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Robustness of a Simple Rule for the Social Cost of Carbon

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Robustness of a Simple Rule for the Social Cost of Carbon

Abstract

The optimal social cost of carbon is in general equilibrium proportional to GDP if utility is logarithmic, production is Cobb-Douglas, depreciation in 100% every period, climate damages as fraction of production decline exponentially with the stock of atmospheric carbon, and fossil fuel extraction does not require capital. The time profile and size of the optimal carbon tax corresponding to this simple formula are not robust to more convex climate damages, smaller elasticities of factor substitution and varying coefficients of relative intergenerational inequality aversion. The optimal timing of energy transitions and the amount of fossil fuel reserves to be locked up in the earth are also not well predicted by this framework. Still, in terms of welfare and global warming the simple formula manages to get quite close to the first best.

JEL-Code: H210, Q510, Q540.

Keywords: social cost of carbon, Ramsey growth, climate damages, energy transitions, stranded fossil, fuel assets, robustness.

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Research highlights:
- The optimal carbon tax is a hump-shaped function of world GDP.
- The optimal carbon tax increases rapidly during the fossil fuel era but peaks only well into carbon-free era.
- A tax rule which sets the carbon tax proportional to current GDP approximates the first-best welfare closely.
- The optimal transition and amount of fossil fuel reserves to be abandoned *in situ* are not well predicted by a proportional tax rule.

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1. Introduction

A tractable model of the optimal carbon tax has been put forward by Golosov et al. (2014) based on a decadal Ramsey growth model and been extended by Hassler and Krusell (2012), Gerlagh and Liski (2012) and Iverson (2013). This model has logarithmic utility, Cobb-Douglas production, 100% depreciation each period, exponential damages, and zero capital intensity of fossil fuel extraction. These are heroic conditions and necessitate a coarse calibration grid, but with them it can be shown that the *social cost of carbon* (*SCC*) is proportional to current GDP and independent of technology. We evaluate the robustness of this simple formula in a more general Ramsey growth model with CES production and CES utility, stock-dependent extraction costs, partial depreciation and more convex climate damages, which allows for endogenous energy transition times and untapped fossil fuel reserves.

2. Ramsey growth and energy transitions

Let social welfare be utilitarian, with per capita utility U depending on per capita consumption C_t / L_t, where L_t is total population (exogenous) and ρ the rate of time preference:

$$(1) \qquad \mathrm{E}_0\left[\sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t L_t U_t(C_t/L_t)\right] = \mathrm{E}_0\left[\sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t L_t\left[\frac{(C_t/L_t)^{1-1/\eta}-1}{1-1/\eta}\right]\right], \quad \rho>0\,.$$

The elasticity of intertemporal substitution equals η. The ethics of climate policy depend on the weight given to future generations (and thus on how small ρ is) and on how small intergenerational inequality aversion is or how difficult it is to substitute current for future consumption per head (how low $1/\eta$ is). Optimal climate policy faces some constraints governing the global economy. First, output at time *t*, $Z(K_t, L_t, F_t, R_t)$, is produced using capital K_t, labor, L_t, fossil fuels (e.g., oil, natural gas and coal), F_t, and renewables (e.g., solar or wind energy), R_t. We allow for imperfect factor substitution, so both fossil fuel and renewable energy are essential in production. Fossil fuel extraction costs, $G(S_t)F_t$, rise as reserves, S_t, fall, $G' < 0$. Renewable energy is supplied infinitely elastically at exogenously decreasing cost, b_t. Technical progress increases productivity in both aggregate and renewable energy production. Climate damages curb output and are captured by the factor $1-\Lambda(T_t)$, $\Lambda' < 0$. Production net of costs of energy production and climate damage is allocated to consumption C_t, investments in manmade capital and depreciation with δ the rate of depreciation:

$$(2) \qquad K_{t+1} = (1-\delta)K_t + \Lambda(T_t)Z(K_t, L_t, F_t, R_t) - G(S_t)F_t - b_t R_t - C_t.$$

The dynamics of fossil fuel reserves are:

$$(3) \qquad S_{t+1} = S_t - F_t = S_0 - \sum_{s=0}^{t} F_s, \quad \sum_{t=0}^{\infty} F_t \le S_0.$$

Golosov et al. (2014) introduce a two-stock carbon cycle where emissions lead to a permanent component E_1 and a transient component E_2 of the stock of carbon in the atmosphere:

$$E_{1t} = E_{1t-1} + \varphi_L F_t, \tag{4}$$

$$E_{2t} = \varphi E_{2t-1} + \varphi_0 (1-\varphi_L) F_t, \tag{5}$$

where φ_L denotes the fraction of emissions that stays permanently in the atmosphere, φ the speed at which the temporary stock of carbon decays, and φ_0 a coefficient to calibrate how much of carbon is returned to the surface of the oceans and earth within a decade. We define temperature, T_t, as deviations from pre-industrial temperature in degrees Celcius. The climate sensitivity, ω, corresponds to the rise in temperature ensuing from a doubling of the total stock of carbon in the atmosphere, E_t:

$$T_t = \omega \ln\left(\frac{E_t}{\overline{E}}\right) / \ln(2), \quad E_t \equiv E_{1t} + E_{2t}, \tag{6}$$

where $\overline{E} = 596.4$ GtC is the IPCC figure for the pre-industrial stock of atmospheric carbon. Using (6) we redefine damages as $D(E_t) \equiv \Lambda\left(\omega \ln\left(\frac{E_t}{\overline{E}}\right) / \ln(2)\right)$. This formulation ignores lags between atmospheric carbon and global warming and the improvements that result from a three-stock carbon cycle, but with these features one can still obtain the linear formula for the *SCC* (Gerlagh and Liski, 2012).

