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Multiple-Project Financing with Informed Trading*

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Abstract

The paper presents an adverse selection-based explanation of the fact that some entrepreneurs choose to finance multiple projects together by issuing a single security and other entrepreneurs decide to finance each project separately. We consider the financing problem of an entrepreneur who has access to two investment projects and needs to raise external financing to undertake these projects in the presence of asymmetric information. The entrepreneur has private information about the quality of the projects and can choose either to finance the projects together by issuing a single security, or to finance the projects separately by issuing two securities, each backed by the cash flows from the corresponding projects. We show that the choice of financing depends on the structure of information available to outside investors. If there are two types of informed traders and each type knows the true value of a different project, the entrepreneur will always choose to finance projects separately. However, if there is only one type of informed trader in the market and she has information about the true value of both projects, then the entrepreneur may, in some circumstances, resort to joint financing.

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1. Introduction

Consider an entrepreneur who wishes to undertake two investment projects and, because of capital constraints, needs to raise the funds in the external financial markets. The value of the projects that he wishes to finance depends on the entrepreneur's quality, and the entrepreneur's quality is private information. Nevertheless, financial markets can form expectations about the average quality of entrepreneurs and will price the securities accordingly. As a consequence, a good quality entrepreneur, whom we will define more precisely later on in the paper, may discover that the market misprices the securities he issues, and he will suffer from dilution. In this economy, some traders may choose to become informed for a cost and receive a signal about the true value of the projects. As a result of informed trading, the price system will become more efficient, partially (or fully) revealing the true quality of the entrepreneur. Under these conditions, the entrepreneur will choose his financing strategy under which the prices reveal more information. However, to maximize his ex-ante expected wealth, should the entrepreneur issue only one security backed up by the two investment projects? Or should the entrepreneur issue two securities, each security backed up by the cash flows of each project?

This paper presents an asymmetric-information-based explanation of the fact that some entrepreneurs choose to finance multiple projects by issuing a single security (what we refer to as "joint financing") while other entrepreneurs decide to finance each project separately ("separate financing"). We investigate a binary version of the Kyle (1985) model with competitive informed traders, and we show that the choice of financing (joint vs. separate) depends on the structure of information available to outside investors.

If potentially-informed traders have access to two signals and each signal reveals the quality of only one project that the entrepreneur has, we show that the entrepreneur will always choose to finance the projects separately. Under separate financing, informed traders trade in the security whose value is very sensitive to their information. As a consequence of this informed trading, the price system becomes more informative and the true value of each project gets embedded in the stock price. If, instead, the entrepreneur finances the projects jointly, by issuing a single security, the value of the security issued turns out to be less sensitive to the information of both types of informed traders. Consequently, informed traders' profits decrease and the number of informed traders, that the model endogenously determines, decreases. We show, as a result, that the price system reveals less information about the quality of the issuer, and the ex-ante wealth of the good-quality entrepreneur is reduced. For these reasons, the issuer will always prefer to finance projects separately.

However, if potentially-informed traders have access to only one signal, and the signal is informative regarding the quality of the entrepreneur (that is, the signal is informative about the quality of each project), we show that in some circumstances, the good-quality entrepreneur (referred as the *type-h* entrepreneur) may resort to joint financing. This is one of the main contributions of the paper and the intuition can be explained as follows. Under separate financing, if informed traders know the true value of both projects and exactly one project turns out to be of bad quality (we refer to this entrepreneur as the mixed type or the *type-m* entrepreneur), then informed traders will concentrate their activity on the one good project, easing the task of the market maker to price the security at fair value. However, if informed

traders learn that the entrepreneur has two good projects, they will spread their trading across the two securities. This trading separation implies that the price system will incorporate less information, and the separation may force the good-quality entrepreneur to issue securities that in equilibrium will be more undervalued (with respect to the *type-m* entrepreneur with only one good project). Notice that, in this situation, separate financing still increases the informativeness of the price system, but the increase benefits only the *type-m* entrepreneur. When the cost of information production is low, the equilibrium amount of informed trading is so high that the mixed type is revealed in the market, and informed traders do not make any profit when only one project is of good quality. The ex-ante profits and, consequently, the equilibrium number of informed traders are adversely affected by the full revelation of the mixed type. This information spillover effect increases the equilibrium degree of under-pricing of the *type-h* entrepreneur so that, in equilibrium, he may prefer joint financing.

Our paper is related to some of the vast literature on security design. Fulghieri and Lukin (2001) develop a model in which a good quality firm needs to raise capital under asymmetric information and adverse selection. They show that the firm prefers to issue a more information sensitive security (namely, equity) rather than issue a security with low sensitivity to private information (debt). By issuing equity, the firm encourages outside investors to produce private information about the project's value. By trading on this information, these specialized informed investors reveal their information to the market. This paper is, however, different from Fulghieri and Lukin (2001) in several respects. They consider the problem of designing a security (debt vs. equity) backed by a project. We consider the problem of choosing the number of projects backing a security.

Habib et al. (1997) argue that to finance projects separately is always the preferable choice. In the context of a rational expectations model à la Grossman and Stiglitz (1980), they show that when several divisions of a firm are spun off into several firms, the price system becomes more informative because it reduces the uncertainty that risk-averse, uninformed investors have about the value of each single division. As a consequence, their expected demand for shares increases, thus increasing firm value. Habib et al.'s results depend crucially on the assumptions on the change in liquidity trading, because the amount of liquidity trading plays an important role in determining equilibrium prices.¹ The difference between their model and our model is that for Habib et al., resorting to separate financing increases both the informativeness of the price system *and* the expected utility of informed traders, thus driving up the price. By contrast, in our model, this latter effect may not materialize when informed traders get a signal on the value of both projects. If the spillover effects are large enough, the price system becomes more informative only for the *type-m* issuer. As a consequence, the degree of adverse selection and the resulting dilution for high quality firms increases. When this situation occurs, the entrepreneur prefers joint financing.

Chowdhry et al. (2002) use a model of security design based on the principle of information aggregation, and show that firms issue different securities to different groups of investors. Each

¹Notice that, in a model with risk-averse agents, an increase in expected prices doesn't necessarily correspond to an increase in expected utility. See Marin and Rahi (2000), and the so-called *Hirshleifer Effect*. More informative prices worsen risk-sharing opportunities for risk-averse agents, thus lowering their expected utility.

firm issues securities that are highly correlated with the private information of the investor to whom the securities are marketed. In this respect, Chowdhry et al.'s paper is similar to ours since we also show that when there are different types of investors, each type being informed about one particular project, it is always optimal for the entrepreneur to finance projects separately and to issue different securities. Our paper is, however, different in two main respects. First, we consider an economy with a different information structure with respect to theirs, and we show how this difference has a profound impact on the results of the model. Second, the number of informed traders (that, in turn, determines the degree of information production) is endogenous in our model.

DeMarzo (2005) considers first a problem of an informed intermediary willing to sell multiple assets in the presence of adverse selection, and always preferring to sell assets separately rather than as a pool in the context of a signalling model; then he also shows that an originator who is uninformed about the true value of the assets will choose to resort to joint financing (to say it in the spirit of the present paper) because pooling will mitigate the underpricing problem similar to the one described by Rock (1986). On the same lines, but in a different setting, Nanda and Narayanan (1999) present a signalling model in which an undervalued firm splits into component businesses in order to obtain cheaper financing. They assume that the market can observe the aggregate cash flows of the firm, but not the divisional ones. If the informativeness of cash flows from different divisions is different, the firm may be undervalued in the market and willing to signal its true value by raising capital through a costly divestiture. Our model is different - and, hence complementary with respect to these models, because the information structure is different. We assume that though insiders of the firm have private information about the quality of the firm, some investors in the market have more precise signals.

Subrahmanyam (1991) and Gorton and Pennacchi (1993) show that issuing baskets of securities can decrease the losses of liquidity traders caused by informed traders. Baskets of securities have lower volatility than do individual securities. Hence, the ability of informed traders to profit from their private information is diminished. In contrast with their papers, in our model, the entrepreneur decides the financing strategy. That is, the decision is supply driven. In their model, this decision is demand driven.

