Voosholz, Frauke

Working Paper
A survey on modeling economic growth. With special interest on natural resource use

CAWM Discussion Paper, Centrum für Angewandte Wirtschaftsforschung Münster, No. 69

Provided in Cooperation with:
Center of Applied Economic Research Münster (CAWM), University of Münster

Suggested Citation: Voosholz, Frauke (2014) : A survey on modeling economic growth. With special interest on natural resource use, CAWM Discussion Paper, Centrum für Angewandte Wirtschaftsforschung Münster, No. 69, CAWM, Münster

This Version is available at:
http://hdl.handle.net/10419/96932

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes.
You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.
If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.
A Survey on Modeling Economic Growth
With Special Interest on Natural Resource Use

Frauke Voosholz*

February 2014

CAWM Discussion Paper No. 69

Abstract

The purpose of this paper is to survey the contributions to economic growth theory. We focus on the basic models and literature that link resource economic and economic growth, in order to reveal the main differences on how the different aspects are incorporated into growth models. As economic science is not a hard science, all economic activities must always be considered against the background of the current economy, the political and social institutions and technical capabilities (as already mentioned by Solow (1985), p. 328). That is why many of the first growth models, fitted to current state when they were developed, are not transferable to remote periods. Furthermore, natural resources as input factors, which influence economic growth, were neglected for a long time, whereas today they are one of the most discussed factors influencing economic growth.

JEL classification: O13, O41, O40, Q20, Q32

Keywords: economic growth, growth theory, historical overview, renewable resources, non-renewable resources

* Dipl. Volksw. Frauke Voosholz, University of Muenster, Institute of Spacial and Housing Economics, Am Stadtgraben 9, 48143 Muenster / Germany, Tel. +49-251-83-22979, f.voos01@wiwi.uni-muenster.de
1 Why is Growth so Important?

There are 215 countries in the world, whose GDPs (gross domestic product) are growing at different rates. Even small differences in the growth rates lead to enormous differences in real terms, even if having the same starting value (after 100 years, a growth rate of 0.7 percent leads to twice the starting GDP, a rate of 1.7 percent leads to an increment of 5.4 times). Some countries, like Qatar and Paraguay, are growing rapidly with an inflation adjusted increase of the GDP of about 15 percent a year, some are not growing at all, others, like Puerto Rico, Anguilla and San Marino, are even dwindling by more than five percent a year (cf. CIA - Central Intelligence Agency (2010)). The GDP does not report in full the real state of the individual welfare of people living in those countries, but it is one of the best possibilities to compare countries quantitatively. Especially the income per person is a useful and highly correlated index to quality of life. Mostly, further indicators have to be added, depending on the question being asked, to get reasonable results.

But why do growth rates differ ever so much? Every country has a different starting point. The supply of labor, the stock of resources, capital and human capital, the level of infrastructure and the climate conditions are some of the main aspects influencing the development of countries. Further aspects affecting the process of the growth rate are the political and institutional environment, the volume of international trade and the current GDP. Growth models try to capture these facts to show the relationships and the development of the variables. They try to reveal how decisions made by economic agents, the value of the variables in the past, present and future as well as specific implications affect the growth rates.

Studies in economic growth are not only done to bother many researchers, they want to make a contribution to solutions on economic-political problems. So it is important to keep in mind the implications, the adequacy to the purpose and the suitability of instruments. Not every problem can be solved with the same methods and even the same problem cannot be solved in the same way in each country because in a specific situation there are a lot of interdependencies between different political issues.

Nevertheless, there is a consent in political and public discussions that only a growth rate above two percent is satisfying; in some countries a satisfying growth rate is even claimed by law, e.g. Germany (cf. Bundesministerium der Justiz (1967) paragraph 1). This is accounted for by the general definition of growth. Growth is understood as a sustained extension of good-production, so that the average household may use more products and achieve a greater level of satisfaction and happiness. The importance of a growth rate not only greater than zero, but even larger is determined by the fact that the growth rate must be higher than the inflation rate to achieve this goal.
Alongside of the direct goal of economic growth there are more indirect goals of economic policies, where economic growth is only a means to an end (cf. Meyer et al. (1998) pp. 1-2). While technical improvement leads to a higher average labor productivity, the employment rate can only be kept constant or be raised if more goods can be distributed. As a consequence economic growth leads towards full employment. Another goal, a fairer income distribution, is usually more easily achievable if you take additional income as basis for the reallocation.

In addition, growth has to be evaluated. It influences many fields that often are not directly included in economic questions. It is relevant which impact it has on the individual utility and on the society. The positive effects have to be balanced with the cost of growth, like for example environmental destruction or less leisure time. Keeping in mind that we are dealing with production functions, where resources are included, it must be guaranteed that not only one generation benefits from economic growth, but that it is long lasting and sustainable. Especially the exhaustion of non-renewable resources must be taken into account. Here technical progress and a substitution progress are important to keep on the economy growing.

The purpose of this paper is to give an overview of the main influences and contributions to the theory of economic growth. Until now there already exists quite a lot of reviews, overviews and surveys dealing with economic growth. They mostly concentrate on special topics or aspects. The growth theory from Harrod to Neumann is already summarized quite detailed by Hahn and Matthews (1964), the endogenous growth theory with all its aspects by Aghion and Howitt (1998), while they even introduce new ideas. Of course many textbooks recapitulate the main aspects of economic growth as well (for ex-

In contrast to this, this paper tries to give a general overview of all topics influencing economic growth theory. It shall be shown, how the different schools of economic thought build up on each other and influence today's dealing with economic growth and its presentation in theoretical literature (see figure 1). This survey is arranged in five main sections and proceeds as follows. First of all a short historical overview of the theory of growth is given. We discuss the historical background of growth theory, followed by the main basic theories of economic growth. In a further section the fundamentals of endogenous growth theory are discussed. Section five introduces factors influencing economic growth, thereby concentrating on resources and the differences in research design. We conclude by giving an outlook to further research requirement.

