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# The Tax-Foundation Theory of Fiat Money

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## Abstract

A government can promote the use of an object as the general medium of exchange by accepting it in tax payments. I prove this old claim in a dynamic model and compare the mechanism to convertibility. The government can often keep its favourite money in circulation even while increasing its quantity and thus causing it to decrease in value. This opens the door for an inflationary policy. Most successful fiat moneys have been acceptable for tax payments, typically due to legal tender laws. Numerous historical failures of fiat moneys are consistent with the theory.

\* dg@drorgoldberg.com. The paper is based on Chapter 1 of my Ph.D. dissertation at the University of Rochester. I thank Per Krusell for his guidance. I benefited from suggestions of Leonardo Auernheimer, Charles Goodhart, Paula Hernandez-Verme, George Selgin, Alan Stockman, Neil Wallace, Randall Wray, Randall Wright, and seminar participants (Rochester, Midwest Macro Meetings, U. of Georgia).

“In practice credible sovereign power—specifically, the ability to enforce the legal tender status of fiat money—is necessary to create the expectations that support a viable fiat money. (Recall that the defeat of the Confederacy rendered Confederate fiat money worthless). ... The reconciliation of theory with ‘facts’ about fiat money remains a central problem in monetary economics.”

Herschel Grossman (1991)

## 1 Introduction

The circulation of inconvertible, intrinsically useless money is a fundamental puzzle in monetary theory. Standard models, such as the random matching model and the overlapping generations model, attribute it to self-fulfilling expectations. In these models there is always an equilibrium in which such money does not circulate because nobody believes that it will. This equilibrium must exist in any model that takes the micro-foundations of such money seriously. Studies have shown that this non-monetary equilibrium can only be eliminated by introducing an external entity, i.e., a government. The government can simply force agents to accept fiat money in trade.<sup>1</sup> Alternatively, if the government itself accepts the money in trade, it can induce agents to do the same.<sup>2</sup> However, the application of either mechanism to modern capitalistic democracies is questionable. In these countries the government’s favourite money is not forced on spot transactions. Although some believe that legal tender laws force money on all transactions, this is clearly not true, as central banks openly admit.<sup>3</sup> As for government sales, in many countries they

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<sup>1</sup>E.g., Lotz and Rocheteau (2002), Selgin (2003).

<sup>2</sup>E.g., Aiyagari and Wallace (1997), Li and Wright (1998).

<sup>3</sup>Legal tender laws are only about settling an obligation that had already come into existence as a result of a previous contract or a tax law. There is nothing illegal in a seller posting an announcement in his shop, saying that he would not accept the domestic legal tender. For official explanations see <http://www.bankofengland.co.uk/banknotes/about/faqs.htm>, <http://www.federalreserve.gov/generalinfo/faq/faqcur.htm#2> . See Goldberg (2008a) for a detailed discussion.

amount to nothing more than sale of postal stamps.

This paper explores another mechanism of government intervention, which appears in every modern legal tender law. The government chooses which objects to accept in tax payments and in turn, affects the value of these objects and their potential to circulate as media of exchange. In order to explore this mechanism, I insert it in a monetary search model. The mechanism is shown to be successful, as long as the government is expected not only to survive but also to maintain an effective tax-collection system. As opposed to convertibility, the government's ability to sustain the money's circulation may be unharmed by excessive money printing.

This theory, succinctly named “the tax-foundation theory” by Ellis (1934, p. 11), had been practiced in China for centuries, reinvented in Massachusetts in 1690, and was briefly discussed by Smith (1776) and Lerner (1947).<sup>4</sup> It is modeled in Starr (1974) in a Walrasian model with a cash-in-advance constraint on consumption, and again in Starr (2003) in a trading post model. In both models, which are static, everyone is taxed with probability one. This paper is novel in its use of a dynamic model to analyze the tax-foundation theory. Thus, it can take into account expectations about the future viability of the government. This is important when analyzing the correlation between a political change and a monetary change. Furthermore, I prove that the theory does not require all agents to be taxpayers at every period. Thus, it is robust to some degrees of tax deferment, tax evasion and tax exemption. The need to reconcile the tax-foundation theory with random matching models is clearly emphasized by Charles Goodhart's critique; he claims that economists are attached to such “nicely constructed models, whatever the facts may be” even though the tax-foundation theory “does far better in explaining and predicting historical reality” (Goodhart [1998], p. 408-9). This paper demonstrates that Goodhart's preferred theory can be well formalized in a variant of the random matching model.

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<sup>4</sup>On China, see von Glahn (1996). On Massachusetts, see Goldberg (2008b). Wray (1998) and Forstater (2006) survey the history of economic thought on this theory.

The mechanism explored here should not be confused with similar ones. It is an ingredient of Knapp's (1905 [1924]) state/chartal theory of money, according to which the government must support the money in many additional ways.<sup>5</sup> The tax-backing theory (Wallace [1981], Sargent [1982], Smith [1985]) and the fiscal theory of the price level focus on the determination of the price level *given* that the money does circulate. In contrast, the present issue is whether any monetary equilibrium is realized *at all*, and the key determinant of that is not the tax rate or deficits, but rather which object is used to pay taxes, and what the penalty is for paying taxes with other objects. To emphasize the different focus of the tax-foundation theory, in my model public deficit can never exist. On the surface, the tax-foundation theory is similar to the legal restrictions theory (Cowen and Kroszner [1994], pp. 148-9). Both theories claim that government-issued fiat money may be valued only because of government regulation. Yet the two theories differ on an important matter as the legal restrictions theory claims that only the government is strong enough to suppress market-created money. The tax-foundation theory has occasionally been seen to imply that markets are too weak to create their own fiat money.

The paper is organized as follows. Section 2 presents the basic model with exogenous prices, a unit upper bound on money holdings, and a fixed money supply. Section 3 introduces a competing outside money or inside money. Section 4 allows multiple money holdings. Section 5 features endogenous prices and an increasing money supply. Section 6 shows that the model can account for important historical episodes. I conclude in Section 7.

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<sup>5</sup>These include: accepting the money for any other payments such as fines, fees, payments for government-produced goods and services, and payments of banks to the central bank; using the money in its purchases; declaring it legal tender in private contractual debts; and fixing the money's exchange rate with the previous domestic money.

## 2 The Basic Model

The goal of this paper is to model the tax-foundation theory in a monetary search model. Such models typically exhibit complete randomness of all meetings between agents; therefore they are also known as random matching models. However, a salient feature of real-life taxation is some lack of randomness. People know when they are going to be taxed and they usually know the terms: how much, where, how and in what medium of payment. They also have enough time to prepare for a tax payment. Some taxes can be completely, and legally, avoided by eschewing certain activities. It is thus important to use a model in which taxation is not completely random.<sup>6</sup> As for the trade meetings, I maintain the same randomness as in random matching models. I do this for the sake of comparison with random matching models of competing mechanisms. In particular, the model is designed to be as close as possible to Li and Wright (1998), who model sales of government goods for fiat money as a way to support that money.

