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Mutual excitation in eurozone sovereign CDS

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Mutual Excitation in Eurozone Sovereign CDS

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Non-Technical Summary

Past crises as well as the recent global financial crisis and the Eurozone sovereign debt crisis have highlighted the fact that extreme events, such as jumps in asset prices, as well as the events that are relevant for the assessment of the default probability of a given debt instrument, including credit rating changes, tend to occur not in isolation but in clusters, both in time as well as across debt issuers, whether those issuers are countries (sovereigns) or firms (corporates).

Credit default swaps (CDS) are derivative instruments which provide insurance against the risk of default by a debt issuer. As such, their spreads provide key information about the cost of such insurance and, consequently, the market's view about the arrival rate of default events. Therefore, they are ideal instruments to investigate default probability itself and cross-excitation both in time and across debt issuers.

This paper proposes and investigates a model designed around a specific feedback mechanism to capture the dependencies among the risk of the different Eurozone countries. The feedback element introduces a propagation dimension from one shock to the next and one sovereign to the next that is not present in common factor models. Motivated by the current Eurozone sovereign debt crisis, we study whether CDS contracts that insure sovereign European debt are priced consistently with the predictions of the model, including the clustering that is apparent in the underlying credit-relevant events.

The analysis of the data reveals i) that Eurozone CDS rates, and hence default intensities, exhibit clusters in time and in space; ii) transmission of shocks that is rapid but not instantaneous; and iii) asymmetry in the extent to which a shock in one sovereign affects the others.

The broad empirical implications of the model are reassuring for its plausibility and realism: Ireland, Portugal, Spain and Italy are the main countries affected by events in Greece, with results varying in strength across the countries, and to a lesser extent France and Germany.

This finding and the analysis surrounding it has important practical implications. Using the estimated model, we perform an impulse-response analysis, a commonly used tool in macroeconometrics, by shocking the system and examining how shocks affecting CDS prices transmit throughout the Eurozone.

We use the results to assess the impact of a capital injection not just on a single country, but also its potential reduction of the spillover to other countries. By inferring the patterns of excitation flowing from one country to another, we can identify the countries where a policy intervention would be most effective, at least with the objective of lowering CDS rates across the board in the Eurozone. Because CDS rates are important inputs to many financial decisions, and they are heavily watched by market participants, it is not unreasonable to view lowering

them as an objective in itself towards restoring confidence in the stability of the Eurozone. In particular, the asymmetry in the way contagion flows means that there is potential for intervention in one CDS market to be more successful than in others. We find for example that a policy intervention in Ireland resulting in an exogenous lowering of the CDS rate there would have been much less effective than a similar policy intervention in Greece.

Mutual Excitation in Eurozone Sovereign CDS

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Abstract

We study self- and cross-excitation of shocks in the Eurozone sovereign CDS market. We adopt a multivariate setting with credit default intensities driven by mutually exciting jump processes, to capture the salient features observed in the data, in particular, the clustering of high default probabilities both in time (over days) and in space (across countries). The feedback between jump events and the intensity of these jumps is the key element of the model. We derive closed-form formulae for CDS prices, and estimate the model by matching theoretical prices to their empirical counterparts. We find evidence of self-excitation and asymmetric cross-excitation. Using impulse-response analysis, we assess the impact of shocks and a potential policy intervention not just on a single country under scrutiny but also, through the effect on cross-excitation risk which generates systemic sovereign risk, on other interconnected countries.

Key words and phrases: CDS; Sovereign risk; Systemic risk; Jumps; Feedback; Hawkes processes; Mutually exciting processes; Impulse-response.

JEL classification: C13; G12.

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1. Introduction

It is well known that extreme events, such as jumps in asset prices, as well as the events that are relevant for the assessment of the default probability of a given debt instrument, including credit rating changes, tend to occur not in isolation but in clusters, both in time as well as across debt issuers, whether those issuers are countries (sovereigns) or firms (corporates). Credit Default Swaps (CDS) are derivative instruments which provide insurance against the risk of default by a debt issuer. As such, their spreads provide key information about the cost of such insurance, and consequently the market's view about the arrival rate of default events.

There is a vast literature on the predictive content embedded in credit spreads (see e.g., Gilchrist and Zakrajšek (2012)) and on quantifying empirically sovereign spreads. Different factors have been identified as explaining these spreads, and the extent of their commonality: see Edwards (1984), Kamin and Von Kleist (1999), Eichengreen and Mody (2000), Duffie et al. (2003), Zhang (2003), Geyer et al. (2004), Pan and Singleton (2008), Remolona et al. (2008), Longstaff et al. (2011) and Ang and Longstaff (2011). Importantly, Ang and Longstaff (2011) have estimated the systemic and idiosyncratic components of sovereign credit spreads embedded in the different CDS spreads, contrasting the systemic risk of states within the U.S. to that of countries within Europe. They find less systemic risk among U.S. states than among European countries, and that the systemic sovereign risk in both cases is more related to financial market variables than to macroeconomic fundamentals.

This paper contributes to this literature by proposing and investigating a different model designed around a specific feedback mechanism to capture the dependencies among the risk of the different Eurozone countries. The feedback element introduces a propagation dimension from one shock to the next and one sovereign to the next that is not present in common factor models. Motivated by the current Eurozone sovereign debt crisis, we study whether CDS contracts that insure sovereign European debt are priced consistently with the predictions of the model, including the clustering that is apparent in the underlying credit-relevant events.¹ The analysis of the data we conduct below reveals that Eurozone CDS rates, and hence default intensities, exhibit clusters in time and in space; transmission of shocks that is rapid but not instantaneous; and asymmetry in the extent to which a shock in one sovereign affects the others. To capture these phenomena, we construct a model that is designed to allow for clusters, but does not impose them, makes the transmission probabilistic rather than certain and instantaneous, allows for asymmetry of transmission, and assess the CDS prices through the viewpoint of the model. All these features are controlled in the model by easily interpretable parameters. By estimating the model using CDS prices, we obtain estimates of the implied values of the parameters controlling these different aspects, and can test for their presence and significance in the data.

Because volatility alone cannot plausibly reproduce the large events that make default possible, jumps are an essential ingredient of the model. But in a typical jump model with arrival rates calibrated to historical data, jumps are inherently rare, occurring perhaps once every few years on average. In addition, because they are based on Lévy processes, such as the Poisson process, the arrival of jumps in a typical model is independent from one period to the next. Hence observing patterns of multiple jumps in close succession over hours or days and across different European markets —that is, observing time and space

¹It should be noted from the onset that while the observed occurrence of default events provides information on the physical probability measure (\mathbb{P}), CDS contracts provide measurements of their rate of arrival under an equivalent martingale measure (\mathbb{Q}). Our analysis in this paper is solely based on the CDS-implied measurements, hence has no implications for the risk premia linking the two.

clusters— would be very unlikely under a standard jump model. This is at odds with what we observe in the data.

To capture these phenomena, we need to leave the class of Lévy processes as driving processes for the credit default intensities. Lévy processes produce independent increments, and as such do not allow for the kind of dependencies along the time series dimension that we are after. The mechanism we employ to generate these clusters in our model is that of *self-* and *cross-excitation*, which constitute *mutual excitation* in time and space, over days and across countries. The general thrust of the model is that a shock in one market feeds back into the shock processes themselves by increasing the probability of successive shocks not only in the affected country but also in other countries. Reasons to expect shocks to raise the arrival rate of future shocks include linkages such as debt of the first country held by banking institutions in the second, flights to quality and self-feeding panics triggered by margin and collateral demands, and ex-post partial mutualization of the risk or outright bailout at the Eurozone level.

The key ingredient we employ to drive the credit default intensity and build our multivariate credit risk framework are Hawkes (1971b) processes. These processes are related to, but different from, doubly stochastic Poisson processes, which are usually adopted in credit modeling, see Duffie and Singleton (2003). In a Hawkes process, the sample paths of the credit default intensities ($\lambda_{i,t}$) are affected by the sample paths of the credit events ($N_{i,t}$) whose intensities are the $\lambda_{i,t}$'s themselves. This feedback element of a mutually exciting process model is the key difference, which we discuss in detail in Section 3.1 below, with the existing literature on CDS contracts and sovereign crises (see the important contributions by Pan and Singleton (2008), Longstaff et al. (2011), and Ang and Longstaff (2011)).²

The estimation methodology we employ is based on matching the theoretical CDS pricing formulae, which we derive in closed-form for our model, with the observed market prices of these contracts. Aït-Sahalia et al. (2010) employed GMM based on the moments of the stock returns in a Hawkes-based model to search for evidence of contagion in stock markets around the world. Here, by contrast, we compute the CDS rates implied by the model and match those to market data. We look for, and find evidence of, self-excitation in CDS rates (from one country to itself) and cross-excitation (from one country to another) that is asymmetric during the Eurozone crisis.

This finding and the analysis surrounding it has important practical implications. Using the estimated model, we perform an impulse-response analysis, a commonly used tool in macroeconometrics, by shocking the system and examining how shocks affecting CDS prices transmit throughout the Eurozone. The feedback element of the model is what makes this analysis possible. We use the results to identify systemically important countries (SICs) within the Eurozone and assess the impact of a capital injection not just on a single country, but also its potential reduction of the spillover to other countries. By inferring the patterns of excitation flowing from one country to another, we can identify the countries where a policy intervention would be most effective, at least with the objective of lowering CDS rates across the board in the Eurozone. Because CDS rates are important inputs of many financial decisions, and are heavily watched by market participants, it is not unreasonable to view lowering them as an objective in itself towards restoring confidence in the stability of the Eurozone.

The rest of this paper is organized as follows: In Section 2, we discuss the data used to conduct our analysis. In Section 3, we present the model, discuss alternatives and some of its properties and derive in

²In work that this paper is also related to, Errais et al. (2010) study the valuation of large portfolios of collateralized debt obligations, with a model in which individual defaults occur with probability one if an underlying Hawkes process exhibits a jump. Our focus by contrast is on the econometrics of CDS rates, within a multivariate mutually exciting setting in which jumps are latent (rather than observable as credit defaults), and on their implications for systemic sovereign risk.

closed-form the CDS pricing formulae for our model. In Section 4, we outline the estimation methodology and present our empirical results. Section 5 contains the impulse-response analysis. Conclusions are in Section 6.

2. The Data

The direct assessment of sovereign debt risk provided by the CDS market constitutes a rich and interesting data source on credit contagion and on related phenomena such as interdependence, spillovers and transmission. A credit default swap (CDS) is a derivative contract providing insurance against the default of a particular entity (see Duffie (1999)). The term “swap” is used to indicate that the agreement specifies that in the case of a “credit event” of the entity³, the buyer of the CDS protection receives a pre-specified contingent amount, usually the face value of the underlying bond minus any recovered amount, while the seller of the CDS protection receives a then-current market value, typically the market value of the defaulted bond. Upon occurrence of a credit event, payment may be through physical or cash settlement. In return for default protection, the CDS protection buyer pays a premium (the CDS spread, CDS rate or CDS price) to the CDS protection seller until maturity of the contract or until a credit event occurs, whichever is earlier. We consider Eurozone sovereign CDS. For sovereign CDS, the bond underlying the contract (the reference obligation) is, usually, a senior external or international debt.

Naturally, sovereign credit ratings are an important determinant of credit spreads. Figure 1 plots the history of Standard and Poor’s (S&P) sovereign ratings for Greece, Portugal and Spain over the period 2000-2011. The figure shows that there are long periods where hardly any rating changes are recorded, followed by periods in which ratings changed in close succession. Figure 2 summarizes some of the key events in the evolution of the Eurozone sovereign debt crisis to date, and shows their impact on the sovereign CDS spreads of several European countries. The (primarily US-centered) banking or credit crisis, which anticipated the Eurozone sovereign debt crisis, reached its apex in the Fall of 2008, with the default of Lehman Brothers in September and its subsequent systemic effects on the financial system.⁴ By the summer of 2012, however the Eurozone sovereign crisis was widespread, as seen in Figure 3, Panel I.

An interesting feature of the data, which our model will seek to replicate, is that clusters of large moves often do not occur exactly simultaneously across countries: it may take some time (of the order of days) for the transmission beyond the initial shock, if at all, to fully realize. To illustrate, the second panel of Figure 3 and the first panel of Figure 4 plot two such episodes. The second panel of Figure 3 represents the Irish and Austrian banking crisis getting slowly reflected in the CDS spread of France, and many other countries. The first panel of Figure 4 shows the downgrade of Portugal on July 5, 2011 as well as the continuing fears of a Greek default, triggering a sell-off in Spanish and Italian sovereign

³According to the International Swap and Derivatives Association (ISDA), “credit events” in the case of corporate credit default swaps include: bankruptcy, obligation acceleration, obligation default, and restructuring. “Credit events” in the case of sovereign credit default swaps means technical (rather than pure / true) default; that is, failure to pay on the coupons or principals, debt restructuring, repudiation and moratorium; see International Swaps and Derivatives Association, Inc. (2003).

⁴Among other effects, Ireland nationalized the Anglo Irish bank in early 2009. As seen in Figure 2, the CDS spread of Ireland increased in early 2009, rapidly followed by increases in the CDS spread of other Eurozone countries. In late 2009, it was revealed that misleading accounting practices had taken place in Greece. The CDS spread of Greece increased dramatically, followed by a series of large upward moves in the sovereign CDS spreads of many other countries: see the first panel of Figure 3. After a series of rating downgrades, and bailouts of Greece (Spring 2010), Ireland (Fall 2010) and Portugal (Spring 2011), the instability reached new heights after the summer of 2011, when both Italy and Spain were downgraded. This spread instability to other European countries, including France and later on Germany, and to many European banks that held large portfolios of Eurozone sovereign debt.

bonds. The figure exemplifies that a large move (jump) in one country (Portugal) is often transmitted to other countries (Italy and Spain), which later on themselves become the sources of further transmissions of shocks. Another interesting feature is that cross-country excitation may be asymmetric (with one country typically exciting other countries without much of the reverse effect). The second panel of Figure 4 provides an example. As described above, in June and early July 2011, Portugal and Greece were at the center of market turmoil, but in July 2011, Italy was at the center of the Eurozone debt market turmoil. However, while increases of Portuguese CDS spreads typically triggered an increase of Italian CDS spreads, the reverse was not (always) observed: for example, on July 8 and 9, 2011, Italian CDS spreads increased strongly but without transmitting to Portuguese CDS spreads.

