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Search costs and risky investment in quality


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Abstract

One striking development associated with the explosion of e-commerce is the increased transparency of sellers’ quality history. In this paper we analyze how this affects firms’ incentives to invest in quality when the outcome of investment is uncertain. We identify two conflicting effects. On the one hand, reducing the consumer’s cost of search for quality exacerbates the negative effects of past poor performance. This increases incentives to invest, leading to higher quality. On the other hand, the fact that a firm, despite its best efforts, may fail to live up to consumers’ more demanding expectations, makes investment less attractive. This discourages investment, leading to lower quality. We show that reducing the search cost leads to higher quality if the initial level of the search cost is sufficiently high but may lead to lower quality if the initial level of the search cost is sufficiently low.

keywords: search, internet search, quality, risky investment

JEL codes: D83, L15
1 Introduction

One striking development associated with the explosion of e-commerce is the increased transparency of sellers’ quality history. Sites like yelp.com, tripAdvisor.com etc. in which past consumers rate their satisfaction from a wide variety of products and services, facilitate comparison of competing vendors by new consumers. Indeed, consumers seem to have become increasingly dependent on such sources. According to a survey by Forrester Research, some 86% of respondents use ratings and reviews for online purchases and 44% go online before buying products in-store. Such comparative information becomes especially important when purchasing online, when there is no opportunity to physically inspect the merchandise as in traditional retailing. Such developments make it more important for a firm to acquire and maintain a good reputation and exacerbates the negative consequences of a sullied one. This suggests that reducing consumers’ cost of becoming informed about firms past performance should increase investment in quality.

However, when the effect of investment on quality is uncertain, lowering the cost of search may also have a countervailing effect. For example, a new restaurant may strive to maintain high standards of hygiene and buy the most expensive ingredients which nevertheless turn out to be contaminated and make its customers ill. Or a manufacturer may invest in a new model with novel features which fails to catch on with consumers. For example, although the Ford motor company invested heavily in launching the Edsel, the model was so unpopular that its name has become synonymous with failure. Similarly, Coca Cola’s investment in ‘new coke’ was rejected by consumers. In such cases, a firm may acquire bad reputation despite its best efforts. A lower search cost exacerbates the negative effect of such an outcome because, the less costly it is for consumers to discover competitors with a better record, the more demanding they become and the less forgiving they are of less than perfect performance.

We explore these issues in a consumer search model for an experience good in which firms choose whether or not to invest in quality. There are two types of firms: competent and incompetent. Investment increases a competent firm’s likelihood of achieving a relatively high quality level, but has no effect on an incompetent firm. Consumers can make inferences about a firm’s type based on its past quality, but only know the past performance of firms from

which they have previously bought. To become informed about the record of other firms they must invest in costly search.

Our main result is the following. If the initial level of the search cost is sufficiently high, reducing it increases the incentives to invest in quality. However, if the initial cost is sufficiently low, reducing it further can lower the incentive to invest. Specifically, suppose that, when the search cost is below some threshold value, consumers are unwilling to buy from firms with a sufficiently poor record. In that case if the initial search cost is below that threshold, reducing it further decreases the incentive to invest, and can lead to lower quality.

The related literature includes, first of all, the vast consumer search literature. In most of this literature, product characteristics are either exogenous or chosen deterministically to cater to heterogenous consumer tastes.\(^2\) By contrast, here all firms strive - but not all succeed - to achieve the highest possible quality, which is sought after by all consumers.

Also related are papers (Horner (2002), Kranton (2003) and Bar-Isaac (2005)) which examine the effects of increasing competition on investment in quality. In those studies, consumers are costlessly informed about the records of all firms\(^3\) whereas here, by contrast, consumers are uninformed and decide whether or not to become better informed.

The feature of our model that lower search costs can lead to lower quality also connects us to a literature showing that better information can lead to worse outcomes (e.g. Moav and Neeman (2010), Dranove, Kessler, McClellan and Satterthwaite (2003)).

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 analyses the equilibrium search behavior of consumers and the pricing decisions of firms. It then considers the firm’s investment in quality and how it depends on the level of the search cost. The conditions under which a change in the search cost affects the existence of an investment equilibrium are examined. Section 4 considers an extension of the model that allows for simple dynamics and shows that the main insights of the base model carry through.

\(^2\)Seminal contributions include, among others, Diamond (1971), Burdett and Judd (1983) and Wolinsky (1986). Some recent contributions are Armstrong, Vickers and Zhou (2009), and Bar-Isaac, Caruana and Cuñat (2009).

\(^3\)In Horner’s model, consumers knows firms’ customer base, which in equilibrium is equivalent to knowing its record.
Model

There is a continuum of firms. Firms are of two types, competent and incompetent, and the proportion of competent firms is $\mu$. Competent firms can improve the quality of their product by investing, as described immediately below, whereas incompetent firms cannot.

At period 0 a competent firm decides whether or not to invest a fixed amount, $I$, which determines the probability of its quality at periods 1 and 2. At both of these periods, a firm may produce at a unit cost of $c$ irrespective of its type and whether or not it invested.

We denote product quality by $\theta$. There are $N$ quality levels, denoted $\theta_1, \ldots, \theta_N$, $0 < \theta_1 < \ldots < \theta_N$, where $\theta_N$ is the highest (most prized) quality, $\theta_{N-1}$ is the second highest (second most prized) quality level, etc. and $\theta_1$ is the lowest (least desirable) quality. If a competent firm invests at period 0, it produces quality $\theta_i$ with probability $\alpha_i^I$ at periods 1 and 2, where the realized quality is i.i.d at each of these periods. If it does not invest, it produces $\theta_i$ with probability $\alpha_i^{NI}$ at periods 1 and 2. An incompetent firm produces quality $\theta_i$ with probability $\alpha_i^{NI}$ at both periods, whether it invested or not.

We make the following standard assumption:

**Assumption 1 (MLRP)**

$$\frac{\alpha_N^I - \alpha_N^{NI}}{\alpha_N^I} > \frac{\alpha_{N-1}^I - \alpha_{N-1}^{NI}}{\alpha_{N-1}^I} > \ldots > \frac{\alpha_1^I - \alpha_1^{NI}}{\alpha_1^I}$$

This assumption means that investment has a greater effect on the probability of achieving a higher quality level than a lower one. Note that this assumption implies that the distribution of qualities of a competent firm that invests first-order stochastically dominates that of a firm that does not invest, i.e. $\sum_{k=1}^l \alpha_k^I < \sum_{k=1}^l \alpha_k^{NI}$ for all $l < N$ (FOSD).

Consumers are repeat buyers and have identical demand at both periods 1 and 2. A consumer’s utility from a quantity $q$ of quality $\theta_i$ is

$$\theta_i v(q)$$

where $v(\cdot)$ is a concave function and $v'(0) = \infty$. If the consumer is uncertain about quality at the time of purchase her utility is $E[\theta] v(q)$ where $E[\theta]$ is the expected quality. The measure of consumers per firm is normalized to one.
Consumers do not observe a firm’s type nor do they observe whether the firm has invested. Consumers also cannot observe quality at the time of purchase but do observe it afterwards. We refer to a firm’s period 1 quality realization as its record and denote it as \( r \); i.e. \( r = k \) if its realized quality in period 1 is \( \theta_k \) for \( k = 1 \ldots N \).

At period 2, a consumer who bought from a firm at period 1 is aware of its record, and can also observe its current price without additional cost. A consumer also knows the distribution of prices and records in the economy, but it is costly for her to determine the price or record of any firm from which she hasn’t previously bought. By paying a search cost \( \gamma \), she learns the current price and quality record of a randomly selected firm. She can learn this information about any number of firms sequentially, by paying the cost \( \gamma \) for each firm sampled.