The social planner maximizes (1) subject to (2)-(6). The Lagrangian reads as follows:

$$L \equiv \sum_{t=0}^{\infty} (1+\rho)^{-t} \left[L_t U_t(C_t / L_t) - \lambda_t \left(K_{t+1} - (1-\delta) K_t - D(E_t) Z(K_t, L_t, F_t, R_t) + G(S_t) F_t + b_t R_t + C_t \right) \right]$$
$$+ \sum_{t=0}^{\infty} (1+\rho)^{-t} \left[\nu_{1,t} (E_{1,t+1} - E_{1,t} - \varphi_L F_t) + \nu_{2,t} \left(E_{2,t+1} - (1-\varphi) E_{2,t} - \varphi_0 (1-\varphi_L) F_t \right) - \mu_t (S_{t+1} - S_t + F_t) \right],$$

where λ_t denotes the shadow value of capital, ν_{1t} and ν_{2t} the shadow disvalue of the permanent and transient stocks of atmospheric carbon, and μ_t the shadow value of in-situ fossil fuel. The efficiency conditions for a social optimum are (appendix A):

$$\frac{C_{t+1} / L_{t+1}}{C_t / L_t} = \left(\frac{1 + r_{t+1}}{1+\rho} \right)^{\eta}, \quad r_{t+1} \equiv D(E_{t+1}) Z_{K_{t+1}} - \delta, \tag{7}$$

$$D(E_t) Z_{F_t} \le G(S_t) + s_t + \tau_t, \quad F_t \ge 0, \quad \text{c.s.}, \tag{8}$$

$$D(E_t) Z_{R_t} \le b_t, \quad R_t \ge 0, \quad \text{c.s.}, \tag{9}$$

$$s_t = -\sum_{\varsigma=0}^{\infty} \left[G'(S_{t+1+\varsigma}) F_{t+1+\varsigma} \Delta_{t+\varsigma} \right], \tag{10}$$

$$\tau_t = -\sum_{\varsigma=0}^{\infty} \left[\left\{ \varphi_L + \varphi_0 (1-\varphi_L)(1-\varphi)^{\varsigma} \right\} \Delta_{t+\varsigma} D'(E_{t+1+\varsigma}) Z(K_{t+1+\varsigma}, L_{t+1+\varsigma}, F_{t+1+\varsigma}, R_{t+1+\varsigma}) \right]. \tag{11}$$

with compound discount factors $\Delta_{t+\varsigma} \equiv \prod_{\varsigma'=0}^{\varsigma}(1+r_{t+1+\varsigma'})^{-1}, \varsigma \geq 0.$

Equation (7) is the Euler equation, where the positive effect of the return on capital (r_{t+1}) on consumption growth is bigger if intertemporal substitution is easier (high η). If fossil fuel is used, equation (8) indicates that its marginal product should equal marginal extraction cost plus the scarcity rent, $s_t \equiv \mu_t / \lambda_t$, plus the *SCC*, $\tau_t \equiv \left[\varphi_L \nu_{1t} + \varphi_0(1-\varphi_L)\nu_{2t}\right] / \lambda_t$. If the marginal product of fossil fuel is below total marginal cost, it is not used. Equation (9) states the equivalent condition for renewable use. Equation (10) follows from the Hotelling rule and gives the scarcity rent of keeping an extra unit of fossil fuel unexploited as the present discounted value of all future reductions in fossil fuel extraction costs. Equation (11) defines the *SCC* as the present discounted value of all future marginal global warming damages from burning an additional unit of fossil fuel. One unit of carbon released from burning fossil fuel affects the economy in two ways: the first part remains in the atmosphere for ever and the second part gradually decays over time at a rate corresponding to roughly 1/300 per year.

The *SCC* is proportional to GDP if utility is logarithmic, $\delta = 1$, $D(E_t) = e^{-\gamma(E_t - \bar{E})}$ with $\gamma > 0$ the climate damage parameter, Cobb-Douglas production function for capital, labor and energy, and extraction does not require capital inputs:

$$(11') \qquad \tau_t = \gamma\left[\left(\frac{1+\rho}{\rho-n}\right)\varphi_L + \left(\frac{1+\rho}{1+\rho-\varphi(1+n)}\right)\varphi_0(1-\varphi_L)\right]GDP_t$$

where $GDP_t = D(E_t)Z(K_t, L_t, F_t, R_t)$ (Golosov et al., 2014), because the ratios of the future capital stock and aggregate consumption to GDP are constant, namely $K_{t+1} = \left[\frac{\alpha(1+n)}{1+\rho}\right]GDP_t$ and $C_t = \left\{1 - \left[\frac{\alpha(1+n)}{1+\rho}\right]\right\}GDP_t$ with α the share of capital in value added (appendix B).

2. Policy Simulation and Optimization

In our simulations time runs from 2010 till 2600 and is measured in decades, t = 0,1,.., 59, so period 0 corresponds to 2010-2020, period 1 to 2020-2030, etc. The final time period is t = 59 or 2600-2610, but we focus on the transitional dynamics in the earlier parts of the simulation.[1] The functional forms and benchmark parameters of our model are discussed in appendix C. We report in table 1 and figure 1 full simulation results for: (*i*) the general model with the *SCC* set to (11), (*ii*) the general model with the *SCC* set to (11′); (*iii*) the general model with the *SCC* set to zero ("laissez faire"); and (*iv*) the simplified model (see end of section 2) with the *SCC* set to (11′).

[1] In simulating the model of Golosov et al (2014) we extend the horizon by 1000 years till 3600 (max t = 159) due to the transitional dynamics' strong dependence on terminal conditions, most notably asymptotic *in situ* depletion $\lim_{\varsigma\to\infty} S_\varsigma = 0$.