Additionally, our model is also related to the vast literature on venture capital. Most of the theoretical literature on venture capital considers the financing and incentive problems facing an entrepreneur trying to finance a single project. Notable exceptions include Kanninen and Keuschnigg (2003), Bernile et al. (2007), Fulghieri and Sevilir (2009), Inderst et al. (2007), and Inderst and Müller (2003). Kanninen and Keuschnigg (2003), and in a similar spirit, Bernile et al. (2007) consider an entrepreneur with multiple identical, risky projects and determine the relation between the venture capital portfolio structure and the effort level. Fulghieri and Sevilir (2009) consider the incentive of a venture-capital firm to concentrate its attention on one or two ventures. Inderst et al. (2007) show that the depth of the entrepreneur's financial pockets may help resolve his financing problems. The common thread of these papers is that the authors investigate the span of a venture capital portfolio, or, as in Inderst et al. (2007), investigate the role of the venture capitalist who is cash constrained in an economy where the span of the venture-capital portfolio is fixed. In this paper, we fix the number of projects and we investigate

the security design problem (i.e. whether an entrepreneur has to issue one or two equity securities) in a model with information production and adverse selection. Of all the research, Inderst and Müller (2003) address a problem that is most similar to ours, although with many differences. They, in fact, study the optimal contracting between individual investors and individual project managers (what they refer to as "decentralized financing"), and between individual investors and a headquarter that runs multiple projects backed by the same security (what they refer to as "headquarter financing"). They model a multi-period economy in which the optimal contract offered is debt, address a moral hazard problem, and explore the relation between financing constraints and the organizational structure. We model a one-period economy in which we restrict our attention to equity, consider the adverse selection problem an entrepreneur faces, and explore the effects of information production on financing.

The remainder of the paper is organized as follows. In Section 2, we describe the economy and introduce the model. In Section 3, we show that if the signal reveals the quality of the project, separate financing always dominates joint financing. In Section 4, we assume that the signal reveals the quality of the entrepreneur, and we show that parameter restrictions exist such that separate financing could be optimal. In Section 5, we discuss the results and conclude the paper. In the Appendix, we include the discussion of a model without information production, and a result of financing when the signal reveals the entrepreneur's quality and markets are integrated.

2. The Model

Consider an economy that lasts for one period, and the choices available to an entrepreneur who wishes to undertake two investment projects and, being capital constrained, needs to raise the funds in the external financial markets.

2.1 Players, Actions, and Events

We assume that all agents in the economy are risk neutral and the riskless interest rate is normalized to 0. Agents in this economy are entrepreneurs, market makers, and outside investors (investors who trade on information, and liquidity traders).

At $t=0$, an entrepreneur has access to two different investment projects that are indexed respectively by $i \in \{1, 2\}$. Project i requires an investment of I_i dollars at time $t=0$ and has a value \tilde{v}_i at time $t=1$. Without loss of generality, we assume that $I_1 = I_2 \equiv I$. The realization of \tilde{v}_i can be either v or 0. If $\tilde{v}_i = v$, then the project is of good quality. If the project yields 0 payoff, it is of bad quality. Outside investors and market makers have a common prior-probability distribution over the quality of both projects, according to which the projects are independently and identically distributed with the probability of a project being of good quality given by $p_1 = p_2 = p$. The NPV of each project is positive: $E[\tilde{v}_1] = E[\tilde{v}_2] = pv > I$. Entrepreneurs have private information about the quality of the projects. Namely, they observe a

random variable \tilde{V} that is the sum of the values of project 1 and 2: $\tilde{V} = \tilde{v}_1 + \tilde{v}_2$.² Therefore, we can view \tilde{V} as the entrepreneur's type, in the sense that it shows the ability of the entrepreneur to find profitable investment opportunities. In this sense, a higher value for \tilde{V} implies better entrepreneurial quality. Hence, we model three types of entrepreneurs, denoted by h , m , and l . *Type-h* entrepreneurs observe that $\tilde{V} = 2v$ and learn that both projects are good; *type-m* entrepreneurs observe that $\tilde{V} = v$ and learn that one project is good and the other is bad, but they do not know which project is bad and which project is good; finally, *type-l* entrepreneurs learn from $\tilde{V} = 0$ that both projects are bad.

The entrepreneur is cash-constrained and needs to raise funds in the financial markets. The set of securities the entrepreneur can issue is restricted to equity that he can sell in the form of an IPO. Given that the entrepreneur has access to two investment projects and the set of securities is restricted to equity, the entrepreneur has two financing strategies available. First, he can raise the required funds by issuing one security backed by the cash flows from both projects. Second, the entrepreneur may decide to finance the projects separately. In this case, each project will belong to a different company and the entrepreneur will raise the funds to finance project i by issuing equity of company i . From now on, we will refer to equity of company i as security i . The entrepreneur will choose the financing strategy that maximizes his expected wealth.

There are two types of outside investors: liquidity traders and informed traders. Liquidity traders exert an exogenous dollar demand that is a random variable. In the case of joint financing, the entrepreneur issues a single security and we denote liquidity traders' demand for this security by \tilde{u}_j . In the case of separate financing, we denote liquidity traders' dollar demand for security i by \tilde{u}_i . We impose the following set of assumptions on liquidity traders' demand.

Assumptions

1. *The dollar demands from liquidity traders \tilde{u}_j and \tilde{u}_i are uniformly distribute respectively in $U[0; A]$ and $U[0; a_i]$;*
2. *Liquidity traders' demands \tilde{u}_1 and \tilde{u}_2 are independent;*
3. $a_1 \leq a_2 \leq a$.

These assumptions simply make the model tractable and allow us to obtain closed-form expressions for the variables of interest. Interestingly, and one of the modeling novelties of the paper, we do not impose any restrictions on the amount of noise trading in the case of joint financing relative to the amount of noise trading under separate financing (that is, we do not need

²This restriction on the information structure of insiders rules out partially pooling equilibria when the entrepreneur with only one good project deviates and raises the funds necessary to finance only the good project.

any restriction on the parameter A). Kyle-style models, as the literature generally use, do not allow to obtain results on multiple-project financing that are robust to changes in the relative amount of informed trading, and use restrictive assumptions about the amount of liquidity traders for each financing strategy. The assumption commonly used in the literature is that the variance of liquidity demand is the same. In the present model, the amount of liquidity traders in the case of joint financing can be greater or smaller than the amount of liquidity trading under separate financing. The results are completely independent of this assumption.

Informed traders are atomistic, act as price takers, and have access to the information-production technology. Each potentially informed trader is endowed with $1 + c_i$ dollars. Before trading takes place, but after the entrepreneur announces his financing choice, each potentially informed trader decides whether to become informed at a cost c . She can, then, spend the remaining dollar by buying a fraction of a single security in the case of joint financing, or fractions of each security in the case of separate financing. Competitive market makers have a prior π on the amount of informed traders. They observe the sum of total dollar demands from liquidity and informed traders, and use Bayes' rule to update their expectations regarding the quality of the securities they market. They, therefore, set prices equal to expected values conditional on the observed total order flow (denoted as ω throughout this paper).

2.2 The Equilibrium Concept

We restrict our attention to pooling equilibria, and more specifically, we consider a pooling equilibrium in which *type-h* entrepreneur chooses the financing strategy that minimizes his dilution costs, and *type-m* and *type-l* pool with *type-h* by mimicking his financing strategy. We eliminate equilibria other than pooling by assuming that the *type-l* chooses the strategy that maximizes the expected share of the projects the issuer retains.³ In this case, *type-h* cannot separate because *type-l* will follow the same strategy. An equilibrium in which *type-h* and *type-l* pool and choose one financing strategy and *type-m* separates and chooses the other financing strategy is not possible for the same reason. Thus, the entrepreneur will raise an amount of funds that is exactly equal to the required investment because the *type-h* entrepreneur will issue securities that are undervalued in the market. In the case of joint financing, the entrepreneur will issue equity in the amount necessary to raise $2I$. When the entrepreneur finances projects separately, he raises I by selling security 1 and raises I by selling security 2. The entrepreneur announces whether he will finance the projects jointly, issuing a single security in the amount of $2I$, or he will issue two securities at the beginning of the period, $t = 0$. In equilibrium, the market maker determines the fraction of equity that the entrepreneur issues and sells to outside investors, denoted as γ in the paper in the case of joint financing, or, the fraction of security i , γ_i , in the case of separate financing. In this economy, we define an equilibrium as follows:

³We have assumed for simplicity that the values of both projects are equal to 0 for the *type-l* entrepreneur, and insiders will have zero expected wealth under any financing strategy. Therefore, they are indifferent between joint and separate financing.

