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## Agglomeration and population ageing in a two region model of exogenous growth

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# VIENNA INSTITUTE OF DEMOGRAPHY

## *Working Papers*

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*Theresa Grafeneder-Weisssteiner and Klaus Prettnner*

## *Agglomeration and population ageing in a two region model of exogenous growth*



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## **Abstract**

This article investigates the effects of introducing demography into the New Economic Geography. We generalize the constructed capital approach, which relies on infinite individual planning horizons, by introducing mortality. The resulting overlapping generation framework with heterogeneous individuals allows us to study the effects of ageing on agglomeration processes by analytically identifying the level of trade costs that triggers catastrophic agglomeration. Interestingly, this threshold value is rather sensitive to changes in mortality. In particular, the introduction of a positive mortality rate makes the symmetric equilibrium more stable and therefore counteracts agglomeration tendencies. In sharp contrast to other New Economic Geography approaches, this implies that deeper integration is not necessarily associated with higher interregional inequality.

## **Keywords**

Agglomeration, New Economic Geography, Trade and Growth, Constructed Capital Model, Population Ageing.

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JEL classification: C61; F12; F15

# Agglomeration and population ageing in a two region model of exogenous growth

Theresa Grafeneder-Weissteiner and Klaus Prettnner

## 1 Introduction

The New Economic Geography literature pioneered by Krugman (1991), Venables (1996) and Krugman and Venables (1995) provided new insights into how transport costs can determine the spatial distribution of economic activity between two regions. These models are mainly characterized by catastrophic agglomeration. Due to circular causality effects encouraging the concentration of industrial activity, there are certain levels of transport costs for which the symmetric equilibrium becomes unstable and the core periphery outcome is the only stable equilibrium. Reciprocal liberalization between initially symmetric regions that strengthens the importance of such circular causality forces thus leads to the agglomeration of productive factors in one region. Puga (1999) set up a model that nested as special cases both the Krugman (1991) framework with labour mobility between regions as well as the vertically linked-industries model of Venables (1996) and Krugman and Venables (1995) without interregional labour mobility. However, the richness of agglomeration features in these models reduced their analytical tractability. Therefore Baldwin (1999) introduced the constructed capital framework with interregional labour and capital immobility but forward-looking agents. His model also features catastrophic agglomeration. The only force determining agglomeration is, however, the difference in the effective capital rental rates between the two regions. A higher rental rate in the home region increases home capital accumulation, whereas capital is decumulated in the foreign region. Circular causality sets in as a higher capital stock also implies higher capital income which in turn raises home expenditures and leads to a further increase in home rental rates. Since neoclassical growth models in the spirit of Solow (1956) and Ramsey (1928) show that capital accumulation is associated with faster growth in the medium run, Baldwin (1999) describes the economy accumulating capital as a growth pole, whereas the other region appears as a growth sink. This illustrates how economic integration in Europe could lead to the development of “rust” and “boom belts”.

In contrast to the Ramsey (1928) framework of one single, infinitely lived, representative agent, on which the constructed capital model heavily relies, agents do not live forever in reality. We therefore generalize Baldwin (1999)’s approach by introducing the possibility of death and thus accounting for finite planning horizons. In doing so, we adopt Blanchard (1985)’s structure of overlapping generations, where heterogeneity among in-

dividuals is due to their date of birth. While still following the lines of intertemporally optimizing agents, this results in a more realistic model incorporating life-cycle decisions and nesting the constructed capital model as a special case. Most notably, it allows us to study the effects of ageing on agglomeration processes between two regions.

Our results indicate that Baldwin (1999)'s agglomeration induced growth finding only applies in the very special case of infinitely lived individuals. In particular, we show that under a more realistic mortality assumption the possibility of the symmetric equilibrium to be unstable is considerably reduced such that agglomeration processes may not set in even if economic integration is promoted up to a high degree. Lifetime uncertainty therefore acts as a dispersion force that countervails the circular causality effects present in our framework.

The paper proceeds as follows. Section 2 presents the structure of the model and the optimization problems of individuals and firms. Section 3 verifies the existence of a symmetric long-run equilibrium and characterizes its properties. Section 4 analyses the stability of this long-run equilibrium with positive mortality and compares them to the results of Baldwin (1999) without mortality. By calibrating the model for reasonable values of the parameters, we complement our analytical findings by numerical illustrations. Finally, section 5 summarizes and draws conclusions for economic policy.

## 2 The model

This section describes how we integrate Blanchard (1985)'s notion of mortality into the constructed capital framework of Baldwin (1999). Consumption and savings behaviour as well as production technologies are introduced and various intermediate findings resulting from profit maximization are presented. In order to be able to analyse the long-run equilibrium, we also derive aggregate law of motions for capital and expenditures.

### 2.1 Basic structure and underlying assumptions

The model consists of two regions or countries, referred to as  $H$  for home and  $F$  for foreign<sup>1</sup>, with symmetric production technologies and preferences of individuals as well as identical labour endowments and demographic structures. Each region has three economic sectors (agriculture, manufacturing and investment) with two immobile factors (labour  $L$  and capital  $K$ ) at their disposal. The homogeneous agricultural good,  $n$ , is produced in a perfectly competitive market with labour as the only input and can be traded between the two regions without any costs. Manufacturing firms are modelled as in the monopolistic competition framework of Dixit and Stiglitz (1977) and therefore produce varieties, denoted as  $m$ , with one unit of capital as fixed input and labour as the variable production factor. A continuum of varieties  $i \in (0, V_H]$  of all manufacturing goods is produced at home, whereas a continuum of varieties  $j \in (0, V_F]$  is manufactured in the foreign region.

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<sup>1</sup>If further distinction is needed, foreign variables are moreover indicated by an asterisk.

In contrast to the agricultural good, trade of manufactures involves iceberg transport costs such that  $\varphi \geq 1$  units of the differentiated goods have to be shipped in order to sell one unit abroad (see for example Baldwin et al. (2003)). In the Walrasian investment sector capital is produced using labour as the only input with a time independent, exogenous unit input coefficient  $F$  where wages are paid out of the individuals' savings. The failure rate of a machine is assumed to be independent of the machine's age. Denoting this failure rate as  $0 < \delta \leq 1$ , and using the law of large numbers, implies that the overall depreciation rate of capital is given by  $\delta$  as well.

As far as the demographic structure of our model economy is concerned, we closely follow Blanchard (1985)'s simplified setting. We assume that at each point in time,  $\tau \in [0, \infty)$ , a large cohort consisting of finitely many individuals is born. The size of this cohort is  $N(\tau, \tau) = \mu N(\tau)$ , where  $0 < \mu \leq 1$  is the constant birth rate and  $N(\tau) \equiv \int_{-\infty}^{\tau} N(t_0, \tau) dt_0$  is total population at time  $\tau$  with  $N(t_0, \tau)$  denoting the amount of individuals born at  $t_0$  for a given point in time  $\tau$ .<sup>2</sup> Consequently, cohorts can be distinguished by the birth date  $t_0$  of their members. Since there is no heterogeneity between members of the same cohort, each cohort can be described by one representative individual, who inelastically supplies his efficiency units of labour at the labour market with perfect mobility across sectors but immobility between regions. The age of the individual is given by  $a = \tau - t_0$  and his time of death is stochastic with an exponential probability density function. In particular, the probability of death is given by the constant, i.e. age independent, parameter  $\mu$  resulting in a surviving probability to age  $\tau - t_0$  of  $e^{-\mu(\tau-t_0)}$ . Since the population size is large, the frequency of dying is equal to the instantaneous mortality rate. Therefore the number of deaths at each instant in time is also  $\mu N(\tau)$ . As this equals, by assumption, the number of births, population size is constant and can be normalized to unity ( $N(\tau)=1$ ). Finally, as in Yaari (1965), a perfect life-insurance company offers actuarial notes, which can be bought or sold by each individual and are cancelled upon the individual's death.

## 2.2 The individual's utility optimization problem

The following discussion refers to the home economy but due to symmetry between the two regions, equivalent equations also hold abroad. Individuals have Cobb-Douglas preferences over the agricultural good and a CES composite of manufacturing goods. Based on the assumptions of section 2.1 the representative individual of cohort  $t_0$  maximizes his expected lifetime utility  $U(t_0, t_0)$ <sup>3</sup> at time  $t_0$ . He therefore chooses at each instant  $\tau > t_0$  consumption of the agricultural good,  $c_n(t_0, \tau)$ , consumption of varieties produced at home,  $c_m^H(i, t_0, \tau)$ , and consumption of varieties produced abroad,  $c_m^F(j, t_0, \tau)$ , according

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<sup>2</sup>In what follows the first time index of a variable will refer to the birth date, whereas the second will indicate a certain point in time.

<sup>3</sup>It is easy to show that the objective in equation (1) can be derived via calculating expected lifetime utility, where the time of death is a random variable with an exponential probability density function parametrized by a constant instantaneous mortality rate  $\mu$ .

to the maximization problem

$$\max_{c_n, c_m^H, c_m^F} \int_{t_0}^{\infty} e^{-(\rho+\mu)(\tau-t_0)} \ln \left[ (c_n(t_0, \tau))^{1-\xi} (c_m^{agg}(t_0, \tau))^{\xi} \right] d\tau, \quad (1)$$

where  $0 < \rho \leq 1$  is the pure rate of time preference,  $0 < \xi < 1$  is the manufacturing share of consumption and

$$c_m^{agg}(t_0, \tau) \equiv \left[ \int_0^{V_H(\tau)} (c_m^H(i, t_0, \tau))^{\frac{\sigma-1}{\sigma}} di + \int_0^{V_F(\tau)} (c_m^F(j, t_0, \tau))^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}$$

represents consumption of the CES composite of manufactured goods with  $\sigma > 1$  denoting the elasticity of substitution between varieties.

Individual savings, defined as income minus consumption expenditures, are converted into capital in the investment sector with a labour input coefficient of  $F$ . Taking this into account, the wealth constraint of a representative individual can be written as

$$\dot{k}(t_0, \tau) = \frac{w(\tau)l(t_0, \tau) + \pi(\tau)k(t_0, \tau) - e(t_0, \tau)}{w(\tau)F} + \mu k(t_0, \tau) - \delta k(t_0, \tau), \quad (2)$$

where  $w(\tau)$  denotes the wage per efficiency unit of labour,  $l(t_0, \tau)$  refers to the efficiency units of labour the representative individual of cohort  $t_0$  supplies,  $\pi(\tau)$  is the capital rental rate,  $k(t_0, \tau)$  the capital stock of an individual and  $e(t_0, \tau)$  are an individual's total expenditures for consumption given by

$$\begin{aligned} e(t_0, \tau) \equiv & p_n(\tau)c_n(t_0, \tau) + \int_0^{V_H(\tau)} p_m^H(i, \tau)c_m^H(i, t_0, \tau)di + \\ & \int_0^{V_F(\tau)} p_{m,\varphi}^F(j, \tau)c_m^F(j, t_0, \tau)dj. \end{aligned}$$

Here  $p_n(\tau)$  is the price of the agricultural good,  $p_m^H(i, \tau)$  the price of a manufactured variety produced at home and  $p_{m,\varphi}^F(j, \tau)$  the price of a manufactured variety produced abroad with the subscript  $\varphi$  indicating the dependence on transport costs.

The particular law of motion for capital given above in equation (2) is based on Yaari (1965)'s full insurance result implying that all individuals only hold their wealth in the form of actuarial notes.<sup>4</sup> Therefore the market rate of return on capital,  $\frac{\pi(\tau)}{w(\tau)F} - \delta$ , has to be augmented by  $\mu$  to obtain the fair rate on actuarial notes (cf. Yaari (1965)).

