Grajek, Michał

Working Paper
Estimating Network Effects and Compatibility in Mobile Telecommunications

ESMT Working Paper, No. 07-001

Provided in Cooperation with:
ESMT European School of Management and Technology, Berlin


This Version is available at:
http://hdl.handle.net/10419/96552

Terms of use:
Documents in EconStor may be saved and copied for your personal and scholarly purposes.
You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.
Estimating Network Effects And Compatibility In Mobile Telecommunications

Michał Grajek, ESMT

ISSN 1866-3494
Abstract

Estimating Network Effects and Compatibility in Mobile Telecommunications

Author(s)*: Michał Grajek, ESMT

I develop a structural demand model for mobile telephone service, which facilitates the identification of network effects and compatibility between networks. Network effects are measured by the dependence of consumer willingness to pay on the installed base of subscribers. Compatibility is measured by the relative extent of cross- and own-network effects. I then estimate the model using quarterly panel data from the Polish mobile telephone industry from 1996-2001 and find strong network effects and—despite full interconnection of the mobile telephone networks—low compatibility. I also show that ignoring network effects leads to an overestimation of elasticity of demand.

Keywords: Structural Econometric Model, Network Effects, Compatibility, Mobile Telecommunications

JEL Classification: C51, D12, L96

* Contact: Michał Grajek (ESMT), Tel: +49 (0)30 21231-1047, Fax: +49 (0)30 21231-1281, Email: grajek@esmt.org.
Financial support from the German Federal Ministry of Education and Research is gratefully acknowledged (project 01AK702A). I would also like to thank Vivek Ghosal, Paul Heidhues, Ulrich Kamecke, Kai Konrad, Tobias Kretschmer, Eugenio Miravete, Lars-Hendrik Röller, and Otto Toivanen for their helpful comments. All remaining errors are my own.
1 Introduction

1.1 Motivation and Contribution

The interest in network economics, where network effects and compatibility between competing network goods play a prominent role, has risen enormously over the last decades. It gave rise to a large body of theoretical literature, which showed that the existence of strong network effects altered market outcomes in many important ways, often leading to market failure. Examples of such market failure include market breakdown due to start-up and hold-up problems (Rohlfs, 1974; Farrell and Gallini, 1988), as well as inability of industries to switch to a superior standard, known as “excess inertia” (Farrell and Saloner, 1985).

From an antitrust perspective, these results make the evaluation of conduct in industries exhibiting network effects more challenging (Economides and White, 1994; Economides, 2004; Gandal, 2002; Röller and Wey, 2003). For instance, in contrast to traditional industries, marginal cost pricing may result in market failure due to the start-up problem in network industries. Hence, introductory pricing below marginal costs may be necessary as a means to achieve a large user network. Yet, it is difficult in practice to distinguish introductory pricing from anticompetitive predatory pricing. At the same time, market dominance does not necessarily imply that the market leader earns super-normal profits. The seemingly high short-run rents may reflect the costs, which were incurred earlier in order to attract the critical mass of consumers.

There are various mechanisms that induce network effects. The network connecting consumers of a given good might be physical, e.g. supported by a telecommunication technology, or virtual, as in hardware/software paradigm (Katz and Shapiro, 1994). The network effects generated by these types of networks are referred to as direct and indirect, respectively. Common to both of them is that network size—typically defined as the total
number of users of the good—ultimately matters for the network good’s valuation by consumers.

One important factor influencing the impact of network effects on market outcomes in the presence of competing network goods is their compatibility. Following the seminal paper by Katz and Shapiro (1985), I define compatibility as a measure, which says to what extent the value of a given network good is influenced by the network size of competing goods. For instance, if the goods are perfectly compatible, consumers of all goods make up a common network and contribute to the network effects at the industry level. In the opposite case of perfect incompatibility, consumers of each good form separate networks and the network effects operate at the firm level.

The importance of compatibility for the network markets’ performance is twofold. First, it directly influences the consumers’ gross consumption benefits, since it expands the size of each network to equal the total membership of all networks. A drawback of this welfare improving effect of compatibility is, however, a loss of variety (Farrell, Saloner, 1986b). Second, compatibility alters the nature of competition between the providers of network goods, thereby indirectly influencing the consumers’ benefits. The providers compete “for the market” under incompatibility, whereas they compete in a more traditional manner “within the market” under compatibility (Besen and Farrell, 1994). Consequently, compatibility tends to relax competition early in the product-life cycle, as it reduces the threat of being driven out of the market. On the other hand, compatibility tends to intensify competition at a later stage, because it makes monopolization of the market less likely (Katz and Shapiro, 1986). Additionally, as pointed out by Farrell and Saloner (1986a), the prospect of winning the market under incompatibility rationalizes strategic product preannouncements and predation, which can have welfare detrimental effects. The early strand of the literature focused on a direct effects model—individual utility increases with the total number of users—viewing it as a reduced form of an indirect effects model. The effects of
(in)compatibility on competition might be, however, substantially different under direct and indirect network effects (Clements, 2004).

It is worth noting that technological compatibility is not sufficient for economic compatibility, as defined above. Although the mobile telephone networks that I study are fully interconnected, it is not a priori clear whether they are compatible or not. One reason for incompatibility involves on-net discounts, which enhance the relative attractiveness of own network to its subscribers. This type of incompatibility has been extensively studied in the ATM markets for instance (Knittel and Stango, 2004, 2006). In the next section, I consider several possible sources of network effects in mobile telecommunications. Some of them imply more compatibility than the others. An estimate of compatibility, therefore, additionally allows for differentiation between various possible sources of network effects.

Although the concept of network effects gave rise to numerous theoretical developments and growing empirical literature, there are still few empirical studies of compatibility. This paper contributes to that literature by providing a structural model of consumer demand for mobile telephone service, which allows for identification of the extent of network effects as well as compatibility. I also provide an empirical test of the model using data from the Polish mobile telephone industry. To the best of my knowledge, this is the first study that empirically tests for the extent of compatibility between competing networks.

The main idea facilitating the identification of network effects in this paper is that network effects give rise to an s-shaped diffusion of subscriptions (Cabral, 1990). These effects are captured by the dependence of consumer willingness to pay on the installed base of subscribers. Together with price changes, the installed base determines the speed of diffusion of mobile telephone service into the population. By estimating structural equations describing the evolution of subscriptions, I am able to disentangle price effect from network effects. Furthermore, in order to identify compatibility on top of the network effects, I make use of the installed base variation at operator level. Intuitively speaking, growing network of competing
operators in the market increases (does not influence) individual utility from joining own network under compatibility (incompatibility). By tracking the operator-level speed of mobile phones’ diffusion, I am then able to separate out own- and cross-network effects and thereby identify compatibility.

I then estimate this model using quarterly panel data from the Polish mobile telephone industry for the period 1996-2001. I find strong network effects, which give rise to an upward-sloping part in the inverse demand function. Simulations of the model using counterfactual values for the structural parameters suggest that the existence of network effects boosts the market size roughly ten times. The estimated degree of compatibility is surprisingly low, which means that subscribers to a given mobile network attach relatively little value to the competing networks’ subscribers. In particular, the mobile networks are fully incompatible in the presence of on-net call discounts.

I also estimate a restricted model, which does not account for network effects. The estimated price elasticity of demand in this model is much higher (in absolute terms) than in the unrestricted model, since it falsely attributes demand responses, which are partly due to network growth, solely to price falls. One conclusion from this exercise, which I also formalize, is that ignoring network effects in the empirical models of fast-growing network industries can lead to substantial overestimation of the elasticity of demand.

