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Working Paper
"Irresponsible lending" with a better informed lender

IMFS Working Paper Series, No. 32

Provided in Cooperation with:
Institute for Monetary and Financial Stability (IMFS), Goethe University Frankfurt am Main

Suggested Citation: Inderst, Roman (2009) : "Irresponsible lending" with a better informed lender, IMFS Working Paper Series, No. 32, Institute for Monetary and Financial Stability, Goethe University Frankfurt am Main, Frankfurt am Main, http://nbn-resolving.de/urn:nbn:de:hebis:30-72920

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ROMAN INDERST

“IRRESPONSIBLE LENDING” WITH A BETTER INFORMED LENDER

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We present a simple model of personal finance in which an incumbent lender has an information advantage vis-à-vis both potential competitors and households. In order to extract more consumer surplus, a lender with sufficient market power may engage in ‘irresponsible’ lending, approving credit even if this is knowingly against a household’s best interest. Unless rival lenders are equally well informed, competition may reduce welfare. This holds, in particular, if less informed rivals can free ride on the incumbent’s superior screening ability.

This article proposes a new framework for analysing household (or consumer) lending. Using its past experience with borrowers in the same local area or borrowers facing similar economic conditions, a sophisticated lender may often have a better estimate of a household’s default probability than the household itself. I derive the conditions where such information asymmetry can lead to ‘irresponsible’ (or ‘too aggressive’ lending), in which case credit is approved even though this is knowingly against a household’s best interest.¹

Admittedly, the more standard approach is to assume that borrowers represent the better informed party. While we do not want to dismiss the importance of borrower adverse selection, the presumption that sophisticated and experienced lenders can estimate the default probability better than individual borrowers may be particularly suitable if borrowers are households. Individual households may not have statistically accurate information about the likelihood of, say, losing their job in an economic downturn or of incurring large medical bills in the future, both of which may cause them to default on a loan.² On the other hand, we do not presume that households’ own estimates are biased on average.

I borrow the assumption of ‘informed lending’ from Inderst and Müller (2006), where a lender is better informed about the expected cash flow from a newly financed project. My current analysis is tailored, instead, towards household finance and my focus is on whether too aggressive lending can occur in equilibrium. This focus is motivated by the particular attention that policy makers have given to this issue. For instance, in the UK various reports and taskforces on consumer lending practices (and, more generally, on the surge of household debt) have brought up

¹ The term ‘irresponsible lending’ is commonly used in the UK, along with ‘aggressive lending’ and ‘predatory lending’, though to a lesser extent.

² Based on survey evidence, Dick and Lehnert (2006) point out that medical problems, followed by divorce and unemployment, are the main causes for personal bankruptcies in the US.
the issue of ‘irresponsible’ lending. In addition, the Financial Service Authority, the UK’s main financial regulator, has recently undertaken a number of investigations into the misselling of financial products, amongst them certain kinds of mortgage products.

Concerns about predatory or abusive lending are equally widespread in the US. Among the criteria that Engel and McCoy (2004) suggest, the criterion that lending is ‘predatory’ or ‘abusive’ if the respective loan does not result in a net benefit to the borrower is closest to the one that I formalise in my model. I define lending as being too aggressive if, given the information that is available to the lender, an approved loan is against the household’s interest.

In my model, an incumbent lender is in a better position to estimate a potential borrower’s default probability than either competitors or the household itself. When the incumbent lender is relatively unconstrained by competition, I find that too aggressive lending can arise even if households perfectly anticipate that the lender is better informed and that he will use his information to his own advantage. Too aggressive lending arises out of the lender’s attempt to extract more of the consumer surplus. However, once competition forces the lender to leave households with a sufficiently high level of surplus, the incumbent lender will no longer be too aggressive. On the contrary, he may now be too conservative. However I put these results into perspective below by discussing how the introduction of lenders’ own agency problem may lead to a qualification of our benchmark.

In light of the ongoing debate about the nature and implications of too aggressive or irresponsible lending, my article thus offers the following contributions. First and possibly most importantly, I show that too aggressive lending can be perfectly rationalised as an equilibrium outcome even if households do not err on average and even if they are not systematically deceived by lenders. Second, my model points to a potential source of too aggressive lending: the information advantage of a lender with market power. However, my model also shows that increased competition from less informed lenders may reduce welfare both because it may make the incumbent lender too conservative and because entrants attempt to free-ride on the incumbent’s superior screening skills.

The rest of this article is organised as follows. In Section 1 I review the related literature. Section 2 introduces the model, which is analysed in Section 3. Section 4 contains a discussion and a comparative analysis, while Section 5 analyses the free-riding problem. Section 6 concludes.

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3 As a remedy, the Griffiths Commission (2005) proposed the introduction of a Statutory Bank Customers Charter, which would replace the Voluntary Banking Code, to which banks now subscribe in the UK. The report by the Department of Trade and Industry ‘Fair, Clear and Competitive – The Consumer Credit Market in the 21st Century’ (DTI, 2005) led to the new Consumer Credit Bill, which will include a definition of ‘unfair lending’ and provisions for how consumers can challenge supposedly unfair agreements.

4 At the level of the European Union, the new Consumer Credit Directive was adopted in 2004 with the aim of harmonising European legislation and protecting consumers better in their credit transactions.

5 For instance, Elliehausen et al. (2005) offer a detailed account of cases and policy responses to potentially abusive mortgage lending practices in the US. At the time of revising this article, many observers linked the ongoing crisis in the US subprime mortgage market to predatory lending practices.

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1. Related Literature

My article relates to the extant literature on household finance; see, for instance, Hynes and Posner (2002) or White (2005) for recent overviews. This literature has identified a number of possible imperfections in the market for credit. In addition to the potential for private information and moral hazard on the part of the borrower, there has also been considerable emphasis on households’ limited understanding of the nature and details of financial products. As discussed by Beales et al. (1981), for instance, one cannot rely on firms to inform and educate consumers properly.\(^6\)

As noted above, my model employs the ‘informed lending’ framework of Inderst and Müller (2006). There, the lender’s superior information relates to the distribution of the perfectly verifiable cash flow from a newly financed project. Consequently, the article’s focus is on the optimal design of the financial security. While the possibility of both too conservative and too aggressive lending has already been recognised there, this article differs from it in a number of aspects. In the current model, the presence of (deadweight) costs of personal bankruptcy can make a lender’s credit policy too aggressive from the point of overall efficiency. Moreover, the possibility that a lender can be too conservative follows from the fact that a household may strategically default, which is not a possibility in Inderst and Müller (2006). The problem of free-riding identified is also novel.

This article and Bond et al. (2005) share two key assumptions from Inderst and Müller (2006), namely that of imperfect competition among lenders and that of an information advantage vis-à-vis borrowers. As in the present article, the main result in Bond et al. (2005) is that the combination of these two features can give rise to ‘predatory lending’, to use their US-focused terminology. My analysis differs, however, in the following aspects. I show that more effective competition, which I model in this case by a change in the sequence of moves, is not always beneficial if it does not go together with a reduction in the incumbent lender’s information advantage. There are two reasons for this. First, competition can give rise to too conservative lending. Second, we show that the possibility of free-riding by less informed lenders may make it unprofitable or even impossible for the informed lender to use his better information to screen borrowers.

Moreover, in my basic model the (deadweight) loss from default arises from personal costs of bankruptcy such as lawyers’ fees or higher future borrowing costs; see Section 2 for a more detailed list. In contrast, in Bond et al. (2005), personal bankruptcy reduces welfare through the seizure of collateralised assets. However, as I show in Section 4.2, unless personal assets are sufficiently ‘lumpy’ or otherwise non-exempt in my model, too aggressive lending would no longer arise once the extent of collateralisation represents a contractual variable.\(^7\)

Only a few other papers have used a framework where lenders are more informed or more sophisticated. In Manove et al. (2001) banks can add value by screening out bad

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\(^6\) The issue that lenders may exhibit insufficient care or may even deceive consumers when selling their products is possibly the one that has received most attention from policy makers, e.g., in the US through the Truth in Lending Act or in the EU through the recently adopted Consumer Credit Directive.