Definition

An equilibrium is given by a measure π of informed traders, an amount γ , and an informed traders' break-even condition such that the following three requirements are satisfied:

- a. Market makers' beliefs about the amount of informed traders in the market are rational, that is, $\pi = \pi^*$;
- b. The fraction of equity that the entrepreneur needs to sell in order to raise the required amount of financing I is given by $\gamma = \frac{I}{E[v|\omega]}$, where ω is the total order flow submitted to the market makers;
- c. The ex-ante expected profits of each informed trader are zero.

Depending on the informativeness of the signal, both market makers and informed traders will play different strategies, and the game they play will lead to different equilibria. In the following section, we assume that two types of potentially-informed traders exist. The first type can learn the true value of the first project, and only of the first project, at a cost c_1 . The second type can learn the true value of the second project, and only of the second project, at a cost c_2 . In other words, when this situation occurs, the signal reveals the quality of the project. In Section 4, we assume that only one signal is available for potentially-informed traders and this signal reveals perfectly the quality of the entrepreneur: if an informed trader acquires the signal, she learns the true value of both projects. We show that the results of the model change significantly when a potentially informed trader can observe the true value of both projects.

Finally, we must notice that in the current economy, but in the absence of information production, the choice of financing becomes irrelevant and the entrepreneur is indifferent between separate and joint financing. This result, that we derive more formally in the Appendix, implies that the different conclusions that we reach in what follows regarding the choice of financing can be attributed solely to the effects of information production.

3. The Signal Reveals the Quality of the Project

In this section, we assume that a trader can spend c dollars and becomes informed about the quality of one project. More precisely, a trader of type 1 can spend c_1 dollars and receives a signal that is informative about the first project. A trader of type 2 can acquire a signal about the second project at a cost c_2 . Without loss of generality, we assume that $c_1 = c_2 = c$. Both signals are perfectly informative: after acquiring the information, an informed trader of type i knows the true value of project i . Let $s_i \in \{g, b\}$ be a signal about the quality of project i received by a trader of type i . If $s_i = g$, the signal is good, and $\tilde{v}_i = v$ at $t = 1$. If $s_i = b$, then the signal is bad and $\tilde{v}_i = 0$ at $t = 1$. In this case, the signal reveals the quality of the project, but not the quality of the entrepreneur.

In what follows, we first derive equilibria for the different choices of financing. Then, we compare these equilibria and choose the one that maximizes the ex-ante expected wealth of the *type-h* entrepreneur.

3.1 Separate Financing

Under separate financing, the entrepreneur offers two securities for sale, each backed by the cash

flows from each project. Each security is sold in the corresponding market. We assume that markets are segmented, that is, market makers in one market cannot observe the total order flow in the other market. In what follows, we derive the equilibrium in one market. The equilibrium in the second market will be identical.

3.1.1. First Market

The dollar demand of liquidity traders in the first market is uniformly distributed on $[0, a]$ with density function given as follows (we omit subscripts for notational simplicity):

$$g(u) = \begin{cases} \frac{1}{a}, & u \in [0, a] \\ 0, & u \notin [0, a] \end{cases} \quad (1)$$

At the beginning of the period, a measure π_s of informed traders decides to become informed. Informed traders are atomistic and act as price takers. If the signal is good, they will buy one dollar worth of the security, and the total dollar demand from informed traders will be π_s . If the signal is bad, they do not submit any order.⁴ The demand of liquidity traders u is realized at the same time. Informed and liquidity traders submit their orders simultaneously to market makers. Market makers observe the total order flow $\omega = \pi_s + u$ and use Bayes' rule to update their probability of the project being good, setting the price for the security equal to the conditional expectation of the cash flows of the project.

Let π_s^M represent market makers' beliefs about the equilibrium amount of informed traders in the market. Given uniformly distributed liquidity demand, Bayesian updating takes a simple form. If the total order flow is less than π_s^M , then market makers learn that the demand from informed traders was zero and, hence, the project is bad. In this case, the issue fails. If the total order flow is greater than π_s^M , but less than a , then the posterior probability of the project being good is

equal to the prior probability p (from Bayes' rule: $\Pr(\tilde{v} = v) = \frac{pg(u - \pi_s)}{pg(u - \pi_s) + (1 - p)g(u)} = p$). If

the total order flow is greater than a , market makers correctly infer that there is positive amount of informed trading in the market and the project is good. In equilibrium, market makers' beliefs are rational. That is, $\pi_s^M = \pi_s$. The proportion of equity the firm has to sell $\gamma = \frac{I}{E[v|\omega]}$ is, then, given by:

$$\gamma(\omega, \pi_s) = \begin{cases} \frac{I}{pv}, & \omega \in [\pi_s, a] \\ \frac{I}{v}, & \omega > a \end{cases} \quad (2)$$

⁴Shortselling is not permitted for informed and liquidity traders. Only market makers are allowed to short sell the security in order to absorb net order flow and avoid rationing.

This amount is, therefore, piecewise constant. When the total order flow is low enough, there is no updating, even if the project is good. However, when the realization of the total order flow is high enough, market makers correctly infer that the project is good (full revelation) and the price of the equity is set equal to its true value. Given the pricing rule (2), the ex-ante expected profits of each informed trader are given by:

$$\Pi(\pi_s) = p \frac{E[\gamma(u + \pi_s, \pi_s)]v - I}{I} - c \quad (3)$$

We can rewrite the last expression as:

$$\begin{aligned} \Pi(\pi_s) &= p \left(\frac{\frac{I}{pv}v - I}{I} \Pr(u < a - \pi_s) + \frac{\frac{I}{v}v - I}{I} \Pr(u > a - \pi_s) \right) - c \\ &= p \frac{\frac{I}{pv}v - I}{I} \Pr(u < a - \pi_s) - c \\ &= (1 - p) \Pr(u < a - \pi_s) - c \end{aligned} \quad (4)$$

where the first equality comes from the fact that the second term in parenthesis is zero. When liquidity demand is high enough, the issuer's type is revealed and informed traders do not make any profits.

Since the probability that the project's type is not revealed is equal to $\Pr(u < a - \pi_s) = 1 - \frac{\pi_s}{a}$, we can derive the equilibrium amount of informed traders π_s^* by equating (4) to zero:

$$\Pi(\pi_s^*) = (1 - p) \left(1 - \frac{\pi_s^*}{a} \right) - c = 0 \quad (5)$$

3.1.2. Expected Wealth of the Entrepreneur

The residual fraction of equity that the entrepreneur with the good project retains depends on the realization of the demand of liquidity traders. If this amount is low, no updating occurs on the probability of the project being good and the issuer suffers from dilution. If the realization of the liquidity demand is high enough, the type of the project is fully revealed (as being good) to the market and the entrepreneur sells the security for its true value. We can express the ex-ante expected wealth of the issuer of the good project as:

$$\begin{aligned} U^G &= \left(v - \frac{I}{p} \right) \Pr(u < a - \pi_s^*) + (v - I) (1 - \Pr(u < a - \pi_s^*)) \\ &= \left(v - \frac{I}{p} \right) \left(1 - \frac{\pi_s^*}{a} \right) + (v - I) \frac{\pi_s^*}{a} \end{aligned} \quad (6)$$

Using the zero profit condition in (5) and simplifying, we obtain that the expected wealth of the

entrepreneur in one market, conditional on the project being good, is given by:

$$U^G = v - I - \frac{cI}{p} \quad (7)$$

The expected wealth of the entrepreneur in (7) consists of two terms. The first term, $v - I$, is the full information value of the project. Intuitively, positive-informed trading occurs only when the value of the project is equal to v . Hence, the entrepreneur is able to obtain the full information value of the project while bearing some costs. The second term, the cost borne by the issuer, is the per dollar cost of information production multiplied by the size of the issue and adjusted for the probability that informed traders can use this information to make profits.

