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SALES TAX COMPETITION AND A MULTINATIONAL WITH A DECREASING MARGINAL COST

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Abstract

Sales tax competition and a multinational with a decreasing marginal cost

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We examine a multinational firm which has a decreasing marginal cost, and the optimal sales tax policies of the regions where that firm operates. We show that the regions set higher sales taxes than those given by a cooperative equilibrium. Each region fails to fully internalize the effects of its tax level on another region's welfare and the incentives for that region's authority. Exponential cost functions which exhibit economies of scale (for example Cobb-Douglas) and linear demand functions satisfy our assumptions. Our results suggest the need to coordinate sales tax levels between countries and between smaller entities, like states in the United States. Smaller regions benefit more from such coordination. Lowering sales taxes in each region increases welfare for all regions, profits for firms, and consumer welfare.

Keywords: tax competition, sales taxes, multinationals, decreasing marginal cost, economies of scale

JEL Classification: F12, F23, H25, H71

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1 Introduction

Tax competition between regions has been extensively investigated, both empirically and theoretically. The main finding is that regions competing for mobile factors under-provide public goods; and tax levels are below optimal.\(^1\)

In this paper, we consider a monopoly firm, which we refer to as a multinational, that sells its product in two regions, A and B, and produces in another region, C.\(^2\) The regions could be countries, or states, or municipalities.\(^3\) We assume that each region has an authority setting that region’s sales tax level. The multinational is assumed to benefit from a diminishing marginal cost of producing in the same factory for both A and B. We take the firm’s location as fixed and analyze its production decision for each region in reaction to local sales tax levels and the relative size of the regions. We then characterize the regions’ sales tax levels set by their authorities at a non-cooperative equilibrium. Finally, we compare the equilibrium tax levels with the cooperative optimum.

We show that, when the firm has a decreasing marginal cost, two non-cooperative regions set higher sales taxes than those set by two optimally cooperating regions.\(^4\) The link connecting the two regions’ tax policies is the nonlinear production cost of the firm. As region A lowers its sales tax, or grows, the monopolist sells more in the region. This reduces the monopolist’s marginal cost, which in turn results in more sales in region B as well. In this case, region A’s consumers effectively subsidize region B’s consumers, as region A collects less tax, but both regions receive more goods. A sales tax increase or a region size decrease brings the opposite result. Raising sales tax in one region exerts a negative externality on the other. In a non-cooperative game, the tax authorities do not fully internalize the effects of changing their sales tax on the other region.

Our results show that there is potential for improving welfare globally by coordinating sales tax levels, and emphasize the importance of international organizations, like the World Trade Organization. Similarly, in the case when the regions are states or counties, the federal government may accomplish this coordination. Coordination between sales tax levels would be beneficial especially for smaller regions, as we show that, if a region is infinitesimally

\(^1\)See Becker and Fuest (2010) for a theoretical argument and Devereux et. al.(2008) for an empirical one. See Wilson (1999) for a review of older literature, starting with Tiebout (1956). Most of the papers deal with country tax competition, but see Braid (1996) and Brueckner and Saaverda (2001) for examples of local competition.

\(^2\)Our arguments would hold if the multinational is based in either A or B, and we assume a negligible transportation cost between A and B.

\(^3\)A more proper term for the firm would be a “multi-regional” instead of a “multinational”, but for simplicity, we use “multinational”, or simply “the firm”.

\(^4\)There are other conditions to ensure the existence of a Nash Equilibrium in the tax setting game between the two countries, and that the firm has a unique solution to its profit maximization problem.
small, its sales tax does not affect a larger foreign market, even though the small region is affected significantly by that larger foreign market.

Most recent estimates of production functions show that firms indeed have decreasing marginal cost. The production function of choice for empirical estimation is Cobb-Douglas, and for this functional form we just need it to exhibit economies of scale. Ackeberg et. al. (2006) show that for a dataset of Chilean plants in four industries – food products, textiles, wood products, and metals – the Cobb-Douglas production function exhibits economies of scale, regardless of the estimation technique used.

Economies of scale are one of the cornerstones of international trade. Previous literature has generally considered only one example of economies of scale: a fixed cost and a constant marginal cost. We instead consider general concave cost functions (diminishing marginal cost). Nonlinear costs are crucial for our results, since the externality between the regions is due to the non-constant marginal cost of production. A constant marginal cost (even with a fixed cost, resulting in economies of scale) leads to no connection between the two markets. There would be no externalities of one region’s tax level on the other.

Many of our assumptions can be relaxed. When the firm’s cost curve is convex (increasing marginal cost), the results are reversed. A higher sales tax in one region leads the firm to produce less for that region, which in turn decreases its marginal cost (since the cost is convex) and thereby increases sales in the other region. In other words, a sales tax of one region imposes a positive externality on the other. In this case, the regions set their sales taxes lower than their cooperative equilibrium levels. The assumption of one monopoly firm in both regions can also be relaxed. While it is impossible to derive as general results as in the monopoly case without further assumptions, we show some evidence that our main result still holds with many firms in Section 5.

The literature provides other explanations to why non-cooperative authorities tax too much. One is the well-known double marginalization problem in vertical tax competition where the federal government is competing with a local tax authority. The other presents itself when two regions tax a multiregional firm and provide tax credits.

