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False consciousness in financial markets: Or is it in ivory towers?

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False Consciousness in Financial Markets: Or is it in Ivory Towers?

By

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December, 2009

#### **Abstract**

In general, models in finance assume that investors are risk averse. An example of such a recent model is the pioneering work of Aumann and Serrano, which presents an economic index of riskiness of gambles which is independent of wealth and holds (as might be understood from the adjective "economic") for exclusively risk averse investors. In their paper, they discuss gambles with positive expected returns which will be accepted or rejected by agents which different levels of risk aversion. The question never asked by the authors (and in most of the finance literature) is: Who is offering these attractive gambles? To arrive at an answer, we extend the Aumann-Serrano risk index in such a way that it accommodates gambles with either positive or negative expectations and is thus suitable for both the risk averse and risk lovers. Once we allow for the existence of risk lovers, it may be shown that in financial markets, many gambles with negative expectations are taken either knowingly or unknowingly so that there are always people that act as if they are risk lovers. The paper concludes with a brief discussion of the implications of our result, in particular that gambling is by no means restricted to the casino or the track.

(The Journal of Gambling Business and Economics, Volume 4, no. 1, 2010, pp. 19-30.)

#### 1 Introduction

The current financial crisis has raised many questions surrounding the arguably excessive risks taken by various agents in financial markets in recent years. After all, it is not often that banks the size of Bear Stearns and Lehmann Brothers go under in the absence of even a rush on banks. And part of the problem is surely the cognitive dissonance that has accompanied the crash. This reaction is largely due to the fact that gambling is supposed to be carried out in casinos and at the track, while investment goes on in the stock, commodity,

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currency, real estate and other "genuine" financial markets, where "investment" implies that the agents are risk averse. An agent is generally defined as risk averse if she would not accept any gamble with zero or negative expected return. And gambles are generally defined as sums or distributions of the products of possible outcomes and the probabilities of their occurrence at a given time. And presumably, were investors provided with accurate expected returns of all gambles offered them in any given market, the vast majority may well demonstrate risk averse behavior, although this turns out to be an empirically un-testable hypothesis. The purpose of this paper is to demonstrate that, as soon as the existence of risk loving agents is permitted in theory, most of the financial markets we know today must contain them in practice.

We do so by showing that for well-defined gambles which are independent of other gambles, if one of the agents (either the one taking or the one offering the gamble) is risk averse, the other must be risk loving. Either that, or both are risk neutral. We call this the **Theorem of Risk Balance**. But if this is so, why do these markets exist and flourish? We begin with one obvious answer which need not be dealt with further in this paper: In many organizations which provide agents for financial markets, the incentive structure may not be consistent with risk averse transactions. Thus, the employee receives a large bonus if his risky bet wins while the risk averse owner of the firm foots the bill if it does not. The rogue traders who have made front page news during the past decade are merely extreme examples, *inter alia*, of this phenomenon at work. Less dramatically, in most cases of managed funds known to the authors, the manager shares in profits from investment (if not also the principle) and does not share in losses.

More important for our purposes is the fact that objective probabilities of outcomes are almost never known in any financial markets (casinos and perhaps lotteries being the ironic exceptions) and the outcomes themselves are rarely known. Thus, if Mary buys a stock today she has no idea what it might be worth next Monday let alone what probabilities to attach to various possible outcomes. And if John sells an option today, he knows that he can gain at most the premium from the sale but has no way to calculate expected losses. This, of course, follows directly from the assumed absence of free lunches in economics. In this case, market agents suffer from false consciousness because, believe as they may that they are risk averse they have no way to accept or reject this hypothesis.

But there is an alternative story that must be considered. Perhaps many participants in financial markets are knowingly risk lovers! In other words, in preference to casinos and the track, they prefer the higher risk gambles offered by, for example, selling naked options. This is an alternative ruled out by academics in economics and finance, perhaps because they truly believe that no one could possibly be risk loving, or could it be that they simply prefer the

calculus of "well-behaved" concave utility functions in their modeling? Either way, in this case, false consciousness would reside in the ivory tower.

