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## The dynamic input-output LSD-model with reduction of idle capacity and modified decision function

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# **Deutsches Institut für Wirtschaftsforschung**

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## **The Dynamic Input-Output LSD-Model with Reduction of Idle Capacity and Modified Decision Function**

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*Dietmar Edler and Tatjana Ribakova*

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# THE DYNAMIC INPUT-OUTPUT LSD-MODEL WITH REDUCTION OF IDLE CAPACITY AND MODIFIED DECISION FUNCTION

by

*Dietmar Edler\* and Tatjana Ribakova\*\**

## 1 Introduction

Since the seminal study by W. Leontief and F. Duchin (Leontief, W. and Duchin, F., 1986) dealing with the future impact of automation on employment in the USA, the new dynamic input-output model, applied in that study<sup>1</sup>, attracts close attention.

The model has been developed and is, at present, generally applied to estimate the future changes in production and employment due to the diffusion of new technologies. Input-output analysis is specifically suited to represent new technologies within a consistent, disaggregated framework, i.e. by incorporating engineering and other primarily micro-oriented data. Besides the ability to cope with the direct and indirect repercussions of new technologies on intermediate goods, the dynamic version of input-output analysis is also capable of modelling their impact on the process of investment. Dealing with the introduction and diffusion of a new technology into the technological structure of an economy as a prototype of a dynamic economic process, it is obvious that dynamic input-output analysis is very attractive for this type of investigation.

Therefore in Germany detailed research was conducted on the basis of the same model in order to evaluate the influence of selected new technologies on employment (e.g. in Edler, D. 1990a; 1990b; Edler, D. et al. 1990 the impact of industrial robots is analyzed). A dynamic input-output model with a different decision function was elaborated and

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<sup>1</sup> The model was firstly published in Duchin, F. and Szyld, D. (1985). Following P. Fleissner (1990) we refer to the model as the LSD (Leontief-Szyld-Duchin)-model.

applied by Kalmbach, P., Kurz, H.D. (1990). Stability of solution and possible ways of its improvement are investigated too (Fleissner, P., 1990; Franke, R., 1988; Kigyossy-Schmidt, E. and Matthes, B., 1988).

Though the stability problem still remains in question, from a practical point of view the LSD-model represents an advanced instrument of long-term analysis of technical change. This view is supported by extensive simulation experiments using German data within and outside the ex post simulation period<sup>2</sup>.

For the ex post period it can be stated, that the model is able to track the actual values of gross output quite well on the sectoral level.

As for investment indicators, we can notice the following peculiarities of solution:

1. Capacity expansion investment is systematically underestimated on the average;
2. Capital replacement investment is overestimated;
3. There exist noticeable fluctuations in gross capital investment which are too volatile in comparison with the actual development. They are induced by strong fluctuations of capacity expansion investment: Years of too intensive investment are coupled with those of too slack one.

The first two features result to a large extent, from the assumption of "no capacity retirement"<sup>3</sup> while the third one is a result of the specific decision function used in the model. We have tried to reduce the above mentioned discrepancies by introducing some new conditions in the original model.

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<sup>2</sup> Several ex ante simulations up to the year 2005 have been performed without stability problems but can not be documented in this paper.

<sup>3</sup> Here and in the following we use the expression "capacity retirement" in the meaning of "reduction of idle capacity".

In the present article, an improved version of the LSD-model is suggested with implicit capacity retirement and a modified decision function. Both concepts will be introduced theoretically in the next two chapters of the paper. In the following section it will be demonstrated how these new concepts influence the simulation capabilities of the model in the ex post period. Finally, some methodical questions will be discussed which are of paramount importance in ex ante simulations.

## 2 Introduction of Capacity Retirement

The original LSD-model as given in Leontief, W. and Duchin, F. (1986), is formulated in the following way:

$$c_i^* (t+\tau) = \min \left[ 1+\delta_i, \frac{x_i (t-1) + x_i (t-2)}{x_i (t-2) + x_i (t-3)} \right]^{\tau+1} \cdot x_i (t-1) \quad (1)$$

$$i = 1, \dots, n$$

$$o (t+\tau) = \max [ 0, c^* (t+\tau) - c (t+\tau-1) ] \quad (2)$$

$$c (t+\tau) = c (t+\tau-1) + o (t+\tau) \quad (3)$$

$$[ I - A (t) - R (t) ] \cdot x (t) = \sum_{\Theta=1}^{\tau} B^{\Theta} (t) \cdot o (t+\Theta) + y (t) , \quad (4)$$

where

- $i$  - index of sector  $i$
- $A(t)$  - matrix of technical coefficients in year  $t$
- $R(t)$  - matrix of capital requirements (for replacement) per unit of output in period  $t$

- $B(t)$  - matrix of capital requirements (for expansion) per unit of output in period  $t$   
 $y(t)$  - vector of final deliveries in period  $t$   
 $x(t)$  - vector of gross output in year  $t$   
 $c(t)$  - production capacity during period  $t$   
 $o(t)$  - increase in production capacity between periods  $t-1$  and  $t$   
 $c^*(t)$  - projected output capacity for (future) period  $t$   
 $\delta_i$  - sector-specific maximum admissible annual rate of capacity expansion  
 $\tau$  - maximum gestation period for capital goods.

Given initial conditions

$$c(t_0), x(t), \quad t = t_0 - \tau - 2, \dots, t_0 - 1,$$

first, the values of  $c^*$ ,  $o$  and  $c$  are calculated for periods  $t_0 + 1$  through  $t_0 + \tau - 1$  using equations (1)-(3).

Then model (1)-(4) is solved for simulation periods  $t = t_0, \dots, T$ . Finally, labor requirements by occupation in period  $t$  are received as

$$l(t) = L(t) \cdot x(t),$$

where  $L(t)$  denotes the matrix of specific labor coefficients by sector and occupation.

It should be noticed, that, in accordance with (3), no reduction of production capacity is possible in the model. In this case the matrices  $R(t) = (r_{ij}(t))$  of capital replacement coefficients should be defined in a way that all capital stock retirement be replaced.

Let's designate the actual retirement of stock of capital good  $i$  in sector  $j$  in  $t^{\text{th}}$  period as  $\bar{r}_{ij}(t)$ .

Then, for the model (1)-(4),



$$r_{ij}(t) = \frac{\bar{r}_{ij}(t)}{\bar{x}_j(t)},$$

where  $\bar{x}(t) = (\bar{x}_j(t))$  denotes the actual gross output in period  $t$ .

In reality, however, capital retirement is not always replaced in full. In particular, in branches where output systematically diminishes, gross capital investment is often less than capital retirement. From that follows that capital retirement obviously is not or is only partially compensated. It results in a strong possibility of capital replacement investment overestimation in the LSD-model.

On the other hand, capacity expansion takes place only if a projected capacity requirement  $c^*(t)$  exceeds the output capacity  $c(t-1)$ . Since  $c(t)$  in the model never decreases, capacity expansion is very likely to be understated<sup>4</sup>.