### 2.1 Environment

Time is discrete. A continuum  $[0, 1]$  of infinitely-lived agents are randomly matched in pairs according to a Poisson arrival rate  $\alpha$ . A fraction  $\gamma \in [0, 1]$  of them are potential government buyers and the rest are private agents. Each private agent derives utility  $U > 0$  from the consumption of one indivisible unit of some goods. It can produce only one type of good and it does not consume it. Production of one unit of any (perishable) good is instantaneous and involves disutility  $C \in (0, U)$ .<sup>7</sup> Production is independent of previous consumption, but agents can produce if and only if they do not hold money.<sup>8</sup> The probability that any private agent consumes the good of its trading partner is  $x > 0$ , and the conditional probability

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<sup>6</sup>For a model of the tax-foundation mechanism with completely random taxation, see Goldberg (2002).

<sup>7</sup>While commodity money is ruled out here by assumption of perishability, I do allow it in Goldberg (2002).

<sup>8</sup>This assumption differs from Li and Wright (1998). It allows agents who were just taxed to resume market activities, while keeping one unit of money as an effective upper bound on their holdings.

that the converse holds too is  $y \geq 0$ . The discount rate is  $r > 0$ .

At present, the only durable object is fiat money called a pound bill, which has a storage cost  $c > 0$  and a fixed supply  $M_{\mathcal{L}} \in [0, \gamma]$ .<sup>9</sup> A fraction  $m_p$  of private agents are endowed with one pound each and are called *private buyers*. The other private agents hold nothing and are called *sellers*. Variables that relate to sellers and real goods have a subscript 0.

The potential government buyers participate in the matching process described above. They have the potential to consume all goods, cannot produce anything, and cannot store any real good. These assumptions, starkly different from Aiyagari and Wallace (1997) and Li and Wright (1998), emphasize that the tax-foundation mechanism, unlike convertibility, can work even for a government which is completely parasitic and cannot credibly promise convertibility into real goods. Among the potential government buyers, I focus on the fraction  $m_g$  of them who hold money (one pound each). These are called *government buyers*. When a government buyer is randomly matched with a seller, the former receives a taste shock. With probability  $x$  it wants to consume the good that the seller can produce. The government buyer then offers its money for the seller's good. It *does not* force the seller to trade. If the money is accepted, the government buyer consumes and becomes a potential government buyer. Being moneyless, it can do nothing until it receives a new pound, as described below.

In addition to all these randomly matched agents, there is another class of government agents, called *tax collectors*. They operate outside of the matching process described above. They are idle during trade meetings and operate right after them.<sup>10</sup> The tax collectors are capable of identifying agents who had just produced, say because these sellers show signs of exhaustion. Tax law in the economy states the following:

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<sup>9</sup>As in Aiyagari and Wallace (1997), this upper bound on money simplifies the analysis because then all the money is held by the government when private agents reject it.

<sup>10</sup>For a similar interaction of private agents with the government after trade, see Shi (2005).

1. Only current income is taxed.
2. An agent who just sold is taxed with probability  $\tau \in (0, 1)$ .
3. The size of the tax payment is the entire income just earned.
4. Tax collectors cannot reject tax payments in pound bills.
5. Tax collectors are allowed to choose whether to accept a tax payment in a real good.
6. Agents whose payment is rejected face a non-monetary punishment  $P > 0$ .<sup>11</sup>

After taxes are collected all the proceeds are transferred to the potential government buyers, with each buyer getting one pound at most. Then a new period begins.

This tax law is different from totally random taxation. Agents know that only a sale will make them eligible for paying the tax. They can choose never to be exposed to the tax. They can also choose to make a sale only if it leaves them well prepared for the tax collector's visit (say, sell for pounds but avoid barter). Nevertheless, some randomness ( $\tau < 1$ ) must remain for the sake of analytical tractability.<sup>12</sup> A good by-product of this limited randomness is that it approximates tax evasion, tax exemption, tax deferment and the fact that income taxes are not paid after every single sale. Note that the only existing models of the tax-foundation theory already have  $\tau = 1$  (Starr [1974, 2003]). Let  $G$  denote the subjective probability that private agents assign to the existence of a taxing government in the next period. Denote the expected probability of being taxed as  $t \equiv \tau G$ .

Section 4 of the tax law specified above is a real legal tender law. Explicitly, it imposes an obligation only on tax collectors, and it says nothing about objects other than pound bills. However, its silence regarding other objects means that tax collectors have full discretion whether to accept such objects

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<sup>11</sup>The punishment can be thought of as beating. It is possible to model it as a fine paid in real goods produced by the offender, as in Soller-Curtis and Waller (2000).

<sup>12</sup>If  $\tau = 1$  was allowed, the only way to keep production incentive-compatible would have been to tax only a fraction of one's income. This would have necessitated an increase in the upper bound on the indivisible money holdings and would have precluded closed-form solutions.



(Section 5 of the tax law). For example, in the U.K., the tax authority chooses to accept checks (to be modeled below) but generally refuses to accept real goods. According to basic legal principles, it is the silence of the U.K. legal tender law regarding real goods that empowers the tax authority to make such a decision.

Finally, note that the government always has a balanced budget in the sense that taxation precedes consumption for all government buyers. With no government borrowing, the controversial fiscal theory of the price level is irrelevant to the current discussion.

## 2.2 Strategies

Trade meetings allow sellers to barter in some cases and sell for money in other cases. Let  $\Pi_0$  be the probability that a random seller agrees to barter. Let  $\Pi_{\mathcal{L}}$  be the probability that a random seller accepts money. Let the probabilities  $\pi_0$  and  $\pi_{\mathcal{L}}$  be the best responses of a maximizing seller who is offered a good it consumes, and a pound, respectively. Let  $s \equiv (1 - \gamma)(1 - m_p)$  be the proportion of sellers. Let  $T_0$  indicate whether tax collectors accept real goods ( $T_0 = 1$ ) or not ( $T_0 = 0$ ). If  $V_0$  and  $V_{\mathcal{L}}$  are the value functions of sellers and private buyers respectively, then

$$rV_0 = \alpha s x y \Pi_0 \cdot \max_{\pi_0} \{U - C + t[T_0(-C) + (1 - T_0)(-P)]\} + \alpha M_{\mathcal{L}} x \cdot \max_{\pi_{\mathcal{L}}} \pi_{\mathcal{L}} [-C + (1 - t)(V_{\mathcal{L}} - V_0)], \quad (1)$$

$$rV_{\mathcal{L}} = -c + \alpha s x \Pi_{\mathcal{L}} (U + V_0 - V_{\mathcal{L}}). \quad (2)$$

In (1) a seller has two interesting matching possibilities. It may have double coincidence of wants with another seller. In this model the choice of barter is not trivial. Barter, which directly yields  $U - C$ , also makes both agents eligible for paying income tax. With probability  $t$  a bartering agent meets a tax collector. If the tax collector agrees to accept the agent's produce then the agent produces. Otherwise, it

faces the punishment  $P$ . If the seller meets any buyer (public or private), it chooses whether to accept a pound. If it does, then it produces and gets  $\mathcal{L}1$ . Again, this makes the seller eligible for taxation. With probability  $t$  it is taxed of all its money so it remains a seller. With probability  $1 - t$  it is not taxed so it becomes a buyer. Note that because agents can choose  $\pi_0 = \pi_{\mathcal{L}} = 0$ , we have  $V_0 \geq 0$ . No matter how large  $P$  is, agents can choose not to be exposed to the punishment.