We restrict attention to CDS contracts with maturities of 5 and 10 years, which are the most frequently traded CDS contracts. Data on CDS spreads are obtained from Datastream, which collects CDS market quotes from industry sources.⁵ Specifically, we consider the daily premia of the dollar-denominated CDS for 7 European countries in the Eurozone; five peripheral countries: Greece (GR), Ireland (IE), Italy (IT), Portugal (PT) and Spain (ES); and two core countries: France (FR) and Germany (DE). The sample period considered is January 1, 2007, until August 31, 2012. Exceptions are Portugal, for which data are unavailable over the period January 1, 2007, until December 14, 2007, and Germany and 10-year CDS of France for which data are unavailable over the period January 1, 2007, until December 20, 2007. Furthermore, for Greece our sample period stops on March 8, 2012, because of the default of Greek debt. In these cases, the lack of availability of the data dictates the subsamples of January 1, 2007 to August, 31, 2012 that we consider.

Table 1 reports summary statistics for the daily sovereign CDS spreads that we analyze, with a maturity of 5 and 10 years, respectively. All CDS spreads are denominated in basis points (bps) and are, therefore, free of units of account. We observe a wide range of prices within the sample. For the 5-year sovereign CDS contracts, the average change of CDS spreads is as low as 0.05bps in the case of Germany, and as high as 27.3bps in the case of Greece. The standard deviation of CDS spread changes highlights further differences: Germany exhibits a daily volatility of spread changes equal to 2.2bps versus Portugal's 22.6bps, and the highest standard deviation corresponds to the contract on Greece, 309.2bps. Skewness is negative, except for Greece and Italy. The 10-year CDS spreads exhibit similar characteristics. For all the CDS contracts, the excess kurtosis is substantially larger than for a Gaussian distribution, consistently with the presence of jumps. The kurtosis of Greece is striking, 181.3; Greece effectively defaulted in the sample period considered. The kurtosis of Ireland, Italy and Portugal is more than double the kurtosis of Germany. The (raw) CDS spreads obtained from Datastream, generically denoted by \bar{s} , are annual-based but paid with the frequency of the coupon bond underlying the contract. For sovereign and corporate bonds in the Eurozone, coupon payment is semi-annual, so upon transforming \bar{s} into $s = 2\log(1 + \bar{s}/2)$, meaning that $\bar{s} = 2[\sqrt{\exp(s)} - 1]$, we obtain continuously compounded and annual-based CDS rates s .

In addition to CDS spread data, our analysis requires interest rate data or, as in our pricing formulae derived below, zero-coupon bond prices. The time t prices of the zero-coupon bonds with maturity T , denoted by $D(t, T)$, are calculated using a standard cubic spline interpolation algorithm (see Longstaff et al. (2005)) from one, three, six, and twelve-month LIBOR rates and two, three, five, seven and ten year Euro swap rates. We collect the LIBOR and Euro swap rate data from Bloomberg. In the (raw) data, interest rates are based on annual compounding. The procedure that determines the zero-coupon bond prices takes this into account and computes directly $D(t, T)$. Overall, looking at a long sample

⁵Until September 30, 2010, the prime industry source is CMA; after September 30, 2010, this is Thomson Reuters.

period reported in the first panel of Figure 5, which shows CDS spread changes of 7 European countries, it is apparent that large CDS spread changes happen in close succession over a few days or weeks, like earthquake aftershocks. It is apparent that large moves cluster both in time (for example, May, 2010, and July-September, 2011) and in space (across different countries). We next turn to the description of the model we employ to capture these phenomena.

3. The Model

We introduce a multivariate credit risk model in which credit default intensities are driven by mutually exciting jump processes (or Hawkes processes, after Hawkes (1971b), see also Hawkes (1971a), Hawkes and Oakes (1974) and Oakes (1975)). The model is best understood by contrasting it with a Poisson process.⁶ Consider $i = 1, \dots, m$ countries. In one of the main examples of a Lévy process, the Poisson process, the probabilities of zero and one jumps in country i over an interval of time of length Δt are

$$\begin{cases} \mathbb{P}[N_{i,t+\Delta t} - N_{i,t} = 0 | \mathcal{F}_t] = 1 - \lambda_i \Delta t + o(\Delta t) \\ \mathbb{P}[N_{i,t+\Delta t} - N_{i,t} = 1 | \mathcal{F}_t] = \lambda_i \Delta t + o(\Delta t) \end{cases} \quad (1)$$

and jump arrivals are independent. The parameter λ_i is the jump intensity, or arrival rate: the higher λ_i , the more likely jumps are to occur.

By contrast, in a mutually exciting process, we have

$$\begin{cases} \mathbb{P}[N_{i,t+\Delta t} - N_{i,t} = 0 | \mathcal{F}_t] = 1 - \lambda_{i,t} \Delta t + o(\Delta t) \\ \mathbb{P}[N_{i,t+\Delta t} - N_{i,t} = 1 | \mathcal{F}_t] = \lambda_{i,t} \Delta t + o(\Delta t) \end{cases} \quad (2)$$

and that country's jump intensity $\lambda_{i,t}$ is stochastic. It will increase in response to recent jumps, that is to changes in that country's and the other countries' jumps themselves. We specify the dynamics of the default intensity as

$$d\lambda_{i,t} = \alpha_i(\lambda_{i,\infty} - \lambda_{i,t})dt + \sum_{j=1}^m \beta_{i,j} dN_{j,t}, \quad i = 1, \dots, m. \quad (3)$$

The (joint) pair (N_t, λ_t) is a Markov process. The jump intensities are stationary Markov processes with $\lambda_{i,t}$ jumping up by $\beta_{i,j}$ whenever a shock in country j occurs, and then mean-reverting back towards $\lambda_{i,\infty}$. Since the jumps' intensities rise in response to the jumps themselves, a feedback element is introduced since in the equation above $\lambda_{i,t}$ increases whenever a jump ($dN_{j,t} = 1$) occurs in one country (j). The feedback comes from the fact that these jump processes $N_{j,t}$'s which increase the jump intensities $\lambda_{i,t}$'s, are the same ones with the $\lambda_{j,t}$'s as their intensities. The increase in each country's jump intensity depends on the past jumps in its country ($\beta_{i,i}$) and in the other countries ($\beta_{i,j}$ for $i \neq j$). And, unlike a variance-covariance matrix, there is no reason to expect that $\beta_{i,j} = \beta_{j,i}$, hence the asymmetry we mentioned above. In order to achieve stationarity, sufficiently strong mean reversion is necessary leading to parameter restrictions (see Section 3.3 below).

⁶A more formal description of the model is postponed until Sections 3.2-3.3 below.

3.1. Comparison with a Common Factor Structure

The framework we employ differs from the Duffie and Singleton (2003), Section 10.7, framework with joint default events, adopted recently by Ang and Longstaff (2011), in important ways. In a doubly stochastic Poisson model, sovereign-specific (idiosyncratic) Poissonian default processes are driven by sovereign-specific intensity processes and a common Poissonian systemic shock process driven by a common intensity process. While these intensity processes are stochastic in their framework and possibly correlated cross-sectionally, they are not affected by the default processes nor by the systemic shock processes: the paths of the intensity processes are given before the Poisson processes are considered; see e.g., Karr (1991).

Using the notation of factor models, consider the model

$$\lambda_{i,t} = a_i + b_i F_t + \varepsilon_{i,t}, \quad (4)$$

with F_t denoting a common factor, $\varepsilon_{i,t}$ the idiosyncratic shock, b_i the factor loading and a_i a constant. The model (4) would have implications that can be made to be quite similar to those of the mutually exciting model: jump intensities for all $i = 1, \dots, m$ would increase together when F_t increases, and if F_t is made to be mean-reverting then so would the $\lambda_{i,t}$'s. The jump intensities would exhibit sensibly the same autocorrelogram pattern in both cases, so the autocovariance of the $\lambda_{i,t}$'s would not be the right moment function to distinguish these two models.

However, the mutually exciting framework significantly departs from the common factor approach in that there is a feedback effect from the jump processes $\{N_{i,t}\}$ to the jump intensities $\{\lambda_{i,t}\}$ and back: it is the jumps themselves that lead to larger jump intensities, which in turn lead to a greater likelihood of further jumps. This feedback effect is absent in the doubly stochastic Poisson framework or the common factor example above. That is, the mutually exciting model predicts that we should observe increases in jump intensities that coincide with the arrival of the jumps themselves, whereas the common factor model does not link the two, so it is the contemporaneous correlation between these two quantities, increments of the jump process and increments of the jump intensities, that would effectively distinguish the two models. Given that these quantities are latent, there would of course be practical difficulties in implementing such a test, especially without high frequency observations, which are not readily available. Further analysis along those lines if and when such data becomes available would certainly be interesting.

While it is difficult to isolate econometrically, the feedback relationship between jumps $\{N_{i,t}\}$ and their intensities $\{\lambda_{i,t}\}$ is likely to capture real phenomena. For example, the jumps themselves very often carry information, are heavily reported in the press, and represent news events that can trigger further jumps. Panics often start that way, and once started, the avalanche can be self-feeding (lines of credit dry up, margin and other collateral demands increase, liquidity shocks occur, etc.) and contagious (if only because risk capital withdraws everywhere as the risk appetite of investors disappears). In that sense, mutual excitation is a natural model for capturing systemic risk that is induced by the functioning of the financial markets themselves, perhaps more so than to capture the systemic risk that arises because of realizations of common macroeconomic factors that affect all the countries together.

A further difference between the different models is that our intensities jump—every time one of the $N_{j,t}$'s jumps—whereas in (4) they evolve continuously as long as the common factor and idiosyncratic shocks do. As a result, jump intensities in our model can experience rapid and sharp increases in times of crisis, which are difficult to reproduce in a continuous model where the buildup is slow and gradual (intensities in the model of Ang and Longstaff (2011) are continuous). Consequently we are able to fit with

the same parameters both the quiet period that preceded the Eurozone crisis, where CDS spreads were very low, and the crisis period where they experienced very quick changes. Although adding a continuous component to the jump intensities would be quite feasible in our model, doing so for the episode we are attempting to capture does not appear to be advantageous relative to the econometric cost imposed by the additional parameters. As we will see below (see in particular Figure 6), the evidence in the Eurozone credit crisis points strongly towards large jumps in intensities.

Recent studies document the role of global factors, in particular VIX, in explaining CDS spreads (Longstaff et al. (2011)). To compare the implications of our model to a common factor model with VIX as common factor, we first compute the correlation between changes in VIX and changes in CDS spreads that exceed the 90-th percentile, which should resemble jumps in CDS spreads. These correlations, which are expected to be large if the two models would be similar, are relatively low: for example, 0.15 for Germany, 0.12 for Ireland, 0.04 for Greece and 0.10 for Portugal. To compare the implications not just for the dynamics of the state variables but also for the CDS spreads, we note that the positive autocorrelation induced by the mutually exciting framework generates periods with positive autocorrelation in CDS spreads. We therefore calculate the (first order) autocorrelation of the large CDS spread changes, using a rolling window of 60 days, and compare it to the autocorrelation pattern of VIX. We find that in some periods the autocorrelation in large CDS spread changes is large, and in the remainder of the sample it is virtually zero, as predicted by our model. By contrast, the autocorrelation of VIX is typically negative and different from the pattern observed for large CDS spread changes.⁷

As we will describe below, one important application we have in mind for the model consists in identifying how shocks transmit in the CDS market from one sovereign to the other, in addition to itself. The feedback mechanism we propose is ideally suited for analyzing this transmission, generating a graph for the pathways that the excitation takes, as well as analyzing the impulse responses to shocking the system, either through an exogenous deepening of the crisis or a policy intervention designed to alleviate it (see Section 5). Such an impulse-response analysis inherently relies on the feedback mechanism that is present in our credit risk model with mutual excitation and is neither applicable nor sensible within the context of a doubly stochastic Poisson model or common factor model.

Note finally that in our mutually exciting case, idiosyncratic and systemic shocks are simultaneously modeled through the point processes $\{N_{i,t}\}$. There is no need to consider separate processes for the two effects, idiosyncratic and systemic, which has the advantage of improving the parsimony of the model. Indeed, in our model, jumps in one of the countries' point processes do not automatically lead to jumps in the other point processes, or to further jumps in the affected country. Through the feedback mechanism, they only lead to an increase in the jump intensities, hence the arrival rate of further jumps, the degree of which is determined by the matrix of β coefficients. The two effects, idiosyncratic and systemic, are therefore captured simultaneously.

Before specializing to our model with mutual excitation in Section 3.3, we first describe in Section 3.2 the basic components of a general default intensity-based CDS pricing model; in other words, how the model for the jump intensities will translate into CDS model prices that can then be matched to the CDS data.

⁷Averaged over the full sample, this autocorrelation is, for example, 0.34 for Greece, 0.04 for Italy, and -0.15 for VIX.