1.2 Literature

Despite the common belief that network effects play an important role in mobile telecommunications, the empirical literature concentrates on the determinants of growth and competitiveness of the industry often neglecting network effects (e.g. Parker and Röller, 1997; Ahn and Lee, 1999; Gruber and Verboven, 2001; Koski and Kretschmer, 2005; Lee et al., 2006). In the context of fixed-line telecommunications, the study by Bousquet and Ivaldi (1997) is probably the first, which empirically tests for the existence of network effects. In
contrast to this study, which focuses on access to telephone service, they concentrate on usage, which seems more relevant in the saturated market that they consider. Consequently, the concept of network externality that they use relies on received calls—subscribers benefit from them without having to pay—rather than on installed base of subscribers. Next, Okada and Hatta (1999) specify the demand for fixed-line and mobile telephone services by adopting the Almost Ideal Demand System. They provide empirical evidence of network effects by showing that the number of mobile subscribers, as a quality measure for telephone service, has a significant positive effect on the share of telecommunications’ expenditures—both mobile and fixed-line—in households’ budgets. More recently, Kim and Kwon (2003) conduct a conditional logit analysis based on a consumer survey. Their analysis reveals that consumers in the Korean mobile telephone market prefer operators with a large number of subscribers, all other things being equal. The authors attribute this size effect, which is in line with the network effects operating at the firm level that I found, to on-net call discounts and quality signaling effects. Fu (2004) reports similar network effects in the Taiwanese mobile phone market. Additionally, it is found there that the extent of network effects is closely linked to the extent of on-net call discount. This work corroborates the previous findings in a structural econometric model estimated on firm-level data.

A separate stream of research investigates factors responsible for the timing and the speed of mobile service diffusion (Gruber and Verboven, 2001; Koski and Kretschmer, 2005). These studies take a reduced-form approach by deploying logistic diffusion model. Liikanen et al. (2004) explicitly account for network effects by including penetration rates of different generations of mobile phones, as well as fixed line phones, in their reduced-form diffusion model. They find evidence of both within-generation and between-generation network effects. This work differs from the previous studies in that the diffusion’s s-shape is explicitly driven by network effects in a structural econometric framework.
Finally, this paper empirically addresses the issue of compatibility, which has been largely unstudied, in particular in telecommunications markets. One reason is the lack of comprehensive and reliable operator-level data. There exist, however, some evidence of the extent and the effects of compatibility from other markets. Berndt et al. (2003) examine the role of network effects in the demand for antineural drugs. They find evidence of the network effects operating at the brand and not the therapeutic class level. In terms of this work, their result means incompatibility between brands in the class of antineural drugs. Other studies used variation in the degree of compatibility to find evidence of network effects in the mainframe computer systems (Greenstein, 1993), the spreadsheets and the data management systems (Gandal, 1994, 1995; Brynjolfsson and Kemerer, 1996), and the ATMs (Knittel and Stango, 2004). None of these studies, however, empirically identifies the degree of compatibility, which I do in this work.

The rest of the paper is organized as follows. Section 2 discusses possible origins of network effects in mobile telecommunications and describes the Polish mobile telephone industry. In section 3, I derive a theoretical model facilitating the empirical analysis. Data, estimation issues, and interpretation of the empirical results are discussed in section 4. Section 5 concludes.

2 Mobile Telecommunications

2.1 Network Effects and Compatibility

In general, the term network effects refers to the dependence of consumer willingness to pay for a given good on the number of users of that good, which is called network size. Given that standard definition, there are several possible mechanisms that could generate network effects in mobile telecommunications. Common to all of the below described mechanisms is that the number of subscribers—the network size—ultimately matters for the
value of mobile telephone service to the consumers. For simplicity, I will refer to all the resulting effects of these mechanisms as network effects bearing in mind that this may overstretch the term.

**Connectivity:** According to the classical direct network effect, consumers value the installed base of subscribers, because they can satisfy more communication needs (Rohlfs, 1974). Since the huge installed base of fixed-line subscribers was already in place when mobile service emerged, it is not clear whether significant network effects could arise because an additional telephone service was offered by the mobile operators. Short message service (SMS), which is available only within mobile network, might help, however, to generate the direct network effect.

**On-net call discounts:** offer another explanation for network effects in mobile telecommunications. The intuition behind these tariff-mediated effects—as they are called in Laffont, Rey, and Tirole (1998)—is as follows: since on-net calls are cheaper than off-net calls, the number of cheap calls that a subscriber to a given operator makes increases with the operator’s installed base of subscribers. All other things being equal, consumers would then prefer large networks over small ones.

**Bandwagon effect:** Another reason why consumers might value network size is their need to buy, consume, and behave like the others, which induces a bandwagon effect as in Leibenstein (1950). The economic consequences of this “desire to join the crowd”, which stems from social interactions, were also studied in some recent economic literature (e.g. Granovetter and Soong, 1986; Becker, 1991; Lindbeck et al., 1999; Schoder, 2000). Consumption of mobile telephone service can be influenced, to some extend, by such conformist behavior, since mobile telecommunications are clearly an important media for social interactions.

**Learning spillovers:** The quality of mobile telephone service can also be a priori unknown to consumers. They could learn about the quality from other consumers, who have
already subscribed to the service. The installed base of customers would then transmit information to the unattached consumers influencing their willingness to pay. This type of learning spillovers could also lead to an s-shaped diffusion of subscriptions (Chamley, 2004, p. 193-210).

 Compatibility: Network effects, as described above, can operate at the industry and the network operator level, which corresponds to full compatibility and full incompatibility between competing networks, respectively. Partial compatibility is also feasible, when the operator-level dominate the industry-level effects, but the latter are still significant. A natural measure of compatibility, which is adopted in this study, is then the relative extent of cross-and own-network effects.

 Beyond the welfare and competitiveness implications, an estimate of compatibility additionally allows for differentiation between various possible sources of network effects. The classical effects due to increased connectivity can be expected to operate at the industry level, as each new subscriber adds to the existing communications possibilities regardless of the operator it subscribes to. In this case, technological compatibility, which facilitates interconnection of the mobile telephone networks, overlaps with economic compatibility. In contrast, on-net call discounts clearly give rise to operator-level network effects, hence incompatibility. To the extent that subscribers’ information about the quality of service concerns mostly the operator that they are subscribed to, the effects raised by information spillovers are also operator specific. Finally, the bandwagon concept could support both types of network effects, depending on the consumers’ reference group.

 2.2 Description of the Market

 Empirical analysis in this paper examines the second-generation (2G) mobile telephone industry that was launched in September 1996 in Poland. The Ministry of
Telecommunications (henceforth, MT)—the regulator of that industry—initially granted two licenses for the provision of mobile telephone service based on the GSM 900 standard. To further intensify competition in the industry, the MT offered a GSM 1800 license to a third provider, which started operations in March 1998. The difference between the two standards, GSM 900 and GSM 1800, is that the latter operates at a higher frequency allowing providers to guaranty more connections per unit of area at the same time. GSM 1800 is, however, more costly to install than GSM 900, as it requires higher density of cellular antenna sites. Thus, the service was offered exclusively in large urban areas in Poland. The entrant became a nationwide provider in March 2000 only after being additionally granted a GSM 900 license. Soon after that, the two incumbents obtained GSM 1800 licenses and initialized the double-frequency service provision as well. Since the focus of the MT was to obtain the desirable market outcomes by promoting competition, there was no price regulation in the industry.

GSM telecommunications turned out to be very successful in Poland unlike its predecessor, which was based on the analog technology standard NMT 450i. The analog service, also known as the first-generation (1G) mobile telecommunications, was introduce in 1992 and reached the size of around 200 thousand subscribers at the time when the 2G service was introduced in 1996. In contrast, four years after its launch, the total number of GSM subscribers amounted to over 3 million and two years later it approached 10 million, which corresponds to 25% of the total population. Obviously, there are differences in the quality between the analogue NMT and the digital GSM standards, but the extreme market outcomes in terms of network size can indicate that, unlike the 2G, the 1G technology was not able to generate a critical mass in the presence of strong network effects.