In (2) a buyer pays the storage cost of money and it can buy a good if the seller it meets accepts money. Following Li and Wright (1998),  $\alpha x$  is henceforth normalized to 1. Assume that agents want to trade if and only if the trade would strictly increase their lifetime utility.

## 2.3 Equilibrium

The analysis is restricted to symmetric, pure strategy, stationary, *non-autarkic* equilibria.

**Definition 1.** A pure monetary equilibrium is an equilibrium in which all sellers accept fiat money but do not barter.

**Definition 2.** A monetary equilibrium is an equilibrium in which all sellers accept fiat money and barter.

**Definition 3.** A non-monetary equilibrium is an equilibrium in which all sellers barter but reject fiat money.

At the heart of the model stand two decisions. First, the tax collectors decide which objects are to be received in tax payments in addition to pound bills. Second, the legislature determines the punishment for those whose tax payment is rejected.

**Definition 4.** An object  $i$ ,  $i \in \{0, \mathcal{L}\}$ , is *tax-receivable* iff  $T_i = 1$ .

The tax law already stated that  $T_{\mathcal{L}} = 1$ .

It is easy to discourage barter (and production in general), by simply setting a high enough probability of taxation. The point of this paper, however, is not to describe such a mechanism, but to relate to the

discrimination regarding tax-receivable objects. The focus here is not on the probability of payment but rather the *medium* of payment. Assumption 1 below ensures that without such discrimination, barter—and therefore the non-monetary equilibrium—can exist.

**Assumption 1.**  $t < U/C - 1$ .

The first result is that if all objects are tax-receivable, money may not circulate at all.

**Proposition 1.** If all objects are tax-receivable, then: (i) the non-monetary equilibrium exists; (ii) the monetary equilibrium exists for some parameter values; (iii) a pure monetary equilibrium does not exist.

**Proof.** (i) Set  $T_0 = \Pi_0 = 1$  and  $\Pi_{\mathcal{L}} = 0$  in (1) and (2). Clearly,  $\pi_{\mathcal{L}} = 0$ , while Assumption 1 implies  $\pi_0 = 1$ . With barter and rejection of fiat money, we have the non-monetary equilibrium. (ii) Set  $T_0 = \Pi_0 = \Pi_{\mathcal{L}} = 1$ . Then  $-C + (1 - t)(V_{\mathcal{L}} - V_0) > 0$  (so  $\pi_{\mathcal{L}} = 1$ ) iff  $c < c_{\exists}$ , where  $c_{\exists} \equiv s(1 - y)U + [sy(1 + t) - \frac{r+s}{1-t}]C$ , and  $s = 1 - \gamma - M_{\mathcal{L}}(1 - t)$ . (iii) Assumption 1 implies  $\pi_0 = 1$ .  $\square$

As usual, fiat money's circulation depends on both its intrinsic properties and agents' beliefs.

## 2.4 Policy

The government may be able to affect existence of equilibria by discriminating between various objects. Specifically, the government can make the pound bill the *only* tax-receivable object.

**Theorem 1.** The government can guarantee the *existence* and *uniqueness* of the pure monetary equilibrium iff  $t > 0$  and the money's storage cost is small enough.

**Proof.** Set  $T_0 = 0$  and  $P = (U - C)/t$ . This implies  $\pi_0 = 0$  so there is no barter. This rules out the non-monetary and monetary equilibria for all parameter values. Setting  $\Pi_{\mathcal{L}} = 1$  implies that  $\pi_{\mathcal{L}} = 1$  iff  $c < c_u$ , where  $c_u \equiv sU - \frac{r+s}{1-t}C$ .  $\square$

Without the policy, money may circulate only if  $c < c_{\exists}$ . With policy, money circulates for sure iff  $c < c_u$ . It is always the case that  $c_{\exists} < c_u$ , which means that in some cases policy enables money to

circulate when it otherwise would not. Looking at  $P$  in the proof, it is clear that the lower the probability of meeting tax collectors in the future, the higher the minimal punishment needs to be.<sup>13</sup>

It is important to compare the tax-foundation mechanism to convertibility. The latter is a commitment of the issuer to convert paper money into gold or any other good or service. In the tax-foundation mechanism the government does not give anything useful for paper money, but it does give something *harmful* if one does *not* have paper money when it is time to pay taxes. One might say that here paper money buys immunity from punishment, or that it is implicitly convertible into immunity. The mechanism is therefore somewhat analogous to convertibility. However, it is not an equivalent. It involves the government by definition, because only the government taxes. Convertibility, on the other hand, has been practiced by both governments and a wide variety of private entities. Close to convertibility are the models of the government as a seller of goods (Aiyagari and Wallace [1997] and Li and Wright [1998]). In both these models and the current one, more interaction with the government implies that more private agents face an exogenous pro-money behaviour, which inspires general circulation of money. The difference is that here the government's crucial role is modeled as a tax collector. In a modern economy the government sells very few goods in the marketplace, but its taxation is considerable.

In fact, this model does not critically depend on any government involvement in, or monitoring of, trade: the government does not convert money into real goods, it does not directly force agents to trade with each other with fiat money (barter in itself is legal), and it does not force sellers to accept fiat money from government buyers. Here, the only role that the government has in trade is a technical one: to return the money that it collects to the economy through its buyers, as real-life governments do.<sup>14</sup>

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<sup>13</sup>I vary only the punishment and keep the tax rate fixed, since tax rates today are usually determined by the fiscal needs of the government, rather than by the need to support monetary equilibria. It is the insistence that these taxes be paid in fiat money and the associated punishment, that can serve monetary equilibria as a positive externality. In colonial America many tax rates were determined so as to deliberately support monetary equilibria (Brock [1975]).

<sup>14</sup>Instead, it could be assumed that the government destroys the collected money and then injects new money by helicopter

Is the policy optimal?

**Proposition 2.** (i) The pure monetary equilibrium has lower welfare than the monetary equilibrium.  
(ii) If  $y$  is small enough, the pure monetary equilibrium is better than the non-monetary equilibrium.

The proof is trivial. The result is a trade-off, very similar to the one in Aiyagari and Wallace (1997). In that model, the government supports money by refusing some barter opportunities. This means that some welfare is lost, compared to a monetary equilibrium without such policy. However, the resulting monetary equilibrium may be better than the non-monetary equilibrium that could have existed if the policy had not existed. The only difference is that the foregone barter opportunities in this model are always between private agents. The policy induces them to give up those trades, but guarantees the use of money. Monetary trade is more likely to be the optimal form of trade if direct barter is difficult (i.e.,  $y$  is small).