3.2. CDS Pricing

Central to many models for CDS spreads is the concept of credit default intensity $\lambda_{i,t}$ (see e.g., Duffie and Singleton (2003)). It is the intensity with which credit events occur in country $i = 1, \dots, m$. Upon the occurrence of a credit event⁸ in country i , meaning that its associated counting process, denoted by $N_{i,t}$, jumps by 1, credit models assume that there is a probability, denoted here by $0 < \gamma_i \leq 1$, of going into default. Let τ_i be the time-of-default. Then, the \mathcal{F}_t -conditional probability of survival to a future time T is given by

$$p(t, T) = \mathbb{E} [\mathbb{P} [\tau_i > T | \mathcal{F}_T] | \mathcal{F}_t] = \mathbb{E} [(1 - \gamma_i)^{N_{i,T} - N_{i,t}} | \mathcal{F}_t], \quad T \geq t. \quad (5)$$

With the convention $0^0 = 1$ and $0^c = 0$, $c > 0$, the expressions in (5) are also valid when $\gamma = 1$. Then

$$p(t, T) = \mathbb{E} [\mathbb{P} [\tau_i > T | \mathcal{F}_T] | \mathcal{F}_t] = \mathbb{P} [N_{i,T} - N_{i,t} = 0 | \mathcal{F}_t]. \quad (6)$$

In the typical doubly stochastic Poisson model, and with $\gamma = 1$, conditional on the path of the intensity processes, given the properties of the Poisson process,

$$\mathbb{P} [\tau_i > T | \{\lambda_{i,s} : t \leq s \leq T\}] = \mathbb{P} [N_{i,T} - N_{i,t} = 0 | \{\lambda_{i,s} : t \leq s \leq T\}] = \exp \left(- \int_t^T \lambda_{i,s} ds \right), \quad (7)$$

hence,

$$p(t, T) = \mathbb{E} [\mathbb{P} [\tau_i > T | \{\lambda_{i,s} : t \leq s \leq T\}] | \mathcal{F}_t] = \mathbb{E} \left[\exp \left(- \int_t^T \lambda_{i,s} ds \right) | \mathcal{F}_t \right], \quad (8)$$

which forms the basis of many credit risk pricing approaches; see Sections 3.4-3.5 in Duffie and Singleton (2003). More generally, with $0 < \gamma \leq 1$,

$$\begin{aligned} \mathbb{P} [\tau_i > T | \{\lambda_{i,s} : t \leq s \leq T\}] &= \mathbb{E} [(1 - \gamma_i)^{N_{i,T} - N_{i,t}} | \{\lambda_{i,s} : t \leq s \leq T\}] \\ &= \sum_{n=0}^{\infty} \frac{1}{n!} \exp \left(- \int_t^T \lambda_{i,s} ds \right) \left((1 - \gamma_i) \int_t^T \lambda_{i,s} ds \right)^n, \end{aligned} \quad (9)$$

hence,

$$\begin{aligned} p(t, T) &= \mathbb{E} [\mathbb{P} [\tau_i > T | \{\lambda_{i,s} : t \leq s \leq T\}] | \mathcal{F}_t] \\ &= \mathbb{E} \left[\sum_{n=0}^{\infty} \frac{1}{n!} \exp \left(- \int_t^T \lambda_{i,s} ds \right) \left((1 - \gamma_i) \int_t^T \lambda_{i,s} ds \right)^n | \mathcal{F}_t \right]. \end{aligned} \quad (10)$$

Let $s_{i,t}^k$ denote the CDS spread of country $i = 1, \dots, m$ at time (day) t with maturity $k = 1, 2$, corresponding to maturities of 5 and 10 years, respectively. It is assumed to be paid continuously. Next to the CDS spread, we assume that there exists a risk-free asset. We denote the associated (continuously compounded) risk-free rate by r_t and the time t price of a zero-coupon bond with maturity T by $D(t, T)$, so that $D(t, T) = \mathbb{E} \left[\exp \left(- \int_t^T r_s ds \right) | \mathcal{F}_t \right]$. We assume independence between the intensity process and the riskless rate.

⁸In line with the existing literature on intensity-based credit default models, we consider jumps to be “credit events” that trigger (increased likelihood of) bonds’ default. The exact legal definition of a default associated to an actual default plays no role in this model.

The CDS contract consists of two legs, the spread leg and the protection leg. The time t present value of the CDS spread leg of country i is given by

$$s_{i,t}^k \int_t^T D(t,s) \mathbb{E} [(1 - \gamma_i)^{N_{i,s} - N_{i,t}} | \mathcal{F}_t] ds, \quad i = 1, \dots, m, \quad (11)$$

with $T - t = 5$ if $k = 1$ and $T - t = 10$ if $k = 2$. The time t present value of the CDS protection leg of country i is given by

$$w_i \int_t^T D(t,s) \mathbb{E} [\gamma_i \lambda_{i,s} (1 - \gamma_i)^{N_{i,s} - N_{i,t}} | \mathcal{F}_t] ds, \quad i = 1, \dots, m, \quad (12)$$

where $1 - w_i$ is the recovery rate.

Under perfectly competitive markets, equating the present value of the CDS spread leg to the present value of the CDS protection leg, gives the expression for $s_{i,t}^k$:

$$s_{i,t}^k = \frac{w_i \int_t^T D(t,s) \mathbb{E} [\gamma_i \lambda_{i,s} (1 - \gamma_i)^{N_{i,s} - N_{i,t}} | \mathcal{F}_t] ds}{\int_t^T D(t,s) \mathbb{E} [(1 - \gamma_i)^{N_{i,s} - N_{i,t}} | \mathcal{F}_t] ds}, \quad i = 1, \dots, m, \quad (13)$$

with $T - t = 5$ if $k = 1$ and $T - t = 10$ if $k = 2$. Next, we specify our credit risk model with mutual excitation and then explicitly calculate within that framework the expectations that appear in (13). Due to a feedback effect from jumps to jump intensities and back, the expressions given in (7), (8), (9) and (10) fail to hold in our model.

3.3. Jump Intensities with Mutual Excitation

We specify the default intensities $\lambda_{i,t}$ and the associated counting processes $N_{i,t}$, $i = 1, \dots, m$, as a multivariate Hawkes process (mutually exciting jump process) with exponential decay, that is (2)-(3). In Ait-Sahalia et al. (2010), where a similar Hawkes model is a component of the asset pricing model we employ to study stock market contagion, we show how (3) follows from the general Hawkes framework with exponential decay of the excitation due to a jump.

We assume that $\lambda_{i,\infty} \geq 0$, $\beta_{i,j} \geq 0$, $\alpha_i > 0$, $i, j = 1, \dots, m$. This ensures that the default intensity processes $\lambda_{i,t}$ are non-negative with probability one. One may verify that, with $\lambda_i = \mathbb{E}[\lambda_{i,t}]$, in vector form, $\Lambda = (I - \Gamma)^{-1} \Lambda_\infty$, with I denoting the identity matrix and $\Gamma = [\beta_{i,j}/\alpha_i]_{i,j=1,\dots,m}$. Positivity and finiteness of Λ ensures stationarity of the model, which we assume henceforth. We note that throughout, we work under the risk-neutral probability measure and not under the physical probability measure. We note further that the two measures are equivalent and that jump times (but not necessarily jump intensities) are equal under both measures.

We recall from Section 3.2 that, conditionally on a jump (“credit event”) in the marginal point process $N_{i,t}$ (“country”), there is a probability of γ_i that country i goes into default. Furthermore, the probability of no default of country i between time t and time T is given by (5). Now, in our model with mutual excitation, upon the occurrence of a jump in one of the marginal point processes, the credit default intensities ramp up, making further jumps in both the affected country and the unaffected countries more likely. The precise effect of a jump in country j on the default intensity of country i , is determined by the parameter $\beta_{i,j}$, $i = 1, \dots, m$. In the absence of further jumps, there is mean reversion, with the credit

default intensity decaying back to $\lambda_{i,\infty}$ at rate α_i . From the CDS pricing perspective, the presence of mutual excitation undermines the validity of the pricing equations (7), (8), (9) and (10), and we now turn to the derivation of the CDS prices in our model. Anticipating these derivations and the empirical analysis that follows, we present in Figure 9 a simulated sample path of the credit default intensities for Greece and Ireland, along with the corresponding 5-year CDS spreads on the basis of estimated parameter values.

3.4. Explicit CDS Pricing Formulae with Mutual Excitation

We compute below the model price (spread) of a CDS contract using the (extended) conditional characteristic function of (N_t, λ_t) , which we derive in closed-form (up to the solution of a system of ODEs). Recall that the time t present value of the CDS spread leg of country i is given by (11) and that the time t present value of the CDS protection leg of country i is given by (12), which jointly determine the model price of a CDS contract through (13). The computation of the expectations in (13) can be carried out in closed-form (up to the solution of a system of ODEs) as a special case of Duffie et al. (2000); see also Errais et al. (2010). (Notice that α, β are (vectors of) parameters and $\alpha(\cdot), \beta(\cdot)$ introduced below are (vectors of) functions.) Indeed, upon adding the stochastic intensity processes to the state vector, we can restrict our model to be part of the class of generalized affine jump-diffusion models (see Appendix B of Duffie et al. (2000)) and compute $\mathbb{E}[(1 - \gamma_i)^{N_{i,T}} | \mathcal{F}_t]$ and $\mathbb{E}[\gamma_i \lambda_{i,T} (1 - \gamma_i)^{N_{i,T}} | \mathcal{F}_t]$ explicitly.

Consider a generalized affine jump-diffusion X in a state space $D \subset \mathbb{R}^{2 \times m}$, defined as a strong solution to the stochastic differential equation

$$dX_t = \mu^X(X_t)dt + \sigma^X(X_t)dW_t^X + \sum_{j=1}^m dJ_{j,t}, \quad (14)$$

where $\mu^X : D \rightarrow \mathbb{R}^{2 \times m}$, $\sigma^X : D \rightarrow \mathbb{R}^{(2 \times m) \times (2 \times m)}$, W^X is a Brownian motion in $\mathbb{R}^{2 \times m}$, and J_i , $i = 1, \dots, m$, are pure jump processes with jump intensities $\lambda_{i,t}^X = \lambda_i^X(X_t)$, for some $\lambda_i^X : D \rightarrow [0, \infty)$, and with fixed jump size distributions on $\mathbb{R}^{2 \times m}$ with jump transforms θ^i . A process X of the form (14) is restricted to be affine, by considering the special case where $\mu^X, \sigma^X \sigma^{X'}$ and λ_i^X are affine on D :

$$\begin{aligned} \mu^X(x) &= K_0 + K_1 x, & (K_0, K_1) &\in \mathbb{R}^{2 \times m} \times \mathbb{R}^{(2 \times m) \times (2 \times m)}, \\ (\sigma^X(x) \sigma^{X'}(x))_{i,j} &= (H_0)_{i,j} + (H_1)_{i,j} \cdot x, & (H_0, H_1) &\in \mathbb{R}^{(2 \times m) \times (2 \times m)} \times \mathbb{R}^{(2 \times m) \times (2 \times m) \times (2 \times m)}, \\ \lambda_i^X(x) &= l_0^i + l_1^i \cdot x, & (l_0^i, l_1^i) &\in \mathbb{R} \times \mathbb{R}^{2 \times m}, \quad i = 1, \dots, m. \end{aligned}$$

As we will see in the proofs below, our Hawkes model with exponential decay can be restricted to be a generalized affine jump-diffusion by setting $X_t = [N_t, \lambda_t]'$ with the corresponding $\mu^X, \sigma^X \sigma^{X'}$ and λ_i^X being affine. Propositions 1 and 3 and Appendix B of Duffie et al. (2000) then yield closed-form expressions for the transform $u \mapsto \mathbb{E} \left[\exp \left(- \int_t^T R(X_s) ds \right) e^{u \cdot X_T} | \mathcal{F}_t \right]$ and extended transform $(u, v) \mapsto \mathbb{E} \left[\exp \left(- \int_t^T R(X_s) ds \right) (v \cdot X_T) e^{u \cdot X_T} | \mathcal{F}_t \right]$, where $R(x) = \rho_0 + \rho_1 \cdot x$ for $(\rho_0, \rho_1) \in \mathbb{R} \times \mathbb{R}^{2 \times m}$. We provide in the Appendix the explicit expressions for the required expectations.

4. Estimation Results

4.1. Estimation Methodology

With the expectations appearing in the expressions for the CDS spreads (13) available in closed-form, up to the solution of a system of ODEs, and with the CDS spread and interest rate data at hand, we can proceed to estimate the model using standard techniques. Specifically, to obtain parameter estimates for our model, we minimize squared pricing errors (model-implied CDS spread \hat{s} minus empirical CDS spread s), over all days and all maturities, that is, we minimize over the parameter set $\Theta = \{\gamma, \alpha, \beta, \lambda_\infty, \lambda\}$:

$$\sum_{i=1}^m \sum_{k=1}^2 \sum_{t=1}^n (\hat{s}_{i,t}^k - s_{i,t}^k)^2, \quad (15)$$

with $i = 1, \dots, m$ corresponding to the countries under consideration, $k = 1, 2$ corresponding to maturities of 5 and 10 years, respectively, and n the number of observation days in our sample. Here, γ, α are m -dimensional vectors, β is an $m \times m$ -dimensional matrix, λ_∞ is an m -dimensional vector, and λ is an $n \times m$ -dimensional matrix. Indeed, since $\lambda_{i,t}$ is \mathcal{F}_t -measurable, it may be treated as a (free) parameter in the CDS model spread at time t , $s_{i,t}$. The minimization problem then produces vectors of intensity estimates, one for each $0 \leq t \leq T$, $\hat{\lambda}_{i,[0,T]}$. (At time t , N_t may be taken to be 0, only $N_T - N_t$ is relevant.)