2 Some Background of Growth Models

We start with the classical economists of the 18th and 19th century, who made important findings in their grand theories, building the basis of further research. Influenced by this, Schumpeter made a first step to a new field of research by his theory of creative destruction and coevally the idea of accelerators and multiplicators was introduced.

2.1 Fundamentals of Growth Theory

The Tableau économique, edued by François Quesney (1694-1774), can be interpreted as first growth model (later translated edition, see Quesney (1965)). Quesney states that only land is net-productive; and the land lords will invest all their income (production Y less consumption C). With constant prices, the amount of goods produced depends only on the consumption rate c, and therefore determines whether production grows, declines or is stationary (formally written $g = 0.5 (c - 0.5) - 2/3 (c - 0.5)^2$, see Eltis (1975) p. 333).

The classical economics, replacing the physiocratic body of thought, had in their
grand theories many aspects which were later used as science base of the first attempts
to describe long-run economic growth more detailed. The main aspects of these grand
theories relevant for economic growth are described shortly in the style of current formal
models. As a basis for the classical economics Adam Smith’s “An Inquiry into the Nature
and Causes of the Wealth of Nations” (Smith (1776)) can be seen. Most principals are
mentioned here, nevertheless there are still many discrepancies between the different

Adam Smith (1723-1790) concentrates mainly on the definition of wealth, how it
originates and why it grows. In contrast to the physiocrates he reveals that the afforded la-
bor of the nation is the source of wealth and not the agricultural primary production. The
efficiency of labor depends on the division of labor, and the division of labor needs func-
tioning markets. Smiths production function may read roughly like this: \( Y = (K, L, T) \)
where T is usable land with \( T \leq T_{\text{max}} \) (see Meyer et al. (1998) pp. 52 ff). As long as
there is a lot of usable land unused, the enlargement of the production is determined by
the capital accumulation and population growth. If at the same time the division of labor
is growing, the returns to scale are growing too. The closer the economy reaches the limit
of population growth \( N_{\text{max}}, \) depending on the natural endowment of the economy (land,
climate, etc.), and has reached a high level of labor division, the more the returns to scale
decrease. In the long-run economies, following Smith, reach a steady state.

In the different editions of “An Essay on the Principle of Population” Thomas
Robert Malthus (1766-1834) mainly develops a population theory, where the popu-
lation growth (geometric series \( n_{t+1} = n_t \cdot 2 \)) is always stronger than the growth of food
production (arithmetic series \( y_{t+1} = y_t + 1 \)) (see Malthus (1798), chapter 2). Therefore
there is a limit to population growth, even under the condition of technological progress.
Savings and as a consequence thereof waiver of consumption were the determining factors
for economic growth. But the risk was that there would be a lack of demand which would
induce a negative cycle. In total, Malthus proceeded on the assumption, that there is a
stationary final state without growth, where the optimal size of population is reached.
But this state would not be achieved for a long time.

A further important classical economist is David Ricardo (1772-1823). In his stud-
ies, compiled in “On the Principles of Political Economy and Taxation” (Ricardo (1821)),
he mainly dealt with the allocation of income between capital, labor and land (cf. Eltis
(1989) p. 202). As long as only fertile soils are used the tenants achieve a higher rent (for
capital). Those profits are normally invested in production, more land can be cultivated,
more workforce employed. At some point in time the less fertile pieces of land have to
be used by the tenants to enlarge the production. Then the tenants have to pay a higher
rent for the best soils to the landowners, which reduces their profits. As long as they
receive a bit more than the minimum rent the tenants will invest and it is possible that
the population of the economy grows or that the workers can enlarge their demand for goods. If the minimum rent is reached the economy reaches a stationary state. The price of goods is controlled by the amount of labor used for the production. To overcome the stationary state, Ricardo advocated international trade, so that every economy specializes on the product where they have an advantage in the costs of production in comparison to the other countries (cf. Heertje and Wenzel (2008) p. 51). The wealth of the economies can be increased in this way.

In 1848 John Stuart Mill (1806-1873) tried to summarize the current state of of the economic theory in his publication “Principles of Political Economy”. He subdivides the economic theory in five categories “Production”, “Distribution”, “Exchange”, “Influence of the progress of society on Production and Distribution” and “Influence of Government” (Mill (1848)). Until today his structure is important in economic research. At the same time he develops further the ideas of earlier acting economists and introduces new modifications. He always tries to keep in touch with the reality and the consequences on political proceedings, by trying to introduce a reform party (cf. Marchi (1989) p. 272). He keeps the idea of a stationary final state, but welcomes it, because it may lead to a fair and cultivated social existence (cf. Holub (2010) p. 233). A further important aspect of Mill’s research was to concentrate on the maximization of utility and fortune in contrast to profit maximization (cf. ibidem pp. 229-230).

Friedrich List (1798-1846) encourages this idea as well. He assumed that each nation has to develop itself, using investments and technological progress, to reach a higher productivity. Prohibitive taxes may be used by nations, which have not reached the maximum level of industrialization yet (cf. List (1841) p. 18).

A important contribution, which is crucial for many growth model, is the fact of aggregation associated with increasing or diminishing returns to scale, which were introduced by Alfred Marshall (1842-1924) (see Romer (1994) p. 14). He summarizes that “every increase of wealth and every increase in the numbers and intelligence of the people increased the facilities for a highly developed industrial organization, which in its turn adds much to the collective efficiency of capital and labor” (Marshall (1980) book 4, chap. 13, paragraph 1).