A particular feature of the Polish mobile market is that the monopolistic NMT operator—a sister company of the fixed-line monopolist—entered the 2G mobile market as the last operator. The preferential terms for the NMT subscribers to join the entrant GSM network partially offset the competitive disadvantage relative to the two established GSM
operators due to a smaller installed base of subscribers and network coverage. Still, in the empirical analysis I account for the inferior quality of the entrant’s service over the first two years of its activity, i.e. until it offered a countrywide coverage.

3 Theoretical Model

3.1 Basic Model of Demand for Mobile Telephone Service

I assume that there are several mobile telephone networks and a normalized to one measure of consumers in a market. Each of the networks is run by a single operator denoted by \( i = 1, 2, \ldots, I \). The number of operators in the market is allowed to change in each time period, which is denoted by the subscript \( t \). The consumer utility derived from subscription to the mobile telephone service in each period is assumed to depend on the individual intrinsic valuation (consumer’s type) and on the network size, which in turn depends on the number of subscribers. Formally, consumer preferences are represented by the willingness-to-pay function \( u(v, x_{i,t-1}) \), where \( v \) is the individual valuation parameter distributed according to some cumulative density function \( F(v) \) and \( x_{i,t-1} \) is the network size of operator \( i \) at the end of the previous period. Furthermore, I assume that \( u(v, x_{i,t-1}) \) is additively separable, strictly increasing, and continuous in \( v \). By construction, the parameter \( v \) establishes a rank ordering of the consumers according to their willingness to pay. I assume that the ranking is invariant with respect to changes in \( x_{i,t-1} \). As a matter of convention, the benefit of subscribing to a network increases with \( v \).

The dependence of willingness to pay on the network size \( x_{i,t-1} \) formalizes the network effects. The introduction of a lag in the perception of the network is crucial to the model. As pointed out by Cabral (1990), it is an equilibrium selection device that gives the unique diffusion path of subscriptions when network effects are strong. The intuition behind this is that the perception lag does not allow consumers to coordinate their subscription decisions,
thus eliminating multiple equilibria due to the coordination problem well known from the
network literature. The additive-separability assumption in turn means that intrinsic—or
stand-alone—value $v$ of the mobile telephone service is independent of the network size. It
can be interpreted, as the value of an option to call to or receive a call from a fixed-line
network. As the fixed-line telephony penetration is relatively stable, $v$ can be treated as
approximately constant and not related to the mobile telephony penetration.

So far, I have not defined what I exactly mean by the network size. Consider two
opposite cases first. If network effects are operator specific, then subscribers to each operator
make up their own networks. Consequently, I then assume that the network size of each
operator equals the sum of its subscribers. In contrast, industry-specific network effects imply
a common network, whose size is given by the total number of subscribers to all operators.
Common and operator-specific network cases correspond to perfect compatibility and perfect
incompatibility, respectively.

In a more general setting, partial compatibility may prevail. In this case, an operator’s
network size is a weighted sum of its own and all other subscribers. Assuming that
compatibility is symmetric, this can be written as

$$x_{i,t} = y_{i,t} + w \sum_{j \neq i} y_{j,t}, \quad (1)$$

where $w \in [0,1]$ measures the degree of compatibility and $y_{i,t-1}$ stands for the number of
subscriber to operator $i$.

At the beginning of each period, consumers decide whether to subscribe to one of the
operators in order to maximize the net utility $u(v, x_{i,t-1}) - p_{i,t}$, where $p_{i,t}$ denotes the price
charged by operator $i$. The outside option of staying out of the market brings zero utility.
Assuming that there are no set-up and termination costs, consumers subscribe whenever the
net utility from subscription is positive.\(^1\) \(v_{i,t}^* = v^*(x_{i,t-1},p_{i,t})\) denotes the type of consumer, who
is indifferent with respect to operator \(i\). \(v_{i,t}^*\) can be obtained from
\[
\begin{align*}
\text{u}(v_{i,t}^*, x_{i,t-1}) &= p_{i,t}. 
\end{align*}
\]

Because I assumed that the willingness-to-pay function is additively separable, the
operator \(i\) for whom \(v_{i,t}^*\) is lowest is the most attractive for all subscribers. The lowest
indifferent type across providers is defined as
\[
\begin{align*}
v_{L,t}^* &= \min_i \{v_{1,t}^*, v_{2,t}^*, \ldots, v_{I,t}^*\}. 
\end{align*}
\]

By construction, all consumers with a type higher than \(v_{L,t}^*\) subscribe. They subscribe to
operators, for whom \(v_{i,t}^* = v_{L,t}^*\). As a tie-breaking rule, I assume that subscribers choose with
equal probability among these operators. I define
\[
\begin{align*}
H_i(v_t^*) &= \begin{cases} 
1 - F(v_{L,t}^*) & \text{if } v_{i,t}^* = v_{L,t}^*, \\
I_{L,t} & \text{otherwise} 
\end{cases} 
\end{align*}
\]
where \(v_t^* = (v_{1,t}^*, v_{2,t}^*, \ldots, v_{I,t}^*)\) is a vector of the indifferent types, \(I_{L,t}\) is the number of
operators for whom \(v_{i,t}^* = v_{L,t}^*\) (active operators) and \(F\) is the distribution function of \(v\). \(H_i\)
equals the number of consumers willing to subscribe to operator \(i\) at time \(t\). Now, the diffusion
of subscriptions to operator \(i\) can be described by the following state equation
\[
\begin{align*}
y_{i,t} &= H_i(v_t^*). 
\end{align*}
\]
In a steady-state (given that all prices stay constant), none of the consumers can increase its
utility by changing the subscription decision, so that the installed base of each operator \(i\) stays
constant over time

\(^1\) These assumptions—in particular the assumption of no termination costs—are generally not true in mobile
telephone markets. I will come back to them and relax the no-termination-costs assumption in the next
subsection.
\[ y_{i,t} = y_{i,t-1}. \]  

(6)

It is worth noting that the steady-state equilibrium of the above model coincides with the equilibrium from a standard one-shot model with fulfilled consumers’ expectations (Rohlfs, 1974; Economides and Himmelberg, 1995).^2

### 3.2 Switching Costs

The above demand model with network effects is probably the most obvious extension of Cabral’s (1990) single network model. In this subsection, I extend this basic model to account for switching costs. This makes the model more realistic at a relatively low cost in terms of additional complexity. The model becomes more realistic not only because switching costs are very relevant to mobile telephone markets, but also because they facilitate persistent differences in equilibrium market shares across active operators. Note that the basic model predicts symmetric market shares as the only feasible outcome.^^3 The switching costs I have in mind are the penalties for premature cancellation of subscriptions and the subscribers’ costs due to the lack of number portability.

One difficulty that arises because of the switching costs is that consumers’ decisions based on current costs and benefits might no longer be optimal. In fact, once consumers have subscribed to an operator they are vulnerable to future price increases. They will also regret having to stay with their operator if another one offers a better deal. Such consideration will lead to a hold-up problem, i.e. unwillingness to subscribe in the first place. I ignore the hold-up problem for the rest of this subsection and will come back to it later when discussing operators’ pricing behavior. In fact, I will assume that operators are able to resolve the hold-up problem by appropriate price commitment.

---

^2 To see that: substitute the expected installed base for the lagged one and drop the time subscripts.