The estimation procedure outlined above may be viewed as an application of GMM (see Hansen (1982)) with only a single moment condition per country and maturity, that in (13). Standard GMM-based testing tools therefore directly apply to our setting. In particular, under standard regularity conditions, with $T_n := n\Delta$, where Δ is the sampling frequency of 1 day ($\approx 1/251$ year), and Θ_0 the true value of Θ , $\sqrt{T_n}(\Theta - \Theta_0)$ converges in law to $N(0, \Omega)$ where $\Omega^{-1} = \Delta^{-1} D' W D (D' W S W D)^{-1} D' W D$. Here, W is a positive definite matrix, to which an initial positive definite weight matrix W_{T_n} is assumed to converge in probability, D is the gradient of the expected moment conditions with respect to the parameters evaluated at the true parameter values, and S^{-1} is the GMM optimal weight matrix.

The following practical issues play a role. We impose the following parameter restrictions: $0 < \gamma_i \leq 1$, $\lambda_{i,t} \geq \lambda_{i,\infty} \geq 0$ and $\alpha_i > \beta_{i,j} \geq 0$, $i, j = 1, \dots, m$. The first two sets of restrictions are necessary conditions for our model to be well-defined; the former restriction means that the probability of going into default, conditionally upon a jump in the underlying counting process, is between 0 and 1 and the latter restriction means that the credit default intensity processes are non-negative with probability one. The third set of restriction(s) is imposed to ensure stationarity of the model; see Section 3.3. These restrictions dictate the domain of Θ . To facilitate identification and enhance parsimony, we impose in the bivariate case the parameter restrictions $\alpha_1 = \alpha_2 =: \alpha$ and $\lambda_{1,\infty} = \lambda_{2,\infty} =: \lambda_\infty$. We treat the loss given default fraction w_i of the par value of the bond as exogenously given. We use $1 - w = w = 0.5$ for all countries; see Ang and Longstaff (2011). As is common, GMM is sensitive to the adopted starting values. To partially remedy this problem, we use parameter estimates obtained from an initial round of estimation as starting values in the next round of estimation. We continue until starting values thus obtained coincide with corresponding parameter estimates and a minimal value function is obtained. In the bivariate case, to facilitate identification, we have made the moment conditions used in GMM relatively comparable in magnitude. Different CDS in (15) are not weighted differently because we have no direct observations on their relative liquidity; should such data be available, the introduction of a weighting scheme is straightforward.

Another distinguishing feature between a doubly stochastic Poisson model and our model with mutual excitation now becomes apparent. In the doubly stochastic Poisson model, and fundamentally different from our model, the only way to assess the sample path of jump intensity changes following the occurrence of a credit event out-of-sample is, in fact, to recalibrate the model based on an enriched sample of CDS spreads. By contrast, our model provides a structural description (feedback mechanism) of the impact of credit events on the jump intensities.

4.2. Empirical Results

We now implement the estimation methodology outlined above on the sovereign CDS data, in a bivariate setting⁹, first considering pairs consisting of Greece and another country. Table 2 reports the parameter estimates. The table also reports the corresponding asymptotic standard errors. We find relatively large, and statistically significantly different from zero, estimated values for the β parameters. These estimates support self-excitation ($\beta_{11} > 0$, $\beta_{22} > 0$) and cross-excitation ($\beta_{12} > 0$, $\beta_{21} > 0$). The effect from Greece to other countries is found to be more pronounced than the reverse effect in that β_{21} is estimated at higher values than β_{12} . Upon comparing the reconstructed moments, obtained by substituting the parameter estimates into (13), to their empirical counterparts, we find that our model fits the data quite well.

More specifically, we observe from Table 2 relatively low estimated values for γ_2 , the conditional probability of default, for France and Germany. Compared to France and Germany, Italy and Spain show somewhat larger estimated values for γ_2 and, in particular, larger estimated values for β_{11} , β_{12} and β_{21} . Ireland has an even larger estimated value for γ_2 . Finally, Portugal shows a relatively large estimated value for λ_∞ , next to relatively large estimated values for all β parameters, including β_{22} .

In Figure 6 we show selected time series plots of the estimated vectors of credit default intensities, $\hat{\lambda}_{i,[0,T]}$, multiplied by the estimated conditional probabilities of default, $\hat{\gamma}_i$, for the pairs (GR,ES) and (GR,IE). Upon further multiplying $\hat{\lambda}_{i,t}\hat{\gamma}_i$ by Δ , we would obtain estimates of the instantaneous probabilities of default, at the leading order in Δ ; see Section 3. We observe low levels of $\hat{\lambda}_{i,t}\hat{\gamma}_i$ prior to 2008 for all countries; and high levels in 2011 and 2012, with Greece reaching its highest level in March 2012, the month in which it actually defaulted. Note the similarity between the left (or right) two panels of the figure.

We next test for the presence of mutual excitation. Econometrically, this comes down to testing the joint hypothesis that all the coefficients of mutual excitation, $\beta_{i,j}$'s, are 0, or that a subset of the $\beta_{i,j}$'s are 0: self- or time-series excitation (diagonal $\beta_{i,i} = 0$), cross-sectional excitation (off-diagonal $\beta_{i,j} = 0$, $i \neq j$), asymmetric excitation ($\beta_{j,i} > 0$ and $\beta_{i,j} = 0$ for a fixed i and all $j \neq i$). Since our framework with mutual excitation nests the standard framework with Poissonian jumps as a special case when all $\beta_{i,j}$'s are 0, these tests also allow us to effectively test our framework against the simpler Poissonian framework with constant credit default intensities, thus without mutual excitation. In the nested Poissonian model, there is no feedback from jumps to jump intensities, meaning that the (constant) jump intensities are not affected by the occurrence of an initial shock. By contrast, in the model with mutual excitation, upon occurrence of an initial shock in country i , the jump intensity of country j ramps up by an amount $\beta_{j,i}$.

⁹Although estimating the model for a larger set of countries together poses no theoretical difficulties, a practical issue is the proliferation of the number of parameters given all the different possible channels for transmission of shocks, the absence of imposed symmetry, unlike a variance-covariance matrix, and the relatively short sample lengths of sovereign CDS data. This can in principle be addressed by imposing strong parameter restrictions, such as $\beta_{ij} = \beta$ for at least some pairs (i, j) , but has the disadvantage of limiting the possibility of cross-country comparisons.

Our estimation methodology is essentially based on standard GMM using the CDS pricing equations as moments and therefore standard GMM-based testing techniques can readily be employed to test these parameter restrictions, or any other combination of parameter restrictions. We consider in particular the following null hypotheses: (i) $\mathcal{H}_0^{(i)} : \beta_{i,j} = 0$, for $i, j = 1, 2$; (ii) $\mathcal{H}_0^{(ii)} : \beta_{i,i} = 0$, for $i = 1, 2$; (iii) $\mathcal{H}_0^{(iii)} : \beta_{i,j} = 0$, for $i, j = 1, 2$, $i \neq j$. Reconsider the asymptotic covariance matrix Ω and the χ^2 statistic that follows from it. We then conduct a Wald chi-square test on the basis of our GMM estimates. We find that the null hypotheses are rejected in all cases at the 99% confidence level. This provides clear statistical support for excitation ($\mathcal{H}_0^{(i)}$ is rejected), self-excitation ($\mathcal{H}_0^{(ii)}$ is rejected) and cross-excitation ($\mathcal{H}_0^{(iii)}$ is rejected). This implies in particular that a Poisson credit default intensity model is rejected when tested against our default intensity model with mutual excitation. We find large chi-square statistics. Note that some of the hypotheses are sequential in nature and hence the sizes of the tests depend on the number of hypotheses tested, leading to an adjustment: see Bhargava (1987) and the references therein.

We do not find evidence of mutual excitation in the subsample of the data ending in the Spring of 2009. Such a phenomenon is typical in a jump setting, with an inherently limited number of jumps being observed. In the model, before jumps start occurring, jump intensities are at or near their values λ_∞ (which are estimated to be low) and stay there. It also means that it would have been difficult to extract from this dataset any advance warning signal of the impending crisis before it actually started. Once the first jump arrives, however, our model has clear implications as each jump foretells more jumps.

Next, we have also analyzed pairs of Ireland and another country. Parameter estimates are reported in Table 3. We again find clear statistical support of self-excitation ($\beta_{11} > 0$, $\beta_{22} > 0$) and cross-excitation ($\beta_{12} > 0$, $\beta_{21} > 0$), except for Germany and Spain, which appear to be less affected by jumps in Ireland. In general, the parameter estimates show that jumps in Ireland have limited effect (and much less than jumps in Greece), on other countries. The estimates also show that the reverse effect of jumps in other countries on Ireland is more pronounced.

5. Systemic Risk in Sovereign CDS: Impulse-Response and Systemically Important Countries

CDS contracts carry important information on market conditions and the perception of default risk. With the model estimated to the data and with the closed-form expressions for CDS spreads that we have derived, important economic questions can be studied. First and foremost, one may assess the impact of a shock in one of the countries on to the other countries, through mutual excitation. Such an analysis uncovers the core essence of systemic sovereign risk. This may be viewed as an impulse-response type analysis assessing the effect (the response) on to the dynamic system of sovereign CDS spreads of an initial shock (the impulse) in a single country. Such analyses are common practice in macroeconomics; see, for example, Hamilton (1995).¹⁰

Specifically, suppose that at time t there is a shock in the default intensity process of one of the countries, meaning that $dN_{i,t} = 1$ for some $i = 1, \dots, m$. In the model, this transmits contagiously to the default intensity processes of the other countries, formally through the system of SDEs (3): upon the occurrence of an initial jump, due to the feedback effect of our model, jump intensities ramp up, which makes further jumps more likely. In our framework, all jump intensities are affected. In particular, the

¹⁰Orthogonalization of the impulse response function, as in VAR, is not considered in our setting.

matrix of β coefficients determines the precise instantaneous impact of the initial shock on to the jump intensities of the other countries. The jump intensities in turn impact the sovereign CDS spreads, formally through the conditional moments in expression (13).

This impulse-response analysis also reveals important information about systemically important countries (SICs) and the question of in which country, if any, to inject capital. Indeed, it seems most effective to inject capital in that country that is systemically most relevant, i.e., the country for which a domestic shock (credit event) has the largest impact on the (domestic and non-domestic) CDS spreads. As an illustration of the impulse-response analysis, the first panel of Table 4 reports the “impulse-response functions,” specifically the changes in the sovereign CDS spreads $s_{i,t}$, $i = 1, \dots, m$, for the various countries under scrutiny, following an initial exogenous shock in Greece. We suppose that prior to the time of occurrence of the shock in Greece all jump intensity processes have reverted to their steady state level, that is, at time t of occurrence of the shock, $\lambda_{i,t} = \lambda_{i,\infty}$, $i = 1, \dots, m$. Further shocks are all endogenously generated by the excitation in the model. The impulse-response functions displayed in the table should be interpreted as follows: an increase of about three percent in the 10-year CDS spread of Greece leads to an increase of about half a percent in the 10-year CDS spreads of France and Germany. The impact on Italy and Spain is larger, about one percent. We note that potential differences in $\lambda_{i,\infty}$, $i = 1, \dots, m$, mean that a comparison across pairs should be made with care. Due to the fact that most of the action (“jumps”) in the data is in the few months prior to the default of Greek debt in March 2012, this (turbulent) part of the sample is most informative for the parameter estimates of the jump model. Jointly with the fact that the model, to remain parsimonious, does not include a continuous Brownian component next to the discontinuous jump component so that all action is attributed to the jump component, this means that the β parameters are estimated at relatively large values. As a result, the reported impulse-responses may be slightly overestimated. A second impulse-response analysis allows to assess also the reverse effect, and the asymmetry in cross-excitation: indeed, the second panel of Table 4 reports the impulse-response functions in pairs with Greece for the various countries under scrutiny, now following an initial shock in the other country. Among other things, we find that a shock in Ireland has relatively little predicted effect on Greece.

In the context of an impulse-response analysis, as an alternative metric to evaluate the effect of an initial shock in one of the sovereigns on to the other sovereigns, we compute the estimated increases in the credit default intensities, captured by the matrix $\hat{\beta}$, following the initial shock, and multiply them by the estimated conditional probabilities of default, $\hat{\gamma}_i$, $i = 1, \dots, m$. This produces an estimate of the increase in the annualized probability of default due to the initial shock. Multiplying this metric by Δ would produce estimates of the increases in instantaneous probabilities of default, at the leading order in Δ . We visualize this (annualized) metric in the second panel of Figure 5 using a “network graph”. Specifically, following an initial shock in Greece, we measure the country-specific estimated increases, $\hat{\beta}_{21}$, in the credit default intensities of the other countries, and multiply by the country-specific estimated conditional probabilities of default, $\hat{\gamma}_2$. The shorter and thicker the arrows, the more exposed is a country to excitation by Greece, according to this metric. We observe from the figure that Italy is most affected by a shock in Greece, followed by Spain. France and Germany are the least affected, while Ireland and Portugal are intermediate. Note that if further shocks in Greece occur beyond the initial shock, credit default intensities ramp up again, by an amount $\hat{\beta}_{11}$ for Greece and by a country-specific amount $\hat{\beta}_{21}$ for the other countries. Each time this happens, the impact on the annualized probability of default of Greece itself, through self-excitation, is in the order of magnitude¹¹ of 1.1, which is to be compared to e.g., 0.67

¹¹Estimates of this metric for Greece vary slightly depending on the specific pair of Greece and another country under

for Italy and 0.36 for France and Germany. And the impact on the annualized probability of default of Greece is quickly amplified upon the occurrence of further shocks, which have become more likely after the initial shock. The ranking from most affected to least affected on the basis of the impact on the annualized probability of default is similar to a ranking on the basis of the impact on 5-year CDS rates. A ranking on the basis of the impact on 10-year CDS rates reveals again that Italy is most affected and France and Germany are the least affected, but produces a slightly different ranking for the intermediate countries Ireland, Portugal and Spain. This change in ranking can be attributed to differences in $\lambda_{i,\infty}$. Our finding that Italy is more affected than Portugal by a shock in Greece might be explained by the perception that Portugal is small enough to save, but that Italy is not (“too-big-to-save”).