The *first-best* outcomes in the general and simplified IAM's, (*i*) and (*iv*), correspond to the solid green and long-dashed blue lines in figure 1, respectively. The forecasts of transitional booms in productivity and population lead to sustained growth in consumption, output, and the capital stock over the next two centuries (as depicted in first, second and third panels in fig.1). In the general IAM the social optimum involves a period of initial fossil fuel use which is phased out halfway this century. After this switch point, energy is supplied from renewable sources. Optimal climate policy limits global warming to 2.3°C above pre-industrial temperature and is implemented through a rapidly rising carbon tax which flattens out and falls as economic growth and emissions taper off and decay of atmospheric carbon starts to dominate.

Table 1: Transition times and carbon budget

	Fossil fuel Only	*Simultaneous use*	*Renewable Only*	*Carbon used*	*Maximum temperature*
First best	2010-2050	2060	2070 –	690 GtC	2.3 °C
Proportional tax	2010-2070	x	2080 –	990 GtC	2.8 °C
"Laissez faire"	2010-2110	x	2120 –	2100 GtC	4.3 °C
Simple IAM	x	2010 –	x	4000 GtC	4.8 °C

In contrast, the simplified IAM features persistent simultaneous use of fossil and renewable energy sources.[2] Under positive discounting, the fossil fuel extraction decreases by the discount factor but technical progress in both energy sectors sustains rising energy production. The absence of extraction costs implies asymptotic depletion of all *in situ* sources (4,000 GtC) and translates into high degrees of global warming of up to 4.8°C (which translate into a loss in output of mere 3.3%). In contrast, in the general IAM with extraction costs cumulative 700 GtC are burnt and 3,300 GtC is locked up *in situ*.

The missing carbon tax under "laissez faire" leads to inefficient allocation of resources with excessive fossil fuel extraction and high global warming damages. This lowers welfare by 17% of today's GDP because it keeps consumption low in early periods of the program to allow for capital accumulation and consumption low in future periods due to high global warming of up to 4.3°C. Under "laissez faire" decarbonisation is delayed until the next century and cumulative emissions are three times the social optimum (2,100 GtC).

[2] The absence of extraction costs in the model of Golosov et al. (2014) required us to keep their assumption of limited (unit) substitutability between fossil and renewable energy sources. Infinite elasticity would imply an unrealistically high initial extraction level and rapid extraction path.

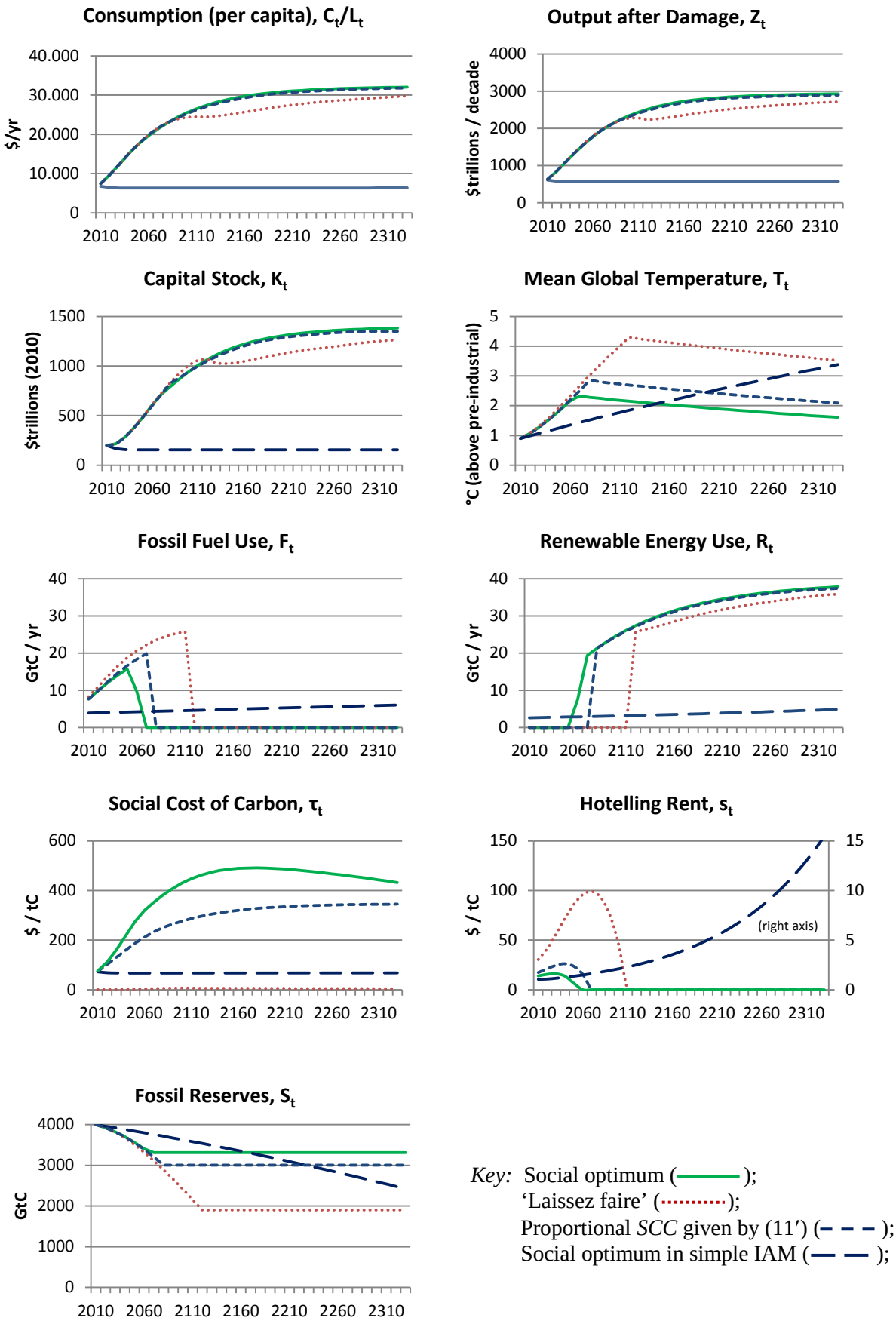
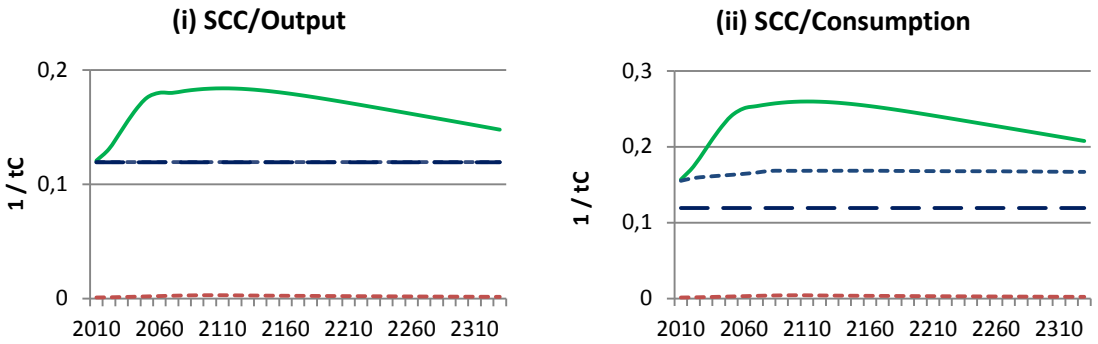