One more policy should be considered, because it relates to the historical discussion below. Suppose that the tax is denominated in goods rather than money. In particular, it is set at one good per taxpayer. In addition to accepting goods, tax collectors also accept money, but only according to its market value. If an agent pays in money and the money's market value is zero ( $\Pi_{\mathcal{L}} = 0$ ) then the tax collector forces this agent to produce one more good as payment. A seller's value function is now

$$rV_0 = sy\Pi_0 \cdot \max_{\pi_0} \pi_0 [U - C(1+t)] + M_{\mathcal{L}} \cdot \max_{\pi_{\mathcal{L}}} \pi_{\mathcal{L}} [-C - t(1 - \Pi_{\mathcal{L}})C + (1-t)(V_{\mathcal{L}} - V_0)]. \quad (3)$$

**Proposition 3.** A policy of accepting money for taxes at market value is ineffective.

**Proof.**  $\pi_0 = 1$  because there is no punishment. For there to be any equilibrium at all, set  $\Pi_0 = 1$  as well. Setting  $\Pi_{\mathcal{L}} = 0$  replicates the qualitative outcome of Proposition 1(i), while  $\Pi_{\mathcal{L}} = 1$  exactly replicates Proposition 1(ii). Proposition 1(iii) holds here too.  $\square$

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drops; the results would remain unchanged.

The key point with regards to government intervention is that it introduces exogenous behaviour, which is unrelated to endogenous market expectations. Once the policy itself succumbs to these expectations and accommodates them, it has no hope of any real impact.

### 3 Competing Currencies

In the real world pound bills compete not only with barter but also with other currencies. I will now turn to examine another kind of outside money (euro, denoted  $e$ ) and inside money (banknote, denoted  $b$ ). Let  $M_i$ ,  $V_i$ ,  $T_i$ ,  $\pi_i$ , and  $\Pi_i$ ,  $i \in \{0, \mathcal{L}, e, b\}$ , be the obvious generalization of the above notation. I consider these competing currencies one at a time.

#### 3.1 Another Fiat Money

Suppose that euro bills have the same physical properties of pound bills, including the storage cost  $c$ . To simplify the analysis, I assume that currency trading of pounds for euros is impossible. Due to the symmetry between agents and between currencies, such trading could not be mutually beneficial in equilibrium, and thus would not occur in any case. The value functions are now determined as follows.

$$rV_0 = sy\Pi_0 \cdot \max_{\pi_0} \{U - C + t[T_0(-C) + (1 - T_0)(-P)]\} + M_{\mathcal{L}} \cdot \max_{\pi_{\mathcal{L}}} [-C + (1 - t)(V_{\mathcal{L}} - V_0)] + \quad (4)$$

$$M_e \cdot \max_{\pi_e} [-C + t(1 - T_e)(-P) + (1 - t)(V_e - V_0)],$$

$$rV_i = -c + s\Pi_i(U + V_0 - V_i), \quad (5)$$

for  $i = \mathcal{L}, e$ .

**Proposition 4.** If all objects are tax-receivable and  $c < c_{\exists}$  there are four equilibria coexisting: a non-monetary equilibrium; a monetary equilibrium with the pound as the unique money; a monetary equilibrium with the euro as the unique money; and a monetary equilibrium with both pounds and euros as money. There is no pure monetary equilibrium.

**Proof.** Essentially identical to the proof of Proposition 1.

Suppose that the government wants to promote the use of pounds as money.

**Theorem 2.** The government can guarantee the *existence* and *uniqueness* of the pure monetary equilibrium with pounds iff  $t > 0$  and the money's storage cost is small enough.

**Proof.** Set in (4)  $T_0 = T_e = 0$ . Setting  $P = (U - C)/t$  is sufficient to rule out barter (and therefore all non-monetary and monetary equilibria). It is also sufficient for ruling out the pure monetary equilibrium in which the euro is the unique money. A possibly higher punishment ( $P = \frac{(1-t)(sU-c)-C(r+s)}{t[r+s+(1-t)M_{\mathcal{L}}]}$ ) is needed to rule out the pure monetary equilibrium in which both pounds and euros are money. This leaves only the pure monetary equilibrium with the pound as the unique money.  $\square$

In general, if there are  $n$  types of intrinsically useless objects with the same low storage cost, then without government intervention there are  $\sum_{x=1}^n \binom{n}{x}$  pure monetary equilibria and the same number of monetary equilibria. The non-monetary equilibrium also exists if the government does not intervene, so overall there are  $2^{n+1} - 1$  equilibria. The government can make any of the  $n$  objects the unique money in a unique pure monetary equilibrium.

### 3.2 Inside Money

Instead of euros let us now assume that there is another entity in the economy called a bank. It has a fixed location which is costlessly accessible to all agents between trading and taxation. The bank is monitored by the government and can therefore make commitments. It has a unique technology which enables it to produce banknotes, which are durable, indivisible and are not consumed. Their advantage over pound

bills is that they have no storage cost. This advantage represents the convenience of banknotes compared with gold coins in the past, and the convenience of checks and electronic money compared with pound bills today.

An agent who holds a pound bill after a trade round can go to the bank, deposit the pound bill, and receive a lighter banknote instead. In return for this service the agent has to produce for the bank.<sup>15</sup> The bank consumes all the real goods. Any agent holding a banknote can go to the bank after a trade round and try to convert it into a pound bill. There is no cost to such conversion. The bank keeps a 100% reserve ratio but might vanish with probability  $1 - R$ , where  $R \in [0, 1)$ . Therefore, conversion succeeds with probability  $R$ .

Let  $\pi_{ij}^S$ ,  $i, j \in \{\mathcal{L}, b\}$ , be the strategy of a seller who just earned some money  $i$  and chooses whether to convert it at the bank into money  $j \neq i$ . Let  $\pi_{ij}^B$  be the similar strategy of a buyer who had money  $i$  before the last trade round and failed to spend it. Due to the increased complexity of the model, it is useful to let  $y = 0$  here, and to assume that buyers cannot swap a pound and a banknote among themselves. These simplifications do not affect the results. The value functions are now

$$rV_0 = (M_{\mathcal{L}} - M_b) \cdot \max_{\pi_{\mathcal{L}}, \pi_{\mathcal{L}b}^S} \pi_{\mathcal{L}} \{-C + (1-t)(V_{\mathcal{L}} - V_0) + \pi_{\mathcal{L}b}^S [-C + t(1-T_b)(-P) + (1-t)(V_b - V_{\mathcal{L}})]\} + \quad (6)$$

$$M_b \cdot \max_{\pi_b, \pi_{b\mathcal{L}}^S} \pi_b \{-C + \pi_{b\mathcal{L}}^S R(1-t)(V_{\mathcal{L}} - V_0) + (1 - \pi_{b\mathcal{L}}^S R)[t(1-T_b)(-P) + (1-t)(V_b - V_0)]\},$$

$$rV_{\mathcal{L}} = -c + s\Pi_{\mathcal{L}}(U + V_0 - V_{\mathcal{L}}) + (1 - s\Pi_{\mathcal{L}}) \cdot \max_{\pi_{\mathcal{L}b}^B} \pi_{\mathcal{L}b}^B (-C + V_b - V_{\mathcal{L}}), \quad (7)$$

$$rV_b = s\Pi_b(U + V_0 - V_b) + (1 - s\Pi_b) \cdot \max_{\pi_{b\mathcal{L}}^B} \pi_{b\mathcal{L}}^B R(V_{\mathcal{L}} - V_b). \quad (8)$$

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<sup>15</sup>The seller can produce only after it gives the pound to the bank and before receiving the banknote.



