considered \( \rho = 0.6 \). However, given the inexistence of a sound literature to motivate the choice for the CES aggregator of capital stocks, we have run the simulations with two alternative values for \( \rho \), namely \( \rho = 0.5 \) and \( \rho = 0.7 \). This parameter changes the bond to bank ratio in the steady state, which is unappealing (0.5 yields a bond/bank ratio of 1.78 and 0.7 a ratio of bond/bank equal to 1.12). The results change only quantitatively, with the effects of our shadow banking system increasing with the steady-state ratio of bond to bank finance.
developed securitized finance under loose regulation and seemingly unlimited leverage in the run-up to the 2007 bust.

We modeled the spread in bond finance according to U.S. data, adopting a two tier approach. First, we calibrated the baseline (normal times) counter-cyclical behavior of the bond spread. Then, we calibrated the dynamics of the spread in periods of over-optimism.

Our model adds realism to the state-of-the-art DSGE model with financial frictions, as *too low for too long* interest rates policy induces an increase in the leverage and bonds issued by the entrepreneurs that resort to the shadow banking system. The outcome of the model when the *too low for too long* interest rates policy is combined with a surge in optimism depends crucially on whether the policy path is anticipated or not by economic agents.

When the policy is anticipated, the model comes a step further to predict a boom and bust, as total credit (bank loans plus bonds), investment and the price of capital increase substantially more than without optimism or in alternative models. However, the price of capital does not feature the gradual build that is typically associated with booms nor the below-average levels typically associated with busts. In turn, when the policy path is unanticipated, the model generates a boom-bust dynamics: (i) output, investment and total credit increase and respond in a hump-shaped pattern; (ii) the price of capital rises steadily while the policy interest rate is too low, and then falls abruptly to below its steady-state level.

Our model thus suggests that the boom and bust in the U.S. during the 2000s has been caused by the combination of three factors – a policy of *too low for too long* interest rates, a mood of excessive optimism and a failure of agents to anticipate the duration of the abnormally favorable macro conditions.

As corollaries, we draw two lessons for monetary policy. First, it seems warranted that spreads in the bond market should be given a more important role in the conduct and monitoring of monetary policy: spreads substantially below historical levels are associated with over-confidence, under-estimation of risk, excessive leverage and an increasing likelihood of a bust – and therefore should trigger some monetary policy tightening. Second, it is crucial that policy-makers build a strong credibility and communicate policies with the maximum transparency: when agents are able to anticipate the policy path, the reaction of financial and macroeconomic variables is faster and maybe larger, but the likelihood of uncontrollable booms and busts seems smaller.
<table>
<thead>
<tr>
<th>FOMC Meeting</th>
<th>FFR target</th>
<th>Statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: 2003-2004</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2003-June</td>
<td>↘ 25 bp</td>
<td>a slightly more expansive monetary policy would add further support for an economy which it expects to improve over time</td>
</tr>
<tr>
<td>from 2003-August until 2003-December</td>
<td>↔ at 1%</td>
<td>policy accommodation can be maintained for a considerable period</td>
</tr>
<tr>
<td>2004-January</td>
<td>↔ at 1%</td>
<td>[the FOMC] can be patient in removing its policy accommodation</td>
</tr>
<tr>
<td>2004-March</td>
<td>↔ at 1%</td>
<td>policy accommodation can be removed at a pace that is likely to be measured</td>
</tr>
<tr>
<td>2004-June</td>
<td>↗ 25 bp</td>
<td>policy accommodation can be removed at a pace that is likely to be measured</td>
</tr>
<tr>
<td>B: 2008-2012</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2008-December</td>
<td>from 1% ↘ to [0; 0.25] %</td>
<td>the Committee anticipates that weak economic conditions are likely to warrant exceptionally low levels of the federal funds rate for some time</td>
</tr>
<tr>
<td>2009-January</td>
<td>↔ at [0; 0.25] %</td>
<td>the Committee anticipates that weak economic conditions are likely to warrant exceptionally low levels of the federal funds rate for some time</td>
</tr>
<tr>
<td>from 2009-March until 2011-June</td>
<td>↔ at [0; 0.25] %</td>
<td>the Committee anticipates that weak economic conditions are likely to warrant exceptionally low levels of the federal funds rate for an extended period</td>
</tr>
<tr>
<td>from 2011-August until 2011-December</td>
<td>↔ at [0; 0.25] %</td>
<td>the Committee anticipates that economic conditions [...] are likely to warrant exceptionally low levels for the federal funds rate at least through mid-2013</td>
</tr>
<tr>
<td>since 2012-January</td>
<td>↔ at [0; 0.25] %</td>
<td>the Committee anticipates that economic conditions [...] are likely to warrant exceptionally low levels for the federal funds rate at least through late 2014</td>
</tr>
</tbody>
</table>

Table 1: FOMC decisions and statements in selected periods of the 2000s
<table>
<thead>
<tr>
<th>Households</th>
<th>Value</th>
<th>Source</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.9875</td>
<td>our calibration</td>
<td>discount factor</td>
</tr>
<tr>
<td>$\psi_L$</td>
<td>(36)</td>
<td>(endogenous)</td>
<td>weight on disutility of labor</td>
</tr>
<tr>
<td>$\sigma_L$</td>
<td>1</td>
<td>CMR</td>
<td>curvature of disutility of labor</td>
</tr>
<tr>
<td>$b$</td>
<td>0.63</td>
<td>CMR</td>
<td>habit persistence in consumption</td>
</tr>
<tr>
<td>$\xi_w$</td>
<td>0.75</td>
<td>Erceg et al. (2000)</td>
<td>fraction of households that cannot reoptimize wage</td>
</tr>
<tr>
<td>$\lambda_w$</td>
<td>1.05</td>
<td>CMR</td>
<td>markup, workers</td>
</tr>
<tr>
<td>$\xi_{w1}$</td>
<td>0.29</td>
<td>CMR</td>
<td>weight of wage indexation to steady-state inflation</td>
</tr>
<tr>
<td><strong>Firms</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.36</td>
<td>Levin et al. (2005)</td>
<td>capital share in the production function</td>
</tr>
<tr>
<td>$\xi_p$</td>
<td>0.75</td>
<td>Erceg et al. (2000)</td>
<td>fraction of firms that cannot reoptimize price</td>
</tr>
<tr>
<td>$\iota_1$</td>
<td>0.16</td>
<td>CMR</td>
<td>weight of price indexation to steady-state inflation</td>
</tr>
<tr>
<td>$\lambda_f$</td>
<td>1.2</td>
<td>CMR</td>
<td>markup, intermediate-good firms</td>
</tr>
<tr>
<td>$S''$</td>
<td>29.3</td>
<td>CMR</td>
<td>curvature of investment adjustment cost function</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.03</td>
<td>CMR</td>
<td>depreciation rate on capital</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.6</td>
<td>our calibration</td>
<td>degree of substitutability between capital services</td>
</tr>
<tr>
<td><strong>Entrepreneurs</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_a^{HR}$, $\sigma_a^{LR}$</td>
<td>18.9</td>
<td>CMR</td>
<td>curvature of capital utilization cost functions</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.15</td>
<td>our calibration</td>
<td>fraction of realized profits lost in bankruptcy</td>
</tr>
<tr>
<td>$\gamma^{HR}$, $W_e,LR$</td>
<td>0.02</td>
<td>CMR</td>
<td>transfer from households</td>
</tr>
<tr>
<td>$\gamma^{LR}$</td>
<td>0.96</td>
<td>our calibration</td>
<td>survival probability of safer entrepreneurs</td>
</tr>
<tr>
<td>$\gamma^{HR}$</td>
<td>0.97</td>
<td>our calibration</td>
<td>survival probability of riskier entrepreneurs</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.2772</td>
<td>our calibration</td>
<td>share of riskier entrepreneurs</td>
</tr>
<tr>
<td><strong>Bond Market</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{\varepsilon}$</td>
<td>510</td>
<td>Chen et al. (2007)</td>
<td>steady-state elasticity of the demand for funds</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>30000</td>
<td>our calibration</td>
<td>sensitivity of elasticity to output gap</td>
</tr>
<tr>
<td>$\rho_x$</td>
<td>0.9</td>
<td>Kurz and Motolese (2011)</td>
<td>persistence in optimism</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>24</td>
<td>our calibration</td>
<td>sensitivity of optimism to entrepreneur’s net worth</td>
</tr>
<tr>
<td>$\chi$</td>
<td>0</td>
<td>our calibration</td>
<td>steady-state level of optimism</td>
</tr>
<tr>
<td><strong>Policy</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{\rho}$</td>
<td>0.88</td>
<td>CMR</td>
<td>interest rate smoothing</td>
</tr>
<tr>
<td>$\alpha_\pi$</td>
<td>1.82</td>
<td>CMR</td>
<td>weight of expected inflation in Taylor rule</td>
</tr>
<tr>
<td>$\alpha_y$</td>
<td>0.11</td>
<td>CMR</td>
<td>weight of output gap in Taylor rule</td>
</tr>
<tr>
<td>$\eta_g$</td>
<td>0.2</td>
<td>CMR</td>
<td>share of government consumption</td>
</tr>
</tbody>
</table>