Figure 1: Simulations of social optimum and "laissez faire" in general and simplified IAM

3. The optimal carbon tax is not proportional to GDP or aggregate consumption

Figure 2 plots the ratio of the *SCC* to both GDP and aggregate consumption.



Key: Social optimum (——); "Laissez faire" (·····);
SCC given by (11′) (– – –); Simplified IAM (— —);

Figure 2: The social cost of carbon as ratio of GDP and aggregate consumption

We observe that the *SCC* of the general IAM (solid green line) is not well described by a constant proportion of GDP or aggregate consumption. The general pattern of the *SCC* is a steeply rising section during the initial phases of fossil fuel use as more carbon emissions push up marginal climate damages whilst during the carbon-free phases the *SCC* falls as a proportion of GDP as a significant part of the stock of carbon in the atmosphere is gradually returned to the surface of the oceans and biomass. The transient population boom increases the initial slope as growth-related emission accelerates. The simple formula (11′) performs even worse when expressed as a fraction of consumption, because the elasticity of substitution different from unity markedly shifts the consumption profile: the short-dashed blue and solid lines in panel (ii) have different levels and slopes.

The short-dashed blue dashed lines in fig. 1 coresond to scenario (*iv*) and uses the *SCC* of the simplified model (11′) in the more general IAM. While the *SCC* is rising too slowly, the approximation is able to internalize the climate externality sufficiently to limit global warming to 2.8°C and reduce the amount of carbon burnt to 1,000 GtC. The policy error in using a proportional policy rule is small, since the welfare loss relative to the first best is only 1% of today's GDP compared with 17% under "laissez faire".

4. Robustness of the optimal social cost of carbon

Figure 3 reports some robustness exercises, all of which suggest that setting the *SCC* according to (11′) is a poor approximation to the optimal *SCC*. The simple formula for the *SCC* is non-optimal and induces almost 50% more fossil fuel to be burnt relative to the first best (see table 1) and more severe climate change.

A higher (lower) social rate of discount ρ leads to a less (more) ambitious climate policy with a lower (higher) *SCC*, later (earlier) phasing in of renewables and less (more) fossil fuel is left *in situ*. A higher elasticity of intertemporal substitution η corresponds to a lower coefficient of intergenerational inequality aversion (which was set to 2 in the baseline scenario) which implies that the *SCC* hurts earlier generations much more than later generations. Since the economy is growing, the social planner is relatively more concerned with fighting global warming than with avoiding big differences in consumption of different generations.

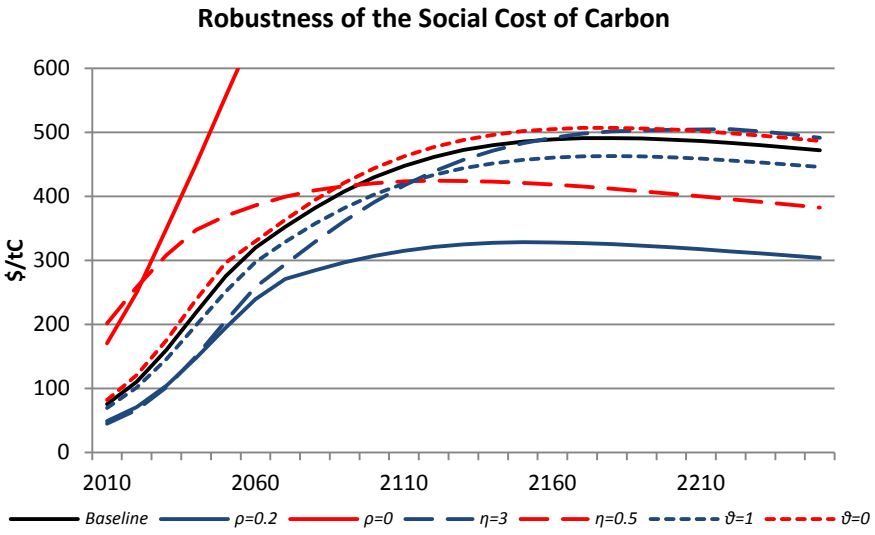


Figure 3: Sensitivity analysis for the time paths of the social cost of carbon

A higher elasticity of substitution between energy and the capital-labor aggregate, ϑ, makes energy demand more sensitive to change in the relative costs of energy. This implies that the economy uses less energy and leaves more fossil fuel *in situ* which lowers the social cost of carbon. Under Leontief production, more fossil fuel is used but the economy switches to renewable energy earlier.