A branch of the literature analyzes the optimal tax policies of different regions while focusing on the principal-agent problems where the regions cannot observe the costs of the multinational. In this context, Bond and Gresik (1996) find that regions tax the firm too much if the taxes are set non-cooperatively. We find parallel results due to a different reason, which is the existence of a firm producing with a decreasing marginal cost. Calzolari (2001)

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continues with the multi-agency approach and relates the regulation of the multinationals to the standard regulation theory. Different from this literature, all the players in our model have full information and there are no principal-agent issues. Gresik (2001) reviews different ways that the multinationals’ income gets taxed, and the implications of these methods.

Another related work is Bulow, Geanakoplos, and Klemperer (1985). Similar to our results, they show that following a positive shock, and thus more sales in one market, the firm sells more (respectively less) in the other market if its cost function exhibits economies (diseconomies) of scale. In our paper, we specify a positive shock as an increase in the relative market size or as a decrease in the sales tax level of a given market. Different from Bulow et. al.(1985), we endogenize the shocks by analyzing the optimal sales tax levels set by the authorities.\(^7\)

In the next section we present the general setup. As a benchmark, we consider the one-region case where we first solve for the monopolist’s optimal reaction to a sales tax in that region, and then characterize the region’s optimal tax policy. We then analyze the two-region case where we solve for the monopolist’s optimal sales in the regions given their sales tax levels. In Section 3, we present and discuss comparative statics of the equilibrium quantities with changes in market conditions in the domestic and the foreign markets. In Section 4, we characterize sales tax levels set by the regions in a non-cooperative equilibrium and compare them with the levels set at cooperative optimum. Section 5 extends our arguments to the case of competing multinationals by an illustrative example. In Section 6, we discuss some of our assumptions and how relaxing them would affect our results. Section 7 concludes our paper. The technical proofs are relegated to the Appendix.

2 Model

A monopolist sells its product in two regions, A and B. Region A’s inverse demand is \(P(q_A)\), where \(q_A\) is the quantity sold in A. To analyze the implications of the relative market size of the two regions, we define region B’s demand as a scaled version of region A’s demand, such that B’s inverse demand is \(\theta P(q_B)\), where \(\theta > 0\).\(^8\) If \(\theta = 1\), then the countries are the same, if \(\theta \to 0\), then A is much bigger than B. We assume that the demand is decreasing in quantity sold, \(P'(\bullet) < 0\), and there is a choke price in each region (prices cannot go to infinity).

The monopolist is allowed to charge different prices across the two regions and we assume

\(^7\)Moreover, we consider a firm with a decreasing marginal cost.

\(^8\)The qualitative results would still be valid if we allowed for different demands in different regions (through substituting \(P_i(q_i)\) by \(P(q_i)\) in the analysis).
that there is no parallel importing. The tax authority in region A sets its sales tax level, 
\( t_A \in (0, 1) \), which results in the total tax revenue of \( t_A P(q_A)q_A \) while leaving the firm a revenue of \( (1 - t_A) P(q_A)q_A \). Similarly, region B’s tax authority determines the sales tax level in region B, \( t_B \in (0, 1) \), which creates \( t_B \theta P(q_B)q_B \) of tax revenue for region B and leaves the firm \( (1 - t_B) \theta P(q_B)q_B \) of revenue from region B. The tax authorities maximize the sum of the consumer surplus and tax receipts in their region.

The timing of the interactions is the following. First, the authorities simultaneously and non-cooperatively set their sales tax levels, \((t_A, t_B)\). After observing \((t_A, t_B)\), the monopolist chooses its level of production for regions A and B, \((q_A, q_B)\). Let \((q^*_A, q^*_B)\) and \((t^*_A, t^*_B)\) denote respectively the equilibrium quantities sold by the monopolist and the equilibrium tax levels set by the authorities.

Suppose that the firm has one factory located in another region with the same transportation cost to A and to B.\(^9\) The firm’s cost function is \( C(q_A + q_B) \), which we assume to be increasing, twice differentiable and concave:

**A1.** Decreasing Marginal Cost: \( C''(q) < 0 \)

A diminishing marginal cost in quantity could be because, e.g., a larger scale enables the firm to learn about more cost efficient technologies or use of its resources.

We focus on an interior solution for the firm’s profit maximization problem where the firm sells positive quantities in both markets, i.e., \( q^*_A, q^*_B > 0 \). To ensure that the profit of the firm is a quasi-concave function in its production for region A and its production for region B, we make the following assumptions: for \( q > 0 \) and \( i = A, B \),

**A2.** Sufficiently Concave Revenue: \( (1 - t_i)[2P'_i(q) + P''_i(q)q] < C''(q) \).

The assumptions above imply concave revenue. In general, log-concave demand functions lead to a concave revenue.

Define \( \overline{t}_i(q) \) as

\[
\overline{t}_i(q) \equiv 1 - \frac{C''(q)}{2P'_i(q) + P''_i(q)q}, \quad \text{for} \quad i = A, B.
\]

Assumption A2 ensures that \( \overline{t}_i(q) > 0 \). An interior solution exists only for sufficiently low values of \( t_i \). For instance, when \( t_i \) goes to one, A2 does not hold. A2 implies that \( \overline{t}_i(q) \in (0, 1) \), and thus, for all \( t_i \in (0, \overline{t}_i(q)) \) there exists an interior solution to the firm’s optimality problem.