Table 2: Model parameters (time unit of model: quarterly)
### Table 3: Steady-State Properties, Model versus U.S. Data

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model</th>
<th>U.S. data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K/Y$</td>
<td>5.46</td>
<td>10.7</td>
</tr>
<tr>
<td>$C/Y$</td>
<td>0.63</td>
<td>0.56</td>
</tr>
<tr>
<td>$I/Y$</td>
<td>0.17</td>
<td>0.25</td>
</tr>
<tr>
<td>$G/Y$</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>leverage ratio = $QK/N$ (^1)</td>
<td>safer</td>
<td>1.26</td>
</tr>
<tr>
<td></td>
<td>riskier</td>
<td>1.35</td>
</tr>
<tr>
<td>bond to bank finance ratio (^2)</td>
<td></td>
<td>[1.21 ; 1.77]</td>
</tr>
</tbody>
</table>

Note. When not specified, the source for U.S. data is CMR and the sample period is 1998Q4-2003Q4.  
\(^1\) CMR compute the leverage as $N/(Q\bar{K} - N)$. We compute the leverage as in Bernanke et al. (1999).  

### Table 4: Interest Rates, Model versus U.S. Data

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model</th>
<th>U.S. data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of return on capital, $R^k$</td>
<td>safer</td>
<td>11.38%</td>
</tr>
<tr>
<td></td>
<td>riskier</td>
<td>8.40%</td>
</tr>
<tr>
<td>Cost of external finance, $Z$</td>
<td>6.81%</td>
<td>[7.1 ; 8.1] %</td>
</tr>
<tr>
<td>Time deposit, $R^e$</td>
<td>5.16%</td>
<td>5.12%</td>
</tr>
<tr>
<td>Cost of bond finance, $R^{coupon}$</td>
<td>5.99%</td>
<td>5.96%</td>
</tr>
</tbody>
</table>

Note. When not specified, the source for U.S. data is CMR and the sample period is 1987Q1-2003Q4.  
\(^1\) Chen et al. (2007) find an average yield spread of AAA bonds over the period 1995-2003 of 84 basis points. Adding this spread to the risk-free rate ($R^e$) gives the value displayed in the table.
Figure 1: Effective federal funds rate in the 2000s
Note. Grey bars denote NBER recessions.
A. Anticipated policy

B. Unanticipated policy

Figure 2: Monetary policy of *too low for too long* interest rates in CMR

Note. Values expressed as percentage deviation from steady-state values. Inflation is expressed as annualized percent deviation from its steady state and the interest rates are expressed as annual percentage points. CMR-FA model: crossed lines. CMR-Simple model: solid lines. Steady state: dashed-dotted lines.
Figure 3: Structure of the model

Figure 4: BAA spread and output gap

Figure 5: Anticipated monetary policy of *too low for too long* interest rates

Note. Values expressed as percentage deviation from steady-state values. Inflation is expressed as annualized percent deviation from its steady state and the interest rates are expressed as annual percentage points. Model in normal times: crossed lines. Model in optimistic times: circled lines. Steady state: dashed-dotted lines. Baseline parameters: see table 2.
Figure 6: Unanticipated monetary policy of too low for too long interest rates
Note. Values expressed as percentage deviation from steady-state values. Inflation is expressed as annualized percent deviation from its steady state and the interest rates are expressed as annual percentage points. Model in normal times: crossed lines. Model in optimistic times: circled lines. Steady state: dashed-dotted lines. Baseline parameters: see table 2.
Appendix A - The complete model

Final-good firms

Perfectly competitive firms produce the final good that is converted into household consumption goods, investment goods, government goods, goods used up in capital utilization and in bank monitoring.

The representative firm produces the final good $Y_t$, using the intermediate goods $Y_{i,t}$, and the production technology $Y_t = \left[ \int_0^1 Y_{i,t}^{\lambda_f} di \right]^{\lambda_f}$, where $\lambda_f, \infty > \lambda_f \geq 1$, is the markup for the intermediate-good firms. The representative firm chooses $Y_{i,t}$ to maximize its profits, taking the output price, $P_t$, and the input prices, $P_{i,t}$, as given. The maximization problem of the representative firm is thus given by:

$$\max_{Y_{i,t}} \quad P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di$$

subject to $Y_t = \left[ \int_0^1 Y_{i,t}^{\lambda_f} di \right]^{\lambda_f}$.

Solving the profit maximization problem yields the following demand function for the intermediate good $i$: $Y_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{\frac{1}{\lambda_f-1}} Y_t$, and perfect competition in the final goods market implies that the price of the final good can be written as:

$$P_t = \left[ \int_0^1 P_{i,t}^{\frac{\lambda_f}{\lambda_f-1}} di \right]^{1-\lambda_f}. \quad (A.1)$$

Intermediate-good firms

Monopolistic competitive firms, indexed by $i \in [0, 1]$, produce differentiated intermediate goods using the following production function:

$$Y_{i,t} = (K_{i,t})^\alpha (L_{i,t})^{1-\alpha}, \quad (A.2)$$

where $0 < \alpha < 1$ and $K_{i,t}$ and $L_{i,t}$ denote, respectively, the capital and labor input for the production of good $i$. The capital input is assumed to be a composite of two entrepreneur-specific bundles of
capital services, \( K_{i,t}^{HR} \) and \( K_{i,t}^{LR} \) which in turn combine the capital services of the individual members of the two entrepreneur sectors, \( K_{i,t}^{HR,r} \) and \( K_{i,t}^{LR,l} \). Formally,

\[
K_{i,t} = \left[ \eta^{1-\rho} (K_{i,t}^{HR})^{\rho} + (1 - \eta)^{1-\rho} (K_{i,t}^{LR})^{\rho} \right]^{\frac{1}{\rho}},
\]

(A.3)

where \( \rho \) denotes the degree of substitutability between the two entrepreneur-specific bundles of capital services and, since all entrepreneurs are identical within each group, \( K_{i,t}^{HR} = \eta K_{i,t}^{HR,r} \) and \( K_{i,t}^{LR} = (1 - \eta) K_{i,t}^{LR,l} \).