3.1.3. Both Markets

In the second market, the expected wealth of the entrepreneur who has a good project is given by the same expression (7). The *type-h* entrepreneur has two projects that are both of good quality. Hence, in the case of separate financing, we have proven the following lemma:

Lemma 1 *When the entrepreneur finances the projects separately, the equilibrium ex-ante expected wealth of the type-h entrepreneur, U_s^h , is given by:*

$$U_s^h = 2U^G = 2(v - I) - \frac{2cI}{p} \quad (8)$$

That is, the equilibrium ex-ante expected wealth of the entrepreneur of the *type-h* is equal to the NPV of the two projects, minus the dilution costs (which are equal to $2cI / p$).

3.2 Joint Financing

In the case of joint financing, the two projects are pooled together and the value of the security that the entrepreneur sells is given by:

$$\tilde{V} = \begin{cases} 2v & \text{with probability } p^2 \\ v & \text{with probability } p(1-p) \\ v & \text{with probability } p(1-p) \\ 0 & \text{with probability } (1-p)^2 \end{cases} \quad (9)$$

The entrepreneur has to raise the amount necessary to finance both projects, that is $2I$. In the previous case (separate financing), a certain number of informed traders were informed about the true value of the securities in each market. But the information they had was homogeneous among traders in that market. In the market for the first security, for example, all informed traders had the same information regarding the first security, and no trader had superior information about the payoff of the second project. The situation is different in the case of joint

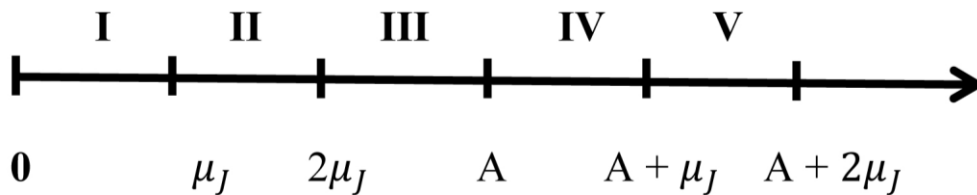
financing: the entrepreneur issues only one security and its payoff depends on the values of both projects. In such a case, two types of informed traders will be trading in equilibrium, and both types have less than perfect information about the true value of the security.

Let μ_{j1}^* and μ_{j2}^* be the equilibrium numbers of informed traders of each type. In the Appendix, we prove the following:

Result *The number of informed traders of the first type is equal to the number of informed traders of the second type. That is, $\mu_{j1}^* = \mu_{j2}^* = \mu_j^*$.*

The demand of liquidity traders is uniformly distributed on $[0, A]$. Market makers, given their rational beliefs, will observe the total order flow and update their expectations about the value of the cash flows. Consider Figure 1. Depending on the total order flow, no updating, full revelation, or partial revelation of the entrepreneur's type will occur.

Figure 1



From Figure 1, we can see that we have five different regions for the total order flow $\omega_j = \mu_{j1} + \mu_{j2} + u_j$. If $\omega_j \in [0, \mu_j^*]$, then market makers infer that both projects are bad and the issue fails. If $\omega_j \in [\mu_j^*, 2\mu_j^*]$, then market makers infer that both projects cannot be good and use Bayes' rule to calculate their expectations regarding the cash flows:⁵

$$E[\tilde{V} | \omega_j \in [\mu_j^*, 2\mu_j^*]] = \frac{2p\Omega \approx p\Theta}{2p\Omega \approx p\Theta \approx \Omega \approx p\Theta} v = \frac{2p}{1 \approx p} v$$

If $\omega_j \in [2\mu_j^*, A]$, then no updating occurs, market makers do not learn any information, and the

⁵Notice that in this case, the firm can be either of *type-m* or *type-l*. Without loss of generality, we assume that the issue fails.

conditional expected value is equal to their prior, i.e.: $E[\tilde{V}|\omega_j \in [2\mu_j^*, A]] = 2pv$. If $\omega_j \in [A, A + \mu_j^*]$, then the market makers infer that both projects cannot be bad and the conditional expected value is given by $E[\tilde{V}|\omega_j \in [A, A + \mu_j^*]] = \frac{2}{2-p}v$. Finally, if the total order flow falls in the region between $A + \mu_j^*$ and $A + 2\mu_j^*$, we have full revelation that both projects are of good type and the conditional expected value will be equal to the true value of the cash flows: that is, $2v$.

Each type's informed traders' profits will depend on the realization of the order flow. The order flow will, in turn, depend on the quality of the projects. If both projects are good, then informed traders of both types will be trading in the market. If, for example, only project 1 is good, then only informed traders who bought a signal about the first project will be present in the market.

Consider the ex-ante expected profits of an informed trader who decides to purchase a signal about the value of the first project. With probability p , she will receive a good signal and will submit a buying order. Her profits are, then, given by:

$$\begin{aligned} \Pi_1(\mu_j^*) = & p^2 \left(\frac{\frac{1}{2pv} 2v - I}{I} \right) \Pr(u_j + 2\mu_j^* \in [2\mu_j^*, A]) \\ & + p^2 \left(\frac{\frac{2-p}{2v} 2v - I}{I} \right) \Pr(u_j + 2\mu_j^* \in [A, A + \mu_j^*]) \\ & + p * (1-p) \left(\frac{\frac{1}{2pv} v - I}{I} \right) \Pr(u_j + \mu_j^* \in [2\mu_j^*, A]) \\ & + p * (1-p) \left(\frac{\frac{2-p}{2v} v - I}{I} \right) \Pr(u_j + \mu_j^* \in [A, A + 2\mu_j^*]) - c \end{aligned} \tag{10}$$

The intuition behind (10) goes as follows. The first two terms represent the per dollar profits when both projects are good. The probability that this condition will be realized is p^2 . The total order flow is, then, equal to $\omega_j = u_j + 2\mu_j^*$ and can fall in regions III, IV, and V of Figure 1. The first term is the per dollar profit when ω_j falls in the third region; the second term is the profit when ω_j falls in the fourth region. When the total order flow falls in the fifth region, the type is fully revealed and the profits of an informed trader are zero. Similarly, the third and the fourth terms of (10) give the per dollar profits when the first project is good, but the second project is bad. The total order flow is, then, $\omega_j = u_j + \mu_j^* \in [\mu_j^*, A + \mu_j^*]$ and can fall in regions II, III, and IV of Figure 1. In region II, the issue is cancelled. The third and the fourth terms represent the profits when the total order flow falls in regions III and IV, respectively. Substituting the values of the probabilities, and simplifying the terms, we can write the zero profit condition for the informed trader as:

$$\Pi_1(\mu_j^*) = \frac{1-p}{2} \left(1 - \frac{\mu_j^*}{A} (2-p^2) \right) - c = 0 \quad (11)$$

From the above zero profit condition, it follows that the equilibrium amount of informed traders μ_j^* is equal to:

$$\frac{\mu_j^*}{A} = \left(1 - \frac{2c}{1-p} \right) \frac{1}{2-p^2} \quad (12)$$

Consequently, the expected wealth of the *type-h* entrepreneur is given by:

$$U_j^h = \left(2v - \frac{2I}{p} \right) \left(1 - \frac{2\mu_j^*}{A} \right) + (2v - 2I(2-p)) \frac{\mu_j^*}{A} + (2v - 2I) \frac{\mu_j^*}{A} \quad (13)$$

The first term corresponds to the wealth of the *type-h* entrepreneur when the total order flow is not informative; the second term, to the case when there is partial revelation; and the third term, to the case when the type is fully revealed to the market. Plugging the expression for $\frac{\mu_j^*}{A}$ from (12) into (13), we obtain the following lemma.

