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# **Unemployment Fluctuations in a Small Open-Economy Model with Segmented Labour Markets: The Case of Canada**

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# **Unemployment Fluctuations in a Small Open-Economy Model with Segmented Labour Markets: The Case of Canada**

**by**

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## Abstract

The recent financial crisis and subsequent recession have spurred great interest in the sources of unemployment fluctuations. Previous studies predominantly assume a single economy-wide labour market, and therefore abstract from differences across sector-specific labour markets in the economy. In Canada, such differences are substantial. From 1991 to 2010, employment in the tradable sector is almost three times as volatile as that in the non-tradable sector, and wages are about twice as volatile. To capture the labour market differences at the sectoral level, I introduce a segmented labour market structure to a medium-scale dynamic stochastic general-equilibrium model with financial and labour market frictions and estimate the model using Canadian data from 1991 to 2010. I find that, in the long run, unemployment fluctuations are mainly driven by the shocks to firms' net worth and production technology in the non-tradable sector and the shocks to the foreign interest rate. In the short run, however, it is the shocks to firms' net worth in the tradable sector that account for about 50 per cent of unemployment fluctuations. I also find that inclusion of the recent financial crisis data in the estimation is crucial for assessing the effects of the financial wealth shocks.

*JEL classification: E32, E44, J6*

*Bank classification: Labour markets; Economic models; Business fluctuations and cycles*

## Résumé

La récente crise financière et la récession qui l'a succédé ont suscité beaucoup d'intérêt à l'égard des sources de fluctuation du chômage. La majeure partie des recherches menées par le passé sont fondées sur l'existence d'un seul marché du travail pour l'ensemble de l'économie et, par conséquent, font abstraction des différences sectorielles entre les marchés du travail. Or, au Canada, ces différences sont importantes. Pour la période allant de 1991 à 2010, la volatilité de l'emploi dans le secteur des biens échangeables a été presque trois fois plus élevée, et la volatilité des salaires, environ deux fois plus grande, que dans le secteur des biens non échangeables. Dans le but de cerner ces différences sectorielles, l'auteure introduit une structure de marché segmentée dans un modèle d'équilibre général dynamique et stochastique de taille moyenne intégrant des frictions financières et un marché du travail soumis à des frictions. Le modèle est estimé au moyen de données canadiennes s'étalant de 1991 à 2010. L'auteure constate qu'à long terme, les fluctuations du chômage sont principalement attribuables aux chocs qui touchent la valeur nette des entreprises et les techniques de production dans le secteur des biens non échangeables, mais aussi aux chocs relatifs aux taux d'intérêt à l'étranger. À court terme, toutefois, les chocs modifiant la valeur nette des entreprises du secteur des biens échangeables rendent compte d'environ 50 % des fluctuations du chômage. L'auteure note également que l'inclusion des données liées à la récente crise financière dans l'estimation s'avère essentielle pour évaluer l'incidence des chocs de richesse financière.

*Classification JEL : E32, E44, J6*

*Classification de la Banque : Marchés du travail; Modèles économiques; Cycles et fluctuations économiques*

# 1 Introduction

The recent financial crisis and the subsequent recession have spurred great interest in searching for the sources of unemployment fluctuations, particularly in exploring the importance of the frictions and shocks in the financial sector for explaining labour market dynamics. Recent work, namely by Christiano, Trabandt and Walentin (2011) and Zhang (2011a and b), has applied the Bayesian maximum-likelihood method to estimate medium-scale dynamic stochastic general equilibrium models in order to assess the contribution of a variety of shocks to unemployment fluctuations, with the focus on financial shocks “relative” to other shocks. Christiano, Trabandt and Walentin (2011) find that the domestic markup shock is the most important shock, explaining more than 20 per cent of unemployment fluctuations in the Swedish economy, and that the financial wealth shock explains about 10 per cent; Zhang (2011a and b) finds that technology, investment and financial wealth shocks account for most of the unemployment fluctuations in the United States and Canada, with financial wealth shocks explaining about 30 per cent of the fluctuations.<sup>1</sup>

In these studies, there is only one aggregate labour market. This not only assumes away the labour market differences across sectors, but also ignores the potential impacts of the sector-specific shocks on unemployment at the aggregate level. In reality, however, economies consist of multiple sectors, and there are significant differences in terms of the dynamics of the key labour market variables across sectors in the Canadian economy: employment in the tradable sector (manufacturing industries) is almost three times as volatile as that in the non-tradable sector (mainly services industries), and wages are about twice as volatile. Moreover, during the recent financial crisis, one distinct feature is that job loss is not evenly distributed across sectors in Canada. The tradable sector has been more affected than others: despite employing only about 20 per cent of the total labour force, it accounted for over one-half of the total job loss. This suggests that the shocks occurring in one sector can be an important source of aggregate unemployment fluctuations. Shocks at the sectoral level require resources to move from contracting to expanding sectors. However, it is usually more difficult for workers who lost jobs in one sector to find jobs in the other sector. As long as job losses occurring in the contracting sectors are not fully offset by the expanding sectors, sector-specific shocks will have an impact on aggregate unemployment variations.<sup>2</sup>