The \( i \)-th firm hires labor and rents capital in competitive markets by minimizing its production costs, taking as given the nominal wage rate, \( W_t \), and the real rental rates of capital, \( r_t^{k,HR} \) and \( r_t^{k,LR} \). The firm \( i \)'s optimal demand for capital and labor services solves the following cost minimization problem:

\[
\min_{\{L_{i,t}, K_{i,t}^{HR}, K_{i,t}^{LR}\}} C(\cdot) = \frac{W_t L_{i,t}}{P_t} + K_{i,t}^{HR} (K_{i,t}^{HR,r})^{\frac{\alpha}{\rho + \alpha (1 - \rho)}} (Y_t) \left[ \frac{\alpha}{\rho + \alpha (1 - \rho)} \right] \frac{\rho}{\rho + \alpha (1 - \rho)},
\]

subject to (A.2) and (A.3). Since all firms \( i \) face the same input prices and since they all have access to the same production technology, real marginal costs \( s_t \) are identical across firms and are given by

\[
s_t = \left[ \frac{\bar{w}_t}{1 - \alpha} \right]^{1-\frac{\alpha}{\rho + \alpha (1 - \rho)}} \left[ \frac{\alpha}{r_t^{k,HR}} (K_{i,t}^{HR,r})^{\rho - 1} \right] \frac{\rho}{\rho + \alpha (1 - \rho)},
\]

where \( \bar{w}_t \) denotes the real wage.

**Price setting**

Prices are determined through a variant of the Calvo's (1983) mechanism. In particular, every firm faces a constant probability, \( 1 - \xi_p \), of reoptimizing its price in any given period, whereas the non-reoptimizing firms set their prices according to the indexation rule \( P_{i,t} = P_{i,t-1} (\bar{\pi})^{\epsilon_1} (\pi_{t-1})^{1-\epsilon_1} \), where \( \bar{\pi} \) represents the steady-state inflation rate, \( \pi_{t-1} = P_{t-1}/P_{t-2} \) is the inflation rate from \( t-2 \) to \( t-1 \) and the parameter \( \epsilon_1, 0 \leq \epsilon_1 \leq 1 \), represents the degree of price indexation to steady-state inflation.

The \( i \)-th firm that optimizes its price at time \( t \) chooses \( P_{i,t} = \bar{P}_{i,t} \) that maximizes the present value of future expected nominal profits:

\[
\max_{P_{i,t}} \Pi_t^{LG} = E_t \sum_{\tau=0}^{\infty} (\beta \xi_p)^\tau \lambda_{i+\tau} [(P_{i,t+\tau} - S_{t+\tau}) Y_{i,t+\tau}]
\]

36
subject to \( Y_{i,t+\tau} = \left( \frac{P_{i,t+\tau}}{P_{r+\tau}} \right)^{\lambda_f} Y_{i+\tau} \),

where \( E_t \) denotes the mathematical expectations operator conditional on information available at time \( t \), \( \lambda_{t+\tau} \) the multiplier in the households’ budget constraint, \( S_{t+\tau} \) the firm’s nominal marginal cost and \( \beta \in (0, 1) \) the discount factor. At the end of each time period, profits are rebated to households.

We consider only the symmetric equilibrium at which all firms choose the same \( \bar{P}_t = \bar{P}_{i,t} \). Thus, from (A.1), the law of motion for the aggregate price index is

\[
P_t = \left\{ (1 - \xi_p) \bar{P}_t^{\frac{1}{1-\xi_p}} + \xi_p \left[ P_{t-1} (\bar{\pi})^{\xi_1} (\pi_{t-1})^{1-\xi_1} \right]^{\frac{1}{1-\xi_p}} \right\}^{1-\lambda_f}.
\]

**Capital producers**

A continuum of competitive capital producers produce the aggregate stock of capital \( \bar{K}_t \). New capital produced in period \( t \) can be used in productive activities in period \( t+1 \). At the end of period \( t \), capital producers purchase existing capital, \( x_{K,t} \), from entrepreneurs and investment goods in the final good market, \( I_t \), and combine them to produce new capital, \( x_{K,t}' \), using the following technology: \( x_{K,t}' = x_{K,t} + A(I_t, I_{t-1}) \). Old capital can be converted one-to-one into new capital, while the transformation of the investment good is subject to quadratic adjustment costs. The function \( A(\cdot) \) summarizes the technology that transforms current and past investment into installed capital.

Investment goods are purchased in the final good market at price \( P_t \). Let \( Q_{\bar{K},t} \) be the nominal price of new capital. Since the marginal rate of transformation between new and old capital is unity, the price of old capital is also \( Q_{\bar{K},t} \). The representative capital producer’s period-\( t \) profit maximization problem is thus given by

\[
\max_{\{I_{t+\tau},x_{K,t+\tau}\}} E_t \sum_{\tau=0}^{\infty} \beta^\tau \lambda_{t+\tau} \left\{ Q_{\bar{K},t+\tau} \left[ x_{K,t+\tau} + A(I_{t+\tau}, I_{t+\tau-1}) \right] - Q_{\bar{K},t+\tau} x_{K,t+\tau} - P_{t+\tau} I_{t+\tau} \right\}.
\] (A.4)

Let \( \delta \) denote the depreciation rate and note that, from (A.4), any value of \( x_{K,t+\tau} \) is profit maximizing. Thus considering \( x_{K,t+\tau} = (1 - \delta) \bar{K}_{t+\tau} \) is consistent with both profit maximization and market clearing.
The first order condition with respect to $I_t$ is:

$$E_t \left[ \lambda_t (Q_{c,t} A_{1,t} - P_t) + \beta \lambda_{t+1} Q_{c,t+1} A_{2,t+1} \right] = 0,$$

where $A_{1,t} = \partial A (I_t, I_{t-1}) / \partial I_t$ and $A_{2,t+1} = \partial A (I_{t+1}, I_t) / \partial I_t$. This is the standard Tobin’s $Q$ equation that relates the price of capital to the marginal costs of producing investment goods.\(^\text{17}\) The aggregate capital stock evolves according to

$$\bar{K}_{t+1} = \eta \bar{K}_{t+1}^{HR,r} + (1 - \eta) \bar{K}_{t+1}^{LR,l} = (1 - \delta) \left[ \eta \bar{K}_{t}^{HR,r} + (1 - \eta) \bar{K}_{t}^{LR,l} \right] + A (I_t, I_{t-1}).$$

**Riskier entrepreneurs and retail banks**

The role of the representative retail bank in the model is to collect time deposits from households in order to finance riskier entrepreneur’s investment project. The bank hedges against credit risk by charging a premium over the risk-free rate at which it can borrow from households. The risk-free rate that the bank views as its opportunity cost to lending is a contractual nominal interest rate that is determined at the time the bank liability to households is issued. Unlike in Bernanke et al. (1999), this rate is not contingent on the shocks that intervene before the entrepreneurial loan matures.

At each point in time there is a continuum of heterogeneous entrepreneurs of total measure $\eta$, indexed by $(HR,r)$. At the end of time $t$, each entrepreneur is characterized by his net worth, $N_{t+1}^{HR,r}$, which is used, in combination with a bank loan, to purchase the time-$(t+1)$ stock of capital, $\bar{K}_{t+1}^{HR,r}$. After the purchase, the entrepreneur experiences an idiosyncratic productivity shock, $\omega_{t+1}^{HR,r}$, which transforms the purchased capital $\bar{K}_{t+1}^{HR,r}$ into $\omega_{t+1}^{HR,r} \bar{K}_{t+1}^{HR,r}$. By assumption, $\omega^{HR,r}$ is independently and identically distributed over time and across entrepreneurs and follows a log-normal distribution, $\ln (\omega^{HR,r}) \sim N (-\frac{1}{2}\sigma^2, \sigma^2)$, where $\sigma$ is the standard deviation of $\ln (\omega^{HR,r})$.