Our basic assumption concerning capacity retirement is that productive capacity, which is systematically "not in use", will be reduced.

Here "systematically" means "during a critical, long enough period". Assume the length of this period to be  $\Psi$  years. Capacity is considered to be "not in use" or "idle" if it is not utilized and is not a part of some normal capacity reserve. Let  $\beta(t) = (\beta_i(t))$  denote a vector of some normative (necessary, rational, desirable, etc.) capacity utilization coefficients in period  $t$ .

Then capacity "not in use" in year  $t$ ,  $\alpha_i = (\alpha_{it})$  can be computed as

$$\alpha_{it} = \max [ 0, c_i(t) - x_i(t) / \beta_i(t) ] \quad , \quad i = 1, \dots, n \quad (5)$$

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<sup>4</sup> Besides, there can be other reasons of expansion investment underestimation in the model. In particular, it may result from the definition of coefficients in  $B$ -matrices.

If we have the series of values

$$\alpha_{t-\Psi}, \dots, \alpha_{t-1},$$

the minimum of them will be a capacity which, by the year  $t$ , remains idle during consequent  $\Psi$  years. We assume here that capacity retirement  $d(t+\tau)$  is equal to this value.

When computing  $d$  for the initial year, the following formula can be used:

$$d(t + \tau) = \min [ \alpha_{t-1} , \dots, \alpha_{t-\Psi} ] \quad (6)$$

For the next years, however, the values  $\alpha_{t-\psi}$   $\psi = 2, \dots, \Psi$  should be first revised. Now they must represent the rest of idle capacity of year  $t-\psi$ , which, by the beginning of current simulation year  $t$  (i.e. by the end of period  $t-1$ ), has not yet been retired. We denote this rest as

$$\alpha_{t-\psi}(t-1).$$

If  $\psi = 1$ , it results in

$$\alpha_{t-1}(t-1) = \alpha_{t-1}. \quad (7)$$

When  $\psi = 2, \dots, \Psi$ , the result is as follows

$$\alpha_{t-\psi}(t-1) = \alpha_{t-\psi} - \sum_{\Theta=t_1}^{t_2} d(\Theta), \quad (8)$$

where  $t_1 = t + \tau - \psi + 1$  and  $t_2 = t + \tau - 1$ .

In general, capacity reduction  $d(t+\tau)$  is introduced by the expression:

$$d(t+\tau) = \min [ \alpha_{t-1}(t-1), \dots, \alpha_{t-\Psi}(t-1) ], \quad (9)$$

with parameters  $\alpha_{t-\psi}(t-1)$ ,  $\psi = 1, \dots, \Psi$  computed according to (5), (7), (8).

Now the LSD-model with capacity retirement can be written as follows:

$$c_i^* (t+\tau) = \min \left[ 1+\delta_i, \frac{x_i (t-1) + x_i (t-2)}{x_i (t-2) + x_i (t-3)} \right]^{\tau+1} \cdot x_i (t-1) \quad (10)$$

$$i = 1, \dots, n$$

$$d (t+\tau) = \min [ \alpha_{t-1} (t-1) , \dots, \alpha_{t-\Psi} (t-1) ] \quad (11)$$

$$o (t+\tau) = \max [ 0, c^* (t+\tau) - (c (t+\tau-1) - d (t+\tau)) ] \quad (12)$$

$$c (t+\tau) = c (t+\tau-1) + o (t+\tau) - d (t+\tau) \quad (13)$$

$$[ I - A (t) - R (t) ] \cdot x (t) = \sum_{\Theta=1}^{\tau} B^{\Theta} (t) \cdot o (t+\Theta) + y (t) , \quad (14)$$

where  $\alpha_{t-\psi} (t-1)$  are determined as stated above.<sup>5</sup>

The initial conditions must now specify values  $c(t)$ ,  $x(t)$ ,  $\beta(t)$  for respective past years. Given these conditions, first, initial values for  $\alpha_{t-\psi}$   $\psi = 1, \dots, \Psi$  are defined in accordance with (5). Then  $d(t_o + 1)$  is computed using formula (6). After that the values of  $c^*$ ,  $o$  and  $c$  can be calculated for period  $t_o + 1$  using equations (10), (12), (13) of the model.

At the beginning of the next year, the new value of  $\alpha_{t-1} (t-1)$  is computed according to (5), (7) and the values  $\alpha_{t-\psi}$   $\psi = 2, \dots, \Psi$  are revised according to (8)<sup>6</sup>. Then the values  $c^*$ ,  $d$ ,  $o$  and  $c$  are defined for the years  $t_o + 1, \dots, t_o + \tau - 1$ .

<sup>5</sup> In this reformulated model equations (10) and (14) are equivalent to equations (1) and (4) in the original model.

<sup>6</sup> For  $\theta < t_o + 1$  the values of  $d(\theta)$  are assumed to be zero.

Now the complete model (5), (7), (8), (10) - (14) can be solved for  $x(t)$  for each period from  $t_0$  through  $T^7$ .

With the introduction of capacity retirement, the method of computing the matrices  $R$  of capital replacement coefficients can be changed as it is no more necessary to replace all the retired capital stock.

Let  $\tilde{G}(t) = (\tilde{g}_{ij}(t))$  be a matrix of actual gross deliveries of capital good  $i$  to sector  $j$  in  $t^{\text{th}}$  period.

Then elements of matrix  $R(t)$  can be obtained as

$$r_{ij}(t) = \frac{\min [ \bar{r}_{ij}(t) , \bar{g}_{ij}(t) ]}{\bar{x}_j(t)} . \quad (15)$$

### 3 Modification of decision function

The fundamental idea underlying decision function (1)-(2) in the original LSD-model is that expansion decisions in each sector rely on its recent past experience.

Within that the recent past experience is represented only by data on gross output, i.e.

$$x(t), t = t_0 - \tau - 2, \dots, t_0 - 1 .$$

In the above paragraph, the formulation of decision function has been changed into (10)-(12) with respect to the reduction of idle capacity. Additionally, we have introduced the vectors  $\beta(t)$  of normative capacity utilization coefficients. It should be noted that only the

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<sup>7</sup> Methodical aspects of estimating  $\beta(t)$  for the years  $t-\Psi, \dots, t-1$ , as well as for the years of the simulation period are discussed in section 4 and in the appendix.

values for past periods (respective to current year of simulation) are used to compute capacity retirement.

With these parameters we can now use, within the same basic idea, additional information and try to improve further on the rule for the expansion decision. We make the following assumptions:

- (a) As long as capacity reserves are "more than sufficient" no capacity expansion is planned, irrespective of conditions (10)-(12).
- (b) If production capacity reserves are "normal", the expansion decision is made in accordance with (10)-(12).
- (c) When capacity reserves become "too small", future capacity expansion is supposed to be necessary, even if that does not follow from (10)-(12). In any case, it cannot be less than the value which ensures normal capacity reserves.

A capacity reserve is considered "more than sufficient", if the current capacity utilization coefficient is less than some average normative value,  $\bar{\beta}$ , computed on the basis of the previous, long enough period .