In order to capture the sectoral differences, I introduce a segmented labour market structure into a multiple-sector small open-economy model with both financial and labour market frictions. I model the financial and labour market frictions similar to Christiano, Trabandt and Walentin (2011) and Zhang (2011a and b): financial frictions are introduced à la Bernanke, Gertler and Gilchrist (1999) – to finance capital acquisition, entrepreneurs in each sector need to pay a risk premium in order to obtain external funds from financiers, and the risk premium depends on entrepreneurs’

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<sup>1</sup>Without explicitly modelling unemployment, Jermann and Quadrini (2012) use the Bayesian likelihood method to estimate a model with financial frictions and shocks using U.S. data from 1984Q1 to 2010Q2, and they find that the financial shocks contribute about 33 per cent of the volatility of labour hours.

<sup>2</sup>See Walsh (2011) for further discussion regarding the implications of imperfect mobility of labour resources for the aggregate economy and monetary policy.

balance-sheet positions. Labour market frictions are modelled using the Mortensen-Pissarides-Diamond framework – which assumes that there are search frictions in the labour market and that unemployment is an equilibrium outcome. The new feature is the segmented labour market structure. There are two separate labour markets in the model: one for the tradable sector, and one for the non-tradable sector. The labour market parameters are sector-specific. As a result, in one sector, the labour market may be tighter than in the other, and the wage contract may be less sticky than in the other. These features, from the unemployed workers' perspective, make the labour market in that sector less frictional, because it is easier for them to find jobs. In addition, the frictions related to labour mobility are modelled in the following way: for workers willing to move across sectors, they first must be separated from jobs in one sector and become unemployed. The separation rate is exogenously given. Once unemployed, they have a chance to search for jobs in the other sector and the probability of finding a job depends on the labour market tightness in that sector.

I estimate the model using Canadian data from 1991Q1 to 2010Q4. The main findings are as follows. First, given that none of the labour market variables are used in the estimation, the model performs particularly well in terms of matching the key features in the labour market: it not only matches the fact that aggregate unemployment is much more volatile than output, but also generates labour market dynamics differences at the sectoral level. In particular, the model matches very well the relative volatility of real wages. It also captures almost half of the relative volatility of employment and output. These sectoral differences are mainly explained by sector-specific technology and financial wealth shocks. Second, the estimation results show that the degrees of frictions in the labour and financial markets are indeed different for the two sectors: compared to the tradable sector, the elasticity of external finance for the non-tradable sector is about ten times larger, and the average wage contract is about twice as long. Third, the financial wealth shocks – the shocks to the net worth of the entrepreneurs – are the most important shocks for explaining the unemployment fluctuations in the Canadian labour market. In the long run, the financial wealth shocks in the non-tradable sector, together with the technology shocks in the non-tradable sector and foreign interest rate shocks, explain about 70 per cent of the unemployment variations. In the short run, it is the financial wealth shock in the tradable sector that explains about half of the unemployment variations. Fourth, I find that it is crucial to include the data from the recent financial crisis in the estimation. Compared to the pre-financial crisis subsample results, the estimated financial shocks based on the full sample are more volatile and the external finance premium is more responsive, suggesting that the data from the recent financial crisis might play an essential role in determining the importance of the financial wealth shocks.

Lastly, I extend the model to include a commodity production sector. Since Canada is a net exporter of commodities, commodity price fluctuations in the world market can lead to fluctuations in the terms of trade, and this can have an important impact on the tradable and non-tradable sectors. The results show that, in the presence of commodity price shocks, financial wealth shocks still remain relatively important, accounting for close to 20 per cent of the unemployment fluctuations.

Table 1: Standard Deviations: Tradable vs. Non-tradable

	Output	Employment	Wages
Tradable	0.0348	0.0186	0.0105
Non-tradable	0.0088	0.0066	0.0061
Relative volatility	3.95	2.81	1.71

The paper is organized as follows. In the next section, I document some empirical facts regarding sectoral differences. In section 3 I describe the model, and in section 4 I discuss the data and estimation strategy. In section 5, I report the estimation results and discuss the effect of the sector-specific shocks on aggregate unemployment fluctuations. In section 6, I extend the model to include a commodity sector. In section 7, I offer some concluding remarks.