^^3 Here I mean symmetry across active operators, i.e. operators who have an installed base of subscribers.
I assume that the switching costs are high enough to completely prevent switching operators. Ignoring the hold-up problem, this alters the subscription demand described in the previous subsection to the extent that only the unattached consumers can feed the diffusion of subscriptions. This can be written as

\[
H_i'(v_t^*, v_{t-1}^*) \equiv \begin{cases} 
\frac{F(v_{L,t-1}^*) - F(v_{L,t}^*)}{I_{L,t}} & \text{if } v_{L,t}^* = v_{L,t}^* \\
0 & \text{otherwise}
\end{cases}.
\] (7)

Now, \(H_i'\) equals the number of the new consumers willing to subscribe to operator \(i\) in period \(t\). The respective state equation describing the diffusion of \(i\)’s subscriptions becomes

\[
y_{i,t} = H_i'(v_t^*, v_{t-1}^*) + y_{i,t-1}. \tag{8}
\]

In contrast to equation (5), equation (8) exhibits substantial inertia. Any asymmetry inherited from the past will not disappear. In particular, the incumbent’s installed base of subscribers will constitute a persistent competitive advantage over the entrant. Substituting definitions (4) and (7) into (8) and rearranging the terms yields\(^4\)

\[
y_{i,t} = H_i(v_t^*) + l_iE_t
\]

where \(l_i\)’s are operator-specific constants and \(E_t\) is an entry indicator function that equals zero in the pre-entry period and one afterwards. One appealing feature of (9) is that it nests two regimes: with and without switching costs. Formal derivation of (9)—shown in the appendix—requires, however, additional constraints on the operators’ pricing behavior. Intuitively, if entry is to be the only event that breaks symmetry, as in (9), then I need to assume that each operator strategically adjusts its price in response to changes in own and competitors’ installed bases in order to win new subscribers in each period. I cover the pricing issues in the next subsection.

\(^4\) See appendix for details. I consider single entry only, which is in line with the data in the empirical section. It is straightforward to extend the model to account for multiple entries.
3.3 Pricing

As mentioned before, switching costs and termination costs might render the consumers’ subscription decisions based on current benefits and prices suboptimal and lead to the hold-up problem. I assume that in order to solve this problem operators credibly commit i) not to increase prices for the installed base of subscribers and ii) to stay active (win new subscribers) in each period.

Note that the first assumption does not ban operators from increasing prices for the new subscribers. It still solves the hold-up problem, as the actual subscribers are no longer vulnerable to price increases. This kind of commitment can be—and actually is—implemented in mobile telephone service contracts. One way to achieve this is to entitle the subscribers to stick to the price (tariff) that they initially subscribed to, regardless of new prices (tariffs) that arise. In this way, a new, higher tariff will not affect the price paid by the existing subscribers. Alternatively, the contracts can offer an option to terminate the subscription without penalty in case of a price increase.

The second assumption means that prices corrected for network size—I call them quasi-hedonic to highlight the analogy to prices corrected for quality—are equal across the operators in each period. This assures that subscribers are indifferent across operators in each period and do not need to consider potential switching costs in the future.

Equal quasi-hedonic prices (hence equal sales) seem a natural outcome of price competition in a symmetric case when all operators simultaneously enter the market (or when networks are fully compatible, so the installed base gives no competitive advantage). If the first entrant had an installed base edge instead, it would be able to undercut (in network-adjusted terms) its competitors and to capture all new customers. This would imply, however,
cutting price on its installed base, which might discourage the first entrant from aggressive competition. In fact, this is the tension that relaxes price competition and facilitates sales by all firms in each period in many switching costs models (see Farrell and Klemperer, 2006, section 2.4, for a recent overview and the citations in there).

4 Empirical Evidence

4.1 Data

The operator-level quarterly panel data used in this study covers prices and number of subscribers in the Polish mobile telephone industry from 1996-2001. The information on prices has been obtained from the Ministry of Telecommunication (MT). It includes all price plans for each provider. In the period studied, all providers used non-linear pricing in the form of multiple price plans from which the customers selected. A plan consists of a monthly fixed charge and a price per minute of usage in addition to some minutes free of charge. A usage price is further diversified according to the time of the day (peak/off-peak hours) and to the termination network (on-net discounts). Following Parker and Röller (1997), I define a single price for each operator as the “best-deal monthly bill” paid for a given constant calling pattern. That is, I assume some calling pattern (including the overall monthly usage and the proportions of the calls at various times of the day and to various networks) and calculate the monthly bills for all available price plans for each provider. The single price for each provider is then the lowest bill among them that reflects the best-deal selection for an average customer. Figure 1 shows the nominal prices for each provider over the studied period.

Nominal prices, as opposed to the real ones, which I use for estimations, exhibit significant

---

5 Note that pricing in the European telecommunications markets is based on the Calling Party Pays principle, as opposed to the Receiving Party Pays principle adopted in the U.S. for instance.

6 Note that the intertemporal price discrimination discussed earlier does not allow the incumbent to charge the installed base more than the new customers; as the subscribers typically can switch to a new tariff whenever they want.
rigidity. Over the first three years, the incumbents kept basically constant prices. This situation first changed at the end of 1999 when the entrant was additionally granted a GSM 900 license, which enabled it to offer countrywide service and adjust the price upwards.

The subscriber figures I use come from the Informa Telecoms & Media’s World Cellular GSM Datapack. The prepaid card users are excluded from the sample, since the mobile telephone prices obtained from the MT do not account for prepaid cards. Figure 2 shows the diffusion of subscriptions in the market.

The theoretical model developed in this paper predicts that the number of new subscribers attracted in each period is equal across operators. This implies that the distance between the operators’ installed bases should be constant over time, which corresponds reasonably well with figure 2. One important exception is the year 2000, in which the first incumbent grew its installed base at a faster pace than its competitors after a major price cut in the first quarter of the year. The second incumbent followed this strategy in 2001, with less success, however. One needs to add that imperfect information about the prices—which actually are much more complex than the proxies depicted in figure 1—is likely to trigger suboptimal choices by some consumers and blur the subscription patterns predicted by the theoretical model.

The theoretical model also implicitly assumes that the subscribers do not switch operators. The respective monthly churn rates reported in the Merrill Lynch’s Global Wireless Matrix on average amount to 1% in 1999 and 2000 and 2.5% in 2001, which suggest that switching is indeed not a massive phenomenon. These churn figures correspond to the entrant, however, and are not available for the incumbents.

---

7 I apply a calling pattern from the OECD Telecommunications Basket Definitions (2000).
8 Because inflation in the late 90’s was still relatively high in Poland (on average 10% per annum over 1996-2001), the real prices fall much faster than the nominal ones.
9 This data has been used in previous studies, e.g. Koski and Kretschmer (2005).
4.2 Functional Specification and Identification

To facilitate the empirical analysis, I need to specify functional forms in the theoretical model. Let consumer \( v \)'s willingness to pay for a subscription to operator \( i \) in each period be given by

\[
u(v, x_{i,t-1}) = v + cx_{i,t-1} + dx^2_{i,t-1},
\]

where \( c \) and \( d \) are parameters determining the extent of network effects. The squared term captures possible nonlinearities, e.g. diminishing marginal network effects usually assumed in the theoretical literature. Consumer types \( v \) are uniformly distributed over \((-\infty, a]\) with some density \( b > 0 \). For convenience, population's size is not normalized to one, as in the theoretical model. In fact, given the specified distribution of types, the population is infinite.

