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Equity Capital, Bankruptcy Risk and the Liquidity Trap*

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Abstract

This paper explains the emergence of liquidity traps in the aftermath of large-scale financial crises, as happened in the US 1930s, Japan 1990s and recently in the US and Europe. The paper introduces a new balance sheet channel that links equity capital to the risk-free interest rate. When equity capital falls, bankruptcy risks rise. Firms become more vulnerable to external shocks, which makes financial disasters more likely to happen. Consequently, demand for safe assets increases, and the interest rate falls to the lower bound. Simulations show that the interest rate may stay at the lower bound for a long time.

Keywords: liquidity trap, financial crisis, rare disasters, equity capital, leverage, bankruptcy risk.

JEL classification: E32, E43, E44, E52, G12, G32.

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1 Introduction

The liquidity trap appeared in the 20th century twice, in Japan during the 1990s and in the US Great Depression in the 1930s. Both economies suffered from large-scale financial crises that were followed by an extended period of economic slump. The short-term interest rate dropped close to zero and stayed there for a long time. Yet it had no impact on investments and production and could not end the recession. Interest rates should have been much lower than zero, which was impossible due to the zero lower bound. Similar conditions have been developing in the US and Europe since 2008 as interest rates have fallen sharply to zero suggesting the revival of the liquidity trap. Interestingly, this third episode also followed a large financial crisis.

The emergence of liquidity traps in the aftermath of financial crises suggests that financial reasons lie at the root of the problem. The present paper offers a theory that relates liquidity traps to financial factors. It establishes a link between equity capital and the risk-free interest rate. Equity capital serves as a shock buffer ensuring that debt contracts such as bonds and deposits could be paid in bad times. When equity capital falls, bankruptcy risk rises. Firms become more vulnerable to external shocks, which makes financial disasters more likely to happen. As a result, demand for safe assets increases, and the risk-free interest rate declines. When equity capital falls too much, the risk-free interest rate drops to its lower bound and the economy slides into a liquidity trap.

In spite of the empirical coincidence of liquidity traps and financial crises, the literature has focused on non-financial explanations for the trap. Keynes (1936) mentioned the excess investment in physical capital taking place in the US and Great Britain following World War I, reducing the return on physical capital "more rapidly than the rate of interest can fall" (p. 219). Referring to the case of Japan, Krugman (1998) attributed the large interest rate decline to weak demand for investments, as agents anticipated low future growth due to Japan's aging population. Jeanne and Svensson (2007) provided a model that formalized Krugman's view, in which the liquidity trap was caused by an anticipated productivity fall. Eggertsson (2006, 2008), Eggertsson and Woodford (2003) and Auerbach and Obstfeld (2005) modeled

the liquidity trap differently by introducing an exogenous shock to agents' intertemporal utility function. In their model, a rise in the time discount factor reduced the real interest rate to zero, generating the conditions of a liquidity trap. A different approach is taken by Benhabib, Schmitt-Grohé and Uribe (2002). They argue that the liquidity trap can result from a monetary policy applying a Taylor rule.

The purpose of this paper is to demonstrate the role of financial factors in creating the liquidity trap, by linking the interest rate decline to the fall in equity capital. Indeed, the previous liquidity traps in the US and Japan have been associated with a significant contraction in equity capital, particularly within the banking sector. Hoshi and Kashyap (2010) estimate that the book value of Japan's bank capital (after adjusting for deferred tax assets and under-reserving) fell 68% between 1996 and 2003. Excluding capital injections by the government, the book ratio of capital-to-assets dropped from 3.3% in 1996 to a negligible level of 0.2% in 2003. The non-bank sector has also suffered from declining equity capital, as evident by the steep rise in bad loans and write-offs (Hoshi 2001, Fukao 2003). Eventually, the chronic shortage of capital has led to a wave of bank failures (Imai 2009, Hoshi and Ito 2004). Massive failures of banks and commercial business occurred also during the Great Depression, indicating that firms equity capital had been falling significantly (Bernanke 1983). Calomiris and Wilson (2004) show that the market capital-to-asset ratio of New York banks declined from 27% to 12% between 1929 and 1933, and the probability of bank default increased.

More recently, banks have lost some $2.2 trillion on securities and loans due to the sub-prime crisis (International Monetary Fund 2010a). Despite extensive government support and some improvement in capital markets, the market value of bank equity capital in 2009 was still half its pre-crisis level (Bank of England 2009). Moreover, non-performing loans have been rising significantly (International Monetary Fund 2010b), reflecting equity depletion of the non-bank sector as well. Hence, the liquidity trap conditions prevailing since 2008 have been associated again with lower equity capital and higher bankruptcy risk.

To relate the interest rate to firms' equity capital, the paper builds on the rare disaster literature developed by Rietz (1988) and Barro (2006). These papers show

that rare economic disasters can have a significant impact on the interest rate, even though the probability of such events is very low. The same principle is applied here. Low equity capital raises bankruptcy risk and therefore increases the probability of a massive wave of defaults. This event can be particularly harmful when it involves the collapse of the banking sector, as was the case during the Great Depression. Such disastrous events are usually very rare, but when equity capital falls they become more probable, thereby inducing a lower risk-free interest rate. The novel feature of the model is that the disaster probability is endogenous and changes according to firms' equity capital (which is also determined endogenously). By contrast, in Rietz (1988) and Barro (2006) the disaster probability is exogenous and fixed so the risk-free rate is constant. Later studies have explored the effect of time-varying disaster risk, but the disaster probability was still determined exogenously (*e.g.* see Gabaix 2008).

In this paper, disasters are defined as a wholesale default of the banking sector. Banks are modelled as corporate entities (firms) that borrow funds from consumers and invest them in risky assets. If the value of the bank's assets falls below its debt, the bank defaults. Since all banks are identical, they all default at the same time, triggering a financial disaster. However, it is not the disaster itself that drives the main results, but rather the probability that it would occur. This probability is determined by banks' capital structure, namely, the ratio of equity capital to debt. Therefore, to model the probability of a financial disaster we have to construct a setup where capital structure is determined in equilibrium and linked to the probability of default.

It is well known that capital structure is indeterminate in the absence of some sort of friction (Modigliani and Miller 1958). The macro literature, pioneered by Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and Holmstrom and Tirole (1997), adopted asymmetric information problems to construct models where firms' capital structure produces macroeconomic effects. This approach is well suited for small firms, where the owners of the firm are also the managers. In these cases there is a large information asymmetry between the shareholders (managers) and the creditors, who are outsiders to the firm. However, when it comes to financial

institutions, the scope of information asymmetry between investors is very limited. In most cases, the shareholders of the bank and the debt holders observe the same information about the bank. Only the bank managers, who usually own a tiny fraction of the bank capital, have an information advantage. Hence, information differences cannot explain the ratio of debt to equity capital chosen by the bank.

A natural way to model the capital structure of financial institutions is by applying risk sharing arguments. Allen and Gale (1988, 1994) have shown that in an incomplete market with heterogeneous agents, firms can raise their market value by selling securities with different risk profiles, *e.g.* equity and debt. If agents vary in their risk preferences but cannot trade risk freely because markets are incomplete, the issuance of heterogeneous securities improves their risk sharing. When applied to banks, this means that more risk averse agents would hold bonds and bank deposits, while less risk averse agents would hold equity shares of the bank. Namely, the shareholders and the bondholders of the bank share risk by holding different claims against the bank assets. Allen and Gale's (1988) model does not require any information asymmetry between shareholders and bondholders. In this respect, it is an attractive way to model banks' capital structure, as both types of investors are mostly outsiders to the bank. Interestingly, the macroeconomic literature has not explored this path yet. Hence, the present paper makes a contribution by studying macroeconomic effects of banks' balance sheets, using the risk-sharing approach to optimal capital structure.

The paper proposes a new balance sheet channel that has not been studied before. The usual balance sheet channel operates through informational frictions that create a linkage between firms' balance sheets and the external finance premium (Bernanke and Gertler 1989, Kiyotaki and Moore 1997, Holmstrom and Tirole 1997). In contrast, the present paper works through the impact of firms' balance sheets on aggregate bankruptcy risk and hence on the probability of financial disasters. Previous studies usually abstract from bankruptcy risk by constructing models where the equilibrium probability of default is zero, *e.g.* see Kiyotaki and Moore (1997), He and Krishnamurthy (2009), Gertler and Karadi (2010) and Jermann and Quadrini (2010). In Bernanke, Gertler and Gilchrist (1999) firms may default, but savers are

able to eliminate this risk and earn a perfectly safe return on their savings. Hence, their model does not contain a precautionary saving motive, which is essential in order to get asset price effects of aggregate risk. The present paper combines elements from the balance-sheet-channel literature with the asset-pricing-rare-disaster literature, in order to generate asset price effects of aggregate bankruptcy risk.

The model provides two main results. First, it shows that liquidity traps are likely to appear when bank capital drops significantly and the probability of bank default rises. This is a novel result in the liquidity trap literature, which has focused mainly on non-financial explanations for the trap. Second, the model suggests that the recovery from a liquidity trap may take a long time, during which banks accumulate new equity capital. The economy exits the trap when banks are well capitalized and the probability of default is sufficiently low. The long duration of the liquidity trap is consistent with past episodes which lasted more than a decade, and the current episode which is already few years long. This cannot be explained by standard business cycle models, in which the interest rate is determined solely by consumption dynamics through an Euler equation. In these models the interest rate tends to rise once the economy starts to recover after an initial negative shock. Hence, the interest rate hits the lower bound only for a short period, as in Gertler and Karadi (2010). The introduction of a disaster effect produces a different result. The interest rate may stay at the lower bound for a long time, as long as the banking sector continues to be poorly capitalized and highly vulnerable to negative shocks.

The paper proceeds as follows: Section 2 describes the model and section 3 derives the conditions for a general equilibrium. Section 4 elaborates on the choice of parameter values used to simulate the model. Then, section 5 presents the model results, and in particular the effect of equity capital on the risk-free interest rate. Section 6 imposes a lower bound on the risk-free interest rate and demonstrates how a large contraction in bank capital can create a liquidity trap. Section 7 concludes.

2 The Model

The basic setup is an overlapping generation model with heterogeneous agents. Agents consume in the first period of life and leave a bequest to their offspring in the second period. Their utility function is defined over consumption and bequest. This type of bequest motive is sometimes called "warm glow preferences" because agents derive direct utility from bequests. Acemoglu (2009) provides a detailed discussion on these models and their application in the literature. The main advantage of this setup is its tractability, as agents optimize over two periods only. This is particularly useful in models with heterogeneous agents and a corporate finance problem, *e.g.* Bernanke and Gertler (1989), Holmstrom and Tirole (1997). The two-period setup simplifies the dynamic dimension of the model and allows to focus the attention on the corporate finance issues, which provide the main insights of the model. The introduction of a bequest motive enriches the dynamic structure of the model and enables to address dynamic issues in a relatively simple framework.