## 2 Sectoral Differences: Some Empirical Facts

### 2.1 Output, employment and wages

Throughout the paper, the tradable sector refers to manufacturing industries, and the non-tradable sector refers to the rest of the economy but excludes agriculture and natural resources.<sup>3</sup> The data used in this section and the estimation section are from Statistics Canada. Output is measured by real GDP and expressed in per capita terms using the civilian population aged 15 and up. Wages are measured using an index of average hourly earnings. All three of the series are logged and detrended using the Hodrick-Prescott (HP) filter with smoothing parameter 1600. Figures 1 and 2 plot the output, employment and wages for the tradable and non-tradable sectors from 1991Q1 to 2010Q4, and they clearly show that all three variables are more volatile in the tradable sector compared to the non-tradable sector.

Table 1 provides the standard deviations of output, employment and wages for the two sectors. The relative volatility in the third row is computed by normalizing the standard deviations of the three variables in the tradable sector on their non-tradable counterparts. Table 1 further quantifies the difference in volatilities across the two sectors: output in the tradable sector is about 4 times as volatile as that in the non-tradable sector, employment is about 3 times, and wages are about 2 times.

## 3 The Model

I consider a small open economy that consists of three sectors: the tradable, non-tradable and imported-goods sectors. Labour markets for the tradable and non-tradable sectors are segmented. In each labour market, employment agencies post vacancies and hire workers seeking jobs in that sector. The sector-specific employment agencies supply labour services to the entrepreneurs in that

<sup>3</sup>The non-tradable sector includes utilities; construction; wholesale and retail trade; transportation; information; finance and insurance; professional services; administrative and support; waste management; education; and health and arts.



sector, who produce sector-specific intermediate goods using labour services and capital. Since entrepreneurs need to borrow to finance capital purchases, they are subject to financial frictions. Entrepreneurs supply intermediate goods to retailers in each sector, who produce final goods. A representative household with a large family structure has a fraction of its members unemployed, and the rest are employed in either the tradable or non-tradable sector. The household consumes, saves in domestic bonds and foreign bonds, pays taxes, and receives profits from retailers in each sector. In addition, there are capital producers, a government that balances the budget and a central bank that implements a simple interest rate rule. In this section, I describe the role of each agent in the model.

### 3.1 Household

Each member in the household performs a number of functions: it consumes, holds both nominal domestic bonds  $B_t$  and foreign bonds  $B_t^*$  (denominated in foreign currency), receives dividends from retailers  $\Pi_t$ , and pays taxes  $T_t$ . At time  $t$ , a fraction of household members are employed,  $n_t$ , and a fraction are unemployed  $u_t = 1 - n_t$ . For the employed household members,  $n_{T,t}$  of them are employed in the tradable sector, and  $n_{N,t}$  in the non-tradable sector. For those who are unemployed,  $u_{T,t}$  of them search for jobs in the tradable sector, and  $u_{N,t}$  of them search for jobs in the non-tradable sector. The employed family members earn nominal wages  $W_{T,t}$  and  $W_{N,t}$ , respectively. The unemployed members receive unemployment benefits  $ub_t$ . Following Andolfatto (1996) and Merz (1995), family members are assumed to be perfectly insured against the risk of becoming unemployed. Thus consumption is the same for each family member. The budget constraint for the representative household is

$$P_t c_t + B_t + \frac{e_t B_t^*}{\kappa_t R_t^*} + T_t \leq W_{T,t} n_{T,t} + W_{N,t} n_{N,t} + ub_t(1 - n_t) + R_{t-1} B_{t-1} + e_t B_{t-1}^* + \Pi_t, \quad (1)$$

where both  $W_t^T$  and  $W_t^N$  are determined by Nash bargaining between employment agencies and workers. The labour supply  $n_{T,t}$  and  $n_{N,t}$  is determined by a search and match process. The nominal exchange rate is denoted by  $e_t$ . The return on the foreign bonds,  $\kappa_t^h R_t^*$ , depends on the foreign interest rate  $R_t^*$  and a country-specific risk premium  $\kappa_t^h$ , which is assumed to be an increasing function of the net foreign-debt-to-GDP ratio:

$$\kappa_t^h = \exp \left( v \frac{e_t \tilde{B}_t^*}{P_t y_t} \right),$$

where  $v > 0$ ,  $y_t$  is real GDP and  $\tilde{B}_t^*$  is the total level of indebtedness of the economy.