Alternatively, the distribution support could be bounded from below to limit the population of consumers and the bound assumed to be low enough in order to avoid the necessity of considering corner solutions, when all consumers subscribe. One needs to stress that the uniform distribution assumption that greatly simplifies the problem may not be innocuous. If the true distribution of consumer valuations was bell-shaped, one would see the diffusion of subscriptions accelerate as the indifference level approaches the average valuation even in absence of any network effects. Ignoring this feature might then introduce a bias in the estimates, as it would appear to be network effects that were causing the acceleration in subscriptions.

Given the above functional forms, diffusion equation (9) becomes

\[
y_{i,t} = I_t E_t + \frac{1}{I_t}(ab - bp_{i,t} + bcx_{i,t-1} + bdx^2_{i,t-1}).
\]

Finally, to obtain the equation I estimate, I substitute (1) into (11) and add the error term
\begin{equation}
    y_{i,t} = \lambda_i E_t + \frac{1}{I_i} \left( x + \beta p_{i,t} + \gamma_1 y_{i,t-1} + \gamma_2 y_{i,t-1}^2 + \gamma_3 y_{i,t-1}^3 + \gamma_4 y_{i,t-1} y_{-i,t-1} + \gamma_5 y_{i,t-1} y_{-i,t-1} + \gamma_6 y_{i,t-1}^2 + \gamma_7 y_{-i,t-1} \right) + \psi_{i,t},
\end{equation}

where \( y_{-i,t-1} \) denotes the sum of subscribers to all operators other than \( i \) in period \( t-1 \). The structural parameters of the model can be identified from coefficients in (12). Simple algebra yields the highest consumer type for the population \( a = -\alpha/\beta \) and density of the distribution of types \( b = -\beta \). The parameter \( \alpha = ab \) can be interpreted as the number of consumers with positive valuation of the mobile telephone service given a zero network size. The network effects parameters \( c \) and \( d \) are \( -\gamma_1/\beta \) and \( -\gamma_2/\beta \), respectively. The compatibility parameter \( w \) is overidentified, since it is identified by \( \gamma_{11}/\gamma_1, \gamma_{21}/2\gamma_2 \), and \( \pm \sqrt{\gamma_{22}/\gamma_2} \). When the network effects are only present at the operator level (incompatible networks, \( w = 0 \)), the coefficients \( \gamma_{11}, \gamma_{21} \) and \( \gamma_{22} \) are equal zero. In the opposite case, when the effects operate at the industry level (fully compatible networks, \( w = 1 \)), the following equalities hold: \( \gamma_{11} = \gamma_1, \gamma_{22} = \gamma_2 \) and \( \gamma_{21} = 2\gamma_2 \). All the intermediate cases with partial compatibility can be easily obtained from the three identifying equalities as well.

For simplicity, \( w \) was treated as a constant in the theoretical part of this paper. It is straightforward to relax this constraint, which might be important, since operators eliminated the tariff-mediated motive of network effects by changing their pricing strategy. Initially, all three operators favored their own subscribers by including on-net call discounts in the tariffs. Two of them eliminated the discounts during the period studied. As a result—and in accordance with the discussion on network and compatibility form section 2—I expect their compatibility to increase. To capture this effect I introduce into (12) the interactions of dependent variables: \( \frac{y_{-i,t-1}}{I_i}, \frac{y_{i,t-1} y_{-i,t-1}}{I_i} \), and \( \frac{y_{i,t-1}^2}{I_i} \) with firm-specific pricing dummies \( D_{i,t} \). These pricing dummies equal one in the periods without on-net discounts in \( i \)'s tariffs, otherwise zero. The change in the operators’ compatibility due to their abandonment of on-net
discounts, denoted by $\Delta w$, can be identified from the estimated coefficients on new interaction variables. The identification is analogous to that of $w$.

Another modification of (12) that I make accounts for the initial period in which service offered by the entrant was limited to only the large urban areas and based exclusively on the GSM 1800 technology. Since one of my main identifying assumptions is that the quasi-hedonic prices are equal across providers in each period, ignoring the inferior quality of the entrant’s service might substantially bias the results. In particular, it might falsely lead toward finding incompatibility, as the smaller installed base of the entrant relative to the incumbents could take over the missing quality effect. To account for this potential problem I include an entrant-specific dummy variable, $C_{i,t}$, which equals one in the transitory period of GSM 1800 (as opposed to the dual mode GSM 900/1800 introduced only later), otherwise zero.

### 4.3 Results and Interpretations

Table 1 shows the estimation results for equation (12) with the above mentioned amendments. The identified structural parameters from theoretical model can be found in table 2. I estimate six regressions, which differ in terms of estimation technique, selection of instruments, and parameter restrictions. The column labels in both tables identify the regressions. Given the limited number of observations, I restrict two out of three coefficients in both the “Cross-network effects” and “Pricing regime” groups exploiting overidentification of the compatibility parameters $w$ and $\Delta w$.

The results in the first four columns of table 1 are obtained by a GMM estimation. Columns (a) - (c) report the estimated coefficients of the model with three alternative sets of

---

10 Other networks operated on the GSM 900 standard, which facilitated national-wide service provision.

11 In particular, the coefficients on $y_{i,t-1}y_{j,t-1}$ and $y_{j,t-1}^2$ were set as $\gamma_{21} = 2\gamma_{12}/\gamma_1$ and $\gamma_{22} = (\gamma_{11}/\gamma_1)^2\gamma_2$ respectively. The coefficients on $D_i y_{i,t-1}$ and $D_j y_{j,t-1}^2$ were set analogously.
instruments. In regression (a) all explanatory variables serve as their own instruments. In regression (b) I use the twice-lagged instead of lagged dependent variable in order to account for a possible autocorrelation in the error term. Additionally, to address possible endogeneity of prices, I instrument them with their lagged values in regression (c). Regression (d) is a benchmark, where it is assumed that there are no network effects in the demand for mobile telephone service. Without network effects, the theoretical structure simplifies to a basic linear demand model. For consistency with the remaining regressions, the benchmark regression (a) is estimated on data aggregated to the industry level, whereby the model is exactly identified.

The Sargan test of overidentifying restrictions in table 1 does not reject the orthogonality of the instruments. Nevertheless, I conduct further robustness checks to account for autocorrelation of the error term indicated by the Godfrey test. The last two columns of table 1 report the results. First, I estimate the model by nonlinear least squares to see whether the results substantially differ from the GMM results. Second, I apply the two-stage procedure devised by Hatanaka (1974) for the models with lagged dependent variables and autoregressive errors.

In general, the estimation results are fairly robust across specifications. All coefficients have expected signs and most of them are statistically significant. Another specification test of the model consists of checking whether the operator-specific effects correspond to their expected values. Given the structure of the theoretical model, these values can be calculated from subscriber data in the last period prior to market entry, as shown in the appendix. The basic intuition behind these calculations is that under high switching costs the installed base of subscribers gives the incumbents a persistent competitive advantage over the entrant. I then expect $\lambda_1$, $\lambda_2$, and $\lambda_3$ to equal $1.342$, $1.164$, and $-2.506$, respectively, which means that the advantage of the first and second incumbents amounts to 385 thousand and 367 thousand subscribers, respectively. The Wald and Langrange multiplier tests do not reject
these parameter restrictions in either of the regressions except (a). This suggests that the
switching costs actually are high enough to prevent massive switching between the operators
and, at the same time, confirms that the model’s specification is correct.

Now I turn to the interpretation of the empirical results. First, they indicate strong
network effects and low compatibility between mobile telephone operators. Parameter \( c \)’s
positive values and the negative values of \( d \), as reported in table 2, correspond to the positive
and diminishing marginal network effects. The estimate of compatibility between operators
(\( w \)) is low. When on-net call discounts are present, compatibility is usually not significantly
different from zero. Without on-net discounts (\( w + \Delta w \)), the measure increases to roughly 0.2,
which means that subscribers still value their own networks five times higher than the
networks of the other operators.