A modern text demonstrates the problem well:<sup>2</sup> To determine whether students understand the difference between speculation (which is presumed compatible with risk aversion, since speculation, unlike gambling, is a socially acceptable feature of financial markets) and gambling (risk loving behavior and a "bad" thing), they are asked whether a US investor who faces exchange rate risk when buying UK T-bills is engaged in speculation or gambling. The last two sentences of their answer – after explaining what is involved in the mechanics of the trade – are: *Therefore, if the investor expects favorable exchange rate movements, the UK bill is a speculative investment. Otherwise, it is a gamble.* But, of course, nowhere in the remaining nine hundred-plus pages of the book, where gambling is never mentioned again, are students informed how to calculate expected returns from investments involving exchange rate risk and thus there is no way of applying this tautology in the real world. And they are not informed because the authors know well that according to financial theory there is no knowable way to predict movements in foreign exchange with any accuracy.

We proceed as follows. In the next section, we take the economic model of riskiness developed by Aumann and Serrano, 2007 (hereafter AS) and extend it to permit the existence of risk lovers. Our purpose is to discover whether there is a connection between risk averse and risk loving traders in financial markets once such a connection is no longer *a priori* ruled out as by AS and most other authors. We shall show that once the index is generalized it does indeed suggest a compelling connection between risk averse and risk loving traders. We call this result the *Theorem of Risk Balance* and conclude the paper with a brief discussion of its meaning and application to real financial markets.

### 2 Generalization of the Aumann and Serrano Index of Riskiness

Following [AS]<sup>3</sup>, we introduce and discuss the notion of a *generalized index of* economic riskiness, with no a priori assumptions about attitudes toward risk. A *utility function* is

<sup>3</sup> For an alternative approach to the derivation of the [AS] index, see Hart (2008).

<sup>&</sup>lt;sup>2</sup> See Brodie, Kane and Marcus (2009), pp. 158, 186.

a strictly monotonic twice continuously differentiable function u defined over the entire line. We normalize u so that

$$u(0) = 0$$
 and  $u'(0) = 1$  (1)

If u is concave then an agent with a utility function u is risk averse, while if u is convex, then an agent with a utility function u is risk lover.

The following definition is due to Arrow (1965 and 1971) and Pratt (1964):

**Definition 1.1** The coefficient of absolute risk of an agent i with utility function  $u_i$  and wealth w is given by:

$$\rho_i(w) = \rho_i(w, u_i) = -u_i''(w)/u_i'(w)$$

Note  $u_i(x)$  is concave in a neighborhood of w if and only if  $\rho_i(w) > 0$ , while if it is convex if and only if  $\rho_i(w) < 0$ .

Along the lines of [AS, Lemma 2,3] we show:

**Lemma 1.2** Let agents i and j have normalized utility functions  $u_i$  and  $u_j$  and Arrow-Pratt coefficients  $\rho_i$  and  $\rho_j$  of absolute risk aversion. Then

- 1. For each  $\delta > 0$ , suppose that  $\rho_i(w) > \rho_j(w)$  at each w with  $|w| < \delta$ . Then  $u_i(w) < u_i(w)$  whenever  $0 \neq |w| < \delta$ .
  - 2. If  $\rho_i(w) \le \rho_i(w)$  for all w, then  $u_i(w) \ge u_i(w)$  for all w.

**Proof.** 1. Let  $|y| < \delta$ . If y > 0. Then by (1),

$$\ln u_i'(y) = \ln u_i'(y) - \ln u_i'(0) = \int_0^y (\ln u_i'(z))' dz = \int_0^y u_i'(z)/u_i'(z) dz$$
$$= \int_0^y -\rho_i(z) dz < \int_0^y -\rho_j(z) dz = \ln u_j'(y)$$

If y < 0 then the inequality is reversed by the same arguments. Thus,  $\ln u_i'(y) \ge \ln u_j'(y)$  whenever  $y \ge 0$ , so also  $u_i'(y) \ge u_j'(y)$  whenever  $y \ge 0$ . So if w > 0 then by (1)

$$u_i(w) = \int_0^w u_i'(y) dy < \int_0^w u_j'(y) = u_j(w)$$

and similarly by using the reverse signs, when w < 0.