\(^7\) There are also differences in the technical analysis between the two papers. While in the present article contracts are designed before private information is observed, Bond et al. (2005) follow the analysis in Villeneuve (2005) and consider a signalling game.
projects, which firms themselves cannot distinguish from good ones. In Garmaise (2001) and Habib and Johnsen (2000), informed investors bring valuable information to a firm. More related to the household finance context, in Bolton et al. (2003), financial advisors know better which products are suitable for households with different investment needs. Other papers have assumed that, in contrast to the firm’s owner, a lender is more sophisticated, as he does not suffer from behavioural biases such as overconfidence or optimism, e.g. de Meza and Southey (1996) and Landier and Thesmar (forthcoming). Finally, it has been recognised in both empirical and theoretical work that due to bounded rationality and costs of search and decision making, borrowers can make errors and are prone to be misled and deceived by lenders; see, e.g., Woodward (2003).

2. The Model

I consider a single household that wants to finance the purchase of an indivisible good with a loan. The household has zero initial wealth. The required expenditure, which for the moment is also equal to the size of the loan, is given by $l > 0$. There are three points of time in our model: $t = 0$, $t = 1$, and $t = 2$. The good must be purchased in $t = 0$. In $t = 1$, the household realises a random income $y$, which for simplicity can only be equal to either $y = 0$ or $y = \bar{y} > 0$. Throughout the article I take $\bar{y}$ to be sufficiently large to cover the contractually stipulated repayment.

For the moment, I also stipulate that the lender is a monopolist. One justification for this is that too aggressive lending has often been associated with market segments where there is little competition. For instance, Engel and McCoy (2004) explicitly mention the lack of competition in what are often spatially segmented local markets for subprime lending as one of the major culprits for abusive or predatory lending practices. Further below I consider competition between a better informed (incumbent) lender and other, less informed lenders.

In $t = 1$ the household may make a repayment to the lender. I make the somewhat extreme assumption, though it is quite standard in the personal finance literature, that future income cannot be credibly pledged. Hence, any positive repayment must be enforced by the threat of default. For instance, in the US until October 2005 debtors had the choice between filing for personal bankruptcy under Chapter 7 or Chapter 13. Under Chapter 7, debtors were not obliged to use future earnings to repay existing debt. Even if a country’s bankruptcy code does not specify such a generous exemption, my analysis may be fully applicable to borrowers whose earnings are hard to ‘verify’, given the nature of their profession or the lack of regular work. Incidentally, the constraint arising from the assumption that not (all) income can be credibly pledged for repayment will not be binding whenever lending is too aggressive but only when the lender is too conservative.

As the good does not yield consumption benefits beyond $t = 2$, the assumption that all repayment must be made in $t = 1$ is not restrictive. The utility derived from the good

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8 Strictly speaking, while creditors were generally able to obtain a court order to garnish debtors’ wages up to a certain limit, the debtor could gain protection from these orders by filing for bankruptcy. Though the 100% exemption of post-bankruptcy earnings (the ‘fresh start’) is quite extreme, partial exemptions of future earnings are quite typical under other bankruptcy codes, in particular since many countries have reformed their laws over the last two decades; see Tabb (2005) for an overview.

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in $t = 2$ is given by $u$. In addition, in the first period and thus before the repayment must be made the household derives the utility $xu$, where $x \geq 0$. Typically, one may think of $x$ as being (relatively) small, unless the loan is meant to finance immediate consumption. Unlike mortgages, I suppose that the loan finances the purchase of some household equipment.

Both the lender and the household are risk neutral. I set the risk-neutral interest rate to zero. In my model, the purpose of the loan is thus not to smooth consumption but to finance the purchase of a long-lived good. Without the purchase in $t = 0$, the respective consumption benefits are lost.

A crucial feature in my model is that the household incurs some additional costs $c > 0$ when defaulting. For instance, Fay et al. (2002) list the loss of other (non-exempt) assets as well as payment of bankruptcy court filing fees and lawyers’ fees as ‘financial costs’ from bankruptcy. In addition, non-pecuniary costs include ‘the cost of acquiring information about the bankruptcy process, higher future borrowing costs, and the cost of bankruptcy stigma’ (Fay et al., 2002, p. 707).

A contract stipulates that in $t = 1$ the household repays the principal $l$ together with interest $rl$. In what follows, it will be more convenient though to work instead with the total repayment requirement $R := l(1 + r)$. As future income cannot be credibly pledged, the loan contract can stipulate an incentive compatible repayment up to

$$R \leq u + \gamma. \quad (1)$$

I now come to the core feature of my model: the lender’s information advantage vis-à-vis the borrower. During the credit approval process the lender privately learns of the household’s ‘type’ $\theta \in \Theta = [\underline{\theta}, \bar{\theta}]$. The type $\theta \in \Theta$ determines the probability $p(\theta)$ with which the household subsequently realises high income. Types are ordered such that high types have a strictly lower probability of subsequent default, $1 - p(\theta)$. Also, it is convenient to assume that $p(\theta)$ is continuous and that $p(\theta) = 0$ and $p(\bar{\theta}) = 1$.

From an ex ante perspective, types $\theta$ have the distribution function $F(\theta)$, which is atomless and has the continuous density $f(\theta) > 0$ over $\theta \in \Theta$. Hence, from an ex ante perspective the household will realise high income with probability

$$P := E[p(\theta) | \theta \in \Theta] = \int_{\underline{\theta}}^{\bar{\theta}} p(\theta)f(\theta) \, d\theta.$$  

This coarse information is all the household has. The more precise information that comes from observing the type $\theta$ is the lender’s private (proprietary) information.

As discussed in the Introduction, the assumption that a sophisticated lender may be in a better position to estimate a borrower’s probability of default should be particularly reasonable when borrowers are households (instead of corporations). As a particular example, take the case of the ‘home credit’ market in the UK. One key feature of this market is the close contact between a household and the lender. This is fostered through frequent collection of instalments, sometimes even weekly, which should substantially reduce the importance of private information on the side of the household, e.g., related to the employment status or health status of household members. On
the other hand, far from being local loan sharks, some of the lenders in the UK’s ‘home credit’ market are large, internationally active public companies with thousands of representatives in the field. As these lenders will clearly mine their own data base and use statistical information when designing their offers, they should have some information advantage vis-à-vis their unsophisticated, often relatively uneducated, clientele.

Before proceeding with the analysis, I provide some comments on my choice of simple loan contracts. The restriction to offering only a single contract \( R \) instead of a menu is without loss of generality as after observing \( \theta \), the lender would always strictly prefer to pick the contract with the highest repayment requirement.\(^{10}\) By the same token, it will also be immediate that the optimal contract would not be renegotiated, given that an increase or decrease in \( R \) would invariably hurt one of the parties.

The final restriction is that after a failure to repay \( R \) the loan will be foreclosed with probability one. It can be shown that this restriction on the feasible contractual set only binds if the lender becomes too aggressive, in which case the condition of ‘no strategic default’ in (1) also does not bind. Here, the lender may want to reduce the burden imposed on a defaulting household as this in turn allows him to raise the required repayment \( R \), while still satisfying the (yet to be formalised) ex ante participation constraint of the borrower. In a sense, this thereby allows the two contracting parties to negotiate around the existing personal bankruptcy provisions, as captured by the cost parameter \( \gamma \). In addition, specifying that a borrower may only be forced into personal bankruptcy with a well-defined and strictly positive probability may be difficult to implement, in particular if the lender resells the loan and the ultimate holder may thus not be in close contact with the borrower.\(^{11}\)

3. Analysis

3.1. The Lender’s Loan Policy

If granted credit, the household will repay \( R \) after realising high income in \( t = 1 \). In the case of low income, the household will default at private costs of \( \gamma \). Hence, if the lender approves credit after observing \( \theta \), the household’s expected net utility from receiving a loan is given by

\[
U(\theta) := xu + p(\theta)(u - R) - [1 - p(\theta)]\gamma.
\]

Likewise, the lender’s expected net profits from approving credit after observing \( \theta \) are

\[
V(\theta) := p(\theta)R - l.
\]

Note that the definition of \( V(\theta) \) does not take into account any possible resale value for the seized good. Observe next that \( V(\theta) \) is continuous and strictly increasing given

\(^{10}\) The choice of deterministic menus is, however, restrictive. More generally, a menu, from which the lender picks a contract after observing \( \theta \), could be made incentive compatible by stipulating different probabilities of subsequently concluding a contract. However, if the respective lottery prescribed not concluding a contract, this would typically not be renegotiation proof.