Let \( E[\theta| r] = \sum_{i=1}^{N} \theta_i \Pr(\theta_i| r) \) be the expected quality of a firm with a record \( r \), where \( \Pr(\theta_i| r) \) is the posterior probability that a firm with record \( r \) provides quality \( \theta_i \) for \( i = 1 \ldots N \) at period 2. Let \( Q_r(p) = \arg \max_Q [E[\theta| r] v(Q) - pQ] \) denote the quantity demanded by a consumer from a monopolist firm with record \( r \) when its price is \( p \). By the concavity of \( v(Q) \), \( Q_r(p) \) is implicitly defined by the first-order condition,

\[
E[\theta| r] v'(Q) - p = 0. \tag{2}
\]

Denote also by \( P_r(Q) = E[\theta| r] v'(Q) \) the inverse demand function, by \( \varepsilon_r(Q) = \frac{P_r(Q)}{P_r(Q)Q} \) the elasticity of demand, and impose the following weak regularity condition

**Assumption 2** \( \varepsilon_r(Q) \) is weakly increasing in \( Q \), for all \( r \).

Let \( S_r(p) = E[\theta| r] v(Q_r(p)) - pQ_r(p) \) the consumer surplus from buying a quantity \( Q_r(p) \).\(^4\) Let \( \pi_r(p) = (p - c)Q_r(p) \) be the per-consumer profit of a monopolist firm with record \( r \) and price \( p \). We assume \( \pi_r(p) \) is single-peaked, and denote \( p^*_r = \arg \max \pi_r(p) \) as the unique monopoly price corresponding to a record \( r \) and \( \pi^*_r \) as the monopoly profit. Finally, let \( p_r \) be the equilibrium price of firms with a record of \( r \). Henceforth we shall refer to a firm with record \( r \) as an "\( r \) firm".

In several places below, we use the following parametric example to illustrate our results.

\(^4\)Since \( v'(0) = \infty \) then \( Q_r(p) > 0 \) and \( S_r(p) > 0 \) for all \( p \) and \( r \).
Example 1  Let $v(Q) = Q^\beta / \beta$, where $0 < \beta < 1$. It is straightforward to show that

\[
Q_r(p) = \left[ \frac{p}{E[\theta | r]} \right]^{\frac{1}{\beta - 1}}
\]

\[
S_r(p) = \frac{1 - \beta}{\beta} \cdot \frac{E[\theta | r]}{p^{\frac{2}{1-\beta}}}
\]

and that $p_r^* = c / \beta$ for all $r$.

3  Analysis

Obviously only competent firms invest. A strategy for a competent firm is: at period 0, whether to invest and, at periods 1 and 2, which price to set. A strategy for a consumer is a search rule which determines which records and prices she accepts without search and which she rejects and continues to search. We characterize Perfect-Bayesian equilibria for this game.

Generally, there exists an equilibrium in which no firms invest;\(^5\) in this equilibrium consumers believe that no firm invests, hence their willingness to pay in period 2 is independent of the firm’s record and hence a firm has no incentive to invest. We focus instead on the more interesting investment equilibria in which all competent firms invest. Henceforth the term "equilibrium" refers to an investment equilibrium.

Proofs that don’t appear in the text are in the appendix.

3.1  Preliminaries

At period 1 no firm has any record, hence all firms charge the same price and hence a firm’s profit (gross of investment) doesn’t depend on whether or not it invests.

At period 2, by Assumption 1 (MLRP), the better a firm’s record, the more likely it is to have invested, hence the greater the consumers’ expected surplus for any quoted price.

Lemma 1  Consider two records $r, \hat{r}$ in $\{1, ..., N\}$ where $r > \hat{r}$. Then

1. $S_r(p) > S_{\hat{r}}(p)$ for all $p$.
2. $S_r(p_r^*) > S_{\hat{r}}(p_{\hat{r}}^*)$.

\(^5\)This is the case if, for example, $\alpha^{N1}$ has full support.
Lemmas 2–4 below derive properties which characterize any equilibrium. Lemma 2 - which is reminiscent of Diamond (1971)- establishes that in equilibrium the period 2 price charged by a firm with the highest record (quality \( N \)) equals the monopoly price.

**Lemma 2** \( p_N = p_N^* \).

Consider a consumer at period 2 who is matched with a firm with record \( r \) and price \( p \). She may buy from that firm, receiving a surplus of \( S_r(p) \). Alternatively she may reject that firm to search at least once more. Let \( S_\gamma \) (derived below) be the *value of search* – the consumer’s expected surplus from the optimal search strategy.\(^6\) Thus a consumer optimally accepts \( p \) without further search if \( S_r(p) \geq S_\gamma \) and otherwise rejects it to search.

**Lemma 3** *For every record* \( r \),

1. If \( S_r(p_r^*) \geq S_\gamma \) then \( p_r = p_r^* \).
2. If \( S_r(c) \leq S_\gamma \) then \( p_r = c \).
3. Otherwise \( p_r \) satisfies \( S_r(p_r) = S_\gamma \).

**Proof.**

1. Suppose to the contrary that \( p_r < p_r^* \). Then \( S_r(p_r) > S_r(p_r^*) \geq S_\gamma \), which implies that firm \( r \) can increase its price (and profits) without losing any customers.

2. For any price \( p > c \), \( S_r(p) < S_\gamma \). Thus such a firm can only sell by pricing below cost and optimally charges a price \( \geq c \). Without loss of generality, \( p_r = c \).

3. Given that \( S_r(p_r) > S_\gamma \), then \( S_r(p_r^*) < S_\gamma \) implies that \( p_r < p_r^* \). Firm \( r \) could then increase its price without losing any customers and increase profit. If \( S_r(p_r) < S_\gamma \), firm \( r \) makes no sales and earns zero profit. But since \( S_r(c) > S_\gamma \), it could make positive profits by lowering its price to a level (above cost) that consumers would accept.

---

\(^6\)It is well known that when the number of firms is infinite, as were assuming, \( S_\gamma \) does not depends on the currently offered quality\( \backslash \)price and on whether the consumer can recall previously rejected prices.
Based on the preceding result, the following lemma establishes that there is a lower threshold \( r(\gamma) \) such that a firm earns positive profit only if its record \( r \geq r(\gamma) \) and an upper threshold \( \bar{r}(\gamma) \) such that a firm whose record \( r \geq \bar{r}(\gamma) \) earns monopoly profit.

**Lemma 4** There exist thresholds \( r(\gamma) \) and \( \bar{r}(\gamma) \), where \( 1 \leq r(\gamma) \leq \bar{r}(\gamma) \leq N \) such that

1. \( \pi_r = \pi^*_r \) if and only if \( r \geq \bar{r}(\gamma) \).
2. \( \pi_r > 0 \) and increasing in \( r \) if and only if \( r \geq r(\gamma) \).

Let \( t_r = \mu \alpha^f_r + (1 - \mu) \alpha_r^{NF} \) denote the proportion of firms with record \( r \). Then by Lemmas 3 and 4, the value of search is \( S_\gamma = \sum_{r < r(\gamma)} t_r S_\gamma + \sum_{r \geq r(\gamma)} t_r S_r (p^*_r - \gamma) \). Rearranging, we obtain:

\[
S_\gamma = \frac{\sum_{r \geq r(\gamma)} t_r S_r (p^*_r - \gamma)}{\sum_{r \geq r(\gamma)} t_r}.
\] (3)

The next Proposition shows that the thresholds \( r(\gamma) \) and \( \bar{r}(\gamma) \), and therefore equilibrium prices are uniquely defined.

**Proposition 1** A unique equilibrium exists.

We can now derive comparative statics of changes in the search cost \( \gamma \) on equilibrium prices and profits.

**Lemma 5**

1. \( \bar{r}(\gamma) \) and \( \underline{r}(\gamma) \) are weakly decreasing step functions of \( \gamma \).
2. \( p_r \) and \( \pi_r \) are weakly increasing in \( \gamma \) for all \( r \) and are strictly increasing if \( r \in \{\underline{r}(\gamma), \ldots \bar{r}(\gamma) - 1\} \).