\(^{17}\) We adopt the following investment adjustment costs function:

$$A (I_t, I_{t-1}) = \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right] I_t,$$

so that $S (1) = S' (1) = 0$ and $S'' (1) = S'' > 0$ in steady state.
Capital utilization decision

At the beginning of period \( t \), the representative entrepreneur provides capital services to intermediate-good firms. Capital services, \( K^H_{t,r} \), are related to the entrepreneur’s stock of physical capital by \( K^H_{t,r} = u^H_{t,r} K^HR_{t} \), where \( u^H_{t,r} \) denotes the level of capital utilization. In choosing the capital utilization rate, the entrepreneur takes into account the increasing and convex utilization cost function \( a\left( u^H_{t,r}\right) \), that denotes the cost, in units of final goods, of setting the utilization rate to \( u^H_{t,r} \).\(^{18}\)

The entrepreneur chooses \( u^H_{t,r} \) solving the following maximization problem:

\[
\max_{u^H_{t,r}} \left[ u^H_{t,r} r^k_{t,r} - a\left( u^H_{t,r}\right) \right] \omega^H_{t,r} \bar{K}^HR_{t} P_t .
\]

After determining the utilization rate of capital and earning rent on it, the entrepreneur sells the undepreciated part to capital producers at price \( Q_{k,t} \). The average nominal gross rate of return on capital across all entrepreneurs is given by

\[
1 + R^k_{t} = \frac{\left[ u^H_{t,r} r^k_{t} - a\left( u^H_{t,r}\right) \right] \omega^H_{t,r} \bar{K}^HR_{t} P_t + (1 - \delta) Q_{k,t}}{Q_{k,t-1} - \omega^H_{t+1} \bar{K}^HR_{t+1}} .
\] (A.5)

Loan decision and the standard debt contract

At the end of period \( t \), the entrepreneur has available net worth, \( N^H_{t+1,r} \), which he uses to finance his capital expenditures, \( Q_{k,t} \bar{K}^HR_{t+1} \). To finance the difference between expenditures and net worth, he borrows an amount \( B^H_{t+1,r} = Q_{k,t} \bar{K}^HR_{t+1} - N^H_{t+1,r} \) from the retail bank.

After the purchase, the entrepreneur experiences an idiosyncratic productivity shock, \( \omega^H_{t+1,r} \), which transforms the purchased capital \( \bar{K}^HR_{t+1} \) into \( \omega^H_{t+1,r} \bar{K}^HR_{t+1} \). Financial frictions arise from asymmetric information between entrepreneur and bank. In particular, the entrepreneur costlessly observes his idiosyncratic shock, whereas the bank must pay a monitoring cost – which represent a fraction \( \mu \), \( 0 < \mu < 1 \), of the entrepreneur’s gross return – to observe it. The optimal financing mechanism is a debt contract which gives the lender the right to all liquidation proceeds in the event of an entrepreneur’s default.\(^{18}\)

\(^{18}\) The functional form that we use is \( a\left( u^H_{t,r}\right) = \frac{r^H_{t,r}}{a^H_{r}} \left[ \exp^{a^H_{r}}\left( u^H_{t,r} - 1\right) - 1\right] \), where \( r^k_{t} \) is the steady-state value of the rental rate of capital, \( a(1) = 0, a''(1) > 0 \) and \( a^H_{r} = a''(1) / a'(1) \) is a parameter that controls the degree of convexity of costs.
At the end of time \( t \), the bank offers a debt contract to the entrepreneur, which specifies the loan amount, \( B_{t+1}^{HR,r} \), and the gross interest rate on the loan, \( Z_{t+1}^{HR,r} \). At time \( t + 1 \), the entrepreneur declares bankruptcy if \( \omega_{t+1}^{HR,r} \) is smaller than the default threshold level, \( \bar{\omega}_{t+1}^{HR,r} \), defined by

\[
\omega_{t+1}^{HR,r} \left( 1 + R_{t+1}^{k,HR,r} \right) Q_{t+1}^{HR,r} K_{t+1}^{HR,r} = Z_{t+1}^{HR,r} B_{t+1}^{HR,r} .
\]  

(A.6)

Therefore, if \( \omega_{t+1}^{HR,r} > \bar{\omega}_{t+1}^{HR,r} \), the entrepreneur pays the lender the amount \( Z_{t+1}^{HR,r} B_{t+1}^{HR,r} \) and keeps the remaining \( \left( \omega_{t+1}^{HR,r} - \bar{\omega}_{t+1}^{HR,r} \right) \left( 1 + R_{t+1}^{k,HR,r} \right) Q_{t+1}^{HR,r} K_{t+1}^{HR,r} \). On the other hand, if \( \omega_{t+1}^{HR,r} < \bar{\omega}_{t+1}^{HR,r} \), the entrepreneur defaults and receives nothing, while the bank monitors the entrepreneur and receives all of the residual net worth \( (1 - \mu) \left( 1 + R_{t+1}^{k,HR,r} \right) \omega_{t+1}^{HR,r} Q_{t+1}^{HR,r} K_{t+1}^{HR,r} \).

The bank raises the funds that are necessary to finance the entrepreneurs activities issuing time deposits to households, and pays them a nominal rate of return \( R_{t+1}^{e} \). Perfect competition in the banking sector implies that the following bank’s zero profit condition holds in each period:

\[
\left[ 1 - F_{t} \left( \omega_{t+1}^{HR,r} \right) \right] Z_{t+1}^{HR,r} B_{t+1}^{HR,r} + (1 - \mu) \int_{0}^{\omega_{t+1}^{HR,r}} \omega^{HR,r} dF \left( \omega^{HR,r} \right) \left( 1 + R_{t+1}^{k,HR,r} \right) Q_{t+1}^{HR,r} K_{t+1}^{HR,r} = (1 + R_{t+1}^{e}) B_{t+1}^{HR,r} ,
\]  

(A.7)

where \( F_{t} \left( \omega^{HR,r} \right) \) is the cumulative distribution function of \( \omega^{HR,r} \). Let \( k_{t+1}^{HR,r} = \frac{Q_{t+1}^{HR,r} K_{t+1}^{HR,r}}{N_{t+1}^{HR,r}} \) denote the ratio of capital expenditures to net worth. Combining (A.6) with (A.7) and using the definition of \( k_{t+1}^{HR,r} \) yields

\[
\left[ \Gamma_{t} \left( \omega_{t+1}^{HR,r} \right) - \mu G_{t} \left( \omega_{t+1}^{HR,r} \right) \right] k_{t+1}^{HR,r} \frac{1 + R_{t+1}^{k,HR,r}}{1 + R_{t+1}^{HR,r}} = k_{t+1}^{HR,r} - 1 ,
\]

where \( G_{t} \left( \omega_{t+1}^{HR,r} \right) = \int_{0}^{\omega_{t+1}^{HR,r}} \omega^{HR,r} dF \left( \omega^{HR,r} \right) \) and \( \Gamma_{t} \left( \omega_{t+1}^{HR,r} \right) = \bar{\omega}_{t+1}^{HR,r} \left[ 1 - F_{t} \left( \omega_{t+1}^{HR,r} \right) \right] + G_{t} \left( \omega_{t+1}^{HR,r} \right) \). The term \( \Gamma_{t} \left( \omega_{t+1}^{HR,r} \right) \) represents the share of entrepreneurial earnings received by the bank and \( \mu G_{t} \left( \omega_{t+1}^{HR,r} \right) \) the expected monitoring costs. Therefore \( 1 - \Gamma_{t} \left( \omega_{t+1}^{HR,r} \right) \) is the share of profits going to the entrepreneur.