\(^{11}\) With an arbitrarily small but strictly positive resale value of the repossessed good it is clearly ex post optimal in my model for a lender to force the borrower into bankruptcy.

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that the same properties hold for \( p(\theta) \). Unless \( p(\theta) R < l \), in which case credit will never be approved, there thus exists a unique threshold \( \theta < \theta^* < \tilde{\theta} \) where \( V(\theta^*) = 0 \), implying that the lender optimally approves credit if and only if \( \theta \geq \theta^* \). Clearly, \( \theta^* \) depends on \( R \), though in what follows I choose to only make this dependency explicit whenever this is necessary to avoid ambiguity.

Before I proceed with the analysis of the market equilibrium, I discuss the benchmark of maximising total surplus, in which case the loan should be approved if

\[
u[a + p(\theta)] \geq [1 - p(\theta)] \gamma + l,
\]

i.e., if the expected consumption benefit is at least equal to the expected costs of bankruptcy plus the initial expenditure \( l \). I refer to this benchmark as the second-best benchmark. Clearly, if \( u(1 + x) > l \) holds (which is what I will assume below), then from \( \gamma > 0 \) it would be first-best optimal always to approve credit and to never let the household default.

To reduce the number of case distinctions but without losing any insights, I want to ensure that it is sometimes but not always (second-best) efficient to grant credit, i.e., that (2) holds only for sufficiently high values of \( \theta \). This is the case if

\[
ax - \gamma < l < u(1 + x).
\]

In this case, I obtain a unique interior second-best cutoff \( \theta < \theta_{SB} < \tilde{\theta} \) from (2): credit should be approved if and only if \( \theta \geq \theta_{SB} \). Under a total welfare standard, the lender is thus too aggressive if \( \theta^* < \theta_{SB} \), implying that credit is granted too often, and too conservative if \( \theta^* > \theta_{SB} \), implying that credit is denied too often. Below I will also discuss a standard of consumer surplus.

3.2. Loan Policy in Equilibrium

Anticipating the lender’s (privately) optimal choice of \( \theta^* \), the household’s expected utility is

\[
E[U(\theta)|\theta \geq \theta^*] := \int_{\theta^*}^{\theta} U(\theta) \frac{f(\theta)}{1 - F(\theta^*)} d\theta.
\]

Optimally, the lender now offers a repayment requirement \( R \) that maximises his expected payoff

\[
V := \int_{\theta^*}^{\theta} V(\theta)f(\theta) ds
\]

subject to the household’s participation constraint

\[
E[U(\theta)|\theta \geq \theta^*] \geq 0.
\]

To solve the model, I must now distinguish between three different cases. In the first case, the consumption benefits obtained before the repayment is due, \( ax \), are not too high as

\[
ax < \gamma.
\]

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Note that if condition (6) did not hold, then even a household that was sure to default (which is not the case in my model) would still prefer to take out a loan as the maximum ‘punishment’ in case of defaulting, $\gamma$, does not exceed the early consumption benefits $zu$.

If $zu < \gamma$ holds, as in condition (6), then the lender can increase $R$ until the household’s participation constraint (5) binds. Moreover, to satisfy (5) the household must realise a strictly positive net utility when income is high and the good is not repossessed. Note next that as the expected default probability is lower for high $\theta$, $U(\theta)$ is strictly increasing in $\theta$. From these two results, namely that, first, (5) binds under the optimal contract and that, second, $U(\theta)$ is strictly increasing, it is then immediate that $U(\theta^*) < 0$ and thus, together with $V(\theta^*) = 0$, that $\theta^* < \theta_{SB}$. Under the optimal contract the lender is thus too aggressive.

Putting it somewhat differently, if the household could share the lender’s information, then for all $\theta \geq \theta^*$ that are close to $\theta^*$ the household would strictly prefer not to borrow, though the lender finds it optimal to approve credit.

The two remaining cases to consider are the case where $zu = \gamma$ holds with equality, for which $\theta^* = \theta_{SB}$, and the case where $zu > \gamma$, for which the lender becomes too conservative as $\theta^* > \theta_{SB}$. To see the intuition for the last case, recall that the maximum that the lender can extract in case of high income is $R = u + c$. For any higher $R$ the household would (strategically) choose to default even after high income as the utility from non-defaulting, $u - R$, is strictly below the costs of defaulting $c$. Where $zu > \gamma$, the household’s participation constraint (5) then remains slack and the lender’s credit approval decision is too conservative.

**Proposition 1.** There is a unique equilibrium. If $zu < \gamma$ holds, then the lender grants credit to households even though this reduces total surplus and is knowingly against their best interest. For $zu > \gamma$, the lender is instead too conservative, while lending is (second-best) efficient if $zu = \gamma$.

At this point it should be noted that granting a loan against the household’s interest as $U(\theta) < 0$ does not necessarily imply that this is inefficient. Conversely, while a loan is efficient when $\theta > \theta_{SB}$ holds, it may still make the household worse off. For the most interesting case with $zu < \gamma$, I can formalise these observations as follows. Given continuity and strict monotonicity of $U(\theta)$, there exists some type $\theta_{SB} < \theta^* < \bar{\theta}$ such that the household would strictly prefer not to be granted credit for all $\theta \in [\theta^*, \theta^*)$. Whether one would describe the lender as being too aggressive for all $\theta \in (\theta_{SB}, \theta^*)$ depends thus on whether one applies a consumer standard or a total welfare standard.

In essence, the finding in Proposition 1 that lending can be too aggressive is the consequence of a monopoly pricing problem. In order to extract the household’s surplus fully, the lender offers a contract that subsequently makes it optimal to approve credit even against the household’s best interest. The lender thereby extracts (in expectation) the surplus that the household makes when there is no default. Extracting the household’s surplus in this way is, however, not welfare neutral. As $\theta^*$ is pushed down below $\theta_{SB}$ total welfare is lower than in the (second-best) benchmark case.

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I conclude this Section with some immediate implications of Proposition 1. From Proposition 1 it follows that lending is more likely to be too aggressive if the utility derived from the purchased good is more ‘back-loaded’ relative to the repayment schedule, which in my model is concentrated on \( t = 1 \). This could be more likely if the loan finances the purchase of relatively long-lived consumer goods or residential property. Also, too aggressive lending is more likely if the costs of personal bankruptcy, \( \gamma \), are higher.

**Corollary 1** The lender is more likely to be too aggressive if personal costs of bankruptcy are higher or if the consumption benefits from the purchased good are more back-loaded.

### 4. Discussion and Comparative Analysis

#### 4.1. An Explanation of Additional Cash Advances

In my model, where there are no preferences for smoothing consumption over time, the household has no need to borrow more than \( l \). Still I find that advancing additional cash, \( a > 0 \), can be optimal for the lender. Casual evidence suggests that such cash-in-advance loans are indeed common for (first or second) mortgages or purchases of durable goods, at least in the UK.

Clearly, advancing \( a \) above the purchase price \( l \) can only be optimal for the lender if the borrower will then still refrain from strategically defaulting in the case of high income. Moreover, even if the lender could tie the loan \( l + a \) to the purchase of the good, realistically he may now risk attracting borrowers who have only a low value \( u \) for the good and thus already anticipate strategically defaulting later. With this limitation in mind, the following result should thus be seen more as a hypothetical benchmark.

**Proposition 2.** If it is possible to advance more than \( l \), namely \( l + a \), and if the lender is otherwise too aggressive, it is optimal to choose \( a > 0 \). If this is feasible, then by setting \( a = \gamma \) the second best can be achieved.

Intuitively, by jointly increasing \( a \) and \( R \), the loan contract smooths out the difference between the borrower’s (net) utility when having high income and thus repaying the loan, \( u + a - R \), and when having low income and thus defaulting, \( a - \gamma \). This reduces the household’s (consumer) surplus while at the same time implementing a higher, more efficient cutoff \( \theta^* \). Though as discussed above this may not always be realistic, the lender can thereby even ensure that the second best is achieved with \( \theta^* = \theta_{SB} \). Intuitively, this is the case if the lender sets \( a \) just equal to the personal bankruptcy costs \( \gamma \) while choosing \( R = u + \gamma \), which taken together ensures that for all \( \theta \) it holds that \( U(\theta) = 0 \).