### 3.2 The cost of search and the incentives to invest

Consider a competent firm’s investment decision. Its profit at period 1 does not depend on whether or not it invested. Denote the expected operating profit at period 2 of a firm which
invests as \( \pi_I(\gamma) \), its expected profit if it does not invest as \( \pi_{NI}(\gamma) \) and by \( s_r \in [0, 1] \) an \( r \)-firm’s market share. Then

\[
\pi_I(\gamma) = \sum_{r=1}^{N} \alpha_r^I s_r \pi_r
\]

\[
\pi_{NI}(\gamma) = \sum_{r=1}^{N} \alpha_r^{NI} s_r \pi_r
\]

Firms with a record worse than \( \underline{r}(\gamma) \) make no sales and thus \( s_r = 0 \) for all \( r < \underline{r}(\gamma) \). Since in equilibrium customers of firms with records \( r \geq \underline{r}(\gamma) \) do not search, such firms have the same share. Thus \( s_r = s(\gamma) \) for \( r \geq \underline{r}(\gamma) \), where \( s(\gamma) = 1 / \sum_{k \geq \underline{r}(\gamma)} t_k \). Since \( s(\gamma) \) decreases when \( \underline{r}(\gamma) \) decreases, it is a decreasing step function of \( \gamma \) as well.

Let \( W(\gamma) \) be the return to investment. That is the difference between the profits of a firm which invest and does not invest, \( \pi_I(\gamma) - \pi_{NI}(\gamma) \). Hence,

\[
W(\gamma) = \sum_{r=\underline{r}(\gamma)}^{N} (\alpha_r^I - \alpha_r^{NI}) s(\gamma) \pi_r(\gamma)
\]

Denote also the per-customer return to investment by

\[
w(\gamma) = \sum_{r=\underline{r}(\gamma)}^{N} (\alpha_r^I - \alpha_r^{NI}) \pi_r(\gamma)
\]

Thus \( W(\gamma) = s(\gamma) w(\gamma) \).

Figure 1 illustrates \( w(\gamma) \), \( s(\gamma) \) and \( W(\gamma) \) for an example with eight qualities for the parametric functions presented in Example 1.\(^7\) Depending on the initial level of the search cost and the size of the change, the return to investment \( W(\gamma) \) can either increase or decrease in the search cost. For example, for initial search cost corresponding to the second highest segment (approximately \( \gamma = 25 \) to \( 120 \)), an increase in the search cost to \( \gamma \leq 120 \) increases the return to investment, while an increase to \( \gamma > 120 \) lowers the return to investment. And if the initial search cost is greater than \( 120 \) then any increase in the search cost lowers the return to investment.

We proceed to analyze the effect of an increase in the search cost on the return to investment. First, observe that MLRP (Assumption 1) implies that there is a quality level \( k_o > 1 \) such that investment increases the likelihood of quality levels greater or equal to \( k_o \) and decreases the likelihood of quality levels below \( k_o \). That is, \( \alpha_l^I - \alpha_l^{NI} > 0 \) for all \( l \geq k_o \) and

\(^7\)For the figure, the following values for the parameters were used: \( \beta = 1/2; \mu = .15; N = 8; \theta = (1,5,10,15,100,200,300,400); \alpha^{NI} = (.25,.2,.1,.05,.05) \) and \( \alpha^I = (.03,.04,.06,.11,.14,.19,.21,.22) \).
The following proposition describes the effect of an increase in the search cost on the return to investment per customer, $w(\gamma)$.

**Proposition 2** Consider an increase in the search cost from $\gamma_1$ to $\gamma_2 > \gamma_1$.

The per-customer return to investment $w(\gamma)$ **decreases** if either:

1. $k_o \geq \pi(\gamma_1)$.

2. $\underline{r}(\gamma_2) \leq k_o < \pi(\gamma_1)$ and \( \sum_{r=\underline{r}(\gamma_2)}^{\pi(\gamma_1)-1} (\alpha_r^l - \alpha_r^{NI}) \leq 0. \)

The per-customer return to investment $w(\gamma)$ **increases** if

3. $k_o \leq \underline{r}(\gamma_2)$

The intuition for part 1 of the proposition is the following. Recall that $r \geq \pi$ earns monopoly profit. Since $\pi(\gamma)$ is decreasing in $\gamma$, then if $k_o \geq \pi(\gamma_1)$, all records above $k_0$ – which are the records that are more likely outcomes if the firm invests than if doesn’t – earn the same profit whether the search cost is $\gamma_2$ or $\gamma_1$. Therefore, an increase in the search cost from $\gamma_1$ to $\gamma_2$ only increases the profits of records which are more likely if the firm doesn’t invest ($r < k_0$) and hence decreases the return on investment.

Conversely, the intuition for part 3 is the following. Recall that $r \leq \underline{r}$ earns zero profit. Since $\underline{r}(\gamma)$ is decreasing in $\gamma$, if $k_o \leq \underline{r}(\gamma_2)$, all records below $k_0$ - which are the records that

\[ \alpha^l - \alpha^{NI}_l < 0 \text{ for all } l < k_o. \]

It follows from MLRP that $\alpha^l_m - \alpha^{NI}_m > 0$ for all $m > l$. 

-It follows from MLRP that $\alpha^l_m - \alpha^{NI}_m > 0$ for all $m > l$. 

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-It follows from MLRP that $\alpha^l_m - \alpha^{NI}_m > 0$ for all $m > l$. 

$\alpha^l - \alpha^{NI}_l < 0 \text{ for all } l < k_o$. The following proposition describes the effect of an increase in the search cost on the return to investment per customer, $w(\gamma)$.

-It follows from MLRP that $\alpha^l_m - \alpha^{NI}_m > 0$ for all $m > l$. 

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are more likely outcomes if the firm doesn’t invest than if it invests – earn the same profit (zero) whether the search cost is \(\gamma_2\) or \(\gamma_1\). Therefore, a change in the search cost from \(\gamma_1\) to \(\gamma_2\) only increases the profits of records which are more likely under investment than under no investment and hence increases the return on investment.

Finally, in part 2 both effects are present and the condition \(\sum_{r \in \mathcal{L}(\gamma_2)} (\alpha_r - \alpha_r^NI) \leq 0\) is sufficient for the first one to dominate.

Consider two levels of the search cost, \(\gamma_1, \gamma_2\) such that \(\gamma_2 > \gamma_1\). The difference between the total returns to investment corresponding to these two search cost is

\[
W(\gamma_2) - W(\gamma_1) = s(\gamma_2)w(\gamma_2) - s(\gamma_1)w(\gamma_1).
\]

Thus, if the change in the search cost has no effect on market share, i.e. \(s(\gamma_1) = s(\gamma_2) = s\), then \(W(\gamma_2) - W(\gamma_1) = s \cdot [w(\gamma_2) - w(\gamma_1)]\). In that case the change in the total return to investment has the same sign as the change in the per-customer measure. Also since \(s(\gamma_2) \leq s(\gamma_1)\), \(W(\gamma_2) - W(\gamma_1) \leq s(\gamma_1) \cdot [w(\gamma_2) - w(\gamma_1)]\) and thus if \(w(\gamma_2) \leq w(\gamma_1)\) then \(W(\gamma_2) \leq W(\gamma_1)\).

For some special cases, the preceding results yields unambiguous implications about the effect of changes in \(\gamma\). First, if all records earn positive profits at \(\gamma_2\), i.e. \(\mathcal{L}(\gamma_2) = 1\), then \(w(\gamma)\) is decreasing in \(\gamma\) (as either part 1 or part 2 of Proposition 2 applies). Thus, \(W(\gamma)\) is decreasing in \(\gamma\) as well.

Another implication is the following:

**Lemma 6** If \(k_o = N\) then \(W(\gamma)\) is decreasing in \(\gamma\), for all \(\gamma\).

**Proof.** If \(k_o = N\), then for all \(\gamma\), part 1 of Proposition 2 applies. Thus \(w(\gamma)\) and therefore \(W(\gamma)\) are decreasing in \(\gamma\).

In other words, if investment increases only the probability of obtaining the highest quality (and decreases the probability of all other records), an increase in the search cost always lowers the incentives to invest. Two special cases in which this holds are the case of two qualities \((N = 2)\) and the case in which investment is deterministic and leads to the firm producing the highest quality for sure.

---

9Note that \(\sum_{r=1}^{\tau(\gamma_1)-1} (\alpha_r - \alpha_r^NI) \leq 0\) is implied by FOSD.
Corollary 1

1. **Two quality levels:** If $N = 2$ then $W(\gamma)$ is decreasing for all $\gamma$.

2. **Deterministic investment:** If $a_N^I = 1$ then $W(\gamma)$ is decreasing for all $\gamma$.