To assess the economic significance of the estimated magnitudes, I simulate the
steady-state demand implied by regression (c)’s estimates. Figure 3 illustrates the results. On
the horizontal axis, there is an aggregated number of subscribers in the industry \( y = \sum_i y_i \).

For simplicity of the exposition, I assume that the three operators are of equal size, hence the
prices across operators are also equal and can be represented by a common price \( p \). The thin
line represents the counterfactual case in which the network effects parameters are set to zero.
There are two important observations based on the simulations in figure 3: i) the network
effects generate the upward-sloping part in the inverse demand function, ii) they boost the
market size by roughly ten times. Both observations suggest that the network effects are
indeed strong in the mobile telephone industry under consideration. Moreover, the upward-
sloping part of the demand has a critical mass interpretation (Rohlfs, 1974; Grajek, 2002).
This in turn points to the start-up problem as a potential concern for the industry. This result
suggests that one reason for the flop of the 1G service in Poland was the inability to attract a
critical mass of subscribers.
Another important result of this study concerns the price elasticity of demand. When I account for network effects, the price coefficient is estimated at roughly -0.06 or -0.03 depending on whether the price variable is instrumented for or not. In contrast, this estimate in the benchmark regression (d) amounts to -0.42. The respective elasticities evaluated at the sample mean amount to 0.6, 1.2, and 8.2. This discrepancy indicates that ignoring network effects leads to a significant overestimation of the price elasticity. Figure 4 illustrates this result showing that the demand functions that account for the network effects are in general steeper than the benchmark. The intuition behind this result is as follows: by ignoring the installed base all changes in the current number of subscriptions are falsely attributed to changes in price only. Since both the rising installed base of subscribers and falling price lead to more subscriptions, if—loosely speaking—price and installed base are negatively correlated, then the price coefficient is biased downward and consequently the price elasticity is overestimated. I formalize this intuition in the appendix.

Finally, I can draw some interesting conclusions based on the identified compatibility between the mobile telephone networks. The estimated low degree of compatibility suggests that harmonizing the technical standards on which the mobile telephone networks operate is not enough to achieve compatibility in the economic sense. Although the 2G mobile telephone industry in Poland coordinated on the GSM 900/1800 standard, subscribers almost exclusively value the installed base of their own operator, leaving the installed base of the competitors unattended. As a result, a strong tendency toward monopolization of the industry prevails, which triggers fierce competition “for the market”. High switching costs, however, which prevent subscribers from massive switching between operators, to some extent relax the competition.

Given the extent of compatibility that I found, one can also discriminate between the possible origins of network effects in mobile telecommunications. As already discussed, some

---

12 Again, the downward-sloping parts of the steady-state demand are the relevant ones.
are more likely to generate operator specific effects (low compatibility), while others might
generate industry specific effects (high compatibility). I am also able to test the tariff-
mediated-network-effects hypothesis more directly, by estimating the change in compatibility
due to the abandonment of on-net call discounts. Our results provide empirical evidence in
favor of the tariff-mediated effects, as $\Delta w$ is both statistically and economically significant.
This hypothesis alone, however, does not explain the identified network effects. Compatibility
in the industry remains low, even without on-net discounts. This suggests that the network
effects in mobile telecommunications might also originate from the transmission of
information about the quality of the service (learning spillovers) and the conformist behavior
of consumers (bandwagon effect).

The empirical results of this paper can be expected to carry over to other mobile
markets operating under similar institutional setup, which is the case for at least all European
markets. A further research is needed, however, to confirm these results and address the
shortcomings of the model, which arise partially due to the limited data.

5 Conclusion and Further Research

In this paper I specify and estimate a structural model of demand for mobile telephone
service. By doing so, I provide a structural way of identifying network effects and
compatibility between competing networks. Operator-level data for the analysis comes from
the (2G) digital mobile telephone industry in Poland. The Polish market is a standard
European mobile telephone market and the empirical results can be expected to hold for other
geographical markets as well.

Our results indicate strong network effects, both in statistical and economic terms. The
regression coefficients and the identified structural parameters measuring the extent of the
effects are statistically highly significant. Simulations of the steady-state demand in the
industry show that the network effects boost the market size by roughly ten times. I also show
that the identified network effects lead to a critical-mass problem in the industry and argue that the failure of the 1G mobile telephone technology in Poland might be due to the inability to generate the critical mass of subscribers.

Next, I find compatibility between competing networks to be low. In particular, when the operators’ pricing strategy involves on-net call discounts, the networks are perfectly incompatible. Without on-net discounts the compatibility increases, but remains low. On the one hand, this result suggests that operators fiercely compete “for the market”, as opposed to the traditional competition mode “within the market”. On the other hand, it allows for differentiation between the possible origins of the network effect. The low compatibility I find is consistent with the learning spillovers and bandwagon hypotheses. I also find direct evidence of the tariff-mediated network effects, as the on-net call discounts matter for the network effects’ strength.

Last but not least, I show that ignoring network effects in empirical demand models can lead to substantial overestimation of the demand elasticity, particularly, when considering demand in new and fast-growing markets.

I recognize, however, that the model developed in this paper relies on restrictive assumptions, which are partially driven by the data limitations. These assumptions include the functional specification, as well as the exogeneity of entry decisions and other structural regime changes. In particular, uniform distribution of valuations and quadratic preferences could be generalized given a richer set of data. For instance, the distribution of valuations could bear on the distribution of income, or other buyer characteristics, in a region and alternative specifications of preferences could be tested. A Cobb-Douglas utility function offers one important alternative for the quadratic, additively separable preferences (see Economides and Himmelber, 1995). Finally, dummy variables that account for the entry, the technology standards offered, and the on-net discounts could be endogenized, as they capture
operators’ decisions—subject to regulation, though—rather than simply regime changes.

Further research is needed in order to address these limitations.

6 Appendix

6.1 Diffusion Equation under Switching Costs

To nest the two regimes—with and without switching costs—in a single equation (9), I substitute definitions (4) and (7) into (8) and rearrange the terms to get

\[ y_{it} = H_i(v_t^*) - H_i(v_{t-1}^*) \frac{I_{Lt}}{I_{Lt}} + y_{i(t-1)}. \]  

(13)

Then I expand this recursive equation as follows

\[ y_{it} = H_i(v_t^*) - H_i(v_{t-1}^*) \frac{I_{Lt}}{I_{Lt}} + H_i(v_{t-1}^*) - H_i(v_{t-2}^*) \frac{I_{Lt-1}}{I_{Lt-1}} + H_i(v_{t-2}^*) + \]

\[ + ... - H_i(v_0^*) \frac{I_{L0}}{I_{L1}} + y_{i0}, \]  

(14)

where \( t = 1 \) indicates the time period when the market starts up, i.e. there were no subscribers and no active operators in \( t = 0 \).

Now I need the assumption of equal quasi-hedonic prices across operators. This assumption assures that each operator is active, i.e. wins new subscribers, in each period. Suppose for the moment that there has been no entry; thus the number of active operators in the market is constant: \( I_{Lt} = I_L \). Then, all the middle terms on the right-hand side (henceforth, RHS) of (14) cancel out and the last two terms equal zero, because prior to the market start-up there were no subscribers. In this case (14) simplifies to (5), i.e. the diffusion equations both with and without switching costs are the same.
Now suppose that there was one-time entry into the market at $t = E$. This means that $I_{L,t}$ increases at $t = E$ and stays constant thereafter. An incumbent operator’s diffusion equation no longer simplifies to (5), it instead becomes

$$y_{it}^{inc} = H_i(v_{it}^*) + H_i(v_{E-1}^*) \left[ 1 - \frac{I_{L,E-1}}{I_{L,E}} \right],$$

(15)

for $t \geq E$. The second term on the RHS of (15) is clearly positive. It is also invariant with respect to any events in $t > E$ and can be treated therefore as an operator-specific constant in the post-entry periods.