In total one could say that these economists were more interested in developments in the long-run than in short-term changes. They mostly assumed that there is a stationary limit to which all initially growing economies convert, accounted for by different mechanisms.
2.2 From the Theory of Long-period Equilibrium to the Theory of Growth

In contrast to the former theories Joseph Alois Schumpeter (1883-1950) sees innovations as the main driving force for economic growth. A constant evolution and modification of products, production processes, new customers and suppliers, and even changes in the political settings lead to new market structures and temporary monopoles (Schumpeter (1934)). At the same time old ideas become inefficient and the economic cycle, the possible finale state and the equilibrium is changed.

Using the idea of Albert Aftalion (1874-1956), the accelerator principle was developed by John Bates Clark (1847-1938) (first published in French 1913, Aftalion (1927)). It states that investment depends on consumption and a constant accelerator \( c \), that is why a change in consumption leads to an over-proportional change in investments, formally \( I = c \cdot \frac{dC}{dt} \) with \( c > 1 \), which leads to economic growth (cf. Clark (1917) p. 223).

In contrast to J. B. Clark, John Mynard Keynes (1883-1946) states that the investments are independently given and the total income is given by \( Y = C + I \). The national income may either be consumed \( C \) or saved for investments \( S \), consequently consumption is a function of the total income. If the marginal propensity to consume \( c' \) is constant the multiplier results from \( \frac{dY}{dt} = \frac{1}{1-c'} \cdot \frac{dI}{dt} \), a change in the amount of investments leads to a change of the total income subject to the multiplier \( \frac{1}{1-c'} \) (Keynes (1936)). If the equilibrium of \( S = I \) is disturbed, additional national investments have to be made to retrieve the equilibrium. But his theory does not lead to suggestions concerning long-run growth, rather to guidance in different phases of the economic cycle (cf. Murad (1962) p. 57).

Karl Marx (1818-1883) uses theoretical relations, where two different economic sectors are considered. An equilibrium is reached if and only if both sectors grow with the same rate. Formally the growth rate \( g \) is defined by the ration of savings and investment rate \( s \) to the capital coefficient \( z \), where \( z \) is determined by the ratio of fixed and variable capital to wages and the income of the capitalists. If \( s \) and \( z \) are constant, continuous balanced growth is possible (see Marx (1962) with an example in chapter 21, section 3, III). Originally Marx states that the composition of capital \( z \) grows, because of the capitalization the capitalists’ income and therefore the total growth rate will sink and finally stagnate. The case of continuous balanced growth is only a random exceptional state.

Being not a specialist in growth theory Allyn Abbot Young (1876–1929) wrote one single paper as a presidential address to the British Association in September 1928 on "Increasing Returns and economic Progress" dealing with the understanding of economic growth (Young (1928)). His main idea is that inventions of any kind, which entail an
economic progress, will lead to changes elsewhere and therefore expedite themselves in a cumulative way. He sees no limits to the process of expansion as long as the demand is elastic and returns of production are increasing (see Young (1928) pp. 533-534). He describes the trend of the growth rate and the differences between countries, which are influenced by the growing size of the market and therefore enhance the economics of scale (cf. Currie (1981) p. 53). It is a theory of self-perpetuating, demand-induced growth. The paper had been neglected for a long time, because it was written at the wrong time (Great depression, followed by the Second World War), when the focus concentrated on different aspects, but especially not on increasing returns (cf. ibidem p. 54).

3 The First Growth Theories

The sector of the post-keynesian growth theory rose, revealing the first separate theories of growth. Today’s theory of growth is mostly influenced by the neoclassical theory, amended by the industrial economics and the macroeconomic contributions to the relevance and design of human capital. Schumpeter’s theory of creative destruction became the starting point for three further research areas, the evolutionary approach, the industrial economic and area of human capital research.

3.1 Postkeynesian Growth Theory

The idea was to combine the ideas of Marx (expended reproduction) and of Keynes (solving the equilibrium by driving consumption with additional investments) and additionally take into account the capacity effect and not only the income effect of investments. Roy Harrod (1900-1978) assumes an exogenous constant saving rate $s = \frac{S}{Y}$ and uses a function for the demand of investments, depending on the investment coefficient $v' = \frac{I_t}{\Delta Y_t}$. $v'$ is determined by the technological progress and the behavior of the firms. From the equilibrium condition $I = S$ follows that the "warranted rate of growth" must be $g^* = \frac{S}{v}$ (see Harrod (1939) p. 18). In contrast to this, Evsey Domar (1914-1997) determines the capacity effect by the marginal capital coefficient $v' = \frac{\Delta K_t}{\Delta Y_t^*}$ (with $\Delta Y_t^* = \text{change in production capacity}$). His model reaches an equilibrium if the investments create as much new capacity as new demand is generated. This is given if the investments grow with the constant rate of $g_I^* = \frac{S}{v}$ (see Domar (1946) p. 145). Determined by these equations of both models, all growth rates must grow at the same rate in an equilibrium $g_Y = g_K = g_L$. The economic moves along on the knife-edge and if the real investment rate differs from the optimal one, it will, following the models, collapse. Unveiling their models short after the Great Depression, the basic ideas were favorably accepted, but do

These ideas are amended by the assumption of constant labor coefficient. It is assumed that factor prices are quite inflexible and substitution of factors is nearly impossible. Therefore a Leontief production function is used, which implies that an equilibrium with full employment of all input factors is only possible, when all factors grow at the same rates. If the growth rate of population $n$ is constant, all other factors also have to grow with the rate of $n$, which is called the natural growth rate in Harrod’s terminology (see Hahn and Matthews (1964) pp. 783-784). It must not equal the “warranted rate of growth”, but if differing from this rate, the level of satisfaction may be over or under fulfilled.