Lemma 2 *The expected wealth of the type-h entrepreneur under joint financing is given by:*

$$U_j^h = 2v - \frac{2I}{2-p^2} \left(3 - 2p + \frac{2c(2-p)}{p} \right) \quad (14)$$

Notice that, as in the case of separate financing, the expected wealth of the *type-h* entrepreneur consists of two terms. However, now the first term, $2v - \frac{(3-2p)}{2-p^2} 2I$, is less than the full information value $(2v - 2I)$, because positive informed trading occurs not only when the issuer is of *type-h*, but also when the issuer is of *type-m*. That is, the price system is less informative. The cost borne by the entrepreneur is, again, the cost of information production per dollar of trade c multiplied by the size of the issue $2I$; however, the probability adjustment is now equal to $\frac{2(2-p)}{2-p^2} \frac{1}{p} > \frac{1}{p}$.

3.3 Choice of Financing When the Signal Reveals the Quality of the Project

We are therefore ready to state our first result:

Proposition 1 *If there are two types of informed traders in the market, and each type is informed about the true quality of only one project, then the entrepreneur will always finance the projects separately.*

Proof. We need to prove that the *type-h* entrepreneur will suffer from less dilution when he finances the projects separately. That is, we need to show that the expected wealth is greater

under separate financing than under joint financing. Using Lemma 2 and the definition of $\frac{\mu_j^*}{A}$ given in (12), we simplify and get:

$$U_j^h = 2v - \frac{2I}{p} \left(1 - \frac{\mu_j^*}{A} (1-p)(2-p) \right) \quad (15)$$

From (12), we have that:

$$\frac{\mu_j^*}{A} < \left(1 - \frac{2c}{1-p} \right) \frac{1}{2-p} \quad (16)$$

Then, using (15) and (16), we obtain:

$$\begin{aligned} U_j^h &< 2v - \frac{2I}{p} \left(1 - \left(1 - \frac{2c}{1-p} \right) (1-p) \right) \\ &= 2v - 2I \left(1 + \frac{2c}{p} \right) < 2(v-I) - \frac{2cI}{p} \\ &= U_s^h \end{aligned} \quad (17)$$

where the first inequality comes from (16), and the last equality comes from Lemma 1. ■

When we have two types of informed traders in the market, each informed about the quality of one project, financing projects separately is a dominating strategy. Pooling projects together reduces the value of information for each type of informed traders and the number of informed traders decreases. Notice, in fact, that by comparing (5) and (12), we see that the condition $\frac{\mu_j^*}{A} < \frac{\pi_s^*}{a}$ holds. That is, the relative amount of informed trading is greater in the case of separate financing than in the case of joint financing. Or equivalently, the price system is more informative in the case of separate financing for a given amount of informed trading. When fewer informed traders trade on their information, the price system becomes less informative, and the degree of adverse selection in the market increases, reducing the expected wealth of the entrepreneur. Therefore, the entrepreneur will always finance the projects separately.

4 The Signal Reveals the Quality of the Entrepreneur

We have assumed so far that two signals are available: one about the first project, and the other about the second project. In this section, we assume that there is only one signal available to potentially-informed traders, and this signal reveals perfectly the entrepreneur's quality. In other words, if an informed trader acquires the signal, she learns the true value of both projects. More formally, the set of available signals consists of one perfectly revealing signal $s = (s_1, s_2)$, where $s_i \in (g, b)$. If $s_i = g$, then project i is good; if $s_i = b$, then project i is bad. A potentially-informed trader can pay a cost c and observes the signal $s \in S = \{(g, g), (g, b), (b, g), (b, b)\}$. It turns out that the results of the model change drastically when the structure of available signals is

different.

4.1 Joint Financing

It is more intuitive to consider first the case when the entrepreneur announces that he will finance both projects by issuing a single security. The payoff structure to this joint firm is given by (9). Let μ_j^* be the number of informed traders in equilibrium. The strategy of each informed trader will now depends on both s_1 and s_2 . If both projects are good, that is, $s = (g; g)$, then the informed trader will buy one dollar worth of security. If both projects are bad, that is, $s = (b, b)$, then the informed trader will not trade. Her decision whether to trade or not when one project is good and the other is bad, i.e. $s \in \{(g, b), (b, g)\}$, will depend on the value of the parameters of the model. We will proceed in the following order. First, we solve the model for the case when informed traders do not buy the security when they learn that $s \in \{(g, b), (b, g)\}$, and then for the case when they do trade when $s \in \{(g, b), (b, g)\}$. Then we state under what parameter values these cases will, indeed, be the equilibria of the model.

Let us first assume that informed traders do not trade when only one project is good. That is, let us assume that each informed trader uses the following equilibrium strategy: buy one dollar worth of the security when both projects are good and do not trade otherwise.

Given this informed traders' strategy, market makers will update their expectations about the value of the cash flows in the following way. If the total order flow ω_j is less than μ_j^* , then both projects cannot be good and the issue fails. If $\omega_j \in [\mu_j^*, A]$, the signal is not informative, and the posterior expected value for the value of the firm is equal to its prior: that is, $E[\tilde{V}|\omega_j] = 2vp$. If the total order flow is greater than A, then both projects are good and the expected value of the firm is given by $E[\tilde{V}|\omega_j] = 2v$. The zero profits condition of the informed traders is given by:

$$\Pi(\mu_j^*) = p^2 \left(\frac{1}{p} - 1 \right) \Pr(u_j + \mu_j^* < A) - c = 0 \quad (18)$$

Informed traders will trade only when both projects are good and will earn positive profits only when the total order flow is below A. The ex-ante expected wealth of the *type-h* entrepreneur is given by:

$$U_{j1}^h = \left(2v - \frac{2I}{p} \right) \Pr(u_j + \mu_j^* < A) + (2v - 2I)(1 - \Pr(u_j + \mu_j^* < A)) \quad (19)$$

Using (18) to express $\Pr(u_j + \mu_j^* < A)$ and substituting it into (19), we find that if, in equilibrium, informed traders trade only when both projects are good, then the ex-ante expected wealth of the *type-h* entrepreneur in the case of joint financing is given by:

$$U_{J1}^h = 2v - 2I \left(1 + \frac{c}{p^2} \right) \quad (20)$$

Note that the result is almost identical to the case of separate financing in the previous section. The only difference is that the cost is divided by p^2 instead of p because the ex-ante probability of submitting an order for an informed trader is now p^2 .

Let us now consider the case when informed traders also submit buying orders when only one project is good. In this case, informed traders will not trade only if both signals are bad. After adjusting the market makers' pricing rule, and following a similar logic as in the previous case, it is straightforward to prove that, if in equilibrium, informed traders do not trade only when both projects are bad, then ex-ante expected wealth of the *type-h* entrepreneur in the case of joint financing is given by:

$$U_{J2}^h = 2v - 2I \left((2-p) + \frac{c}{p} \right) \quad (21)$$

The first term in parenthesis is greater than the first term in (20), reflecting that positive informed trading reveals less information. The second term is less than $\frac{c}{p^2}$ because the probability of positive informed trading is higher. Note that $U_{J1}^h = U_{J2}^h$ when $c = p^2$. This value turns out to be the threshold such that for all values of c below this value, a positive-informed trading equilibrium occurs only when the issuer is of *type-h*, and for all values of c above this threshold, we find a positive-informed trading equilibrium when the issuer is either *type-h* or *type-m*. To prove the following lemma, we have only to check that informed traders' strategies are, indeed, optimal, given market makers' beliefs.