Thus a necessary condition for the return to investment $W(\gamma)$ to increase with the search cost $\gamma$ is that $k_o < N$. That is, investment has to increase not only the probability of producing the highest quality, $N$, but also the probabilities of producing some of the intermediate qualities. To allow for more richer effects we henceforth assume:

**Assumption 3** $k_o < N$

Propositions 3 and 4 derive effects of changes in the search cost on the return to investment under Assumption 3.

**Proposition 3 (Return to investment decreasing in search cost)** There exists $\overline{\gamma}$ such that for all $\gamma_1$ and $\gamma_2$, $\gamma_2 \geq \gamma_1$ in the interval $[\overline{\gamma}, \infty)$, $W(\gamma_2) \leq W(\gamma_1)$.

The intuition for this result is as follows: Suppose that the search cost is sufficiently high that all records above $k_0$ sell at the monopoly price while some records below $k_0$ sell above cost, but below the monopoly level. Then, a reduction of the search cost reduces the profitability of records which are more likely without investment but doesn’t affect the profitability of records which are more likely under investment. In this case, a moderate reduction of the search cost reduces the profitability of not investing more than the profitability of investment, which increases the incentive to invest.

**Proposition 4 (Return to investment increasing in search cost)** Suppose that

$$S_{k_0-1}(c) < S_N(p_N^*)$$

Then there exists $\underline{\gamma} \geq 0$ such that for all $\gamma_1$ and $\gamma_2$, $\gamma_2 \geq \gamma_1$ in the interval $(0, \underline{\gamma})$ such that $W(\gamma_2) \geq W(\gamma_1)$.
The intuition for this results is as follows: Under condition (7), there is a level of the search cost, \( \gamma \), such that if the search cost is below this level, only records greater or equal to \( k_0 \) (the records which are more likely under investment than without investment) are viable - and at least some of them are priced below the monopoly level - while records below \( k_0 \) are out of business. In that case, a further reduction of the search cost only reduces the profitability of records greater than \( k_0 \) (by reducing the equilibrium price of those records) but has no effect on records below it (since their profitability is already zero), thus lowering the profitability of investment. This lowers the incentive to invest.

Condition (7) holds if (i) the probability that a firm with a record \( k_0 - 1 \) is incompetent is relatively high and (ii) the difference between the expected utilities from a product of a competent firm and of an incompetent firm is sufficiently large. To further investigate the plausibility of (7), consider its implication for the parameterizations presented in Example 1. Based on the calculations stated there,

\[
S_N(p^*_N) - S_{k_0-1}(c) = \frac{1 - \beta}{\beta} \cdot \frac{1}{c^{1-\beta}} \cdot \left[ \beta^{1-\beta} E[\theta|N]^{1-\beta} - E[\theta|k_0-1]^{1-\beta} \right]
\]

where \( 0 < \beta < 1 \). Hence, condition (7) obtains if and only if \( \frac{E[\theta|N]}{E[\theta|k_0-1]} > \beta^{-\beta} \). As \( \lim_{\beta \to 0} \beta^{-\beta} = \lim_{\beta \to 1} \beta^{-\beta} = 1 \), and as \( E[\theta|N] > E[\theta|k_0-1] \) (as is shown in Lemma A.1), the condition is certainly satisfied if \( \beta \) is sufficiently high or sufficiently small. Furthermore, \( \beta^{-\beta} \) is maximized at \( \beta = 1/e \) giving \( \beta^{-\beta} = 1.445 \). Hence, if \( E[\theta|N] > 1.445 \cdot E[\theta|k_0-1] \), condition (7) holds for all \( \beta \).

To summarize, Propositions 3 and 4 imply the following: If the initial search cost is sufficiently high, a reduction of the search cost increases the incentive to invest. Under reasonable conditions, if the initial level of the search cost is sufficiently low, a further reduction of the search cost reduces the incentive to invest.

The preceding analysis can now be used to determine how the changes in the search cost affects actual quality. Specifically, consider two levels of the search cost \( \gamma', \gamma'' \) such that

\[
W(\gamma') \geq I > W(\gamma'')
\]

where \( I \) is the cost of investment. Assume that the investment equilibrium obtains whenever it exists. Then if the search cost is equal to \( \gamma' \) an investment equilibrium exists in which all
competent firms invest. And if \( \gamma \) is equal to \( \gamma'' \) an investment equilibrium does not exist and no firm invests. Thus if the search cost changes from \( \gamma' \) to \( \gamma'' \) average quality in the market decreases and if the search cost changes from \( \gamma'' \) to \( \gamma \) average quality increases.

4 Extension with dynamic reputation building

The preceding analysis may be applied to a more dynamic version in which a firm’s reputation develops cumulatively over more than one period. In this version, there are only two product qualities, \( \theta_L \) (low) and \( \theta_H \) (high), \( \theta_L < \theta_H \). As before, there are competent and incompetent firms and the proportion of competent firms is \( \mu \). Competent firms first decides whether or not to invest \( I \) and then the market is open for periods 1 through \( T \), where \( T \geq 3 \). Realized quality at each period is i.i.d. While in the base model the number of possible records corresponds to the number of quality levels, here, the number of possible records corresponds to the number of possible quality histories, which increases over time. Let \( \delta \) be the probability with which a competent firm that invests produces high quality in each period and \( \beta \) the probability with which a firm that doesn’t invest or is incompetent produces high quality at each period, where \( \beta < \delta \). Because quality is i.i.d across periods, a firm’s record in each period is the number of times in the past in which it has produced high quality (\( \theta_H \)).

In Appendix B we show how the analysis of the base model may be applied to this version for the simplest case, \( T = 3 \). In particular, it is shown that lowering the search cost always increases the return to investment if the probability of success under no investment is sufficiently high (\( \beta > 0.5 \)) or if the difference between the probabilities of success under investment and no investment is large enough (\( \delta > 1 - \beta \)). In other cases lowering the search cost may reduce the return to investment.
A Proofs

The proof of Lemma 1 builds on the following auxiliary lemma (A.1), which shows that the expected quality of a firm is higher, the better its record.

In the text, just ahead of Proposition 2, we argue that there exists a quality level \( k_0 > 1 \) such that \( \alpha_i^l - \alpha_i^{NI} > 0 \) for all \( l \geq k_0 \) and \( \alpha_i^l - \alpha_i^{NI} < 0 \) for all \( l < k_0 \).

**Lemma A.1** \( E[\theta|m] > E[\theta|l] \) if and only if \( m > l \).

**Proof.** Assume that \( m > l \). Note that

\[
\Pr(\theta_i|m) = \Pr(\theta_i|l) \alpha_i^G + \Pr(\theta_i|l) \alpha_i^{NI} = \Pr(\theta_i|m) \alpha_i^G + [1 - \Pr(\theta_i|l)] \alpha_i^{NI} \tag{8}
\]

where

\[
\Pr(\theta_i|l) = \frac{\mu \alpha_i^l}{\mu \alpha_l + (1 - \mu) \alpha_r^{NI}} = \frac{\mu \alpha_i^l}{\alpha_l - (1 - \mu) (\alpha_l - \alpha_r^{NI})} = \frac{\mu}{1 - (1 - \mu) \cdot \frac{\alpha_l - \alpha_r^{NI}}{\alpha_i^l}}.
\]

By MLRP (Assumption 1) this implies that \( \Pr(\theta_i|m) > \Pr(\theta_i|l) \). Hence, for \( i \geq k_0 \), as \( \alpha_i^l - \alpha_i^{NI} > 0 \), \( \Pr(\theta_i|m) > \Pr(\theta_i|l) \), whereas for \( i < k_0 \), the reverse is true. Next,

\[
E[\theta|m] - E[\theta|l] = \sum_{i=1}^{N} \theta_i \cdot [\Pr(\theta_i|m) - \Pr(\theta_i|l)]
\]

\[
= \sum_{i=1}^{N} \theta_i (\alpha_i^l - \alpha_i^{NI}) [\Pr(\theta_i|m) - \Pr(\theta_i|l)]
\]

\[
= \sum_{i=1}^{k_0-1} \theta_i (\alpha_i^l - \alpha_i^{NI}) [\Pr(\theta_i|m) - \Pr(\theta_i|l)] + \sum_{i=k_0}^{N} \theta_i (\alpha_i^l - \alpha_i^{NI}) [\Pr(\theta_i|m) - \Pr(\theta_i|l)]
\]