\(^9\)Alternatively, the factory could be located in either A or B, with negligible transportation costs to the other region.
We also assume that the welfare function that each region maximizes is concave in the region’s tax level, so that we can solve each region’s maximization problem using the standard first-order conditions. This is a standard assumption, with one caveat in our model: unfortunately, it involves numerous higher order derivatives of optimal quantities sold in each region, and it is displayed in a somewhat more manageable form in the Appendix. For an optimal tax problem’s second-order condition to hold, the firm’s margin in that region cannot be decreasing too fast in the region’s tax level, and the optimal quantity sold in the region cannot be too convex in the region’s tax level.

2.1 Benchmark: one-region case

We present the case of one region, say region A, where the monopolist is selling its product and paying a unit tax, $t_A$, over its sales revenue, so the firm’s profit is the difference between its after tax revenue and the cost:

$$\Pi(q_A) = (1 - t_A) P(q_A) q_A - C(q_A).$$

The firm chooses its optimal quantity to maximize its profit:

$$\max_{q_A} \Pi(q_A) \tag{1}$$

The first-order condition of (1) gives us the optimal quantity, $q_A^*(t_A)$, which satisfies

$$(1 - t_A) [P(q_A^*) + P'(q_A^*) q_A^*] = C'(q_A^*) \tag{2}$$

The maximum profit is achieved when the firm equates its marginal revenue to its marginal cost. We verify that the second order condition is satisfied by A2.Considering the impact of the sales tax on the firm’s activity, we obtain the following result:

**Lemma 1** The monopolist sells less when the sales tax level increases ($\frac{\partial q_A^*}{\partial t_A} < 0$).

**Proof.** By applying the Implicit Function Theorem to equilibrium condition (2), we show that the optimal quantity is decreasing in the tax level:

$$\frac{dq_A^*}{dt_A} = \frac{P(q_A^*) + P'(q_A^*) q_A^*}{(1 - t_A) [2P'(q_A^*) + P''(q_A^*) q_A^*] - C''(q_A^*))} \tag{3}$$

$$= \frac{C'(q_A^*)}{(1 - t_A) SOC_A} < 0,$$
since \( P(q_A^*) + P'(q_A^*)q_A^* = \frac{C(q_A^*)}{1-t_A} > 0 \) by condition (2) and the denominator is negative by the second-order condition.

The tax authority sets a sales tax which maximizes the sum of the consumer welfare of the region and its tax revenue:

\[
\max_{t_A} W_A(t_A) = \int_0^{q_A^*} P(q) dq - (1-t_A)P(q_A^*)q_A^*. 
\]

The optimal tax level, \( t_A^* \), is characterized by the first-order condition:

\[
P(q_A^*)q_A^* + \left\{ P(q_A^*) - (1-t_A) \left[ P(q_A^*) + P'(q_A^*)q_A^* \right] \right\} \frac{dq_A^*}{dt_A} = 0 \quad \text{(4)}
\]

Substituting the monopolist’s equilibrium condition, (2) and the derivative of its optimal quantity, (3), into equation (4) leads to the optimality condition for \( t_A^* \):

\[
P(q_A^*)q_A^* + \left[ P(q_A^*) - C'(q_A^*) \right] \frac{C'(q_A^*)}{(1-t_A) SOC_A} = 0. \quad \text{(5)}
\]

## 2.2 Two-region case

We now analyze our base model when the firm sells its product in both regions. The firm’s profit is then

\[
\Pi(q_A, q_B) = (1-t_A) P(q_A)q_A + (1-t_B) \theta P(q_B)q_B - C(q_A + q_B), \quad \text{(6)}
\]

where the first term on the right hand side is the after-tax revenue from region A, the second term is the after-tax revenue from region B and the third term is the total production cost. The firm takes the sales tax levels \( t_A \) and \( t_B \) as given, and chooses quantities \( q_A \) and \( q_B \) to maximize its profit. In equilibrium the monopolist sells positive quantities \( q_A^* \) and \( q_B^* \) such that:

\[
(1-t_A) \left[ P(q_A^*) + P'(q_A^*)q_A^* \right] = C'(q_A^* + q_B^*) \quad \text{(7)}
\]

\[
(1-t_B) \theta \left[ P(q_B^*) + P'(q_B^*)q_B^* \right] = C'(q_A^* + q_B^*). \quad \text{(8)}
\]

To derive explicit equations for the optimal quantities as functions of tax levels \( t_A \) and \( t_B \), one needs to specify the cost and demand functions.\(^\text{10}\)

Using the implicit definitions \( q_A^*(t_A, t_B, \theta) \) and \( q_B^*(t_A, t_B, \theta) \), we now derive some comparative statics.

\(^{10}\)It is also necessary to verify that the cost and demand specifications satisfy our assumptions.
3 The effect of market conditions on optimal quantities

3.1 Changes in the domestic market

Considering the impact of a sales tax level on the monopolist’s sales in that region, we obtain
the following result:

**Proposition 1** If a region has a higher sales tax level, the firm sells less in that region
\((\frac{\partial q^*_A}{\partial t_A}, \frac{\partial q^*_B}{\partial t_B} < 0)\).

The proposition generalizes the result of the one-region case to the case of two regions:
increasing the sales tax level in a region leads to less sales in that region.

When \(\theta\) is very close to zero, the relative market size of region B goes to zero, the
firm sells nearly zero in region B and sells \(q^*_A(t_A)\) in region A which is given by the one-
region equilibrium condition, (2). In other words, when \(\theta\) approaches zero, the equilibrium
quantities of the two-region case approach the corresponding equilibrium quantities of the
one-region case. This is intuitive, but it does not prevent B’s tax rate from influencing A, even
when \(\theta\) goes to zero. This is the case if a marginal change in B’s production still influences
the firm’s marginal cost. Below we show that our assumption of an interior solution rules
this case out.