The model not only predicts the extent to which credit default intensities are affected instantaneously, following an initial shock in one of the countries, but also provides information on their decay over time. The rate of decay is captured by $\hat{\alpha}_i$, $i = 1, \dots, m$. In the left and upper right panels of Figure 7, we plot the decay over time of the increase in the annualized probability of default metric discussed above, which starts at a country-specific value of $\hat{\beta}_{21}\hat{\gamma}_2$. We distinguish between three cases: (i) an initial shock in Greece at time $t = 0$ and no further shocks; (ii) an initial shock in Greece at time $t = 0$, another shock in Greece at time $t = 10$ and a final shock in Greece at time $t = 16$; (iii) an initial shock in Greece at time $t = 0$ and a policy intervention in Greece at time $t = 10$, reducing jump intensities (hence CDS spreads) by $0.5\hat{\beta}_{11}$ in Greece and a country-specific $0.5\hat{\beta}_{21}$ in the other countries; time is measured in days. The decay for France and Germany is virtually indistinguishable. Therefore we have not included Germany in the plots.

The analysis of the decay over time of the increase in the annualized probability of default metric can also easily be translated to the decay over time of changes in CDS spreads, which shows similar patterns. This is illustrated in the lower right panel of Figure 7, where we measure the impact of an initial shock in Greece on the 10-year CDS spreads of the other countries and assess the rates of decay of the changes in CDS spreads. We assume an initial shock in Greece at time $t = 0$ and no further shocks. We suppose again that prior to the time of occurrence of the shock in Greece all jump intensity processes have reverted to their steady state level, and note again that potential differences in $\lambda_{i,\infty}$, $i = 1, \dots, m$, mean that a comparison across pairs should be made with care.

Next, we have carried out the same analysis for pairs of Ireland and another country. The corresponding impulse-response functions are in Table 5; the decay over time of the increase in the annualized probability of default metric are in the left and upper right panels of Figure 8; and the impact of an initial shock in Ireland on the 10-year CDS spreads of the other countries are in the lower right panel of Figure 8. The impulse-responses show most noticeably that jumps in Ireland have much less impact on Eurozone countries than jumps in Greece.

6. Conclusions

We proposed a model for the comovements of CDS rates in the Eurozone by capturing the excitation phenomenon, both in time and across countries, then estimated the model. We found evidence for self-excitation and asymmetric cross-excitation. As a prime application of our model, we have carried out an impulse-response analysis, a commonly used tool in macroeconometrics, by shocking the system and examining how shocks transmit throughout the Eurozone. The broad empirical implications of the model

consideration.

are reassuring for its plausibility and realism: Ireland, Portugal, Spain and Italy are the main countries affected by events in Greece, with results varying in strength across the countries, and to a lesser extent France and Germany. The model makes it possible to analyze the likely results of a policy intervention in the CDS market attempting to push rates down. Given the patterns contagion through the excitation mechanism, where would an intervention be most likely to have positive externalities on other countries? The asymmetry in the way contagion flows means that there is potential for intervention in one CDS market to be more successful than in others. We find for example that a policy intervention in Ireland resulting in an exogenous lowering of the CDS rate there would have been much less effective than a similar policy intervention in Greece.

Appendix: Expected Values in CDS Prices

A. The Univariate Case

We first consider the univariate self-exciting case:

$$dX_t = d \begin{pmatrix} N_t \\ \lambda_t \end{pmatrix} = \begin{pmatrix} 0 \\ \alpha(\lambda_\infty - \lambda_t) \end{pmatrix} dt + \begin{pmatrix} 1 \\ \beta \end{pmatrix} dN_t. \quad (\text{A.1})$$

As discussed above, computing CDS rates requires the computation of certain transforms, which we now state. The following results provide the expressions of the required transforms:

Proposition 1.

$$\mathbb{E} [(1 - \gamma)^{N_T} | \mathcal{F}_t] = e^{\alpha(t) + \beta_1(t)N_t + \beta_2(t)\lambda_t},$$

with

$$\begin{aligned} \dot{\alpha}(t) &= -\alpha\lambda_\infty\beta_2(t), \quad \dot{\beta}_1(t) = 0, \\ \dot{\beta}_2(t) &= \alpha\beta_2(t) - \left(e^{\log(1-\gamma) + \beta\beta_2(t)} - 1 \right) = \alpha\beta_2(t) - \left((1-\gamma)e^{\beta\beta_2(t)} - 1 \right), \\ \alpha(T) = \beta_2(T) &= 0, \quad \beta_1(T) = \log(1-\gamma) = \beta_1(s), \quad t \leq s \leq T. \end{aligned}$$

Corollary 1. *In the case of a Poisson process, corresponding to $\beta = 0$, we have that $\mathbb{E} [(1 - \gamma)^{N_T} | \mathcal{F}_t] = e^{-\gamma\lambda(T-t)}(1 - \gamma)^{N_t}$, as desired.*

Proof. We have $K_0 = (0, \alpha\lambda_\infty)'$, $K_1 = \begin{pmatrix} 0 & 0 \\ 0 & -\alpha \end{pmatrix}$ and $H_0 \equiv 0$, $H_1 \equiv 0$, $l_0 = 0$, $l_1 = (0, 1)'$.

Furthermore, $\theta(c) = \exp(c_1 + c_2\beta)$. We are interested in computing the transform $\mathbb{E} [(1 - \gamma)^{N_T} | \mathcal{F}_t]$. Hence $\rho_0 = 0$, $\rho_1 = (0, 0)'$, $u = (\log(1 - \gamma), 0)'$. Application of Proposition 1 of Duffie et al. (2000) then yields the result. \square

Proposition 2.

$$\mathbb{E} [\gamma\lambda_T(1 - \gamma)^{N_T} | \mathcal{F}_t] = e^{\alpha(t) + \beta_1(t)N_t + \beta_2(t)\lambda_t} (A(t) + B(t)\lambda_t),$$

with

$$\begin{aligned} -\dot{A}(t) &= \alpha\lambda_\infty B(t), & -\dot{B}(t) &= B(t)(-\alpha + (1-\gamma)\beta e^{\beta\beta_2(t)}), & \alpha(T) &= \beta_2(T) = A(T) = 0, \\ \beta_1(T) &= \log(1-\gamma) = \beta_1(s), & t \leq s \leq T, & & B(T) &= \gamma, \end{aligned}$$

where $\alpha(t)$ and $\beta(t)$ satisfy the ODEs given in Proposition 1.

Corollary 2. *In the case of a Poisson process, corresponding to $\beta = 0$, we have that $\mathbb{E}[\gamma\lambda_T(1-\gamma)^{N_T}|\mathcal{F}_t] = \gamma\lambda e^{-\gamma\lambda(T-t)}(1-\gamma)^{N_t}$, as desired.*

Proof of Proposition 2. We are now interested in the extended transform $\mathbb{E}[\gamma\lambda_T(1-\gamma)^{N_T}|\mathcal{F}_t]$. We have $v = (0, \gamma)'$. Application of Proposition 3 of Duffie et al. (2000) then yields the result. \square

B. The Bivariate Case

Next, we consider the bivariate mutually exciting case:

$$dX_t = d \begin{pmatrix} N_{1,t} \\ N_{2,t} \\ \lambda_{1,t} \\ \lambda_{2,t} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \alpha_1(\lambda_{1,\infty} - \lambda_{1,t}) \\ \alpha_2(\lambda_{2,\infty} - \lambda_{2,t}) \end{pmatrix} dt + \begin{pmatrix} 1 \\ 0 \\ \beta_{1,1} \\ \beta_{2,1} \end{pmatrix} dN_{1,t} + \begin{pmatrix} 0 \\ 1 \\ \beta_{1,2} \\ \beta_{2,2} \end{pmatrix} dN_{2,t}. \quad (\text{B.2})$$

We obtain the following results:

Proposition 3.

$$\mathbb{E}[(1-\gamma_1)^{N_{1,T}}|\mathcal{F}_t] = e^{\alpha(t)+\beta_1(t)N_{1,t}+\beta_3(t)\lambda_{1,t}+\beta_4(t)\lambda_{2,t}},$$

with

$$\begin{aligned} \dot{\alpha}(t) &= -\alpha_1\lambda_{1,\infty}\beta_3(t) - \alpha_2\lambda_{2,\infty}\beta_4(t), & \dot{\beta}_1(t) &= \dot{\beta}_2(t) = 0, \\ \dot{\beta}_3(t) &= \alpha_1\beta_3(t) - \left(e^{\log(1-\gamma_1)+\beta_{1,1}\beta_3(t)+\beta_{2,1}\beta_4(t)} - 1 \right) = \alpha_1\beta_3(t) - \left((1-\gamma_1)e^{\beta_{1,1}\beta_3(t)+\beta_{2,1}\beta_4(t)} - 1 \right), \\ \dot{\beta}_4(t) &= \alpha_2\beta_4(t) - \left(e^{\beta_{1,2}\beta_3(t)+\beta_{2,2}\beta_4(t)} - 1 \right), \\ \alpha(T) &= \beta_2(T) = \beta_2(s) = \beta_3(T) = \beta_4(T) = 0, & \beta_1(T) &= \log(1-\gamma_1) = \beta_1(s), \quad t \leq s \leq T. \end{aligned}$$

Proof of Proposition 3. We have $K_0 = (0, 0, \alpha_1\lambda_{1,\infty}, \alpha_2\lambda_{2,\infty})'$,

$$K_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -\alpha_1 & 0 \\ 0 & 0 & 0 & -\alpha_2 \end{pmatrix},$$

and $H_0 \equiv 0$, $H_1 \equiv 0$, $l_0^1 = 0$, $l_0^2 = 0$, $l_1^1 = (0, 0, 1, 0)'$, $l_1^2 = (0, 0, 0, 1)'$. Furthermore, $\theta^1(c) = \exp(c_1 + \beta_{1,1}c_3 + \beta_{2,1}c_4)$ and $\theta^2(c) = \exp(c_2 + \beta_{1,2}c_3 + \beta_{2,2}c_4)$. We are interested in computing the transform

$\mathbb{E}[(1 - \gamma_1)^{N_{1,T}} | \mathcal{F}_t]$ and we have $\rho_0 = 0$, $\rho_1 = (0, 0, 0, 0)'$, $u = (\log(1 - \gamma_1), 0, 0, 0)'$. Application of Proposition 1 of Duffie et al. (2000) and Appendix B then yields the result. \square

Proposition 4.

$$\mathbb{E}[(1 - \gamma_2)^{N_{2,T}} | \mathcal{F}_t] = e^{\tilde{\alpha}(t) + \tilde{\beta}_2(t)N_{2,t} + \tilde{\beta}_3(t)\lambda_{1,t} + \tilde{\beta}_4(t)\lambda_{2,t}},$$

with in this case

$$\begin{aligned} \dot{\tilde{\alpha}}(t) &= -\alpha_1\lambda_{1,\infty}\tilde{\beta}_3(t) - \alpha_2\lambda_{2,\infty}\tilde{\beta}_4(t), \quad \dot{\tilde{\beta}}_1(t) = \dot{\tilde{\beta}}_2(t) = 0, \\ \dot{\tilde{\beta}}_3(t) &= \alpha_1\tilde{\beta}_3(t) - \left(e^{\beta_{1,1}\tilde{\beta}_3(t) + \beta_{2,1}\tilde{\beta}_4(t)} - 1 \right), \\ \dot{\tilde{\beta}}_4(t) &= \alpha_2\tilde{\beta}_4(t) - \left(e^{\log(1-\gamma_2) + \beta_{1,2}\tilde{\beta}_3(t) + \beta_{2,2}\tilde{\beta}_4(t)} - 1 \right) = \alpha_2\tilde{\beta}_4(t) - \left((1 - \gamma_2)e^{\beta_{1,2}\tilde{\beta}_3(t) + \beta_{2,2}\tilde{\beta}_4(t)} - 1 \right), \\ \tilde{\alpha}(T) &= \tilde{\beta}_1(T) = \tilde{\beta}_1(s) = \tilde{\beta}_3(T) = \tilde{\beta}_4(T) = 0, \quad \tilde{\beta}_2(T) = \log(1 - \gamma_2) = \tilde{\beta}_2(s), \quad t \leq s \leq T. \end{aligned}$$

Proof of Proposition 4. We are now interested in computing the transform $\mathbb{E}[(1 - \gamma_2)^{N_{2,T}} | \mathcal{F}_t]$. Hence $\rho_0 = 0$, $\rho_1 = (0, 0, 0, 0)'$, $u = (0, \log(1 - \gamma_2), 0, 0)'$ and Proposition 1 of Duffie et al. (2000) and Appendix B yields the result. \square

Proposition 5.

$$\mathbb{E}[\gamma_1\lambda_{1,T}(1 - \gamma_1)^{N_{1,T}} | \mathcal{F}_t] = e^{\alpha(t) + \log(1-\gamma_1)N_{1,t} + \beta_3(t)\lambda_{1,t} + \beta_4(t)\lambda_{2,t}} (A(t) + B_3(t)\lambda_{1,t} + B_4(t)\lambda_{2,t}),$$

with

$$\begin{aligned} -\dot{A}(t) &= \alpha_1\lambda_{1,\infty}B_3(t) + \alpha_2\lambda_{2,\infty}B_4(t), \\ -\dot{B}_3(t) &= -\alpha_1B_3(t) + (1 - \gamma_1)\beta_{1,1}e^{\beta_{1,1}\beta_3(t) + \beta_{2,1}\beta_4(t)}B_3(t) + (1 - \gamma_1)\beta_{2,1}e^{\beta_{1,1}\beta_3(t) + \beta_{2,1}\beta_4(t)}B_4(t), \\ -\dot{B}_4(t) &= -\alpha_2B_4(t) + \beta_{1,2}e^{\beta_{1,2}\beta_3(t) + \beta_{2,2}\beta_4(t)}B_3(t) + \beta_{2,2}e^{\beta_{1,2}\beta_3(t) + \beta_{2,2}\beta_4(t)}B_4(t), \\ \alpha(T) &= \beta_2(T) = \beta_2(s) = \beta_3(T) = \beta_4(T) = A(T) = B_1(s) = B_2(s) = B_4(T) = 0, \quad t \leq s \leq T, \\ \beta_1(T) &= \log(1 - \gamma_1) = \beta_1(s), \quad t \leq s \leq T, \quad B_3 = \gamma_1, \end{aligned}$$

where $\alpha(t)$ and $\beta(t)$ satisfy the ODEs presented in Proposition 3.