2. In parallel to the first part, with i and j interchanged, strict inequalities are replaced by weak inequalities and the restriction to  $|w| < \delta$  is eliminated. QED

Let agent i have utility function  $u_i$ , and let w be a real number. We say that i accepts g at w if  $Eu_i(w+g) > u_i(w)$ , where E stands for "expectation", otherwise, i rejects g at w. We show:

**Proposition 1.3** Let agents i and j have normalized utility functions  $u_i$  and  $u_j$  and Arrow-Pratt coefficients  $\rho_i$  and  $\rho_j$  of absolute risk aversion. If  $\rho_i(w_i) > \rho_j(w_j)$  then there is a gamble g that j accepts at  $w_i$  and i rejects at  $w_i$ .

**Proof.** Without loss of generality assume  $w_i = w_j = 0$ , so  $\rho_i(0) > \rho_j(0)$ . Since  $u_i$  and  $u_j$  are twice continuously differentiable it follows that there is a  $\delta > 0$  so that  $\rho_i(w) > \rho_j(w)$  for all  $|w| < \delta$ . Moreover, for  $\delta$  small enough  $u_i$  and  $u_j$  are each either concave or convex in the interval  $(-\delta, \delta)$ . It follows from Def.1.1 that if  $u_j$  is concave then  $\rho_i(w) > \rho_j(w) > 0$  and so  $u_i$  is concave as well, and if  $u_i$  is convex so is  $u_j$ . Now, by Lemma 1.2.1,

$$u_i(w) < u_j(w)$$
 whenever  $0 \neq |w| < \delta$  (2)

(I) Assume first that  $u_j$  is concave. Choose  $\varepsilon$  with  $0 \le \varepsilon \le \delta/2$ . For  $0 \le x \le \varepsilon$  and k = i, j, set

$$f_k(x) := \frac{1}{2}u_k(x-\varepsilon) + \frac{1}{2}u_k(x+\varepsilon)$$

By (2),

$$f_i(x) < f_j(x)$$
 for all  $x$  (3)

By (3), concavity of  $u_i$ , and (1),

$$f_i(0) < f_j(0) < u_j(0) = 0.$$

On the other hand, by monotonicity of the utilities,

$$f_i(\varepsilon) = \frac{1}{2}u_i(2\varepsilon) > \frac{1}{2}u_i(0) = 0$$

Since  $f_i$  is continuous, it follows that  $f_i(y) = 0$  for some  $0 < y < \varepsilon$  and so by (3),  $f_j(y) > 0$ . So if  $\eta > 0$  is sufficiently small then  $y - \eta > 0$  and

$$f_i(y-\eta) < 0 < f_i(y-\eta)$$

Let g be the half-half gamble yielding  $-\varepsilon + y - \eta$  or  $\varepsilon + y - \eta$  Then

$$Eu_i(g) = f_i(y-\eta) < 0 < f_i(y-\eta) = Eu_i(g)$$

Hence j accepts g at 0 while i rejects it.

(II) Now assume that  $u_i$  is convex. For k = i, j, define  $\widetilde{u}_k$  by

$$\widetilde{u}_{\iota}(x) = -u_{\iota}(-x)$$

Then

$$\mathcal{U}_{Q}(0) = 0$$
,  $\mathcal{U}_{k}(x) = u \not\in -x$ ,  $\mathcal{U}_{k}(0) = 1$  and  $\mathcal{U}_{k}(x) = -u \not\in -x$ .