In appendix A we solve the individual's utility optimization problem by applying a three stage procedure. In the first stage the dynamic savings-expenditure decision is analysed. Stage two deals with the static optimal consumption allocation between the CES composite and the agricultural good and in stage three individuals decide upon the amount of consumption they allocate to each of the manufactured varieties. Altogether

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<sup>4</sup>Two interpretations of the capital accumulation process are therefore possible. Either each individual itself converts its savings into capital and then leaves it to the insurance company or savings are immediately transferred to the insurance company which converts them into machines by employing workers.

this leads to the following demand functions for the agricultural good and for each of the manufactured varieties

$$c_n(t_0, \tau) = \frac{(1 - \xi)e(t_0, \tau)}{p_n(\tau)}, \quad (3)$$

$$c_m^H(i, t_0, \tau) = \frac{\xi e(t_0, \tau)(p_m^H(i, \tau))^{-\sigma}}{\left[ \int_0^{V_H(\tau)} (p_m^H(i, \tau))^{1-\sigma} di + \int_0^{V_F(\tau)} (p_{m,\varphi}^F(j, \tau))^{1-\sigma} dj \right]}, \quad (4)$$

$$c_m^F(j, t_0, \tau) = \frac{\xi e(t_0, \tau)(p_{m,\varphi}^F(j, \tau))^{-\sigma}}{\left[ \int_0^{V_H(\tau)} (p_m^H(i, \tau))^{1-\sigma} di + \int_0^{V_F(\tau)} (p_{m,\varphi}^F(j, \tau))^{1-\sigma} dj \right]} \quad (5)$$

as well as to the consumption Euler equation for the representative individual of cohort  $t_0$

$$\frac{\dot{e}(t_0, \tau)}{e(t_0, \tau)} = \frac{\pi(\tau)}{Fw(\tau)} - \delta - \rho. \quad (6)$$

As first shown by Yaari (1965) the representative individual's Euler equation with fully insured lifetime uncertainty is identical to the Euler equation when no lifetime uncertainty exists, i.e. the individual Euler equation does not include the mortality rate.

### 2.3 Aggregate expenditures and capital

Due to the overlapping generations structure resulting from the introduction of mortality into the constructed capital framework, our model set-up does not feature *one* single representative individual. In order to be able to analyse the long-run equilibrium of the economy as well as its stability properties it is therefore necessary to derive the aggregate law of motions of capital and consumption expenditures. The capital stock of the economy at a certain point in time  $t$  is the aggregate of individual capital stocks integrated over all birth dates. Analogous definitions apply to consumption expenditures and the available efficiency units of labour. These aggregation rules are formally given by

$$K(t) \equiv \int_{-\infty}^t k(t_0, t) N(t_0, t) dt_0, \quad (7)$$

$$E(t) \equiv \int_{-\infty}^t e(t_0, t) N(t_0, t) dt_0, \quad (8)$$

$$L(t) \equiv \int_{-\infty}^t l(t_0, t) N(t_0, t) dt_0, \quad (9)$$

where  $K(t)$  is the aggregate capital stock,  $L(t)$  refers to the total amount of available efficiency units of labour and  $E(t)$  denotes aggregate consumption expenditures. Equivalent equations hold for the foreign region.

Using the demographic assumptions described in section 2.1 we can exactly trace the size  $N(t_0, t)$  of any particular cohort over time. A cohort born at time  $t_0$  is of size  $\mu e^{-\mu(t-t_0)}$  at time  $t \geq t_0$  as the probability of surviving to time  $t$  equals  $e^{-\mu(t-t_0)}$  and the initial size



of the cohort is  $\mu$ . Substituting for  $N(t_0, t)$  in equation (8) therefore yields

$$E(t) \equiv \mu \int_{-\infty}^t e(t_0, t) e^{-\mu(t-t_0)} dt_0. \quad (10)$$

The aggregate Euler equation directly follows from equation (10) by differentiating it with respect to  $t$  and then substituting for  $\dot{e}(t_0, t)$  from the individual Euler equation (6) and for  $e(t, t)$  and  $E(t)$  from the corresponding expressions derived in appendix B where we describe the various aggregation steps in detail.<sup>5</sup> It is given by

$$\frac{\dot{E}(t)}{E(t)} = -\mu(\rho + \mu)Fw(t)\frac{K(t)}{E(t)} + \frac{\pi(t)}{w(t)F} - \rho - \delta \quad (11)$$

$$= -\mu \frac{E(t) - e(t, t)}{E(t)} + \frac{\dot{e}(t_0, t)}{e(t_0, t)}. \quad (12)$$

This aggregate Euler equation, modified for the existence of overlapping generations of finitely lived agents, is identical to the individual Euler equation given in expression (6) except for an additional correction term resulting from the distributional effects caused by the turnover of generations (cf. Heijdra and van der Ploeg (2002), chapter 16). Optimal consumption expenditure growth is the same for all generations but older generations have a higher consumption expenditure level than younger generations because they are wealthier. Since newborns with no capital holdings continually replace dying old generations, aggregate consumption expenditure growth is smaller than individual consumption expenditure growth. The correction term on the right hand side of equation (11) therefore describes the difference between average consumption expenditures<sup>6</sup> and consumption expenditures by newborns as shown in equation (12). As  $E(t) - e(t, t)$  is unambiguously positive, a higher mortality rate decreases aggregate consumption expenditure growth. This is intuitively clear as a higher  $\mu$  implies a higher generational turnover and therefore a higher (negative) impact of the correction term, and it is also consistent with the life-cycle savings literature (see for example Gertler (1999)).

Similarly, the aggregate law of motion for the capital stock can be obtained. Rewriting equation (7) in analogy to equation (10) and then differentiating it with respect to  $t$  yields

$$\dot{K}(t) = \left[ \frac{\pi(t)}{w(t)F} - \delta \right] K(t) + \frac{w(t)L(t)}{w(t)F} - \frac{E(t)}{w(t)F}, \quad (13)$$

where we applied the same steps as in the derivation for the aggregate Euler equation shown in appendix B.<sup>7</sup> Compared to the law of motion for individual capital there appears no term featuring the mortality rate  $\mu$ . This captures the fact that  $\mu K(t)$  does not represent aggregate capital accumulation but is a transfer - via the life insurance company - from

<sup>5</sup>Those aggregation steps closely follow the ones described by Heijdra and van der Ploeg (2002) in chapter 16.

<sup>6</sup>Since we normalized total population size to 1, total consumption expenditures  $E(t)$  are equal to average consumption expenditures.

<sup>7</sup>In particular, we substituted for  $\dot{k}(t_0, t)$  from equation (2).

individuals who died to those who survived within a given cohort. As a consequence, aggregate capital accumulates at a rate  $\frac{\pi(t)}{w(t)F} - \delta$ , whereas individual capital attracts the actuarial interest rate  $\frac{\pi(t)}{w(t)F} + \mu - \delta$  for surviving individuals (cf. Heijdra and van der Ploeg (2002), chapter 16).

Summarizing, the mortality rate  $\mu$  enters the law of motion for the individual capital stock but disappears in the corresponding aggregate law of motion. This is in sharp contrast to the Euler equation, where we have seen that  $\mu$  does not show up at the individual level but is part of the aggregate consumption expenditure growth rate.

## 2.4 Production technology and profit maximization

Profit maximization in the manufacturing and agricultural sector closely follows Baldwin (1999) and yields various intermediate results that simplify the subsequent analysis of the long-run equilibrium. In particular, the way the manufacturing sector is modelled allows us to derive the rental rate of capital as a function of home and foreign capital stocks and expenditures.

### 2.4.1 Agricultural sector

The homogeneous agricultural good, which can be interpreted as food, is produced according to the following constant returns to scale production function

$$Y_n(t) = \frac{1}{\alpha} L_n(t), \quad (14)$$

where  $Y_n(t)$  denotes output of the agricultural sector,  $L_n(t)$  represents aggregate labour devoted to agricultural production, and  $\alpha$  is the unit input coefficient in the production of agricultural goods. Free trade of the agricultural good between home and foreign equalizes its price as long as each of the two regions produces some  $Y_n(t)$ . This can be shown to hold if  $\xi$ , the manufacturing share of consumption, is not too large (cf. Baldwin (1999)) which will be assumed from now on. Profit maximization under perfect competition implies that firms charge the following price which equals marginal costs

$$p_n(t) = \alpha w_n(t). \quad (15)$$

Since labour is perfectly mobile across sectors the wage rate in the economy  $w(t)$  satisfies

$$w_n(t) = w_m(t) = w_{inv}(t) = w(t), \quad (16)$$

where  $w_n(t)$ ,  $w_m(t)$  and  $w_{inv}(t)$  denote wages in the agricultural, manufacturing and investment sector. Therefore equation (15) pins down equilibrium wages which are equalized across regions due to free trade. Moreover, by choice of units for agricultural output,  $\alpha$  can be set to one implying that the wage rate is equal to the price of the agricultural good.

Finally, choosing the agricultural good as numeraire leads to

$$w(t) = w^*(t) = 1. \quad (17)$$

#### 2.4.2 Manufacturing sector

Each firm in the Dixit and Stiglitz (1977) monopolistically competitive manufacturing sector produces a different output variety using labour as variable and one variety-specific machine as fixed input. This machine originates from the investment sector and is equivalent to one unit of capital. Due to the fixed costs, firms face an increasing returns to scale production technology with an associated cost function

$$\pi(t) + w(t)\beta Y_m(i, t), \quad (18)$$

where  $\beta$  is the unit input coefficient for efficiency units of labour,  $Y_m(i, t)$  is total output of one manufacturing good producer and the capital rental rate  $\pi(t)$  represents the fixed cost. Since we have variety specificity of capital and free entry into the manufacturing sector driving pure profits down to zero, this capital rental rate is equivalent to the Ricardian surplus, i.e. the operating profit of each manufacturing firm. In particular, the insurance companies, which hold all the capital (cf. section 2.2), rent their capital holdings to the manufacturing firms and can fully extract all profits.

Defining<sup>8</sup>  $P_m(t) \equiv \int_0^{V_H(t)} (p_m^H(i, t))^{1-\sigma} di + \int_0^{V_F(t)} (p_{m,\varphi}^F(j, t))^{1-\sigma} dj$  and  $P_m^*(t) \equiv \int_0^{V_F(t)} (p_m^H(j, t))^{1-\sigma} dj + \int_0^{V_H(t)} (p_{m,\varphi}^F(i, t))^{1-\sigma} di$  and recognizing that each individual firm has mass zero and hence does not influence the price indexes  $P_m$  and  $P_m^*$ , leads to the following maximization problem for each firm at time  $t$ <sup>9</sup>

$$\begin{aligned} \max_{p_m^H, p_{m,\varphi}^F} \quad & (p_m^H(i, t) - w(t)\beta) \left( \int_{-\infty}^t c_m^H(i, t_0, t) N(t_0, t) dt_0 \right) \\ & + (p_{m,\varphi}^F(i, t) - w(t)\varphi\beta) \left( \int_{-\infty}^t c_m^{H*}(i, t_0, t) N^*(t_0, t) dt_0 \right) \\ \text{s.t.} \quad & c_m^H(i, t_0, t) = \frac{\xi e(t_0, t) (p_m^H(i, t))^{-\sigma}}{P_m(t)} \\ & c_m^{H*}(i, t_0, t) = \frac{\xi e^*(t_0, t) (p_{m,\varphi}^F(i, t))^{-\sigma}}{P_m^*(t)}. \end{aligned} \quad (19)$$

<sup>8</sup>Note that  $p_m^{*H}(i, t) = p_{m,\varphi}^F(i, t)$  and  $p_m^{*F}(j, t) = p_m^H(j, t)$  due to symmetry between the two regions, where  $p_m^{*H}(i, t)$  is the price of a good manufactured in the home economy but sold in the foreign region.

<sup>9</sup>We ignore fixed costs in the derivations here as they do not influence the first order conditions. Therefore we just maximize operating profits defined as revenues from selling the variety to the home and foreign region minus variable production costs (taking into account the effect of transport costs).

Carrying out the associated calculations shown in appendix C gives expressions for optimal prices

$$p_m^H(i, t) = \frac{\sigma}{\sigma - 1} w(t) \beta, \quad (20)$$

$$p_{m,\varphi}^F(i, t) = \frac{\sigma}{\sigma - 1} w(t) \beta \varphi. \quad (21)$$

Therefore the profit maximization problem yields the familiar rule that prices are equal to a constant mark-up over marginal costs which decreases in  $\sigma$ . This implies that a higher elasticity of substitution reduces the market power of manufacturing firms. Moreover, mill pricing is optimal, i.e. the only difference between prices in the two regions is due to transport costs (cf. Baldwin et al. (2003)).