#### 4.2. Borrowing Against the Household’s Assets

Households’ costs of bankruptcy could also arise as some assets must be sold below their respective value to the household. I first assume that the level of collateralisation can be freely chosen. Hence, the contract can now specify that in case of default the
household loses assets of personal value \( c \), which have liquidation value \( \beta c \) with \( \beta < 1 \).
With assets of intrinsic value \( c \) at stake in case of default, the household’s utility is now
\[
U(\theta) = xu + p(\theta)(u - R) - [1 - p(\theta)](\gamma + c),
\]
while the lender’s payoff is
\[
V(\theta) = p(\theta)R + [1 - p(\theta)]\beta c - l.
\]
Moreover, for given \( c \) it is now second-best efficient to approve credit whenever
\[
u(x + p(\theta)) \geq [1 - p(\theta)](\gamma + c(1 - \beta)) + l,
\]
which takes into account that the value \( c(1 - \beta) \) is destroyed when liquidating the assets. This gives the following results.

Proposition 3. Suppose that it is possible to back a loan with some of the borrower’s (otherwise exempt) assets and that the respective collateral \( c \) can be freely chosen. Then if the lender is too aggressive at \( c = 0 \) (i.e., if \( xu < \gamma \)), it is uniquely optimal to set \( c = 0 \). Instead, if the lender is too conservative at \( c = 0 \) (i.e., if \( xu > \gamma \)), then the optimal contract sets \( c > 0 \) just sufficiently high so as to extract all consumer surplus. Given the optimal choice of \( c > 0 \) in the latter case, the lender’s subsequent decision whether to approve credit is second-best efficient: \( \theta^* = \theta_{SB} \).

The insight in Proposition 3 is thus that endogenous costs of bankruptcy that arise from the inclusion of assets that must be liquidated below value in case of bankruptcy are different from exogenous costs of bankruptcy, which are captured by \( \gamma \). As Proposition 3 shows, if it is possible to adjust the amount of assets that are used as collateral continuously, then assets will only be pledged if the lender is too conservative, not if the lender is too aggressive.

However, in some cases the inclusion of assets is clearly either exogenous, as they are non-exempt, or can only be undertaken in a ‘lumpy’ way. Intuitively, given that the household’s loss from defaulting is then equal to \( c + \gamma \), the lender will be too aggressive if \( xu < c + \gamma \) holds.

4.3. Policy Responses

My model with its novel feature that a lender may be better informed than households may shed new light on some of the policy recommendations that have been discussed in the literature on consumer lending. To start with, my results so far point to market power as a potential culprit for too aggressive lending. A policy option may thus be to impose an upper boundary \( \bar{R} \) on the total repayment.\(^{12}\)

Proposition 4. If the lender was previously too aggressive, then the imposition of a (not too low) cap \( R \leq \bar{R} \) would increase the household’s expected utility and welfare. Otherwise, if the lender was too conservative, the cap would reduce welfare but would have an ambiguous effect on consumer surplus.

\(^{12}\) One way to impose such a constraint on lenders could be to introduce or to strengthen an existing usury law. For instance, in the US the state of Carolina has recently modified its usury law provisions to curb predatory lending practices. Stopping short of imposing a cap on loan rates, the legislation contains a definition of ‘high cost home loans’, on which special requirements are imposed.
Imposing some not too excessive cap on $R$ will increase consumer surplus for two reasons if, otherwise, the lender is too aggressive. First, a lower $R$ directly benefits the household if it does not default. Second, as $\theta^*$ is pushed upwards, the set of types $\theta$ at which the household receives a loan against its best interest shrinks. If the lender is already too conservative, however, then a lower value of $R$ makes the lender even more conservative, creating a trade-off for the household between a lower repayment requirement and a lower likelihood of obtaining credit in the first place.

Researchers – more so than policy makers – have also frequently discussed the possibility of letting debtors waive some of their rights in case of default, in particular the right to have their debt discharged by filing for personal bankruptcy. However Aghion and Hermalin (1990) have argued that from a welfare perspective, it can be beneficial to force privately informed borrowers to ‘pool’ at less onerous terms in case of default. With a better informed lender, the implications are as follows.

**Proposition 5.** Suppose we allow households to waive their ‘rights’ and, thereby, make it possible to pledge all future income credibly. Then the household is made strictly worse off if the lender’s contract requires such a waiver. Such a clause is only imposed if the lender is otherwise too conservative, though it will not fully eliminate the lender’s conservatism.

The possibility of defaulting strategically imposes a cap on $R$, which if it binds ensures that not all consumer surplus can be extracted by the lender. If this is possible, the lender’s offer would force the household to waive his rights. Somewhat surprisingly, the lender will then still be too conservative, which would clearly not be the case without private information on $\theta$.

4.4. *The Role of Monopoly Power*

As the arguments following Proposition 1 suggest, the result that lending can be too aggressive should be sensitive to the lender’s degree of market power. I explore this in two steps. While I investigate a fully specified model of competition in more detail below, for the moment I take a simpler route to capture competitive pressure. In the spirit of the literature on contestable markets, as in Baumol and Willig (1981), I now suppose that the lender’s offer must maximise the household’s expected payoff. This is the case if it generates

$$U_{\text{max}} := \max_R \left[ \int_{\theta^*}^{\theta^*} U(\theta)f(\theta)d\theta \right].$$  

(7)

Note that in (7) the repayment $R$ affects the borrower’s expected utility both directly via the utility $u - R$ in the case of high income and indirectly via the threshold $\theta^*$. The only change to the lender’s programme is then to replace the borrower’s participation constraint (5) by the requirement that

$$\int_{\theta^*}^{\theta^*} U(\theta)f(\theta)d\theta \geq U_{\text{max}}.$$  

(8)

13 Though this is a common theme in the literature – see, e.g., Schwartz (1997) – I am not aware of any country where such waivers would be legally enforceable.

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This gives the following result.

**Proposition 6.** If the lender’s offer must satisfy the new participation constraint (8), then the lender is always too conservative ($h^*/C > h_{SB}$).

Once the lender must leave the household with a sufficiently high surplus, the lender ceases to be too aggressive. Instead, if the constraint imposed by the market is sufficiently strong, then the lender may now even become too conservative.

Taking the interpretation of a contestable market somewhat literally, a comparison of Propositions 1 and 6 suggests that the likelihood with which a given household receives credit is higher under an unconstrained monopoly. This somewhat counterintuitive result comes, however, with an important caveat. My model does not contain a standard monopoly pricing problem. This could, for instance, arise if households were privately informed about their consumption benefit from the asset. By choosing a higher $R$, an unconstrained monopolist would then exclude some households, creating less credit and a higher deadweight loss.

4.5. A Model of Competition

In this Section, I consider a simple model of competition where the current ‘incumbent’ lender faces competition from entrants. Crucially, entrants’ information is only as precise as that of households.

The analysis of games with simultaneous offers from differentially informed lenders is typically plagued with non-existence of equilibria in pure strategies. My way around this is to stipulate that a household who, after observing both the incumbent’s and the entrants’ offers, turns to a particular lender is forced to take up the respective loan offer once he has been approved. (Compare, however, the analysis in Section 5.)