The contract determines the division of the expected profits between borrower and lender. In particular, the optimal contract maximizes the entrepreneur’s expected return at time \( t + 1 \) subject to the zero
profit condition on banks. The optimal contracting problem may be written in the following way:

$$\max_{\{k_{t+1}, \omega_{t+1}\}} E_t \left\{ \left[ 1 - \Gamma_t \left( \hat{\omega}_{t+1} \right) \right] \frac{1 + R_t^{k_{t+1}}}{1 + R_t^{k_{t+1}}} k_{t+1} \right\}$$

subject to

$$\Gamma_t \left( \hat{\omega}_{t+1} \right) \left( \bar{\omega}_{t+1} \right) k_{t+1}^{HHR, r} \frac{1 + R_t^{k_{t+1}}}{1 + R_t^{k_{t+1}}} = k_{t+1}^{HHR, r} - 1 .$$

**Entrepreneurial net worth**

After the loan contract received in $t - 1$ is settled, the entrepreneurial equity, $V_t^{HHR, r}$, is given by

$$V_t^{HHR, r} = \left( 1 + R_t^{k_{HHR}} \right) Q_{\bar{k}', t-1} \bar{K}_t^{HHR, r} - \left[ 1 + R_t^{k_{HHR}} \left( \frac{\mu_{0}^{\omega_{HHR, r}} \omega^{HHR, r} dF_{t-1} \left( \omega^{HHR, r} \right) \left( 1 + R_t^{k_{HHR}} \right) Q_{\bar{k}', t-1} \bar{K}_t^{HHR, r}}{Q_{\bar{k}', t-1} \bar{K}_t^{HHR, r} - N_t^{HHR, r}} \right) \right] \left( Q_{\bar{k}', t-1} \bar{K}_t^{HHR, r} - N_t^{HHR, r} \right).$$

Equity depends on the profits accumulated from the entrepreneur’s activities. The first term represents the proceeds from selling undepreciated capital to capital producers, plus the rental income of capital, net of the costs of utilization (see equation A.5). The term in squared brackets represents the gross rate of return paid by entrepreneur on time-$(t - 1)$ loans.

At this point, to ensure that entrepreneur does not accumulate enough net worth to be fully self-financed, CMR assume that there is a constant probability of death. Namely, in each period entrepreneur exits the economy with probability $1 - \gamma^{HHR}$. In this case, entrepreneur rebates his equity to households in a lump-sum way:

$$\text{transfer to households} = (1 - \gamma^{HHR}) V_t^{HHR, r} .$$

To keep the population constant, $1 - \gamma^{HHR}$ entrepreneurs are born each period. Entrepreneurial net worth, $N_{t+1}^{HHR, r}$, combines the total equity and a transfer received from households, $W_{t}^{e, HHR, r}$, and is given by

$$N_{t+1}^{HHR, r} = \gamma^{HHR} V_{t}^{HHR, r} + W_{t}^{e, HHR, r} .$$

A feature of the debt contract is that entrepreneurs with no net worth receive no loans. Thus, if newborn entrepreneurs receive no transfers, they would have zero net worth and would therefore not be able to purchase any capital. The same happens with the fraction of entrepreneurs who are bankrupt due to a low realization of $\omega$. To avoid this situation, the $1 - \gamma^{HHR}$ entrepreneurs who are born and the $\gamma^{HHR}$ who survive receive the subsidy $W_{t}^{e, HHR, r}$ from households.
Households

There is a continuum of infinitely lived risk averse households, indexed by \( j \in [0, 1] \). Each household consumes, supplies a differentiated labor input and allocates his savings between riskless time deposit and corporate bonds. As households differ in hours worked and in income, one would expect that they would also differ in consumption and asset allocations. However, each household \( j \) is assumed to hold state-contingent securities that provide insurance against household-specific wage-income risk. As a result, households are homogeneous with respect to consumption and asset holdings in equilibrium. Therefore, in what follows, consumption and saving decisions are not indexed by \( j \).\(^{19}\)

Consumption and saving decisions

The instantaneous utility function of a given household is separable in consumption and hours worked and given by:

\[
u(\cdot) = \log (C_{t+\tau} - bC_{t+\tau-1}) - \psi_L \frac{h_{j,t+\tau}^{1+\sigma_L}}{1+\sigma_L},
\]

where \( C_t \) denotes the household consumption at time \( t \) and \( h_{j,t} \) denotes its hours worked in period \( t \).

The parameter \( b > 0 \) measures the degree of external habit formation in consumption, \( \sigma_L > 0 \) is the inverse of the Frisch elasticity of labor supply and \( \psi_L > 0 \) is a preference parameter that affects the disutility of supplying labor.

At the end of period \( t \), household allocates his savings into time deposits, \( T_t \), and corporate bonds, \( CB_t \). At the end of period \( t+1 \), time deposits pay a riskless rate of return equal to \( R_{t+1} \), while the rate of return on corporate bonds is \( R_{t+1} \). We assume that both rates are known when household makes his saving decision and are not contingent on the realization of period-(\( t+1 \)) monetary policy shock.

The household budget constraint at time \( t \), written in nominal terms, is given by

\[
(1 + R_{t}^e) T_{t-1} + (1 + R_{t}^F) CB_{t-1} + W_{j,t} h_{j,t} \\
+ (1 - \gamma^{LR}) (1 - \eta) V_{t}^{LR,l} + (1 - \gamma^{HR}) \eta V_{t}^{HR,r} \\
+ \Pi_{t}^{GF} + \Pi_{t}^{FB} + NCS_{t} - CB_{t} - T_{t} - P_{t} C_{t} - W_{t} - Lump_{t} \geq 0,
\]

\(^{19}\) See Erceg et al. (2000) for a discussion about the existence of state-contingent securities.
where $W_{j,t}$ is the wage earned by the household $j$, $NCS_{j,t}$ represents the net payoff of the state contingent securities that the $j^{th}$ household purchases to insulate itself from the uncertainty associated with the ability to re-optimize its wage, $\Pi_t^{IGF}$ and $\Pi_t^{IB}$ are the profits received from, respectively, intermediate-good firms and investment banks, $(1 - \gamma^{HR}) \eta V_{t}^{HR,r}$ are the lump-sum transfers received from riskier entrepreneurs who exit the economy, $(1 - \gamma^{LR})(1 - \eta)V_{t}^{LR,l}$ are the lump-sum transfers received from safer entrepreneurs who exit the economy, $W^e_t$ is the total transfer payment to entrepreneurs and $Lump_t$ are lump-sum taxes paid to finance government expenditures.