In (6) the probability of meeting a pound holder depends on how many pounds are stored at the bank (i.e., how many banknotes are outstanding). A seller who just earned a pound can exchange it for a banknote by producing for the bank. If it is later taxed and the tax collector refuses to accept banknotes then it is punished. If it accepts a banknote in trade (second line of (6)) it can try to redeem it at the bank for a pound bill. The motivation for this is the possibility that banknotes are not tax-receivable. Not only a recent seller can deposit a pound at the bank: in (7) a buyer who fails to buy with a pound can deposit it. In (8), a buyer holding a banknote can try to convert it into a pound. Since barter is ruled out by assumption, only pure monetary equilibria are possible.

**Proposition 5.** If all objects are tax-receivable, then: (i) the pure monetary equilibrium with pounds exists for some parameter values. (ii) for another set of parameter values there is a pure monetary equilibrium in which banknotes circulate and are used in tax payments, while all the pound bills—although not rejected in trade—are actually always at the bank. (iii) if pounds are rejected in trade the pure monetary equilibrium with banknotes may or may not exist.

**Proof.** (i) Set in (6)-(8)  $T_b = \Pi_{\mathcal{L}} = 1$ ,  $\Pi_b = 0$ . Banknotes are rejected in trade ( $\pi_b = 0$ ) iff  $c \leq sU - \frac{(R+r)[r+s+(1-t)M_{\mathcal{L}}]M_{\mathcal{L}}C}{[r+(1-t)M_{\mathcal{L}}][(1-t)(1-R)M_{\mathcal{L}}-R(1+r)]}$  and  $R < \frac{(1-t)M_{\mathcal{L}}}{(1-t)M_{\mathcal{L}}+1+r}$ . It is optimal to convert banknotes into pounds ( $\pi_{b\mathcal{L}}^S = \pi_{b\mathcal{L}}^B = 1$ ) iff  $c < sU - \frac{sM_{\mathcal{L}}C}{r+(1-t)M_{\mathcal{L}}}$ . This condition is sufficient to prevent depositing of pounds ( $\pi_{\mathcal{L}b}^S = \pi_{\mathcal{L}b}^B = 0$ ). The rest is exactly as in Proposition 1(ii). (ii) Set  $T_b = \Pi_{\mathcal{L}} = \Pi_b = 1$ . Banknotes are accepted in trade iff  $sU > \frac{r+s}{1-t}C$ .<sup>16</sup> Pounds are deposited by sellers iff  $c > \left(\frac{r+t}{1-t} - s\right)C$ . This condition is sufficient for a buyer with a pound to deposit it. It is not optimal to convert banknotes into pounds. (iii) With pounds rejected in trade, equations (6) and (8) are essentially the same as (1) and (2), only that the notation "b" replaces " $\mathcal{L}$ " everywhere, there is no barter, and there is no storage cost. Thus Proposition 1 applies.  $\square$

**Theorem 3.** The government can guarantee the existence and uniqueness of the pure monetary

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<sup>16</sup>This is the same condition from Proposition 1(ii), only here  $y = 0$  and banknotes have no storage cost.

equilibrium with pounds iff  $t > 0$  and the money's storage cost is small enough.

**Proof.** Set  $T_b = 0$  and  $\Pi_{\mathcal{L}} = 1$ . It is easy to see in (6) that regardless of the value of  $\Pi_b$ , a high enough  $P$  results in rejection of banknotes in trade and no depositing of pounds. The rest is the same as in Theorems 1 and 2.  $\square$

As with any piece of paper, the circulation of a banknote or a check depends on agents' beliefs. The tax law does not specifically address these banknotes, therefore tax collectors have full discretion whether to accept them. If they choose to reject them, they can drive the banknotes out of circulation, but they can also choose to accept them and not disrupt this efficient use of a lighter medium of exchange.<sup>17</sup>

## 4 Multiple Money Holdings

An obvious shortcoming of the model is that agents never hold more than one object at a time. One might suspect that allowing more flexibility would allow agents to diversify their portfolios or at least be flexible about what they accept in payment. Assume then that agents can produce only if they hold up to one object of any type. This effectively increases the upper bound on money holdings from one unit to two units. Prices are still fixed for now at 1.

The point can be made by considering the economy with barter and only pound bills as potential money. Let  $m_i$ ,  $i \in \{0, \mathcal{L}, \mathcal{L}2\}$ , be the fraction of private agents holding  $i$ . For  $i, j \in \{0, \mathcal{L}\}$ , let  $\Pi_{ij}$  be the probability that a random seller holding  $i$  accepts  $j$ , and let  $\pi_{ij}$  be the best response of an agent who holds  $i$  and is offered  $j$ . Let  $V_{\mathcal{L}2}$  be the value function of an agent holding  $\mathcal{L}2$ . The value functions are

$$rV_0 = (1 - \gamma)(m_0\Pi_{00} + m_{\mathcal{L}}\Pi_{\mathcal{L}0})y \cdot \max_{\pi_{00}}\{U - C + t[T_0(-C) + (1 - T_0)(-P)]\} + \quad (9)$$

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<sup>17</sup>The assumption  $R < 1$  is critical. If the costless redemption is also riskless, then agents never reject banknotes even if banknotes are not tax-receivable. The reason is timing: agents can go to the bank between trade and taxation and convert a banknote into a tax-receivable pound bill.

$$[M_{\mathcal{L}} - (1 - \gamma)m_{\mathcal{L}2}] \cdot \max_{\pi_{0\mathcal{L}}} \pi_{0\mathcal{L}} [-C + (1 - t)(V_{\mathcal{L}} - V_0)],$$

$$rV_{\mathcal{L}} = -c + (1 - \gamma)(m_0\Pi_{00} + m_{\mathcal{L}}\Pi_{\mathcal{L}0})y \cdot \max_{\pi_{\mathcal{L}0}} \pi_{\mathcal{L}0} [U - C + t(V_0 - V_{\mathcal{L}})] + \quad (10)$$

$$[M_{\mathcal{L}} - (1 - \gamma)m_{\mathcal{L}2}] \cdot \max_{\pi_{\mathcal{L}\mathcal{L}}} \pi_{\mathcal{L}\mathcal{L}} [-C + (1 - t)(V_{\mathcal{L}2} - V_{\mathcal{L}})] +$$

$$(1 - \gamma)(m_0\Pi_{0\mathcal{L}} + m_{\mathcal{L}}\Pi_{\mathcal{L}\mathcal{L}})(U + V_0 - V_{\mathcal{L}}),$$

$$rV_{\mathcal{L}2} = -2c + (1 - \gamma)(m_0\Pi_{0\mathcal{L}} + m_{\mathcal{L}}\Pi_{\mathcal{L}\mathcal{L}})(U + V_{\mathcal{L}} - V_{\mathcal{L}2}). \quad (11)$$

In the first line of (9) a seller meets a barter partner who has either  $\mathcal{L}0$  or  $\mathcal{L}1$ . The second line is a meeting with a buyer who has at least  $\mathcal{L}1$ . In (10) an agent with  $\mathcal{L}1$  has three interesting matches. First, it may meet a barter partner. In this case, if it makes a sale it may be taxed and then it ends up with no money. Second, it may make a monetary sale and accumulate another pound. Third, it may spend the money in shopping. In (11) the agent suffers the storage cost twice because it holds two pound bills. It can only buy.