Using the first order conditions of the maximization problem formulated in equation (19) (see appendix C as well as Baldwin (1999) for details of the following derivations) and defining

$$s_H^H(t) \equiv \frac{p_m^H(t) \int_{-\infty}^t c_m^H(t_0, t) N(t_0, t) dt_0}{\int_{-\infty}^t \xi e(t_0, t) N(t_0, t) dt_0}, \quad (22)$$

$$s_H^F(t) \equiv \frac{p_{m,\varphi}^F(t) \int_{-\infty}^t c_m^{H*}(t_0, t) N^*(t_0, t) dt_0}{\int_{-\infty}^t \xi e^*(t_0, t) N^*(t_0, t) dt_0} \quad (23)$$

as the share of a domestic firm in the home and in the foreign market with equivalent definitions holding for  $s_F^F(t)$  and  $s_F^H(t)$  gives rental rates in the home and foreign region as

$$\pi(t) = \frac{\xi}{\sigma} \left[ \int_{-\infty}^t s_H^H(t) e(t_0, t) N(t_0, t) dt_0 + \frac{\xi}{\sigma} \int_{-\infty}^t s_H^F(t) e^*(t_0, t) N^*(t_0, t) dt_0 \right], \quad (24)$$

$$\pi^*(t) = \frac{\xi}{\sigma} \left[ \int_{-\infty}^t s_F^F(t) e^*(t_0, t) N^*(t_0, t) dt_0 + \frac{\xi}{\sigma} \int_{-\infty}^t s_F^H(t) e(t_0, t) N(t_0, t) dt_0 \right]. \quad (25)$$

Again using mill pricing and redefining global quantities and regional share variables leads to the final expressions for regional rental rates<sup>10</sup>

$$\pi = \underbrace{\left( \frac{\theta_E}{\theta_K + \phi(1 - \theta_K)} + \frac{(1 - \theta_E)\phi}{\phi\theta_K + 1 - \theta_K} \right)}_{Bias} \left( \frac{\xi E^W}{\sigma K^W} \right), \quad (26)$$

$$\pi^* = \underbrace{\left( \frac{1 - \theta_E}{1 - \theta_K + \phi\theta_K} + \frac{\theta_E\phi}{\phi(1 - \theta_K) + \theta_K} \right)}_{Bias^*} \left( \frac{\xi E^W}{\sigma K^W} \right), \quad (27)$$

where  $\phi \equiv \varphi^{1-\sigma}$  is a measure of openness between the two regions with  $\phi = 0$  indicating prohibitive trade barriers and  $\phi = 1$  free trade. World expenditures are defined as  $E^W \equiv$

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<sup>10</sup>We ignore time arguments here.

$E + E^*$  and the world capital stock as  $K^W \equiv K + K^*$  with  $\theta_K$  and  $\theta_E$  being the respective home shares of these quantities, i.e.  $\theta_K \equiv \frac{K}{K+K^*}$  and  $\theta_E \equiv \frac{E}{E+E^*}$ . As expected, these rental rates are identical to those derived in Baldwin (1999)'s constructed capital model, since the introduction of mortality does not change the production side of the economy. Analogously to Baldwin (1999), the terms labelled *Bias* and *Bias*<sup>\*</sup> can be interpreted as the bias in national sales, i.e. *Bias* measures the extent to which a home variety's sales exceed the world average sales per variety. Additionally, these terms describe the impact of production and expenditure shifting on profits. In the symmetric case with  $\theta_K = 1/2$  and  $\theta_E = 1/2$ , shifting expenditure to home ( $d\theta_E > 0$ ) raises  $\pi$  and lowers  $\pi^*$  since it increases the market size at home. Production shifting<sup>11</sup> to home ( $d\theta_K > 0$ ), on the other hand, has the opposite impact as it increases competition in the home market. It can be shown that lowering trade costs weakens the magnitude of both effects but erodes the local competition effect more rapidly. Consequently, agglomerative tendencies gain weight as integration between the two regions is increased. The crucial question to be investigated in the following sections is whether these agglomerative tendencies are strong enough to create a core periphery outcome with all capital, and thus all manufacturing firms, located in one region which is the case in the constructed capital model of Baldwin (1999).

### 3 Long-run equilibrium

The dynamics of this neoclassical growth model with overlapping generations are fully described by the following four dimensional system in the variables  $E$ ,  $E^*$ ,  $K$  and  $K^*$  whose equations were derived in section 2.3 and are given by<sup>12</sup>

$$\dot{K} = \left[ \frac{\xi}{\sigma F} \left( \frac{E}{K + \phi K^*} + \frac{\phi E^*}{\phi K + K^*} \right) - \delta \right] K + \frac{L}{F} - \frac{E}{F}, \quad (28)$$

$$\dot{E} = -\mu(\rho + \mu)FK + E \left[ \frac{\xi}{\sigma F} \left( \frac{E}{K + \phi K^*} + \frac{\phi E^*}{\phi K + K^*} \right) - \rho - \delta \right], \quad (29)$$

$$\dot{K}^* = \left[ \frac{\xi}{\sigma F} \left( \frac{E^*}{K^* + \phi K} + \frac{\phi E}{\phi K^* + K} \right) - \delta \right] K^* + \frac{L}{F} - \frac{E^*}{F}, \quad (30)$$

$$\dot{E}^* = -\mu(\rho + \mu)FK^* + E^* \left[ \frac{\xi}{\sigma F} \left( \frac{E^*}{K^* + \phi K} + \frac{\phi E}{\phi K^* + K} \right) - \rho - \delta \right]. \quad (31)$$

Here we used that the equilibrium wage rate is equal to one in both regions and we already substituted for the rental rates from equations (26) and (27).<sup>13</sup> For the special case of  $\mu = 0$  this system of equations is exactly identical to the one obtained by Baldwin (1999) with an infinitely lived representative agent.

<sup>11</sup>Note that the number of varieties in the home region,  $V_H(t)$ , is equal to the capital stock at home,  $K(t)$ , as one variety exactly requires one unit of capital as fixed input (analogously  $K^*(t) \equiv V_F(t)$ ). This implies that capital accumulation in one region is tantamount to firm creation.

<sup>12</sup>We again suppress time arguments here.

<sup>13</sup>Note also that due to the assumption of symmetric regions we have  $L = L^*$  and  $\mu = \mu^*$  as well as  $F = F^*$ ,  $\delta = \delta^*$ ,  $\rho = \rho^*$ ,  $\xi = \xi^*$  and  $\sigma = \sigma^*$ .

A long-run equilibrium of this model characterized by the steady-state values  $\bar{E}$ ,  $\bar{K}$ ,  $\bar{E}^*$  and  $\bar{K}^*$  must fulfil the system with the left hand side set equal to zero. It can be verified<sup>14</sup> that the symmetric outcome with  $K = K^*$  and  $E = E^*$  has this property with the steady-state values given by<sup>15</sup>

$$\bar{E}_{sym} = \frac{L\sigma \left( \sigma\delta^2 + \rho\sigma\delta - 2\mu(\mu + \rho)(\sigma - \xi) + \delta\sqrt{\sigma}\sqrt{\sigma(\delta + \rho)^2 + 4\mu(\mu + \rho)\xi} \right)}{2(\delta\sigma + (\mu + \rho)(\sigma - \xi))(\delta\sigma + \mu(\xi - \sigma))}, \quad (32)$$

$$\bar{K}_{sym} = \frac{\delta L\sigma(\sigma + \xi) + L\sqrt{\sigma}(\sigma - \xi) \left( \rho\sqrt{\sigma} - \sqrt{\sigma(\delta + \rho)^2 + 4\mu(\mu + \rho)\xi} \right)}{2F(\delta\sigma + (\mu + \rho)(\sigma - \xi))(\delta\sigma + \mu(\xi - \sigma))}. \quad (33)$$

Investigating how these steady-state values of consumption expenditures and capital react to varying mortality rates reveals some interesting features. The signs of the corresponding derivatives with respect to  $\mu$  are, however, analytically ambiguous. We therefore evaluated them at the following parameter values:  $\mu = 0.0125$  resulting in a life expectancy of 80 years<sup>16</sup>,  $\delta = 0.05$  implying that capital depreciates on average after 20 years,  $\rho = 0.015$ , which is the value used by Auerbach and Kotlikoff (1987), and  $L = 1$  and  $F = 2$ . Since there is considerable disagreement about the parameter values of  $\sigma$  and  $\xi$  in the literature, we used a wide range of plausible values in our numerical calculations. As far as the former is concerned, a plausible lower bound is  $\sigma = 2$  as in Baldwin (1999). Most authors, however, consider  $\sigma \approx 4$  (cf. Bosker and Garretsen (2007), Brakman et al. (2005), Krugman (1991), Krugman and Venables (1995), Martin and Ottaviano (1999) and Puga (1999)). In order to allow for all possibilities we choose as an upper bound  $\sigma = 8$ . With respect to  $\xi$ , which in fact describes the share of consumption expenditures for the good produced under increasing returns to scale (relative to the good produced under constant returns to scale), Head and Mayer (2003), Bosker and Garretsen (2007) and Puga (1999) consider a value of  $\xi = 0.1$ , Baldwin (1999) and Krugman (1991) set  $\xi = 0.3$ , Krugman and Venables (1995) choose  $\xi = 0.6$  and Martin and Ottaviano (1999) set  $\xi = 0.8$ . We therefore consider a possible parameter range of  $0.1 \leq \xi \leq 0.9$  to account for this wide spread.<sup>17</sup>

Figure 1 and 2 reveal that for those parameter ranges the derivative of  $\bar{E}_{sym}$  with respect to  $\mu$  is positive, whereas the derivative of  $\bar{K}_{sym}$  is negative.<sup>18</sup> Consequently,

<sup>14</sup>This and most other results were derived with Mathematica. The corresponding files are available from the authors upon request.

<sup>15</sup>Solving the system for the symmetric equilibrium values in fact yielded two solution pairs. As one of them gives negative equilibrium expenditures for plausible parameter values we restrict our attention to the economically meaningful solution pair.

<sup>16</sup>Since the probability of death during each year equals 0.0125, average life expectancy is  $\frac{1}{0.0125}$ .

<sup>17</sup>Recall, however, that production of the agricultural good in both regions requires  $\xi$  to be sufficiently small.

<sup>18</sup>We also investigated the derivatives for varying mortality rates. Assuming  $0.008 \leq \mu \leq 0.025$  leading to a life expectancy between 40 and 120 years, and still considering the aforementioned values for the other parameters, does not change our findings.

Figure 1: Derivative of  $\bar{E}_{sym}$  with respect to  $\mu$

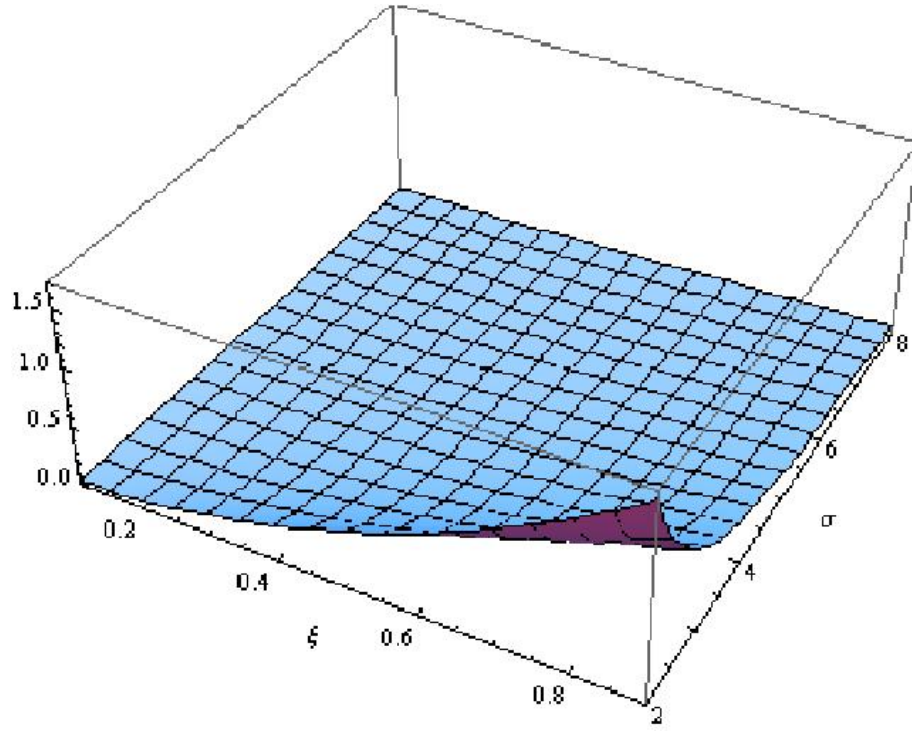
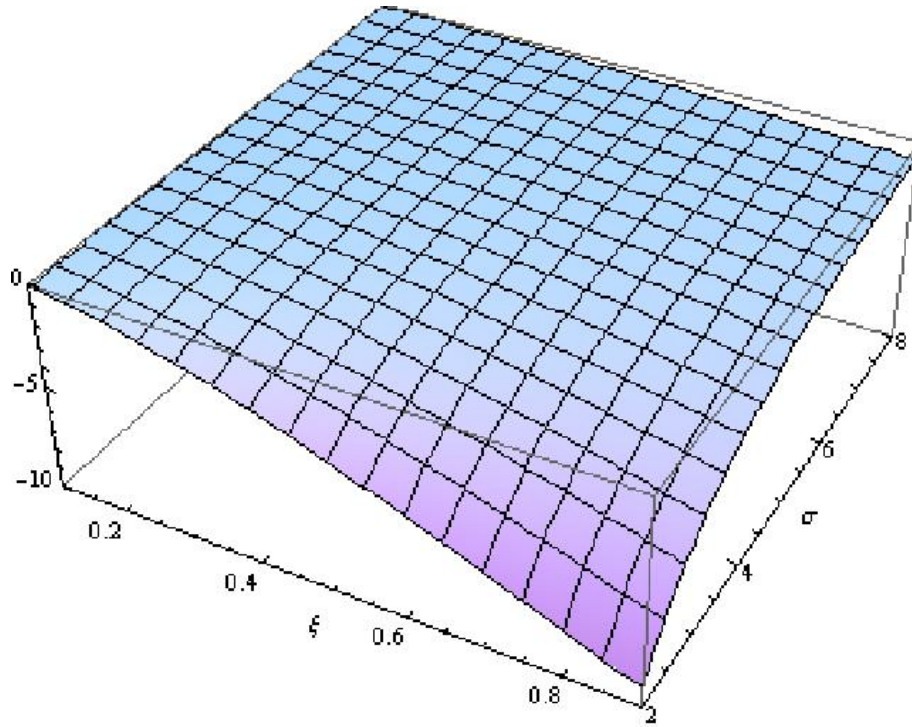


Figure 2: Derivative of  $\bar{K}_{sym}$  with respect to  $\mu$



a decrease in the mortality rate increases the equilibrium capital stock and decreases equilibrium expenditures, which is intuitively clear as a lower mortality rate increases the proportion of elderly to young individuals. Since the former hold more capital, the aggregate capital stock increases. This result is again consistent with the life-cycle savings literature (cf. Gertler (1999), Futagami and Nakajima (2001) and Zhang et al. (2003)).