As \( \alpha_i^l - \alpha_i^{NI} < 0 \) for \( i < k_0 \) and as \( \theta_i \) is increasing in \( i \),

\[
\sum_{i=1}^{k_0-1} \theta_i (\alpha_i^l - \alpha_i^{NI}) [\Pr(\theta_i|m) - \Pr(\theta_i|l)] > \theta_{k_0} \sum_{i=1}^{k_0-1} (\alpha_i^l - \alpha_i^{NI}) [\Pr(\theta_i|m) - \Pr(\theta_i|l)].
\]

Similarly, as \( \alpha_i^l - \alpha_i^{NI} > 0 \) for \( i \geq k_0 \)

\[
\sum_{i=k_0}^{N} \theta_i (\alpha_i^l - \alpha_i^{NI}) [\Pr(\theta_i|m) - \Pr(\theta_i|l)] \geq \theta_{k_0} \sum_{i=k_0}^{N} (\alpha_i^l - \alpha_i^{NI}) [\Pr(\theta_i|m) - \Pr(\theta_i|l)].
\]
Hence
\[
E \left[ \theta | m \right] - E \left[ \theta | l \right] > \theta_k \cdot \sum_{i=1}^{N} \left( \alpha_i^I - \alpha_i^{NI} \right) \left( \Pr \left( I | m \right) - \Pr \left( I | l \right) \right)
\]
\[
= \theta_k \cdot \left( \Pr \left( I | m \right) - \Pr \left( I | l \right) \right) \sum_{i=1}^{N} \left( \alpha_i^I - \alpha_i^{NI} \right) = 0.
\]

\textbf{Proof of Lemma 1.}

1. Given records \( r, \hat{r} \) such that \( r > \hat{r} \),

\[
S_r(p) = E \left[ \theta | r \right] v \left( Q_r(p) \right) - pQ_r(p)
\geq E \left[ \theta | r \right] v \left( Q_r(p) \right) - pQ_r(p) > E \left[ \theta | \hat{r} \right] v \left( Q_{\hat{r}}(p) \right) - pQ_{\hat{r}}(p) = S_{\hat{r}}(p).
\]

2. Observe that \( S_r(p^*_r) = E \left[ \theta | r \right] v \left( Q^*_r \right) - p^*_r Q^*_r \) and that \( p^*_r = P_r(Q^*_r) = E \left[ \theta | r \right] v' (Q^*_r) \). Hence

\[
S_r(p^*_r) = E \left[ \theta | r \right] \left( v \left( Q^*_r \right) - v' (Q^*_r) Q^*_r \right).
\]

As \( v(Q) - v'(Q) Q \) is increasing in \( Q \) (its derivative is \(-v''(Q) > 0\), \( S_r(p^*_r) \) is increasing in \( r \), as \( Q^*_r \) is. This can be most readily be seen from the first-order condition for the monopoly problem, which sets the marginal revenue, \( MR_r(Q) = dP_r(Q) / dQ \) equal to the marginal cost \( c \). As \( MR_r(Q) = E \left[ \theta | r \right] \left( v'(Q) + v''(Q) Q \right) \) is increasing in \( r \) and decreasing in \( Q \) (which follows from the second-order condition of the firm’s maximization problem), \( Q^*_r \) is increasing in \( r \).

\textbf{Proof of Lemma 2.} First note that for all \( r, p_r \leq p^*_r \); otherwise an \( r \) firm could lower its price without losing customers and increase its profit. Suppose \( p_N < p^*_N \). If \( S_N(p_N) \geq S_r(p_r) \) for all \( r < N \), then an \( N \) firm can slightly increase its price without inducing its customers to search, increasing thereby the profit per customer (by concavity of the profit function). Thus \( S_N(p_N) \leq S_r(p_r) \) for at least one \( r < N \) and let \( k = \arg\max_{r < N} \{ S_r(p_r) \} \). Then \( p_k = p^*_k \); otherwise a \( k \) firm could increase its price slightly without losing customers and increase profit. But then, since \( p_N < p^*_N \), \( S_N(p_N) > S_N(p^*_N) > S_k(p^*_k) = S_k(p_k) \), a contradiction. This completes the proof. □
Proof of Lemma 4.

1. Define \( \bar{r}(\gamma) \) as the lowest record such that \( S_r(p_r^*) \geq S_\gamma \). By Lemma 2, \( S_N(p_N^*) \geq S_\gamma \) and thus \( \bar{r}(\gamma) \leq N \) exists. For \( r < \bar{r}(\gamma) \), \( S_r(p_r^*) < S_\gamma \) and therefore a firm with such a record can only make sales if \( p_r < p_r^* \). For \( r \geq \bar{r}(\gamma) \), by Lemma 1, \( S_r(p_r^*) \geq S_{\bar{r}(\gamma)}(p_{\bar{r}(\gamma)}^*) \geq S_\gamma \) and thus \( p_r^* \) maximizes the profits of such firms.

2. Observe first that a consumer will buy from a firm with a record \( r \) and price \( p \) if and only if \( S_r(p) \geq S_\gamma \). Let \( r' \) be defined as the highest value of \( r \) such that \( S_r(c) \leq S_\gamma \). If \( r' \) does not exist let \( \bar{r}(\gamma) = 1 \) and in that case any record can earn positive profit by charging a price slightly above \( c \). If \( r' \geq 1 \) does exist, then define \( \bar{r}(\gamma) \equiv r' + 1 \). From Lemma 1, for all \( r < \bar{r}(\gamma) \), \( S_r(c) \leq S_{r'}(c) \leq S_\gamma \) and hence a firm with this record cannot earn positive profits. And for all \( r \geq \bar{r}(\gamma) \), \( S_r(c) \geq S_{\bar{r}(\gamma)} > S_\gamma \) and thus a firm with this record can earn positive profit by charging a price slightly above \( c \).

Clearly \( \pi_r^* \) is increasing in \( r \). For \( r \in \{ \bar{r}(\gamma), \ldots, \tau(\gamma) - 1 \} \), \( 0 < \pi_r < \pi_r^* \). Since, in this range, \( S_r(p_r) = S_\gamma \) and as \( S_r(p) \) is increasing in \( r \) then \( p_r \) is increasing in \( r \). Finally, by the first-order condition (2), \( Q_r(p) \) is strictly increasing in \( r \). Thus \( \pi_r \) is increasing in \( r \) as well.

Proof of Proposition 1. Uniqueness is proved as follows: for any \( j \leq N \), define

\[
S^{(j)}_{\gamma} = \frac{\sum_{r=j}^{N} t_r S_r(p_r^*) - \gamma}{\sum_{r=j}^{N} t_r} \tag{9}
\]

For future reference note that if \( \tau \leq j \), then \( S^{(j)}_{\gamma} \) is the expected consumer surplus from the following search strategy: search until a \( r \geq j \) is found.