Subsection 2.1 presents the optimization problem of the agents, which is fairly standard. Agents have different risk preferences. Some agents are risk neutral and others are risk averse. Markets are incomplete so the only assets available are equity and bonds issued by firms, which must be held in non-negative amounts (no short sales). The supply of equity and bonds is modelled in subsection 2.2, which presents the corporate finance problem of the firm. Finally, subsection 2.3 introduces the definition of a financial disaster and derives the distribution of asset returns. Equilibrium is solved in section 3.

2.1 Agents

There are two types of agents denoted A and B. The quantity of each type is normalized to 1. Both types live for two periods, consume in the first period and bequeath their wealth to newly born agents of the same type in the second period. The only difference between the two types is their utility from bequest. Type A agents have a linear utility from bequest while type B have an isoelastic (CRRA) utility. Hence, type A are indifferent to portfolio risk but type B are not. The

heterogeneity in risk aversion will produce heterogeneity in financial assets, namely, equity and bonds. In equilibrium, the risk-averse agents (type B) will hold bonds while the risk-neutral (type A) will hold equity[1].

Young agents of type A born in period t maximize the following utility function:

$$U_t^A = c_t^A + \beta E_t W_{t+1}^A, \quad 0 < \beta < 1 \tag{1}$$

subject to the budget constraints:

$$c_t^A = W_t^A - b_{t+1}^A - e_{t+1}^A - m_{t+1}^A, \tag{2}$$

$$W_{t+1}^A = b_{t+1}^A R_{t+1}^b + e_{t+1}^A R_{t+1}^e + m_{t+1}^A. \tag{3}$$

Superscript A stands for type A agents, c_t^A denotes consumption in period t of a single consumption good, W_t^A is wealth inherited from an old type A agent, and W_{t+1}^A denotes bequest in $t+1$. The variable b_{t+1}^A denotes bonds acquired in period t which pay the (state dependent) gross return R_{t+1}^b in $t+1$. Similarly, e_{t+1}^A denotes the value of equity shares acquired in period t paying a return R_{t+1}^e in $t+1$.

In addition to equity and bonds, agents can also save in storage. Storage is denoted by m because its role in this model resembles money in monetary models. It imposes a lower bound on the risk-free interest rate as storage is a perfectly safe asset with zero net return. In what follows, I will study two different cases. The first case assumes that saving in storage is not allowed so the risk-free interest rate is unbounded. The second case introduces a lower bound on the risk-free rate by allowing agents to save in storage. Comparing the two cases will enable to study the effects of the lower bound.

I assume that agents cannot borrow or make short sales of securities, so the holdings of bonds and stocks must be non-negative. This assumption provides a role for corporate finance, because the Modigliani-Miller Theorem (Modigliani and Miller 1958) does not hold under this condition in the presence of bankruptcy risk

[1]The same type of heterogeneity in risk-aversion can be found in Allen and Gale (1988), Bernanke, Gertler and Gilchrist (1999) and Gale and Özgür (2005).

(Hellwig 1981). Moreover, the assumption of no short sales ensures that firms behave competitively, as discussed in Allen and Gale (1991)[2]. The no-short-sales assumption implies also that bequests are non-negative because the returns on equity and bonds are non-negative (see below).

The optimization problem of type A agents yields the following first order conditions:

$$1+\phi = \beta E_t R^b_{t+1} \quad \geq \quad \text{if } b^A_{t+1} = 0, \tag{4}$$

$$1+\phi = \beta E_t R^e_{t+1} \quad \geq \quad \text{if } e^A_{t+1} = 0, \tag{5}$$

$$1+\phi = \beta \quad \geq \quad \text{if } m^A_{t+1} = 0, \tag{6}$$

where ϕ denotes the Lagrange multiplier associated with the constraint $c^A_t \geq 0$. It is clear from (6) that type A agents never hold storage ($m^A_{t+1} = 0$) since $\beta < 1$ and $\phi \geq 0$.

The FOC imply that type A agents save by holding the asset with the highest expected return, but only if this asset provides an expected gross return of at least β^{-1}. If the expected gross return is higher than β^{-1} they save all their wealth, and if it equals β^{-1} they may consume some of their wealth. Finally, if the expected gross return is lower than β^{-1}, type A agents consume all their wealth leaving no bequest.

Turning now to agents of type B, these agents have the following utility function:

$$U^B_t = c^B_t + \delta E_t \frac{\left(W^B_{t+1}\right)^{1-\theta}}{1-\theta}, \quad \theta > 1. \tag{7}$$

Their utility function differs from type A agents in the utility from bequest which has a constant relative risk aversion (CRRA)[3]. Hence, type B agents are risk-averse

[2]Allen and Gale (1991) show that when short sales are allowed, a small firm can have a large impact on equilibrium. By issuing a new security that does not exist in the market and cannot be spanned by other securities, the firm can change the consumption possibilities of the agents. Even if the amount of the issued security is negligible, the possibility to short sell it by all agents implies that it can have a non-negligible effect on agents' portfolios.

[3]It is possible to assume that agents differ also in their utility from consumption, which yields very similar results. The advantage of the current setting is that it generates a stationary wealth

with respect to their future wealth.

Type B agents maximize (7) subject to the same budget constraints (2) and (3) (where superscript B substitutes A). The first order conditions of this problem are given by:

$$1+\psi = \delta E_t \frac{R^b_{t+1}}{(W^B_{t+1})^\theta} \quad \geq \quad \text{if } b^B_{t+1}=0, \tag{8}$$

$$1+\psi = \delta E_t \frac{R^e_{t+1}}{(W^B_{t+1})^\theta} \quad \geq \quad \text{if } e^B_{t+1}=0, \tag{9}$$

$$1+\psi = \delta E_t \frac{1}{(W^B_{t+1})^\theta} \quad \geq \quad \text{if } m^B_{t+1}=0, \tag{10}$$

where ψ is the Lagrange multiplier of the constraint $c^B_t \geq 0$. These FOC yield the following saving rule for type B agents:

$$W^B_t - c^B_t = \min\left(\left\{\delta E_t(\Pi^B_{t+1})^{1-\theta}\right\}^{\frac{1}{\theta}},\ W^B_t\right), \tag{11}$$

where $\Pi^B_{t+1} \equiv \frac{W^B_{t+1}}{W^B_t - c^B_t}$ denotes the total return on agents B savings[4]. Hence, when the inherited wealth of type B agents is less than $\left\{\delta E_t(\Pi^B_{t+1})^{1-\theta}\right\}^{\frac{1}{\theta}}$, type B agents save all their wealth. When their wealth exceeds that level, they consume the surplus.

2.2 Firms

The corporate sector is composed of firms that exist for two periods in an overlapping generation pattern[5]. At a later stage of the paper I interpret firms as financial institutions (banks), but at this point they are modelled more generally, as representatives of the entire corporate sector (including banks). In the first period of their lifetime firms buy assets and issue equity and bonds. The assets yield a stochastic

distribution, which is harder to obtain when heterogeneity among agents is too large.

[4]To get (11), multiply (8), (9) and (10) by b^B_{t+1}, e^B_{t+1} and m^B_{t+1}, respectively, and sum them together to have $\left(W^B_t - c^B_t\right)(1+\psi) = \delta E_t\left(W^B_{t+1}\right)^{1-\theta}$. When c^B_t is positive $\psi = 0$ so $W^B_t - c^B_t = \left\{\delta E_t(\Pi^B_{t+1})^{1-\theta}\right\}^{\frac{1}{\theta}}$, which yields (11).

[5]This assumption is made for exposition clarity. It has no effect on the results.

return realized in period $t+1$, which is identical for all firms. Hence, risk is aggregate as it cannot be diversified away. When asset returns are realized, bondholders and shareholders are paid off. Firms might default on their debts if the realized value of their assets is lower than the promised debt. Since all firms are identical, bankruptcy means a collapse of the entire corporate sector, which is defined below as a financial disaster. In this sense, interpreting firms as banks is consistent with the model assumptions, because a failure of the entire banking system is likely to create a serious financial disaster.

The composition of equity and bonds determines the probability that the firm will default. Therefore, the main issue is how firms choose their optimal capital structure. The construction of the optimization problem of the firm follows Allen and Gale (1988). In their model, agents have heterogeneous risk preferences but markets are incomplete, so risk cannot be traded freely. Firms can increase their market value by issuing securities that vary in their payment structure, *e.g.* equity and bonds. This improves agents' risk sharing, because agents can share risk by holding assets with a different risk profile. Namely, agents that are more risk averse would hold safer assets.

To model the firm's problem, suppose that each firm is established by an entrepreneur. The entrepreneur buys the firm assets and sells claims against these assets in the form of equity and bonds. Hence, the decision about the capital structure of the firm is made by the entrepreneur. It is shown below that the identity of the entrepreneur has no effect on the firm optimal decision. Similarly, selling the firm to other agents would not change its optimal decision (see below). Once the entrepreneur sells all the securities of the firm to new investors his role in the firm terminates.

For simplicity, assume that any agent can become an entrepreneur and establish a new firm. This assumption will not change the optimization problem of the agents, because in equilibrium the gain of the entrepreneur is zero, as shown below. For the clarity of exposition, variables that are controlled by the entrepreneur are denoted by subscript i, and the other variables are denoted by a time subscript. The terms "firm" and "entrepreneur" are used interchangeably because the firm is fully owned

by the entrepreneur when it decides on its optimal capital structure.

Let a_i be the assets acquired by firm (entrepreneur) i. These assets yield a stochastic return of R^a_{t+1} paid in the next period $(t+1)$. Note that asset returns are identical for all firms so they are independent of i. The market value of the firm's assets in period $t+1$ is given by $a_i R^a_{t+1}$.