**Proposition 6.** (i) The government can guarantee that the non-monetary equilibrium does not exist iff  $t > 0$ . (ii) For some parameter values the government can guarantee the existence of a monetary equilibrium. (iii) The government cannot guarantee the existence of a pure monetary equilibrium.

**Proof.** (i) Set  $T_0 = 0$ . By setting  $P = (U - C)/t$  as before, moneyless agents will not barter ( $\pi_{00} = 0$ ). (ii) Also set  $\Pi_{00} = 0$ ,  $\Pi_{\mathcal{L}0} = \Pi_{0\mathcal{L}} = \Pi_{\mathcal{L}\mathcal{L}} = 1$ . There is no closed-form solution because the distribution of money is too complicated. However, it can be verified that indeed  $\pi_{\mathcal{L}0} = \pi_{0\mathcal{L}} = \pi_{\mathcal{L}\mathcal{L}} = 1$  for the following parameter values:  $U = 5$ ,  $C = 1$ ,  $M_{\mathcal{L}} = .5$ ,  $r = .01$ ,  $\gamma = .1$ ,  $t = .25$ ,  $y = .3$ ,  $c = .01$ . This means

that agents always accept money, but they barter only if they already have money. (iii) No matter how high the punishment is, an agent who already has  $\mathcal{L}1$  may still barter. The reason is apparent from (10): such an agent barterers ( $\pi_{\mathcal{L}0} = 1$ ) iff  $U - C + t(V_0 - V_{\mathcal{L}}) > 0$ . It cannot be punished. Although it does not earn money during this barter sale, it already has a pound bill to begin with, so it can use that bill to pay the tax.  $\square$

These results obviously generalize to any larger upper bound on money holdings. For the same reason, the government cannot guarantee equilibria without other outside or inside moneys. An agent who already holds a pound bill may trade in any other way and pay that bill as a tax. On the other hand, the government can at least make sure that the moneyless agents accept pound bills in sales. The result is that the government's favourite money still circulates, but not exclusively.

## 5 Endogenous Prices

Following Li and Wright (1998), I proceed by making goods divisible, while keeping everything else as in Section 2. Along with checking the robustness of the previous results, this also serves to show the robustness of the tax-foundation mechanism to money printing.

### 5.1 Robustness of Results

As in Li and Wright (1998) and similar papers, a private agent now derives utility  $u(q) \geq 0$  from consuming a quantity  $q$  of one of its preferred goods, and the cost of producing a quantity  $q$  is  $q$ . Also  $u(0) = 0$ ,  $u'(q) > 0$  and  $u''(q) < 0$  for all  $q > 0$ . In meetings between sellers the bargaining power is equal, so both sides produce the efficient  $q^*$  which satisfies  $u'(q^*) = 1$ . In meetings between buyers and sellers, the buyers (whether private or public) make take-it-or-leave-it offers. The quantity produced in all other matches is denoted  $Q$ . The only new notation, compared with the literature, is  $q^t$ , which is the quantity

that may be produced for the tax collector after a barter meeting. The value functions are now

$$rV_0 = sy\Pi_0 \cdot \max_{\pi_0} \{u(q^*) - q^* + t[T_0(-q^t) + (1 - T_0)(-P)]\} + M_{\mathcal{L}} \cdot \max_{\pi_{\mathcal{L}}} [-q + (1 - t)(V_{\mathcal{L}} - V_0)], \quad (12)$$

$$rV_{\mathcal{L}} = -c + s\Pi_{\mathcal{L}}[u(Q) + V_0 - V_{\mathcal{L}}]. \quad (13)$$

Assume that  $q^t = V_{\mathcal{L}} - V_0$ . This makes the utility loss from tax identical for all taxpayers, regardless of how they pay.

**Proposition 7.** If all objects are tax-receivable, then: (i) the non-monetary equilibrium always exists. (ii) there are two monetary equilibria, which exist only for some parameter values. (iii) there is no pure monetary equilibrium.

**Proof.** (i) and (iii) are as in Proposition 1. (ii) Set  $T_0 = \Pi_0 = \Pi_{\mathcal{L}} = 1$ . The bargaining rule implies  $q = (1 - t)(V_{\mathcal{L}} - V_0)$ , which is strictly positive iff  $c < s\{u(Q) - y[u(q^*) - q^*]\}$ . A buyer's offer is  $q(Q) = \max \left[ (1 - t) \frac{s\{u(Q) - y[u(q^*) - q^*]\} - c}{r + s(1 - yt)}, 0 \right]$ .  $\square$

These equilibria are shown in Figure 1, where an equilibrium is any intersection of the offer with the 45 degrees line (i.e.,  $q(Q) = Q$ ).

The uniqueness result of Theorem 1 is replaced by something weaker.

**Proposition 8.** The government can guarantee the existence of two pure monetary equilibria, and that no other equilibria exist, iff  $t > 0$  and the money's storage cost is small enough.

**Proof.** Setting  $P = [u(q^*) - q^*]/t$  and  $T_0 = 0$  eliminates barter. Setting  $\Pi_{\mathcal{L}} = 1$  results in equilibrium iff  $c < su(Q)$ . A buyer's offer is  $q(Q) = \max \left[ (1 - t) \frac{su(Q) - c}{r + s}, 0 \right]$ .  $\square$

The offer has the same shape as in Figure 1.

## 5.2 Money Printing

I will now show that the government can maintain the circulation of its favourite money while increasing its supply and decreasing its value. Suppose that the government makes an unannounced, once-and-for-all money injection at the end of a period. The money supply doubles and is given in proportional transfers, so that all agents, including government buyers, either have  $\mathcal{L}0$  or  $\mathcal{L}2$ .<sup>18</sup>

For  $i \in \{\mathcal{L}, \mathcal{L}2\}$ , let  $n_i$  be the probability that a seller meets a buyer who offers  $i$  (note that the buyer might hold  $\mathcal{L}2$  and yet offer only  $\mathcal{L}1$ ), let  $q_i$  be the quantity demanded by such a buyer, let  $Q_i$  be the quantity produced in all other matches for  $i$ , let  $\Pi_i$  be the probability that a random seller accepts a payment  $i$ , and let  $\pi_i$  be the best response of a seller who is offered a payment  $i$ . The value functions are

$$rV_0 = sy\Pi_0 \cdot \max_{\pi_0} \pi_0 \{u(q^*) - q^* + t[T_0(-q^t) + (1 - T_0)(-P)]\} + \quad (14)$$

$$n_{\mathcal{L}} \cdot \max_{\pi_{\mathcal{L}}} \pi_{\mathcal{L}} [-q_{\mathcal{L}} + (1 - t)(V_{\mathcal{L}} - V_0)] + n_{\mathcal{L}2} \cdot \max_{\pi_{\mathcal{L}2}} \pi_{\mathcal{L}2} [-q_{\mathcal{L}2} + (1 - t)(V_{\mathcal{L}2} - V_0)],$$

$$rV_{\mathcal{L}} = -c + s\Pi_{\mathcal{L}}[u(Q_{\mathcal{L}}) + V_0 - V_{\mathcal{L}}], \quad (15)$$

$$rV_{\mathcal{L}2} = -2c + s\{(1 - \Pi_{\mathcal{L}})\Pi_{\mathcal{L}2}[u(Q_{\mathcal{L}2}) + V_0 - V_{\mathcal{L}2}] + \quad (16)$$

$$\Pi_{\mathcal{L}}(1 - \Pi_{\mathcal{L}2})[u(Q_{\mathcal{L}}) + V_{\mathcal{L}} - V_{\mathcal{L}2}] + \Pi_{\mathcal{L}}\Pi_{\mathcal{L}2} \cdot \max[u(Q_{\mathcal{L}2}) + V_0 - V_{\mathcal{L}2}, u(Q_{\mathcal{L}}) + V_{\mathcal{L}} - V_{\mathcal{L}2}]\}.$$

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<sup>18</sup>Recall that throughout the paper agents are not physically prevented from holding more than  $\mathcal{L}1$  at a time. Only restriction on their production capabilities prevents such accumulation in most of the paper. I maintain the assumption (relaxed in the previous section) that only moneyless agents can produce.