Later the ideas of Harrod and Domar were developed further to the “Harrod-Domar-Theory”. The main improvement was that the investments were divided into induced investments $I_{ind}$ and autonomous investments $I_{aut}$. The induced investments are determined by the accelerator principle $I_{ind} = v' \cdot \frac{dY}{dt}$ if $\frac{dY}{dt} > 0$, and the autonomous ones grow with an exogenous constant rate (cf. Hamberg and Schultz (1961) p. 60-62).

### 3.2 Neo-keynesian Growth Theory

At the same time as Harrod Paul Samuelson (1915-2009) developed a quite similar model (Samuelson (1939)). He adds governmental expenditures as an other input factor of the national income, but it is set constantly to $G = 1$. Depending on the relation of consumption and private investments he reveals four different states of the economy, either growing but with decreasing growth rate, reaching a steady state, oscillating around zero income or a steadily growing state.

Nicolas Kaldor (1908–1986) uses a consumption function, in which he distinguishes between two categories, the wage income and the capital income, as basis for consumption. He assumes that the categories have different saving rates, the capital income recipients’ one is higher than the wage income recipients’ one. The total savings $S$ are the arithmetical average of both categories of savings, weighted with the income quotes. To achieve the equilibrium of $S = I$, income has to be shifted from capital income to wage income, if $S$ is higher than the optimal level, or the other way around, if $S$ is lower than the optimal level (Kaldor (1957)).
3.3 Neoclassical Growth Theory

To overcome the criticisms of the post-keynesian growth theories, a new approach was developed. In contrast to Harrod and Domar Robert Merton Solow (born 1924) uses a aggregated production function, where substitution between labor and capital is possible and therefore the capital coefficient becomes variable (Solow (1956)). But the returns of scale are still assumed to be constant and the savings and consumption rates are still exogenous given. The main equation describes the change of the per capita capital intensity with respect to time \( \dot{k} = s \cdot f(k) - (n + \delta) \cdot k \), where \( n + \delta \) describes the effective discount rate. The model describes the growth rate during the adjustment to the steady state. Differences from the optimal capital intensity are autocratically adjusted. Balanced growth is reached if \( s \cdot f(k^*) = (n + \delta) \cdot k^* \), with a constant \( k^* \). In balanced growth \( Y, K, C \) all have to grow with the same rate of \( n \). Trevor Swan (1918-1989) developed a quite similar model using different sets of illustrations (see Swan (1956)). That is why models using this approach refer to the Solow-Swan-Model (cf. Diamand and Spencer (2008)).

Solow did not specify the size of the saving rate \( s \). With a high \( s \) the economic reaches a higher income, but consumption at the same time is quite low; with a small \( s \) there is a lot of consumption but capital intensity is quite low and therefore production and income are quite low. The golden rule of accumulation was developed by different economists, including Edmund Phelps (born 1933). They state, that the optimal capital intensity in the balanced growth has to maximize per capita consumption. The optimal saving rate than, is determined by \( s = \frac{dY}{dt} \cdot \frac{k}{y} \), in other words savings must equal the income of interests (see Phelps (1961) pp. 641-642). To reach the balanced growth, savings may differ from the optimal rate to reach the optimal stock of capital.

An increase of the income in the steady state is only possible, if there is exogenous technological progress \( A \), which leads to a shift in production and therefore in the level of the value of the output (cf. Solow (1957) p. 313). Technological progress may influence the production function in different ways. Hicks-neutral technological progress propagates the amount of products produced with the same amount of input factors \( Y = A \cdot F(K, L) \). In contrast to this harrod-neutral technological progress leads to growing efficiencies on labor input. Producing the same amount of output needs less input of labor \( Y = F(K, A \cdot L) \). Just like this solow-neutral technological progress affects the efficiency of capital input (for further details see Hahn and Matthews (1964) pp. 825-832). Compatible to the neoclassical concept is only the harrod-neutral technological progress, because with a constant saving rate, the capital coefficient has to be constant as well (cf. Uzawa (1961) pp. 117-118).

In chronological perspective Frank P. Ramsey (1903-1930) would have been the
beginning of this chapter but his ideas were neglected until the 1960s (see Barro and Sala-i Martin (1995) p.10). Already in 1928 Ramsey published his ideas on how the optimal amount of savings is determined, so that the total (undiscounted) utility of the economic is maximized (see Ramsey (1928) p. 544). Furthermore he applied techniques of dynamic optimization to encourage his idea. Mainly David Cass (1937–2008) and Tjalling Koopmans (1910-1985) picked up this idea and developed it to the well-known Ramsey-Rule. The difference to Solow’s model is that they endogenize the determination of the optimal amount of savings and explicitly model the consumer side (see Cass (1965) and Koopmans (1963)). Koopmanns uses a discounted utility function with an infinite time horizon to determine the optimal path of consumption and investment, under the individuals’ budget constraint (see Koopmans (1963) pp. 22-23). Specific information on the behavior of the optimal saving rate can only be given with additional assumptions concerning the utility and the production function (see Cass (1965) p. 238), but generally it can be said, that the saving rate is now determined by the interest rate and is not given exogenously. Beginning with a given stock of capital, there only exists one optimal allocation path to reach the steady state, where stock of capital grows with the same intensity as the labor input. Keeping in mind, that the utility is now discounted, the optimal stock of capital per capita in the steady state is smaller than in the Solow model.