A capacity reserve is considered to be "too small", if capacity utilization exceeds all the values of normative parameters  $\beta$  in the recent past years (or, otherwise stated, if it exceeds the maximum of them,  $\beta^{max}$ ).

Otherwise, the capacity reserve is regarded as "normal".

$\bar{\beta}$  is computed as the moving arithmetic average over a period of  $\sigma$  years, so that

$$\bar{\beta} (t-1) = \frac{1}{\sigma} \sum_{\theta=t-\sigma}^{t-1} \beta (\theta) , \quad (16a)$$

while  $\beta^{max}$  is obtained as

$$\beta^{max} (t-1) = \max [\beta (t-1), \beta (t-2), \dots, \beta (t-\lambda)] , \quad (16b)$$

where  $\lambda$  is the number of recent past years taken into consideration.

Now equations (11), (12) of decision function can be modified as follows:

$$o_i(t+\tau) = \begin{cases} 0, & \text{if } x_i(t-1) / c_i(t-1) < \bar{\beta}_i \cdot (t-1) \\ \max [0, \bar{c}_i^*(t+\tau) - (c_i(t+\tau-1) - d_i(t+\tau))] , & \text{otherwise} \end{cases} \quad (17)$$

$$i = 1, \dots, n$$

where

$$\bar{c}_i^*(t+\tau) = \begin{cases} \max [c_i^*(t+\tau), x_i(t-1) / \beta_i^{\max}(t-1)] , & \\ \quad \text{if } x_i(t-1) / c_i(t-1) > \beta_i^{\max}(t-1) & \\ c_i^*(t+\tau) , & \text{otherwise} \end{cases} \quad (18)$$

$$i = 1, \dots, n$$

Finally, we use the idea of flexible accelerator (Edler, 1990b), which has proved to be effective in experiments with the LSD-model. The essence of this approach is, that the implementation period of investment is made smoothed, thus contributing noticeably to general smoothness of solution<sup>8</sup>.

Let  $p_k$ ,  $k = 0, \dots, K$ , be some non-negative parameters, and

$$\sum_{k=0}^K p_k = 1 \quad .$$

A capacity increment  $o(t+\tau)$ , once planned in accordance with the decision function, is then assumed to be put into operation gradually, during  $K$  years:

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<sup>8</sup> Simulations using the LSD-model with flexible accelerator as applied to the German economy are described in detail in Edler (1990b). There the effect of flexible accelerator on measures of goodness of fit in ex post simulation is demonstrated.

$$o(t+\tau) = \sum_{k=0}^K p_k \cdot o(t+\tau+k) \quad (19)$$

On the other hand, the effective capacity expansion in period  $t+\tau$ ,  $\bar{o}(t+\tau)$ , is now formed as a sum of the component parts of increments planned for recent years:

$$\bar{o}(t+\tau) = \sum_{k=0}^K p_k \cdot o(t+\tau-k) \quad (20)$$

Now we can write the resultant formulation of the modified LSD-model in the following way:

$$c_i^*(t+\tau) = \min \left[ 1 + \delta_i, \frac{x_i(t-1) + x_i(t-2)}{x_i(t-2) + x_i(t-3)} \right]^{\tau+1} \cdot x_i(t-1), \quad (21)$$

$$i = 1, \dots, n$$

$$\bar{c}_i^*(t+\tau) = \begin{cases} \max [c_i^*(t+\tau), x_i(t-1) / \beta_i^{\max}(t-1)], \\ \quad \text{if } x_i(t-1) / c_i(t-1) > \beta_i^{\max}(t-1) \\ c_i^*(t+\tau), \quad \text{otherwise} \end{cases} \quad (22)$$

$$i = 1, \dots, n$$

$$d(t+\tau) = \min [\alpha_{t-1}(t-1), \dots, \alpha_{t-\Psi}(t-1)] \quad (23)$$

$$o_i(t+\tau) = \begin{cases} 0, & \text{if } x_i(t-1) / c_i(t-1) < \bar{\beta}_i(t-1) \\ \max [0, \bar{c}_i^*(t+\tau) - (c_i(t+\tau-1) - d_i(t+\tau))] , & \text{otherwise} \end{cases} \quad (24)$$

$$i = 1, \dots, n$$

$$\bar{o}(t+\tau) = \sum_{k=0}^K p_k \cdot o(t+\tau-k) \quad (25)$$

$$c(t+\tau) = c(t+\tau-1) + \bar{o}(t+\tau) - d(t+\tau) \quad (26)$$

$$[I - A(t) - R(t)] \cdot x(t) = \sum_{\Theta=1}^{\tau} B^{\Theta}(t) \cdot \bar{o}(t+\Theta) + y(t), \quad (27)$$

with variables  $\alpha$  computed at the beginning of each simulation period,  $t$ , as

$$\alpha_{i, t-\psi} = \max [0, c_i(t-\psi) - x_i(t-\psi) / \beta_i(t-\psi)] \quad (28a)$$

$$i = 1, \dots, n, \quad \psi = 1, \dots, \Psi$$

$$\alpha_{i, t-1}(t-1) = \alpha_{i, t-1} \quad (28b)$$

$$\alpha_{i, t-\psi}(t-1) = \alpha_{i, t-\psi} - \sum_{\Theta=t_1}^{t_2} d(\Theta), \quad (28c)$$

$$\psi = 2, \dots, \Psi, \quad t_1 = t+\tau-\psi+1, \quad t_2 = t+\tau-1.$$

$\bar{\beta}(t-1)$  is calculated using (16a),  $\beta^{max}(t-1)$  is obtained from (16b) and  $R(t)$  is defined in accordance with (15).<sup>9</sup> To solve the model (21)-(28), first, the same initialization procedure can be used as for the model (5), (7), (8), (10)-(14).

#### 4 Ex post simulations with the original and improved LSD-model for the period 1975 - 1986

The theoretical considerations presented in the last two sections have been developed on the background of extensive experimental simulations with various versions of the

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<sup>9</sup> In the model described by (21)-(28), the equations (21) and (27) are equivalent to equations (1) and (4) of the original model.



dynamic input-output model, starting with the original LSD-model as shown in (1)-(4). In a process of gradual change of the model, where theoretical and empirical considerations mutually influenced each other, we finally resulted in formulation (21)-(28).

To demonstrate the effects of our theoretical modifications, the results of ex post simulations for the Federal Republic of Germany will be presented using both the original LSD (1)-(4) and the improved LSD-model (21)-(28). The years 1975 through 1986 were taken as simulation period, because for this period fully comparable actual data are available (see for a detailed description of the database Stäglin, Edler, Schintke, 1992).

When computing  $\beta^{\max}$  and  $\alpha$  in the model (21)-(28), we assumed periods considered being equal to three years ( $\Psi, \lambda=3$ ) in order to benefit from the recent past experience, but also to avoid random fluctuations. To calculate  $\bar{\beta}$  we took the period approximating the length of a business cycle ( $\sigma=7$ ).