In (14), in addition to the usual possibility of barter, there is also the possibility of meeting a buyer who offers  $\mathcal{L}1$  for  $q_{\mathcal{L}}$ , and a buyer who offers  $\mathcal{L}2$  for  $q_{\mathcal{L}2}$ . Meeting with a tax collector results in a total loss of that monetary income, whether it is  $\mathcal{L}1$  or  $\mathcal{L}2$  (in accordance with the tax law). In (16), the buyer might meet a seller who accepts only  $\mathcal{L}2$ , a seller who accepts only  $\mathcal{L}1$ , or a seller who accepts either  $\mathcal{L}1$  or  $\mathcal{L}2$ . In the latter case, the buyer chooses how much to pay.

**Theorem 4.** The government can still guarantee the existence of two pure monetary equilibria, and that no other equilibria exist, iff  $t > 0$  and the money's storage cost is small enough. All payments are of  $\mathcal{L}2$ . The threshold storage cost is smaller than in Proposition 8. The price level is almost exactly twice as it was before the increase in the quantity of money.

**Proof.** Setting  $P = [u(q^*) - q^*] / t$  and  $T_0 = 0$  eliminates barter. Setting  $\Pi_{\mathcal{L}} = 0$  and  $\Pi_{\mathcal{L}2} = 1$  results in equilibrium iff  $c < su(Q)/2$ . A buyer's offer is  $q(Q) = \max \left[ (1 - t) \frac{su(Q) - 2c}{r + s}, 0 \right]$ .  $\square$

The only change is that after the increase in the quantity of money all buyers carry twice as much money. Every trade involves a payment of  $\mathcal{L}2$ . Pound bills need to be even lighter now to sustain the pure monetary equilibria. The added cost also affects the bargaining outcome and thus the curve is a bit lower than in Figure 1. Depending on which equilibrium is realized, the output per meeting can be slightly higher or lower than before. Given that almost the same quantity of real goods is traded for twice as much money, money is almost neutral. The important point is that guaranteeing the circulation of money is almost as easy with more money than with less money. Put differently, the transfer is neutral not only in terms of allocations, as could be expected, but also in terms of the strength of the monetary equilibria and the policy's viability. Of course, even these minor changes in allocations can be avoided if the government orders that every two pound bills must be converted into one "new pound" bill.

Section 2 shows that the tax-foundation mechanism is similar to convertibility in its ability to guarantee *circulation*. However, it cannot guarantee the *value* of fiat money (Woodward [1995], p. 929); because unlike convertibility, the tax-foundation mechanism enables the money supply to be entirely dis-

cretionary. The government can print money at will, causing higher prices, higher nominal incomes—and thus higher *nominal* tax obligations. The price of immunity from the tax collectors’ punishment increases proportionately, and therefore the money’s acceptability is sustained.

While some see this “indeterminacy” of the price level as a fatal flaw for a theory of money (e.g., Ellis [1934], Klein [1974]), abusive governments have realized that “indeterminacy” is their ultimate victory: they can guarantee that their money is acceptable (as with convertibility), even though they can print more of it at will. The tax-foundation mechanism allows a government to have its cake and eat it too. Therefore, “indeterminacy” of the price level is not grounds to ignore the tax-foundation theory, but rather a crucial reason for economists to understand it.

## 6 Monetary History

I will now turn to discuss the model’s application in the real world. Parts of the tax law described in Section 2 are in the legal tender law of every modern country. Every such country declares at least one type of fiat money to be legal tender for taxes (and contractual debts). Usually only one type of fiat money—that issued by the domestic government—has this status. The tax collecting authority cannot reject tax payments made in the various objects that belong to this type of money (i.e., various coins and notes of different denominations). As for all other objects, such as checks, used cars, grain, or Monopoly money, the legal tender law is silent. Standard legal principles dictate that this silence grants the tax-collecting authority complete discretion on whether to accept such objects in tax payments. It just so happens that most tax-collecting authorities accept checks but reject anything else.

Casual observation clearly favours the hypothesis that there is a relation between what is declared legal tender and what circulates as money. Given political and economic stability, the domestic legal tender is indeed typically the general medium of exchange. Countries and currency unions seem to be



able to change currency at will, as recently seen in Iraq and the European Union. As Goodhart (1998) notes, money tends to follow political unification and disintegration of federations (e.g., German and Soviet currencies, 1871-1991).

It is easy to find cases of fiat moneys that have failed even though they were receivable for taxes. The following discussion explicates such cases within the theoretical framework of the model outlined above.

1. Non-Unique Legal Tender: the greenbacks of the American Civil War were rejected in the West Coast (Mitchell [1903]) even though they were legal tender for taxes. Their failure is consistent with the theory because gold was also considered legal tender. Proposition 1 shows that if all objects are receivable for tax payments ( $T_i = 1 \forall i$ ), a fiat money equilibrium always coexists with an equilibrium without fiat money. More recently, the American public has rejected the half-dollar coin, the Susan B. Anthony dollar, and the two-dollar bill. According to Proposition 4, if two types of fiat money have the same storage cost and both are receivable for taxes, then either one of them or both may circulate. Since people do not have to pay taxes in coins and notes of particular denominations, the failure of some denominations does not contradict the theory. Thus, in both cases the model features multiple equilibria. Indeed, other evidence points to the opposite behaviour: during the American Revolution gold was hoarded while the paper continental was used in trade (Calomiris [1988]); and the recent Sacagawea golden dollar coins were happily received by sellers but were hoarded instead of being spent (Lotz and Rocheteau [2002]). The same goes for rejection of slightly damaged notes in modern India (Jordan [1996]).