When considering the effect of the mortality rate on the steady-state consumption expenditure share<sup>19</sup>,  $\frac{\bar{E}_{sym}}{\delta K_{sym} + E_{sym}}$ , even analytical results can be derived. This share is obtainable from the ratio of the equilibrium capital stock to the equilibrium expenditures<sup>20</sup>

$$\frac{\bar{K}_{sym}}{\bar{E}_{sym}} = \frac{2\xi}{F(\delta\sigma + \rho\sigma + \sqrt{\sigma}\sqrt{\sigma(\delta + \rho)^2 + 4\mu(\mu + \rho)\xi})}, \quad (34)$$

which obviously depends negatively on the mortality rate. The steady-state consumption expenditure share therefore increases with  $\mu$ . This again illustrates that a higher mortality rate decreases savings and therefore increases consumption relative to income.

## 4 Symmetric equilibrium *stability* -

### The impact of introducing mortality on catastrophic agglomeration

New Economic Geography models emphasize that reciprocal liberalization between initially symmetric regions leads to catastrophic agglomeration. In this section we show that the introduction of mortality considerably reduces this possibility of the symmetric equilibrium to be unstable such that agglomeration processes may not set in even if economic integration is promoted up to a high degree.

#### 4.1 Analytical results

The stability properties of the symmetric long-run equilibrium for varying trade costs and mortality rates are analysed by following the classical approach (cf. Barro and Sala-i-Martin (2004)) of linearising the non-linear dynamic system given in equations (28), (29), (30) and (31) around the symmetric equilibrium and then by evaluating the eigenvalues of the corresponding  $4 \times 4$  Jacobian matrix

$$J_{sym} = \begin{pmatrix} J1 & J2 \\ J3 & J4 \end{pmatrix}, \quad (35)$$

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<sup>19</sup>This share is defined as equilibrium consumption expenditures divided by steady-state income, where steady-state income is the sum of replacement investment,  $\delta K$  (equal to savings in steady-state), and consumption expenditures.

<sup>20</sup>Simply calculate  $\frac{1}{\frac{\delta K_{sym}}{\bar{E}_{sym}} + 1}$ .



where the four symmetric  $2 \times 2$  sub-matrices  $J_i$  for  $i = 1, \dots, 4$  are given in appendix D. Solving the characteristic equation yields the following four eigenvalues

$$eig1 = \frac{1}{2}(real_1 - \sqrt{rad_1}), \quad (36)$$

$$eig2 = \frac{1}{2}(real_1 + \sqrt{rad_1}), \quad (37)$$

$$eig3 = \frac{1}{(\phi + 1)^2 \sqrt{\sigma}}(real_2 - \sqrt{rad_2}), \quad (38)$$

$$eig4 = \frac{1}{(\phi + 1)^2 \sqrt{\sigma}}(real_2 + \sqrt{rad_2}), \quad (39)$$

where

$$\begin{aligned} real_1 &\equiv \frac{A}{\sqrt{\sigma}} - \delta, \\ rad_1 &\equiv \left( \frac{A}{\sqrt{\sigma}} + \delta \right)^2 + \frac{(\sigma - \xi) \left( (A + B)^2 + 4\mu(\mu + \rho)\xi \right)}{\sigma\xi}, \\ real_2 &\equiv 3\phi A + A - \sqrt{\sigma} \left( \delta(2\phi^2 + \phi + 1) + (\phi - 1)\phi\rho \right), \\ rad_2 &\equiv \frac{\left( A(\phi - 1) + (\delta(\phi - 1) + \phi(\phi + 3)\rho)\sqrt{\sigma} \right)^2 + (\phi + 1)(\phi\sigma + \sigma + \phi\xi - \xi) \left( (A + B)^2(\phi - 1)^2 + 4\mu(\phi + 1)^2(\mu + \rho)\xi \right)}{\xi}, \end{aligned}$$

with the parameter clusters  $A \equiv \sqrt{\sigma(\delta + \rho)^2 + 4\mu(\mu + \rho)\xi}$  as well as  $B \equiv (\delta + \rho)\sqrt{\sigma}$ . The signs and nature of these eigenvalues fully characterize the system's local dynamics around the symmetric equilibrium. Analytically investigating them<sup>21</sup> thus results in lemma 1.

**Lemma 1.** *Eigenvalue 3 is decisive for the local stability properties of the symmetric equilibrium. A positive eigenvalue 3 implies instability, a negative one saddle path stability.*

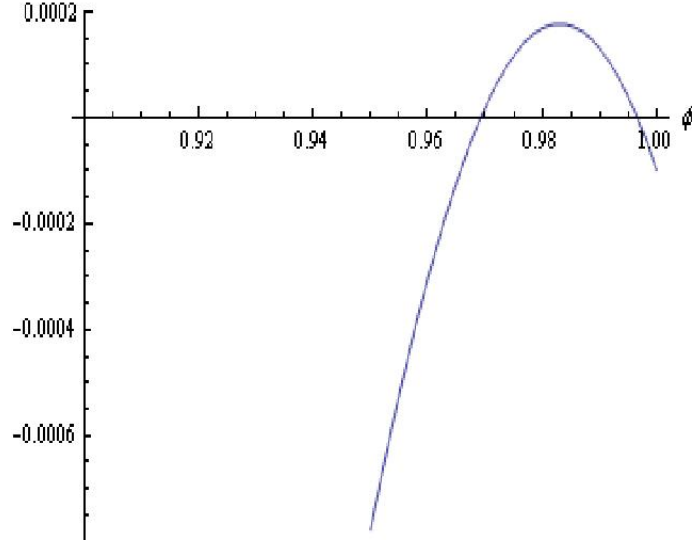
*Proof.* By investigating the expressions for the eigenvalues it is first easily established that all of them are real. This holds since their radicals are non-negative for  $\sigma > \xi$  which is true for all parameter ranges considered<sup>22</sup>. Convergence to or divergence from the symmetric equilibrium is therefore monotonic.

As there are two jump variables  $E$  and  $E^*$ , saddle path stability prevails if and only if there are two negative eigenvalues. If fewer than two eigenvalue are negative, the system is locally unstable. By inserting the expression for A, it is easily established that  $real_1 > 0$ . We can therefore immediately conclude that eigenvalue 2 is positive for all parameter values. In order to find out the sign of eigenvalue 1, we compare the real term with the corresponding term under the radical. The square of the former is smaller than the latter, implying that eigenvalue 1 is always negative. It remains to investigate the signs of eigenvalues 3 and 4. Again we first check whether the real part is non-negative for all

<sup>21</sup>In order to get a first idea about the signs and nature of the eigenvalues, we also calibrated the model and investigated the eigenvalues numerically. The corresponding findings are presented in appendix D.

<sup>22</sup>Recall the parameter ranges  $\sigma > 1$ ,  $0 < \delta \leq 1$ ,  $0 < \rho \leq 1$ ,  $0 < \mu \leq 1$ ,  $0 < \xi < 1$  and  $0 \leq \phi \leq 1$  which imply that  $A > 0$  and  $B > 0$ .

Figure 3: **Eigenvalue 3 for varying trade costs given  $\mu = 0.0001$**



parameter values. By inserting the expression for A,  $real_2$  can be rewritten as

$$real_2 = \underbrace{-\sqrt{\sigma}\delta(2\phi^2 + \phi + 1)}_{term1} + \underbrace{\sqrt{\sigma}(1 - \phi)\phi\rho}_{term2} + \underbrace{(1 + 3\phi)\sqrt{\sigma(\delta + \rho)^2 + 4\mu(\mu + \rho)\xi}}_{term3}. \quad (40)$$

All three terms are increasing in  $\rho$ ,  $\xi$  and  $\mu$  but react differently to changes in  $\phi$ ,  $\delta$  and  $\sigma$ . In order to show that  $real_2$  is nevertheless non-negative for all parameter values we set  $\rho$ ,  $\xi$  and  $\mu$  close to zero resulting in the “worst”, i.e. most negative, outcome with respect to these parameters and then check for which values of  $\phi$ ,  $\delta$  and  $\sigma$  the above real part is still positive by solving the corresponding system of inequalities. We find that this holds for the whole parameter space implying that the fourth eigenvalue is definitely positive. Summarizing, we have established that eigenvalue 2 and 4 are always positive, whereas eigenvalue 1 is always negative. This proves the crucial role of the third eigenvalue.  $\square$

Having demonstrated that changes in the parameter values, and in particular of the mortality rate, can only influence the stability properties of the symmetric equilibrium via eigenvalue 3, it is immediate to investigate this eigenvalue more thoroughly. Figure 3 plots eigenvalue 3 as a function of  $\phi$  for  $\mu = 0.0001$  given our choice of the most plausible values of the other parameters ( $\rho = 0.015$ ,  $\delta = 0.05$ ,  $\xi = 0.3$  and  $\sigma = 4$ ). The graph shows that, depending on the level of trade costs, eigenvalue 3 is either positive or negative.<sup>23</sup> The crucial question, however, is whether changes in the mortality rate also influence the sign of eigenvalue 3.

<sup>23</sup>The numerical investigation of eigenvalue 3 in appendix D also reveals that it is impossible to come up with a definite sign for the whole parameter space.

**Proposition 1.** *The sign of eigenvalue 3 and hence saddle path stability of the symmetric equilibrium depends on the mortality rate.*

*Proof.* To prove this proposition, we use the concept of the critical level of trade costs  $\phi_{break}$ . This threshold value identifies the degree of openness where eigenvalue 3 changes its sign and therefore where the stability properties of the symmetric equilibrium change (cf. where eigenvalue 3 crosses the horizontal axis in figure 3). To analytically obtain  $\phi_{break}$  we set the expression for the third eigenvalue equal to zero and solve the resulting equation. This yields two solutions for  $\phi_{break}$  as functions of the other parameters.<sup>24</sup> Since these two critical levels in particular also depend on the mortality rate, proposition 1 holds.  $\square$

## 4.2 The impact of mortality on catastrophic agglomeration

Using the results obtained in the previous section 4.1 we are now ready to investigate the effects of ageing on catastrophic agglomeration. Most notably, we will show that a realistic mortality rate rules out the possibility of instability of the symmetric equilibrium in the constructed capital model of Baldwin (1999).

Figure 4, which plots the contour lines of eigenvalue 3 for varying  $\mu$  and  $\phi$ <sup>25</sup> given our choice of the most plausible values of the other parameters ( $\rho = 0.015$ ,  $\delta = 0.05$ ,  $\xi = 0.3$  and  $\sigma = 4$ ), illustrates that there only exists a very small range of combinations of  $\mu$  and  $\phi$  where the sign of the third eigenvalue is positive. This instability region is characterized by parameter combinations inside the contour line=0 which yield a non-negative eigenvalue 3. Higher transport costs<sup>26</sup> or a higher mortality rate decrease the value of eigenvalue 3 rather quickly. Only in case of an implausibly low mortality rate it is possible to find critical values of transport costs within which the symmetric equilibrium becomes unstable and agglomeration can set in.