Using the idea from Samuelson, Peter Diamond (born 1940) introduced the overlapping generations in neoclassical growth theory (Samuelson (1958) and Diamond (1965)). He extended the set-up of Cass and Koopmans by introducing individuals with finite time horizon. Individuals are continuously dying and new agents are born. Individuals have different saving rates in different stages of life. It is assumed that individuals save an amount of their income in the first period of life, which generates the capital stock in the next period. During the second period of life, they receive income from the capital stock, which is completely consumed. Depending on the time preferences the decision on the saving rates has to be made, taking into consideration the growing population and the growing demand.

John von Neumann (1903-1957) develops a linear economic model of growth. His main contribution was the mathematical approach to prove the existence and uniqueness of an equilibrium of an expanding economy. In contrast to the other models, Neumann (1945) uses a production function, where the goods are produced from each other and a decision has to be made, which of the different available production technologies should be used. The important parameter is to define with which intensity the chosen technologies should be used to maximize total production (cf. Neumann (1945) p. 2). The fact of producing and developing these technologies is not considered. The difficulty is to find the set, where the demand for all goods (for production and consumption) and the supply of these goods, is balanced. The equilibrium is reached if the economy expands without any change in the production structure (cf. Hahn and Matthews (1964) p. 855).
3.4 Evolutionary Theory

In complement to the neoclassical framework, in the 1980s a new approach of explaining the technological change influencing economic growth was developed. The evolutionary theory designs the economic process similar to the biological evolution. There is one market and the ongoing competition between products, companies and even economic systems leads to the situation, that only those will survive which fit the needs best and adjust themselves best to changing conditions. Building upon the ideas of Friedrich von Hayek (competition as discovery of new possibilities (see v. Hayek (1969))) and of Schumpeter (1883-1950) (theory creative destruction (see chapter 2.2 of this paper)), it is possible, that changes and innovations will cause the destruction of ideas and products. The evolutionary theory was established as a separate research field after the critique of Nelson and Winter on the basic question of how firms and industries change overtime, giving an answer with evolutionary theory background (see Nelson and Winter (1982)).

4 Endogenous Growth Theory

The neoclassical approaches were quite unsatisfactory because the main variable explaining economic growth, the rate of technological progress, was exogenous given. The results did not fit to the empirical data and the insight in economics that, technological progress depends on economic decisions. Furthermore, if the rate of technological progress is exogenous given, it must be the same for all countries, and differences between countries would vanish. A new field of research, a new neoclassical approach or the endogenous growth theory, developed, trying to explain technological progress endogenously. The main problem was how to deal with the possibility of increasing returns, keeping in mind, that under these conditions not all factors can be paid with the marginal factor product (see Aghion and Howitt (1998) p. 23). The different ideas to overcome these problems are already sorted by Romer (1994) and described relatively detailed by Jones and Manuelli (1997). In the next sections the main aspects are explained, concentrating on the authors’ different assumptions.

4.1 Leading Towards Endogenous Growth

Young can be seen as one of the forerunners of endogenous growth (see chapter 2.2 of this paper). He already insists on the assertion that growth is endogenous and cumulative and the main source of growth is growth itself (cf. Sandilands (2000) p. 309). Already
Nordhaus (born 1941) stated, that it is not satisfying or rather not correct to say that economic growth is induced by technological progress, because it cannot be determined what leads to economic growth. It is more precise to say, that the growth of the input factors cannot explain the growth of the output and that there have to be other driving forces (see Nordhaus (1969) p. 18). As a consequence he claims for theories that explain the generation and transmission of new knowledge and therefore are able to define the forces leading to productivity changes. In an first attempt to archive this aim, Nordhaus defines the technological change as depending on the number of inventions made, which in turn depend on amount of output devoted to invention. A new result is, that a rising technological change leads to reduction of the capital stock. The problem of this model is, that there has to be population growth in a first step to start the process of a growing number of inventions. A growing amount of output in real terms must be devoted to invention production, because diminishing returns are assumed; otherwise technology will stagnate (ibidem p.23). Karl Shell (born 1938) introduced an invention sector in almost the same manner, thereby concentrating on the organizational matters as monopolies and the properties of pure public goods (Shell (1973)).

4.2 Endogenous Growth Using Capital Accumulation

One of the simplest ideas to overcome the exogenous explanation of economic growth is by relaxing some assumptions of the Solow model. Now the marginal productivity of capital converges to a positive value $A > 0$ and therefore production becomes a linear equation depending on capital, in the Cobb-Douglas case this reads $Y = AK$. This kind of model is better known as the AK-approach. Sergio Rebelo (born 1959) chooses this approach in his basic growth model, using a very wide capital definition including both physical and human capital (cf. Rebelo (1991) pp. 502-503). In this case growth is professed by always positive marginal returns to capital, which seems to be not realistic. As in the linear Neumann Model, technological progress is completely left out of consideration. Larry Jones and Rodolfo Manuelli add to the production function a further term to incorporate further input factors, $Y = AK + f(K, L)$ (cf. Jones and Manuelli (1990) p. 1014). But these factors cannot be essential to production, because otherwise the assumption of steady positive values of the marginal productivity is harmed. It might decrease in the beginning, but with rising capital intensity it convergences to $A$ again.

4.3 Technological Progress Using Spillover Effects

The main idea is, that there must be a difference between production seen from the standpoint of an individual firm to the standpoint of the whole economy. In the individual view, the state of the technology is exogenous given, and even the progress might seem to
be exogenous, because discoveries may be accidental and not always based on individual decisions. But in the total view the technological progress is endogenous because it is influenced by the decisions and efforts made by individuals (cf. Romer (1994) pp. 13-14). There are externalities from the individual behavior to the total stock of knowledge and reverse.