The firm issues claims against its assets in the form of equity and bonds. The total face value of the issued bonds is denoted X_i. Namely, the firm promises to pay its bondholders in period $t+1$ a fixed sum of X_i consumption goods. Hence, X_i is the promised payout to all bondholders. If $a_i R^a_{t+1} < X_i$ the value of the firm assets is less than the value of its debt so the firm defaults. In this case, bondholders receive the entire assets of the firm. In short, bondholders are paid $\min\left(X_i, a_i R^a_{t+1}\right)$. Shareholders get the remaining assets of the firm, after bondholders are paid. Hence, shareholders are paid $\max\left(a_i R^a_{t+1} - X_i, 0\right)$. Let $Y^j\left(a_i, X_i\right)$ denote the total payoff to the holders of security j of firm i, given the firm's assets (a_i) and debt (X_i). The index $j \in \{e, b\}$ can be an equity or a bond. Then, the payoffs to the bondholders and the shareholders are defined by the following functions, respectively:

$$\begin{aligned} Y^b\left(a_i, X_i\right) &= \min\left(X_i, a_i R^a_{t+1}\right), \\ Y^e\left(a_i, X_i\right) &= \max\left(a_i R^a_{t+1} - X_i, 0\right). \end{aligned} \tag{12}$$

The entrepreneur is interested in maximizing his gain from buying the firm assets and selling the firm securities. Let $V^e\left(a_i, X_i\right)$ and $V^b\left(a_i, X_i\right)$ denote the market value of equity and bonds (respectively) issued by firm i. These market values would depend on the capital structure of the firm, represented by the variables a_i and X_i. The target of the entrepreneur is to maximize his net gain (or net market value) by solving the following problem:

$$\max_{a_i, X_i} \quad V^e\left(a_i, X_i\right) + V^b\left(a_i, X_i\right) - a_i. \tag{13}$$

The expression $V^e + V^b$ is the market value at which the securities of the firm are

sold, and a_i is the cost of buying the assets of the firm. Hence, (13) maximizes the entrepreneur's gain from buying assets and selling securities.

The question is how to construct the functions $V^e(a_i, X_i)$ and $V^b(a_i, X_i)$. These functions provide the market value of equity and bonds for each choice of a_i and X_i. One possibility is to look at similar securities that circulate in the market and learn their prices. But what if the firm wants to set a_i and X_i at levels that do not exist in the market? How would the firm gauge the effect of its new capital structure on the market value of its securities? In a complete market the answer is simple. The value of any security is similar for all agents, because marginal rates of substitution between future and current consumption are equalized across all agents for each state. In this case, the pricing rule of one agent (say, the entrepreneur) represents the pricing of the entire market. However, when markets are incomplete this is not the case anymore. Agents may have different valuations for the same security, so it is necessary to construct the pricing function of the market. This pricing function is sometimes called conjecture or perception (see references in Bisin, Gottardi and Ruta 2009), because the firm cannot observe the market value for all possible choices of a_i and X_i, so it has to conjecture it.

Allen and Gale (1988) use a price conjecture originally proposed by Makowski (1983). According to this conjecture, the market value of a new security is determined by the highest price that any agent is willing to pay for buying a small amount of it. The finance literature has proposed other conjectures, which are discussed in detail by Bisin, Gottardi and Ruta (2009). The virtue of Makowski's conjecture is that it maintains the assumption that the firm behaves competitively. Namely, the firm takes market prices of all types of securities as given (Makowski 1983). In addition, if the firm is established by several entrepreneurs, they will all unanimously agree with the objective function (13), because they will all have the same price conjectures. Moreover, subsequent shareholders will also unanimously agree with the same objective function[6]. These properties ensure that the optimal choice of the firm is independent of the identity of its shareholders. Hence, trading the shares of this firm will not change its capital structure.

[6]References for these results are in Makowski (1983) and Bisin, Gottardi and Ruta (2009).

To present Makowski's price conjecture in the context of the present model, consider security j issued by firm i, where $j \in \{e, b\}$ is an equity share or a bond. As before, $Y^j(a_i, X_i)$ denotes the total payoffs to the holders of security j in period $t+1$. Let $V^{j,A}$ be the value of security j, at which type A agents are indifferent between buying and not buying a small amount of it. Namely, at that specific value type A agents would choose to buy exactly zero amount of the security. The corresponding first order condition of type A agents with respect to security j must hold with equality, because they are indifferent between buying and not buying the security. Hence, $V^{j,A}$ should satisfy the following first order condition of a type A agent:

$$1 + \phi = \beta E_t \frac{Y^j(a_i, X_i)}{V^{j,A}}.$$

The expression under the expectation term on the RHS is the return on security j in $t+1$, provided that its value in period t is $V^{j,A}$. Namely, this is the ratio between the payoff in $t+1$ to the holders of the security (Y^j) and the cost of buying the security in period t when its market value is priced by type A agents ($V^{j,A}$). Hence, this first order condition is similar to (4)-(5). We can now solve for $V^{j,A}$ and get:

$$V^{j,A}(a_i, X_i) = \frac{\beta}{1+\phi} E_t Y^j(a_i, X_i) \qquad \forall j \in \{e, b\}. \tag{14}$$

To understand (14), note that in equilibrium type A agents hold only assets with an expected return of $(1+\phi)/\beta$. Hence, they are indifferent between buying and not buying any asset that provides the same expected return. This implies that they will price any asset at a value that guarantees an expected return of $(1+\phi)/\beta$. This value is given by (14).

In a similar way we can derive the value of security j at which type B agents are indifferent between buying and not buying a small amount of it. Denote this value function by $V^{j,B}(a_i, X_i)$. This function is derived from the first order conditions of type B agents (8)-(9):

$$V^{j,B}\left(a_i, X_i\right) = \frac{\delta}{1+\psi} E_t \frac{Y^j\left(a_i, X_i\right)}{\left(W^B_{t+1}\right)^\theta} \qquad \forall j \in \{e, b\}\,. \tag{15}$$

Since all information is public, the firm knows the prices that agents A and B are willing to pay for a small amount of security j. Namely, the firm knows $V^{j,A}$ and $V^{j,B}$. Therefore, the firm expects to sell its securities for the maximum of the two bids. Thus, the market value of any security j conjectured by the firm would be:

$$V^j\left(a_i, X_i\right) = \max\{V^{j,A}\left(a_i, X_i\right), V^{j,B}\left(a_i, X_i\right)\} \qquad \forall j \in \{e, b\}\,. \tag{16}$$

For example, if type A agents are willing to pay more than type B for a small amount of equity shares of firm i, the market value of the shares would be determined by the bid of type A agents, which is $V^{e,A}$. On the other hand, the bonds of the firm might be valued more by type B agents, so their market value would be $V^{b,B}$.

The pricing formula (16) is identical to the way prices are set in a competitive market for goods. The equilibrium price is always the highest price for which a marginal amount of the good can be sold. Specifically, this is the maximum across all agents of the value at which they are indifferent between buying and not buying a small amount of the good. This is how (16) was constructed. Note that (16) is a general pricing function of all securities, whether they circulate in equilibrium or not. When a certain security does not circulate in equilibrium, its market value is determined exactly the same way, except that the market clears at zero supply for that specific security. For further discussion see Allen and Gale (1988). The firm takes the conjectured security prices as given, and picks a_i and X_i that maximize its objective function (13).

The complete optimization problem of the firm is defined by (13)-(16). Note that the objective function (13) exhibits constant returns to scale, because the payoff function $Y^j\left(a_i, X_i\right)$ is CRS for equity and bonds, see (12). The CRS property implies that entrepreneurs gain zero profits. Namely, in equilibrium the market value of the

securities of the firm is equal to the market value of the firm assets:

$$a_i = V^e\left(a_i, X_i\right) + V^b\left(a_i, X_i\right).$$

Since this is an equilibrium result, it is identical for all firms. Hence, we can write the same equation for the entire corporate sector, where a_{t+1} is the total amount of assets acquired by all firms in period t, and e_{t+1} and b_{t+1} denote the market value of all equity and bonds issued in period t (subscripts refer to the period that payoffs are made):

$$a_{t+1} = e_{t+1} + b_{t+1}. \tag{17}$$

Note that e_{t+1} and b_{t+1} are also the total savings of the two types of agents in equity shares and bonds. Hence, agents' savings in equity and bonds are equal in equilibrium to firms' assets.

Another useful result is the equilibrium returns on equity and bonds. The returns are defined by the ratio of period $t+1$ payoff to period t market value. The payoffs to equity holders and bond holders are denoted Y^e and Y^b and defined in (12). Dividing them by the equilibrium market values e_{t+1} and b_{t+1}, respectively, provides the following returns:

$$\begin{aligned} R^e_{t+1} &= \frac{\max\left(R^a_{t+1} - x_{t+1}, 0\right)}{1-\lambda_{t+1}}, \\ R^b_{t+1} &= \frac{\min\left(x_{t+1}, R^a_{t+1}\right)}{\lambda_{t+1}}, \end{aligned} \tag{18}$$

$$\text{where } x_{t+1} \equiv \frac{X_{t+1}}{a_{t+1}} \text{ and } \lambda_{t+1} \equiv \frac{b_{t+1}}{a_{t+1}}.$$

The variables x_{t+1} and λ_{t+1} are two closely related measures of firm leverage. Variable x_{t+1} denotes the ratio between the payoff that the firm promises to make to

its bondholders and the firm assets. Variable λ_{t+1} is slightly different. It is the ratio between the amount borrowed (b_{t+1}) and the firm assets. Note that the promised interest rate on the bonds is equal to x_{t+1}/λ_{t+1}. In the rest of the paper I will refer to λ_{t+1} as the firm leverage.

2.3 Asset returns and financial disasters

Uncertainty is introduced into the model by assuming that returns on the firm assets (R^a_{t+1}) are random. The risk of asset returns is modelled as an aggregate risk. Namely, all firms earn the same return on their assets. Aggregate risks come in many forms, such as productivity shocks, demand shocks, asset price fluctuations, inflation shocks and so on. The source of the risk is less important for the purposes of this paper. The main focus is on its impact on the probability of default. Hence, to keep the analysis simple I assume that asset returns fluctuate randomly without modelling the specific source of fluctuation[7].

The distribution of R^a_{t+1} is modelled as follows. Let $u_{t+1} \sim \text{LN}(g, \sigma^2)$ be a log-normally distributed variable. This is the only exogenous shock in the model. It provides the return on assets when firms are solvent. Using the notation $x_{t+1} \equiv X_{t+1}/a_{t+1}$, firms are solvent whenever $R^a_{t+1} \geq x_{t+1}$. Hence, $R^a_{t+1} = u_{t+1}$ when $u_{t+1} \geq x_{t+1}$. By contrast, when $u_{t+1} < x_{t+1}$ firms cannot be solvent. In this case firms default. We call this state a financial disaster. Financial disasters reduce asset returns by a fixed parameter d. Hence, in financial disasters asset returns fall to $u_{t+1}(1-d)$.