5. Conclusion

The *SCC* is not well described by a constant proportion of GDP or aggregate consumption as suggested by Golosov et al. (2014). The correct *SCC* rises steeply during the initial phases of fossil fuel use as more carbon emissions push up marginal damages of global warming whilst during the carbon-free phases it falls as a proportion of GDP due to decay of the transient part of atmospheric carbon. This pattern is consistent over a broad range of key parameters. Although the *SCC* put forward by Golosov et al. (2014) rises too slowly, the approximation is able to internalize the climate externality sufficiently to limit global warming to 2.8°C and reduce the amount of carbon burnt to 1,000 GtC. The policy error in using the simple formula for the *SCC* is small: the welfare loss relative to the first best is only 1% of today's GDP compared to a loss of 17% of today's GDP under "laissez faire". The simple formula curbs global warming from 4.3 °C under "laissez faire" to 2.8 °C , which

is not far from the first best which curbs temperature to 2.3 °C. Although the simple formula induces sub-optimal energy transition times and amounts of untapped fossil fuel, we conclude that a policy approximating the *SCC* closely is able to avoid the most deleterious damages and is thus a lot better than doing nothing.

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Appendix A: Necessary optimality conditions for the general model

Necessary conditions for a social optimum are:

(A.1) $U'(C_t / L_t) = (C_t / L_t)^{-1/\eta} = \lambda_t,$

(A.2) $D(E_t)Z_{F_t} \le G(S_t) + \left[\mu_t + \varphi_L \nu_{1t} + \varphi_0(1-\varphi_L)\nu_{2t} \right] / \lambda_t, \quad F_t \ge 0, \quad \text{c.s.},$

(A.3) $D(E_t)Z_{R_t} \le b_t, \quad R_t \ge 0, \quad \text{c.s.},$

(A.4) $(1-\delta + D(E_{t+1})Z_{K_{t+1}})\lambda_{t+1} = (1+\rho)\lambda_t,$

(A.5) $\mu_{t+1} = (1+\rho)\mu_t + G'(S_{t+1})F_{t+1}\lambda_{t+1},$

(A.6) $\nu_{1t+1} = (1+\rho)\nu_{1t} - D'(E_{t+1})Z_{t+1}\lambda_{t+1},$

(A.7) $(1-\varphi)\nu_{2t+1} = (1+\rho)\nu_{2t} - D'(E_{t+1})Z(K_{t+1}, L_{t+1}, F_{t+1}, R_{t+1})\lambda_{t+1}.$

Equations (A.1) and (A.4) give (7) and equations (A.2) and (A.3) yield (8) and (9) with $s_t \equiv \mu_t / \lambda_t$ and $\tau_t \equiv \left[\varphi_L \nu_{1t} + \varphi_0(1-\varphi_L)\nu_{2t}\right] / \lambda_t$. Equations (A.4) and (A.5) give (10). Finally, using (A.6), (A.7) and (A.4), the *SCC*, defined in final good units, can be found to equal (11).

Appendix B: Necessary optimality conditions for the simple model

We use a version of Golosov et al. (2014) which conflates coal, oil and natural gas into one fossil fuel source, F_t. The *SCC* is derived assuming that $U(C_t / L_t) = \ln(C_t / L_t)$, $\delta = 1$, $D(E_t) = e^{-\gamma(E_t - \bar{E})}$, $Z_t = A_t L_{0t}^{1-\alpha-\upsilon} K_t^{\alpha} H_t^{\upsilon}$, and $H_t = \left(\kappa_1 F_t^{\chi} + \kappa_2 R_t^{\chi}\right)^{1/\chi}$ with $\varepsilon \equiv 1/(1-\chi) > 0$ the elasticity of factor substitution. Only labor is needed to extract fossil fuel or produce renewable energy: $F_t = A_{Ft} L_{Ft}$ and $R_t = A_{R,t} L_{R,t}$. Labor is perfectly mobile between the final goods and energy sectors:

(B.1) $L_t = L_{0t} + F_t / A_{Ft} + R_t / A_{Rt}.$

where L_t denotes the exogenous aggregate supply of labor. The carbon cycle is still given by (3)-(5).

After substitution of L_{0t} the Lagrangian function for this problem is:

$$
\begin{aligned}
L \equiv & \sum_{t=0}^{\infty} (1+\rho)^{-t} \left\{ L_t \ln(C_t / L_t) - \mu_t (S_{t+1} - S_t + F_t) \right\} \\
& + \sum_{t=0}^{\infty} (1+\rho)^{-t} \left\{ \nu_{1t} \left[E_{1t} - E_{1t-1} - \varphi_L F_t \right] + \nu_{2t} \left[E_{2t} - \varphi E_{2t-1} - \varphi_0(1-\varphi_L) F_t \right] \right\} \\
& - \sum_{t=0}^{\infty} (1+\rho)^{-t} \lambda_t \left\{ K_{t+1} + C_t - e^{-\gamma(E_{1t} + E_{2t} - \bar{E})} A_{0t} \left(L_t - F_t / A_{Ft} - R_t / A_{Rt} \right)^{1-\alpha-\upsilon} K_t^{\alpha} \left[\left(\kappa_1 F_t^{\chi} + \kappa_2 R_t^{\chi} \right)^{1/\chi} \right]^{\upsilon} \right\},
\end{aligned}
$$