The representative household takes its consumption and saving decisions so as to maximize the expected lifetime utility subject to its intertemporal budget constraint. The optimization problem is given by

$$\max_{\{C_{t+\tau}, T_{t+\tau}, CB_{t+\tau}\}} \mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^\tau \left[ \log (C_{t+\tau} - bC_{t+\tau-1}) - \psi_L \frac{h_{j,t+\tau}^{1+\sigma_L}}{1+\sigma_L} \right]$$

subject to

$$(1 + R_{t+\tau}^e) T_{t-1+\tau} + (1 + R_{t+\tau}^F) CB_{t-1+\tau} + W_{j,t+\tau} h_{j,t+\tau}$$
$$+ (1 - \gamma^{LR})(1 - \eta)V_{t+\tau}^{LR,l} + (1 - \gamma^{HR}) \eta V_{t+\tau}^{HR,r}$$
$$+ \Pi_{t+\tau}^{IGF} + \Pi_{t+\tau}^{IB} + NCS_{t+\tau} - CB_{t+\tau} - T_{t+\tau}$$
$$- P_{t+\tau} C_{t+\tau} - W^e_{t+\tau} - Lump_{t+\tau} \geq 0 .$$

The first order conditions with respect to $T_t$, $CB_t$ and $C_t$ are, respectively,

$$\lambda_t = \beta (1 + R_{t+1}^e) E_t (\lambda_{t+1})$$
(A.9)

$$\lambda_t = \beta (1 + R_{t+1}^F) E_t (\lambda_{t+1})$$
(A.10)

$$P_t \lambda_t = \frac{1}{(C_t - bC_{t-1})} - \beta b \frac{1}{(C_{t+1} - bC_t)} ,$$
(A.11)

where $\lambda_t$ is the Lagrange multiplier associated to the households’ budget constraint. Equation (A.9) represents the standard Euler equation. The right hand side of (A.11) is the marginal utility of consumption, taking into account habit persistence. Comparing (A.9) and (A.10), it must hold that $R_{t+1}^F = R_{t+1}^e \forall t$, i.e. the return on corporate bonds equals the return on time deposits, which in turn is equal to the central bank nominal interest rate. This result is due to the assumption that both interest rates are known when household makes his optimal decision and are not contingent on the realization of period-$(t + 1)$ monetary policy shock.
Labor supply and wage setting

Each household is a monopolistic supplier of a differentiated labor service, $h_{j,t}$, to the production sector. Labor services are bundled together using the aggregator function $L_{i,t} = \left[ \int_0^1 (h_{j,t})^{\lambda_w} dj \right]^{\lambda_w}$, where $\lambda_w$, $\infty > \lambda_w \geq 1$, represents the wage markup. The demand curve for the $j^{th}$ household specialized labor services is $h_{j,t} = \left( \frac{W_{j,t}}{W_t} \right)^{\lambda_w} - \lambda W_{j,t}$, and the aggregate nominal wage, $W_t$, is given by

$$W_t = \left[ \int_0^1 (W_{j,t})^{1-\lambda_w} dj \right]^{1-\lambda_w}.$$  \hspace{1cm} (A.12)

In each period, a fraction $\xi_w$ of households cannot reoptimize their wages and, therefore, set their wages according to the indexation rule $W_{j,t} = W_{j,t-1} (\bar{\pi})_{t-1}^{1-\xi_w}$, where $i_{t-1}, 0 \leq i_{t-1} \leq 1$, represents the degree of wage indexation to steady-state inflation rate. The fraction $1 - \xi_w$ of reoptimizing households set their wages by maximizing

$$\max_{W_{j,t}} E_t \sum_{\tau=0}^{\infty} (\beta \xi_w)^\tau \left[ \psi_{1+\sigma_L} \frac{h_{j,t,\tau+1}}{1+\sigma_L} + \lambda_{t+\tau} W_{j,t,\tau+1} h_{j,t,\tau+1} \right]$$

subject to $h_{j,t} = \left( \frac{W_{j,t}}{W_t} \right)^{1-\lambda_w} L_{i,t}$.

We only consider the symmetric equilibrium in which all households choose the same $\bar{W}_t = W_{j,t}$. Thus, given (A.12), the law of motion of the aggregate wage index is given by

$$W_t = \left\{ (1 - \xi_w) \bar{W}_t^{1-\lambda_w} + \xi_w \left[ W_{t-1} (\bar{\pi})_{t-1}^{1-\xi_w} \right] \right\}^{1-\lambda_w}.$$  

Resource constraint

The aggregate resource constraint is

$$C_t + I_t + GC_t + \eta \mu \int_0^{\omega_t} \omega dF(\omega) \left( 1 + R_t^{HR} \right) \frac{Q_t^{HR}}{K_t^{HR}} + \eta \left( u_t^{HR} \right) K_t^{HR} + (1 - \eta) a \left( u_t^{LR} \right) K_t^{LR} = Y_t.$$  

Government expenditures, $GC_t$, are determined exogenously as a constant fraction, $\eta_g$, of final output:
\(GC_t = \eta_g Y_t\) and are financed by lump-sum taxes to the households. The fourth term represents final output used by banks in monitoring riskier entrepreneurs, and the last two terms on the left hand side capture capital utilization costs.

### Aggregate variables and market clearing conditions

Aggregate net worth \((N_{t+1}^{TOT})\) is defined as

\[
N_{t+1}^{TOT} = \eta N_{t+1}^{HR, r} + (1 - \eta) N_{t+1}^{LR, l}.
\]

Total credit \((B_{t+1}^{TOT})\) is defined as the sum of bank loans and bonds issued and is given by:

\[
B_{t+1}^{TOT} = \eta B_{t+1}^{HR, r} + (1 - \eta) B_{t+1}^{LR, l}.
\]

The capital rental market clearing conditions are:

\[
\int_0^1 K_t^{HR} di = \int_0^1 K_t^{LR} di = \eta K_t^{HR, r} = \eta u_t^{HR, r} K_t^{HR, r}
\]

and

\[
\int_0^1 K_t^{LR} di = \int_0^1 (1 - \eta) K_t^{LR, l} = (1 - \eta) u_t^{LR, l} K_t^{LR, l}.
\]

Loan and bond market clearing conditions are, respectively, \(T_t = \eta B_t^{HR, r}\) and \(CB_t = (1 - \eta) B_t^{LR, l}\).

The market clearing condition in the labor market is:

\[
L_t = \int_0^1 \left\{ \left[ \int_0^1 (h_{j,t}) \frac{d\omega}{\omega} \right]^{\lambda_w} dj \right\} di.
\]

Finally, the total transfer from households \((W^e_t)\) to entrepreneurs must satisfy

\[
W^e_t = \eta W_t^{e, HR, r} + (1 - \eta) W_t^{e, LR, l}.
\]
Appendix B - Model solution

This appendix reports the details on how we solved the model. The solution strategy involves linearization around the model’s nonstochastic steady state. We first solve numerically the model, for the steady state, using the computational procedure described later in this appendix. We then employ the Dynare software package to compute the first-order Taylor series approximation of the equilibrium conditions in the neighborhood of the steady state.

In what follows, we adopt the following scaling notation:

\[ q_t = \frac{Q_t}{p_t}, \quad \lambda_{t,l} = \lambda_l p_t, \quad w_{t,LR,l} = \frac{W_{t,LR,l}}{p_t}, \quad w_{t,HR,r} = \frac{W_{t,HR,r}}{p_t}, \]

\[ n_{HR,r}^{t+1} = \frac{N_{HR,r}^{t+1}}{p_t}, \quad n_{LR,l}^{t+1} = \frac{N_{LR,l}^{t+1}}{p_t}. \]

The equations that characterize the model’s equilibrium, expressed in scaled form, are listed below.