Similar conclusions are obtained from investigating how the critical level of trade costs reacts to changes in the mortality rate. Without mortality, i.e.  $\mu = 0$ , and the parameter values assumed by Baldwin (1999), i.e.  $\rho = \delta = 0.1$ ,  $\xi = 0.3$  and  $\sigma = 2$ , the two critical levels of trade costs are  $\phi_{break1} = 0.860465$  and  $\phi_{break2} = 1$ .<sup>27</sup> In between those values, i.e. for sufficiently low levels of trade costs, the symmetric equilibrium is unstable and catastrophic agglomeration does occur. Allowing  $\mu$  to be positive, however, and still assuming the same values as Baldwin (1999) for the other parameters, establishes that  $\phi_{break1}$  increases, while  $\phi_{break2}$  decreases with  $\mu$  (cf. figure 5). The range where the symmetric equilibrium is unstable clearly shrinks (in figure 3 an increase in  $\mu$  would thus shift eigenvalue 3 downwards). A higher mortality rate therefore stabilizes the symmetric equilibrium and prevents the two regions from unequal development. In particular, we can

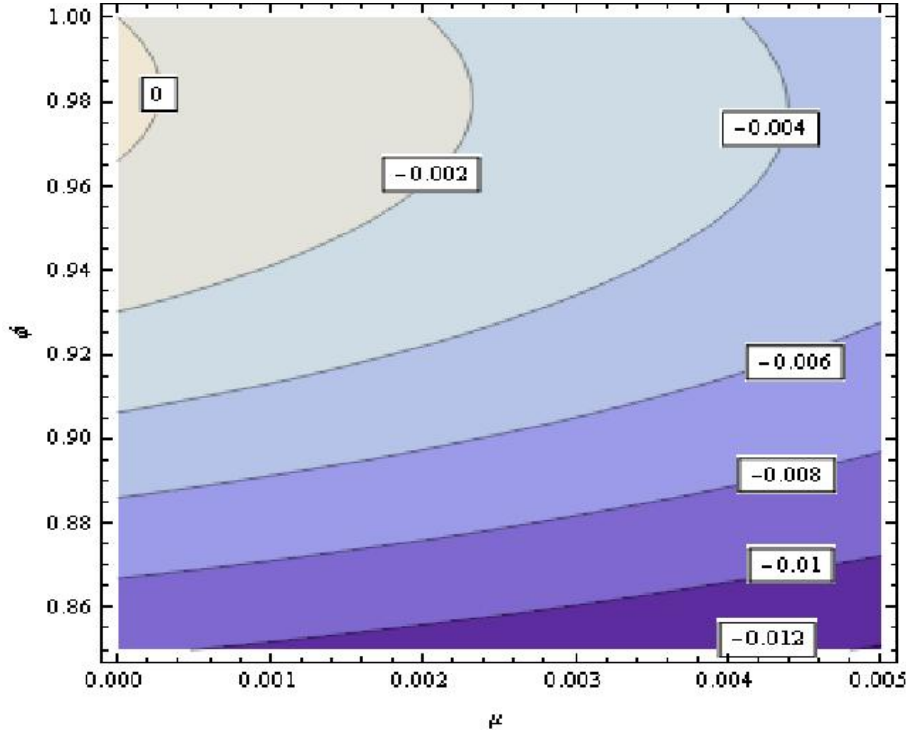
<sup>24</sup>As the expressions are rather cumbersome they are not presented here but available upon request.

<sup>25</sup>Note that we plot this figure only for  $\mu > 0.005$  and  $\phi > 0.85$  which indicates how small the instability region relative to the whole parameter range is.

<sup>26</sup>Remember that a lower value of  $\phi$  is equivalent to higher transport costs.

<sup>27</sup>These values are exactly identical to those obtained by Baldwin (1999).

Figure 4: Contour plot of eigenvalue 3



establish that for  $\mu > 0.00395$ , implying a (plausible) life expectancy of less than approximately 250 years, there exists no level of trade costs such that the symmetric equilibrium is unstable (i.e. the downward shift in figure 3 is such that eigenvalue 3 does not cross the horizontal axis anymore where it would become positive<sup>28</sup>).<sup>29</sup> In sharp contrast to Baldwin (1999)'s catastrophic agglomeration result, our model thus predicts the symmetric outcome to be the dominating one even in the presence of high economic integration. The introduction of finitely lived individuals profoundly stabilizes the symmetric equilibrium.

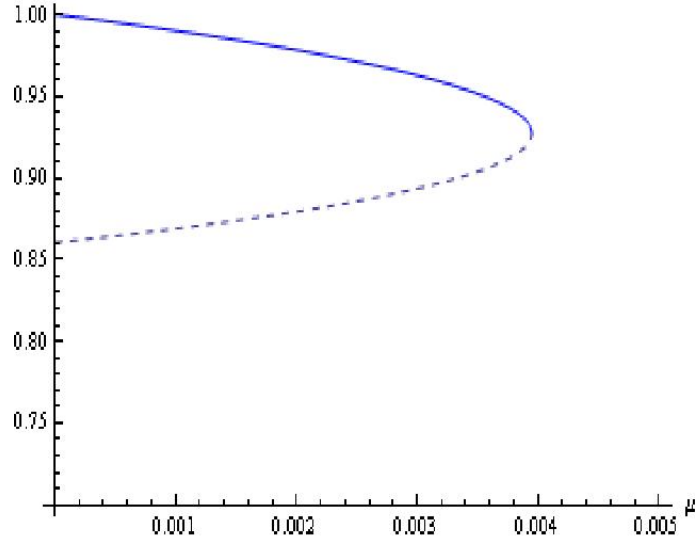
### 4.3 Economic intuition

One immediate question refers to how and why the mortality rate influences the forces in our model that determine the stability properties of the symmetric equilibrium. As shown by Baldwin (1999), the formal stability analysis pursued in section 4.1 yields the same results as compared to a more informal way of checking the stability of the symmetric equilibrium. This informal way is based on investigating how an exogenous perturbation of the home share of capital,  $\theta_K$ , influences the profitability of home-based firms relative to

<sup>28</sup>Note that we have already plotted figure 3 for a very low mortality rate  $\mu = 0.0001$  to illustrate a case where it crosses the axis.

<sup>29</sup>We also performed these simulations with respect to the critical level of trade costs for other parameter ranges, in particular for our choice of the most plausible values,  $\rho = 0.015$ ,  $\delta = 0.05$ ,  $\xi = 0.3$  and  $\sigma = 4$ . In this case the critical mortality rate, above which the symmetric equilibrium is always stable, is given by  $\mu = 0.00028$ . This implies that if we (realistically) assume a life expectancy of less than approximately 3500 years we never get instability.

Figure 5:  $\phi_{break1}$  (dashed) and  $\phi_{break2}$  (solid) as a function of  $\mu$



foreign-based firms. A positive impact would imply instability as even more firms would locate in the home region. Conducting this more informal stability analysis can yield valuable insights into the forces fostering or weakening agglomeration in our model.

The main mechanism behind all agglomeration tendencies is the following demand linked circular causality between consumption expenditures and capital: If expenditures in one region are exogenously increased, this implies higher profits such that more firms enter the market, leading to a higher capital stock in the region. The higher capital stock is associated with higher income and therefore further increases expenditures and profitability. This agglomeration force was first introduced by Baldwin (1999) and is due to the endogeneity of capital in his model. It hinges critically on the immobility of capital as only in this case capital income cannot be repatriated to its immobile owners and therefore increases the region's own income. In our model with capital immobility it is, however, indeed the case that the equilibrium value of consumption expenditures depends, via this income effect, on the capital stock. If stability of the symmetric equilibrium is now investigated by the informal approach mentioned above, this particular equilibrium reaction must be taken into account. Therefore the sign of the derivative  $(d\pi/d\theta_K)_{equ} \equiv d\pi[\theta_K, \bar{\theta}_E[\theta_K]; \phi]/d\theta_K$ <sup>30</sup> evaluated at the symmetric equilibrium should be checked, where  $\bar{\theta}_E[\theta_K]$  gives equilibrium expenditures as a function of the capital stock. As already mentioned, stability requires this derivative to be negative since then, as Baldwin (1999) puts it, “...if a unit of capital ‘accidentally’ disturbed symmetry, the ‘accident’ pushes capital’s rental rate below its steady-state level in the ‘receiving’ nation (home). This induces home savers/investors to allow  $K$  to erode back to its pre-shock level. More-

<sup>30</sup>We add the subscript  $(...)_{equ}$  to indicate that equilibrium reactions of  $\theta_E$  are taken into account in this derivative in contrast to the pure local competition effect  $\frac{d\pi}{d\theta_K}$  described in section 2.4.2.

over, since  $d\pi[\dots]/d\theta_K > 0$  means  $d\pi^*[\dots]/d\theta_K < 0$ , foreign savers react in the opposite direction.” (Baldwin, 1999, p. 263). To gain more insights about the forces at work we rewrite the derivative as

$$\left(\frac{d\pi}{d\theta_K}\right)_{equ} = \frac{d\pi}{d\theta_K} + \frac{d\pi}{d\theta_E} \frac{d\bar{\theta}_E}{d\theta_K}. \quad (41)$$

The first term on the right hand side of this equation,  $\frac{d\pi}{d\theta_K}$ , represents the anti-agglomerative local competition effect which is negative and was already described in section 2.4.2. The second term is the pro-agglomerative circular causality force. It captures the fact that shifting production increases capital income in the receiving nation and therefore expenditures, i.e.  $\frac{d\bar{\theta}_E}{d\theta_K} > 0$ . This in turn increases profits since  $\frac{d\pi}{d\theta_E} > 0$ , as was also mentioned in section 2.4.2, and therefore induces further capital accumulation. Since both, the negative local competition effect,  $\frac{d\pi}{d\theta_K}$ , and the positive effect of expenditures on profits,  $\frac{d\pi}{d\theta_E}$ , are independent of the mortality rate (cf. section 2.4.2), the introduction of ageing can only influence stability via the effect of production shifting on expenditures, captured by the term  $\frac{d\bar{\theta}_E}{d\theta_K}$ . In order to confirm our stability result with respect to  $\mu$  obtained by investigating the eigenvalues, this derivative must decrease in the mortality rate such that higher mortality weakens the agglomerative force and therefore increases stability. To put it differently, the mortality rate determines how much weight is given to the agglomeration force as compared to the dispersion force. The comparative statics result of section 3 suggest some possible lines of explanation why the production shifting effect on expenditures should decrease in the mortality rate.<sup>31</sup> There we have shown that for plausible parameter values a higher  $\mu$  implies a larger proportion of young and poor compared to old and wealthy individuals. Aggregate equilibrium expenditures are therefore already higher and do not react that much to production shifting. This effect seems to dominate all the other forces that work in favour of an increase of  $\frac{d\bar{\theta}_E}{d\theta_K}$  in  $\mu$ , e.g. the increase in equilibrium profits resulting from a higher mortality rate.<sup>32</sup>

## 5 Concluding remarks

The model in this paper generalizes the constructed capital framework of Baldwin (1999) by allowing for more realistic demographic structures. In particular, incorporating finite planning horizons makes it possible to investigate the impacts of population ageing on agglomeration tendencies of economic activities. We show that in the case of reasonable mortality rates, even very low levels of trade costs do *not* lead to catastrophic agglomeration. Introducing mortality therefore stabilizes the symmetric equilibrium and acts as a force that promotes a more equal distribution of productive factors between two regions.

<sup>31</sup>Doing the informal stability analysis of Baldwin (1999) is impossible in our model set-up due to the fact that the additional expression  $-\mu(\rho + \mu)FK$  appears in the aggregate Euler equations as an additive term and makes them highly non-linear. As a consequence the production shifting effect on expenditures,  $\frac{d\bar{\theta}_E}{d\theta_K}$ , is not obtainable.

<sup>32</sup>The positive dependence of equilibrium profits on the mortality rate can be shown easily. It indicates that a higher mortality rate amplifies the expenditure shifting effect that is associated with shifting production (capital) to the other region.

From the point of view of economic policy, the most important insight is that, in sharp contrast to other New Economic Geography approaches, our model does not necessarily associate deeper integration with higher interregional inequality. In particular, we have shown that plausible parameter values are far away from supporting core-periphery outcomes. Consequently, there is no need to impose trade barriers like tariffs and quotas to increase transport costs in order to avoid de-industrialization of one region. Especially in the case of Europe this implies that there is no tradeoff between the two most important targets of the European Union: integration on the one hand and interregional equality on the other hand. Instead, the implementation of appropriate policies to achieve one objective does not interfere with the realization of the other goal.