Proof of Proposition 5. We are interested in the extended transform $\mathbb{E}[\gamma_1\lambda_{1,T}(1 - \gamma_1)^{N_{1,T}} | \mathcal{F}_t]$. Application of Proposition 3 of Duffie et al. (2000) and Appendix B with $u = (\log(1 - \gamma_1), 0, 0, 0)'$, $v = (0, 0, \gamma_1, 0)'$ then yields the result. \square

Proposition 6.

$$\mathbb{E}[\gamma_2\lambda_{2,T}(1 - \gamma_2)^{N_{2,T}} | \mathcal{F}_t] = e^{\tilde{\alpha}(t) + \log(1-\gamma_2)N_{2,t} + \tilde{\beta}_3(t)\lambda_{1,t} + \tilde{\beta}_4(t)\lambda_{2,t}} (\tilde{A}(t) + \tilde{B}_3(t)\lambda_{1,t} + \tilde{B}_4(t)\lambda_{2,t}),$$

with in this case

$$\begin{aligned}
-\dot{\tilde{A}}(t) &= \alpha_1 \lambda_{1,\infty} \tilde{B}_3(t) + \alpha_2 \lambda_{2,\infty} \tilde{B}_4(t), \\
-\dot{\tilde{B}}_3(t) &= -\alpha_1 \tilde{B}_3(t) + \beta_{1,1} e^{\beta_{1,1} \tilde{\beta}_3(t) + \beta_{2,1} \tilde{\beta}_4(t)} \tilde{B}_3(t) + \beta_{2,1} e^{\beta_{1,1} \tilde{\beta}_3(t) + \beta_{2,1} \tilde{\beta}_4(t)} \tilde{B}_4(t), \\
-\dot{\tilde{B}}_4(t) &= -\alpha_2 \tilde{B}_4(t) + (1 - \gamma_2) \beta_{1,2} e^{\beta_{1,2} \tilde{\beta}_3(t) + \beta_{2,2} \tilde{\beta}_4(t)} \tilde{B}_3(t) + (1 - \gamma_2) \beta_{2,2} e^{\beta_{1,2} \tilde{\beta}_3(t) + \beta_{2,2} \tilde{\beta}_4(t)} \tilde{B}_4(t), \\
\tilde{\alpha}(T) &= \tilde{\beta}_1(T) = \tilde{\beta}_1(s) = \tilde{\beta}_3(T) = \tilde{\beta}_4(T) = \tilde{A}(T) = \tilde{B}_1(s) = \tilde{B}_2(s) = \tilde{B}_3(T) = 0, \quad t \leq s \leq T, \\
\tilde{\beta}_2(T) &= \log(1 - \gamma_2) = \tilde{\beta}_2(s), \quad t \leq s \leq T, \quad \tilde{B}_4 = \gamma_2,
\end{aligned}$$

where $\tilde{\alpha}(t)$ and $\tilde{\beta}(t)$ satisfy the ODEs presented in Proposition 4.

Proof of Proposition 6. We are now interested in the extended transform $\mathbb{E} [\gamma_2 \lambda_{2,T} (1 - \gamma_2)^{N_{2,T}} | \mathcal{F}_t]$. Application of Proposition 3 of Duffie et al. (2000) and Appendix B with $u = (0, \log(1 - \gamma_2), 0, 0)'$, $v = (0, 0, 0, \gamma_2)'$ yields the result. \square

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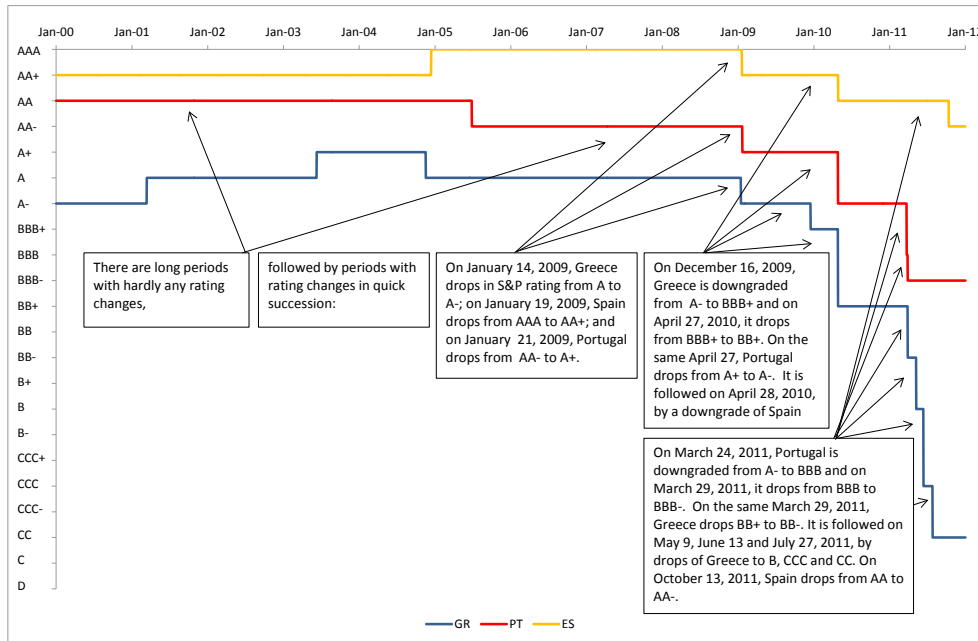


Fig. 1. S&P sovereign ratings for Greece, Portugal and Spain.

This figure plots the history of Standard and Poor's (S&P) sovereign ratings for Greece (GR), Portugal (PT) and Spain (ES) over the period January 1, 2000, to December 31, 2011.

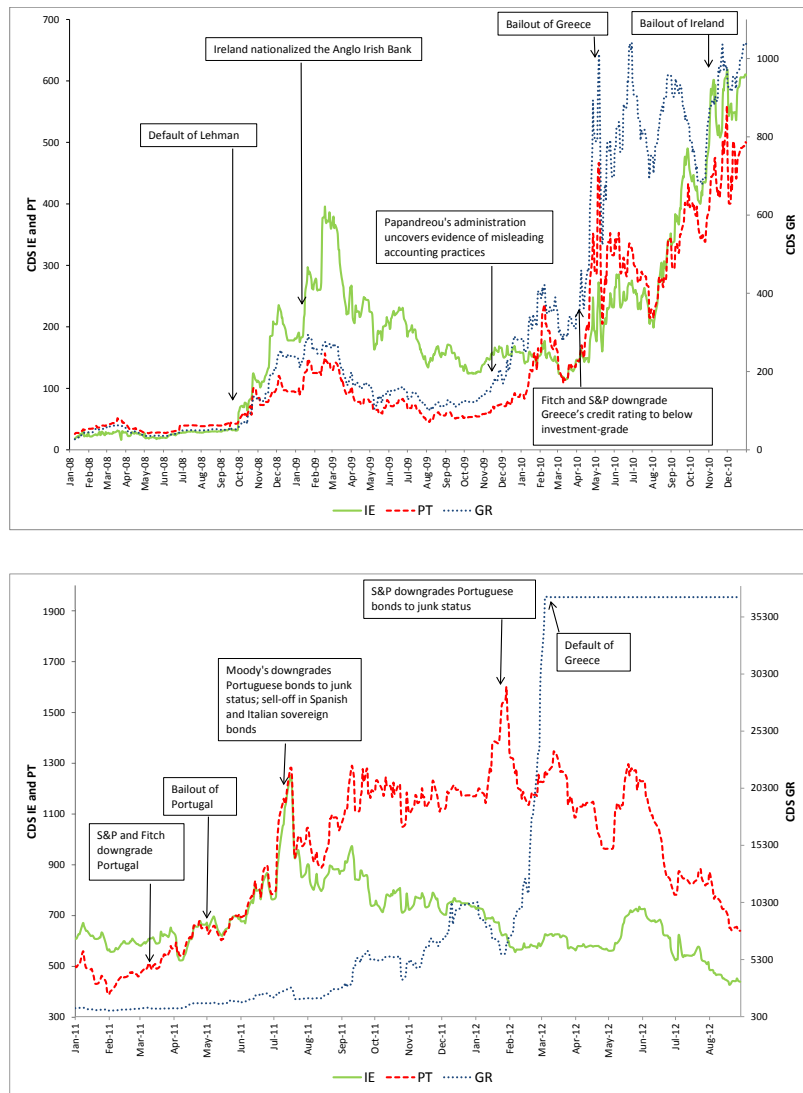


Fig. 2. CDS Ireland, Portugal and Greece.

This figure plots the 5-year CDS spreads (in bps) of Ireland (IE), Portugal (PT) and Greece (GR) from January 2008 until December 2010 (Panel I) and from January 2011 until August 2012 (Panel II). Data are daily. Source Datastream.

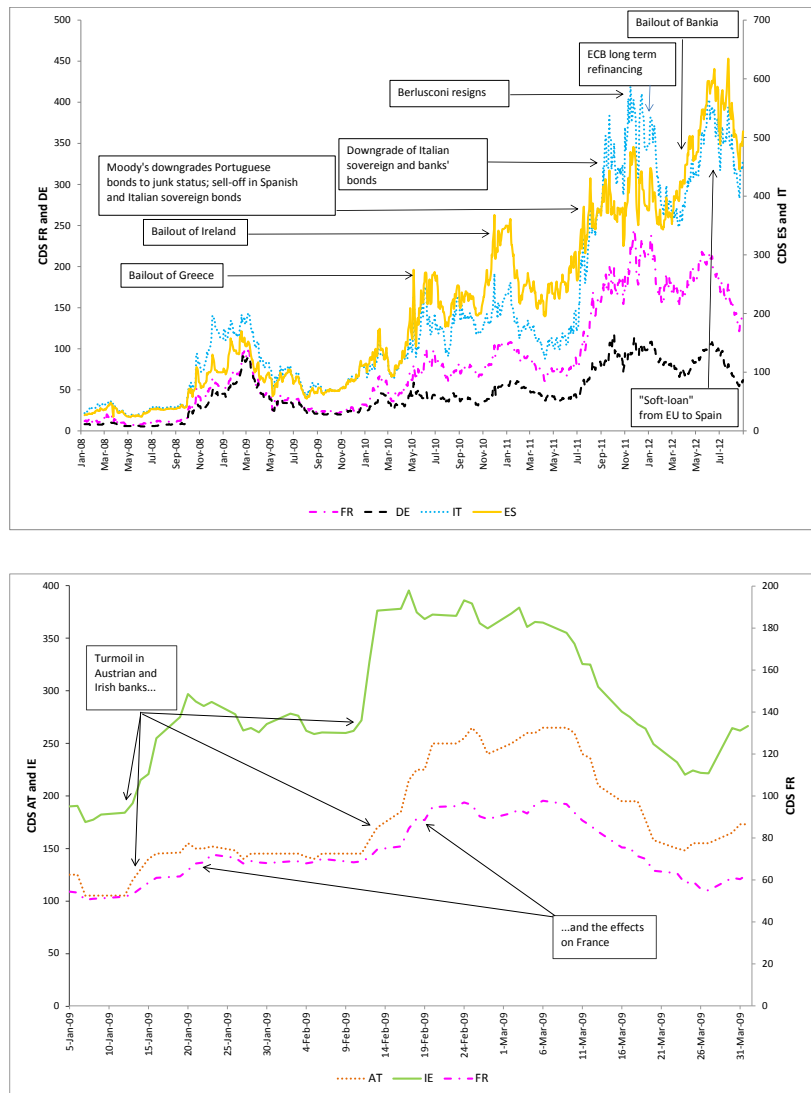


Fig. 3. CDS France, Germany, Italy and Spain; and CDS Austria, Ireland and France.

The first panel of this figure plots the 5-year CDS spreads (in bps) of France (FR), Germany (DE), Italy (IT) and Spain (ES) from January 2008 until August 2012. Data are daily. Source Datastream.

The second panel of this figure plots the 5-year CDS spreads (in bps) of Austria (AT), Ireland (IE) and France (FR) over the period March 5, 2009, to March 31, 2009. It shows that the Austrian and Irish banking crises get reflected with some delay into French CDS spreads. Data are daily. Source Datastream.



Fig. 4. CDS Italy, Spain and Portugal; and CDS Portugal and Italy.

The first panel of this figure plots the 5-year CDS spreads (in bps) of Italy (IT), Spain (ES) and Portugal (PT) over the period July 4, 2011, to July 22, 2011. It shows Moody’s downgrade of Portuguese bonds to junk status and the effect of a sell-off in Spanish and, with some delay, Italian sovereign bonds. Data are daily. Source Datastream.

The second panel of this figure plots the changes in the 5-year CDS spreads (in bps) of Portugal (PT) and Italy (IT) over the period July 17, 2011, to July 29, 2011. Solid arrows show “symmetric jumps”, dashed arrows show “asymmetric jumps”, indicating the presence of asymmetric cross-excitation. Data are daily. Source Datastream.