Moreover, by (2),

$$\tilde{u}_i(w) < \tilde{u}_i(w)$$
 whenever  $0 \neq |w| < \delta$ 

Since  $\tilde{u}_i$  is concave in the interval  $(-\delta, \delta)$ , we are in the same situation as in (I), with i and j interchanged, thus there is a gamble g so that

$$E\widetilde{u}_{j}(g) < 0 < E\widetilde{u}_{i}(g)$$

Take the gamble -g, then

$$Eu_i(-g) = -E\widetilde{u}_i(g) < 0 < -E\widetilde{u}_j(g) = Eu_j(-g)$$

Hence j accepts -g at 0 while i rejects it.

(III) Finally assume  $u_i$  is concave and  $u_j$  is convex. This case is the simplest. Let g be the half-half gamble yielding  $-\varepsilon$  or  $\varepsilon$ . Then

$$Eu_i(g) < 0 < Eu_i(g)$$

Thus i rejects g and j accepts it.

For arbitrary  $w_i$  and  $w_j$ , define  $u_i(x)$  by  $u_i^*(x+w_i)$  and  $u_j^*(x)=u_j(x+w_j)$  and apply the following to  $u_i^*$  and  $u_j^*$ . QED

**Definition 1.4** Call i at least risk averse or no more risk loving than j (written  $i \succeq j$ ) if for all levels  $w_i$  and  $w_j$  of wealth, j accepts at  $w_j$  any gamble that i accepts at  $w_i$ . Call i more risk averse or less risk loving than j (written  $i \succ j$ ) if  $i \succeq j$  and  $j \upharpoonright i$ .

As a corollary of Prop.1.3 we have:

**Corollary 1.5** Given agents i and j, then

$$i \ge j \square r_i(w_i)^3 r_j(w_j)$$

for all  $w_i$  and  $w_i$ .

**Proof.** Assume  $i \ge j$  and assume there are  $w_i$  and  $w_j$  with  $\rho_i(w_i) < \rho_j(w_j)$ . By Prop.1.3, there is a gamble g that i accepts and j rejects, a contradiction. So  $\rho_i(w_i) \ge \rho_j(w_j)$  for all  $w_i, w_j$ .

Assume now  $\rho_i(w_i) \ge \rho_j(w_j)$  for all  $w_i, w_j$ . We wish to show that for all  $w_i$  and  $w_j$  and any gamble g, if i accepts g at  $w_i$  then j accepts g at  $w_j$ . Without loss of generality assume  $w_i = w_j = 0$ . Then Lemma 1.2.2 with i and j interchanged implies  $u_j(w) \ge u_i(w)$  for all w. Hence  $Eu_j(g) \ge Eu_i(g)$  for all g implying the desired result. QED

**Definition 1.6** An agent is said to have Constant Absolute Risk (CAR) utility function if his normalized utility function u(x) is given by

$$u_{\alpha}(x) = \begin{cases} \alpha^{-1}(1 - e^{-\alpha x}), & \alpha \neq 0 \\ x & \alpha = 0 \end{cases}$$

If  $\alpha > 0$  then the agent is risk-averse with a CARA utility function, while if  $\alpha < 0$  then the agent is risk-loving with a CARL - Constant Absolute Risk-Loving - utility function . If  $\alpha = 0$  then the agent is risk neutral. The notion of ``CAR" is justified since for any  $\alpha$ , the coefficient of absolute risk  $\rho$  defined in Def.1.1, satisfies  $\rho(w) = \alpha$  for all w, that is, the

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<sup>&</sup>lt;sup>4</sup> Note that in [AS] the above is defined for risk averse agents only, and is denoted by "i is at least as risk averse as j".

Arrow-Pratt coefficient is a constant that does not depend on w. We have thus a sheaf of functions  $u_{\alpha}$  satisfying for all x:

$$u_{\alpha}(x)$$
 is continuous at  $\alpha = 0$ .