However, introducing mortality was only a first step in making Baldwin (1999)'s constructed capital model more realistic. The assumption of a constant mortality rate adopted for the sake of analytical tractability is still at odds with reality. Using age dependent mortality rates is therefore one possible line for future research. Moreover, it would be worthwhile to consider the effects of varying mortality rates between regions. In such a setting one could investigate how differences in mortality rates are linked to differences in capital accumulation rates, again a question of high relevance for economic policy.

## Appendix

### A The individual's utility optimization problem

Suppressing time arguments in the optimization procedure the current value Hamiltonian for the individual's optimization problem can be written as

$$H(e, k, \lambda, t) = \ln \left[ \frac{e}{P} \right] + \lambda \left( \frac{wl + \pi k - e}{wF} + \mu k - \delta k \right) \quad (42)$$

where  $P$  is the perfect price index translating expenditures into indirect utility.<sup>33</sup> The first order conditions of the problem associated with equation (42) are given by

$$\frac{\partial H}{\partial e} \doteq 0 \Rightarrow \frac{1}{e} = \frac{\lambda}{Fw}, \quad (43)$$

$$\frac{\partial H}{\partial k} \doteq (\rho + \mu)\lambda - \dot{\lambda} \Rightarrow \frac{\dot{\lambda}}{\lambda} = -\frac{\pi}{Fw} + \rho + \delta, \quad (44)$$

$$\frac{\partial H}{\partial \lambda} \doteq \dot{k} \Rightarrow \frac{wl + \pi k - e}{wF} + \mu k - \delta k = \dot{k} \quad (45)$$

and the standard transversality condition. Taking the time derivative of equation (43) under the assumption that  $w$  is time independent<sup>34</sup> and combining it with equation (44) yields the consumption Euler equation for the representative individual

$$\frac{\dot{e}}{e} = \frac{\pi}{Fw} - \delta - \rho.$$

The static problem of dividing consumption between the manufacturing composite and the agricultural good for fixed consumption expenditure  $e$  can be formulated as

$$\begin{aligned} & \max_{c_m^{agg}, c_n} (c_n)^{1-\xi} (c_m^{agg})^\xi \\ \text{s.t. } & p_n c_n + p_m^{agg} c_m^{agg} = e, \end{aligned} \quad (46)$$

where  $p_m^{agg}$  is an appropriate price index which can be shown to equal a weighted average of the two Dixit and Stiglitz (1977) price indexes at home and foreign with the foreign price index being augmented by transport costs. Setting up the Lagrangian as

$$\ell(c_n, c_m^{agg}, \lambda_a) = (c_n)^{1-\xi} (c_m^{agg})^\xi + \lambda_a (e - p_n c_n - p_m^{agg} c_m^{agg}) \quad (47)$$

<sup>33</sup>This price index can be obtained from the solution to the optimization problem in stage two and three.

<sup>34</sup>Section 2.4.1 shows that this indeed holds as the wage rate is pinned down by the price of the agricultural good which is chosen to be the numeraire of the economy.



and solving for the first order conditions yields

$$\frac{\partial \ell}{\partial c_n} \doteq 0 \Rightarrow (1 - \xi)(c_n)^{-\xi}(c_m^{agg})^\xi = \lambda_a p_n, \quad (48)$$

$$\frac{\partial \ell}{\partial c_m^{agg}} \doteq 0 \Rightarrow (c_n)^{1-\xi} \xi (c_m^{agg})^{\xi-1} = \lambda_a p_m^{agg}, \quad (49)$$

$$\frac{\partial \ell}{\partial \lambda_a} \doteq 0 \Rightarrow p_n c_n + p_m^{agg} c_m^{agg} = e. \quad (50)$$

Manipulating these first order conditions leads to unit elastic demands for the agricultural good and the CES composite of manufactured goods given by

$$\begin{aligned} c_n &= \frac{(1 - \xi)e}{p_n} \\ c_m^{agg} &= \frac{\xi e}{p_m^{agg}}. \end{aligned} \quad (51)$$

Taking into account the Cobb-Douglas specification of utility in these two goods it is no surprise that a fraction  $\xi$  of income used for consumption is spent on manufactures and a fraction  $1 - \xi$  on the agricultural good.

In the last stage the static problem of distributing manufacturing consumption among different varieties for fixed manufacturing consumption expenditure  $\xi e$  can be formulated as

$$\begin{aligned} \max_{c_m^H(i), c_m^F(j)} & \left[ \int_0^{V_H} (c_m^H(i))^{\frac{\sigma-1}{\sigma}} di + \int_0^{V_F} (c_m^F(j))^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}} \\ \text{s.t.} & \int_0^{V_H} p_m^H(i) c_m^H(i) di + \int_0^{V_F} p_{m,\varphi}^F(j) c_m^F(j) dj = \xi e. \end{aligned} \quad (52)$$

Setting up the Lagrangian as

$$\begin{aligned} \ell(c_m^H(i), c_m^F(j), \lambda_m) &= \left[ \int_0^{V_H} (c_m^H(i))^{\frac{\sigma-1}{\sigma}} di + \int_0^{V_F} (c_m^F(j))^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}} + \\ & \lambda_m \left[ \xi e - \int_0^{V_H} p_m^H(i) c_m^H(i) di - \int_0^{V_F} p_{m,\varphi}^F(j) c_m^F(j) dj \right] \end{aligned} \quad (53)$$

and solving for the first order conditions yields<sup>35</sup>

$$\begin{aligned} \frac{\partial \ell}{\partial c_m^H(i)} \doteq 0 \quad \Rightarrow \quad & \frac{\sigma}{\sigma-1} \left[ \int_0^{V_H} (c_m^H(i))^{\frac{\sigma-1}{\sigma}} di + \int_0^{V_F} (c_m^F(j))^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{1}{\sigma-1}} \\ & \times \frac{\sigma-1}{\sigma} (c_m^H(i))^{-\frac{1}{\sigma}} = \lambda_m p_m^H(i), \end{aligned} \quad (54)$$

$$\begin{aligned} \frac{\partial \ell}{\partial c_m^F(j)} \doteq 0 \quad \Rightarrow \quad & \frac{\sigma}{\sigma-1} \left[ \int_0^{V_H} (c_m^H(i))^{\frac{\sigma-1}{\sigma}} di + \int_0^{V_F} (c_m^F(j))^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{1}{\sigma-1}} \\ & \times \frac{\sigma-1}{\sigma} (c_m^F(j))^{-\frac{1}{\sigma}} = \lambda_m p_{m,\varphi}^F(j), \end{aligned} \quad (55)$$

$$\frac{\partial \ell}{\partial \lambda_m} \doteq 0 \quad \Rightarrow \quad \int_0^{V_H} p_m^H(i) c_m^H(i) di + \int_0^{V_F} p_{m,\varphi}^F(j) c_m^F(j) dj = \xi e. \quad (56)$$

Recalling the definition of  $c_m^{agg}$  given below equation (1) these first order conditions can be rewritten as

$$c_m^{agg} \left[ \int_0^{V_H} (c_m^H(i))^{\frac{\sigma-1}{\sigma}} di + \int_0^{V_F} (c_m^F(j))^{\frac{\sigma-1}{\sigma}} dj \right]^{-1} (c_m^H(i))^{-\frac{1}{\sigma}} = \lambda_m p_m^H(i), \quad (57)$$

$$c_m^{agg} \left[ \int_0^{V_H} (c_m^H(i))^{\frac{\sigma-1}{\sigma}} di + \int_0^{V_F} (c_m^F(j))^{\frac{\sigma-1}{\sigma}} dj \right]^{-1} (c_m^F(j))^{-\frac{1}{\sigma}} = \lambda_m p_{m,\varphi}^F(j). \quad (58)$$

Isolating  $c_m^H(i)$  and  $c_m^F(j)$  on the left hand side, then multiplying both sides by  $p_m^H(i)$  or  $p_{m,\varphi}^F(j)$  and finally integrating over all varieties yields

$$\begin{aligned} & \int_0^{V_H} p_m^H(i) c_m^H(i) di = \\ & \frac{\lambda_m^{-\sigma} \int_0^{V_H} (p_m^H(i))^{1-\sigma} di \left[ \int_0^{V_H} (c_m^H(i))^{\frac{\sigma-1}{\sigma}} di + \int_0^{V_F} (c_m^F(j))^{\frac{\sigma-1}{\sigma}} dj \right]^{-\sigma}}{(c_m^{agg})^{-\sigma}}, \\ & \int_0^{V_F} p_{m,\varphi}^F(j) c_m^F(j) dj = \\ & \frac{\lambda_m^{-\sigma} \int_0^{V_F} (p_{m,\varphi}^F(j))^{1-\sigma} dj \left[ \int_0^{V_H} (c_m^H(i))^{\frac{\sigma-1}{\sigma}} di + \int_0^{V_F} (c_m^F(j))^{\frac{\sigma-1}{\sigma}} dj \right]^{-\sigma}}{(c_m^{agg})^{-\sigma}}. \end{aligned}$$

Adding these two expressions, using the budget constraint from above, and isolating  $\lambda_m$  gives the following equation for the Lagrange multiplier, i.e. the shadow price of manu-

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<sup>35</sup>Note that this is in fact a variational problem.

facturing consumption,

$$\lambda_m = \frac{(\xi e)^{-\frac{1}{\sigma}} c_m^{agg} \left[ \int_0^{V_H} (p_m^H(i))^{1-\sigma} di + \int_0^{V_F} (p_{m,\varphi}^F(j))^{1-\sigma} dj \right]^{\frac{1}{\sigma}}}{\left[ \int_0^{V_H} (c_m^H(i))^{\frac{\sigma-1}{\sigma}} di + \int_0^{V_F} (c_m^F(j))^{\frac{\sigma-1}{\sigma}} dj \right]}. \quad (59)$$

Plugging this expression back into equations (57) and (58) finally leads to the demands for all varieties given by

$$\begin{aligned} c_m^H(i) &= \frac{\xi e (p_m^H(i))^{-\sigma}}{\left[ \int_0^{V_H} (p_m^H(i))^{1-\sigma} di + \int_0^{V_F} (p_{m,\varphi}^F(j))^{1-\sigma} dj \right]}, \\ c_m^F(j) &= \frac{\xi e (p_{m,\varphi}^F(j))^{-\sigma}}{\left[ \int_0^{V_H} (p_m^H(i))^{1-\sigma} di + \int_0^{V_F} (p_{m,\varphi}^F(j))^{1-\sigma} dj \right]}. \end{aligned}$$

## B Aggregation over individuals

The aggregate Euler equation can be derived as follows.<sup>36</sup> Taking the time derivative of aggregate consumption expenditures given in equation (10) yields

$$\begin{aligned} \dot{E}(t) &= \mu e(t, t) + \mu \int_{-\infty}^t \dot{e}(t_0, t) e^{-\mu(t-t_0)} + e(t_0, t) (-\mu) e^{-\mu(t-t_0)} dt_0 \\ &= \mu e(t, t) - \mu E(t) + \mu \int_{-\infty}^t \dot{e}(t_0, t) e^{-\mu(t-t_0)} dt_0, \end{aligned} \quad (60)$$

where we used the definition of aggregate consumption expenditures in going from the first to the second line. To arrive at the final aggregate Euler equation it is necessary to derive optimal consumption expenditures  $e(t, t)$  of newborns in the planning period  $t$  and the aggregate consumption expenditure rule  $E(t)$ . To achieve this we reformulate the individual's optimization problem as follows. In line with equation (1) the expected utility  $U(t_0, t)$  at an arbitrary point in time  $t$  of a consumer born at time  $t_0 \leq t$  is given by

$$U(t_0, t) \equiv \int_t^\infty e^{-(\rho+\mu)(\tau-t)} \ln \left( \frac{e(t_0, \tau)}{P(\tau)} \right) d\tau, \quad (61)$$

where we again used the perfect price index  $P$  translating expenditures in indirect utility (cf. appendix A). The law of motion of capital given in equation (2) can be rewritten as

$$\begin{aligned} \dot{k}(t_0, \tau) &= \frac{w(\tau)l(t_0, \tau) + \pi(\tau)k(t_0, \tau) - e(t_0, \tau)}{w(\tau)F} + \mu k(t_0, \tau) - \delta k(t_0, \tau) \\ &= \left( \frac{\pi(\tau)}{w(\tau)F} + \mu - \delta \right) k(t_0, \tau) + \frac{l(t_0, \tau)}{F} - \frac{e(t_0, \tau)}{w(\tau)F}. \end{aligned} \quad (62)$$