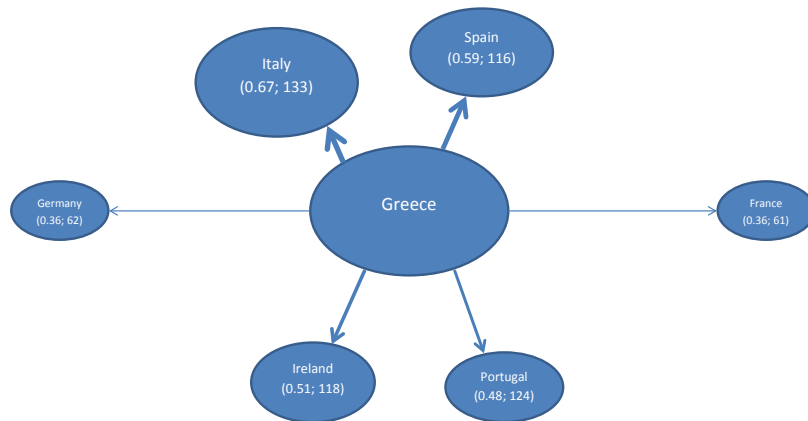
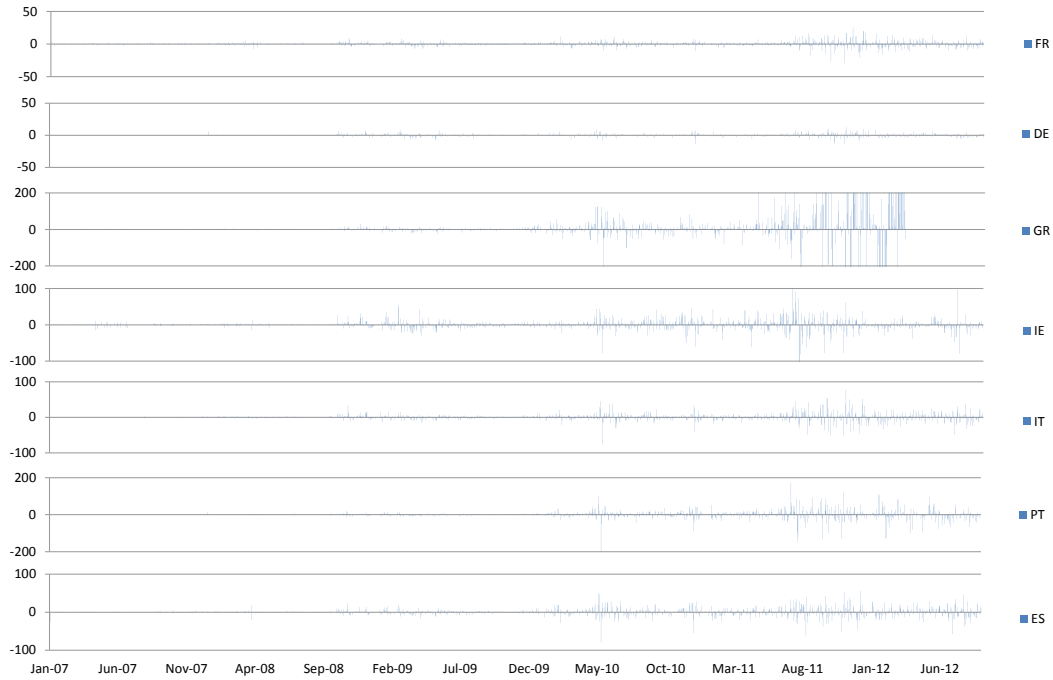


Fig. 5. Sovereign CDS; and Impulse-Response Analysis in a Network Graph.

The first panel of this figure plots the changes in the 5-year CDS spreads (in bps) of France (FR), Germany (DE), Greece (GR), Ireland (IE), Italy (IT), Portugal (PT) and Spain (ES). Sample period FR, IE, IT and ES: January 1, 2007, to August 31, 2012. Sample period DE: December 21, 2007, to August 31, 2012. Sample period GR: January 1, 2007, to March 8, 2012. Sample period PT: December 17, 2007, to August 31, 2012. Data are daily. Source Datastream.

The second panel of this figure visualizes the impulse-response analysis. Following an initial exogenous shock in Greece, we measure the country-specific estimated increases, $\hat{\beta}_{21}$, in the credit default intensities of the other countries, and multiply by the country-specific estimated conditional probabilities of default, $\hat{\gamma}_2$. This produces an estimate of the increase in the annualized probability of default due to the initial shock. The shorter and thicker the arrows, the more exposed is a country to excitation by Greece, according to this metric, which is included as the first number between brackets. If one would multiply this metric by Δ , one would obtain estimates of the increases in instantaneous probabilities of default, at the leading order in Δ . We also include, as the second number between brackets, the impact on the 10-year CDS spreads (in bps) following an initial shock in Greece; see the first panel of Table 4.

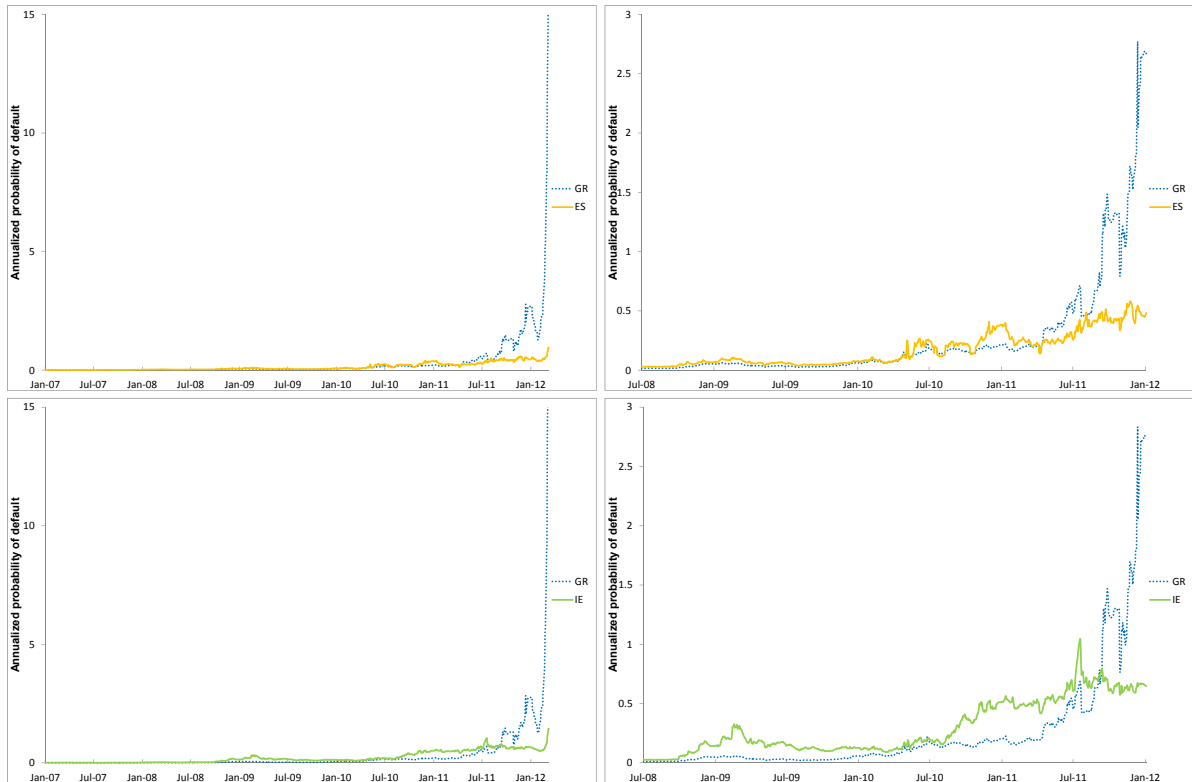


Fig. 6. Estimated Default Intensities Greece and Spain, and Greece and Ireland.

The top left and lower left panels of this figure plot the estimated time series of the credit default intensities, $\hat{\lambda}_{i,[0,T]}$, multiplied by the estimated conditional probabilities of default, $\hat{\gamma}_i$, for the pairs (GR,ES) and (GR,IE). The resulting metric provides estimates of the annualized probabilities of default. Multiplying this metric by Δ produces estimates of the instantaneous probabilities of default, at the leading order in Δ . The top right and lower right panels of this figure plot the same estimated time series as the top left and lower left panels of this figure but from July 1, 2008, up to January 1, 2012, only, and zoomed in.

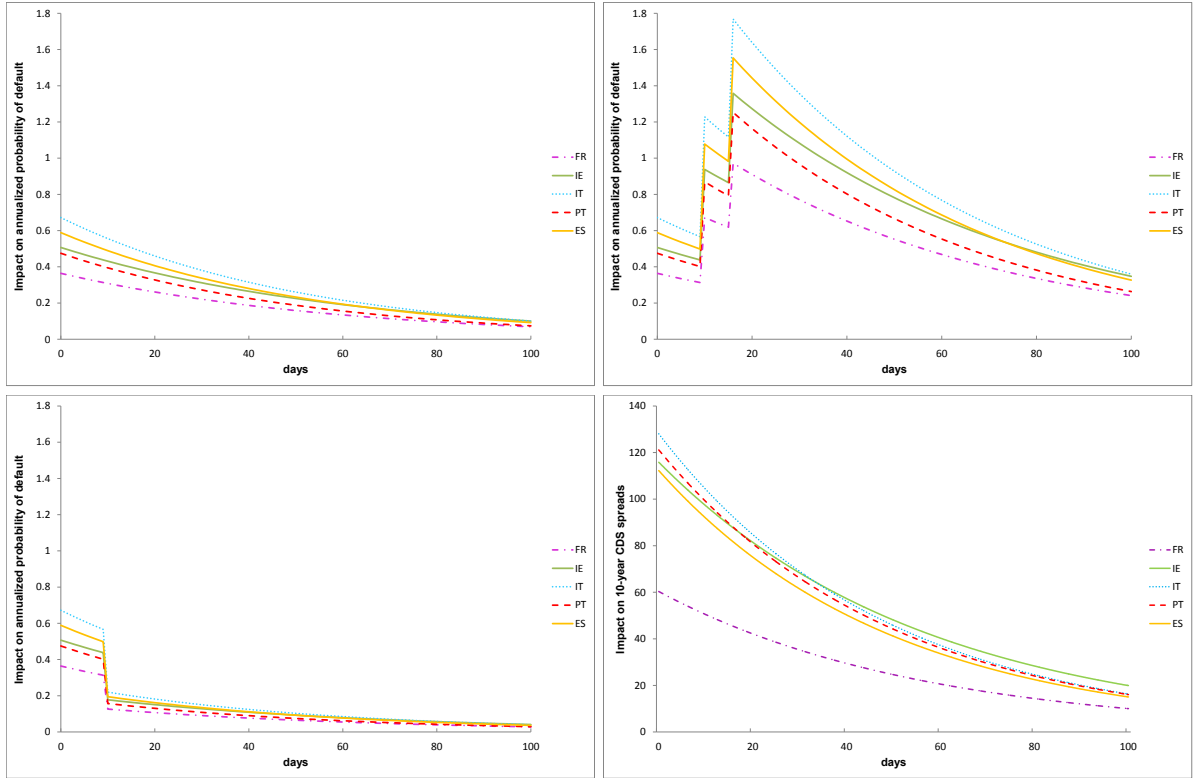


Fig. 7. Impulse-Response Analysis along the Time Dimension: Shock in Greece.

This figure visualizes the impulse-response analysis along the time dimension. Following an initial shock in Greece, we measure in the left and upper right panels the country-specific estimated increases, $\hat{\beta}_{21}$, in the credit default intensities of the other countries, multiply by the country-specific estimated conditional probabilities of default, $\hat{\gamma}_2$, and assess the rate of decay of the resulting metric. In the top left panel of this figure, we assume an initial shock in Greece at time $t = 0$ and no further shocks. In the top right panel of this figure, we assume an initial shock in Greece at time $t = 0$, another shock in Greece at time $t = 10$ and a final shock in Greece at time $t = 16$. In the lower left panel of this figure, we assume an initial shock in Greece at time $t = 0$ and a policy intervention in Greece at time $t = 10$, reducing jump intensities (hence CDS spreads) by $0.5\hat{\beta}_{11}$ in Greece and a country-specific $0.5\hat{\beta}_{21}$ in the other countries. The metric provides estimates of the increases in the annualized probabilities of default due to the shock. If one would multiply the metric by Δ , one would obtain estimates of the increases in instantaneous probabilities of default, at the leading order in Δ . The lower right panel of this figure visualizes the impulse-response analysis along the time dimension in terms of changes in CDS spreads (in bps). Following an initial shock in Greece, we measure the impact on the 10-year CDS spreads of the other countries and assess the rates of decay of the changes in CDS spreads. We assume an initial shock in Greece at time $t = 0$ and no further shocks. Prior to the occurrence of the shock the intensity processes have reverted to their steady state level.