The assumption of a disaster effect d follows the rare-disaster literature of Barro (2006) and Rietz (1988). In the present model, disasters are states where firms default. Since all firms are identical they all default at the same time. When this (rare event) occurs asset returns fall sharply. The reasons for this fall are various. First, when all firms default production is severely damaged as firms enter bankruptcy proceedings. Second, credit costs may rise significantly due to the failure of the

[7] A possible interpretation of this framework is an AK model, where firms invest in physical capital (K) that provides a random return independent of K.

banking system, impairing production and profits. Third, financial institutions may be forced to fire sell their assets, generating a severe asset price meltdown. For these reasons, asset returns are expected to fall during a financial disaster. The total disaster effect is captured by parameter d.

The distribution of R^a_{t+1} can be summarized as follows:

$$R^a_{t+1} = \begin{cases} u_{t+1} & u_{t+1} \geq x_{t+1} \quad \text{(normal times)} \\ (1-d)u_{t+1} & u_{t+1} < x_{t+1} \quad \text{(financial disasters)} \end{cases} \tag{19}$$

where $u_{t+1} \sim \text{LN}(g, \sigma^2)$ and d is a fixed parameter.

The realization of u_{t+1} determines if firms are solvent or not. When $u_{t+1} \geq x_{t+1}$ firms are solvent and when $u_{t+1} < x_{t+1}$ firms default. Hence, the probability of a financial disaster is given by $F(x_{t+1})$, where F denotes the CDF of u_{t+1}. Having the distribution of R^a_{t+1}, we can describe the exact distributions of R^b_{t+1} and R^e_{t+1} using (18)-(19). These distributions are presented in Table 1.

The distribution of R^a_{t+1} is taken by the firm as given. Hence, at the firm level the disaster probability is exogenous. However, at the aggregate level the disaster probability is endogenous as it depends on firms' capital structure (x_{t+1}). If firms choose a safer structure, namely a lower x_{t+1}, disasters are less likely to occur. However, a small firm has no effect on the disaster probability. Even if a firm picks a completely safe capital structure with no debt at all, a financial disaster would still occur if all other firms default[8].

3 General Equilibrium

Equilibrium is attained when agents' consumption and asset allocation are optimal given the distributions of bond and stock returns, and the capital structure of the firm is optimal given security prices defined by (14)-(16). The general equilib-

[8]For that matter, a financial disaster should be more precisely defined as a default of almost all firms, but since firms are atomistically small the difference is negligible.

rium is solved in the first two subsections of this chapter. Subsection 3.1 solves the equilibrium portfolio choices of the two agents and shows that type A agents save in equity shares only and type B agents save in bonds and storage only. This result is used in subsection 3.2 to solve the remaining equilibrium variables and present the full system of equilibrium conditions. The last subsection derives the equilibrium risk-free interest rate, which will be important for the liquidity trap analysis in the following sections.

3.1 Agents' portfolio choices

Proposition 1 describes the optimal portfolios of agents A and B prevailing in equilibrium. The rest of this section provides the proof.

Proposition 1 *A general equilibrium is attained when: (a) type A agents do not hold bonds; (b) type B agents do not hold equity; and (c):*

$$E_t \frac{R^e_{t+1}}{(W^B_{t+1})^\theta} = E_t \frac{R^b_{t+1}}{(W^B_{t+1})^\theta}. \tag{20}$$

To prove Proposition 1, it is sufficient to show that when the conditions of the proposition hold, the first order conditions of the two types of agents are satisfied and firms obtain their optimal capital structure.

Condition (20) implies that type B agents are indifferent (at the margin) between equity and bonds, see (8)-(9). Hence, not holding equity is optimal for them. This proves that the FOC of type B agents hold.

Let $\bar{W}^B_{t+1}$ denote the bequest value of type B agents when firms are solvent, namely, when $R^e_{t+1} > 0$. Note that $\bar{W}^B_{t+1}$ is fixed, because type B agents save only in bonds and storage. Hence, when firms are solvent bonds are fully paid so the bequest of type B agents is at its maximum value. Therefore, we can rewrite (20) as follows:

$$\frac{E_t R^e_{t+1}}{\left(\bar{W}^B_{t+1}\right)^\theta} = E_t \frac{R^b_{t+1}}{\left(W^B_{t+1}\right)^\theta}.$$

Since $\bar{W}^B_{t+1} \geq W^B_{t+1}$ we get:

$$E_t R^e_{t+1} \geq E_t R^b_{t+1}. \tag{21}$$

Hence, not holding bonds is optimal for type A agents because the expected return on bonds is (weakly) lower than the expected return on equity. This completes the proof that the FOC of the two types of agents hold under Proposition 1.

I turn now to show that under the conditions of Proposition 1, the capital structure of the firm is also optimal. To do so, we have to show that the firm cannot raise its market value by picking a different capital structure. Due to constant returns to scale, the main control variable of the firm is the ratio of debt to total assets, denoted $x_i \equiv X_i/a_i$. Hence, the capital structure of the firm can be represented by x_i.

For exposition clarity, consider a firm with an asset value of 1. The firm will be denoted by i, so its asset value is $a_i = 1$. To save notation, I will henceforth denote dependence on capital structure by the single variable x_i, instead of the two previously used variables (a_i, X_i). As before, subscript i denotes variables that are controlled by firm i and subscript t (or $t+1$) denotes equilibrium values which are taken as given by the firm. For instance, the debt to asset ratio of firm i is denoted x_i, whereas the equilibrium debt to asset ratio is denoted x_{t+1}. To prove the optimality of the firm capital structure, I will show that the firm cannot raise its net market value by choosing $x_i \neq x_{t+1}$.

First, consider the market value of the other firms in the market. These firms have the equilibrium capital structure x_{t+1}. The equity shares of these firm are priced at the same value by the two types of agents, namely:

$$V^{e,A}(x_{t+1}) = V^{e,B}(x_{t+1}).$$

This follows directly from Proposition 1, which states that type A agents save in equity shares and type B agents are indifferent between buying and not buying shares.

Using (14) and (15) for $V^{e,A}(x_{t+1})$ and $V^{e,B}(x_{t+1})$ we get:

$$\frac{\beta}{1+\phi}E_t Y^e\left(x_{t+1}\right) = \frac{\delta}{1+\psi}E_t\frac{Y^e\left(x_{t+1}\right)}{\left(W_{t+1}^B\right)^\theta}, \tag{22}$$

where $Y^e\left(x_{t+1}\right)$ denotes the payoff to the firm shareholders as a function of the firm capital structure defined by (12). Recall that when $Y^e > 0$ the firm is solvent and bondholders are fully paid. We denoted the bequest of type B agents in these states by the fixed value $\bar{W}_{t+1}^B$. We can substitute this value in the RHS of (22), because $W_{t+1}^B = \bar{W}_{t+1}^B$ whenever Y^e is non-zero. This yields the following equilibrium result:

$$\frac{\beta}{1+\phi} = \frac{\delta}{\left(\bar{W}_{t+1}^B\right)^\theta\left(1+\psi\right)}. \tag{23}$$

Now, consider any security j issued by firm i with capital structure x_i. Agents A and B would price this security at $V^{j,A}\left(x_i\right)$ and $V^{j,B}\left(x_i\right)$, respectively. These values are defined by (14) and (15). It can be shown that[9]:

$$V^{j,B}\left(x_i\right) \geq V^{j,A}\left(x_i\right) \quad \forall j \in \{e, b\}. \tag{24}$$

Namely, type B agents price any security at a value that is at least as high as the pricing of type A agents.

To get the intuition of this result, note that the RHS of (23) provides the (equilibrium) marginal rate of substitution (MRS) between current consumption and future wealth of type B agents in states where firms are solvent. In these states, the wealth of type B agents is $\bar{W}_{t+1}^B$ so their MRS is constant. The MRS of type A agents is always constant and appears on the LHS of (23). Hence, the two types of agents have the same (constant) MRS across all solvent states. This implies that securities that pay only in solvent states are priced similarly by the two types of agents. However, in financial disasters firms default and the MRS of type B agents is larger because their wealth is lower. Hence, any security that pays in disaster states would have a higher value for type B agents than for type A agents. Therefore, type B agents

[9]The proof is as follows: Since $W_{t+1}^B \leq \bar{W}_{t+1}^B$, (15) implies that $V^{j,B} \geq \frac{\delta}{\left(\bar{W}_{t+1}^B\right)^\theta\left(1+\psi\right)}E_t Y^j$. Substitute (23) and use (14) to get (24).

price any security at a (weakly) higher value than type A agents, because it can provide hedging against disaster states (if it pays in disaster states).

Substituting (24) in (16) shows that the market value of any security is determined by the pricing function of type B agents. Hence, a small firm that issues equity and bonds would always expect to get the highest price by selling its securities to type B agents. Therefore, the total market value of the firm securities conjectured by the firm is given by the pricing of type B agents:

$$V^{e,B} + V^{b,B} = \frac{\delta}{1+\psi} E_t \frac{R^a_{t+1}}{(W^B_{t+1})^\theta}. \tag{25}$$

This result implies that the market value conjectured by the firm is independent of its capital structure. Namely, the firm cannot raise its market value by choosing $x_i \neq x_{t+1}$, so x_{t+1} is optimal for the firm. It is important to note that this is not a corollary of the Modigliani-Miller Theorem. It holds only under the conditions of Proposition 1, which state that type A agents do not hold bonds and type B agents do not hold equity. If agents allocate their savings differently, (25) would no longer hold, as shown in the technical appendix. The appendix further shows that Proposition 1 is the only equilibrium solution for agents' portfolio choices.

3.2 The complete system of equilibrium conditions

We can now summarize the system of equilibrium conditions and discuss how to solve the variables of the system. The starting point is Proposition 1, which states that type A agents do not hold bonds and type B do not hold equity shares, namely $e_{t+1} = e^A_{t+1}$ and $b_{t+1} = b^B_{t+1}$. We also know from (6) that type A never hold storage so $m^A_{t+1} = 0$. These results provide the following equilibrium conditions:

$$W_t^A - c_t^A = (1 - \lambda_{t+1}) a_{t+1}, \tag{26}$$

$$W_t^B - c_t^B = \frac{\lambda_{t+1} a_{t+1}}{1 - \mu_{t+1}}, \tag{27}$$

$$W_t^B - c_t^B = \min\left(\left\{ \delta E_t (\Pi_{t+1}^B)^{1-\theta} \right\}^{\frac{1}{\theta}}, W_t^B \right). \tag{28}$$

The variable λ_{t+1} was introduced earlier. It denotes the ratio of bonds to assets, *i.e.* $\lambda_{t+1} \equiv b_{t+1}/a_{t+1}$. This variable will be used below to analyse the dynamics of the model. It will be named the firm leverage, hence the notation λ. The new variable $\mu_{t+1} \equiv m_{t+1} / \left(W_t^B - c_t^B \right)$ denotes the share of storage in the savings of type B agents. The total return on the savings of type B agents is denoted $\Pi_{t+1}^B = (1 - \mu_{t+1}) R_{t+1}^b + \mu_{t+1}$.