Maximizing with respect to C_t and K_t yields $L_t / C_t = \lambda_t$ and $(1+\rho)\lambda_{t-1} = \alpha\lambda_t Y_t / K_t$ which gives $\frac{\alpha}{1+\rho}\frac{C_{t-1}}{L_{t-1}} = \frac{K_t}{Y_t}\frac{C_t}{L_t}$, where $Y_t \equiv D(E_t)Z_t$. It follows that $\frac{\alpha}{1+\rho}\frac{L_t}{L_{t-1}}\frac{C_{t-1}}{Y_{t-1}} = \left(1-\frac{C_{t-1}}{Y_{t-1}}\right)\frac{C_t}{Y_t}$. Assuming constant population growth *n*, i.e., $L_t = (1+n)L_{t-1}$, we have (cf. Brock and Mirman, 1972):

(B.2) $$K_{t+1} = \left[\frac{\alpha(1+n)}{1+\rho}\right]Y_t, \quad C_t = \left\{1-\left[\frac{\alpha(1+n)}{1+\rho}\right]\right\}Y_t.$$

Maximizing with respect to the permanent and transient stocks of atmospheric carbon, we get $\gamma Y_t \lambda_t = \frac{\gamma L_t}{1-\alpha(1+\rho)^{-1}(1+n)} = \nu_{1t} - \frac{\nu_{1t+1}}{1+\rho} = \nu_{2t} - \frac{\varphi\nu_{2t+1}}{1+\rho}$, which gives:

(B.3a) $$\nu_{1t} = \frac{\gamma L_t}{\left[1-(1+\rho)^{-1}(1+n)\right]\left[1-\alpha(1+\rho)^{-1}(1+n)\right]}, \quad \text{and}$$

(B.3b) $$\nu_{2t} = \frac{\gamma L_t}{\left[1-\varphi(1+\rho)^{-1}(1+n)\right]\left[1-\alpha(1+\rho)^{-1}(1+n)\right]}.$$

Making use of (B.2) and (B.3a) and (B3b) we get the *SCC*, $\tau_t \equiv \left[\varphi_L\nu_{1t} + \varphi_0(1-\varphi_L)\nu_{2t}\right]/\lambda_t$ as (11′). The *SCC* is thus proportional to world GDP (usual output after damage). The *SCC* as fraction of GDP is higher if society is more patient (lower ρ), damages are more sensitive to the stock of atmospheric carbon (higher γ), the temporary component of atmospheric carbon decays more quickly (higher φ), and population growth is bigger. If all carbon stays in the atmosphere forever ($\varphi = 1$), equation (11′) simplifies to $\tau_t = \left(\frac{1+\rho}{\rho-n}\right)\varphi_L^*\gamma Y_t$, where $\varphi_L^* \equiv \varphi_L + \varphi_0(1-\varphi_L)$. Since global warming is the only market failure, the social optimum can be realized in a market economy by setting the optimal carbon tax exactly equal to the *SCC* and refunding the revenues in a lump-sum fashion to the private sector.

Maximizing with respect to F_t, and R_t yields $A_{Rt}\upsilon\frac{Y_t}{H_t}\kappa_R\left(\frac{H_t}{F_{Rt}}\right)^{1-\chi}\lambda_t = w_t\lambda_t$,

$\upsilon\frac{Y_t}{H_t}\kappa_1\left(\frac{H_t}{F_t}\right)^{1-\chi}\lambda_t = \mu_t + \eta_t + \frac{w_t}{A_{1t}}\lambda_t$ or alternatively:

(B.4) $$\upsilon\frac{Y_t}{H_t}\kappa_1\left(\frac{H_t}{F_t}\right)^{1-\chi} = \frac{w_t}{A_{Ft}} + s_t + \tau_t, \quad \upsilon\frac{Y_t}{H_t}\kappa_2\left(\frac{H_t}{F_{Rt}}\right)^{1-\chi} = \frac{w_t}{A_{Rt}},$$

where the *wage* is $w_t = (1-\alpha-\upsilon)\frac{Y_t}{L_{0t}}$ and the *scarcity rent of oil* is $s_t \equiv \mu_t / \lambda_t$. The first part of (B.4) states that the marginal product of oil is set to the sum of the labor cost involved in extracting oil, the

scarcity rent of oil and the social cost of carbon. The second part of (B.4) states that the marginal product of renewable energy equals the labor cost of producing it. The labor cost is less if efficiency of labor is higher and if the wage is less. Equations (B.4) can be solved for the two types of energy use (labor use in fossil and renewable production), aggregate energy use and world GDP as functions of the scarcity rent of oil, the social cost of carbon, the capital stock and the total stock of atmospheric carbon given aggregate labor supply and the technology parameters.

Maximizing with respect to S_t gives $(1+\rho)\mu_{t-1} = \mu_t$ and the Hotelling rule:

(B.5) $$s_t = \frac{\alpha Y_t}{K_t} s_{t-1}.$$

This rule states that the scarcity rent of oil should grow at the rate of interest for society to be indifferent between keeping an extra barrel of oil in the ground and extracting an extra barrel of oil. Since extraction costs of fossil fuel do not become infinitely large as oil and gas reserves vanish, the optimum is characterized by asymptotic depletion of these reserves.

The "laissez-faire" outcome does not internalize climate externalities and thus corresponds to a zero carbon tax, $\tau_t = 0$. It follows from (B.4) and (B.5) that $s_t = \upsilon \frac{Y_t}{H_t} \kappa_1 \left(\frac{H_t}{F_t} \right)^{1-\chi} - \frac{w_t}{A_{Rt}} = \frac{\alpha Y_t}{K_t} s_{t-1}$.