- Investment bank
  - elasticity (normal times)
    \[ \varepsilon_{t+1}^{normal} = \bar{\varepsilon} + \alpha_1 (Y_t - \bar{Y}) \]  
    (B.1)
  - coupon rate (normal times)
    \[ 1 + R_{t+1}^{coupon,normal} = \frac{\varepsilon_{t+1}^{normal}}{\varepsilon_t} \frac{1}{\varepsilon_{t+1}^{normal}} (1 + R_t^{e}) \]  
    (B.2)
  - spread (normal times)
    \[ spread_{t+1}^{normal} = \frac{1}{\varepsilon_{t+1}^{normal}} - 1 (1 + R_t^{e}) \]  
    (B.3)
  - optimism
    \[ \chi_t = \rho\chi_{t-1} + (1 - \rho) \left[ \bar{\chi} + \alpha_2 \left( n_{LR,l}^{t+1} - n_{LR,l}^{t} \right) \right] \]  
    (B.4)
  - coupon rate (with optimism)
    \[ 1 + R_{t+1}^{coupon,optimistic} = \frac{\varepsilon_{t+1}^{normal}}{\varepsilon_t} \frac{(1 + \chi_t)}{(1 + \chi_t)} \]  
    (B.5)
• Intermediate-good firms

- arbitrage condition for the choice of capital services

\[
\frac{r_{k,HR}^t}{r_{k,LR}^t} = \left( \frac{u_{t}^{HHR,r} K_{t}^{HHR,r}}{u_{t}^{LR,l} K_{t}^{LR,l}} \right)^{\rho - 1}
\] (B.6)

- two measure of marginal costs

\[
s_t = \frac{\rho}{\rho + \alpha (1 - \rho)} \left[ \frac{\bar{w}_t}{1 - \alpha} \right]^{-\rho + \alpha (1 - \rho)} \left[ \frac{\alpha}{u_{t}^{HHR,r} K_{t}^{HHR,r}} \right]^{\rho - 1} \left[ \left( \frac{u_{t}^{HHR,r} K_{t}^{HHR,r}}{\rho} \right) \left( \frac{Y_t}{\rho + \alpha (1 - \rho)} \right) \right]^{\frac{1}{\rho - 1}}
\] (B.7)

\[
s_t = \frac{r_{k,HR}^t}{\alpha \left( \frac{\bar{K}_t}{\bar{K}_t} \right)^{1 - \alpha} \left( 1 - \eta \left( \frac{u_{t}^{HHR,r} K_{t}^{HHR,r}}{\rho} \right)^{\rho} + (1 - \eta) \left( \frac{u_{t}^{LR,l} K_{t}^{LR,l}}{\rho} \right)^{\rho} \right)]^{\frac{1}{\rho - 1}}
\] (B.8)

where

\[
K_t = \left[ \eta \left( \frac{u_{t}^{HHR,r} K_{t}^{HHR,r}}{\rho} \right)^{\rho} + (1 - \eta) \left( \frac{u_{t}^{LR,l} K_{t}^{LR,l}}{\rho} \right)^{\rho} \right]^\frac{1}{\rho}
\] (B.9)

• Capital producers

- first order condition with respect to investment

\[
\lambda_{n,t} q_t \left[ 1 - \frac{S''}{2} \left( \frac{I_{t}}{I_{t-1}} - 1 \right)^2 - S'' \frac{I_{t}}{I_{t-1}} \left( \frac{I_{t}}{I_{t-1}} - 1 \right) \right] - \lambda_{n,t} + \beta \lambda_{n,t+1} q_{t+1} S'' \left( \frac{I_{t+1}}{I_{t}} - 1 \right)^2 \left( \frac{I_{t+1}}{I_{t}} - 1 \right) = 0
\] (B.10)

- aggregate stock of physical capital

\[
\eta \bar{K}_{t+1}^{HHR} + (1 - \eta) \bar{K}_{t+1}^{LR,l} = (1 - \delta) \left[ \eta \bar{K}_{t}^{HHR} + (1 - \eta) \bar{K}_{t}^{LR,l} \right] + \left[ 1 - \frac{S''}{2} \left( \frac{I_{t}}{I_{t-1}} - 1 \right)^2 \right] I_{t}
\] (B.11)

• Riskier entrepreneur and retail bank
- first order condition with respect to capital utilization

\[ r_t^{k,HR} = a' \left( u_t^{HR,r} \right) \]  

(B.12)

- definition of rate of return on capital

\[ 1 + R_t^{k,HR} = \frac{\pi_t}{q_{t-1}} \left\{ \left[ u_t^{HR,r} r_t^{k,HR} - a \left( u_t^{HR,r} \right) \right] + (1 - \delta) q_t \right\} \]  

(B.13)

- standard debt contract

\[ E_t \left\{ [1 - \Gamma_t (\bar{\omega}_{t+1})] \frac{1 + R_t^{k,HR}}{1 + R_t^{e}} + \right. \]

\[ \left. \frac{\Gamma_t' (\bar{\omega}_{t+1})}{\Gamma_t (\bar{\omega}_{t+1}) - \mu G_t (\bar{\omega}_{t+1})} \left[ \Gamma_t (\bar{\omega}_{t+1}) - \mu G_t (\bar{\omega}_{t+1}) \right] \frac{1 + R_t^{k,HR}}{1 + R_t^{e}} - 1 \right\} = 0 \]  

(B.14)

- zero profit condition for bank

\[ [\Gamma_t (\bar{\omega}_t) - \mu G_t (\bar{\omega}_t)] \frac{q_{t-1} R_t^{k,HR,r}}{n_t^{HR,r}} \frac{1 + R_t^{k,HR}}{1 + R_t^{e}} = \frac{q_{t-1} R_t^{k,HR,r}}{n_t^{HR,r}} - 1 \]  

(B.15)

- net worth

\[ n_{t+1}^{HR,r} = \gamma^{HR} q_{t-1} R_t^{k,HR,r} \left[ R_t^{k,HR} - R_t^{e} - \mu \int_0^{\bar{\omega}_t} \omega dF_t (\omega) \left( 1 + R_t^{k,HR} \right) \right] \]

\[ + \gamma^{HR} n_t^{HR,r} \frac{R_t^{k,HR,r}}{\pi_t} (1 + R_t^{e}) + w_{t}^{e,HR,r} \]  

(B.16)

- Safer entrepreneur

- definition of rate of return on capital

\[ 1 + R_t^{k,LR} = \frac{\pi_t}{q_{t-1}} \left\{ \left[ u_t^{LR,l} r_t^{k,LR} - a \left( u_t^{LR,l} \right) \right] + (1 - \delta) q_t \right\} \]  

(B.17)

- first order condition with respect to capital utilization

\[ r_t^{k,LR} = a' \left( u_t^{LR,l} \right) \]  

(B.18)
- first order condition with respect to capital

\[ q_t \left(1 + \beta R^\text{coupon}_{t+1} \right) = \beta E_t \left\{ u_{t+1}^{LR,l} \left[ \beta q^{k,LR}_{t+1} - a \left( u_{t+1}^{LR,l} \right) + (1 - \delta) q_{t+1} \right] \right\} \]  (B.19)

- net worth

\[ n_{t+1}^{LR,l} = \gamma^{LR} \frac{Q_t - 1}{\pi_t} n_{t}^{LR,l} \left( R^{k,LR}_t - R^\text{coupon}_t \right) \]

\[ + \frac{\gamma^{LR}}{\pi_t} (1 + R^\text{coupon}_t) n_{t}^{LR,l} + u_{t}^{c,LR,l} \]  (B.20)

- Households

- first order condition with respect to time deposits

\[ \lambda_{n,t} = \frac{\beta}{\pi_{t+1}} (1 + R^e_{t+1}) \lambda_{n,t+1} \]  (B.21)

- first order condition with respect to consumption

\[ \lambda_{n,t} = \frac{1}{(c_t - bc_{t-1})} - \beta b \frac{1}{(c_{t+1} - bc_t)} \]  (B.22)

- Aggregate resource constraint and production function

\[ C_t + I_t + \eta \left[ \mu \int_0^\infty \omega dF (\omega) \left(1 + R^{k,HR}_t \right) \frac{Q_{t-1}^H R^{HR,r}_t}{\pi_t} \right] \]

\[ + \eta a \left( u_t^{HR,r} \right) k_t^{HR,r} + (1 - \eta) a \left( u_t^{LR,l} \right) k_t^{LR,l} = (1 - \eta_g) Y_t \]  (B.23)

\[ Y_t = K_t^{\alpha_h} h_t^{1-\alpha} \]  (B.24)