**Proposition 2** If one region’s market size is nearly zero, the changes in sales tax level in
that region do not affect the other region \(\lim_{\theta \to 0} \frac{\partial q^*_A(t_A,t_B,\theta)}{\partial t_A} = \frac{\partial q^*_A(t_A)}{\partial t_A}\).

This result shows that if a region is very small compared to some others, its tax policy
does not affect the other regions, even when there is a firm with non-linear costs selling its
product in all regions. On the other hand, a relatively big region’s tax policy has a significant
impact on the smaller regions.

3.2 Externalities between the regions

The relative size of region B to region A, \(\theta\), affects the optimal quantity sold in region A,
since the firm has a non-linear cost of production:

**Proposition 3** The firm sells more in region A if region B’s market is larger \(\left(\frac{\partial q^*_A}{\partial \theta} > 0\right)\).

When the firm’s marginal cost is decreasing, if one market becomes bigger, the monop-
olist produces more overall, lowering its marginal cost of production, and thus selling more
quantities in both regions. The quantity for region A shifts due to a change in an external
circumstance, the relative size of market B (\(\theta\)). As a result of a variation in \(\theta\), region A’s
consumer welfare moves to the same way as quantity.
Corollary 1  The firm sells less in region A if region B increases its sales tax level \( \frac{\partial q_A^*}{\partial t_B} < 0 \).

If region B’s authority sets a higher sales tax, the firm is going to produce less for region B. This increases the firm’s marginal cost, and thus the firm sells less in region A as well. Besides the effect on own consumers, increasing the sales tax level in one region decreases the other region’s consumer welfare, and thereby imposes a negative externality on the other region’s consumers.

Corollary 2  Increasing the sales tax level in one region decreases the consumer welfare in the other region.

One region’s tax authority exerts an externality on the other region when setting its sales tax level. When the regions set their tax policies non-cooperatively, this externality cannot be internalized, which leads to a suboptimal outcome.

4 Equilibrium of tax competition and comparison with the cooperative outcome

The welfare of region A (and respectively B) is the sum of the consumer welfare and the tax receipts of the region:

\[
W_A(t_A, t_B) = \int_0^{q_A^*} P(q) dq - (1-t_A)P(q_A^*)q_A^* \\
W_B(t_A, t_B) = \int_0^{q_B^*} \theta P(q) dq - (1-t_B)\theta P(q_B^*)q_B^*
\]

where \( q_A^* \) and \( q_B^* \) are the equilibrium sales of the firm in region A and B, respectively, as functions of tax levels \( t_A, t_B \) (given by equations (7) and (8)). Each tax authority faces a familiar trade-off while choosing its sales tax level – lowering the tax level induces the firm to produce more, but reduces the tax receipts.

Region A’s authority picks a sales tax level, \( t_A \), to maximize its welfare:

\[
\max_{t_A} W_A(t_A, t_B).
\]  

Region A’s best-response tax level to region B’s tax, \( t_A^{BR}(t_B) \), is the solution of

\[
\frac{\partial W_A}{\partial t_A} = P(q_A^*)q_A^* + \frac{\partial W_A}{\partial q_A} \frac{\partial q_A^*}{\partial t_A} = 0 \\
= P(q_A^*)q_A^* + \left\{ P(q_A^*) - (1-t_A) \left[ P(q_A^*) + P'(q_A^*)q_A^* \right] \right\} \frac{\partial q_A^*}{\partial t_A} = 0.
\]
Substituting in the firm’s equilibrium condition, (7), we get the simplified equilibrium condition characterizing $t^{BR}_A(t_B)$:

$$\frac{\partial W_A}{\partial t_A} = P(q^*_A)q^*_A + [P(q^*_A) - C'(q^*_A + q^*_B)] \frac{\partial q^*_A}{\partial t_A} = 0 \quad (10)$$

Symmetrically, authority B sets its tax level by maximizing

$$\max_{t_B} W_B(t_A, t_B),$$

and so its best-response tax level to region A, $t^{BR}_B(t_A)$, is characterized by

$$\frac{\partial W_B}{\partial t_B} = \theta P(q^*_B)q^*_B + \frac{\partial \partial W_B}{\partial q^*_B} \frac{\partial q^*_B}{\partial t_B} = 0 \quad (11)$$

$$= P(q^*_B)q^*_B + [P(q^*_B) - C'(q^*_A + q^*_B)] \frac{\partial q^*_B}{\partial t_B} = 0.$$

We assume that a region’s tax level has a higher impact on the quantity sold in that region and on its first-order effect than the other region’s tax level, respectively,

A3. $|\frac{\partial q^*_i}{\partial t_i}| > |\frac{\partial q^*_i}{\partial t_{-i}}|$ and $|\frac{\partial^2 q^*_i}{\partial t_i \partial t_{-i}}| > |\frac{\partial^2 q^*_i}{\partial t_i^2}|$, for $i = A, B$.

This seems like a natural assumption when the countries are symmetric, however it might be violated when one of the countries is sufficiently larger than the other, so that it virtually determines the firm’s overall production and its marginal cost.

Assumption A3 ensures that the equilibrium is stable, which means that the best-response functions’ slopes are below one (in absolute value) in the neighborhood of an equilibrium candidate (see Appendix). The non-cooperative equilibrium tax levels are given by the intersection of the best-response functions: $t^*_A = t^{BR}_B(t^{BR}_B(t_A))$ and $t^*_B = t^{BR}_A(t^*_A)$.