Furthermore, I now want to distinguish between households with different, commonly known ex ante characteristics. For this we introduce a real-valued index $\xi \in [\underline{\xi}, \bar{\xi}]$ for the distribution function $F_\xi(\theta)$, where the respective densities $f_\xi(\theta)$ are continuous in $\xi$ for all $\theta$ and satisfy $f_\xi(\theta) > 0$ for all $\theta \in \Theta$ and $\xi \in (\underline{\xi}, \bar{\xi})$. Higher $\xi$ are ‘good news’ in a standard sense: $\xi$ shifts the distribution in the sense of the Monotone Likelihood Ratio Property (MLRP) such that $f_{\xi'}(\theta)/f_\xi(\theta)$ is everywhere strictly increasing in $\theta$ where $\xi' > \xi$. MLRP implies that the household’s ex ante probability of having high income,

$$p_\xi := \int_0^\theta p(\theta)f_\xi(\theta)\,ds,$$

is strictly increasing in $\xi$. Moreover, for very low $\xi$ (almost) all probability mass is put on the lowest type $\bar{\theta} = \bar{\theta}^\xi$, while for very high $\xi$ (almost) all probability mass is put on the highest type $\underline{\theta} = \underline{\theta}^\xi$. Formally $p_\xi \to p(\underline{\theta}) = 0$ as $\xi \to \underline{\xi}$ and $p_\xi \to p(\bar{\theta}) = 1$ as $\xi \to \bar{\xi}$.15

14 See, for instance, the discussion in von Thadden (2004). See also Bolton et al. (2003), who therefore stipulate that lenders move sequentially.

15 This also uses the fact that all $p(\theta)$ are finite. Besides continuity of all $f_\xi$ (and that MLRP holds), I do not need to make further assumptions on the nature of convergence.
Define next the threshold \( \xi \) at which without the incumbent lender’s superior information the surplus from making a loan would just be zero. That is, at \( \xi = \hat{\xi} \)

\[
u(a + p_\xi) = (1 - p_\xi)\gamma + l.
\]

As entrants make their offers to unscreened borrowers, they compete themselves down to\(^{17}\)

\[
R^E = R^\ominus_\xi := \frac{l}{p_\xi},
\]

implying that a borrower’s expected payoff would be

\[
U^\ominus_\xi := \nu(a + p_\xi) - (1 - p_\xi)\gamma + l.
\]

Note that from (9), \( U^\ominus_\xi > 0 \) only if \( \xi > \hat{\xi} \). The characterisation of an equilibrium is now rather intuitive. For \( \xi < \hat{\xi} \) entrants do not impose a constraint on the lender. Proposition 1 entails that in this case the lender is too aggressive. At the other extreme, where \( \xi \) is high, the entrants’ offer becomes very attractive. In fact, as \( \xi \to \tilde{\xi} \) such that \( p_\xi \to p(\tilde{\xi}) \) entrants are willing to lower the required repayment down to

\[
R^E = R^\ominus_\xi \to \lim_{\xi \to \tilde{\xi}}\frac{l}{p(\tilde{\xi})}.
\]

As there is also little to be gained by screening, the incumbent must match this offer, in which case the chosen \( \theta^* \) clearly exceeds \( \theta_{SB} \).

**Proposition 7.** In the game with competition by simultaneous offers, for \( a_u < \gamma \) it is now only for borrowers with commonly known characteristics \( \xi < \xi^{**} \), where \( \xi < \xi^{**} < \hat{\xi} \), that the incumbent lender’s cutoff is too low, while for all \( \xi > \xi^{**} \) it is too high. That is, lending is only too aggressive for borrowers whose alternative offer from the market is sufficiently unattractive. For \( a_u > \gamma \) the incumbent is too conservative for all \( \xi \), while for \( a_u = \gamma \) lending is second-best efficient for \( \xi < \hat{\xi} \) and too conservative for \( \xi > \hat{\xi} \).

5. Free-riding and Competition

In the game with simultaneous moves, entrants could not free-ride on the incumbent by poaching approved households. To analyse this possibility, I now allow entrants to move after the incumbent.

Before fully characterising the equilibrium outcome, I show first that competition is now always sufficiently strong to rule out too aggressive lending. This can be easily seen by arguing to a contradiction. Suppose thus to the contrary that \( \theta^* < \theta_{SB} \) in which case a rejected household would strictly prefer not to be granted credit under the incumbent’s offer. Hence, when only marginally undercutting the incumbent, an entrant’s offer attracts only approved households with type \( \theta \geq \theta^* \) and is thus profitable.\(^{18}\)

\(^{16}\) Existence of \( \hat{\xi} \) follows from my assumptions on the limits where \( \xi \to \hat{\xi} \) and \( \xi \to \tilde{\xi} \) together with MLRP and continuity of all \( f_\xi \), where the latter implies continuity of \( p_\xi \).

\(^{17}\) As is standard, I suppose in what follows that for \( \xi < \hat{\xi} \) entrants do not offer a lower \( R^E \) even if they know that this would still not attract borrowers.

\(^{18}\) Importantly, this argument and the following results all rely on the fact that households correctly update their beliefs after being either approved or rejected by the incumbent. If households, instead, naively believe that lenders are not more sophisticated than they are themselves, I have shown in my working paper Inderst (2005) that even entrants may then engage in too aggressive lending, namely to those households who were previously rejected by the better informed incumbent.

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The following cases now arise in equilibrium.

Case I: In this case, the free-riding problem is extreme and there will be no lending in equilibrium. Here, for any offer $R^I$ that would be attractive to households, entrants can always make a marginally better offer so as to poach an approved household.

Case II: In this case, the incumbent can make a ‘defensive offer’ that ensures that entrants cannot free ride by poaching approved households. The incumbent’s offer is now chosen such that any lower offer $R^E < R^I$ would not allow entrants to break even as a rejected household would also take up the offer. The optimal defensive offer is uniquely determined and, for given $\xi$, denoted by $R^D_{\xi}$. (A formal definition is provided in the Proof of Proposition 8.)

Case III: In this case, the incumbent’s offer is determined by the constraint that entrants are willing to offer a repayment requirement as low as $R^E_{\xi}$, as defined in (10), if this attracts a household irrespective of whether its credit was approved by the incumbent. Case III has two subcases. In the first subcase, Case IIIa, only the incumbent will make a loan in equilibrium – just as in the previous Case II. In Case IIIb, entrants offer a contract that only attracts a rejected household. Consequently, in Case IIIb a household will borrow with probability one – either from the incumbent (and at a lower rate) or from the entrants (and at a higher rate).

When do the different cases arise? As the household’s ex ante probability of default decreases, i.e., as $\xi$ increases, we gradually move from Case I to Case II and finally to Case III. This is intuitive. To see this, take first the two extreme cases: Case I and Case IIIb.

For low $\xi$, a rejected household will be quite pessimistic about his probability of default. This makes it easy for entrants to undercut the incumbent while still ensuring that only an approved household takes up their offer. At the other extreme is Case IIIb, which arises for high $\xi$. Here, even a rejected borrower has now a sufficiently low expected probability of default to make an offer financially viable for entrants, though the required repayment will be higher than in the incumbent’s offer. In the intermediate cases, that is in Case II and in Case IIIa, only households whose application was approved by the incumbent receive credit. The household’s expected probability of default is neither sufficiently high to give rise to the aforementioned extreme form of free-riding nor sufficiently low to make it profitable for entrants to only target a rejected household.

Proposition 8. Suppose that under sequential competition the incumbent lender moves first. Then as $\xi$ increases, thus making it less likely that the household will default, we move successively from Case I to Case IIIb. Formally, there exist three cutoffs $\xi' < \xi < \xi''$ such that Case I applies for $\xi \in (\xi', \xi'')$, Case II applies for $\xi \in (\xi', \xi''')$, Case IIIa applies for $\xi \in (\xi''', \xi''')$, and Case IIIb applies for $\xi \in (\xi''', \xi')$.\(^{19}\)

In Case IIIb, where a household always receives a loan either from the incumbent or the entrant, the informed lender no longer performs a socially valuable screening function. The only use of its better information is to allow him to extract strictly positive profits from the market, which reduces a household’s ex ante utility. As is easy to show, a

\(^{19}\) Note that I exclude the boundary cases where $\xi = \xi'$ and $\xi = \xi''$, which have degenerate probability distributions. Also, when $\xi$ takes the value of one of the three thresholds there are multiple equilibria.
household with a higher $\theta$ will gain from the presence of the better informed lender, though a household with a lower $\theta$ will be worse off.

6. Conclusion

I show that even if households are perfectly rational, better informed lenders can engage in too aggressive lending if they enjoy sufficient market power. In this case, too aggressive lending arises naturally from a lender’s attempt to extract more of the consumer surplus.