The diffusion equation for the entrant is not the same as for the incumbent, as it was inactive before period $E$ ($y_{it}^{ent}$). Its subscriber base develops as follows

$$y_{it}^{ent} = H_i(v_{it}^*) - H_i(v_{E-1}^*) \frac{I_{L,E-1}}{I_{L,E}},$$

(16)

for $t \geq E$. In contrast to (15), the second term on the RHS of (16) is negative. I conclude therefore that the incumbent has a fixed competitive advantage over the entrants.

It is straightforward to show that the operator-specific effects caused by entry sum up to zero. If $A$ and $B$ denote the number of incumbents and entrants, respectively, then the sum of the effects will be

$$AH_i(v_{E-1}^*) \left[ 1 - \frac{I_{L,E-1}}{I_{L,E}} \right] - BH_i(v_{E-1}^*) \frac{I_{L,E-1}}{I_{L,E}} =$$

$$= AH_i(v_{E-1}^*) \left[ 1 - \frac{A}{A + B} \right] - BH_i(v_{E-1}^*) \frac{A}{A + B} =$$

$$= \left( A - \frac{A^2}{A + B} - \frac{AB}{A + B} \right) H_i(v_{E-1}^*) = 0.$$
6.2 Direction of Bias in the Estimated Price Elasticity of Demand

This subsection formalizes the intuition behind the direction of bias in the estimated price elasticity of demand when ignoring network effects. Suppose that a correctly specified linear demand model reads

\[ Y = X_1\beta_1 + X_2\beta_2 + \epsilon, \]  

where \( y \) is an \( N \times 1 \) vector of quantities, \( X_1 \) is an \( N \times K \) matrix, which contains \( N \) observations on each out of the \( K \) basic explanatory variables, and \( \beta_1 \) is a \( K \times 1 \) vector of the corresponding parameters. \( X_2 \) denotes an \( N \times L \) matrix of variables responsible for network effects and \( \beta_2 \) is an \( L \times 1 \) vector of parameters. \( \epsilon \) stands for an \( N \times 1 \) vector of disturbances and \( E[\epsilon] = 0 \). I allow for heteroscedasticity and autocorrelation of disturbances by letting the covariance matrix be \( E[\epsilon\epsilon'] = \Sigma \). Suppose further that some variables in \( X_1 \) are not orthogonal to \( \epsilon \) and a valid set of instruments for them is included in \( Z \), which is an \( N \times M \) matrix, and \( M \geq K \).

If one regresses \( y \) on \( X_1 \) without including \( X_2 \), our GMM estimate of \( \beta_1 \) is then

\[
\hat{\beta}_1^{GMM} = \left[ X_1'ZS^{-1}Z'X_1 \right]^{-1}X_1'ZS^{-1}Z'y = \left[ X_1'ZS^{-1}Z'X_1 \right]^{-1} \left[ X_1'ZS^{-1}Z'(X_1\beta_1 + X_2\beta_2 + \epsilon) \right] = \beta_1 + \left[ X_1'ZS^{-1}Z'X_1 \right]^{-1} \left[ X_1'ZS^{-1}Z'X_2 \right]\beta_2 + \left[ X_1'ZS^{-1}Z'X_1 \right]^{-1} \left[ X_1'ZS^{-1}Z'\epsilon \right],
\]

where \( S \) is a matrix from the sample data that converges in probability to the same matrix as \( \frac{1}{N}Z'\Sigma Z \). In particular, if disturbances \( \epsilon \) are uncorrelated, \( S \) could be the White (1980) estimator; otherwise it could be the Newey and West (1987) estimator.

28
To assess asymptotic properties of the estimator in (19), I first need to assume that
\[
\frac{1}{N} Z' \Sigma Z, \frac{1}{N} X_1' X_1 \text{ and } \frac{1}{N} X_2' X_2 \text{ converge in probability to positively definite matrixes, and}
\]
that \( \frac{1}{N} Z' X_1 \) and \( \frac{1}{N} Z' X_2 \) converge in probability to nonzero matrixes. Then

\[
\text{plim } b_{1}^{\text{GMM}} = \beta_1 + \text{plim} \left[ \left( \frac{1}{N} X_1' Z \right) S^{-1} \left( \frac{1}{N} Z' X_1 \right) \right] \beta_2 + \text{plim} \left[ \left( \frac{1}{N} X_1' Z \right) S^{-1} \left( \frac{1}{N} Z' X_2 \right) \right] \beta_2 + \text{plim} \left[ \left( \frac{1}{N} X_1' Z \right) S^{-1} \left( \frac{1}{N} Z' \varepsilon \right) \right].
\]

(20)

If \( Z \) is indeed a proper set of instruments for \( X_1 \) in model (18), then \( E[z_n \varepsilon_n] = 0 \), where the subscript \( n = 1, 2, \ldots, N \) depicts the \( n \)-th row of the corresponding matrix, so that

\[
\text{plim} \frac{1}{N} Z' \varepsilon = 0 \text{ and the third term on the RHS of (20) vanishes. However, the second term on the RHS of (20) does not vanish, which means that the estimator is inconsistent. Multiplying out the second term on the RHS of (20) yields a vector of persistent biases. The direction of bias on the price coefficient is of particular interest. Suppose for simplicity that } X_2 \text{ contains just one variable—the lagged dependent variable, i.e. installed base in our setting—so that } \beta_2 \text{ is a scalar. If network effects are on average positive, then } \beta_2 \text{ will be positive. The vector of biases in (20) is then a product of a positive scalar and probability limit of a string of matrix computations, which can be recognized as an estimator of some parameter vector } \gamma \text{ from the following relation}
\]

\[
X_2 = X_1 \gamma + \nu.
\]

(21)

If \( Z \) again contains a proper set of instruments, than the estimator of \( \gamma \) is consistent and the direction of bias on the price coefficient in the original demand model, which ignores network effects, is determined by the sign of parameter on price in (21). A negative correlation—negative partial correlation if \( X_1 \) contains more variables than just constant and
price—between the price and the installed base then implies a downward bias on the price coefficient, hence an overestimation of price elasticity of the misspecified demand. This direction of the bias is especially likely in the new and fast-growing markets, where the installed base increases and the price tends to decrease over time.
References


Figure 1. Best-deal prices in the Polish mobile telephone industry (in zloties)*

* In the period studied 1 U.S. dollar = 4 Polish zloties.
Figure 2. Subscribers in the Polish mobile telephone industry
Figure 3. Simulations of the aggregated steady-state demand (in millions of subscribers)
Figure 4. Aggregated steady-state demand (in millions of subscribers)
Table 1. Demand for mobile telephone services: Estimation results