Lemma 3 *If there is a positive amount of informed traders in equilibrium, then for all values of c such that $c \leq p^2$, informed traders trade only when both projects are good, and the ex-ante expected wealth of the type-h entrepreneur in the case of joint financing is given by:*

$$U_{J1}^h = 2v - 2I \left(1 + \frac{c}{p^2} \right) \quad (22)$$

and for all values of c such that $c > p^2$, informed traders do not trade only when both projects are bad, and the ex-ante expected wealth of the type-h entrepreneur in the case of joint financing is given by:

$$U_{J2}^h = 2v - 2I \left((2-p) + \frac{c}{p} \right) \quad (23)$$

4.2 Separate Financing

When the entrepreneur announces that he will finance the two projects separately, informed traders' strategies change in the following way. If only one project is good, then she will trade in this security. If both projects are bad, she will not trade. If both projects are good, she will trade in the security where expected profits are higher. In equilibrium, the expected profits will be the same, and an informed trader will be indifferent between trading in the first or the second security so that half of the informed traders will trade in the first security and the other half will trade in the second security.⁶ As a result, "excessive" informed trading will occur when the entrepreneur is *type-m* and has only one good-quality project, and less informed trading will occur when the entrepreneur is of *type-h*. As we will show, the *type-h* entrepreneur is worse off because of this information spillover effect.

Note that the equilibrium strategy the informed traders use now depends on the values of both projects. Accordingly, we can reasonably assume that market makers for a security, correctly anticipating the behavior of informed players in equilibrium, will observe the total order flow in the other security to learn additional information when setting the pricing rule. In this section, we will start our analysis assuming that markets are segmented and the market makers cannot observe the total order flow in the other market.⁷

We can write the ex-ante expected profits of each informed trader as (we, again, omit subscripts for notational simplicity):

$$\Pi(\mu_j^*) = p^2 \left(\frac{1}{p} - 1 \right) \Pr \left(u + \frac{\mu_s^*}{2} < a \right) + 2p(1-p) \left(\frac{1}{p} - 1 \right) \Pr(u + \mu_s^* < a) - c \quad (24)$$

where the first term represents the trading profits when both projects are good and the number of informed traders in each security is $\frac{\mu_s^*}{2}$, and the second term represents the trading profits when only one project is good and all informed traders trade in this security. Note that, in the above calculations, we assumed that $\Pr(u + \mu_s^* < a) > 0$. In other words, when the equilibrium amount

⁶We implicitly assume that informed traders cannot split their orders. The informed trader then randomizes between trading in the first and the second markets when both projects are good. The probability of choosing each market is the same and equal to 0.5.

⁷For the sake of completeness, in the Appendix we relax this assumption, allowing markets to be integrated, and we show that 1. the main results of this section holds (i.e. for some parameters of the problem, the entrepreneur may resort to separate financing) and 2. contrary to intuition, the expected wealth of the *type-h* entrepreneur can be lower than in the case of segmented markets.

of informed trading is low enough, there is a positive probability to earn non-zero profits, even when only one project is good. From (24), the equivalent condition is $c > \frac{p(1-p)}{2} \equiv c^*$. If the cost of information production is lower than the above threshold c , then the good project is perfectly revealed when the second project is bad, and all informed trading is concentrated in the market with the good project. From now on, we assume that $c < c^*$ and we consider only the case when informed traders do not make any profits when the issuer is of *type-m*. Under this assumption, the effect of the information spillover is of important magnitude, and the degree of adverse selection in the case of separate financing may become so high that the issuer will prefer to finance projects jointly.

If $c < c^*$, the second term of (24) disappears, and the zero profit condition becomes:

$$\begin{aligned}\Pi(\mu_s^*) &= p^2 \left(\frac{1}{p} - 1 \right) \Pr \left(u + \frac{\mu_s^*}{2} < a \right) - c \\ &= p(1-p) \left(1 - \frac{\mu_s^*}{2a} \right) - c = 0\end{aligned}\tag{25}$$

The expected wealth of the *type-h* entrepreneur from financing the first and the second project is the same. To calculate his expected wealth, we consider only one security and multiply this expression by two. That is, the ex-ante expected wealth of the *type-h* entrepreneur becomes:

$$\begin{aligned}U_s^h &= 2 \left(\left(2v - \frac{I}{p} \right) \Pr \left(u + \frac{\mu_s^*}{2} < a \right) + (v - I) \left(1 - \Pr \left(u + \frac{\mu_s^*}{2} < a \right) \right) \right) \\ &= 2v - 2I \left(1 + \frac{1-p}{p} \left(1 - \frac{\mu_s^*}{2a} \right) \right) \\ &= 2v - 2I \left(1 + \frac{c}{p^2} \right)\end{aligned}\tag{26}$$

where the third equality is obtained by substituting (25). We have, therefore, proved that:

Lemma 4 *If a positive amount of informed trading occurs in equilibrium and the cost of information production is below c^* , then ex-ante expected wealth of the type-h entrepreneur in the case of separate financing is given by:*

$$U_s^h = 2v - 2I \left(1 + \frac{c}{p^2} \right)\tag{27}$$

4.3 The Choice of Financing

Whether the entrepreneur chooses separate or joint financing depends, again, on the difference in expected wealth.

Proposition 2 *If there is positive amount of informed trading in equilibrium, the entrepreneur's financing choice will depend on the information production costs:*

1. *When the cost of information production c is low enough, $c \leq \min\{p^2, c^*\}$, then the entrepreneur is indifferent between joint and separate financing.*
2. *When the cost of information production c satisfies $c \in (p^2, c^*)$, then the entrepreneur will finance projects jointly.*

Proof. Let us first consider the case when $c \leq p^2$. In this case, under joint financing and the strategies played in equilibrium, informed traders trade only when both projects are good. Using Lemma 4 and Lemma 5, and more precisely, from (22) and (27), we observe that $U_{J1}^h = U_S^h$, and the entrepreneur is indifferent between joint and separate financing. Now let us consider the case when $c^* \geq c > p^2$, so that the information production cost is at an intermediate level. We assume here that $c^* = \frac{p(1-p)}{2} > p^2$, which is equivalent to $p < \frac{1}{3}$. Using (23) and (27), we can write the difference between U_{J2}^h and U_S^h as:

$$\frac{1}{2I}(U_{J2}^h - U_S^h) = \left(\frac{c}{p^2} - 1\right)(1-p) > 0 \quad (28)$$

given that $c > p^2$. ■

The intuition behind this result is simple. Under both separate and joint financing, informed traders make positive profits only when both projects are good. Recall, in fact, that under joint financing, given that $c \leq p^2$, positive informed trading occurs only when both projects are good; under separate financing when $c \leq c^*$, the *type-m* issuer does not make any profits. As a result, the relative value of the demand from informed traders is the same for both financing strategies. Given market segmentation, the equilibrium in each security market under separate financing is simply a scaled-down version of the equilibrium under joint financing. However, when the information production cost is in the stated range, we have two countervailing forces at play on the relative amount of informed trading. On the one hand, under the parameter restrictions, the *type-h* entrepreneur faces less adverse selection under joint financing. Intuitively, in the case of separate financing when the cost of information production is low enough, informed traders make profits only when both projects are good. However, when the projects are financed jointly, informed traders make profits not only when both project are good, but also when only one project is good. That is, informed traders make profits in more states of the world than in the case of separate financing. The ability to generate profits in more states of the world increases the relative amount of informed trading under joint financing, thus decreasing the dilution costs that cause the issue. On the other hand, under joint financing, positive informed trading occurs not only when the entrepreneur is of *type-h*, but also when he is of *type-m*. Consequently, the market makers cannot separate *type-h* from *type-m* entrepreneurs, even after the high realization of the total order flow reveals that there is a positive amount of informed trading. That is, the market will realize that the entrepreneur is either of *type-h* or of *type-m*, and the entrepreneur of *type-h* continues to be underpriced. From *type-h* entrepreneur's point of view, the inability of the market makers to separate types decreases the relative impact of informed trading on the

informativeness of the price system, thus increasing the dilution costs he will bear. The relative magnitude of these two effects determines the equilibrium financing strategy. It turns out that for $c \in (p^2, c^*)$, the first effect dominates the second, and the entrepreneur prefers joint financing.