As discussed in detail in the Introduction, the claim that some lending is ‘irresponsible’ or ‘predatory’ is frequently made both in the US and Europe. My contribution is to show that such lending practices can be perfectly rational even if households do not make systematic errors. Moreover, I link the prevalence of too aggressive lending to market power. In my model, too aggressive lending may fully disappear even if competitive pressure is exerted only by lenders that do not share the incumbent lender’s information advantage. However, this may not increase consumer surplus or welfare. First, I show that competition can make lenders too conservative. Second, the possibility of free-riding on the incumbent may make it impossible to harness his better information.

My model is simple in concept. The main justification for this is to focus the analysis on the novel feature: the information gap between the (incumbent) lender and the household. For instance, an extension to more than two periods together with risk aversion could provide further insights into the optimal contract design for personal loans. A less obvious, though in my view potentially more important, shortcoming of my analysis is that in reality personal loan contracts, including mortgages, are often solicited by third-party agents, whose interests may diverge substantially from the institution that approves or ultimately holds the loan. In fact, cases of too aggressive lending seem to be more often than not associated with (asserted) misbehaviour of these agents such as brokerage firms.20 In a related model of expert advice, Inderst and Ottaviani (2007) consider such opportunistic behaviour of agents. One of the key insights is that more competition may then lead to more ‘misselling’: the act where a household follows advice and takes out a loan even though this is not in its own best interest.

Appendix: Proofs

Proof of Proposition 1: It is helpful to restate the lender’s programme fully. The lender chooses $R$ to maximise his expected payoff $V$ subject to

(i) the borrower’s participation constraint (5), where the threshold $\theta^*$ is chosen (expost) optimally by the lender such that $V(\theta^*) = 0$, and subject to

(ii) the constraint that $R \leq \min\{u + \gamma, \bar{y}\}$.

As noted in the main text, I always assume that $\bar{y}$ is sufficiently high such that the constraint $R \leq \bar{y}$ will not bind. (Formally, this is the case whenever $\bar{y} > u + \gamma$.)

20 See, for instance, the introductory examples in Renuart (2004) for the US.

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Suppose first that (6) holds such that \( xu < \gamma \), which by the argument in the main text implies that \( u - R > -\gamma \) as otherwise (5) would not be satisfied. I show next that (5) binds at an optimum. To see this, note first that by continuity of \( p(\theta) U(\theta) \) and \( V(\theta) \) are also continuous in \( \theta \). This also implies that \( \theta^* \), which is defined by \( V(\theta^*) = 0 \), is continuous in \( R \) such that finally the borrower’s expected utility \( E[U(\theta) \mid \theta \geq \theta^*] \) is also continuous in \( R \). As the lender’s payoff \( V \) is strictly increasing in \( R \), it is thus uniquely optimal for the lender to choose the highest possible \( R \) such that \( E[U(\theta) \mid \theta \geq \theta^*] = 0 \) is satisfied with equality. (Note that \( E[U(\theta) \mid \theta \geq \theta^*] \) may not be monotonic in \( R \) over the relevant range of \( R \).) \( U(\theta^*) < 0 \) and \( \theta^* < \theta_{SB} \) follows then from the arguments in the main text.

Suppose next that \( xu \geq \gamma \). In this case, the unique optimal contract for the lender is to extract as much as possible in \( t = 1 \) by setting \( R = u + \gamma \). As is easily checked, this implies that (5) then holds with equality for \( xu = \gamma \), while (5) is slack for \( xu > \gamma \). When \( xu = \gamma \), \( U(\theta) = 0 \) for all \( \theta \in \Theta \) and thus, in particular, at \( \theta = \theta_{SB} \) implying that \( V(\theta_{SB}) = 0 \) and thus \( \theta^* = \theta_{SB} \). When \( xu > \gamma \), \( U(\theta) = xu - \gamma > 0 \) for all \( \theta \in \Theta \) and thus, in particular, at \( \theta = \theta_{SB} \), implying that \( V(\theta_{SB}) < 0 \) and thus by strict monotonicity and continuity of \( V(\theta) \) that \( \theta^* > \theta_{SB} \).

**Proof of Proposition 2:** I can restrict consideration to the case where with \( a = 0 \) lending would be too aggressive. Hence, if I substitute from \( V(\theta^*) = 0 \) into the binding participation constraint, I have a unique value \( \theta^* < \theta_{SB} \) satisfying the requirement

\[
\int_{\theta^*}^{\theta} \left\{ p(\theta) \left[ u - \frac{l}{p(\theta)} \right] - \left[ 1 - p(\theta) \right] \gamma \right\} f(\theta)d\theta = 0. \tag{11}
\]

Note that uniqueness follows immediately as from \( p(\theta) u - \left[ 1 - p(\theta) \right] \gamma - l < 0 \) the left-hand side is strictly increasing in \( \theta^* \). If we allow for \( a > 0 \), implying that \( \theta^* \) is determined by \( V(\theta^*) = 0 \) with \( V(\theta) := p(\theta) R - (l + a) \), then by analogy to (11)

\[
\int_{\theta^*}^{\theta} \left\{ p(\theta) \left[ u + a - \frac{l + a}{p(\theta)} \right] + \left[ 1 - p(\theta) \right] (a - \gamma) \right\} f(\theta)d\theta = 0. \tag{12}
\]

As long as \( \theta^* < \theta_{SB} \), the left-hand side of (12) is again strictly increasing in \( \theta^* \). Denoting the derivative by \( D > 0 \), then, from (12)

\[
\frac{d\theta^*}{da} = \frac{\int_{\theta^*}^{\theta} \left\{ p(\theta) - p(\theta^*) \right\} f(\theta)d\theta}{D} > 0. \tag{13}
\]

As the lender extracts all surplus, it is thus optimal to increase \( a \) further as long as still \( \theta^* < \theta_{SB} \) holds. Through setting \( a = \gamma \) and thereby ensuring that \( U(\theta) = 0 \) holds for all \( \theta \), this ultimately gives \( \theta^* = \theta_{SB} \).

**Proof of Proposition 3:** For \( xu > \gamma \), with \( c = 0 \) the household realises \( U(\theta) = xu - \gamma > 0 \) for all \( \theta \). Recall that the lender’s expected payoff is strictly increasing in \( R \). Consequently, it is always optimal for the lender to increase \( R \) as far as possible, that is until either (5) or \( R \leq u + \gamma + c \) becomes binding such that \( c = xu + \gamma \) and \( R = u + \gamma + c \). This implies \( U(\theta) = 0 \) for all \( \theta \in \Theta \) and thus \( \theta^* = \theta_{SB} \) where \( \theta_{SB} \) depends on \( c \) according to \( p(\theta_{SB}) u(1 + c) = \left[ 1 - p(\theta_{SB}) \right] (\gamma + c(1 - \beta)) \).

Suppose next that \( xu < \gamma \). I show that \( c = 0 \) is then uniquely optimal as, otherwise, \( \theta^* < \theta_{SB} \) would be even further pushed down. Denote now \( \theta^* (R, c) \) and \( \mu(\theta) := p(\theta) u - \left[ 1 - p(\theta) \right] \gamma \). Substituting the definition of \( \theta^* (R, c) \) into the borrower’s binding participation constraint (5), gives

\[
\int_{\theta^*(R,c)}^{\theta} \mu(\theta)f(\theta)d\theta = (R - \beta c) \int_{\theta^*(R,c)}^{\theta} \left\{ p(\theta) - p(\theta^* (R, c)) \right\} f(\theta)d\theta. \tag{14}
\]
Compare now some \((R, c)\) with \(c = 0\) and another contract \((R', c')\) with \(c' > 0\) and \(R' < R\), where in both cases \((14)\) is satisfied with equality. \(\theta^*(R, c') < \theta^*(R, c)\) follows then from \(R - \beta c > R' - \beta c'\) and \((14)\), together with \(\mu(\theta) < 0\) for all \(\theta < \theta_{SB}\) and the fact that \(\theta^*(R, c) < \theta_{SB}\) which in turn follows from Proposition 1 and as, by assumption, \(zu < \gamma\).