To see this, we need to show that for all x,  $\lim_{a \oplus 0} u_a(x) = x$ , where we consider the two sided limit. Now, if x = 0 then for all a,  $u_a(x) = x$  and if  $x \neq 0$  then

$$\lim_{\alpha \to 0} \alpha^{-1} (1 - e^{-\alpha x}) = \lim_{\alpha \to 0} x e^{-\alpha x} = x$$

Given any  $\beta$ , observe that

$$Eu_{-\beta}(g) = -Eu_{\beta}(-g) \tag{4}$$

Indeed, assume g results in  $\{x_1, \dots x_n\}$  with respective probabilities  $p_1, \dots p_n$ . Then

$$Eu_{-\beta}(g) = \sum p_i u_{-\beta}(x_i) = \sum p_i (-\beta)^{-1} (1 - e^{\beta x_i}) = -Eu_{\beta}(-g)$$

The CARA version of following proposition is proved in [AS, Prop.4.1]. We state here the general case.

**Proposition 1.7** An agent i has CAR utility function if and only if for any gamble g and any two wealth levels, i either accepts g at both wealth levels, or rejects g at both wealth levels.

**Proof.** Any CAR utility function  $u_{\alpha}$  accepts g if and only if  $-Ee^{\alpha(g+w)} > -e^{-\alpha w}$ , that is if and only if  $Ee^{-\alpha g} > 1$  which is independent of w. Conversely, assume an agent i so that  $\rho_i(w_*) < \rho_i(w_*)$  for wealth levels  $w, w_*$ . If  $\rho_i(w) > \rho_i(w_*) > 0$  then we can follow the proof at [AS]. The proof there is based on the formula  $\rho(w) = \lim_{\delta \to 0} (p_{\delta}(w) - 1/2)/\delta$  where  $p_{\delta}(w)$  is that p for which i is indifferent at w between taking and not taking the gamble yielding  $\pm \delta$  with probabilities p and 1-p respectively. (This formula can be found in e.g. Aumann and Kurz (1977), Section 6). It is then used to construct another gamble which is rejected by i at w but accepted at  $w_*$ .

If  $\rho_i(w) < 0$ , then as in 1.3, define  $\widetilde{u}_i(x) = -u_i(-x)$ . Then  $\rho_i(\widetilde{u}_i, w)$  is positive and we have a gamble g accepted at one level and rejected at the other for  $\widetilde{u}_i$ . Replacing g by -g concludes the proof for this case.

If  $\rho_i(w_*) < 0 < \rho_i(w)$  then for  $\delta$  small enough a half-half gamble resulting  $\pm \delta$  will be accepted at  $w_*$  and rejected at w. QED

The next theorem verifies the existence of the general index for the following class of gambles. A gamble g is *gameable* if it results in possible losses and possible gains. If g has a continuous distribution function, then it is gameable if it is bounded from above and below, that is, its distribution function is truncated.

**Theorem 1.8** Let g be a gameable gamble. Then there exists a unique number  $\alpha$ , so that, for any wealth, a person with utility function  $u_{\alpha}$  is indifferent between taking and not taking g. In other words, the CAR utility function  $u_{\alpha}$  satisfies for all x,

$$Eu_{\alpha}(g+x) = u_{\alpha}(x).$$

Moreover,  $\alpha$  is positive (negative) if and only if Eg is positive (resp. negative),

**Proof.** Define a map  $f(\alpha)$  by

$$f(\alpha) = 1 - Ee^{-\alpha g} \tag{5}$$

Since g is gameable it follows first that  $f(\alpha)$  is defined for all  $\alpha$ , and then, since it results positive and negative values, we have  $\lim_{\alpha \to \infty} f(\alpha) = -\infty$  and  $\lim_{\alpha \to \infty} f(\alpha) = -\infty$ .

Now.

(i) 
$$f(0) = 0$$
 (ii)  $f'(0) = Eg$  (iii)  $f''(\alpha) < 0 \,\forall \alpha$ 

By (iii) f is concave, hence has at most two roots, one of which is zero. If Eg > 0 then f increases at 0, hence the second root  $\alpha$  is positive. If Eg < 0 then f decreases at 0, hence the second root is negative. If Eg = 0 then  $\alpha = 0$  is the only root.