5. Conclusion

In this paper, we present an asymmetric-information rationale of why some entrepreneurs choose to finance multiple projects by issuing a single security (joint financing) and other entrepreneurs decide to finance each project separately (separate financing). We show that in a model with asymmetric-information regarding the quality of the projects and *a fortiori* the quality of the securities sold by the entrepreneur, but without information production, the choice of financing is never important and the entrepreneur is indifferent between joint and separate financing. However, we show that when we add information production, the entrepreneur's financing choice will depend on the structure of the signals available to potentially-informed traders. If there are more signals available to potentially-informed traders and each signal reveals the quality of only one project that the entrepreneur has, we show that the entrepreneur will always choose to finance the projects separately. However, if there is only one signal available to potentially-informed traders, and the signal is informative regarding the quality of the entrepreneur (that is, the signal is informative about the quality of each project), we show that in some circumstances, the entrepreneur may resort to joint financing.

6. Appendix

6.1 A Simple Model Without Information Production

In this section, we construct a simple model and show that under the assumption we spell out in the paper, but in the absence of information production, the entrepreneur is indifferent between separate and joint financing. This initial step is important because it shows that we can attribute the results on the choice of financing that we find in the main body of the paper solely to the effects of information production. For what follows, an entrepreneur has access to two projects that are independently and identically distributed. Project i requires investment I_i at the beginning of the period and pays off \tilde{V}_i at the end of the period. The value of \tilde{V}_i can be either good or bad: $\tilde{V}_i \in \{V_i^G, V_i^B\}$. The entrepreneur has no cash and can finance either both projects together, or each project separately by issuing securities in the market. For simplicity, we restrict the set of available securities to equity. The NPV of each project is positive, given all public information available to the market. Only the entrepreneur, however, knows the true value of the projects. Consider an entrepreneur who has to finance two projects which are both of good quality. If he decides to finance them together by issuing only one security, in order to raise the required financing $I = I_1 + I_2$, then he has to sell the proportion γ of equity as follows:

$$\gamma = \frac{I_1 + I_2}{E[\tilde{V}_1] + E[\tilde{V}_2]} \quad (29)$$

If, instead, he decides to finance the projects separately, then he has to sell a fraction:

$$\gamma_i = \frac{I_i}{E[\tilde{V}_i]} \quad (30)$$

of each project. Define W_J and W_S as the expected wealth to the entrepreneur with two good projects in the case of joint and separate financing. We can write these figures respectively as:

$$W_J = (1 - \gamma)(V_1^G + V_2^G) \quad (31)$$

and

$$W_S = (1 - \gamma_1)V_1^G + (1 - \gamma_2)V_2^G \quad (32)$$

It is straightforward to show that the difference in expected wealth is equal to the following expression:

$$W_J - W_S = \frac{I_1 I_2}{E[\tilde{V}_1] E[\tilde{V}_2]} \left(\frac{V_1^G}{E[\tilde{V}_1]} - \frac{V_2^G}{E[\tilde{V}_2]} \right) \quad (33)$$

If, under separate financing, an entrepreneur has to sell a higher proportion of the project which is more underpriced, then he will prefer to resort to joint financing. When the entrepreneur chooses to finance his projects together instead of financing them separately, he sells each dollar of the cash flows from the first project at a higher price and each dollar of the cash flows from the second project at a lower price as compared to the case of separate financing. If, in addition, $\gamma_1 = \gamma_2 = \gamma$, then these two effects exactly offset each other and the entrepreneur is indifferent between the choices of financing. If $\gamma_1 > \gamma_2$, the first effect dominates the second, and the entrepreneur finances the projects together. If $\gamma_1 < \gamma_2$, the second effect is stronger, and he finances the projects separately. That is, under a quite general model, the relative underpricing of each project plays an important role in the choice of financing. When, however, we impose the additional parameter restrictions that we included in the main model, and more precisely, a. $I_1 = I_2 \equiv I$; b. the quality of the project can be either good or bad (with realization v or 0, respectively); and c. $p_1 = p_2 = p$, we find that we can re-write (29), and (30) respectively as:

$$\frac{I_1 I_2}{E[\tilde{V}_1] E[\tilde{V}_2]} = \frac{2I}{2pv} = \frac{I}{pv}$$

and

$$\frac{I_i}{E[\tilde{V}_i]} = \frac{I}{pv} \quad (34)$$

The difference in expected wealth between the two different financing choices is equal to:

$$W_J - W_S = (1 - \gamma)2v - (1 - \gamma)2v = 0 \quad (35)$$

and the choice of financing is never relevant. What we showed is that under the assumptions spelled out in the paper, but in an economy without information production, the choice of financing (joint vs. separate financing) is never important.

6.2 Result. The number of informed traders of the first type is equal to the number of informed traders of the second type. That is, $\mu_{j1}^ = \mu_{j2}^* = \mu_j^*$.*

Proof. Without loss of generality, let us assume that $\mu_{j1}^* \leq \mu_{j2}^*$. Then, the market makers will update their expectations about V after observing the total order ω_j in the following way.

1. When $\omega_j \in [0, \mu_{j1})$, then both projects are bad, and the issue fails;

2. When $\omega_j \in [\mu_{j_1}, \mu_{j_2})$, either both projects are bad, or project 2 is bad;
3. When $\omega_j \in [\mu_{j_2}, \mu_{j_1} + \mu_{j_2})$, both projects cannot be good;
4. When $\omega_j \in [\mu_{j_1} + \mu_{j_2}, A)$, no updating will occur;
5. When $\omega_j \in [A, A + \mu_{j_1})$, both projects cannot be bad;
6. When $\omega_j \in [A + \mu_{j_1}, A + \mu_{j_2})$, project 1 is good;
7. When $\omega_j \in [A + \mu_{j_2}, A + \mu_{j_1} + \mu_{j_2})$, both projects are good.

The ex-ante expected profits of the informed trader with private information on the first project are given by:

$$\begin{aligned} \pi_1(\phi_{j_1}, \phi_{j_2}) &= p \left(1 - \frac{\phi_{j_1} - \phi_{j_2}}{A} \right) \left(\frac{\phi_{j_1}}{A} \right) \left(\frac{1-p}{1-p} \right) \left(\frac{\phi_{j_2} - \phi_{j_1}}{A} \right) \\ &\quad + p \left(\frac{\phi_{j_2} - \phi_{j_1}}{A} \right) \left(\frac{\phi_{j_1}}{2A} \right) \left(\frac{1}{2} \right) \left(1 - \frac{\phi_{j_1} - \phi_{j_2}}{A} \right) \\ &\quad + p \left(\frac{\phi_{j_1}}{2A} \right) \end{aligned}$$

and the profits of the informed trader of the second type are given by:

$$\begin{aligned} \pi_2(\phi_{j_1}, \phi_{j_2}) &= p \left(1 - \frac{\phi_{j_1} - \phi_{j_2}}{A} \right) \left(\frac{\phi_{j_1}}{A} \right) \left(\frac{1-p}{1-p} \right) \left(\frac{\phi_{j_2} - \phi_{j_1}}{A} \right) \\ &\quad + p \left(\frac{1}{1-p} - 1 \right) \left(\frac{\phi_{j_2} - \phi_{j_1}}{A} \right) \left(\frac{1-p}{2p} - 1 \right) \left(\frac{\phi_{j_1}}{2A} \right) \\ &\quad + p \left(\frac{1}{2p} - 1 \right) \left(1 - \frac{\phi_{j_1} - \phi_{j_2}}{A} \right) \left(\frac{\phi_{j_1}}{2A} \right) \end{aligned}$$

Subtracting the second expression from the first, and equating to zero, we have:

$$\pi_1(\phi_{j_1}, \phi_{j_2}) - \pi_2(\phi_{j_1}, \phi_{j_2}) = \left(\frac{1-p}{1-p} \right) \frac{\phi_{j_2} - \phi_{j_1}}{A} = 0$$