**Proposition 4** If the two regions coordinate their tax policies, they set lower taxes than if they do not cooperate.

When one of the regions increases its tax level, the firm decreases its production level, which in turn increases the marginal cost. As we have shown before, this leads to a decrease in quantity sold in the other region as well, and higher prices. Overall, the regions do not internalize the externality imposed on the other region through the decreasing marginal cost function, and thus set tax levels which are too high, when setting them non-cooperatively.
5  N multinationals

Introducing strategic competition of two (or more) firms in each region makes it impossible to derive general results on comparative statics without making more restrictive assumptions. The issue that makes the derivation particularly tricky is the fact that with imperfect competition tax rate pass-through might be higher than one (see Anderson et.al. (2001) and papers cited within).

Below, we present a numerical result for Cournot competition between $N$ symmetric multinational firms, facing identical linear demands in both markets. The demand function is given by $P(Q) = 2 - Q$, where $Q$ is the total quantity in the market, and each firm has a cost function of $C(Q) = Q^{0.9}$.

Suppose there are $N$ symmetric multinationals operating in both markets, playing a standard Cournot game in both A and B. There is no entry. The game has two stages: authorities of A and B simultaneously decide on taxes, and then the firms play a Cournot game in each market.

In the first stage, authority A maximizes the welfare of region A:

$$\max_{t_A} W_A(t_A) = \int_0^{Q_A^*} P(q) dq - (1 - t_A) P(Q_A^*) Q_A^*, \quad (12)$$

and authority B simultaneously maximizes the symmetric function with respect to $t_B$.

In the second stage, each firm, say firm $i$, simultaneously chooses the quantity it sells in A, $q_{iA}$ and the quantity it sells in B, $q_{iB}$, given $t_A$ and $t_B$, to maximize its profit:

$$\Pi(q_{iA}, q_{iB}) = (1 - t_A) P(Q_A) q_{iA} + (1 - t_B) P(Q_B) q_{iB} - C(q_{iA} + q_{iB}), \quad (13)$$

where the total quantity in region A is $Q_A = \sum_{i=1}^{N} q_{iA}$ and for region B is $Q_B = \sum_{i=1}^{N} q_{ib}$.

In Figure 1, solid line and dashed line represent, respectively, the non-cooperative (Nash) and cooperative equilibrium tax levels as a function of the number of firms in each market. This simulation illustrates that our qualitative result holds in this particular example of Cournot competition with N firms: the competitive tax levels are too high compared to their cooperative levels. The pattern of equilibrium taxes decreasing with the number of firms does not hold with different demand specifications. Similarly, the gap between the two regimes does not always increase with the number of firms. We leave a more complete treatment of symmetric oligopoly, i.e., competition between symmetric multinationals, and the interesting case of an asymmetric oligopoly, i.e., competition between a multinational and a local firm, for future research.
6 Discussion

We assume one monopoly firm selling in two regions, and cost as a function of the sum of quantities in both regions in order to illustrate our results in as simple a setup as possible. Chen and Ross (2007) examine a model of a duopoly with increasing marginal costs competing in two markets. They find that multimarket contact is anticompetitive in that setting. Their results, and our simulations, lead us to believe that our findings would not change substantially if there are several multinational firms. Examining that in more detail might be worthwhile, particularly the case of multinational firms competing against local firms. However, the functional form generality of our framework will have to be sacrificed.

We assume that the firm’s cost is a function of the sum of the quantities produced for the two regions. We conjecture that the qualitative results would still be valid under a more complex cost specification with less interdependence between the production costs of the two regions. With a less interdependent cost structure, the inter-region effects are
correspondingly weakened, but as long as the production decisions that the firm makes for the two regions are strategic complements, our findings hold.

Another concern might be aggregation across industries, assuming that the sales tax has to be the same for all industries. A region could be a big market for one industry, but a small market for another. With a common (for all industries) sales tax, the region is over-taxing all industries, although the over-taxing is less severe for the small market industry. The optimal solution is to separately account for all industries, and have agreements with all other relevant regions. However, the relevant regions are not the ones that the region does a lot of trade with. The relevant regions are the ones that are buying the same goods from the same third region.

We assume that the multinational firm is based in a third region, and therefore the tax authorities of the two regions maximize the sum of their consumers’ welfare and tax revenue. Instead, suppose that the multinational is based in one of these two regions and the region authority cares about the multinational’s profits as well as the consumer welfare and the tax revenue of its region. That region then internalizes some of the effects of its tax level on the other region, and therefore does not raise the sales tax as high as in the case where the multinational is based somewhere else.

If the firm has a convex cost (increasing marginal cost), the results are reversed. Selling more in the domestic market implies selling less in the foreign market, and thus one region’s sales tax level exerts a positive externality on the other region’s consumers. As a result, the regions set their sales taxes lower than their cooperative equilibrium levels.