As for parameters  $p_k$ ,  $k=0, \dots, K$ , first, we assumed the least prolongation of an implementation period ( $K=1$ ). Second, we carried out experimental tests for various combinations of values for  $p_k$ .<sup>10</sup> Finally, values of  $(p_0, p_1)$  equal to (0.6, 0.4) were chosen for our simulation experiments.

It was a more complicated problem to obtain some reasonable estimates of normative capacity utilization coefficients,  $\beta(t)$ . Here we employed an approach based on the idea and methods of measuring potential output.<sup>11</sup>

The estimated potential output of each sector  $i$ ,  $P_i(t)$ , was received by applying a capital-vintage model of production, developed in the German Institute for Economic Research

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<sup>10</sup> See Edler (1990b), where the effects of different combinations of parameters  $p_k$  are investigated for  $K = 2$ .

<sup>11</sup> For a survey on analytical methods of measuring potential output see Görzig (1989).

(Görzig, 1985). With these data, coefficients  $\beta_i(t)$  were computed as a relation of actual gross output,  $\bar{x}_i(t)$ , to a potential one:

$$\beta_i(t) = \frac{\bar{x}_i(t)}{P_i(t)}, \quad i = 1, \dots, n$$

In Figures 1 and 2, the summary simulation results for gross output, gross investment, replacement investment and capacity expansion investment are shown for both models, in each case compared with actual data.

In the modified LSD-model gross capital investment dynamics is much more smoothed. At the same time the model gives better approximation of actual data in most simulation periods (Figure 1). To a large extent, this feature is due to a smoothed, more realistic time path of capacity expansion investment, as well as to a better estimation of replacement investment (Figure 2).

To judge the goodness of fit in the ex post period, some statistical measures<sup>12</sup> are presented in Table 1.

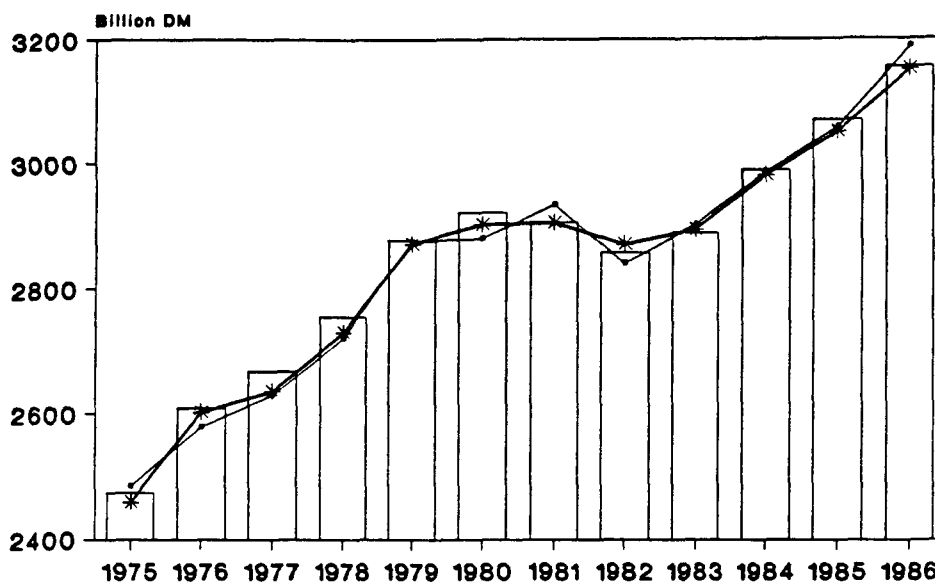
**Table 1:** Ex Post Simulation Errors for Period 1975-1986 in p.c.  
- Macroeconomic Totals for Gross Output, Gross Investment  
Replacement Investment and Capacity Expansion Investment -

	Gross output		Gross investment		Replacement investment		Capacity expansion investment	
	Original LSD-model	Improved LSD-model	Original LSD-model	Improved LSD-model	Original LSD-model	Improved LSD-model	Original LSD-model	Improved LSD-model
Root mean square percentage error (RMSPE)	0.0026	0.0017	0.0434	0.0275	0.0328	0.0018	0.1455	0.0749
Mean percentage error (MPE)	0.0030	0.0036	0.0538	0.0603	-0.0953	0.0037	0.3439	0.1614
Theil's U	0.0045	0.0028	0.0750	0.0484	0.0582	0.0029	0.2786	0.1396

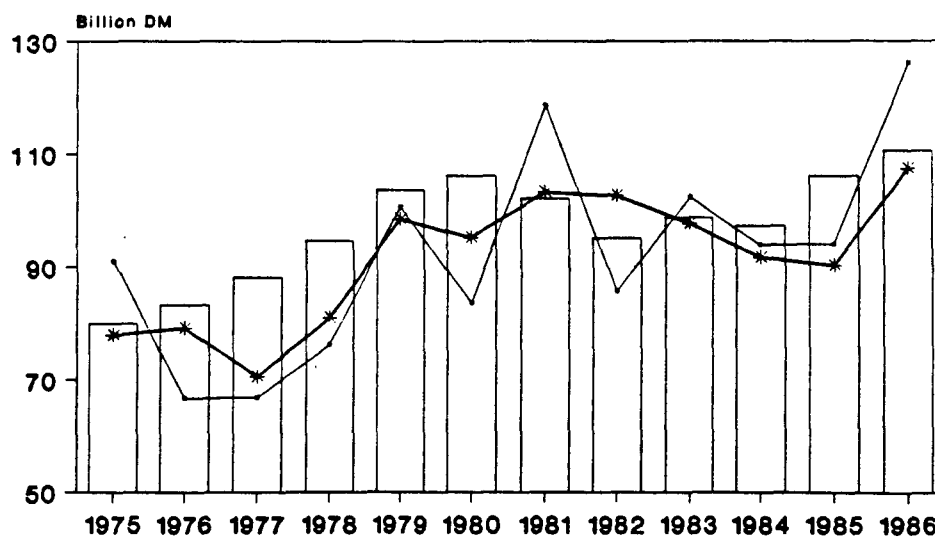
<sup>12</sup> Mean percentage error (MPE) is used as a rough indicator, to show whether the level of the respective variable during the whole simulation period is over- or underestimated. Root mean square percentage error (RMSPE) and Theil's U are usual measures for goodness of fit between actual and simulated values.