Abstracting from extraction costs of oil and gas ($A_{Ft} \to \infty$), supposing a Cobb-Douglas production for the energy aggregate ($\chi \to 0$), abstracting from technological progress in renewable energy production ($A_{Rt} \to A_R$), we get $F_t = (1+\rho)^{-1}(1+n)F_{t-1}$. Imposing the condition that asymptotically *in situ* reserves are depleted, we obtain $F_t = \left[1 - \left(\frac{1+n}{1+\rho} \right) \right] S_t = \left(\frac{\rho - n}{1+\rho} \right) \left(\frac{1+n}{1+\rho} \right)^t S_0$, so that extraction of oil and gas occurs more rapidly if the discount rate is high and population growth is small. For any given current rate of oil extraction the rate of future oil extraction is higher in the social optimum than under "laissez faire". Hence, the former postpones oil extraction and carbon emissions compared to "laissez faire", especially if carbon decay is slow and society is patient (small φ, low ρ).

Appendix C: Functional forms, calibration and computational implementation

In the simulations we use iso-elastic utility function $U(C/L) = \frac{(C/L)^{1-1/\eta} - 1}{1 - 1/\eta}$. We set the elasticity of intertemporal substitution to η = ½ and thus intergenerational inequality aversion to 2. The pure rate of time preference ρ is set to 10% per decade which corresponds to 0.96% per year.

We employ an extraction technology of the form $G(S) = \gamma_1 (S_0 / S)^{\gamma_2}$, where γ_1 and γ_2 are positive constants. This specification implies that reserves will not be fully be extracted; some fossil fuel

remains untapped in the crust of the earth. Extraction costs are calibrated to give an initial share of energy in GDP between 5%-7% depending on the policy scenario. This translates to fossil production costs of \$350/tC (\$35/barrel of oil), where we take one barrel of oil to be equivalent to 1/10 ton of carbon. This gives approximately $G(S_0) = \gamma_1 = 0.75$. The IEA (2008) long-term cost curve for oil extraction gives a doubling to quadrupling of the extraction cost of oil if another 1000 GtC are extracted. Since we are considering all carbon-based energy sources (not only oil) which are more abundant and cheaper to extract, we assume a more doubling but less than quadrupling of production costs if a total 3000 GtC is extracted. With $S_0 = 4000$ GtC,[3] this gives $\gamma_2 = 0.75$.[4] In general, we assume very low extraction costs and a high initial stock of reserves.

The unit cost of renewable energy is calibrated to the percentage of GDP necessary to generate all energy demand from renewables. Under a Leontief technology, with $\vartheta \to 0$, energy demand is σZ_t with Z_t potential, pre-damage output and σ the carbon intensity of output. The cost of generating all energy carbon free is $\sigma Z_t b_t / Z_t = \sigma b_t$. Nordhaus (2008) assumes that it costs 5.6% of GDP to achieve this. We take double this number $\sigma b_1 = 0.12$ (i.e. we assume 12%) or, with $\sigma = 0.62$ as derived below, $b_1 = 2$. In the future this cost falls to current prices of fossil energy (with energy amounting to about 5% of GDP), that is b_t approaches 0.8). We assume that exogenous technical progress lowers the unit cost at a falling rate starting at a reduction of 1% per year. Specifically, $b_t = 0.8 + 1.2e^{-0.1t}$. This calibration is done for a Leontief technology. We assume that for a more general technology the same parameter values can be applied. Our calibration assumes that renewable energy is initially very expensive and falls to current levels only in the very long run. This, together with the assumption about fossil energy, biases the model against rapid de-carbonization.

The initial capital stock is set to 200 (US\$ trillion), which is taken from Rezai and van der Ploeg (2013). We set δ to be 0.5 per decade, which corresponds to a yearly depreciation rate of 6.7%.

Population in 2010 (L_1) is 6.5 billion people. Following Nordhaus (2008) and UN projections population growth is given by $L_t = 8.6 - 2.1e^{-0.35t}$. Population growth starts at 1% per year and falls below 1% percent per decade within six decades and flattens out at 8.6 billion people. Without loss of generality the efficiency of labor $A_t^L = 3 - 2e^{-0.2t}$ starts out with $A_1^L = 1$ and an initial Harrod-neutral rate of technical progress of 2% per year. The efficiency of labor stabilizes at 3 times its current level.

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[3] Stocks of carbon-based energy sources are notoriously hard to estimate. IPCC (2007) assumes in its A2-scenario that 7000 $GtCO_2$ (with 3.66 tCO2 per tC this equals 1912 GtC) will be burnt with a rising trend this century alone. We roughly double this number to get our estimate of 4000 GtC for initial fossil fuel reserves. Nordhaus (2008) assumes an upper limit for carbon-based fuel of 6000 GtC in the DICE-07.

[4] Since $G(1000)/G(4000) = (4000/1000)^{\gamma_2} = 4^{\gamma_2}$ and $4^{0.75} = 2.8$.

Output before damages is $Z_t = \left[(1-\beta)\left(AK_t^{\alpha}(A_t^L L_t)^{1-\alpha} \right)^{1-1/\vartheta} + \beta(\frac{F_t + R_t}{\sigma})^{1-1/\vartheta} \right]^{\frac{1}{1-1/\vartheta}}$, $\vartheta \geq 0, 0 < \alpha < 1$ and $0 < \beta < 1$. This is a constant-returns-to scale CES production function in energy and a capital-labor composite with ϑ the elasticity of substitution, β the share the parameter for energy, and σ the carbon intensity of output. The capital-labor composite is defined by a constant-returns-to-scale Cobb-Douglas function with α the share of capital, *A* total factor productivity and A_t^L the efficiency of labor. The two types of energy are perfect substitutes in production. Damages are calibrated so that they give the same climate damages for the initial levels of output and mean temperature. It is convenient to rewrite production before damages as $Z_t = Z_0 \left[(1-\beta)\left(\frac{AK_t^{\alpha}(A_t^L L_t)^{1-\alpha}}{Z_0} \right)^{1-1/\vartheta} + \beta\left(\frac{F_t + R_t}{\sigma Z_0} \right)^{1-1/\vartheta} \right]^{\frac{1}{1-1/\vartheta}}$.