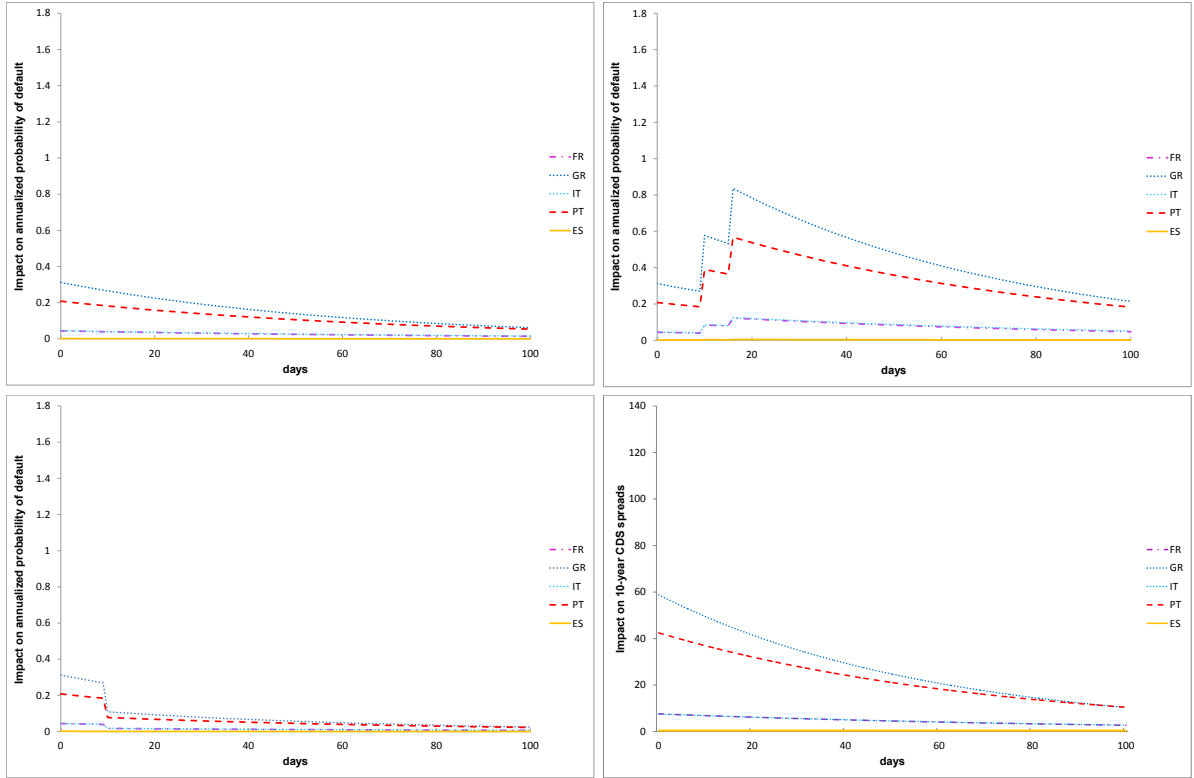


Fig. 8. Impulse-Response Analysis along the Time Dimension: Shock in Ireland.

This figure visualizes the impulse-response analysis along the time dimension. Following an initial shock in Ireland, we measure in the left and upper right panels the country-specific estimated increases, $\hat{\beta}_{21}$, in the credit default intensities of the other countries, multiply by the country-specific estimated conditional probabilities of default, $\hat{\gamma}_2$, and assess the rate of decay of the resulting metric. In the top left panel of this figure, we assume an initial shock in Ireland at time $t = 0$ and no further shocks. In the top right panel of this figure, we assume an initial shock in Ireland at time $t = 0$, another shock in Ireland at time $t = 10$ and a final shock in Ireland at time $t = 16$. In the lower left panel of this figure, we assume an initial shock in Ireland at time $t = 0$ and a policy intervention in Ireland at time $t = 10$, reducing jump intensities (hence CDS spreads) by $0.5\hat{\beta}_{11}$ in Ireland and a country-specific $0.5\hat{\beta}_{21}$ in the other countries. The metric provides estimates of the increases in the annualized probabilities of default due to the shock. If one would multiply the metric by Δ , one would obtain estimates of the increases in instantaneous probabilities of default, at the leading order in Δ . The lower right panel of this figure visualizes the impulse-response analysis along the time dimension in terms of changes in CDS spreads (in bps). Following an initial shock in Ireland, we measure the impact on the 10-year CDS spreads of the other countries and assess the rates of decay of the changes in CDS spreads. We assume an initial shock in Ireland at time $t = 0$ and no further shocks. Prior to the occurrence of the shock the intensity processes have reverted to their steady state level.

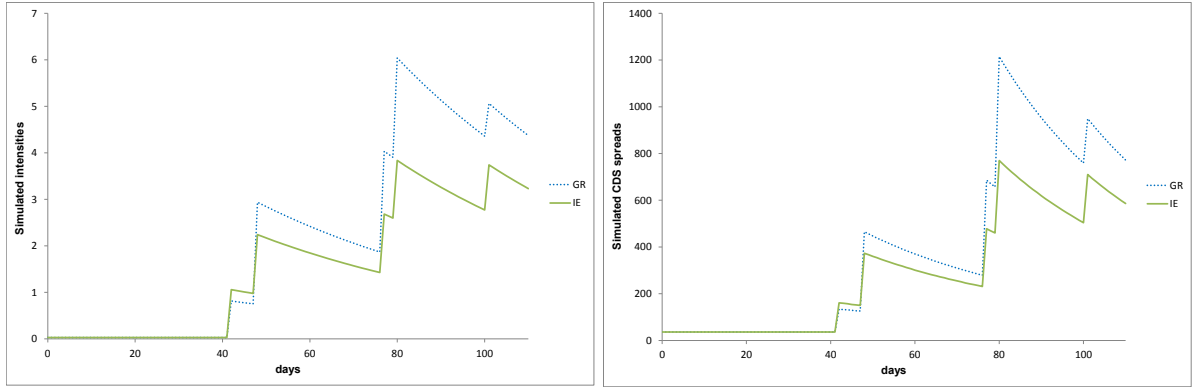


Fig. 9. Simulated Sample Paths.

This figure plots two simulated sample paths of the credit default intensities for Greece and Ireland (left panel) along with the associated 5-year CDS spreads (in bps, right panel), using estimated parameter values.

Table 1. Summary Statistics CDS Spreads.

This table reports summary statistics for daily changes (measured in bps) of sovereign CDS spreads for France (FR), Germany (DE), Greece (GR), Ireland (IE), Italy (IT), Portugal (PT) and Spain (ES). The mean levels of the spreads themselves are also reported (in bps). Sample period IE, IT, ES, 5-year CDS FR: January 1, 2007, to August 31, 2012. Sample period GR: January 1, 2007, to March 8, 2012. Sample period PT: December 17, 2007 to August 31, 2012. Sample period DE and 10-year FR: December 21, 2007, to August 31, 2012.

5-year CDS Spread Daily Changes							5-year CDS Spread Level
Country	Mean	Std. Dev.	Skew.	Exc. Kurt.	N. Obs.		Mean
France (FR)	0.09	3.66	-0.15	11.18	1480		67.05
Germany (DE)	0.05	2.18	-0.15	7.64	1226		45.23
Greece (GR)	27.30	309.16	10.83	181.31	1354		1243.00
Ireland (IE)	0.29	14.04	-0.67	18.34	1480		298.86
Italy (IT)	0.31	10.21	0.35	19.00	1480		166.45
Portugal (PT)	0.51	22.58	-0.74	17.18	1230		422.36
Spain (ES)	0.33	9.74	-0.30	10.09	1480		173.97
10-year CDS Spread Daily Changes							10-year CDS Spread Level
Country	Mean	Std. Dev.	Skew.	Exc. Kurt.	N. Obs.		Mean
France (FR)	0.13	3.96	-0.25	9.55	1226		90.65
Germany (DE)	0.07	2.23	-0.14	7.04	1226		55.45
Greece (GR)	21.42	278.98	8.90	147.21	1354		1012.95
Ireland (IE)	0.29	14.04	-0.67	18.34	1480		246.42
Italy (IT)	0.29	9.44	0.09	11.26	1480		168.30
Portugal (PT)	0.45	21.37	-0.81	19.91	1230		319.95
Spain (ES)	0.33	9.29	-0.36	9.88	1480		172.72

Table 2. Parameter Estimates for the Bivariate Hawkes Credit Default Intensity Model: Pairs with Greece. This table reports the GMM parameter estimates for the 8 parameters of the bivariate Hawkes credit default intensity model with annual-based sovereign CDS spreads measured in absolute amounts; asymptotic standard errors are in parentheses. *, **, and *** indicate significance at the 95%, 97.5%, and 99.5% confidence levels, respectively. We consider seven series: FR; DE; GR; IE; IT; PT; and ES.

1	GR	GR	GR	GR	GR	GR
2	FR	DE	IE	IT	PT	ES
α	4.3*** (0.03)	4.2*** (0.04)	4.2*** (0.03)	4.9*** (0.04)	4.8*** (0.01)	4.8*** (0.04)
β_{11}	2.4*** (0.07)	2.2*** (0.09)	2.2*** (0.07)	2.9*** (0.10)	2.9*** (0.03)	2.8*** (0.10)
β_{12}	0.9*** (0.06)	0.7*** (0.13)	0.8*** (0.08)	1.5*** (0.12)	1.4*** (0.02)	1.3*** (0.11)
β_{21}	1.4*** (0.12)	1.3*** (0.15)	1.3*** (0.09)	2.1*** (0.16)	1.9*** (0.04)	1.9*** (0.15)
β_{22}	1.0*** (0.16)	1.0*** (0.32)	1.0*** (0.15)	0.9*** (0.20)	1.7*** (0.03)	1.0*** (0.21)
λ_∞	0.06*** (0.001)	0.02*** (0.001)	0.03*** (0.001)	0.02*** (0.001)	0.23*** (0.001)	0.02*** (0.001)
γ_1	0.50*** (0.001)	0.47*** (0.002)	0.39*** (0.002)	0.40*** (0.002)	0.45*** (0.001)	0.41*** (0.002)
γ_2	0.26*** (0.008)	0.28*** (0.015)	0.39*** (0.005)	0.32*** (0.007)	0.25*** (0.001)	0.31*** (0.007)
RMSE	0.029	0.029	0.026	0.026	0.029	0.026

Table 3. Parameter Estimates for the Bivariate Hawkes Credit Default Intensity Model: Pairs with Ireland. This table reports the GMM parameter estimates for the 8 parameters of the bivariate Hawkes credit default intensity model with annual-based sovereign CDS spreads measured in absolute amounts; asymptotic standard errors are in parentheses. *, **, and *** indicate significance at the 95%, 97.5%, and 99.5% confidence levels, respectively. We consider seven series: FR; DE; GR; IE; IT; PT; and ES.

1	IE	IE	IE	IE	IE	IE
2	FR	DE	GR	IT	PT	ES
α	2.9*** (0.02)	2.8*** (0.01)	4.2*** (0.03)	2.8*** (0.01)	3.5*** (0.01)	2.6*** (0.01)
β_{11}	0.6*** (0.03)	0.3*** (0.02)	1.0*** (0.15)	0.4*** (0.02)	0.8*** (0.01)	0.3*** (0.01)
β_{12}	1.0*** (0.03)	0.8*** (0.01)	1.3*** (0.09)	0.8*** (0.01)	0.9*** (0.01)	0.7*** (0.01)
β_{21}	0.1*** (0.02)	0.01 (0.01)	0.8*** (0.08)	0.1*** (0.02)	0.7*** (0.01)	0.01 (0.02)
β_{22}	0.9*** (0.04)	0.9*** (0.01)	2.2*** (0.07)	0.8*** (0.01)	1.0*** (0.01)	0.8*** (0.01)
λ_∞	0.04*** (0.001)	0.07*** (0.001)	0.03*** (0.001)	0.03*** (0.001)	0.25*** (0.001)	0.05*** (0.001)
γ_1	0.46*** (0.001)	0.57*** (0.001)	0.39*** (0.005)	0.58*** (0.001)	0.48*** (0.001)	0.60*** (0.001)
γ_2	0.33*** (0.002)	0.29*** (0.001)	0.39*** (0.002)	0.32*** (0.001)	0.29*** (0.001)	0.27*** (0.001)
RMSE	0.002	0.002	0.026	0.002	0.004	0.002

Table 4. Impulse-Response Functions: I.

This table reports impulse-response functions. The first panel measures the changes in sovereign CDS spreads (in absolute amounts) following an initial shock in Greece. The second panel measures the changes in sovereign CDS spreads (in absolute amounts) following an initial shock in the other country. Prior to the occurrence of the shock the intensity processes have reverted to their steady state level. We consider seven series: FR; DE; GR; IE; IT; PT; and ES.

1 2	GR FR	GR DE	GR IE	GR IT	GR PT	GR ES
5-year CDS country 1	0.057	0.044	0.030	0.045	0.075	0.045
5-year CDS country 2	0.0098	0.010	0.019	0.022	0.016	0.019
10-year CDS country 1	0.038	0.027	0.019	0.028	0.061	0.028
10-year CDS country 2	0.0061	0.0062	0.012	0.013	0.012	0.012
5-year CDS country 1	0.019	0.013	0.0098	0.019	0.033	0.018
5-year CDS country 2	0.0057	0.0064	0.013	0.0090	0.012	0.0087
10-year CDS country 1	0.012	0.0081	0.0059	0.012	0.027	0.011
10-year CDS country 2	0.0035	0.0039	0.0076	0.0055	0.0091	0.0053

Table 5. Impulse-Response Functions: II.

This table reports impulse-response functions. The first panel measures the changes in sovereign CDS spreads (in absolute amounts) following an initial shock in Ireland. The second panel measures the changes in sovereign CDS spreads (in absolute amounts) following an initial shock in the other country. Prior to the occurrence of the shock the intensity processes have reverted to their steady state level. We consider seven series: FR; DE; GR; IE; IT; PT; and ES.

1 2	IE FR	IE DE	IE GR	IE IT	IE PT	IE ES
5-year CDS country 1	0.010	0.0085	0.013	0.011	0.020	0.0087
5-year CDS country 2	0.0012	0.0001	0.0098	0.0012	0.0059	0.0001
10-year CDS country 1	0.0061	0.0055	0.0076	0.0067	0.016	0.0054
10-year CDS country 2	0.0008	0.0001	0.0059	0.0008	0.0043	0.0001
5-year CDS country 1	0.019	0.027	0.019	0.025	0.023	0.027
5-year CDS country 2	0.0079	0.0064	0.030	0.0072	0.0079	0.0050
10-year CDS country 1	0.012	0.017	0.012	0.016	0.018	0.017
10-year CDS country 2	0.0048	0.0039	0.019	0.0043	0.0057	0.0030

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