Condition (26) states that type A agents hold all their savings in equity shares. Note that the value of equity shares issued in period t is $e_{t+1} = (1 - \lambda_{t+1}) a_{t+1}$. Condition (27) states that type B agents hold a share $1 - \mu_{t+1}$ of their savings in bonds (the value of bonds is $b_{t+1} = \lambda_{t+1} a_{t+1}$). The total savings of type B agents are given by (28) which replicates (11).

In addition, we have three more equilibrium conditions:

$$E_t \frac{R_{t+1}^e}{(\Pi_{t+1}^B)^\theta} = E_t \frac{R_{t+1}^b}{(\Pi_{t+1}^B)^\theta}, \tag{29}$$

$$E_t \frac{R_{t+1}^b}{(\Pi_{t+1}^B)^\theta} \geq E_t \frac{1}{(\Pi_{t+1}^B)^\theta} \qquad = \text{when } \mu_{t+1} > 0, \tag{30}$$

$$E_t R_{t+1}^e \geq \frac{1}{\beta} \qquad = \text{when } c_t^A > 0. \tag{31}$$

Condition (29) follows from (20), after multiplying both sides of (20) by $\left(W_t^B - c_t^B \right)^\theta$. Condition (30) follows from (8) and (10), where (8) holds with equality because type B agents hold bonds. Finally, condition (31) replicates (5). The distributions of R_{t+1}^e, R_{t+1}^b and R_{t+1}^a are described in Table 1.

It is convenient to describe the system with the new variable μ, which denotes the share of storage in agent B savings, instead of the level of storage m. Thus, the state variables of the system are a_t, λ_t, μ_t and x_t, which are determined one period in advance. Together with the realization of u_t we can solve the six variables of the system: c_t^A, c_t^B, a_{t+1}, λ_{t+1}, μ_{t+1} and x_{t+1} through the six conditions (26) to (31). To do so, we first have to calculate R_t^e and R_t^b through (18). Then we can calculate the wealth inherited by the two types of agents by adding the returns on their parents' portfolios, which yields:

$$W_t^A = (1-\lambda_t)a_t R_t^e$$
$$W_t^B = \frac{\lambda_t a_t}{1-\mu_t}\left\{(1-\mu_t)\,R_t^b + \mu_t\right\}.$$

In the next step, we express the expectation terms that appear in the system in terms of λ_{t+1}, x_{t+1} and μ_{t+1}. Then we get a system of six conditions with exactly six variables to solve.

The appendix shows that a solution exists for the set of parameter values that are used to simulate the model (discussed in section 4). When storage is ruled out (the no-storage case) the solution is unique. When storage is allowed (the full model), there can be at most three equilibria, but only one of them is economically plausible. This equilibrium is studied in the analysis below. The other two equilibria requires that type A agents consume all (or almost all) their wealth in one period. This could be optimal due to the assumption that utility from consumption is linear. Any upper bound on the utility from consumption or declining marginal utility would rule out these two special equilibria. For further discussion see the technical appendix.

3.3 The risk-free interest rate

The risk-free interest rate is the return on a perfectly safe asset. This rate will be used in the next section to define a liquidity trap as a state where the risk-free rate is at its lower bound. Normally, the quantity of safe assets in equilibrium is zero

because bonds usually bear some positive probability of default. The only exception is a liquidity trap equilibrium where agents hold storage (as storage is perfectly safe). Nevertheless, safe assets always have a price in equilibrium, even if their quantity is zero. Result (24) implies that the market value of any security is determined by the pricing function of type B agents. Hence, we can derive the risk-free rate through (15).

Consider a perfectly safe asset that pays one consumption good in period $t+1$ in all states. Denote the market value of this risk-free asset by V_t^f and the return on this asset by R_{t+1}^f, where $R_{t+1}^f = 1/V_t^f$. As explained, the market value of this asset is derived from (15). Using the first order condition (8) which holds with equality, it can be shown that R_{t+1}^f is equal to:

$$R_{t+1}^f = \frac{E_t \frac{R_{t+1}^b}{(W_{t+1}^B)^\theta}}{E_t \frac{1}{(W_{t+1}^B)^\theta}}. \tag{32}$$

Conditions (8) (with equality), (10) and (32) yield the following Lower Bound constraint:

$$R_{t+1}^f \geq 1. \tag{33}$$

This constraint is equivalent to (30) so it is already part of the equilibrium conditions summarized in the previous subsection. As we shall see, the Lower Bound will be binding in a liquidity trap situation, in which agents prefer to hold positive stocks of storage.

4 The Choice of Parameter Values

Before getting into the model results, I describe in this section the choice of parameter values used to simulate the model. The main parameters to be discussed are g, σ and d, which affect the distribution of asset returns (R_{t+1}^a). These returns are paid in $t+1$ for assets acquired in period t. The time period is interpreted as

one year, so the distribution of R^a_{t+1} refers to annual asset returns. In addition to the distribution of R^a_{t+1}, I explain the choice of θ which is the relative risk-aversion coefficient of type B agents. The parameters β and δ are less important for the results and discussed briefly in section 6.

The variable that corresponds to R^a_{t+1} is the aggregate return on firm assets, as the model assumes away idiosyncratic risks. Since aggregate returns on all firms are rarely available, I apply the model to banking firms. Banks serve as a good approximation of the firms in the model for three reasons. First, banks hold highly diversified portfolios and are thus substantially exposed to aggregate shocks. Second, the banking sector is usually the largest issuer of debt contracts (variable b_{t+1} in the model), mainly in the form of bank deposits[10]. Hence, the main risk of default embedded in households' bond portfolio is the default of the banking sector. Finally, the empirical literature provides ample evidence on the adverse effects of bank failures, *e.g.* see Bernanke (1983) and Dell'Ariccia, Detragiache and Rajan (2008). This evidence supports the assumption of a disaster effect in the event of a systemic bank default.

I start by evaluating the parameters g and σ which denote the mean and standard deviation of $\log u$. The random variable u is the return on bank assets (R^a) in normal times, as shown in (19). Table 2 presents annual bank returns in 30 OECD countries during the period 1980-2003. The data is aggregated at the country level, so these statistics reflect aggregate variations. The table presents the return on bank assets and the return on bank equity, corresponding to R^a and R^e. A note of caution is in order. The variable R^a that appears in the model is different from the accounting term Return on Assets (ROA), which is a common measure of bank profitability. The accounting ROA measures bank profits to total assets. Its model counterpart is the term $R^a - x$. Hence, the difference between the accounting ROA and the variable R^a is interest payments (as a ratio of total assets), represented in the model by x. Therefore, the R^a figures presented in table 2 were generated by adding interest

[10]Beck, Demirgüç-Kunt and Levine (1999) show that the asset size of the banking sector is around 55% of GDP in high-income countries, whereas the size of the private bond market is only 20%. In low income countries the differences between the two sectors are even larger.

payments (as a ratio of total assets) to the accounting ROA (before taxes)[11]. The result was adjusted to year end CPI inflation and transformed to log of the gross return.

The mean and standard deviation of $\log R^a$ in the OECD sample are 0.015 and 0.053, respectively (table 2). A large portion of the variation in R^a is due to inflation shocks, because bank assets are mostly nominal. However, banks hedge against inflation shocks by issuing liabilities that are also nominal. Hence, the standard deviation of R^a in the full sample overstates the actual risks that banks are exposed to, because banks hedge against some of these risks (the inflation shocks). Furthermore, the sample includes some bank crisis periods, which are modelled separately by the parameter d (though not all the bank crises in the sample fully comply with the definition of a "financial disaster"). To cope with these problems (mainly the inflation shock bias), I exclude from the sample periods of large inflation shocks[12] and bank crises. Then, the mean and standard deviation of $\log R^a$ change to 0.026 and 0.029, which are taken as proxies of g and σ, respectively.

The third parameter to be evaluated is d, which is the additional loss on firm assets accrued in times of financial disaster. In the context of banking firms, a financial disaster is defined as a complete failure of the banking sector. The closest example of such a rare event is the massive wave of US bank failures during the Great Depression. Out of the 25 thousand banks operating in 1929, about 9 thousand banks suspended operation during the years 1930-1933. Depositors of those suspended banks lost approximately 20% of their money (Board of Governors of the Federal Reserve System 1943). Moreover, the liquidation of insolvent banks took around 4 to 5 years (Treasury Department, Comptroller of the Currency 1940), so depositors' money was practically frozen for a long period of time. I take these figures as indicating the expected loss to bank depositors during financial disasters and

[11] More precisely, the accounting ROA equals $(R^a - 1) - \lambda\left(\bar{R} - 1\right)$, where $\bar{R}$ is the gross interest rate promised by the bank, namely $\bar{R} \equiv x/\lambda$. The term $\lambda\left(\bar{R} - 1\right)$ equals interest payments as a ratio of total bank assets. Hence, R^a can be reconstructed by adding interest payments to the accounting ROA.

[12] Large inflation shocks are defined as changes in annual CPI inflation that are larger than 3 percentage points

calibrate parameter d at 0.2. This means that debt holders (depositors) lose *at least* 20% of their wealth in the event of a financial disaster.

Finally, the parameter θ denotes the relative risk-aversion coefficient of agent B utility from bequest. The asset pricing literature usually calibrates this coefficient in the range of 2 to 5 (Barro 2006). In this literature the relative risk-aversion coefficient refers to consumption (and not wealth) in an infinitely-lived agent model. However, the Bellman representation of these models is similar to the present model, where the risk aversion coefficient relates to future wealth. It should be noted that the asset pricing literature attach the relative risk aversion coefficient to the entire population (the representative agent), while in the present paper it refers only to part of the population (the bondholders). Nevertheless, the application of the model to OECD banks implies that type B agents (the bondholders) hold almost the entire aggregate wealth, because equity capital is only 6% of total bank assets (table 2). Hence, I approximate θ by drawing from the the asset pricing literature. The parameter is calibrated at $\theta = 3$, as in Barro (2006).

5 Results Under No Storage

The results of the model are presented under two alternative assumptions. In this section I assume that agents are not allowed to invest in storage so μ (or equivalently m) is always zero. In this case there is no lower bound on the risk-free interest rate so the liquidity trap does not arise. The next section imposes the lower bound by removing the no-storage assumption. This change generates the conditions of a liquidity trap, which occurs when equity capital is too low. Comparing the results of the two sections demonstrates the economic consequences of the lower bound.