1. Suppose that an equilibrium with \( \tau = k \) exists. Then \( S^{(j)}_{\gamma} < S^{(k)}_{\gamma} \) for every \( j \) such that \( j < k \).
Proof: Observe that we can express $S^{(j)}_\gamma$, using the following recursive formula.

\[
S^{(j)}_\gamma = \frac{t_j S_j \left( p^*_j \right) + \sum_{r=j+1}^{N} t_r S_r \left( p^*_r \right) - \gamma}{\sum_{r=j}^{N} t_r}
\]

\[
= \frac{t_j}{\sum_{r=j}^{N} t_r} S_j \left( p^*_j \right) + \sum_{r=j+1}^{N} t_r \left[ \frac{\sum_{r=j+1}^{N} t_r S_r \left( p^*_r \right) - \gamma}{\sum_{r=j+1}^{N} t_r} \right]
\]

and hence

\[
S^{(j)}_\gamma = \frac{t_j}{\sum_{r=j}^{N} t_r} S_j \left( p^*_j \right) + \sum_{r=j+1}^{N} t_r S^{(j)}_{\gamma}
\]

(10)

Applying (10) repeatedly we obtain for any $k > j$,

\[
S^{(j)}_\gamma = \frac{\sum_{r=j}^{k-1} t_r S_r \left( p^*_r \right) + \sum_{r=k}^{N} t_r S^{(k)}_{\gamma}}{\sum_{r=j}^{N} t_r}
\]

Since an equilibrium with $\tau = k$ exists, it follows from Lemmas 3 and 4 that for $r \leq k - 1$, $S_r \left( p^*_r \right) < S^{(k)}_{\gamma}$. Thus

\[
S^{(j)}_\gamma < \frac{\sum_{r=j}^{k-1} t_r S^{(k)}_{\gamma} + \sum_{r=k}^{N} t_r S^{(k)}_{\gamma}}{\sum_{r=j}^{N} t_r} = S^{(k)}_{\gamma}
\]

2. The equilibrium value of $\tau$ is unique.

Proof: Suppose there are two equilibria. One in which $\tau = k$ and another where $\tau = j < k$. Comparing (3) and (9) shows that in the latter equilibrium $S_\gamma = S^{(j)}_\gamma$. Consider the following strategy: search until a firm with record $r \geq k$ is found (i.e. reject all records less than $k$). In either equilibrium, the expected surplus from following this strategy is $S^{(k)}_{\gamma}$. But by step 1, $S^{(j)}_\gamma < S^{(k)}_{\gamma}$, which means that the equilibrium search strategy in the $\tau = j$ equilibrium is not optimal, a contradiction. By the same argument, an equilibrium with $\tau > k$ cannot exist.

3. Equilibrium prices are uniquely determined.

Proof: Given $\bar{\tau}$, $p_r = p^*_r$ for all $r \geq \bar{\tau}$, which, by step 2, is unique. Also given $\bar{\tau}$, $S_\gamma$ is uniquely defined by (3) and thus, since $S_r \left( p \right)$ are monotonically decreasing in $p$, then $p_r$ for $r < \bar{\tau}$, are uniquely determined by Lemma 3. This completes the proof that the equilibrium is unique.
Existence is proved by construction, using the following algorithm: Set \( r = N \) and calculate \( S_\gamma^{(N)} \). If \( S_{N-1} (p_{N-1}^*) < S_\gamma^{(N)} \), the unique equilibrium has \( r = N \) and prices are uniquely determined by Lemma 3. Otherwise, set \( r = N - 1 \), calculate \( S_\gamma^{(N-1)} \) and proceed as above. If the process reaches \( r = 2 \) and \( S_1 (p_1^*) > S_\gamma^{(2)} \) then \( p_k = p_k^* \) for all \( k = 1, \ldots, N \).

**Proof of Lemma 5.**

1. First, since \( \bar{r} (\gamma) \) and \( \underline{r} (\gamma) \) are defined on the integers they are step functions. Consider some \( \gamma_1, \gamma_2 \) such that \( \gamma_1 < \gamma_2 \). Then, as argued in the Proof of Proposition 1, the value of search corresponding to \( \gamma_1 \) and \( \gamma_2 \) are respectively \( S_{\gamma_1}^{(\bar{r}(\gamma_1))} \) and \( S_{\gamma_2}^{(\bar{r}(\gamma_2))} \). Suppose that \( \bar{r} (\gamma_2) > \bar{r} (\gamma_1) \). If \( \gamma = \gamma_1 \), consider the consumer search strategy: search until \( r \geq \bar{r} (\gamma_2) \) is found. As argued in the Proof of Proposition 1, the expected surplus from this strategy is \( S_{\gamma_1}^{(\bar{r}(\gamma_2))} \). Since \( S_{\gamma_1}^{(\bar{r}(\gamma_1))} \) is the surplus from the equilibrium search strategy, \( S_{\gamma_1}^{(\bar{r}(\gamma_2))} \leq S_{\gamma_1}^{(\bar{r}(\gamma_1))} \). However, from the proof of Proposition 1 it follows that \( S_{\gamma_1}^{(\bar{r}(\gamma_2))} > S_{\gamma_1}^{(\bar{r}(\gamma_1))} \) if \( \bar{r} (\gamma_2) > \bar{r} (\gamma_1) \), a contradiction. This proves that \( \bar{r} (\gamma) \) is weakly decreasing in \( \gamma \).

Recall from the proof of Lemma 4 that \( \underline{r} (\gamma) \) is the lowest value of \( r \) such that \( S_r (c) > S_\gamma \). Thus if \( S_\gamma \) is decreasing in \( \gamma \) then \( \underline{r} (\gamma) \) must be weakly decreasing. To prove that \( S_\gamma \) is decreasing in \( \gamma \), recall from the Proof of Proposition 1 that \( S_{\gamma_2} = S_{\gamma_2}^{(\bar{r}(\gamma_2))} \) and \( S_{\gamma_1} = S_{\gamma_1}^{(\bar{r}(\gamma_1))} \). As \( \bar{r} (\gamma_2) \leq \bar{r} (\gamma_1) \), \( S_{\gamma_2}^{(\bar{r}(\gamma_2))} \leq S_{\gamma_2}^{(\bar{r}(\gamma_1))} \) \( S_{\gamma_1}^{(\bar{r}(\gamma_1))} \), where the first inequality follows from the proof of Proposition 1 and the second inequality follows directly from (9). Thus \( S_{\gamma_2} < S_{\gamma_1} \) if \( \gamma_2 > \gamma_1 \).

2. Consider first an increase in \( \gamma \) that does not change \( \underline{r} (\gamma) \) and \( \bar{r} (\gamma) \). For \( r < \underline{r} (\gamma) \) or \( r \geq \bar{r} (\gamma) \), \( p_r \) does not change. For \( r \in \{ \underline{r} (\gamma), \ldots, \bar{r} (\gamma) - 1 \} \), \( S_r (p_r) = S_\gamma \) and thus as \( S_\gamma \) is decreasing in \( \gamma \), \( p_r \) is strictly increasing in \( \gamma \). As \( p_r < p_r^* \), and as \( p_r^* > 0 \) for \( p_r < p_r^* \), \( p_r \) is decreasing in \( \gamma \). If only \( \underline{r} (\gamma) \) decreases, \( S_\gamma \) does not change and so for all \( r \) for which previously \( p_r > c \) there is no change in price while for all \( r \) which were previously less than \( \underline{r} (\gamma) \), either \( p_r \) increases or does not change. Finally if \( \bar{r} (\gamma) \) decreases, we argued above that \( S_\gamma \) decreases, and so all prices either decrease or stay the same.

\[ \blacksquare \]
The next three auxiliary lemmas (A.2-A.4) are used in the proof of Proposition 2 (part 2).

**Lemma A.2** Suppose that there are \( l, m \) such that \( \overline{r} \leq l < m < \overline{r} \), then \( Q_i = Q_i (p_l) > Q_m (p_m) = Q_m \).

**Proof.** As \( \overline{r} \leq l < m < \overline{r} \), we have \( S_l (p_l) = S_m (p_m) = S_i \). Thus

\[
S_l (p_l) = E[\theta l] \cdot v (Q_l) - p_l Q_i = E[\theta m] \cdot v (Q_m) - p_m Q_m = S_m (p_m)
\]

In addition, from the first-order conditions to the consumer’s problem \( p_i = E[\theta l] \cdot v’ (Q_l) \) and \( p_m = E[\theta m] \cdot v’ (Q_m) \). Hence

\[
E[\theta l] \cdot [v (Q_l) - v’ (Q_l) Q_i] = E[\theta m] \cdot [v (Q_m) - v’ (Q_m) Q_m]
\]

Because \( E[\theta m] > E[\theta l] \), therefore \( v (Q_l) - v’ (Q_l) Q_i > v (Q_m) - v’ (Q_m) Q_m \) and therefore \( Q_i > Q_m \), as the function \( v (Q) - v’ (Q) Q \) is increasing in \( Q \) (its derivative is \( -v'' (Q) > 0 \)).