To show the last part note that if  $\alpha \neq 0$  we have by definition

$$Eu_{\alpha}(g) = \alpha^{-1}(1 - Ee^{-\alpha g}) = 0$$

It follows that for all x,

$$Eu_{\alpha}(x+g) = \alpha^{-1}(1 - Ee^{-\alpha(g+x)}) = \alpha^{-1}(1 - e^{-\alpha x}Ee^{-\alpha g}) = u_{\alpha}(x)$$

Also if  $\alpha = 0$ , then by the proof above necessarily Eg = 0 and so  $Eu_0(g) = Eg = 0$  and  $Eu_0(g+x) = x = u_0(x)$ . QED

**Remark** As pointed out by Schulze (2008), [hereafter Sc], for an unbounded distribution function u(x), the map  $f(\alpha)$  is not necessarily defined for all  $\alpha$ . (In [Sc, Ex 3] it is defined for  $\alpha=0$  only). In this case we cannot apply the proof of Th. 1.8. In [Sc] it is shown that  $Ee^{-\alpha g}$  is the Laplace transform of u(x). Since we consider both positive and negative values of  $\alpha$ , we use the two-sided Laplace transform. Thus  $f(\alpha)$  is defined for all real  $\alpha$  in the region of convergence of u(x). If this region of convergence is wide enough, then the proof is still applicable. The question, which distributions admit the appropriate range of convergence, is beyond the scope of this paper.

**Definition 1.9** Given a gamble g, denote the number  $\alpha$  obtained in Th.1.8 by the upper limit of taking g.

Note that if we replace g by Ng then the upper limit of Ng is the corresponding root of (5) where g is replaced by Ng, and thus equals  $N^{-1}\alpha$ .

**Remark 1.10** Given a gamble g where Eg > 0, let its upper limit  $\alpha$  be as in Def.1.9. Then  $\alpha^{-1}$  is the index of riskiness of g as defined in [AS].

The notation upper limit is justified by the following corollary.

**Corollary 1.11** *Let*  $\alpha$  *be the upper limit of taking a gamble* g. *Then:* 

1. If Eg > 0 then all CARL accept g and a CARA person with a utility function  $u_{\beta}$  accepts g if and only if

$$0 < \beta < \alpha$$

2. If Eg < 0 then all CARA reject g and a CARL person with a utility function  $u_{\beta}$  accepts g if and only if

$$\beta < \alpha < 0$$

3. If E(g) = 0 the all CARA people reject g while all CARL people accept g.

**Proof.** 1. Assume Eg>0 and let  $0<\beta<\alpha$ . Note that for all w,  $a=r(w,u_a)$  and  $\beta=\rho(w,u_\beta)$  as defined in Def.1.1. By Lemma 1.2.2,  $\beta<\alpha$  implies  $u_\beta(x)>u_\alpha(x)$  for all x>0. Hence by definition of the upper limit  $\alpha$ ,

$$Eu_{\beta}(g) > Eu_{\alpha}(g) = u_{\alpha}(0) = 0$$

This implies that a CARA person with a utility function  $u_{\beta}$  accepts g. Similarly, if  $\beta > \alpha$  then a  $\beta$ -CARA person will not accept g.