- Conditions associated with Calvo sticky prices and wages

\[ \lambda_{n,t} Y_t + \beta \xi_p \left( \frac{\pi_{t+1}^{l-1}}{\pi_{t+1}} \right) \frac{1}{\lambda_f} F_{p,t+1} - F_{p,t} = 0 \]  (B.25)

\[ \lambda_{n,t} Y_t \lambda_f s_t + \beta \xi_p \left( \frac{\pi_{t+1}^{l-1}}{\pi_{t+1}} \right) \frac{1}{\lambda_f} K_{p,t+1} - K_{p,t} = 0 \]  (B.26)

\[ h_t \frac{\lambda_{n,t}}{\lambda_w} + \beta \xi_w \left( \frac{1}{\pi_{t+1}^{l-1}} \right) \frac{1}{\pi_{t+1}^{r-1}} F_{w,t+1} - F_{w,t} = 0 \]  (B.27)
\[ h_t^{1+\sigma_L} + \beta \xi_w \left[ \frac{\pi_t^{1-\sigma_L}}{\pi_{t+1}^{\frac{1}{\sigma_L}}} \right]^{1-\lambda_w} \frac{\lambda_w{(1+\sigma_L)}}{1-\lambda_w} K_{w,t+1} - K_{w,t} = 0 \] (B.28)

\[ K_{p,t} = F_{p,t} \left[ 1 - \xi_p \left( \frac{\pi_t^{-1}}{\pi_{t+1}^{-\lambda_f}} \right) \frac{1}{1-\xi_p} \right]^{1-\lambda_f} \] (B.29)

\[ K_{w,t} = F_{w,t} \left[ 1 - \xi_w \left( \frac{\pi_t^{1-\sigma_L}}{\pi_{t+1}^{\frac{1}{\sigma_L}}} \right) \frac{1}{1-\xi_w} \right]^{1-\lambda_w{(1+\sigma_L)}} \] (B.30)

- Other variables

- External finance premium

\[ P_t^e = \bar{\omega}_{t+1} \left( 1 + R_{t+1}^{k,HR} \right) \frac{q_t K_{t+1}^{HR,r}}{q_t K_{t+1}^{HR,r} - n_{t+1}^{HR,r}} - (1 + R_{t+1}^e) \] (B.31)

- Contractual, no-default interest rate on entrepreneurial debt

\[ Z_t = \bar{\omega}_{t+1} \left( 1 + R_{t+1}^{k,HR} \right) \frac{q_t K_{t+1}^{HR,r}}{q_t K_{t+1}^{HR,r} - n_{t+1}^{HR,r}} \] (B.32)

- Aggregate net worth

\[ n_{t+1}^{TOT} = \eta n_{t+1}^{HR,r} + (1 - \eta) n_{t+1}^{LR,l} \] (B.33)

- Bond amount

\[ B_{t+1}^{LR,l} = q_t K_{t+1}^{LR,l} - n_{t+1}^{LR,l} \] (B.34)

- Bank loans

\[ B_{t+1}^{HR,r} = q_t K_{t+1}^{HR,r} - n_{t+1}^{HR,r} \] (B.35)

- Safer entrepreneur’s leverage

\[ lev_{t+1}^{LR,l} = q_t \frac{K_{t+1}^{LR,l}}{n_{t+1}^{LR,l}} \] (B.36)

- Riskier entrepreneur’s leverage

\[ lev_{t+1}^{HR,r} = q_t \frac{K_{t+1}^{HR,r}}{n_{t+1}^{HR,r}} \] (B.37)
- Total credit (bank loans + bonds)

\[
B_{t+1}^{TOT} = \eta B_{t+1}^{HR,r} + (1 - \eta) B_{t+1}^{LR,l}
\]  

\(B.38\)

- Monetary policy rule

\[
R_t = \tilde{\rho} R_{t-1} + (1 - \tilde{\rho}) \left[ R^c + \alpha_\pi (\pi_{t+1} - \bar{\pi}) + \alpha_y (Y_t - \bar{Y}) \right] + \varepsilon_t^{MP}
\]  

\(B.39\)

**Steady state**

The strategy used for computing the steady state in this model follows the approach used by Christiano et al. (2003). We set one of the endogenous variables of the model to a value that seems reasonable based on empirical evidence, making this variable exogenous in the steady-state calculation. We then move a model’s exogenous variable into the list of variables that are endogenous in the steady-state calculation. This approach allows us to simplify the problem of computing the steady state.

We set the steady-state rental rate of capital of the riskier entrepreneur, \(r^{k,HR}\), to 0.0504, in line with the value used by CMR, and we choose the parameter \(\psi_L\) in (A.8) as endogenous variable. The set of endogenous variables is:

\[
\pi_t, s_t, I_t, \bar{\omega}_t, R_t^{k,HR}, R_t^{k,LR}, \bar{K}_t^{HR,r}, \bar{K}_t^{LR,l}, K_t, n_t^{HR,r}, n_t^{LR,l}, q_t, \lambda_{n,t}, C_t, \bar{w}_t, h_t,
\]

\[
r_t^{k,LR}, R^c_t, F_{p,t}, F_{w,t}, K_{p,t}, K_{w,t}, Y_t, \psi_t, u_t^{HR,r}, u_t^{LR,l},
\]

\[
\varepsilon_t^{normal}, R_t^{coupon,normal}, R_t^{coupon,optimistic}, \chi_t, spread_t^{normal},
\]

\[
P_{ext}^{z}, Z_t, B_t^{HR,r}, B_t^{LR,l}, B_t^{TOT}, lev_t^{HR,r}, lev_t^{LR,l}, n_t^{TOT},
\]

and the equations available for computing the steady-state value for these variables are (B.1)-(B.39).

As in Woodford (2003), steady-state inflation is set to zero, that is, \(\bar{\pi} = 1\). From (B.12) and (B.18) \(u^{HR,r} = u^{LR,l} = 1\) and \(\chi = 0\) from equation (B.4). Solve for \(R^c\) and \(q\) using (B.21) and (B.10).

Use (B.1) to compute \(\varepsilon^{normal}\). Solve for the steady state of \(R_t^{coupon,normal}, R_t^{coupon,optimistic}\) and \(spread_t^{normal}\) using, respectively, (B.2), (B.5) and (B.3). Take the ratio of (B.25) and (B.26) to obtain the value for \(s\). Equation (B.19) can be used to obtain \(r^{k,LR}\), and then substitute it in (B.17) to get the value for \(R_t^{k,LR}\).
Now we set $r^{k,HR} = 0.0504$ and solve for $R^{k,HR}$ using (B.13). Then solve the non-linear system composed by equations (B.14)-(B.16) to obtain the values for $n^{HR,r}, \bar{\omega}$ and $\bar{K}^{HR,r}$. From (B.6) we get the value for $\bar{K}^{LR,l}$. Solve for $n^{LR,l}$, $K_t$ and $I$ using (B.20), (B.9) and (B.11), respectively. Solve (B.8) for $h$ and then (B.24) for $Y$. Then use (B.7) and (B.23) to solve for $\bar{w}$ and $C$. Get $\lambda_n$ using (B.22). Equations (B.25), (B.27) and (B.28) can be used to obtain $F_p$, $F_w$ and $K_w$. It then follows from (B.29) that $K_p = F_p$. Finally, solve for $\psi_L$ using (B.30). The remaining variables are trivial functions of the structural parameters and other steady-state values and are computed using equations (B.31)-(B.38).

In these calculations, all variables must be positive, and $\bar{K}^{HR,r} > n^{HR,r} > 0$, $\bar{K}^{LR,l} > n^{LR,l} > 0$ and $Z > R_{\text{coupon}}$. 

52
References


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