From equation (62) the individual's lifetime budget can be derived. First both sides

<sup>36</sup>The derivations shown in this appendix closely follow the ones described by Heijdra and van der Ploeg (2002) in chapter 16.

of the equation are multiplied by  $e^{-R^A(t,\tau)} \equiv e^{-\int_t^\tau \left(\frac{\pi(s)}{w(s)F} + \mu - \delta\right) ds}$  and rearranged to

$$\left[ \dot{k}(t_0, \tau) - \left( \frac{\pi(\tau)}{w(\tau)F} + \mu - \delta \right) k(t_0, \tau) \right] e^{-R^A(t,\tau)} = \left[ \frac{l(t_0, \tau)}{F} - \frac{e(t_0, \tau)}{w(\tau)F} \right] e^{-R^A(t,\tau)}. \quad (63)$$

Observing that the left hand side of equation (63) is  $d \left[ k(t_0, \tau) e^{-R^A(t,\tau)} \right] / d\tau$  by applying Leibnitz's rule to recognize that  $dR^A(t, \tau)/d\tau = \frac{\pi(\tau)}{w(\tau)F} + \mu - \delta$  and integrating over the interval  $[t, \infty)$  yields

$$\int_t^\infty d \left[ k(t_0, \tau) e^{-R^A(t,\tau)} \right] = \int_t^\infty \left[ \frac{l(t_0, \tau)}{F} - \frac{e(t_0, \tau)}{w(\tau)F} \right] e^{-R^A(t,\tau)} d\tau.$$

This expression can be solved to

$$\lim_{\tau \rightarrow \infty} k(t_0, \tau) e^{-R^A(t,\tau)} - k(t_0, t) e^{-R^A(t,t)} = HW(t) - \int_t^\infty \frac{e(t_0, \tau)}{w(\tau)F} e^{-R^A(t,\tau)} d\tau, \quad (64)$$

where we defined  $HW(t) \equiv \int_t^\infty \frac{w(\tau)l(t_0, \tau)}{w(\tau)F} e^{-R^A(t,\tau)} d\tau$  denoting human wealth of individuals in capital units consisting of the present value of lifetime wage income using the annuity factor  $R^A(t, \tau)$  for discounting. Note that  $e^{-R^A(t,t)} = 1$  and that the first term on the left hand side represents “terminal capital holdings”. These holdings must be equal to zero because first, the insurance company will ensure their nonnegativity, and second, it is suboptimal for an individual to have positive terminal assets as there is neither a bequest motive nor satiation from consumption. Taking this into account yields the following solvency condition

$$\lim_{\tau \rightarrow \infty} e^{-R^A(t,\tau)} k(t_0, \tau) = 0, \quad (65)$$

which prevents an individual from running a Ponzi game against the life-insurance company. The No-Ponzi-Game condition can be inserted in equation (64) to obtain the individual's lifetime budget restriction

$$k(t_0, t) + HW(t) = \int_t^\infty \frac{e(t_0, \tau)}{w(\tau)F} e^{-R^A(t,\tau)} d\tau. \quad (66)$$

The present value of an individual's consumption expenditure plan in capital units must be equal to the sum of human wealth in capital units and capital holdings (=total wealth). Evaluating the lifetime budget constraint at  $t = t_0$  shows that the discounted sum of lifetime labour earnings must equal discounted consumption expenditures.<sup>37</sup> This implies, from investigating the law of motion for capital, that discounted savings are equal to discounted accumulated profits, i.e. savings are only used for reallocating consumption across lifetime.

Maximizing expected utility given in equation (61) subject to the budget constraint in

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<sup>37</sup>Note that capital holdings of newborns  $k(t_0, t_0)$  are zero by assumption (no bequests).

equation (66) yields the following first order condition

$$\frac{1}{e(t_0, \tau)} e^{-(\rho+\mu)(\tau-t)} = \lambda(t) \frac{1}{w(\tau)F} e^{-R^A(t, \tau)}, \quad \tau \in [t, \infty), \quad (67)$$

where  $\lambda(t)$  represents the marginal expected lifetime utility of wealth.<sup>38</sup> Individuals should therefore plan consumption expenditures in a way such that the appropriately discounted marginal utility of expenditures and wealth are equated.

Applying equation (67) for the planning period ( $\tau = t$ ) yields  $e(t_0, t) = \frac{w(t)F}{\lambda(t)}$ . Using this result and then substituting for  $\lambda(t)$  also from the first order condition in equation (67) helps to establish the following equality

$$\begin{aligned} \int_t^\infty e(t_0, t) e^{-(\rho+\mu)(\tau-t)} d\tau &= \int_t^\infty \frac{w(t)F}{\lambda(t)} e^{-(\rho+\mu)(\tau-t)} d\tau \\ &= Fw(t) \int_t^\infty \frac{e(t_0, \tau)}{Fw(\tau)} e^{-R^A(t, \tau)} d\tau. \end{aligned}$$

Integrating out and using the lifetime budget constraint of equation (66) finally yields consumption expenditures  $e(t_0, t)$  in the planning period  $t$

$$\begin{aligned} \frac{e(t_0, t)}{\rho + \mu} \left[ -e^{-(\rho+\mu)(\tau-t)} \right]_t^\infty &= Fw(t)[k(t_0, t) + HW(t)] \\ e(t_0, t) &= (\rho + \mu)Fw(t)[k(t_0, t) + HW(t)]. \end{aligned} \quad (69)$$

The above equation clearly shows that optimal consumption expenditures in the planning period  $t$  in capital units,  $\frac{e(t_0, t)}{Fw(t)}$ , are proportional to total wealth with the marginal propensity to consume out of total wealth being constant and equal to the “effective” rate of time preference  $\rho + \mu$ .

Using this expression for optimal consumption expenditures in the definition of aggregate consumption expenditures in equation (10) yields the following very simple aggregate consumption expenditure rule

$$\begin{aligned} E(t) &\equiv \mu \int_{-\infty}^t e^{-\mu(t-t_0)} (\rho + \mu) Fw(t) [k(t_0, t) + HW(t)] dt_0 \\ &= (\rho + \mu) Fw(t) \mu \left[ \int_{-\infty}^t e^{-\mu(t-t_0)} k(t_0, t) dt_0 + \int_{-\infty}^t e^{-\mu(t-t_0)} HW(t) dt_0 \right] \\ &= (\rho + \mu) Fw(t) [K(t) + HW(t)], \end{aligned} \quad (70)$$

where the aggregate capital stock is defined in equation (7) and can be rewritten in anal-

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<sup>38</sup>Differentiating this first order condition with respect to  $\tau$ , inserting the expression for  $\lambda(t)$  also obtainable from this first order condition and simplifying yields the following Euler equation

$$\frac{\dot{e}(t_0, \tau)}{e(t_0, \tau)} = \frac{\pi(\tau)}{w(\tau)F} - \rho - \delta + \frac{\dot{w}(\tau)}{w(\tau)}. \quad (68)$$

With time-invariant wages (cf. section 2.4.1) this Euler equation is exactly the same as the one obtained in equation (6).

ogy to aggregate consumption expenditures given in equation (10). Moreover it is easily established that  $\mu HW(t) \left[ \frac{e^{-\mu(t-t_0)}}{\mu} \right]_{-\infty}^t = HW(t)$ .<sup>39</sup>

Finally we modify equation (60) by substituting for  $e(t, t)$  and  $E(t)$  from the derived expressions of equation (69) evaluated at birth date  $t^{40}$  and equation (70) as well as for  $\dot{e}(t_0, t)$  from the individual Euler equation given in expression (68). Dividing by  $E(t)$  then gives the aggregate Euler equation

$$\begin{aligned} \frac{\dot{E}(t)}{E(t)} &= -\mu(\rho + \mu)Fw(t)\frac{K(t)}{E(t)} + \\ &\quad \frac{\mu}{E(t)} \int_{-\infty}^t e(t_0, t) \left[ \frac{\pi(t)}{w(t)F} - \rho - \delta + \frac{\dot{w}(t)}{w(t)} \right] e^{-\mu(t-t_0)} dt_0 \\ &= -\mu(\rho + \mu)Fw(t)\frac{K(t)}{E(t)} + \frac{\pi(t)}{w(t)F} - \rho - \delta + \frac{\dot{w}(t)}{w(t)} \\ &= -\mu \frac{E(t) - e(t, t)}{E(t)} + \frac{\dot{e}(t_0, t)}{e(t_0, t)}, \end{aligned}$$

where in the second line we used again the definition of aggregate consumption expenditure from equation (10) and the term  $\dot{w}(t)/w(t)$  disappears in the case of time invariant wages (cf. section 2.4.1).

## C The manufacturing firm's profit maximization problem - Derivation of rental rates

Inserting optimal demands for varieties into operating profits leads to the following expression to be maximized

$$\begin{aligned} &(p_m^H(i, t) - w(t)\beta) \left( \int_{-\infty}^t \frac{\xi e(t_0, t)(p_m^H(i, t))^{-\sigma}}{P_m(t)} N(t_0, t) dt_0 \right) + \\ &(p_{m,\varphi}^F(i, t) - w(t)\varphi\beta) \left( \int_{-\infty}^t \frac{\xi e^*(t_0, t)(p_{m,\varphi}^F(i, t))^{-\sigma}}{P_m^*(t)} N^*(t_0, t) dt_0 \right), \end{aligned}$$

<sup>39</sup>This aggregation property of consumption expenditures is due to the fact that we assumed a constant probability of death implying an age independent marginal propensity to consume out of total wealth (see equation (69)).

<sup>40</sup>Note again that  $k(t, t) = 0$  and newborns therefore consume a fraction of their human wealth at birth, i.e.  $e(t, t) = (\rho + \mu)Fw(t)HW(t)$ .

whose derivatives with respect to  $p_m^H(i, t)$  and  $p_{m,\varphi}^F(i, t)$  are set equal to zero to yield the first order conditions

$$\begin{aligned} 0 &= \frac{(1-\sigma)(p_m^H(i, t))^{-\sigma} \xi e(t_0, t)}{P_m(t)} N(t_0, t) \\ &\quad + \frac{\sigma(p_m^H(i, t))^{-\sigma-1} \xi e(t_0, t)}{P_m(t)} w(t) \beta N(t_0, t), \\ 0 &= \frac{(1-\sigma)(p_{m,\varphi}^F(i, t))^{-\sigma} \xi e^*(t_0, t)}{P_m^*(t)} N^*(t_0, t) \\ &\quad + \frac{\sigma(p_{m,\varphi}^F(i, t))^{-\sigma-1} \xi e^*(t_0, t)}{P_m^*(t)} w(t) \beta \varphi N^*(t_0, t). \end{aligned}$$

Rearranging and simplifying gives optimal prices

$$\begin{aligned} p_m^H(i, t) &= \frac{\sigma}{\sigma-1} w(t) \beta, \\ p_{m,\varphi}^F(i, t) &= \frac{\sigma}{\sigma-1} w(t) \beta \varphi. \end{aligned}$$

Using the above first order conditions in the definition of operating profits yields

$$\pi(t) = \frac{p_m^H(t)}{\sigma} \left( \int_{-\infty}^t c_m^H(t_0, t) N(t_0, t) dt_0 \right) + \frac{p_{m,\varphi}^F(t)}{\sigma} \left( \int_{-\infty}^t c_m^{H*}(t_0, t) N^*(t_0, t) dt_0 \right),$$

where an equivalent equation holds in the foreign region. Note that the variety index  $i$  can be dropped since prices and therefore profits are equal for all firms. Applying the definitions of the share variables  $s_H^H(t)$ ,  $s_H^F(t)$ ,  $s_F^F(t)$ , and  $s_F^H(t)$  given above in equations (22) and (23) leads to the following expressions for the rental rates

$$\begin{aligned} \pi(t) &= \frac{\xi}{\sigma} \left( \int_{-\infty}^t s_H^H(t) e(t_0, t) N(t_0, t) dt_0 + \int_{-\infty}^t s_H^F(t) e^*(t_0, t) N^*(t_0, t) dt_0 \right), \\ \pi^*(t) &= \frac{\xi}{\sigma} \left( \int_{-\infty}^t s_F^F(t) e^*(t_0, t) N^*(t_0, t) dt_0 + \int_{-\infty}^t s_F^H(t) e(t_0, t) N(t_0, t) dt_0 \right). \end{aligned}$$

Inserting optimal demands from equation (4) into the definitions for the share variables and again using mill pricing from equations (20) and (21) yields the share variables as functions of home and foreign capital stocks as well as of the level of transport costs<sup>41</sup>

$$s_H^H(t) = \frac{1}{K(t) + \phi K^*(t)}, \quad (71)$$

$$s_H^F(t) = \frac{\phi}{K(t) \phi + K^*(t)}. \quad (72)$$

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<sup>41</sup>Note that the number of varieties in the home region  $V_H(t)$  is equal to the capital stock at home  $K(t)$  as one variety exactly requires one unit of capital as fixed input (analogously  $K^*(t) \equiv V_F(t)$ ).