We do not explicitly restrict the regions to set positive taxes. Indeed, this restriction is not necessary for our arguments. Our conclusion with negative taxes (subsidies) would be that the regions would subsidize more if they were cooperating. In the equilibrium of our oligopoly simulation, we frequently encounter sizable negative taxes (heavy subsidies) in, especially if the marginal cost diminishes particularly quickly (strong economies of scale). Subsidizing competing firms with quickly diminishing marginal costs is a good solution for the regions, since this leads to higher efficiency, and so lower consumer prices.\footnote{With quickly diminishing marginal costs, driving some firms out of business might, of course, be optimal. We kept the number of firms fixed in our simulations.}

We left two major related issues out of our paper for purposes of tractability. The first issue is parallel imports – consumers or other firms buying the good in one region and selling it to another region where they compete with the original producer. We assume no parallel imports. The likely effect of allowing parallel imports in our model would be dampening the cross-region externalities. The second one is per unit, as opposed to ad valorem, taxes. As a line of papers (see Delipalla and Keen (1992) for example) shows, there is a difference
between the two in imperfectly competitive environments. We expect that our results still hold with a per-unit tax. We chose ad valorem tax because it dominates specific tax from the perspective of social welfare.

7 Conclusion

This paper shows that there is an opportunity to improve consumer welfare and tax receipts globally through coordinating national sales tax policies. The same is true on other levels – for example, there should be more cooperation between the states within the United States. When a firm with a decreasing marginal cost sells its product in several regions, lowering sales taxes cooperatively leads to an increase in welfare for all regions, higher profits for the firms, and higher consumer welfare – a win-win-win situation.

We argue that the suboptimality of non-cooperative sales taxes is due to the independent tax authorities not internalizing the externality of their tax level on other regions buying the same good. These externalities originate from the fact that firms have nonlinear production costs, and sell in many regions. The sign of these externalities, and thus whether the taxes are too high or too low, depends on whether firms have decreasing or increasing marginal costs. When multinationals enjoy decreasing marginal costs, a region’s sales tax exerts a negative externality on other regions buying the same product and thereby non-cooperative taxes are set too high compared to their cooperative levels. The opposite result holds if multinationals have increasing marginal costs. The magnitude of these externalities and how far they are from the optimum depends on several factors, like the degree of nonlinearity in multinationals’ production costs, feasibility of parallel-imports between the regions, and the degree of competition among multinationals or between multinationals and local firms. We show that the first factor reinforces these externalities and increases the extend of suboptimality, whereas we conjecture that the other factors might mitigate the externalities.
References


Appendix

The proof of the monopolist’s second-order conditions

Claim. Assumptions 1-3 imply that the second order conditions for the firm’s maximization problem hold.

To maximize the monopolist’s profit (6), we derive the optimality conditions with respect to $q_A$ and $q_B$:

\[
\frac{\partial \Pi}{\partial q_A} = (1 - t_A) [P(q^*_A) + P'(q^*_A)q^*_A] - C'(q^*_A + q^*_B) = 0, 
\]

\[
\frac{\partial \Pi}{\partial q_B} = (1 - t_B) \theta [P(q^*_B) + P'(q^*_B)q^*_B] - C'(q^*_A + q^*_B) = 0. 
\]

Differentiating equations (14) and (15), we get the Hessian matrix for the second order conditions:

\[
H = \begin{bmatrix}
\frac{\partial^2 \Pi}{\partial q_A^2} & \frac{\partial^2 \Pi}{\partial q_A \partial q_B} \\
\frac{\partial^2 \Pi}{\partial q_B \partial q_A} & \frac{\partial^2 \Pi}{\partial q_B^2}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
(1 - t_A) [2P'(q^*_A) + P''(q^*_A)q^*_A] - C''(q^*_A + q^*_B) \\
-C''(q^*_A + q^*_B)
\end{bmatrix}
\]

\[
(1 - t_B) \theta [2P'(q^*_B) + P''(q^*_B)q^*_B] - C''(q^*_A + q^*_B)
\]

The second-order conditions are satisfied if and only if

(i) $\frac{\partial^2 \Pi}{\partial q_A^2} \frac{\partial^2 \Pi}{\partial q_B^2} - \left( \frac{\partial^2 \Pi}{\partial q_A \partial q_B} \right)^2 > 0$, 

(ii) $\frac{\partial^2 \Pi}{\partial q_A^2} < 0$, 

(iii) $\frac{\partial^2 \Pi}{\partial q_B^2} < 0$.

It is then straightforward to check that Assumptions 1-3 are sufficient for the second-order conditions.

Assumptions to ensure that the regions’ second-order conditions hold and that the equilibrium of the regions’ game is stable

Concavity of welfare maximization ($\frac{\partial^2 W^*_A}{\partial t^2_A} < 0$): 

\[
SOC_A: \frac{\partial^2 W^*_A}{\partial t^2_A} = \frac{\partial q^*_A}{\partial t_A} \frac{C'}{1 - t_A} + \frac{\partial q^*_A}{\partial t_A} \frac{\partial (P(q^*_A) - C')}{\partial t_A} + \frac{\partial^2 q^*_A}{\partial t^2_A} (P(q^*_A) - C') < 0. 
\]
Ensuring that the equilibrium is stable (the best-response functions’ slopes are below 1):

\[ \frac{\partial t_A^*}{\partial t_B} = -\frac{\partial^2 W_A^*/\partial t_A \partial t_B}{\partial^2 W_A^*/\partial t_A^2}. \] (18)

Differentiating equation (10) with respect to \( t_B \) for the numerator and \( t_A \) for the denominator we get:

\[ \frac{\partial t_A^*}{\partial t_B} = -\frac{\frac{\partial^2 q_A^*}{\partial t_A \partial t_B} C'}{1-t_A} + \frac{\partial q_A^*}{\partial t_A} \frac{\partial}{\partial t_B} (P(q_A^*) - C') + \frac{\partial^2 q_A^*}{\partial t_A \partial t_B} \left( P(q_A^*) - C' \right). \] (19)

Comparing the numerator and the denominator term-by-term, we can see that the fraction is less than 1 in absolute value if the effects of a country’s tax level on domestic consumption, the firm’s margin, and consumption’s response to tax changes are stronger than the effects of the foreign region’s tax policy on the same variables. We ensure this by A3. Note that this assumption holds when the regions are close in size, or symmetric, however, as one of the regions becomes sufficiently larger than the other, it determines a significant part firm’s production level and thereby its marginal cost, in which case these conditions might not hold anymore.