Figure 1:

### Ex Post Simulation 1975 - 1986 - Original and Improved LSD-Model - Gross Output



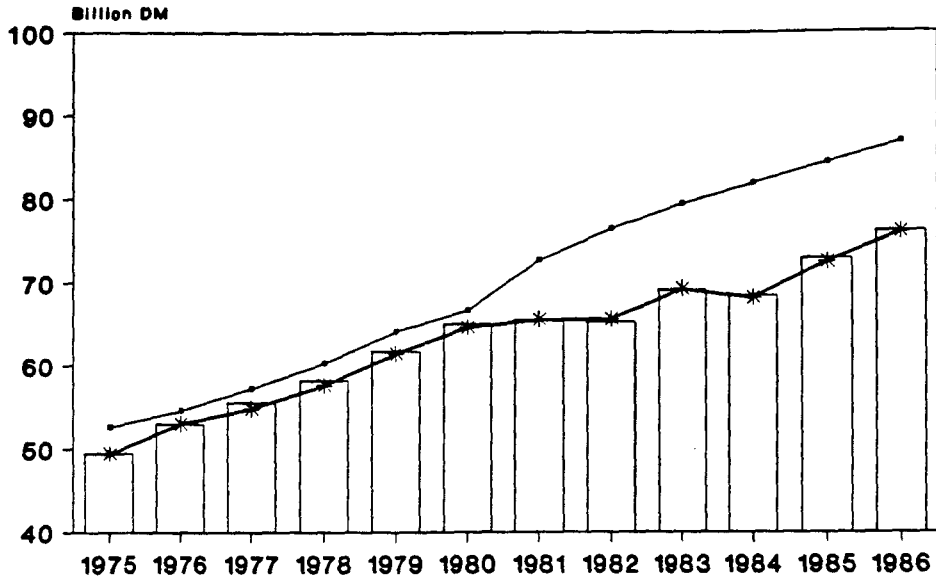
### Gross Investment



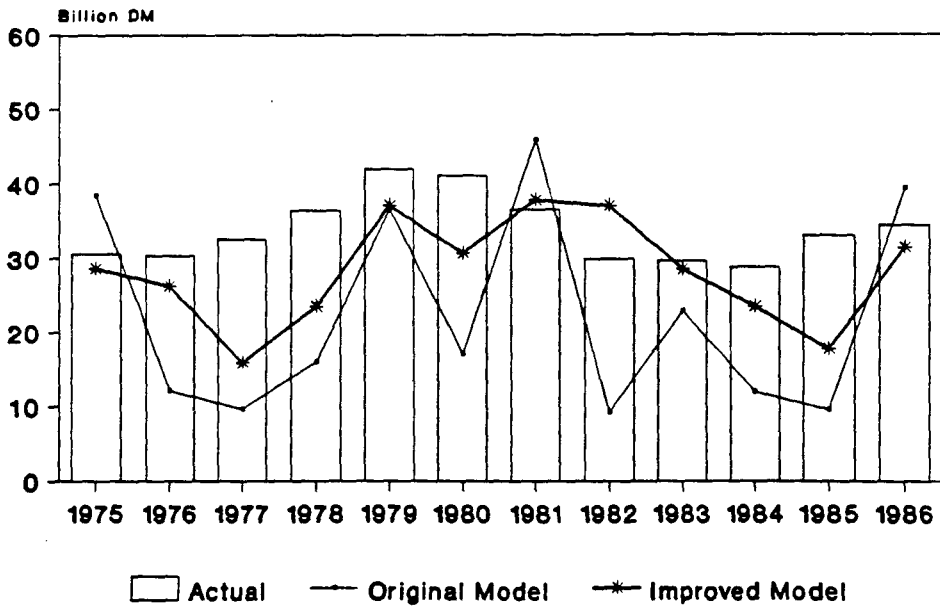
□ Actual    — Original Model    \* Improved Model

Figure 2:

### Ex Post Simulation 1975 - 1986 - Original and Improved LSD-Model - Replacement Investment



### Capacity Expansion Investment



□ Actual    — Original Model    \* Improved Model

As can be seen from Table 1, the measures for goodness of fit for gross output and gross investment are noticeably better in the solution produced by the improved model (21)-(28). At the same time characteristics of the mean level (MPE) have the minor degradation compared to those of the original LSD-model.

As for gross investment it should be pointed out that, in the original model, satisfactory values for MPE and RMSPE result from mutually compensated significant errors in its (overestimated) replacement and (underestimated) expansion parts.

The model (21)-(28) gives a considerably improved solution for both components of gross investment. The goodness of fit is systematically much better for replacement investment, as the new version allows for computation of more correct capital retirement. At the same time, the modified decision function better describes the process of capacity expansion. Thus, all error indicators for capacity expansion investment are about two times smaller for the improved model compared to the original one.

The results reported so far refer to the macroeconomic totals of the respective variables. They are compiled by summing up the results on the sectoral level, for which the functional relations of the model are defined. For a thorough comparison of both models it is, therefore, of special interest, how the behaviour of each model version can be evaluated on the sectoral level. The sectoral ex post simulation errors of gross output, gross investment, replacement investment and capacity expansion investment are compared in Tables 2 and 3. The data indicate whether the (original) LSD-model or the improved one gains better results. Additionally for each variable its respective share of the total is documented, in order to give a hint of its "importance".

In general it can be stated, that at the sectoral level, also, the improved model has better goodness of fit in a large majority of cases. This is especially true when looking at RMSPE, where the improved model clearly dominates the original LSD-model for all four variables. The better fit is most obvious for the variables describing the investment process, the part of the model on which we concentrated our effort. As for MPE, there

**Table 2:** Sectoral Ex Post Simulation Errors  
for Gross Output and Gross Investment  
- Comparison of Original and Improved LSD-Model -

Sector	Gross Output			Gross Investment		
	RMSPE	MPE (1)	Share of sector (2) in p.c.	RMSPE	MPE (1)	Share of sector (2) in p.c.
1 Agricultural products and forestry	=	=	2.35	+	+	4.80
2 Electric power, steam, hot water	+	-	1.81	+	++	6.74
3 Gas	-	-	0.52	+	-	0.31
4 Water	+	+	0.17	-	-	0.25
5 Products of mining	+	+	0.96	+	-	1.30
6 Chemical products (incl. nuclear fuel)	+	-	4.83	+	-	4.64
7 Refined petroleum products	+	+	2.06	+	++	0.92
8 Plastic products	+	+	1.23	++	-	1.21
9 Rubber products	+	+	0.34	-	-	0.52
10 Stones and clays	=	-	0.96	-	+	0.98
11 Ceramic products	+	-	0.11	+	+	0.12
12 Glass and glass products	+	+	0.31	+	-	0.40
13 Iron and steel	+	-	2.63	++	++	1.63
14 Non-ferrous metals	+	-	0.91	+	-	0.48
15 Foundry products	+	++	0.44	++	-	0.31
16 Products of drawing plants, cold rolling mills	+	++	1.05	++	++	0.82
17 Structural metal products, rolling stock	+	-	0.67	+	+	0.35
18 Machinery and equipment (excl. electrical)	+	+	4.07	++	++	3.98
19 Office machinery, computing equipment	+	-	0.71	+	+	1.06
20 Road vehicles	+	-	4.81	+	+	6.39
21 Ships, boats and floating structures	-	+	0.17	++	++	0.08
22 Aircraft and spacecraft	+	-	0.32	++	++	0.20
23 Electrical machinery, equipment and appl.	+	+	3.78	++	-	5.60
24 Precision and optical instr., clocks, watches	-	--	0.56	+	-	0.60
25 Tools and finished metal products	+	+	1.18	+	+	1.04
26 Musical instruments, toys, sport goods etc.	-	-	0.22	+	-	0.18
27 Wood	+	+	0.31	+	++	0.19
28 Wood products	+	+	0.85	-	--	0.71
29 Pulp, paper, and -board	+	+	0.49	++	++	0.68
30 Products of paper and -board	-	-	0.58	+	-	0.51
31 Products of printing and duplicating	-	-	0.86	+	++	0.80
32 Leather and leather products, footwear	-	+	0.23	++	++	0.10
33 Textiles	-	-	0.99	++	++	0.90
34 Wearing apparel	-	=	0.64	+	++	0.20
35 Food products	-	=	5.84	++	++	3.20
36 Construction	=	-	5.74	-	++	2.50
37 Wholesale trade, etc., recovery	-	-	4.61	-	-	2.37
38 Retail trade	=	=	3.42	-	-	2.87
39 Railway transport	+	+	0.42	++	++	1.46
40 Water transport, ports, etc.	+	-	0.38	+	-	1.39
41 Communication services	-	+	1.48	+	+	6.75
42 Other transport services, n.e.c.	=	-	2.46	-	+	3.80
43 Banking	-	+	2.49	+	-	1.85
44 Insurance (excl. social security funds)	-	+	1.16	+	+	0.52
45 Renting of real estate	+	+	5.22	=	=	0.00
46 Hotels and restaurants, homes and hostels	-	=	1.65	-	+	0.84
47 Education, research, culture and publishing	-	=	1.32	+	+	4.13
48 Health and veterinary market services	-	=	1.56	+	+	3.88
49 Other market services, n.e.c.	-	-	6.98	+	-	10.89
50 Government and social security	-	=	11.66	-	++	3.82
51 Private non-profit institutions, dom. services	-	=	1.49	+	+	0.73
Total	+	-	100.00	++	-	100.00