One of the first endogenous growth models is "Increasing Returns and Long-Run Growth" (1986) introduced by Paul M. Romer (born 1955). He models technological progress as depending on the total amount of investments to knowledge $A(R)$, which is determined by individual decisions of each competitive firm $j$. Investments can be either done in the firms’ capital or in individual research, increasing private knowledge $R_j$. Investing one unit into research, enlarges the stock of knowledge not only by one unit, but rather more, depending on the current stock of private knowledge (cf. Romer (1986) p. 1019). The output of the individual firms then reads $Y_j = A(R) \cdot F(R_j, K_j, L_j)$, depending on the total stock of knowledge and the individual inputs, including investments to knowledge (cf. Romer (1994) p. 15). To prevent the production function from increasing returns, the private stock of knowledge is assumed to be rival, and therefore has to be enlarged to achieve more output. The spillovers from the private stocks of knowledge $R_j$ to the total stock of knowledge $R$ describe the endogenization of the technological progress. For the individual firm, the stock of total knowledge is exogenously given, but it is determined by individual decisions.

Another early and quite similar model was developed by Kenneth J. Arrow (born 1921). He stated that the technological progress is influenced by the investments to capital in each period and called it the experience generated by using the goods acquired by the capital stock (cf. Arrow (1962) p. 157 and p. 172). The production function changes to $Y_j = A(K) \cdot F(K_j, L_j)$. In this case the endogenization of the technological progress is a by-product of the production process and the choice of the investment rate, and not directly influenced by an individual decision. Robert Lucas (born 1937) developed a model likewise, distinguishing between physical capital $K$ and human capital $H$. In this case, there are two individual investment decisions. Investments in human capital is generated by devoting time to learning and reaching a higher skill level (cf. Lucas (1988) p. 18). The technological progress is determined by the individual investments into human capital, therefore $Y_j = A(H) \cdot F(K_j, H_j)$ (see additionally for the whole section and especially the equations Romer (1994) p. 15).

The idea of the following models was to explain the differences of income per capita in different countries. To explain the different growth rates the assumption that the same technological opportunities are available at all times for everyone must be dropped (see Romer (1994) p. 4). He also gives one of the first solutions to model this. He predicates that the technological progress depends positively on the the amount of capital
in the respective country and negatively on the amount of labor. The negative effect results from the fact that, with a higher amount of labor it becomes unattractive to drive technological progress in labor saving innovations $A(K, L) = K^\gamma L^{-\gamma}$. Using a Cobb-Douglas production function, as $Y = A(K, L) \cdot K^\alpha \cdot L^{1-\alpha}$, the total output reads $Y = K^{1-\beta}L^\beta$ with $\beta = \alpha - \gamma$. Now the size of $\beta$ is determining, if and to what extend different economies converge (Barro and Sala-i Martin (1992)). Additionally they explain the differences in the possible steady state by further variables included in $A$ like for example the school enrollment rates and political revolutions (ibidem p. 242, Table 3).

Mankiw et al. (1992) assume that the technological level is the same in all economies, but that there is a further input factor of human capital which represents the additional variables which were also used by Robert J. Barro (born 1944) and Xaviar Sala i Martin (born 1962) (Mankiw et al. (1992) p. 11).

### 4.4 Innovation as Engine of Growth

A further approach in endogenous growth theory, which is mainly used today, is to model explicitly technological innovation and investments to research and development. There are two different manners how this influences the production, either by increasing the variety of products or by improving the quality of products. A characteristic of these models is that they take into account that the firms will only invest in research and development, if there is a chance to use the innovation exclusively and that they have at least a temporary monopoly.

One of the best known models is introduced by Paul Romer in 1990. Using the production variety theory of Avinash Dixit (born 1944) and Joseph E. Stiglitz (born 1963), he creates a model where innovations lead to new product or new technologies; the variety of products is increasing (horizontal innovation) (Dixit and Stiglitz (1977), Romer (1990)). To overcome the critique, that obsolescence is not borne in mind, vertical innovations, where the qualities of products and processes are improved and therefore old products substituted, were considered (Grossman and Helpman (1991) and Aghion and Howitt (1992)). Gene Grossman (born 1955) and Elhanan Helpman (born 1946) introduce quality ladders for each product, which are followed during the innovation process. Whereas Philippe Aghion (born 1956) and Peter Howitt (born 1946) follow Schumpeter’s idea of creative destruction (see chapter 2.2 of this paper).

The main setup of these models is quite similar. The economy consists of three sectors, one producing innovations, the second producing intermediates and the third one manufacturing the final output. Using human capital, a distinction between working people’s individual knowledge $H$, which is rival in use, and the technological level $A$, which is non-rival, has to be made (see Romer (1990) p. S79). In the research sector a
share of human capital $H_A$ and the current stock of knowledge are used to produce new ideas and templates (in the case of Romer) or new prototypes with better qualities (in the other case), $\dot{A} = F(A, H_A)$. Than a different firm in the sector of the intermediates bids, to get the right to combine the output of the research sector with further investment goods (foregone output) to produce goods required by the final production $x_i$. In this case the intermediates producer has at least a temporary monopoly and the innovations can be paid (see Aghion and Howitt (1992) p. 328). The remainder of human capital $H_Y$, labor $L$ and physical capital $K$, which consist of the different intermediates $x_i$ are the input used for final production, therefore total output is $Y = F(H_Y, L, \sum_{i=0}^{\infty} x_i)$.

The growth engine of these models is that there are sectors which use their own output as input. In the research sector mainly labor or a share of human capital is used as direct input, combined with the total stock of the technical knowledge of the society influencing the productivity of the used labor. The output of the research sector is the new technical knowledge which is added to the current stock. Additionally the assumption of non-decreasing returns to the direct or indirect inputs is needed to generate positive growth per capita (see Groth (2007) p. 130). Due to the fact that Schumpeter was one of the first to have the idea, that a temporary monopoly is essential to motivate innovations and that new inventions may make old technologies inefficient, growth models incorporating this are called models of neo-Schumpeterian growth (see Aghion and Howitt (1998) chapter 2).