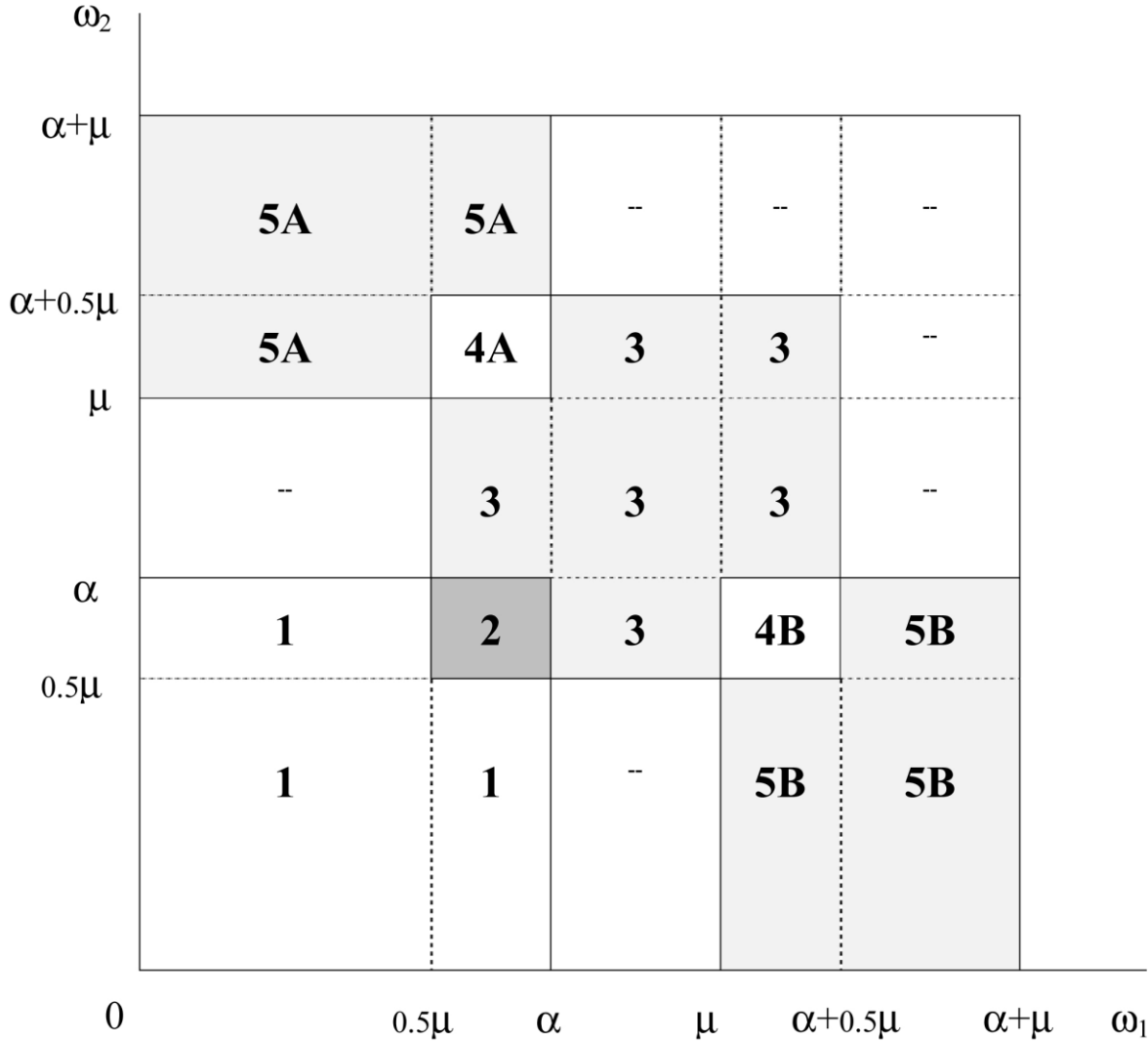
and it follows that the amount of informed traders of different types must be the same in equilibrium. ■

6.3. The Signal Reveals the Quality of the Entrepreneur: The Case of Separate Financing when Markets are not Segmented

We relax the assumption that markets are segmented and assume that market makers can observe the total order flows in both securities. Common intuition would suggest that the *type-h* entrepreneur is likely to be worse off when market makers can observe the total order flow in only one security, but cannot observe the total order flow in the other one. That is, if market makers are able to observe the total order flows in both securities, they may be able to infer more information and will price securities more accurately. We show that the above intuition is not always correct, and we give an example when a *type-h* entrepreneur may be worse off when the markets are integrated.

The strategy of an informed trader will remain the same as in the case of segmented markets: she will trade in the security backed up by the good project and will not trade in the other one when only one project is good; she will not trade in any security if the projects are both bad; and she will randomize between securities with 0.5 probability if both projects are good. The total dollar demand from informed traders in security i will then be 0 if project i is bad, $\frac{\mu_s}{2}$ if both projects are good, and μ_s if project i is good and the other project is bad. Here again we assume, for the sake of simplicity, that the equilibrium measure of informed traders μ_s is greater than a , so informed traders do not make any profits when only one project is good and the information spillover effect is strong. Market makers will update their beliefs about the quality of the projects after observing the total order flows in both securities denoted as $\omega = (\omega_1, \omega_2)$. Their posterior beliefs will depend on the values of both \mathcal{Y}_1 and \mathcal{Y}_2 (see Figure 2).

Figure 2



Proposition 3 *If $\frac{p^2 v}{p^2(1-p)^2} < I$, then the ex-ante expected wealth of the type-h entrepreneur is higher when markets are segmented.*

Proof. Let us assume that market makers can observe the total order flow in both markets $\omega = (\omega_1, \omega_2)$. They will use the following updating rule, given the equilibrium strategy of informed traders. If ω falls in region 1 of Figure 2, then both projects are bad. If ω falls in region 2, then either both projects are bad or both projects are good, and the issue fails. If ω falls in region 3, then both projects are good. If ω falls in region 4A, then the second project cannot be bad. If ω falls in region 4B, then the first project cannot be bad. If ω falls in region 5A, then project 2 is good and project 1 is bad. Finally, if ω falls in region 5B, then project 1 is good and project 2 is bad.

Informed traders will make positive profits from trading when both projects are good and when only ω falls in regions 4A or 4B. Their expected profits are given by:

$$E[\pi_S^U] = p^2 \left(\frac{1}{p} - 1 \right) \left(1 - \frac{K^*}{2a} \right)^2 - c \geq 0$$

The expected wealth of the *type-h* entrepreneur becomes:

$$U_{S,int}^h = 2 \frac{p(I-c)}{p} - \frac{2Ic}{p^2} - \frac{2c}{p(1-p)} = 2I - \frac{2c}{p} - \frac{2c}{p(1-p)}$$

Using (26) and simplifying, we have that: $U_{S,seg}^h - U_{S,int}^h = 2c \left(\frac{pv-I}{p(1-p)} \right) > 0$, and the entrepreneur is better off when markets are segmented. ■

The intuition of this result goes as follows: market makers will update their beliefs about the quality of the projects after observing the total order flows in both markets $\omega = (\omega_1, \omega_2)$. Their posterior beliefs will depend on the values of both ω_1 and ω_2 . For example, if ω falls in region 1 in Figure 2, the market makers infer that both projects are bad and both issues will fail. Notice that now the price system is more informative compared to the case when markets were segmented. However, the *type-h* issuer can be worse off as a result of the increase in the informativeness of the price system. When ω falls in region 2, market makers learn that the issuer cannot be of *type-m*. If *type-m* was strongly underpriced, learning that the issuer is not of *type-m* may bring the expectation of the market makers about the values of the projects to the point where it is less than the amount of the required investment I and the issue fails (this condition is satisfied when $\frac{p^2v}{p^2(1-p)^2} < I$, that is, when the expectation of the value of the project conditional on the fact that the type of the issuer is not m is less than the amount of required investment I). Thus, the *type-h* entrepreneur will not be able to undertake the projects in some states of the world because he will be "pooled" with the *type-l* entrepreneur by market makers. This possibility of pooling is why the *type-h* entrepreneur may be better off under market segmentation.

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Salvatore Cantale, Dmitry Lukin/*The Journal of Entrepreneurial Finance* 16 (2012)

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