(1) comparison of absolute values, (2) actual values in 1986  
+ error of original model is greater than error of improved model, ++ error of original model exceeds that of improved model more than 1.5 times, = errors are equal, - error of improved model is greater than error of original model, -- error of improved model exceeds that of original model more than 1.5 times

**Table 3: Sectoral Ex Post Simulation Errors  
for Replacement Investment and Capacity Expansion Investment  
- Comparison of Original and Improved LSD-Model -**

Sector	Replacement investment			Capacity expansion investment		
	RMSPE	MPE (1)	Share of sector (2) in p.c.	RMSPE	MPE (1)	Share of sector (2) in p.c.
1 Agricultural products and forestry	++	++	6.55	+	-	0.93
2 Electric power, steam, hot water	-	-	5.89	+	++	8.61
3 Gas	+	-	0.32	+	-	0.27
4 Water	++	++	0.33	-	-	0.07
5 Products of mining	+	++	1.56	+	+	0.72
6 Chemical products (incl.nuclear fuel)	++	++	5.82	-	-	2.01
7 Refined petroleum products	++	++	1.27	-	-	0.13
8 Plastic products	+	-	1.03	+	-	1.60
9 Rubber products	++	++	0.64	+	+	0.25
10 Stones and clays	++	++	1.40	-	-	0.05
11 Ceramic products	++	++	0.16	+	-	0.02
12 Glass and glass products	++	++	0.46	++	-	0.28
13 Iron and steel	++	++	2.28	-	-	0.19
14 Non-ferrous metals	++	++	0.62	+	++	0.17
15 Foundry products	++	++	0.40	+	+	0.13
16 Products of drawing plants, cold rolling mills	++	++	1.03	++	++	0.35
17 Structural metal products, rolling stock	++	++	0.47	+	++	0.10
18 Machinery and equipment (excl. electrical)	++	++	4.16	++	++	3.60
19 Office machinery, computing equipment	+	++	1.01	+	-	1.17
20 Road vehicles	+	+	5.36	+	+	8.66
21 Ships, boats and floating structures	++	++	0.12	+	-	0.01
22 Aircraft and spacecraft	++	+	0.16	++	++	0.31
23 Electrical machinery, equipment and appl.	-	++	3.72	++	-	9.75
24 Precision and optical instr., clocks, watches	+	-	0.51	+	-	0.81
25 Tools and finished metal products	++	-	1.22	+	+	0.64
26 Musical instruments, toys, sport goods etc.	+	++	0.21	+	++	0.13
27 Wood	++	++	0.26	-	-	0.03
28 Wood products	++	++	0.91	-	-	0.29
29 Pulp, paper, and -board	++	++	0.73	++	++	0.58
30 Products of paper and -board	+	++	0.60	+	-	0.30
31 Products of printing and duplicating	++	-	0.80	+	++	0.79
32 Leather and leather products, footwear	++	++	0.13	=	=	0.02
33 Textiles	++	++	1.25	+	+	0.12
34 Wearing apparel	++	++	0.27	-	++	0.04
35 Food products	++	++	4.51	++	-	0.30
36 Construction	++	++	3.54	-	-	0.19
37 Wholesale trade, etc., recovery	++	++	3.06	-	-	0.84
38 Retail trade	++	++	3.91	-	-	0.58
39 Railway transport	++	++	2.03	-	-	0.19
40 Water transport, ports, etc.	++	++	1.97	++	+	0.10
41 Communication services	++	++	3.78	+	+	13.33
42 Other transport services, n.e.c.	++	++	4.29	-	-	2.71
43 Banking	+	++	1.53	+	-	2.57
44 Insurance (excl. social security funds)	+	++	0.27	+	+	1.08
45 Renting of real estate	=	=	0.00	-	=	0.00
46 Hotels and restaurants, homes and hostels	++	++	1.17	-	++	0.10
47 Education, research, culture and publishing	=	++	3.07	+	+	6.48
48 Health and veterinary market services	+	-	3.20	+	+	5.37
49 Other market services, n.e.c.	++	++	8.21	+	-	16.81
50 Government and social security	++	++	3.01	+	++	5.61
51 Private non-profit institutions, dom. services	++	++	0.79	-	-	0.58
Total	++	++	100.00	++	++	100.00

(1) comparison of absolute values, (2) actual values in 1986

+ error of original model is greater than error of improved model, ++ error of original model exceeds that of improved model more than 1.5 times, = errors are equal, - error of improved model is greater than error of original model, -- error of improved model exceeds that of original model more than 1.5 times

is a distinct improvement for gross investment and capital replacement investment, and a relation close to "fifty-fifty" for gross output and capacity expansion investment.

It is interesting to demonstrate how the introduction of capacity reduction and the modified decision rule for capacity expansion effect the ability of the improved LSD-model to balance the development of output and production capacity. For this reason the differences between productive capacity decisions obtained from the original and the modified LSD-model are illustrated. As a background, the dynamics of gross output of each model is shown. Some leading sectors of manufacturing and trade are selected (Figure 3) as well as those with a marked downward production trend (Figure 4).

In the improved model version, the output capacity is much more flexible, now responding not only to the upswing, but also to prolonged downward tendencies in production. It can be stated that, in most cases, the relatively better correspondence between production capacity and gross output takes place in the improved LSD-model.