**Lemma A.3** For \( r \) in \( \{\overline{r}, \ldots, \overline{r} - 1\} \), \( \frac{p_r - c}{p_r} \cdot \varepsilon_r (Q_r) \) is decreasing in \( r \)

**Proof.** Note first that we can write

\[
\varepsilon_r (Q) = \frac{P_r (Q)}{[dP_r (Q) / dQ] \cdot Q},
\]

where \( P_r (Q) \) is the inverse demand function given a record \( r \). Substituting \( P_r (Q) = E[\theta r] \cdot v’ (Q) \), we obtain

\[
\varepsilon_r (Q) = \frac{E[\theta r] \cdot v’ (Q)}{E[\theta r] \cdot v'' (Q) \cdot Q} = \frac{v’ (Q)}{v'' (Q) \cdot Q}.
\]

Thus, \( \varepsilon_r (Q) \) is invariant of the record \( r \) and depends only on the quantity \( Q \).

Now, let \( l, m \) be such that \( \overline{r} \leq l < m < \overline{r} \). Hence

\[
\frac{p_m - c}{p_m} \cdot \varepsilon_m (Q_m) < \frac{p_m - c}{p_m} \cdot \varepsilon_m (Q_l) = \frac{p_m - c}{p_m} \cdot \varepsilon_l (Q_l) < \frac{p_l - c}{p_l} \cdot \varepsilon_l (Q_l)
\]

where the first inequality follows from Assumption 2, and the fact that \( Q_l > Q_m \) (Lemma A.2) and the second inequality follows as \( (p - c) / p \) is increasing in \( p \), \( p_l < p_m \) and \( \varepsilon_l (Q_l) < 0 \).
Lemma A.4  Consider two levels of the search cost $\gamma_2 > \gamma_1$. Then $\pi_r (\gamma_2) - \pi_r (\gamma_1)$ is decreasing in $r$.

Proof. For $r \in \{1, \ldots, \bar{r} - 1\}$,

$$
\frac{\partial \pi_r}{\partial \gamma} = \pi'_r (p_r) \cdot \frac{\partial p_r}{\partial \gamma} = \left[ Q_r (p_r) + (p_r - c) Q'_r (p_r) \right] \frac{\partial p_r}{\partial \gamma}.
$$

Recall that $\frac{\partial \pi_r}{\partial \gamma} > 0$. Differentiating the equation $S_r (p_r) = S^*_\gamma$, which defines $p_r$ implicitly, we obtain

$$
\frac{\partial p_r}{\partial \gamma} = \frac{\partial S^*_\gamma}{\partial \gamma} \left/ S'_r (p_r) \right.. 
$$

Note that $S_r (p_r) = \max_Q E [\theta | r] \cdot v (Q) - p_r Q$, and thus, by the envelope theorem, $S'_r (p_r) = -Q_r (p_r)$. Thus

$$
\frac{\partial \pi_r}{\partial \gamma} = -\frac{\partial S^*_\gamma}{\partial \gamma} \cdot \left[ 1 + (p_r - c) \frac{Q'_r (p_r)}{Q_r (p_r)} \right]
$$

$$
= -\frac{\partial S^*_\gamma}{\partial \gamma} \cdot \left[ 1 + \frac{p_r - c}{p_r} \cdot \varepsilon_r (Q_r) \right]
$$

where $\varepsilon_r (Q_r)$ is the equilibrium price elasticity of the demand for an $r$ firm. As $\frac{\partial S^*_\gamma}{\partial \gamma} < 0$, it follows from Lemma A.3 that $\frac{\partial \pi_r}{\partial \gamma}$ is decreasing in $r$. Finally, $\pi_r (\gamma_2) - \pi_r (\gamma_1) = \int_{\gamma_1}^{\gamma_2} \frac{\partial \pi_r}{\partial \gamma}$ is decreasing in $r$ as well. □

Proof of Proposition 2. Recall that $r(\gamma)$ and $\pi(\gamma)$ are both decreasing in $\gamma$. Since for $r \geq r(\gamma_1)$, $\pi_r (\gamma_1) = \pi_r (\gamma_2) = \pi^*$ and for $r \leq r(\gamma_2)$, $\pi_r (\gamma_1) = \pi_r (\gamma_2) = 0$,

$$
w (\gamma_2) - w (\gamma_1) = \sum_{r=1}^{r(\gamma_1)-1} (\alpha^I_r - \alpha^NI_r) \cdot (\pi_r (\gamma_2) - \pi_r (\gamma_1))
$$

1. If $k_o \geq r(\gamma_1)$, $\alpha^I_r - \alpha^NI_r < 0$ for all $r$ in the summation term above. As $\pi_r (\gamma_2) \geq \pi_r (\gamma_1)$ for all $r$ it follows that $w (\gamma_2) - w (\gamma_1) \leq 0$.

2. Provided $l(\gamma_2) \leq k_o < r(\gamma_1)$ one can write $r (\gamma_2) - r (\gamma_1)$ as

$$
w (\gamma_2) - w (\gamma_1) = \sum_{r=l(\gamma_2)}^{k_o-1} (\alpha^I_r - \alpha^NI_r) (\pi_r (\gamma_2) - \pi_r (\gamma_1)) + \sum_{r=k_o}^{r(\gamma_1)-1} (\alpha^I_r - \alpha^NI_r) (\pi_r (\gamma_2) - \pi_r (\gamma_1))
$$

As $\pi_r (\gamma_2) - \pi_r (\gamma_1)$ is decreasing in $r$ (as proved in Lemma A.4),

$$
w (\gamma_2) - w (\gamma_1) \leq \sum_{r=l(\gamma_2)}^{k_o-1} (\alpha^I_r - \alpha^NI_r) (\pi_{k_o} (\gamma_2) - \pi_{k_o} (\gamma_1)) + \sum_{r=k_o}^{r(\gamma_1)-1} (\alpha^I_r - \alpha^NI_r) (\pi_{k_o} (\gamma_2) - \pi_{k_o} (\gamma_1))
$$

$$
= (\pi_{k_o} (\gamma_2) - \pi_{k_o} (\gamma_1)) \cdot \sum_{r=l(\gamma_2)}^{r(\gamma_1)-1} (\alpha^I_r - \alpha^NI_r) \leq 0.
$$
3. 

\[ w(\gamma_2) - w(\gamma_1) = \sum_{r=\bar{\gamma}(\gamma_2)}^{N} (\alpha_r^I - \alpha_r^{NI}) \pi_r(\gamma_2) - \sum_{r=\bar{\gamma}(\gamma_2)}^{N} (\alpha_r^I - \alpha_r^{NI}) \pi_r(\gamma_1) \]

Since \(\pi_r(\gamma_2) \geq \pi_r(\gamma_1)\), and as \(\alpha_r^I - \alpha_r^{NI} > 0\), since \(k_o \leq \bar{\gamma}(\gamma_2)\), it follows that \(w(\gamma_2) \geq w(\gamma_1)\).

**Proof of Proposition 3.** Let \(\bar{\gamma}\) be the smallest value of \(\gamma\) such that if \(\gamma \geq \bar{\gamma}\) either \(\pi_r > 0\) for every \(r\) (i.e. \(\bar{\gamma}(\gamma) = 1\)) or \(\pi_r = \pi_r^+\) for all \(r \geq k_o\) (i.e. \(k_o \geq \bar{\gamma}(\gamma)\)) or both. In the first case, since \(k_o > 1\), \(\bar{\gamma}(\gamma_2) < k_o\). Moreover, it follows from FOSD that \(\sum_{r=1}^{K} (\alpha_r^I - \alpha_r^{NI}) \leq 0\) for all \(K\). Thus, either part 1 or part 2 of Proposition 2 applies. In the second case part 1 of Proposition 2 applies. In either case, \(w(\gamma_2) \leq w(\gamma_1)\) for any \(\gamma_1\) and \(\gamma_2\) such that \(\gamma_2 \geq \gamma_1\) in this interval, and thus \(W(\gamma_2) \leq W(\gamma_1)\). ■