### 4.5 Further Endogenous Growth Models

Today the theory of endogenous growth is the main theory used to analyze economic growth. It is used as basis for further research in many fields and focusing on many different concrete aspects (see for an overview on the different aspects in endogenous growth theory Jones and Manuelli (1997)). Researchers try to incorporate more input factors into the production function to reach more realistic results and to reveal different coherences between growth and the assumptions made to model these input factors. For example, the human capital sector is more specified by incorporating inelastic supply, leisure time, education or unemployment. An other field of economic research incorporates the government as further producing sector producing public goods or as political institution, making different requirements or giving assistance. Different market structures and sizes and even open economics, including international trade, are considered as well. In recent years quite a lot of research deals with the effect of natural resources, keeping sustainability and environmental facts in mind (see next section).

Different sectors are added to reflect the economy of one country or region with the intention to receive more realistic models. The different sectors react differently on
economic decisions, and different production coefficients are assumed (for an introduction to multisector growth models see Roe et al. (2010)). The rising complexity of these models require computational methods. They are introduced to economic growth theory in order to solve these problems and make numerical simulations possible. Additionally not only deterministic models are solved; stochastic parameters are introduced. Doing so, the normal uncertainty of economic development and further uncontrollable influencing factors are incorporated. Now the expected utility becomes the object of maximization (see Heer and Maßner (2009) pp. 27-33 for an example of an stochastic Ramsey model).

A further field of research deals with supplementary empirical studies, which are made to confirm or refuse these expansions to the models and to verify the accompanying assumptions. Testing the theoretical results in comparison to real data sets leads to better assumptions on the different variables. Today more and more data are gathered and edited for research purpose. With the help of today’s computer capacities it is possible to test huge sets of data and compare the results of different sets of variables to each other. Additionally the value of the different variables may be estimated. It is possible to isolate unusual and non-recurrent events, and therefore future predictions become more stalwart, as far as they are determined by the past.

5 Natural Resources and Growth

In recent decades the inter-temporal allocation of exhaustible resources has been a frequently discussed topic, especially in the context of sustainable economic growth. In this context sustainability is defined as, leading to intergenerational justice, meaning that future generations may not be put into a worse position. Concomitant it does not mean that all resources have to be untouched and cannot be used, rather should all consequences of the use (like finiteness and environmental impact) and all substitution possibilities be taken into account (for a further definition see Aghion and Howitt (1998) p. 155-157). During the last years the use of renewable resources is as well discussed critically, because of the competition of land used for cultivation of renewable energy resources and the land used for food production.

5.1 Beginning of Resource Theory

In the beginnings of economic theory, as it is known today, economists were confronted with a completely different set of production. Agricultural products were the main goods produced by using natural resources. The concern that resources would not last forever and might be fully exploited, was of no vital significance. In the classical perspective,
land was the main scare factor, which would lead to diminishing returns and economic stagnation (see Ricardo chapter 2.1 and cf. Groth (2007) p. 127). This period is followed by the first systematic approaches to resource theory, keeping in mind both nonrenewable resources and renewable resources. In 1849 Martin Faustmann (1844-1876) formulates one of the first optimal rules for reforestation (timber as renewable resource), while trying to determine the maximum value of forest floor (Faustmann (1849)); but it was not until 1921, that Bertil Ohlin (1899-1979) formulated the mathematical conditions describing this rule (Ohlin (1995)). An early approach to the economics of nonrenewable resources, which is unknown by most, is evolved by Gray in 1913. He points out the importance of conservation especially of resources which are exhausted by use and non-restorable after exhaustion and the conflict between present and future use (cf. Gray (1913) pp. 499-501). His work is followed by Harrold Hotelling’s (1895-1973) famous work on the economics of exhaustible resources where he revealed the still used Hotelling rule. He provided a rule to maximize the present value of all future profits for the owner of an exhaustible supply (cf. Hotelling (1931) p. 140). The amount extracted has to be chosen in a way that the price of the resource increases at the rate of interest. This leads to the result, that the stock of the resource is not extracted completely as soon as possible.

In the beginning, neoclassical economics dealing with economic growth almost completely neglected natural resources. Until the 1970s the pessimistic Malthusian view, that the limited stock of non-renewable resources would restrict the economic growth, was dominant. Especially the predictions of the Club of Rome emphasized these negative predictions (see Meadows et al. (1972)). By introducing resources as essential input to production in the neoclassical growth framework, this view could be overcome to some extend, because economic growth now was seen as possible as long as there was a sufficient rise in the technical progress. Until than land was seen as the restrict factor for growth (see Ricardo in chapter 2.2 of this paper). Land is introduced as a non-reproducible factor, where the same amount is available in each period (cf. Rebelo (1991) p. 502). Using land as an input factor in the Solow model (see chapter 3.3 of this paper), leads to the results that the growth rates of output and technological progress now differ from each other. The growth rate of total output is now depending on the exogenous technological growth less the production coefficient of land $\beta$ times the population growth $n$

$$gY = g(P, -\beta \cdot n)$$

(cf. Jones (2002) pp. 172-173). In this model land exhibits diminishing returns because it is used as a fixed factor; technological progress, if high enough, may compensate this.