5.1 Firm leverage and asset prices

The leverage of the firm is defined by the ratio of bonds to total assets. It is denoted by λ, where $\lambda = b/a$. Figure 1 depicts the effect of λ on three variables: the expected return on equity, the bond interest rate and the risk-free interest rate. The

figure is derived through (29), which provides the asset pricing condition prevailing in equilibrium. Under the no-storage assumption ($\mu = 0$), this condition provides a unique solution to x for each λ (see technical details in the appendix). Having x we can calculate the expected return on equity (ER^e) and the risk-free interest rate (R^f). We can also calculate the interest rate on bonds which is denoted $\bar{R}$. It is defined by the ratio of the promised payoff to current market value of bonds, namely $\bar{R} \equiv X/b = x/\lambda$.

The main result shown in Figure 1 is the negative impact of leverage (λ) on the risk-free interest rate (R^f). This is the first novel result of the model. It links the interest rate to firms' capital structure. Thus, when equity capital falls and leverage rises, the risk-free interest rate declines. Note that in standard business cycle models, the risk-free interest rate is determined by an Euler equation, in which financial factors play no role. The present model suggests that financial factors may also be relevant and can change the dynamics of the interest rate. This point is discussed further in section 6.

To understand the source of the leverage effect on the risk-free rate, Table 3 provides further details on the other variables of the model for different leverage rates. The variable of interest which drives the results of the model is the probability of a financial disaster. Financial disasters occur in this model when all firms default. This happens when $u < x$, so the disaster probability is simply $\Pr(u < x)$. To calculate the disaster probability, I first solve x for each λ and then use the log-normal distribution of u to derive $\Pr(u < x)$. Consider for instance a leverage of 0.94, which corresponds to an equity-asset ratio of 6 percent observed in OECD banks. The disaster probability in this case is 1.6 percent. When leverage rises to 0.95, the disaster probability rises to 3.8 percent, and the risk-free interest rate falls from 1.9 to 1.0 percent.

The disaster probability affects the risk-free interest rate in two ways. First, there is a general effect on the mean return of the underlying asset, namely ER^a. When disasters become more probable ER^a falls due to the disaster effect d on the distribution of R^a [13]. Table 3 shows that ER^a drops 0.4 percentage points when

[13]To see this, note that the distribution of R^a in (19) implies that $E \log R^a = g + P \cdot \log(1-d)$,

the disaster probability rises from 1.6 to 3.8 percent. The effect on ER^a spreads out to other assets through equilibrium conditions, and thus affects also the risk-free interest rate.

The second effect comes from the convexity of the marginal utility of bequest of type B agents. Due to this convexity, the utility loss of type B agents in states of low bequest is not offset by states of high bequest. Hence, these agents have a natural demand for safe assets, which can hedge against states of financial disasters where bequests are low. A rise in the disaster probability raises the demand for safe assets as a hedge against disaster events, reducing the risk-free interest rate. In the above example, the risk-free rate falls 0.9 percentage points when leverage rises from 0.94 to 0.95. Part of this fall is due to the 0.4 percentage point decline in ER^a and the rest is due to the rise in the demand for safe assets.

Table 4 shows how different parameter values affect these results. The upper panel presents the effect of σ, which is the standard deviation of $\log u$. Higher σ implies higher volatility of asset returns. Overall, it produces a larger disaster probability (denoted P in the table) and a lower risk-free rate (R^f). This is because higher return volatility implies a higher probability of default (holding other variables constant). Note that type A agents are better off, because the expected return on equity is now higher. This reflects the fact that higher risk (*i.e.* larger σ) increases type B agents' demand for insurance against this risk. Since type A agents provide the insurance, they are better off.

The middle panel in Table 4 presents the impact of different values of d (the disaster effect). A rise in d also raises the volatility of asset returns (for a given λ). Hence, type A agents are again better off because the demand for insurance is higher. Thus, the expected return on equity should rise. To get this result, $\bar{R}$ must fall because ER^e is decreasing with $\bar{R}$ (holding λ constant). The default probability is equal to $\Pr(u < x)$ where $x = \lambda\bar{R}$. Hence, a lower interest rate $\bar{R}$ implies a lower default probability (given λ). The same effect applies when the degree of risk-aversion is rising (see the lower panel of Table 4), which also raises the demand for insurance. In both cases, the higher demand for insurance implies a lower risk-free interest rate

where P is the disaster probability.

and a lower default probability (because interest rates are lower).

5.2 Dynamics

I turn now to analyse the dynamics of the model under the no-storage assumption ($\mu = 0$). The dynamics is governed by the saving rules of the two types of agents. Type A agents save in equity capital and type B save in bonds. The accumulation of wealth of the two types of agents will determine the assets of the firm as well as the leverage (*i.e.* the ratio of bonds to total assets). Hence, the model dynamics will be presented through the evolution of two variables: assets (a) and leverage (λ), depicted by the vertical and horizontal axes in Figure 2.

Equilibrium condition (31) requires that $ER^e \geq \beta^{-1}$. Otherwise, type A agents consume all their wealth so there is no equity capital at all. Since ER^e is rising with λ (see Figure 1), there is a specific leverage rate for which $ER^e = \beta^{-1}$. Denote this rate by λ^S. It is depicted in Figure 2 by the vertical solid line at λ^S. The model can be in equilibrium only in the range $\lambda \geq \lambda^S$ in which $ER^e \geq \beta^{-1}$.

The second line in Figure 2 (denoted BB) is derived from the following condition:

$$\left\{\delta E\left(R^b\right)^{1-\theta}\right\}^{\frac{1}{\theta}} = \lambda a. \tag{34}$$

This condition holds when $W_t^B \geq \left\{\delta E_t(\Pi_{t+1}^B)^{1-\theta}\right\}^{\frac{1}{\theta}}$, see (27) and (28). In this case, type B agents inherit more wealth than they wish to save, so they save $\left\{\delta E_t(\Pi_{t+1}^B)^{1-\theta}\right\}^{\frac{1}{\theta}}$ and consume the rest. The LHS of (34) denotes the total amount of savings of type B agents in this case[14] (*i.e.* when inheritance is high). The RHS of (34) provides the total amount of bonds issued by firms which is equal in equilibrium to the savings of type B agents. It can be shown that the LHS of (34) is a function of λ. Thus, condition (34) represents a line on the (λ, a) plane. It is depicted in Figure 2 by the BB line.

[14]Note that under the no-storage assumption type B agents save only in bonds, so the total return on their savings is $\Pi_{t+1}^B = R_{t1}^b$ which is used in (34).

To understand the BB line, consider a certain leverage rate λ. For this leverage rate we have a certain distribution of bond returns (R^b). Hence, we can calculate the maximum amount of savings that type B agents wish to save, which is provided by the LHS of (34). If the initial wealth of type B agents is sufficiently high to allow them to save that amount, we get an equilibrium point on the BB line. If their initial wealth is too low they save less than the expression on the LHS of (34). Hence, firm assets are lower and we get a point below the BB line. An equilibrium point above the BB line is not possible, because it implies that type B agents do not save the optimal amount defined in (28). Hence, the equilibrium point must be in the area below or on the BB line in the range $\lambda \geq \lambda^S$.

The general tendency of the model is to converge to point S. When the model is in the area below the BB line and to the right of λ^S, both agents save all their wealth so a is likely to rise (unless the realization of u_t is very low, see below). Normally, the return on equity would be higher than the interest rate on bonds ($R_t^e > \bar{R}_t$) so shareholders get a higher return on their wealth than bondholders. Hence, equity capital would rise faster than bonds, and the model would move north-west towards S. Once the model reaches point S it stays there as long as u_t is sufficiently large so that $R_t^e, R_t^b > 1$. This implies that both types of agents have sufficient wealth to save the same amounts they have inherited, which means that the model stays at point S.

Suppose the model is at point S. In period 0 the economy is hit by a negative shock, namely, a low realization of u_t (asset return). If u_t is sufficiently low, firms incur a loss that reduces the wealth of their shareholders. This happens when $R_t^e < 1$, *i.e.* when $u_t < 1 + x_t - \lambda_t$. When $R_t^e < 1$, equity capital falls because the wealth of type A agents (the shareholders) shrinks. A new equilibrium is obtained with lower equity capital and higher leverage. Hence, the model deviates from point S to another point Q, which may be below the BB line or exactly on this line, as is the case in Figure 2.

As a result of the rise in firm leverage, the disaster probability also rises and the risk-free interest rate falls (as shown in Figure 1). Following the initial shock, shareholders start to re-accumulate their wealth so equity capital is starting to rise

and the model is moving to the left. As equity capital is rising, disaster probability is falling and the risk-free interest rate is rising back to its initial level. Eventually, the model returns back to point S.

6 The Lower Bound and the Liquidity Trap

I now remove the no-storage restriction to allow agents to save in storage ($\mu \geq 0$). The availability of storage imposes a lower bound on the risk-free interest rate. When the risk-free interest rate is above the lower bound, agents do not hold storage so $\mu_t = 0$ in equilibrium. Hence, the results of the previous section prevail. Conversely, when the risk-free interest rate declines below the lower bound, agents shift some of their wealth to storage so $\mu_t > 0$. In this case, the results of the previous section change, because the assumption of zero storage is no longer consistent with equilibrium conditions. Hence, the first step is to draw the line between equilibria with $\mu_t = 0$ and those with $\mu_t > 0$. This is done by finding the leverage rate (denoted λ^{LB}) at which $R^f = 1$ and $\mu = 0$. Figure 1 depicts the risk-free interest rate for varying levels of λ under the assumption of no-storage ($\mu = 0$). Hence, λ^{LB} is simply the intersection of the R^f line with the horizontal dotted line at 1. It is marked also in Figure 2 by the vertical dotted line at λ^{LB}.

The LB line divides the space in Figure 2 into two areas. In the left area the risk-free interest rate is above the lower bound, hence the lower bound is not binding and $\mu = 0$. The interesting area is to the right of the LB line. In this range, the risk-free interest rate would be below the lower bound if agents could not invest in storage. But since now agents are able to hold storage, they shift some of their wealth from risky investments to storage ("hoarding cash"), yielding $\mu > 0$. In what follows I study the effect of the lower bound on the model dynamics and compare the results to the case of no lower bound.