The same conclusion seems to be valid for the whole economy. On the one hand, total capacity expansion investment dynamics is much closer to the actual one, and cumulative underestimation of this investment during the whole simulation period amounts to 16 %, against 34 % in the original model. On the other hand, the excessive capacity in the whole economy has decreased by 2 %, from original 13.3 to 11.3 % in the improved model.

## **5 Conclusion**

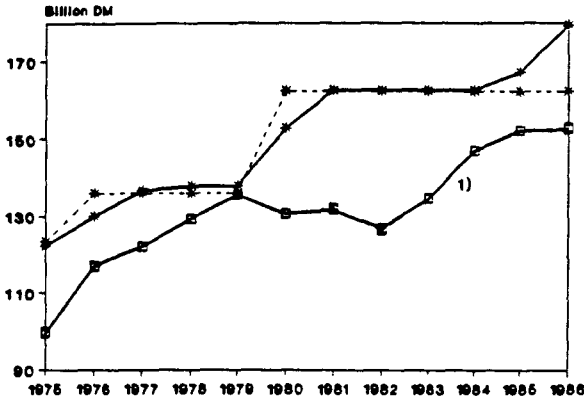
Adequate modeling of the investment process remains one of the main methodical problems of input-output analysis within a dynamic framework. From this point of view, the descriptive approach proposed by Leontief, Duchin, Szyld, appears to be improvable without losing any of its significant gains. This intention, however, requires a further complication of the model by introducing new parameters of capacity reserves.



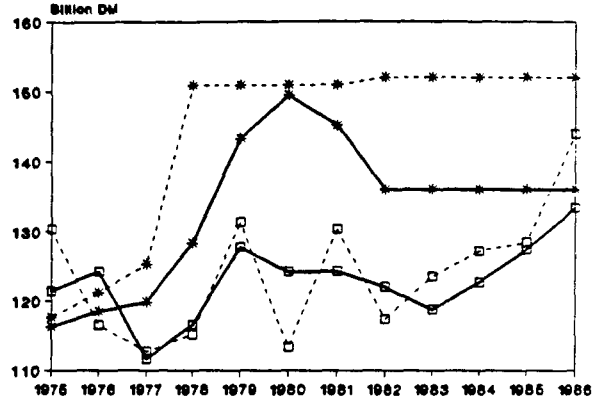
Figure 3:

Simulated Gross Output and Production Capacity in Selected Sectors

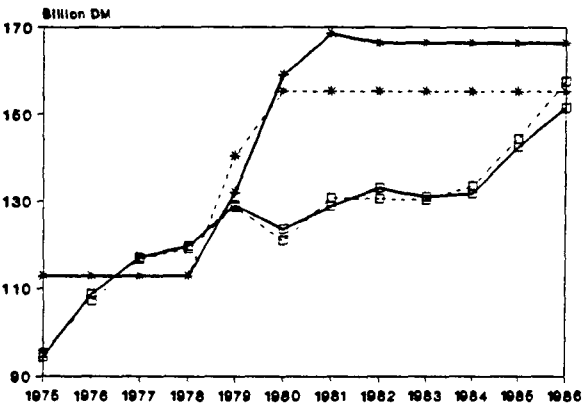
Chemical products



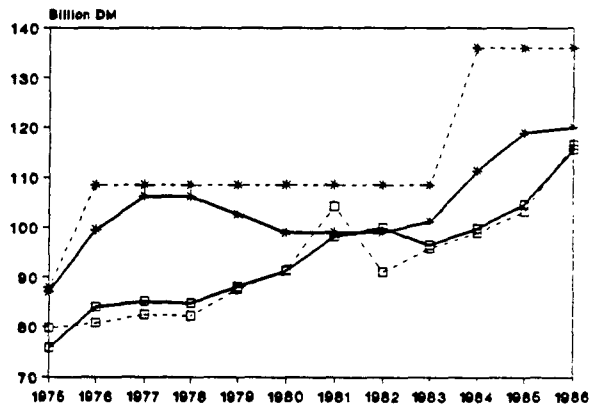
Machinery and Equipment



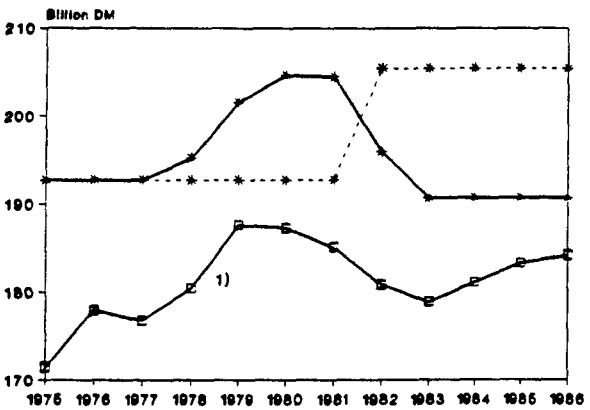
Road vehicles



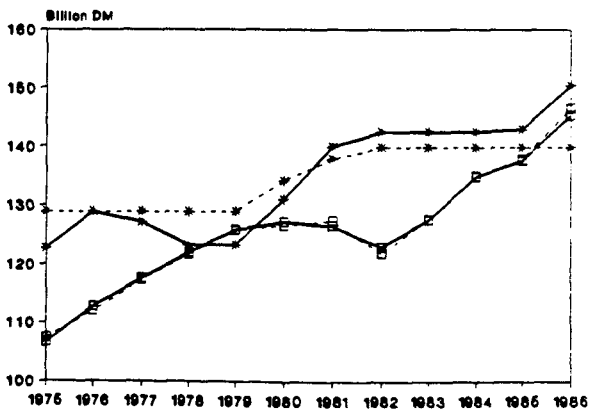
Electrical machinery and equipment



Food products



Wholesale trade, etc., recycling



1) Output for both models coincides graphically

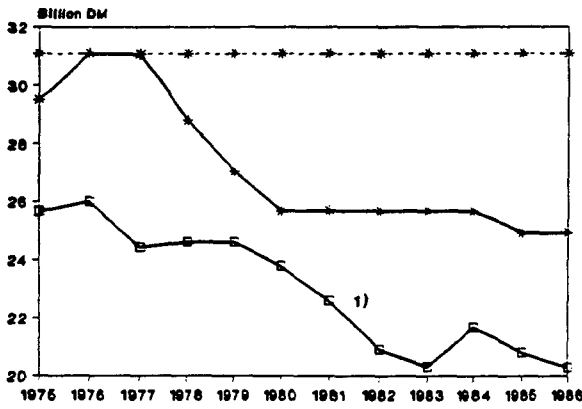
—●— Output (Improved)  
-○- Output (Original)

—\*— Capacity (Improved)  
-·- Capacity (Original)