**Proof of Proposition 1.**

We present the proof for \( \frac{\partial q_A^*}{\partial t_A} < 0 \), the derivation for \( \frac{\partial q_B^*}{\partial t_B} < 0 \) is symmetric.

In the previous section we derive the optimality conditions, (7) and (8), which define \( q_A^*(t_A, t_B, \theta) \) and \( q_B^*(t_A, t_B, \theta) \) implicitly. Treating \( t_B \) and \( \theta \) as constants and taking the total derivatives of the optimality conditions give

\[ (1 - t_A) \left[ 2P'(q_A^*) + P''(q_A^*)q_A^* \right] dq_A^* - [P(q_A^*) + P'(q_A^*)q_A^*] dt_A = (dq_A^* + dq_B^*) C''(q_A^* + q_B^*). \] (20)

\[ (1 - t_B) \left[ 2P'(q_B^*) + P''(q_B^*)q_B^* \right] dq_B^* = (dq_A^* + dq_B^*) C''(q_A^* + q_B^*). \] (21)

By solving the latter equations together, we get

\[ \frac{\partial q_A^*}{\partial t_A} = \frac{C''(q_A^* + q_B^*)}{1 - t_A} - \frac{\partial^2 \Pi}{\partial q_A^2} - \left[ C''(q_A^* + q_B^*) \right]^2. \] (22)

The derivative \( \frac{\partial q_A^*}{\partial t_A} \) is negative because \( \frac{\partial^2 \Pi}{\partial q_A^2} \frac{\partial^2 \Pi}{\partial q_B^2} - [C''(q_A^* + q_B^*)]^2 > 0 \) and \( \frac{\partial^2 \Pi}{\partial q_B^2} < 0 \) by the second-order conditions (see (16)).

**Proof of Proposition 2.**
Consider $\frac{\partial q_A^*}{\partial t_A}$, which we derive in the proof of 1

$$\frac{\partial q_A^*}{\partial t_A} = C'(q_A^* + q_B^*) \frac{\frac{\partial^2 \Pi}{\partial q_A^2}}{1 - t_A} - \frac{\frac{\partial^2 \Pi}{\partial q_B^2}}{\left[C''(q_A^* + q_B^*)\right]^2}.$$

Dividing the nominator and denominator by $\frac{\partial^2 \Pi}{\partial q_B^2}$ gives

$$\frac{\partial q_A^*}{\partial t_A} = C'(q_A^* + q_B^*) \frac{1}{1 - t_A} \frac{\frac{\partial^2 \Pi}{\partial q_A^2} - \left[C''(q_A^* + q_B^*)\right]^2}{\left[C''(q_A^* + q_B^*)\right]^2}.$$

If $\lim_{\theta \to 0} \frac{\partial^2 \Pi}{\partial q_B^2} = -\infty$, we show that

$$\lim_{\theta \to 0} \frac{\partial^2 \Pi}{\partial q_B^2} = -\infty,$$

we have

$$\frac{\partial^2 \Pi}{\partial q_B^2} = \theta (1 - t_B) \left[2P'(q_B^*) + P''(q_B^*)q_B^*\right] - C''(q_A^* + q_B^*).$$

From the proof the Hessian of the monopolist’s problem, we have

$$\frac{\partial^2 \Pi}{\partial q_B^2} = \theta (1 - t_B) \left[2P'(q_B^*) + P''(q_B^*)q_B^*\right] - C''(q_A^* + q_B^*).$$

We know that $C''(q_A^* + q_B^*)$ is finite, and thus, by A2, $P''(q_B^*)q_B^*$ cannot be positive infinity. Let’s examine $\theta P'(q_B^*)$ closer. By the optimal quantity for country B, from 8, we have

$$(1 - t_B) \theta [P(q_B^*) + P'(q_B^*)q_B^*] = C'(q_A^* + q_B^*).$$

Since everything else is finite, $\theta P'(q_B^*)q_B^*$ must be finite as well. Given that $\lim_{\theta \to 0} q_B^* = 0$ and $P' < 0$, $\lim_{\theta \to 0} \theta P'(q_B^*) = -\infty$, and thus $\lim_{\theta \to 0} \frac{\partial^2 \Pi}{\partial q_B^2} = -\infty$.