**Proof of Proposition 4.** Note that, as \(\gamma\) goes to zero, \(\bar{\gamma}(\gamma) \to N\) and thus by (3), \(S_\gamma \to S_N(p_N^*)\). Thus, if (7) obtains, there exists \(\hat{\gamma}\) such that, for all \(\gamma \leq \hat{\gamma}\), a consumer will reject \(r < k_o\) at any price greater or equal to \(c\). Thus, for all \(\gamma \leq \hat{\gamma}\), \(\pi_r = 0\) for all \(r < k_o\) (i.e. \(k_o \leq \bar{\gamma}(\gamma)\)). It thus follows from Proposition 2 part 3 that \(w(\gamma_2) \geq w(\gamma_1)\) for any \(\gamma_1\) and \(\gamma_2\) such that \(\gamma_1 \leq \gamma_2 < \hat{\gamma}\). Finally, let \(\underline{\gamma}\) be largest value of \(\gamma \leq \hat{\gamma}\) such that \(\bar{\gamma}(\gamma)\) (and thus \(s(\gamma)\)) is constant on \((0,\underline{\gamma}]\). For any \(\gamma_1\) and \(\gamma_2\) in \((0,\underline{\gamma}]\) such that \(\gamma_2 \geq \gamma_1\), \(W(\gamma_2) \geq W(\gamma_1)\). ■

**B Analysis of extended model**

In addition to the assumptions discussed in the text of Section 4, it proves convenient, as discussed below, to assume that firms are unaware of their type; that is, at the time of investment a firm believes it is competent with probability \(\mu\). Thus, since ex ante all firms are identical, in an investment equilibrium, all firms invest, not just the competent ones.

Denote a firm’s record at periods 2 and 3 respectively by \(r_2\) and \(r_3\). At period 2 there are two possible records which we denote by \(L\) (a low quality outcome at period 1) and \(H\) (a high quality outcome at period 1). With respect to period 3, the number of possible records depends on which of two possible equilibrium scenarios obtain. One possible scenario is that firms which produced low quality at period 1 remain viable at period 2. Thus, under this scenario, there are three possible records at period 3: \(LL\) (low quality at both periods 1 and
2), LH (low quality at one of the periods and high quality at the other), and HH (high quality at each of the periods). The alternative scenario is that firms which produce low quality at period 1 are not viable at period 2. For the sake of brevity we shall consider only the former scenario here. Thus \( r_2 \in \{L, H\} \) and \( r_3 \in \{LL, LH, HH\} \).

Consider period 3. The analysis of this period is exactly the same as the base model when \( N = 3 \).

Using our previous notation, the vector which describes the probabilities of the possible records at period 3 of a competent firm which invests is denoted \( (\alpha_{LL}^I, \alpha_{LH}^I, \alpha_{HH}^I) = [(1 - \delta)^2, 2\delta (1 - \delta), \delta^2] \) and that of a firm which doesn’t invest or is incompetent is denoted as \( (\alpha_{LL}^{NI}, \alpha_{LH}^{NI}, \alpha_{HH}^{NI}) = [(1 - \beta)^2, 2\beta (1 - \beta), \beta^2] \). Let \( \Pi_3^{NI} \) denote the the ex ante expected profit at period 3 – as evaluated at the investment period 0 – of a firm which doesn’t invest.

\[
\Pi_3^{NI} = \sum_{r_3 \in \{LL, LH, HH\}} \alpha_{r_3}^{NI} \pi_{r_3}.
\]

where \( \pi_{r_3} \) is the period 3 profit of a firm with a record \( r_3 \). Let \( \Pi_3^I \) denote the ex ante expected profits at period 3 of a firm which does invest. With probability \( \mu \) it is competent, in which case its expected profit is \( \sum_{r_3 \in \{LL, LH, HH\}} \alpha_{r_3}^I \pi_{r_3} \) and with the complementary probability its profit is \( \Pi_3^{NI} \). Thus

\[
\Pi_3^I = \mu \cdot \sum_{r_3 \in \{LL, LH, HH\}} \alpha_{r_3}^I \pi_{r_3} + (1 - \mu) \cdot \sum_{r_3 \in \{LL, LH, HH\}} \alpha_{r_3}^{NI} \pi_{r_3}.
\]

Now consider period 2. The vectors of probabilities for investing and non investing firms respectively at this period are \( (\alpha_{L}^I, \alpha_{H}^I) = [1 - \delta, \delta] \) and \( (\alpha_{L}^{NI}, \alpha_{H}^{NI}) = [1 - \beta, \beta] \). \(^{11}\)

For sake of brevity we omit the full details of how prices are determined in period 2. The analysis is very similar to that of period 3, with the exception that the consumer’s search decision accounts for both her surplus at period 2 and the effect on her future surplus at period 3. We also note that it possible for \( p_L < c \). A firm may take a loss at period 2 in order to retain its costumers in period 3 (at which time its record might improve to LH). \(^{12}\)

\(^{10}\)MLRP is satisfied since

\[
\frac{\delta^2 - \beta^2}{\delta^2} > \frac{(1 - \delta) - \beta (1 - \beta)}{\delta (1 - \delta)} > \frac{(1 - \delta)^2 - (1 - \beta)^2}{(1 - \delta)^2} \iff 1 - \left( \frac{\beta}{\delta} \right)^2 > 1 - \frac{\beta (1 - \beta)}{\delta (1 - \delta)} > 1 - \left( \frac{1 - \beta}{1 - \delta} \right)^2
\]

which is satisfied as \( \frac{\beta}{\delta} < 1 < \frac{1 - \beta}{1 - \delta} \).

\(^{11}\)Again MLRP is satisfied.

\(^{12}\)Note that, because a firm is uninformed about its type, selling below cost is not a signal of type.
Denote the ex ante period 2 profit, as evaluated at period 0, of a firm which doesn’t invest and one which does, respectively, as $\Pi_2^{NI}$ and $\Pi_2^I$. Then, by the preceding arguments,

$$\Pi_2^{NI} = \sum_{r_2 \in \{L,H\}} \alpha_{r_2}^{NI} \pi_{r_2}$$

$$\Pi_2^I = \mu \cdot \sum_{r_2 \in \{L,H\}} \alpha_{r_2}^{I} \pi_{r_2} + (1 - \mu) \cdot \sum_{r_2 \in \{L,H\}} \alpha_{r_2}^{NI} \pi_{r_2}$$

Finally in period 1 $\Pi_1^I = \Pi_1^{NI}$.

**Return to investment**

Let $W$ be the total return to investment over periods 2 and 3 of a firm which invests. Then $W$ is equal to

$$W = \Pi_1^I + \Pi_2^I + \Pi_3^I - (\Pi_1^{NI} + \Pi_2^{NI} + \Pi_3^{NI}) = \Pi_2^I - \Pi_2^{NI} + \Pi_3^I - \Pi_3^{NI} = W_2 + W_3$$

where $W_t = \Pi_t^I - \Pi_t^{NI}$ is the portion of the total return to investment received in period $t$. Note that, for $t = 2, 3$, $\Pi_t^I - \Pi_t^{NI} = \mu \cdot \sum_{r_t} (\alpha_{r_t}^I - \alpha_{r_t}^{NI}) \pi_{r_t}$ is identical, up to the proportion factor $\mu$, to the equivalent term in the base model.

We can apply the results obtained in the base model to examine the effect of changes in the search cost in this dynamic setup. Since at period 2 there are only two possible records, we know from the analysis of Section 3 that $W_2$ is decreasing in $\gamma$ everywhere. Thus, $W$ decreases in $\gamma$ if $W_3$ is decreasing in $\gamma$ and can increase in $\gamma$ only if $W_3$ increases in $\gamma$. At period 3, there are three possible records $LL, LH, HH$. Suppose $2\delta (1 - \delta) < 2\beta (1 - \beta)$, that is, investment increases the probability of $HH$ and lowers the probability of $LL$ and $LH$. This case is the equivalent to the case $k_0 = N$ in the base model, where we know from Corollary 1 that the return to investment is decreasing in $\gamma$. Also, from the base model we know that the return to investment is decreasing in $\gamma$ if the $\pi_{LL} > 0$. So in both these cases $W_3$ is decreasing in $\gamma$. From Proposition 4, it follows that $W_3$ increases in $\gamma$ for sufficiently small $\gamma$ if: (i) $\beta < 0.5$ and $\delta < 1 - \beta$ and (ii) $\pi_{LL} = 0$.

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13Because $\Pi_t^I - \Pi_t^{NI}$ is proportional to the similar term in the base model, the results may be directly applied.
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