We set the share of capital to $\alpha = 0.35$, the energy share parameter to $\beta = 0.05$, and the elasticity of factor substitution to $\vartheta = 1$. World GDP in 2010 is 63 $trillion. The energy intensity of output σ is calibrated to current energy use. In the Leontief case energy demand (only fossil fuel initially) is $F_0 = \sigma D_0 Z_0$. With carbon input equal to 8.36GtC in 2010, we obtain $\sigma = (8.36 / 2.13) / 63 = 0.062$. Finally, given $A_1^L = 1$ we can back out *A* = 34.67.

Following Golosov et al. (2014), the decay rate for the transient stock of atmospheric carbon is $\varphi = 0.0228$ and $\varphi_L = 0.2$, so that 20% of carbon emissions stay up 'forever' in the atmosphere and the remainder has a mean life of about 300 years The parameter $\varphi_0 = 0.393$ is calibrated so that about half of the carbon impulse is removed after 30 years. We set $E_0^P = 103$ GtC and $E_0^T = 699$ GtC. We suppose an equilibrium climate sensitivity of $\omega = 3$.

Nordhaus (2008) supposes that with global warming of 2.5° C damages are 1.7% of world GDP and uses this to calibrate: $D(T_t) = \frac{1}{1 + 0.00284 T_t^2} = \frac{1}{1 + (T_t / 18.8)^2}$. Weitzman (2010) argues that global warming damages rise more rapidly at higher levels of mean global temperature. With climate damages equal to 50% of world GDP at 6° C and 99% at 12.5° C, Ackerman and Stanton (2012) calibrate $D(T_t) = \frac{1}{1 + (T_t / 20.2)^2 + (T_t / 6.08)^{6.76}}$. The extra term in the denominator captures potentially catastrophic losses at high temperatures.

Computational implementation

The transversality condition for the model is $\lim_{t \to \infty} e^{-\rho t}(\lambda_t K_t + \mu_t S_t + \eta_{1t} E_t^P + \eta_{2t} E_t^T) = 0$. In our simulations we solve the model for finite time and use the turnpike property to approximate the infinite-horizon problem. All equilibrium paths approach the steady state quickly such that the

turnpike property renders terminal conditions essentially unimportant. We allow for continuation stocks to reduce the impact of the terminal condition on the transitions paths in the early periods of the program. We use the computer program GAMS and its optimization solver CONOPT3 to solve the model numerically. The social planner optimum in which the externality is taken into account fit the program structure readily. To solve for the "laissez faire" equilibrium paths, we adopt the iterative approach discussed in detail in Rezai (2011). Briefly, to approximate the externality scenario, the aggregate economy is fragmented into *N* dynasties. Each dynasty has *1/N*th of the initial endowments and chooses consumption, investment and energy use in order to maximize the discounted total utility of per capita consumption. The dynasties understand the contribution of their own emissions to the climate change, but take carbon emissions of others as given. The climate dynamics are affected by the decisions of all dynasties. This constitutes the market failure.

It might seem easier to simply assume that there is one dynasty that ignores the externality but this would not be a rational expectation equilibrium. The externality problem is not an optimization but an equilibrium problem. The CONOPT3 solver of GAMS is powerful in solving maximization problems and it is more efficient to adopt an iterative routine in which a planner of a fragmented economy solves an optimization problem representatively than to attempt solving the equilibrium conditions directly. Given our specifications, the computation of the equilibrium problem takes less than one minute. To introduce this approximate externality, we make the following adjustments to the initial stocks $K(0) = K_0 / N, S(0) = S_0 / N$ and $L(0) = L_0 / N$. All production and cost functions are homogeneous of degree 1 and therefore invariant to *N*. The introduction of the pollution externality only requires a modification of the transition equation of atmospheric carbon to include emissions regarded as exogenous by each dynasty:

(4) $$E_{1t} = E_{1t-1} + \varphi_L (F_t + F_t^{exg}),$$

(5) $$E_{2t} = \varphi E_{2t-1} + \varphi_0 (1 - \varphi_L)(F_t + F_t^{exg}).$$

Under "laissez faire" dynasties essentially play a dynamic non-cooperative game, which leads to a Nash equilibrium in which each agent forecasts the paths of emissions correctly and all agents take the same decisions as all dynasties are identical. Equilibrium requires $F_t^{exg} = (N-1)F_t$. Under "laissez faire" the planner only adjusts her controls to take into account the effects of her own decisions (i.e. 1/*N*th of the climate externality). If $N = 1$ the externality is internalized and we obtain the social optimum. As $N \to \infty$, we obtain the "laissez faire" outcome characterized in section 2.

Following Rezai (2011), the numerical routine starts by assuming a time path of emissions exogenous to the dynasty's optimization, F_t^{exg}, at an informed guess. GAMS solves for the representative dynasty's welfare-maximizing investment, consumption, and energy use choices conditional on this level of exogenous emissions. $(N-1)$ times the dynasty's emission trajectory, F_t, defines the time

profile of exogenous emissions in the next iteration, $F_t^{exg\,\prime}$. The routine is repeated and F_t^{exg} updated until the difference in the time profile between iterations meets a pre-defined stopping criterion. In the reported results iterations stop if the deviation $\left|(N-1)F_t / F_t^{exg} - 1\right|$ in each time period is at most 0.001%.

We set $N = 400$ to account for the fact that in the present world economy, the externality in the market of GHG emissions is already internalized to a very small extent through the imposition of carbon taxes or tradable emission permits and non-market *regulation* (e.g. through the Kyoto Protocol or the establishment of the European Union Emission Trading Scheme). In our "laissez faire" simulations, the dynastic planner takes into account less than 0.25% of global emissions.