Still the problem was, that in the neoclassical framework, technological progress was seen as exogenous and therefore definitive answers could not be given. The stepping up of efforts in research including the dealing with natural resource was driven by the oil crisis in the mid-1970s. In an symposium issue of Review of Economic Studies in 1974, the basic articles on considering non-renewable resources $R$ in economic growth models
were published (for an summery see Krautkraemer (1998) pp. 2091-2095). They are all starting from the same main assumptions and then concentrate on different aspects (cf. Groth (2007) pp. 135-142). A neoclassical production function with constant returns to scale and exogenous technological progress is used. There is a fixed amount of cumulative resource extraction for all times. If population growth is considered, the largest consumption per head, which lasts to eternity, must be found, keeping in mind the finiteness of the non-renewable resource (cf. Solow (1974) p. 35). There are different possibilities to do so; the non-renewable resource may be substituted by capital or other resources, there might be resources-augmenting technical progress, or increasing returns to scale (cf. Neumeyer (2000) p. 307).

There is only one state where the capital-output ratio $K/Y$ and the ration of used resource to resource stock $R/S$ is constant. It can only be reached, if the rate of technical progress is greater than the share of the resource multiplied by the rate of population growth, which means resource augmenting technological progress (Stiglitz (1974a,b)). In case the economy departs from this path, there exists a finite time horizon after which consumption stops growing and the resource is fully exhausted. Furthermore it is not certain, whether resource augmenting technical progress is permanently possible and is sufficient to replace the declining stock of the resource (cf. Neumeyer (2000) p. 325). Dasgupta and Heal (1974) use a quite different approach to solve the same problem. They use a CES production function; in the case of the substitution parameter $\sigma \leq 1$ the non-renewable resources are essential to production. The production and consequently the consumption level first rises, but tends to zero during time. If $\sigma > 1$, there is no problem, because the resource is inessential for production. To overcome the result of declining output, technical change is introduced at a specific date $T$, when a perfect substitute for the non-renewable resource, in form of a flow of services at a constant rate, is discovered (cf. Dasgupta and Heal (1974) p. 7). At point $T$ the economy switches, and production and consumption rises again.

5.2 Including Resources in Economic Growth Models Today

Again the approach, that is starting from the assumption that technological progress is only exogenously given, is not satisfying. That is why models with endogenous technological progress and including natural resources were developed. The different basic ideas are summarized by Aghion and Howitt (1998), chapter 5. On the one side they incorporate a non-renewable resource to the AK-model (see chapter 4.2 of this paper) and to the Schumpeterian approach (see chapter 3.4 of this paper); on the other side, they include the state of the environment. In all approaches innovations and technological progress are used as a necessary input in order to be able to reach an optimal sustainable growth path. It is possible; but the question is, how it can be reached and which institutions
have to be introduced (cf. Aghion and Howitt (1998) pp. 164-165). Grimaud and Rougé (2003) develop a model using the approach of Aghion and Howitt (1992) incorporating a non-renewable resource and using labor either for final production or for research. Under certain conditions an equilibrium growth path can be revealed, but the outcome may be positive or negative, depending on the discount rate of the economy in relation to the parameters of the research function (cf. Grimaud and Rougé (2003) p. 452). Further approaches were made for example by Scholz and Ziemes (1999) and Schou (2000). The basic results of these models are all quite similar to those, when no non-renewable resource is used, because the resources do not enter the growth engine, the research sector (cf. Groth (2007) p. 150). An idea to solve these critique is presented by Groth (2005). He introduces the non-renewable resource not only into the production function of the output but also as input to the research activity (see ibidem pp. 3-4). In this case there must be population growth and a high elasticity of output to knowledge to compensate for the declining resource input.

It seems so, that when using renewable resources in an economic growth model instead, there are no big differences to a model where resources are not considered. As long as the regeneration capacity of the resource \( \eta_A \) is higher than the demand of the resource for production, and no further input or output is observed, this is true (cf. Aznar-Márquez and Ruiz-Tamarit (2005) p. 180). But in the context of renewable resources further extensions have to be kept in mind. Depending on the chosen production function, a higher input of a resource leads to an higher output and therefore to higher growth. It would be reasonable to use as much renewable resource as possible, but in the context of sustainable growth, the regeneration capacity is a limiting factor. Additionally a further influencing factor is the allocation of labor between harvesting the resource and its use in the final output sector (cf. Turnovsky and Eliasson (2002) pp. 5-6). Another approach is to model the sensitiveness of the resource regeneration to environmental pollution arising from final good production (cf. Tahvonen and Kuuluvainen (1991) pp. 651-652). But pollution may also be generated by the production and use of the resources (both by renewable or by non-renewable). Furthermore it is discussed if a growing use of renewable resources, and therefore an expansion of production is after all desirable, because the land used for resource production is in competition to the land used for food production (cf. Kuhn et al. (2013) p. 471).

6 Outlook

We can see that for a long time the basic theories for economic growth were subject to enormous changes, influenced by the different stages of the societies. Because the existing theories were not satisfying in the new context or did not fit to undergone experiences,
**Fig. 2:** Summery of the presented growth theories

The old theories were developed further. Figure 2 shows a summery of the presented growth theories. Since the endogenous approach to economic growth theory, science concentrated on the extension of these models. In endogenous growth theory, not only the direct influence of agents’ economic decisions, being either consumers, firms or the state, made at time $t$, on growth is revealed. Furthermore the implicit effects on the other agents, variables determined by the past and exogenous unplanned effects are considered (cf. Novales et al. (2010) pp. 27-28). In the beginning growth theory was almost entirely a macroeconomic theory. Today the structure of growth models is completely microeconomic founded, to include specific and detailed assumptions. Especially with the usage of computer technologies, it is possible to solve even more complex models. Today’s research is focused on the incorporation of more aspects influencing economic growth, either in a positive direction or limiting economic growth.

Since the beginning of the 21st century, non-renewable but also renewable resource...
become more and more an important input factor to models of economic growth. In the background of scare resource, sustainable development and environmental issues it is essential to integrate the economic resource theory into growth theory to reveal the influences on economic growth.

References