**Proof of Proposition 4:** Recall that for \(zu < \gamma\) the participation constraint \((5)\) is satisfied with equality. I now consider a marginal reduction in \(R\), which by the arguments in Proposition 1 leads to a marginal increase in \(\theta^*\). Given that \(\theta^* < \theta_{SB}\) holds under the (uncapped) monopoly offer, this strictly increases total surplus. To see that the household’s utility also strictly increases, note that \(U(\theta)\) is strictly decreasing in \(R\) for all \(\theta \in \Theta\) and that \(U(\theta) < 0\) holds for all \(\theta\) sufficiently close to \(\theta^* < \theta_{SB}\).

For \(zu > \gamma\) where \(\theta^* > \theta_{SB}\), a reduction in \(R\) leads to a further increase in \(\theta^*\) and thus reduces total surplus. Differentiating \(\int_0^\theta U(\theta)f(\theta)d\theta\) next w.r.t. \(R\), gives
\[
-\int_0^\theta p(\theta) \frac{f(\theta)}{1 - F(\theta)} d\theta - U(\theta_{SB}) \frac{d\theta^*}{dR},
\]
where after implicit differentiation of \(p(\theta^*)R - l = 0\) I can substitute \(d\theta^*/dR < 0\). The derivative \((15)\) reveals that the impact is in general ambiguous.

**Proof of Proposition 5:** If the repayment constraint was not binding previously as \(zu \leq \gamma\), then for the lender there is no need to impose the additional clause. If instead \(zu > \gamma\) and if the lender can now increase \(R\) further until the household’s participation constraint \((5)\) binds, then this is optimal and implies also that \(R > u + \gamma\). Consequently, \(U(\theta)\) becomes strictly decreasing, implying \(U(\theta^*) > 0\), which together with \(V(\theta^*) = 0\) finally implies that still \(\theta^* > \theta_{SB}\).

**Proof of Proposition 6:** Note first that by the argument from Proposition 1, the household’s participation constraint binds: \(\int_0^\theta U(\theta)f(\theta)d\theta = U_{\max}\). Recall also that \(\theta^*\) is strictly decreasing and continuous in \(R\). As long as \(\theta^* < \theta_{SB}\), \(\int_0^\theta U(\theta)f(\theta)d\theta\) is strictly decreasing in \(R\), which follows as \(U(\theta)\) is strictly decreasing in \(R\) for all \(\theta \in \Theta\) and as \(U(\theta) < 0\) holds for all \(\theta\) sufficiently close to \(\theta^*\).

It thus remains to show that a further (marginal) decrease of \(R\) at \(\theta^* = \theta_{SB}\) is also optimal. This follows immediately from the envelope theorem. Formally, at \(\theta^* = \theta_{SB}\)
\[
\frac{d}{dR} \left[ \int_0^{\theta^*} U(\theta)f(\theta)d\theta \right] = -\int_0^{\theta^*} p(\theta)f(\theta)d\theta - U(\theta_{SB}) \frac{d\theta^*}{dR},
\]
where I can substitute \(U(\theta_{SB}) = 0\).

**Proof of Proposition 7:** We focus on the case where \(zu < \gamma\). (Those with \(zu > \gamma\) and \(zu = \gamma\) then follow immediately.) As entrants make the offer characterised in \((10)\), the incumbent’s offer is the highest value at which
\[
\int_{\theta^*}^\theta U(\theta)p(\theta)f(\theta)ds \geq \max\left( U_{\zeta}^{\theta^*}, 0 \right)
\]
just holds with equality.\(^{22}\) For \(\zeta \leq \xi\), where the right-hand side of \((16)\) is zero, from Proposition 1 \(\theta_{\xi}^* < \theta_{SB}\). I argue next that \(\theta_{\xi}^* > \theta_{SB}\) must hold for all sufficiently high \(\xi\). For this note first that, as
\[^{21}\]\(I\) can substitute \(d\theta^*/dR < 0\) from implicitly differentiating the definition of \(\theta^*\), from which it holds that \(p(\theta^*)R - l = 0\).
\[^{22}\]As I am only interested in the case where the incumbent can make a sufficiently attractive offer, I can focus on the case where \((16)\) can be satisfied. Generally, as the incumbent always extracts positive profits, for high \(\xi\) this need not hold.

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argued in the main text, for $\xi \to \xi^*$, $R_\xi^* \to 1/p(\bar{\theta})$ such that the right-hand side of (16) converges to $u[x + p(\bar{\theta})] - [1 - p(\bar{\theta})] \gamma - L$. As for $\theta^* \leq \theta_{SB}$ the offer of the incumbent must be bounded from below by $1/p(\theta_{SB}) > 1/p(\bar{\theta})$, (16) cannot be satisfied for all sufficiently high $\xi$ unless $\theta^* > \theta_{SB}$.

The asserted threshold $\xi^*$ then exists if I can show that $\theta^*_\xi$ is strictly increasing in $\xi$, which holds if $R^*_\xi$ is strictly decreasing. I prove this by showing first that holding $R^*_\xi$ constant as $\xi$ is marginally increased, (16) would no longer be satisfied. Define the entrants’ offer as $U^E_\xi(\theta) := xu + p(\theta)(u - R^*_\xi) - [1 - p(\theta)] \gamma$ and the incumbent’s offer as $U^I_\xi(\theta) := U(\theta)$ for $\theta \geq \theta^*_\xi$ (evaluated at $R = R^*_\xi$) and $U^I_\xi(\theta) := 0$ for $\theta < \theta^*_\xi$. Next, define $\Delta_\xi(\theta) := U^E_\xi(\theta) - U^I_\xi(\theta)$, which is equal to $p(\theta)(R^*_\xi - R^2)$ for $\theta \geq \theta^*_\xi$ and equal to $U^I_\xi(\theta)$ for $\theta < \theta^*_\xi$. Note that $\Delta_\xi(\theta)$ is strictly increasing in $\theta$ and that $\int_{\theta^*_\xi}^{\theta_{SB}} \Delta_\xi(\theta)f_\xi(\theta)\,d\theta = 0$ holds from the binding constraint (16). As $\xi$ is increased while holding both contracts constant, (16) is no longer satisfied. Formally, this holds as $F_\xi$ satisfies strict First-Order Stochastic Dominance (as implied by MLRP). The assertion follows then after observing that also the true value of $R^2$ strictly decreases, which even further increases the right-hand side of (16).

Proof of Proposition 8: It is now sometimes helpful to make the dependency of the lender’s cutoff on the loan contract explicit by writing $\theta^*(R)$ and to denote the household’s utility by $U(\theta, R)$. Note now first that by offering $R^E = R^2 - \varepsilon$ for any $\varepsilon > 0$ entrants can always attract an approved household. An offer $R^2$ will in turn not be strictly preferred by a rejected household if

$$\int_{\theta}^{\theta^*(R^2)} U(\theta, R^2) \frac{f_\xi(\theta)}{F_\xi(\theta^*(R^2))} \,d\theta \leq 0. \quad (17)$$

Taken together, it is thus not possible for entrants to make an offer that is strictly preferred by an approved but not so by a rejected household, if and only if the incumbent’s offer satisfies

$$\int_{\theta}^{\theta^*(R^2)} U(\theta, R^2) \frac{f_\xi(\theta)}{F_\xi(\theta^*(R^2))} \,d\theta \leq 0. \quad (18)$$

I argue first that the left-hand side of (18) is strictly decreasing in $R^2$. To see this, note that a higher $R^2$ strictly reduces $U(\theta,R^2)$ for all $\theta \geq \theta^*(R^2)$. Moreover, as a higher $R^2$ also reduces $\theta^*(R^2)$, the assertion follows as the conditional probability of non-defaulting in case $\theta \geq \theta^*(R^2)$ is strictly increasing in $\theta^*(R^2)$. Note next that from $p(\bar{\theta}) = 0$ the left-hand side of (18) is surely strictly negative for all $\theta^*$ sufficiently close to $\bar{\theta}$ (and thus for all sufficiently high $R^2$). I must distinguish between different cases. In the first case, the left-hand side of (18) is strictly positive at $R^2 = 1/p(\bar{\theta})$, i.e., it holds that for given $\xi$

$$p_\xi \left[ u - \frac{L}{p(\bar{\theta})} \right] - (1-\xi) \gamma > 0. \quad (19)$$

In this case, there is a unique value of $R^2$ at which the left-hand side of (18) is just equal to zero. I denote this value for given $\xi$ by $R^2_{\xi}$, which clearly satisfies $\theta^*(R^2_{\xi}) > \theta_{SB}$.