2. Regime Transition: the correlation observed between a paper money's circulation and the existence of its issuing regime is highly positive but not perfect. Saddam Hussein's money circulated in Iraq half a year after his regime collapsed. On the other hand, Germans abandoned their paper money a few months before the Nazi surrender (Einzig [1966], p. 299). Yet these facts are still in line with the theory. The propositions and theorems above that show the government's power to promote its favourite currency include the condition  $t > 0$ , where  $t \equiv \tau G$  is the expected probability of being taxed in the

future by a government that accepts this currency (recall that  $\tau$  is the probabilistic tax rate and  $G$  is the government's survival probability). In such cases of extreme political instability it is not the policy of the residing government that is of issue, but rather the subjective expectations regarding the future government and its policies (Goodhart [1998], King [2004]). Iraqis expected the Coalition Provisional Authority to convert Hussein's money into its new legal tender; therefore, there was no reason to reject it. Russian acceptance of the dead Czar's money during the chaotic hyperinflation of the 1920s (Friedman [1992], p. 11-12) can also be explained by a belief that whoever ends up in power would either convert that money or accept it in various payments. The hasty abandonment of German money can be seen to reflect expectations that the Allies would treat Germans harshly and not conduct such a conversion.<sup>19</sup>

3. Ineffective Tax System: the mechanism may fail if the tax system is not functioning (Wray [1998], p. 36), i.e.,  $\tau = 0$ . This may be the case even if the government is expected to remain in power ( $G > 0$  still results in  $t \equiv \tau G = 0$ ). It is particularly true for new regimes because it takes time to establish effective tax systems that detect and punish tax offenders. This may explain the failure of Japanese money right after Japan occupied Taiwan (Li and Wright [1998]), and the problems of new fiat moneys in disintegrated federations where tax collection was not important beforehand (e.g., the Confederacy during the American Civil War and formerly communist countries). This situation is similar to the loss, or expected loss, of the Treasury's gold holdings under convertibility. With no immunities to sell to taxpayers, the fiat money becomes unbacked. Alternatively, the government may denominate the tax in one unit of account and accept another money as well. If it accepts the other money according to market value rather than at a pre-determined exchange rate, there is nothing to prevent the collapse of the other money, as shown in Proposition 2. An equilibrium in which both the market and the tax authority deem money valueless always exists. This actually happened with the continental currency of

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<sup>19</sup>Alternatively, perhaps they expected to be treated in the same manner that they treated some occupied peoples. The predictions of monetary theory regarding fiat money in finite horizon economies are well-known.

the American Revolution (Calomiris [1988], p. 59) and the *mandat* currency of the French Revolution (Nussbaum [1950], p. 50).

4. Non-exclusive Circulation: in many countries the legal tender currency circulates side by side with foreign currency. It is especially common during high inflation, when people prefer a foreign, stable currency whenever possible. As shown in Proposition 6, when agents can hold more than  $\mathcal{L}1$ , sometimes they might choose to conduct transactions in other ways. They may do so as long as they already hold the minimum they need for tax payments in the domestic currency. This is an important constraint on the government's power and the extent to which it can encourage the use of its favourite money in trade. However, it does not contradict the theory. Occasionally, agents do demand the domestic currency because they need it for tax payments. Perhaps it is this demand that keeps the price level finite even after people realize that their government is bent on hyperinflation (Bruno [1993], p. 8).

## 7 Conclusion

In the real world, there is more to government-issued fiat money than intrinsic uselessness and inconvertibility: its acceptance in tax payments is guaranteed. Generally, no other objects can be forced on the tax authority. The main contribution of this paper is the use of a monetary search model to prove in a dynamic economy that receivability for taxes can make fiat money the general medium of exchange. Other contributions are comparison of this mechanism to convertibility and exploration of its limitations.

There are many ways for a government to promote the circulation of its paper money. The ideal way would assure money-holders that their money could be put to good use, be robust to inflation in the sense that the money will not be completely abandoned, and also be easy and cheap to implement. The tax-foundation mechanism could very well prove ideal. Its assurance that the money can be put to good use was shown to be somewhat equivalent to convertibility. Its robustness to inflation was also shown:

circulation can be maintained even while prices increase. As for implementation, this method is also the cheapest. There is no need to obtain and store gold. There is no need to monitor market transactions. There is no need to conduct searches for illegal currencies. Given that the government collects taxes in any case, it can easily promote any money simply by insisting on accepting only that money.

As mentioned in the Introduction, the tax-foundation theory is part of Knapp's (1905) state theory of money. The German economist celebrated the fact that the government determined what was money and he entirely disregarded issues such as the quantity of money and inflation. His theory was very popular in Germany in the early twentieth century and was viewed by German policy-makers as a license to print money. The disastrous results are well-known. In contrast, this paper aims to promote an understanding of this menace and its limitations so that we can better protect ourselves from it.

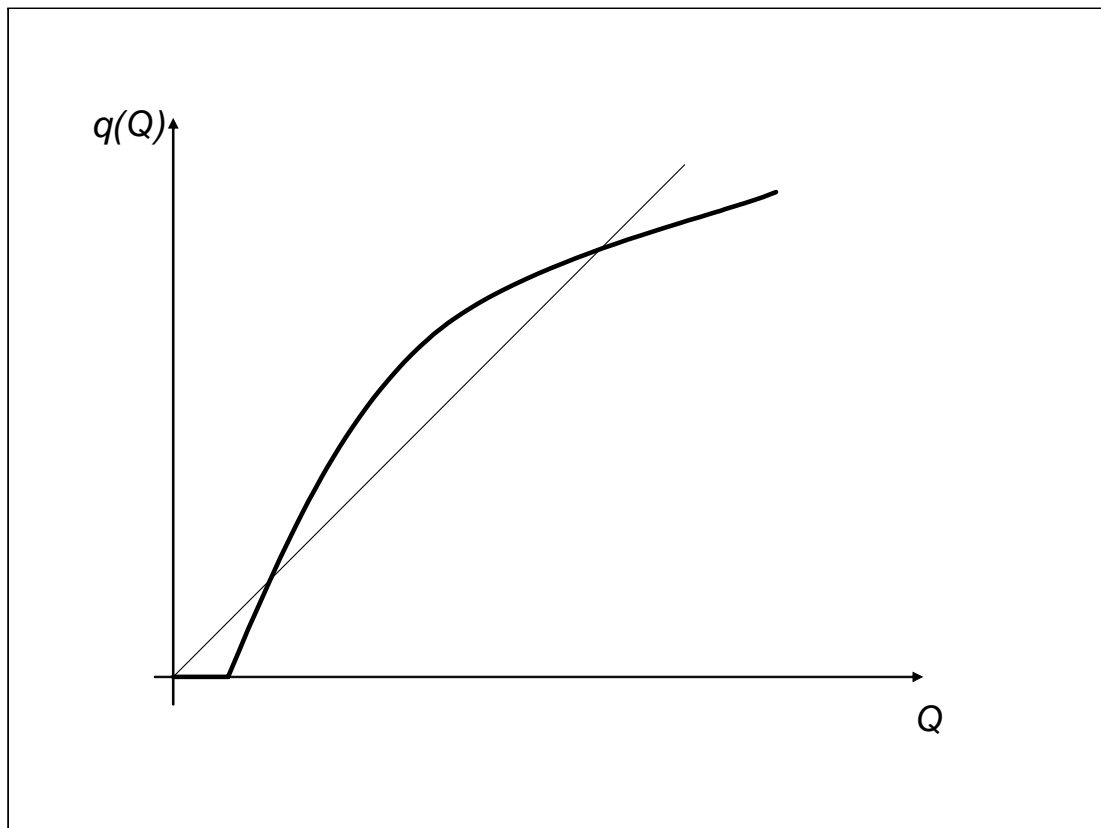
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Figure 1: Equilibria with Endogenous Prices





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