2. If Eg < 0 then by Th.1.8,  $\alpha < 0$ , and for  $\beta < \alpha < 0$  we have  $0 < -\alpha < -\beta$ . Since E(-g) > 0 this implies by (4) and part 1:

$$-Eu_{\beta}(g) = Eu_{-\beta}(-g) < Eu_{-\alpha}(-g) = -Eu_{\alpha}(g) = 0$$

Hence  $Eu_{\beta}(g) > 0$  and a CARL person with a utility function  $u_{\beta}$  accepts g. QED

We propose here a general index of economic riskiness. Given a gamble g and its upper limit  $\alpha$ , define Q(g) by

$$Q(g) = e^{-\alpha} \tag{6}$$

It is straightforward to check the following properties:

**Theorem 1.12** The generalized index Q(g) given in (6) satisfies:

- 1. Q(g) > 0 for all g.
- 2. If Eg > 0 then Q(g) < 1 and if Eg < 0 then Q(g) > 1. When Eg = 0 then Q(g) = 1.
  - 3.  $Q(Ng) = Q(g)^{1/N}$ . In particular

$$Q(-g) = Q(g)^{-1}$$

**Proof.** 1. is clear. 2. follows directly from Th.1.8.

3. By Remark 1.9, the upper limit of taking Ng is  $N^{-1}\alpha$ , where  $\alpha$  is the upper limit of taking g. Hence by (7),

$$Q(Ng) = e^{-N^{-1}\alpha} = Q(g)^{1/N}$$

**QED** 

#### 3 Discussion and Conclusions

Property 3 of Theorem 1.12 may be termed the *Risk Balance Theorem*<sup>5</sup> and its implications are simply put: One can think of any financial transaction as someone buying a gamble, which at the same time must be sold by someone else. Thus, while one party in the transaction faces g, the other faces -g. In any transaction, if buying is very risky, selling must have little risk, and vice versa. So long as g is not dependent upon other gambles (as, for instance, selling a naked call to an agent who does not hold the underlying), one of the agents must accept a gamble with negative expected return. The case of transacting an asset of zero expected return is also interesting, albeit perhaps less so, as one could argue that buying such an asset is not particularly riskier or less risky than selling it.

At first glance, one might even be tempted to argue that, risk neutral agents aside, half of all traders in financial markets must be risk lovers, but this would be to assume that one gamble is independent of all others and this is clearly false. Thus, suppose that John sells Mary 1000 out-of-the-money put options on IBM stock for the end of next year. Suppose further, that John somehow knows that his gamble has a positive expected return for the entire period until expiry. He is hence accepting a gamble with expected positive return with respect to this trade. But what of Mary? Her trade has a negative expected return, but does that make Mary a risk lover? Not necessarily. If she owns 1000 IBM shares she has simply bought insurance against a sharp fall in the market and her overall investment (purchase of stock *and* puts) may well have a positive expected return, making her risk averse *on average*. Accordingly, our result *vis a vis* trades will not be as dramatic as it may seem. But to the extent that most traders in commodity markets never take deliver or produce the commodity in which they trade and to the extent that most traders in options markets do not hold or sell the underlying asset, our argument carries some weight. In other words, throughout the derivation of the risk balance result the implicit assumption was made that we are dealing either with a risk lover or a risk averse agent whereas

<sup>5</sup> The authors wish to thank an anonymous referee for this suggestion and its elaboration.

in reality many investors put together a portfolio whose separate elements may have variously positive and negative expectations.<sup>6</sup>

Yet this is to assume – as we have done so far – that the parameters of each gamble, the payouts and their concomitant probabilities, are known. And while this is true, agents are aware of the expectations of different gambles and, if rational<sup>7</sup>, will only ever knowingly behave as risk lovers. But, in truth, probabilities are almost never known in traditional financial markets and payouts only occasionally, so that traders may be victims of false consciousness, believing themselves to be investors (and thus, by definition, risk averse) while in reality accepting gambles with negative returns.

We are unable to distinguish between these cases since we require well-defined and gameable gambles for the derivation of risk balance and thus the question of how many genuine risk lovers are to be found in financial markets is ultimately an empirical one. It may, however, be safely concluded that the number is far from that assumed in the ivory tower, namely zero.

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<sup>6</sup> The classic paper dealing risk loving and risk averse behavior by the same agent, albeit in a very different context, is Friedman and Savage (1948).

<sup>&</sup>lt;sup>7</sup> A consideration of the connection between the arguments presented here and those underlying behavioral finance is beyond the scope of this paper.

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