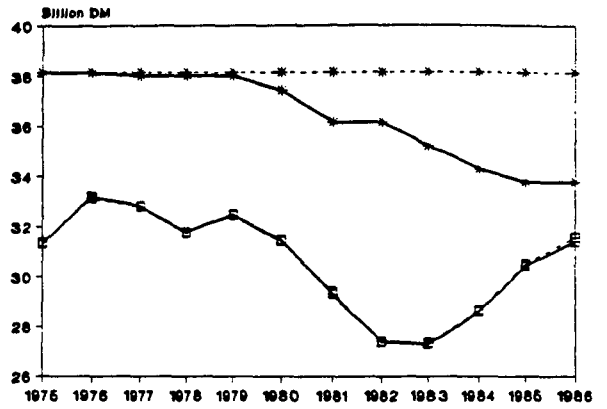
Figure 4:

Simulated Gross Output and Production Capacity in Selected Sectors

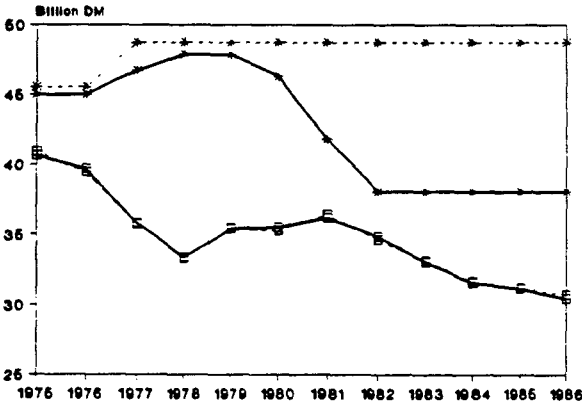
Wearing apparel



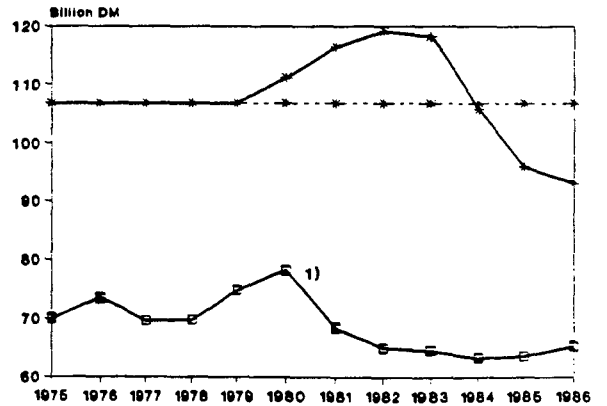
Textiles



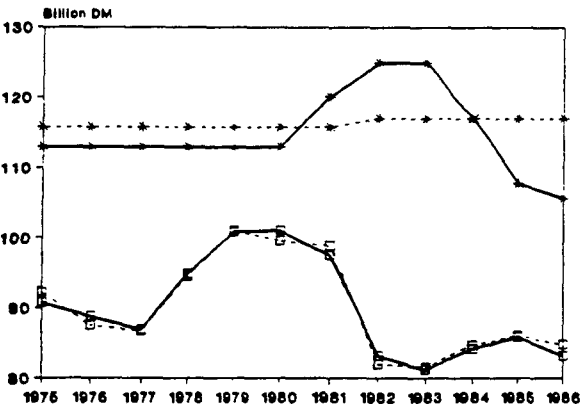
Products of mining



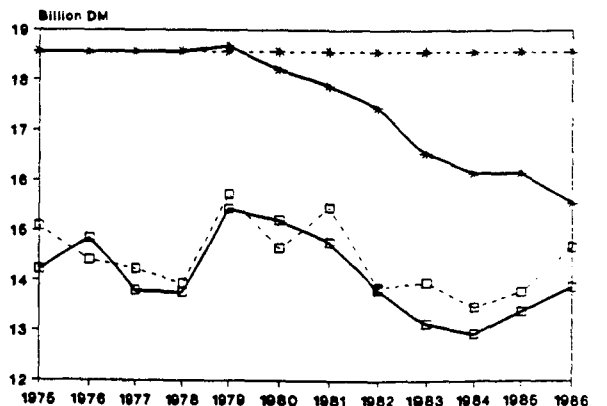
Refined petroleum products



Iron and steel



Foundry products



1) Output for both models coincides graphically

—○— Output (improved)  
- - -○- - - Output (Original)

—□— Capacity (improved)  
- - -□- - - Capacity (Original)

Using the concept of capacity reserve, we, first, elaborated the method of describing capacity retirement within the model. Thus, we could renounce the unrealistic hypothesis of its absence in the original LSD formulation. Second, this concept, being further developed, has helped us to specify a more accurate rule for expansion investment decisions. Estimating the values of capacity reserves' parameters, we have applied the experience and techniques of measuring the sectoral potential output.

In order to test the goodness of fit for the improved model, ex post simulations were conducted for the economy of the FRG (1975-1986). It could be shown that the improved model is better in tracing the actual development of gross output, gross investment and capacity expansion investment. This is true on the aggregate as well as on the sectoral level.

## APPENDIX

### **Estimation of parameters in ex ante simulations: possible approaches**

Using the mean values of  $\beta$  (over some ex post periods) seems to be the most simple way to solve this problem in long-term prognostic calculations. In this case, one is usually interested in average growth tendencies but not in yearly fluctuations induced by the current phase of a business cycle.

In fact, however, this method is often far from being satisfactory, as the series of  $\beta$  can have a marked trend, and extrapolation of a trend is hardly possible for capacity utilization coefficients.

The basic idea when estimating  $\beta$  in ex ante simulations, is to use the same approach as in ex post verification. The principle possibility follows from the fact that only the values of  $\beta$  for the previous years are needed in the modified model in order to get a solution for each current year. Thus, each time, evaluating  $\beta$ , we have at our disposal all the (simulated) data on the respective past years.

Though the capital vintage model cannot be directly applied to derive a potential output in this case (as it demands more detailed data than those available from the dynamic model) some simplified approach, such as that applied by the German Council of Economic Advisers (CEA 1987) can be used.

In the CEA approach the capital stock is regarded as the only limiting factor for potential output. The productivity of capital at full employment level, i.e. the potential productivity of capital, is assumed to develop with a constant growth rate. It is found by applying regression analysis on the logarithms of measured productivity of capital (actual data) and can easily be extrapolated. Dividing capital stock by the potential productivity of capital yields potential output.

In the model (21)-(28) the capital stock  $G_j$  in a sector  $j$  for each year of simulation period can be computed as

$$G_j(t) = \sum_i b_{ij}(t) \cdot x_j(t)$$

Let  $\rho_i(t)$  be the productivity of capital in sector  $i$  in year  $t$ . According to CEA,

$$q_i(t) = (1 + \gamma_i) \cdot q_i(t-1),$$

thus, knowing  $\rho_i(t_0-1)$  and  $\gamma_i$ , one can compute  $\rho_i(t)$ ,  $t = t_0, \dots, T$ .

The initial values of  $\rho_i(t_0-1)$  can be obtained from actual information. After that, potential output,  $P_i(t-1)$ , is calculated as follows:

$$P_i(t-1) = G_i(t-1) / q_i(t-1), \quad t = t_0, \dots, T$$

and, using the same approach, as above

$$\beta_i(t-1) = \frac{x_i(t-1)}{P_i(t-1)}.$$

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