Using them in equations (24) and (25) gives the final expressions for rental rates

$$\begin{aligned}\pi &= \underbrace{\left( \frac{\theta_E}{\theta_K + \phi(1 - \theta_K)} + \frac{(1 - \theta_E)\phi}{\phi\theta_K + 1 - \theta_K} \right)}_{Bias} \left( \frac{\xi E^W}{\sigma K^W} \right), \\ \pi^* &= \underbrace{\left( \frac{1 - \theta_E}{1 - \theta_K + \phi\theta_K} + \frac{\theta_E\phi}{\phi(1 - \theta_K) + \theta_K} \right)}_{Bias^*} \left( \frac{\xi E^W}{\sigma K^W} \right).\end{aligned}$$

## D Intermediate results for the stability analysis

The Jacobian matrix  $J_{sym}$ , which is evaluated at the symmetric equilibrium and given in equation 35, has the following entries  $J_i$  for  $i = 1, \dots, 4$ :

$$J1 = \frac{1}{2(\phi + 1)\sqrt{\sigma}} \begin{pmatrix} A(\phi + 2) - B\phi & (A + B)\phi \\ (A + B)\phi & A(\phi + 2) - B\phi \end{pmatrix}, \quad (73)$$

$$J2 = \begin{pmatrix} \frac{-F(A+B)^2(\phi^2+1)}{4(\phi+1)^2\xi} - F\mu(\mu + \rho) & \frac{-(A+B)^2F\phi}{2(\phi+1)^2\xi} \\ -\frac{(A+B)^2F\phi}{2(\phi+1)^2\xi} & \frac{-F(A+B)^2(\phi^2+1)}{4(\phi+1)^2\xi} - F\mu(\mu + \rho) \end{pmatrix}, \quad (74)$$

$$J3 = \frac{1}{F(\phi + 1)\sigma} \begin{pmatrix} \xi - (\phi + 1)\sigma & \phi\xi \\ \phi\xi & \xi - (\phi + 1)\sigma \end{pmatrix}, \quad (75)$$

$$J4 = \begin{pmatrix} \frac{\phi(A+\rho\sqrt{\sigma}) - \delta(\phi^2 + \phi + 1)\sqrt{\sigma}}{(\phi+1)^2\sqrt{\sigma}} & \frac{-(A+B)\phi}{(\phi+1)^2\sqrt{\sigma}} \\ -\frac{(A+B)\phi}{(\phi+1)^2\sqrt{\sigma}} & \frac{\phi(A+\rho\sqrt{\sigma}) - \delta(\phi^2 + \phi + 1)\sqrt{\sigma}}{(\phi+1)^2\sqrt{\sigma}} \end{pmatrix}, \quad (76)$$

with the parameter clusters  $A \equiv \sqrt{\sigma(\delta + \rho)^2 + 4\mu(\mu + \rho)\xi}$  as well as  $B \equiv (\delta + \rho)\sqrt{\sigma}$ .

In order to get a first insight into the nature and signs of the eigenvalues of  $J_{sym}$ , we calibrated the model using the parameter values  $\rho = 0.015$  and  $\delta = 0.05$  and allowing the elasticity of substitution and the manufacturing share of consumption to vary within the ranges  $2 \leq \sigma \leq 8$  and  $0.1 \leq \xi \leq 0.9$ . Figures 6, 7, 8 and 9 illustrate the numerical investigation of the signs of the eigenvalues for  $\sigma = 4$ ,  $\xi = 0.3$  and varying  $\mu$  and  $\phi$ .<sup>42</sup>

First, the figures suggest that all eigenvalues are real for the chosen parameter space. Moreover, figures 6, 7 and 9 show that the first eigenvalue is always negative, whereas the second and fourth are always positive. This result is independent of the level of transport costs and the mortality rate. Saddle path stability of the symmetric equilibrium therefore seems to crucially depend on the third eigenvalue by requiring it to be negative. As can be seen from the 3D plot in figure 8 there only exists a very small range of combinations of low  $\mu$  and high  $\phi$  where the sign of the third eigenvalue is positive. One is therefore tempted to conclude that with a sufficiently high mortality rate, the symmetric equilibrium is stable for all levels of transport costs.

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<sup>42</sup>We also conducted the same simulations for other values of  $\sigma$  and  $\xi$  within the considered range. Overall we find that our findings with respect to the signs of the eigenvalues are insensitive to changes in those parameters.



Figure 6: **Eigenvalue 1**

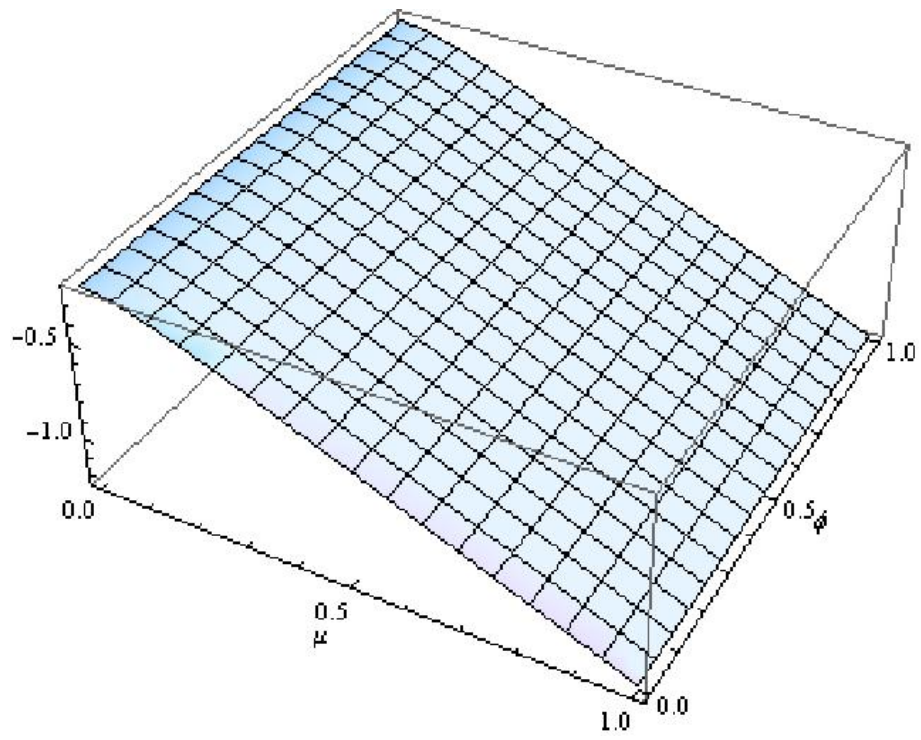


Figure 7: **Eigenvalue 2**

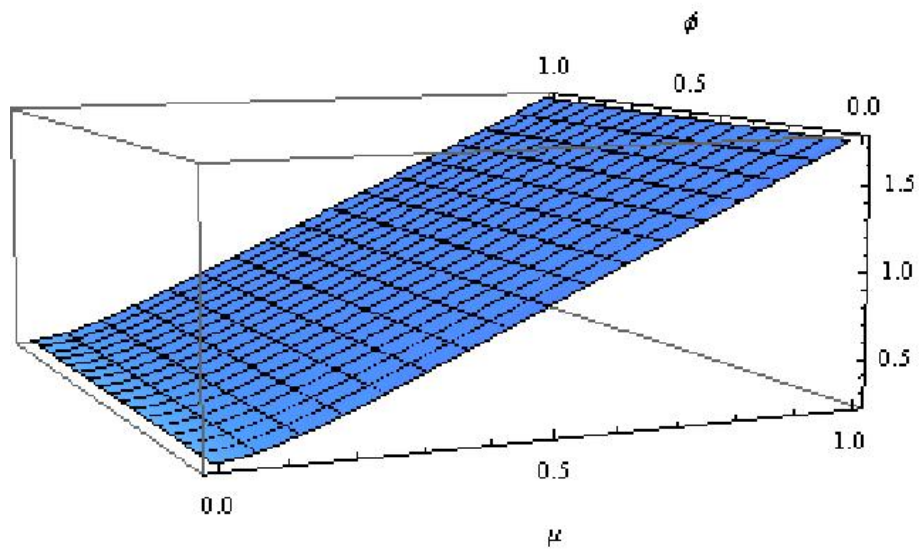


Figure 8: **Eigenvalue 3**

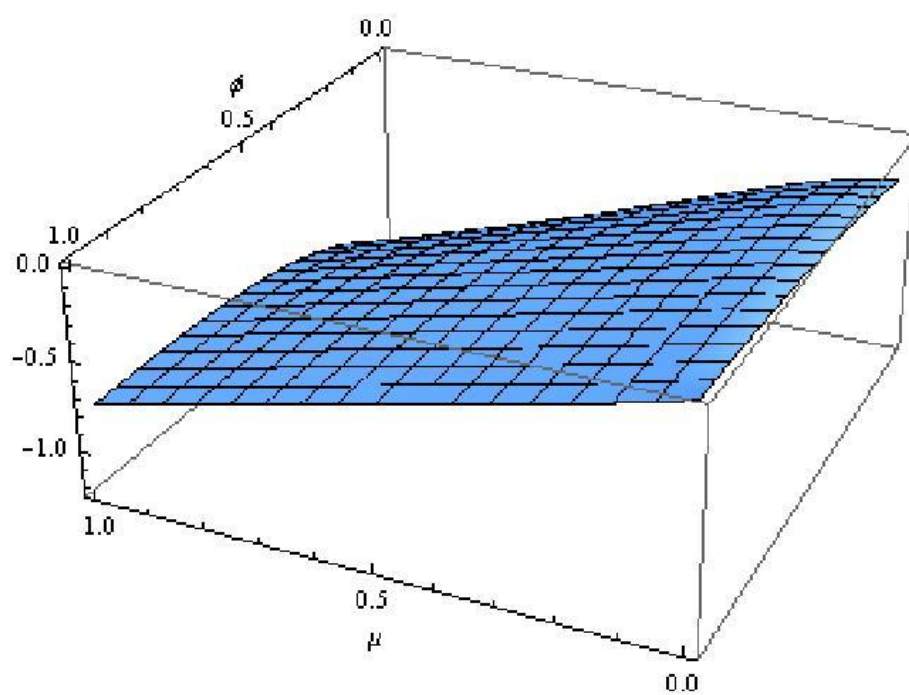
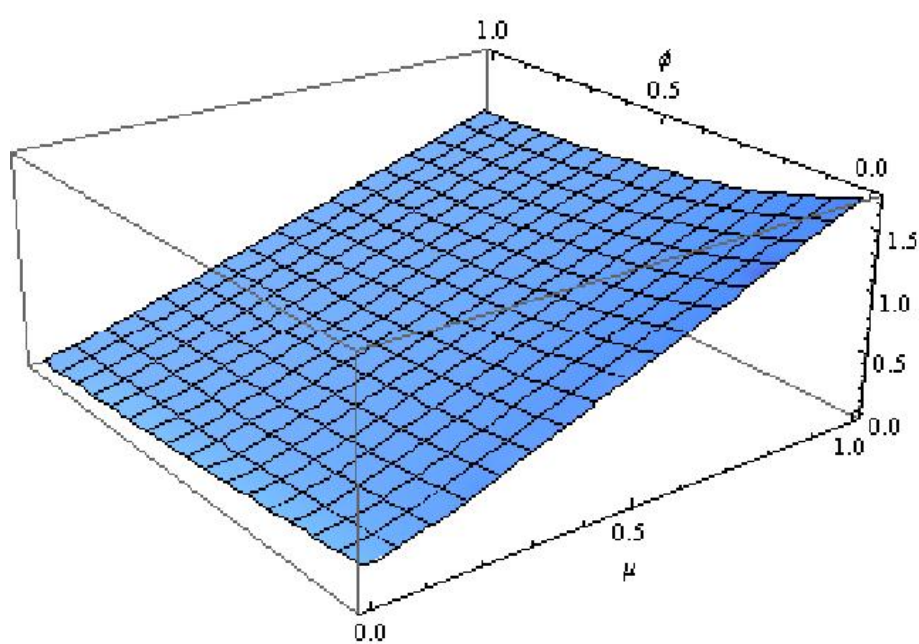


Figure 9: **Eigenvalue 4**



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