Suppose that the economy is at point S, at which $E_t R^e_{t+1} = \beta^{-1}$. Now, consider a large negative shock to u_t yielding $R^e_t < 1$, and assume that in the absence of a lower bound the model would move to point Q. Clearly, point Q is not an equilibrium when storage is allowed, since it lies to the right of the LB line. In this range, the risk-free

interest rate is below the lower bound if $\mu = 0$. Hence, at point Q bondholders strictly prefer storage over bonds, as bonds are perceived to be too risky. Therefore, bondholders reduce their bond holdings and shift some of their wealth to storage (yielding $\mu > 0$). As a result, firm assets and firm leverage decline. Equilibrium is attained at point L denoting a liquidity trap solution.

The comparison between points Q and L demonstrates the economic contraction associated with the liquidity trap. Point Q presents the model response to the shock in the absence of a lower bound on the risk-free interest rate, whereas point L describes the Lower Bound effect. If we interpret the variable a_t as a portfolio of real projects, then point L reflects a contraction in the stock of real projects. Similarly, we can interpret a_t as the stock of bank loans, so that point L involves a credit crunch. In any case, at point L agents prefer to hoard cash (storage) over making productive (albeit risky) investments, so the stock of productive investments declines, as evident by the transition from point Q to L.

Figure 3 provides a detailed description of the model response to an initial shock of one standard deviation to $\log u$. Namely, in period 0 the net return on assets $(R_0^a - 1)$ is equal to -0.3% and for the other periods the net return is 2.6%. The model starts at point S at which $E_t R_{t+1}^e = \beta^{-1}$. The parameter β is calibrated at 1.036^{-1}, which ensures that at point S the leverage (λ^S) is 0.94 (the mean in the OECD bank sample). The parameter δ has no effect except for scaling agents' wealth, so it is normalized to 1.

The first graph on the upper-left of Figure 3 presents bank assets (a_t) as a ratio of their initial level (a_0). When storage is allowed (the solid line), bank assets respond to the initial shock by contracting 21 percent. This contraction is demonstrated by the transition from S to L in Figure 2. By comparison, when storage is not allowed bank assets fall only 3 percent (the dashed line in the upper-left of Figure 3 and the transition from S to Q in Figure 2). The reason for this difference is the steep rise in storage depicted in the upper-right graph of Figure 3. The share of storage in type B savings (variable μ_t) rises from zero to 17 percent, thereby reducing the supply of funds available to banks.

Following the initial shock and the rise in storage holdings, storage starts to

decline gradually for the next 10 periods until it returns back to zero. Hence, the liquidity trap in this simulation lasts 10 years, during which the risk-free interest rate is at its lower bound (see the lower-left graph of Figure 3). The slow recovery from the liquidity trap stems from the low rate of growth of type A wealth (the shareholders). These agents lose 46 percent of their wealth in the first period ($R_0^e = 0.54$), and then gain an average of 2.9 percent each period for the next 10 periods.

The actual length of the liquidity trap depends on how the economy performs following the initial shocks. If negative shocks keep coming in, the trap would take longer. On the other hand, positive shocks would shorten the trap. For instance, if we temporarily raise bank income in period 1 by 10% (relative to the previous simulation), the duration of the liquidity trap decreases from 10 to 7 years. Since leverage is very high, positive shocks to bank income have a large effect on the return on equity. Thus, shareholders are able to recover their wealth much faster. But this, of course, works also in the opposite direction.

Another factor that can affect the duration of the trap is the volatility of asset returns. Volatility of asset returns plays an important role in the model because it determines the disaster probability, *i.e.* $\Pr(u < x)$. Table 4 presents the disaster probability and the risk-free interest rate for different volatility levels and leverage rates. Volatility is measured by σ which is the standard deviation of $\log u$. In general, larger volatility raises the disaster probability and lowers the risk-free interest rate. Hence, we can imagine that a negative shock to the economy that creates a severe recession would also change the expected volatility of asset returns. For instance, government bonds may become riskier as fears of a sovereign default rise during a prolonged recession. Since banks usually hold large amounts of government bonds, higher risk of government default implies a larger σ. In this case, the disaster probability increases and that would lengthen the liquidity trap.

It is interesting to compare these results with Gertler and Karadi (2010). Gertler and Karadi (2010) study a DSGE model with a leverage constraint on bank assets, derived from a moral hazard problem. Their simulations with a lower bound constraint produce a contraction in bank equity capital of around 50%, similar in magnitude to the simulation presented here in Figure 3. The interesting difference

regards the duration of the liquidity trap, namely, for how long the lower bound is binding. In their simulations, the interest rate hits the lower bound for several quarters only, compared to a period of several years in the present simulation. One of the reasons for this difference is the disaster effect, which is absent from Gertler and Karadi (2010). In standard business cycle models, as the one studied by Gertler and Karadi (2010), the risk-free interest rate is governed by consumption dynamics (adjusted for habit formation). Hence, once the economy starts to stabilize following the initial shock, the interest rate tends to rise. By contrast, in the present model the risk-free interest rate is strongly affected by the probability of bank default. If bank leverage is high, bankruptcy risk is also high and the interest rate remains at the lower bound, even if the economy is slowly recovering.

The long duration of the liquidity trap is consistent with current and past traps. The current episode, which started with the collapse of Lehman Brothers in 2008, is already several years long. In Japan, the interest rate has been practically zero since October 1995 (Eggertsson and Woodford 2003) to these days. Similarly, the three month yield on US Treasury Bills has been less than 0.5% from May 1932 to June 1947 (Board of Governors of the Federal Reserve System 1943, 1976). These long periods of zero interest rate can be explained, at least partially, by the story told in this paper[15].

7 Conclusions

This paper provides a theory that relates the liquidity trap to banks' capital structure. The paper shows that a fall in banks' equity capital can reduce the risk-free interest rate to the lower bound, creating a liquidity trap. This is a novel result, as previous studies have provided mainly non-financial explanations for the trap. It is also consistent with empirical facts showing that liquidity traps have been associated with a significant contraction in equity capital and a rise in bankruptcy risk, in particular within the banking sector.

[15]During the 1940s, the interest rate has probably been affected also by the risks associated with WWII, see Barro (2006).

The model suggests that a recovery from a liquidity trap may take a long time, which is also consistent with past and present episodes of liquidity traps. This cannot be explained by standard business cycle models, in which the interest rate is determined solely by consumption dynamics through an Euler equation. The introduction of a disaster effect working through banks' balance sheets produces a different result. The interest rate stays at the lower bound as long as the banking sector continues to be poorly capitalized and highly vulnerable to negative shocks.

An important element of the model is the assumption that households and firms take the disaster probability as given. Namely, they do not consider the impact of their financial decisions on the likelihood of financial disasters. However, their aggregate decisions do affect the disaster probability because they determine firm leverage and hence aggregate bankruptcy risk. Higher leverage exposes firms to higher probability of default, which raises the likelihood of having a financial disaster. In other words, financial decisions taken at the household and firm level have an externality effect, through their impact on the probability of financial disasters. This result justifies government intervention to reduce bankruptcy risks, especially during financial crises where these risks rise steeply.

One way to reduce bankruptcy risks is through government injection of equity capital into the corporate sector. Nevertheless, if the shortage in equity capital is too large, the required amount of public support might put the government itself at risk. Thus, instead of having a risky corporate sector, we might get a risky public sector, so the disaster probability would still be high. An alternative way is to use tax incentives to encourage firms to issue more equity capital and households to buy this capital. Assessing the effects of these policy measures is beyond the scope of the paper. However, the model proposed in this paper may be useful to study these issues, which I leave for future research.

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Table 1: The return distribution of assets, bonds and stocks in equilibrium

	State (u_{t+1})	Asset returns (R^a_{t+1})	Bond returns (R^b_{t+1})	Stock returns (R^e_{t+1})
Normal:	$u_{t+1} \geq x_{t+1}$	u_{t+1}	$\frac{x_{t+1}}{\lambda_{t+1}}$	$\frac{u_{t+1} - x_{t+1}}{1 - \lambda_{t+1}}$
Disaster:	$u_{t+1} < x_{t+1}$	$(1-d)u_{t+1}$	$\frac{(1-d)u_{t+1}}{\lambda_{t+1}}$	0

Table 2: Bank Financial Statistics in OECD Countries (1980-2003)*

		Return on Bank Assets[1] ($\log R^a$)		Return on Bank Equity[2] ($\log R^e$)		Equity to Assets[3] ($1-\lambda$)
	N	mean	s.d.	mean	s.d.	mean
All years	549	0.015	0.053	0.070	0.171	0.060
Non-crisis years	517	0.019	0.043	0.081	0.134	0.060
Bank crisis years	32	−0.044	0.126	−0.113	0.427	0.055
Excl. inflation shocks[4]	434	0.025	0.030	0.082	0.150	0.059
Non-crisis years	414	0.026	0.029	0.091	0.128	0.059
Bank crisis years	20	0.014	0.057	−0.092	0.348	0.053

* Source: OECD, Bank Profitability: Financial Statements of Banks (2004 edition).
Data is annual at the country level. Countries included: US (1980-2003), UK (1985-2003), Austria (1988-2003), Belgium (1982-2003), Denmark (1980-2003), France (1989-2003), Germany (1980-2003), Italy (1985-2003), Luxembourg (1980-2003), Netherlands (1993-2003), Norway (1980-2003), Sweden (1980-2003), Switzerland (1980-2003), Canada (1983-2003), Japan (1980-2003), Finland (1980-2003), Greece (1980-2003), Iceland (1982-2003), Ireland (1996-2003), Portugal (1980-2003), Spain (1980-2003), Turkey (1986-2003), Australia (1987-2003), New Zealand (1991-2003), Mexico (1989-2003), Korea (1991-2001), Czech Republic (1994-2003), Slovak Republic (1998-2003), Hungary (1995-2003), Poland (1994-2003).
Bank crisis years are taken from Dell'Ariccia, Detragiache and Rajan (2008).

[1] The return on bank assets in this table is different from the accounting measure of ROA (see explanation in the main text). It is calculated by annual profits before interest expenses and taxes over (previous year end) total assets. The result is adjusted for (year end) CPI inflation and transformed to log of gross return.

[2] The return on bank equity is calculated by annual profits before taxes over (previous year end) equity capital. The result is adjusted for (year end) CPI inflation and transformed to log of gross return.

[3] Equity capital to total assets, year end.

[4] This sub-sample excludes years where the inflation rate has increased or decreased by more than 3 percentage points compared to the previous year.