Claim 1. If (19) does not hold, it is not possible for the incumbent to make an offer $R^2$ that, given the entrants’ optimal response, ensures that an approved household stays with the incumbent. If (19) holds, then $R^2 = R^2_{\xi}$ is the highest possible offer that will ensure that entrants cannot make a counteroffer that only attracts an approved household.

When does (19) hold?

Claim 2. There exists a threshold $\xi < \xi^* < \xi^*$ such that (19) holds if and only if $\xi > \xi^*$.

Proof. Recall first that $p_\xi$ is strictly increasing in $\xi$. Moreover, as $p_\xi \to p(\bar{\theta}) = 0$ for $\xi \to \xi^*$, the converse of (19) holds strictly for all sufficiently low $\xi$. Moreover, by definition of $\xi$ (19) holds at $\xi = \xi^*$. As the left-hand side of (19) is also continuous in $\xi$ (by continuity of $p_\xi$), there is thus a
unique value \( \tilde{\xi} < \tilde{\xi}' < \tilde{\xi}'' \) at which (19) holds with equality, while it holds strictly for all \( \tilde{\xi} > \tilde{\xi}' \) and the converse of it holds strictly for all \( \tilde{\xi} < \tilde{\xi}' \).

For all \( \tilde{\xi} \) satisfying (19), I next compare \( R_N^N \) with \( R_N^\circ \).

**Claim 3.** There exists a threshold \( \tilde{\xi} < \tilde{\xi}' < \tilde{\xi}'' \) such that \( R_N^N = R_N^\circ \) holds at \( \tilde{\xi} = \tilde{\xi}' \), while \( R_N^N < R_N^\circ \) holds for \( \tilde{\xi}' < \tilde{\xi} < \tilde{\xi}'' \) and \( R_N^N > R_N^\circ \) holds for \( \tilde{\xi} > \tilde{\xi}'' \).

**Proof.** Note first that \( R_N^N = R_N^\circ \) holds if and only if \( p[0^\prime(N^N_N)] = p_L \), while \( R_N^N < R_N^\circ \) \( (R_N^N > R_N^\circ) \) holds if and only if \( p[0^\prime(N^N_N)] > p_L \) \( (p[0^\prime(N^N_N)] < p_L) \). By definition of \( \xi \), \( R_N^\circ \) is strictly decreasing in \( \xi \) (and continuous by continuity of \( p_L \)). I argue next that \( R_N^N \) is strictly increasing in \( \xi \). To see this, note first that for some fixed \( R' \) and thus fixed \( 0^\prime(R') \) the term in (18) is strictly increasing in \( \xi \). This follows from the MLRP of \( F_L \) and as \( U(\theta,R') \) is strictly increasing in \( \theta \). Given that (18) is also strictly decreasing in \( R' \) (holding \( \xi \) fixed this time), we thus have that a higher \( \xi \) must lead to a higher \( R_N^N \). (Recall that for \( R' = R_N^N \) (18) is satisfied with equality.) That \( R_N^\circ \) is also continuous in \( \xi \) follows next from continuity of all \( f_L(\theta) \).

Hence, to conclude the proof of Claim 3 it remains to show that \( p[0^\prime(N^N_N)] > p_L \) holds for all \( \xi \) close to \( \tilde{\xi} \) while \( p[0^\prime(N^N_N)] < p_L \) holds for all \( \xi \) close to \( \tilde{\xi}'' \). This follows as by definition of \( R_N^N \) for \( \xi \to \tilde{\xi} \) that \( 0^\prime(R^N_N) \to 0 \), while for \( \xi \to \tilde{\xi}'' \), \( p_L - p(\theta) > p[0^\prime(N^N_N)] \). QED

From Claim 3, for all \( \tilde{\xi}' < \tilde{\xi} < \tilde{\xi}'' \) the unique equilibrium offer of the incumbent is \( R' = R_N^\circ \). Given that \( R_N^N < R_N^\circ \) holds in this case, entrants can also not profitably target only the rejected household.

To complete the proof of Proposition 8, it thus remains to consider the case where \( \tilde{\xi} > \tilde{\xi}'' \). There, the incumbent can no longer offer \( R_N^N \) and make strictly positive profits. In this case, entrants could counter with an offer \( R^E = R_N^\circ - \varepsilon \) for some \( \varepsilon > 0 \), which would attract all households and, given that \( \tilde{\xi}'' > \tilde{\xi} \), would allow entrants to break even. Hence, for all \( \tilde{\xi} > \tilde{\xi}'' \) the incumbent’s offer cannot be higher than \( R_N^\circ \). Given that \( R_N^N < R_N^\circ \) it is also not possible in this case for entrants to poach only the approved household, implying that the unique equilibrium offer for the incumbent is now \( R' = R_N^\circ \).

For the entrants’ offer there are now two subcases to distinguish (namely, Cases IIIa and IIIb). If it is feasible for entrants to make an acceptable offer to a rejected household, then competition ensures that

\[
R^E_1 = \int_1^\infty \frac{f_L(\theta)}{F_L[0^\prime(R_N^\circ)]} p(\theta) d\theta, \tag{20}
\]

where I substituted \( R' = R_N^\circ \). Defining

\[
p_L^R := \int_1^\infty \frac{f_L(\theta)}{F_L[0^\prime(R_N^\circ)]} p(\theta) d\theta,
\]

the entrants’ offer from (20) is only acceptable to a rejected household.

---

23 To be precise, partial differentiation of (13) shows that a sufficient condition for this to hold is that \( f_L(\theta)/F_L(\theta) \) is strictly increasing in \( \xi \) for all interior \( \theta \). This holds in turn if, for two given \( \xi_1 > \xi_2 > f_L(\theta)/F_L(\theta) > f_L(\theta)/F_L(\theta) \). To see that this is finally implied by MLRP, note that from \( f_L(\theta) > 0 \) for all \( \theta \in \Theta \), \( f_L(\theta)/F_L(\theta) \to f_L(\theta)/f_L(\theta) \) as \( \theta \to 0 \), while clearly for \( \theta = 0 \) MLRP requires that \( f_L(\theta)/f_L(\theta) > f_L(\theta)/F_L(\theta) \). Note next that at any \( \theta \) where \( f_L(\theta)/f_L(\theta) = f_L(\theta)/F_L(\theta) \) holds with equality, the derivative of the right-hand side is just zero (the sign is given by \( F_L(\theta)f_L(\theta) - f_L(\theta)f_L(\theta) \)), while the derivative of the left-hand side is strictly positive by MLRP. (I use differentiability only for convenience at this point.) Hence, I have shown that for all \( \theta > 0 \) the graph of \( f_L(\theta)/f_L(\theta) \) must always lie above that of \( F_L(\theta)/F_L(\theta) \).
\[ p^R_n u + (1 - p^R_n) \gamma \geq l. \] (21)

Consequently, whenever (21) holds strictly, a household that was previously rejected by the incumbent receives a loan from an entrant. If the converse of (21) holds strictly, then a rejected household does not take out a loan.

I finally ask when condition (21) holds. Observe first that the left-hand side of (21) is strictly increasing in \( \xi \). To see this, recall that \( R^o_\xi \) is strictly decreasing such that \( 0^+ (R^o_\xi) \) is strictly increasing. Consequently, together with the MLRP of \( F_\xi \) the conditional probability \( p^R_\xi \) is strictly increasing in \( \xi \) (footnote 22). By continuity of all \( f_\xi \) it is also continuous in \( \xi \). Next, at \( \xi = \xi'' \) by definition \( R^i = R^N = R^o \), implying that \( R^i < R^o \) would not attract the rejected household and thus (21) cannot hold. Hence, a unique threshold \( \xi = \xi''' \) with \( \xi'' < \xi''' < \xi'' \) at which (21) holds with equality exists if (21) holds for all sufficiently high \( \xi \). This, however, follows as for \( \xi \to \xi', p_{\xi} \to p(\bar{\theta}) \) such that \( R^o_{\xi} \to 1/p(\bar{\theta}) \) and \( \theta^* \to \bar{\theta} \).

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Submitted: 26 May 2006
Accepted: 22 February 2008

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