**Proof of Proposition 3.**

Treating $t_A$ and $t_B$ as constants, and taking the total derivatives of the optimality conditions, (7) and (8), result in

$$\begin{align*}
(1 - t_A) \left[2P'(q_A^*) + P''(q_A^*)q_A^*\right] dq_A^* & = (dq_A^* + dq_B^*) C''(q_A^* + q_B^*) \quad (23) \\
(1 - t_B) \left\{ \theta [2P'(q_B^*) + P''(q_B^*)q_B^*] dq_B^* + [P(q_B^*) + P'(q_B^*)q_B^*] d\theta \right\} & = (dq_A^* + dq_B^*) C''(q_A^* + q_B^*). \quad (24)
\end{align*}$$

Using the definitions of $\frac{\partial^2 \Pi}{\partial q_A^2}$ and $\frac{\partial^2 \Pi}{\partial q_B^2}$ (see the proof of the second-order conditions above),
we rewrite the latter equations:

\[ \frac{\partial^2 \Pi}{\partial q^2} dq_A^* = dq_B^* C''(q_A^* + q_B^*) \]

\[ \frac{\partial^2 \Pi}{\partial q^2} dq_B^* + (1 - t_B) [P(q_B^*) + P'(q_B^*)q_B^*] d\theta = dq_A^* C''(q_A^* + q_B^*) \]

After substituting the optimality condition (8), \((1 - t_B) [P(q_B^*) + P'(q_B^*)q_B^*] = \frac{C'(q_A^* + q_B^*)}{\theta}\), into the latter equality, the solution of the two equations gives

\[ \frac{dq_A^*}{d\theta} = -\frac{C'(q_A^* + q_B^*)}{\theta} \frac{C''(q_A^* + q_B^*)}{\frac{\partial^2 \Pi}{\partial q_A^2} \frac{\partial^2 \Pi}{\partial q_B^2} - [C''(q_A^* + q_B^*)]^2} \]

(25)

Since \( \frac{\partial^2 \Pi}{\partial q_A^2} \frac{\partial^2 \Pi}{\partial q_B^2} - [C''(q_A^* + q_B^*)]^2 > 0 \) by the second-order condition (i) (see (16)) and \( C'(.) > 0 \) by assumption, we conclude that \( \frac{dq_A^*}{d\theta} \) is of the opposite sign of \( C''(q_A^* + q_B^*) \).

**Proof of Corollary 1**

From the view point of the firm, the effect of \( t_B \) is the opposite of the effect of \( \theta \). Symmetric to (25), we derive

\[ \frac{dq_A^*}{dt_B} = \frac{C'(q_A^* + q_B^*)}{1 - t_B} \frac{C''(q_A^* + q_B^*)}{\frac{\partial^2 \Pi}{\partial q_A^2} \frac{\partial^2 \Pi}{\partial q_B^2} - [C''(q_A^* + q_B^*)]^2} \]

(26)

Since \( \frac{\partial^2 \Pi}{\partial q_A^2} \frac{\partial^2 \Pi}{\partial q_B^2} - [C''(q_A^* + q_B^*)]^2 > 0 \) by the second-order condition (i) of the firm’s problem (see (16)) and \( C'(.) > 0 \) by assumption, we conclude that \( \frac{dq_A^*}{dt_B} \) is of the same sign as \( C''(q_A^* + q_B^*) \).

**Proof of Proposition 4**

If the two regions’ tax authorities coordinated their policies, they would set their tax levels by

\[ \max_{t_A, t_B} [W_A(t_A, t_B) + W_B(t_A, t_B)] \]

Cooperative equilibrium taxes, \((t_A^C, t_B^C)\), is the solution to the two first-order conditions:

\[ \frac{\partial W_A}{\partial t_A} + \frac{\partial W_B}{\partial t_A} = 0 \]

\[ \frac{\partial W_A}{\partial t_B} + \frac{\partial W_B}{\partial t_B} = 0 \]

Consider the value of the first-order condition with respect to \( t_A \) at competitive tax levels,
\( t_A^* \) and \( t_B^* \):

\[
\frac{\partial W_A}{\partial t_A} \bigg|_{t_A=t_A^*, t_B=t_B^*} + \frac{\partial W_B}{\partial t_A} \bigg|_{t_A=t_A^*, t_B=t_B^*}
\]

From (10), the first derivative is zero. Since the welfare of region B depends on the tax level A only through its equilibrium quantity, we have

\[
\frac{\partial W_B}{\partial t_A} \bigg|_{t_A=t_A^*, t_B=t_B^*} = \left( \frac{\partial W_B}{\partial q_B} \frac{\partial q_B^*}{\partial t_A} \right) \bigg|_{t_A=t_A^*, t_B=t_B^*}
\]

Moreover, from (11), we get

\[
\frac{\partial W_B}{\partial q_B} \bigg|_{t_A=t_A^*, t_B=t_B^*} = -\frac{\theta P(q_B^*)q_B^*}{\frac{\partial q_B^*}{\partial t_A}}
\]

and therefore

\[
\left( \frac{\partial W_B}{\partial q_B} \frac{\partial q_B^*}{\partial t_A} \right) \bigg|_{t_A=t_A^*, t_B=t_B^*} = -\frac{\theta P(q_B^*)q_B^*}{\frac{\partial q_B^*}{\partial t_A}} \frac{\partial q_B^*}{\partial t_A} < 0
\]

if and only if \( \frac{\partial q_B^*}{\partial t_A} < 0 \), i.e., if and only if the cost function is concave (by Corollary 1). This implies that whenever the cost is concave, we have

\[
\left( \frac{\partial W_A}{\partial t_A} + \frac{\partial W_B}{\partial t_A} \right) \bigg|_{t_A=t_A^*, t_B=t_B^*} < 0
\]

and therefore, starting from the competitive tax levels, the regions would prefer to lower the tax level of region A at cooperative equilibrium: \( t_A^C < t_A^* \). Symmetrically, we can prove that \( t_B^C < t_B^* \) if and only if the cost is concave.
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