<table>
<thead>
<tr>
<th>Variables</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic demand</td>
<td>Intercept</td>
<td>4.989***</td>
<td>3.740*</td>
<td>11.245***</td>
<td>86.967***</td>
<td>6.692</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.031)</td>
<td>(1.934)</td>
<td>(2.261)</td>
<td>(3.207)</td>
<td>(3.989)</td>
</tr>
<tr>
<td></td>
<td>$P_{it}$</td>
<td>-0.027***</td>
<td>-0.027***</td>
<td>-0.061***</td>
<td>-0.422***c</td>
<td>-0.034*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.005)</td>
<td>(0.009)</td>
<td>(0.012)</td>
<td>(0.020)</td>
<td>(0.019)</td>
</tr>
<tr>
<td>Own network effects</td>
<td>$y_{i,t-1}$</td>
<td>2.512***</td>
<td>3.939***</td>
<td>2.974***</td>
<td>-</td>
<td>2.387***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.427)</td>
<td>(1.033)</td>
<td>(0.557)</td>
<td></td>
<td>(0.417)</td>
</tr>
<tr>
<td></td>
<td>$\gamma_{i,t-1}$</td>
<td>-0.014</td>
<td>-0.041</td>
<td>-0.030</td>
<td>-</td>
<td>-0.014</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.007)</td>
<td>(0.026)</td>
<td>(0.011)</td>
<td></td>
<td>(0.007)</td>
</tr>
<tr>
<td>Cross-network effects</td>
<td>$y_{i,t}$</td>
<td>0.369**</td>
<td>-0.164</td>
<td>0.147</td>
<td>-</td>
<td>0.378*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.137)</td>
<td>(0.348)</td>
<td>(0.266)</td>
<td></td>
<td>(0.187)</td>
</tr>
<tr>
<td>Pricing regime</td>
<td>$D_{i,t}$</td>
<td>0.238**</td>
<td>0.063</td>
<td>0.431***</td>
<td>-</td>
<td>0.268**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.086)</td>
<td>(0.124)</td>
<td>(0.112)</td>
<td></td>
<td>(0.115)</td>
</tr>
<tr>
<td>Technology (GSM900/1800)</td>
<td>$C_{i,t}$</td>
<td>2.889*</td>
<td>0.530</td>
<td>3.080*</td>
<td>-</td>
<td>2.438*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.392)</td>
<td>(1.448)</td>
<td>(1.713)</td>
<td></td>
<td>(1.259)</td>
</tr>
<tr>
<td>Entry fixed effects</td>
<td>$\lambda_1$</td>
<td>1.165**</td>
<td>0.500</td>
<td>1.047***</td>
<td>-</td>
<td>1.888***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.782)</td>
<td>(1.198)</td>
<td>(0.332)</td>
<td></td>
<td>(0.425)</td>
</tr>
<tr>
<td></td>
<td>$\lambda_2$</td>
<td>1.144</td>
<td>0.255</td>
<td>0.909**</td>
<td>-</td>
<td>1.334**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.757)</td>
<td>(1.051)</td>
<td>(0.361)</td>
<td></td>
<td>(0.394)</td>
</tr>
<tr>
<td></td>
<td>$\lambda_3$</td>
<td>-4.303**</td>
<td>-0.071</td>
<td>-5.148**</td>
<td>-</td>
<td>-4.448**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.683)</td>
<td>(2.507)</td>
<td>(2.422)</td>
<td></td>
<td>(1.862)</td>
</tr>
<tr>
<td>Estimation method</td>
<td>GMM</td>
<td>GMM</td>
<td>GMM</td>
<td>GMM</td>
<td>NLS</td>
<td>Hatanaka</td>
</tr>
<tr>
<td>Instruments :</td>
<td>Lag(2) subscribers</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td>Lag(1) prices</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Number of observations</td>
<td>56</td>
<td>56</td>
<td>56</td>
<td>20</td>
<td>56</td>
<td>54</td>
</tr>
<tr>
<td>Test of the over-ident. restrictions</td>
<td>8.74(11)</td>
<td>6.37(11)</td>
<td>8.01(11)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Godfrey’s serial correlation test</td>
<td>10.82(1)**</td>
<td>13.18(1)**</td>
<td>13.01(1)**</td>
<td>8.16(1)**</td>
<td>12.28(1)**</td>
<td>12.24(1)**</td>
</tr>
<tr>
<td></td>
<td>1.73(1)</td>
<td>7.56(1)**</td>
<td>6.76(1)**</td>
<td>1.92(1)</td>
<td>9.54(1)**</td>
<td>6.19(1)**</td>
</tr>
<tr>
<td></td>
<td>18.68(1)**</td>
<td>19.14(1)**</td>
<td>18.55(1)**</td>
<td>18.54(1)**</td>
<td>15.98(1)**</td>
<td></td>
</tr>
<tr>
<td>Test of the entry fixed effects</td>
<td>Wald: 11.3***</td>
<td>Wald: 2.46</td>
<td>Wald: 3.68</td>
<td>-</td>
<td>Wald: 2.69</td>
<td>Wald: 6.19</td>
</tr>
</tbody>
</table>

*** denotes significance at 1% level, ** at 5% level, and * at 10% level; standard errors in parentheses.

a For consistency with the results of other estimations, the observations in (d) are aggregated to the market level.
b Coefficients on lagged first-stage residuals are suppressed.
c Aggregated price is calculated as the average price across providers weighted by their market shares.
d Restrictions on regression coefficients that overidentify structural parameters are imposed.
e The test statistics are $\chi^2$-distributed under the null hypotheses; degrees of freedom in parentheses.
f Test against $H_0$: ($\lambda_1 = 1.342$, $\lambda_2 = 1.164$, $\lambda_3 = -2.506$); 3 degrees of freedom.
Table 2. Identified structural parameters of demand for mobile telephone services

<table>
<thead>
<tr>
<th>Parameters</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distribution of consumer types</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( a )</td>
<td>186.25***</td>
<td>140.14***</td>
<td>183.56***</td>
<td>205.77***</td>
<td>196.46***</td>
<td>177.58***</td>
</tr>
<tr>
<td></td>
<td>(18.36)</td>
<td>(39.44)</td>
<td>(14.94)</td>
<td>(3.11)</td>
<td>(18.48)</td>
<td>(23.19)</td>
</tr>
<tr>
<td>( b )</td>
<td>0.027***</td>
<td>0.027***</td>
<td>0.061***</td>
<td>0.423***</td>
<td>0.034*</td>
<td>0.064**</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.009)</td>
<td>(0.012)</td>
<td>(0.020)</td>
<td>(0.019)</td>
<td>(0.022)</td>
</tr>
<tr>
<td><strong>Network effects</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( c )</td>
<td>93.79***</td>
<td>147.57**</td>
<td>48.55***</td>
<td>-</td>
<td>70.08</td>
<td>34.31</td>
</tr>
<tr>
<td></td>
<td>(22.81)</td>
<td>(62.07)</td>
<td>(11.22)</td>
<td></td>
<td>(43.72)</td>
<td>(20.55)</td>
</tr>
<tr>
<td>( d )</td>
<td>-0.539*</td>
<td>-1.539</td>
<td>-0.496**</td>
<td>-</td>
<td>-0.414</td>
<td>-0.308</td>
</tr>
<tr>
<td></td>
<td>(0.298)</td>
<td>(1.056)</td>
<td>(0.177)</td>
<td></td>
<td>(0.291)</td>
<td>(0.243)</td>
</tr>
<tr>
<td><strong>Compatibility</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( w )</td>
<td>0.147*</td>
<td>-0.042</td>
<td>0.050</td>
<td>-</td>
<td>0.159</td>
<td>0.308</td>
</tr>
<tr>
<td></td>
<td>(0.074)</td>
<td>(0.078)</td>
<td>(0.098)</td>
<td></td>
<td>(0.102)</td>
<td>(0.258)</td>
</tr>
<tr>
<td>( \Delta w )</td>
<td>0.095**</td>
<td>0.016</td>
<td>0.145**</td>
<td>-</td>
<td>0.112*</td>
<td>0.230*</td>
</tr>
<tr>
<td></td>
<td>(0.044)</td>
<td>(0.035)</td>
<td>(0.061)</td>
<td></td>
<td>(0.059)</td>
<td>(0.128)</td>
</tr>
</tbody>
</table>

*** denotes significance at 1% level, ** at 5% level, and * at 10% level; standard errors in parentheses.