Table 3: Dependency of the main variables on firm leverage under no-storage ($\mu = 0$)

Leverage λ	Asset Return ER^a	Equity Return ER^e	Bond Return ER^b	Risk Free Rate R^f	Disaster Probability
0.92	1.026	1.028	1.026	1.026	0.002
0.93	1.026	1.030	1.025	1.024	0.006
0.94	1.024	1.036	1.023	1.019	0.016
0.95	1.020	1.049	1.018	1.010	0.038
0.96	1.012	1.074	1.009	0.995	0.077
0.97	0.998	1.117	0.995	0.970	0.146
0.98	0.975	1.194	0.971	0.935	0.261
0.99	0.933	1.340	0.929	0.888	0.466

The table presents equilibrium values of the model variables for different leverage rates, under the no-storage assumption ($\mu = 0$). The parameter values are $g = .026$, $\sigma = .029$, $d = .2$ and $\theta = 3$, see section 4 for details.

Table 4: Dependency of disaster probability (P), expected equity return (ER^e), bond interest rate ($\bar{R}$) and risk-free rate (R^f) on parameter values.

λ	P	ER^e	$\bar{R}$	R^f	P	ER^e	$\bar{R}$	R^f
		$\sigma = 0.019$				$\sigma = 0.029$		
0.93	0.000	1.027	1.027	1.027	0.031	1.046	1.026	1.013
0.94	0.001	1.027	1.027	1.026	0.055	1.061	1.026	1.003
0.95	0.004	1.029	1.026	1.025	0.092	1.084	1.026	0.988
0.96	0.016	1.036	1.026	1.020	0.147	1.121	1.026	0.968
0.97	0.053	1.058	1.026	1.005	0.227	1.176	1.028	0.942
		$d = 0.1$				$d = 0.3$		
0.93	0.006	1.028	1.027	1.026	0.006	1.033	1.026	1.021
0.94	0.017	1.031	1.027	1.024	0.016	1.043	1.026	1.011
0.95	0.039	1.037	1.027	1.020	0.035	1.065	1.025	0.994
0.96	0.083	1.050	1.027	1.014	0.071	1.104	1.025	0.965
0.97	0.158	1.072	1.028	1.003	0.131	1.172	1.024	0.924
		$\theta = 2$				$\theta = 4$		
0.93	0.006	1.028	1.027	1.025	0.006	1.033	1.026	1.023
0.94	0.017	1.031	1.027	1.021	0.016	1.043	1.026	1.018
0.95	0.039	1.037	1.027	1.013	0.036	1.063	1.025	1.007
0.96	0.083	1.050	1.027	1.000	0.072	1.100	1.025	0.989
0.97	0.159	1.072	1.028	0.977	0.133	1.166	1.024	0.963

The table presents the effect of parameter values on the disaster probability, the expected return on equity, the bond interest rate and the risk-free rate, under the no-storage assumption ($\mu = 0$). Parameter values in each panel are equal to the benchmark values in Table 3 except for a single parameter stated at the top of the panel.

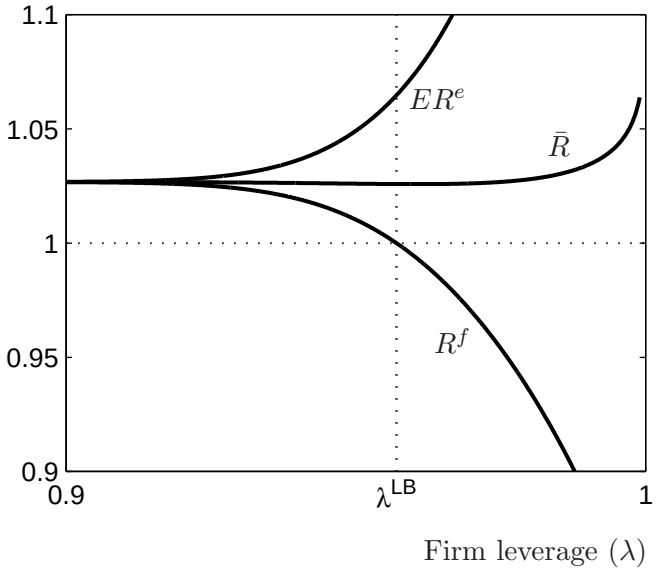

Figure 1: The dependency of asset returns on firm leverage under no-storage ($\mu = 0$)

The figure shows how asset returns are affected by firm leverage, under the no-storage assumption ($\mu = 0$). The figure presents the expected return on equity (ER^e), the interest rate on bonds ($\bar{R}$) and the risk-free rate (R^f) as functions of firm leverage (λ). The parameter values are $g = .026$, $\sigma = .029$, $d = .2$ and $\theta = 3$, see section 4 for details.

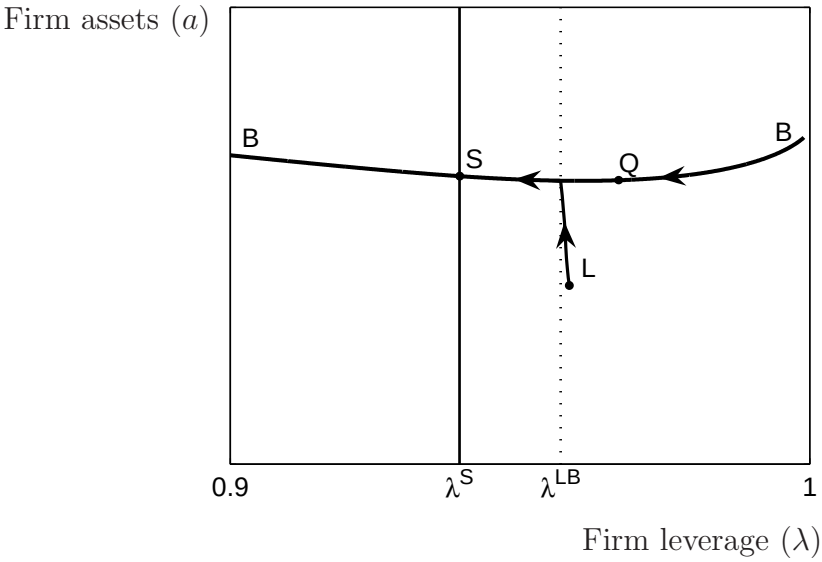

Figure 2: The dynamics of firm assets and leverage

The figure presents the model dynamics on the (λ, a) plane, starting at point S. Point Q presents the model response to one s.d. fall in $\log u$ under no-storage (no lower bound). Point L describes the model response when there is a lower bound on the risk-free rate. The parameter values are $g = .026$, $\sigma = .029$, $d = .2$ and $\theta = 3$, see section 4 for details. The parameter β is calibrated at 1.036^{-1} to ensure that $\lambda^S = .94$, namely, the model starts from the OECD leverage rate, see section 6.

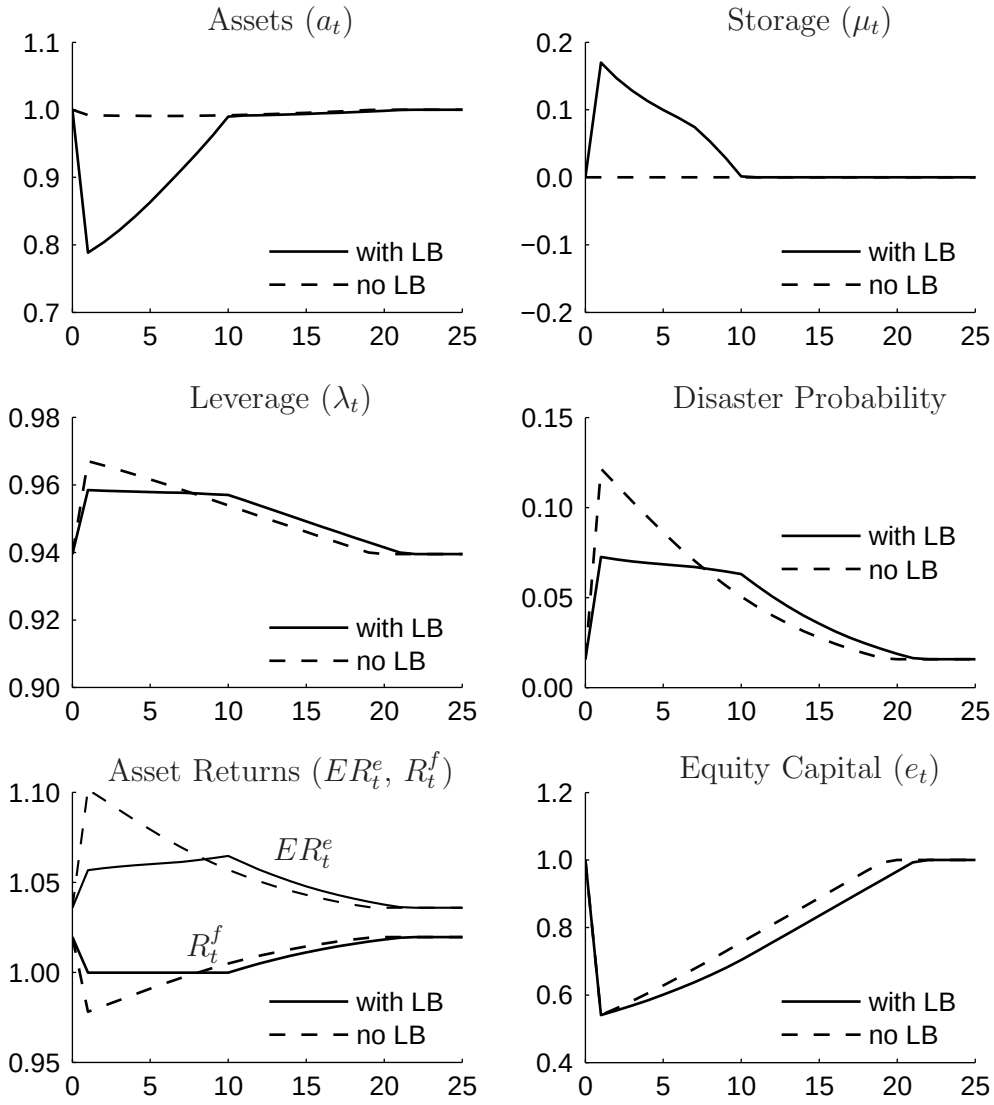

Figure 3: Impulse response functions with and without a lower bound (LB) on the risk-free interest rate

The figure depicts the dynamic response of the model variables to one standard deviation fall in $\log u$. Parameter values are as in Figure 2. The variables a_t and e_t are presented as ratios of their initial levels. The other variables are presented at their natural units. Solid lines present the model response when storage is allowed, so the risk-free interest rate has a lower bound (LB). Dashed lines present the model response when storage is not allowed, so the risk-free interest rate is not bounded (no LB). The periods are presented on the horizontal axis.