Given the budget constraint equation (1), the representative household chooses  $c_t$ ,  $B_t$  and  $B_t^*$  to

maximize the lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

with

$$u(c_t) = \mu_t \frac{c_t^{1-\sigma}}{1-\sigma},$$

where  $c_t$  is the consumption of final goods in period  $t$  and  $\mu_t$  is a preference shock that follows

$$\log \mu_t = \rho_\mu \log \mu_{t-1} + \epsilon_t^\mu, \quad \epsilon_t^\mu \sim i.i.d. N(0, \sigma_{\epsilon^\mu}^2).$$

The first-order conditions yield

$$\lambda_t \mu_t = \beta E_t \left[ \frac{R_t \lambda_{t+1} \mu_{t+1}}{\pi_{t+1}} \right], \quad (2)$$

$$\lambda_t \mu_t = \beta E_t \left[ \frac{R_t^* \kappa_t^h s_{t+1} \lambda_{t+1} \mu_{t+1}}{s_t \pi_{t+1}^*} \right], \quad (3)$$

where  $\pi_t = P_t/P_{t-1}$  is the CPI inflation rate and  $s_t = e_t P_t^*/P_t$  is the real exchange rate.  $P_t^*$  is a foreign price index. Equations (2) and (3) imply the uncovered interest rate parity condition:

$$\frac{R_t}{\kappa_t^h R_t^*} = \frac{e_{t+1}}{e_t}.$$

### 3.2 Employment agencies

Following Christiano, Trabandt and Waletin (2011), I model employment agencies as intermediaries between the representative household (who supply labour) and entrepreneurs (who demand labour to produce wholesale goods).<sup>4</sup> The labour market is modelled using a standard search framework. On the one hand, the employment agencies post vacancies and bargain over wages with workers; on the other, they combine labour supplied by households into homogeneous labour services and supply them to entrepreneurs at a competitive price. The labour market is segmented. Thus, employment agencies in the tradable sector (non-tradable) post vacancies only in the tradable (non-tradable) sector. Unemployed workers need to decide which sector to search. In equilibrium, searching for jobs in each sector gives the same expected payoff.

In the beginning of period  $t$ , in each sector  $i$ , employment agency  $j$  employs  $n_{i,t}(j)$  workers, and posts  $v_{i,t}(j)$  vacancies to attract new workers. The total number of vacancies and the number of employed workers are denoted as  $v_{i,t} = \int v_{i,t}(j) dj$  and  $n_{i,t} = \int n_{i,t}(j) dj$ . The number of unemployed

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<sup>4</sup>This leaves the equilibrium conditions associated with the production of wholesale goods unaffected, although the labour market is frictional.

workers at the beginning of period  $t$  is

$$\begin{aligned} u_t &= 1 - n_t = 1 - n_{T,t} - n_{N,t} \\ &= u_{T,t} + u_{N,t}. \end{aligned}$$

The number of new hires  $m_{i,t}$  is given by a standard Cobb-Douglas aggregate matching technology,

$$m_{i,t} = \mu_{i,m} u_{i,t}^{\sigma_m} v_{i,t}^{1-\sigma_m},$$

where  $\mu_{i,m}$  is the parameter governing matching efficiency. The probability that a firm fills a vacancy in period  $t$ ,  $q_{i,t}^l$ , is given by

$$q_{i,t}^l = \frac{m_{i,t}}{v_{i,t}}.$$

Similarly, the probability that a searching worker finds a job,  $s_{i,t}^l$ , is given by

$$s_{i,t}^l = \frac{m_{i,t}}{u_{i,t}}.$$

Both firms and workers take  $q_{i,t}^l$  and  $s_{i,t}^l$  as given. In each period, a fraction  $1 - \rho_i$  of the existing workforce  $n_{i,t}$  exogenously separates from the firms. The total labour force is the sum of the number of surviving workers and the new matches:

$$n_{i,t+1} = \rho_i n_{i,t} + m_{i,t}.$$

I define the hiring rate,  $x_{i,t}(j)$ , as the ratio of new hires,  $q_{i,t}^l v_{i,t}(j)$ , to the existing workforce,  $n_{i,t}(j)$ :

$$x_{i,t}(j) = \frac{q_{i,t}^l v_{i,t}(j)}{n_{i,t}(j)}.$$

The value of the employment agency  $F_{i,t}(j)$  is

$$F_{i,t}(j) = p_{i,t}^l n_{i,t}(j) - \frac{W_{i,t}(j)}{p_t} n_{i,t}(j) - \frac{\kappa_i}{2} x_{i,t}(j)^2 n_{i,t}(j) + \beta E_t \Lambda_{t,t+1} F_{i,t+1}(j),$$

where  $\frac{\kappa_i}{2} x_{i,t}(j)^2 n_{i,t}(j)$  is the quadratic costs of adjusting employment, and  $\beta E_t \Lambda_{t,t+1}$  is the employment agency's discount rate with  $\Lambda_{t,t+1} = c_t / c_{t+1}$ . At any time, the employment agency chooses the hiring rate  $x_{i,t}(j)$  to maximize  $F_{i,t}(j)$ , given the existing employment stock  $n_{i,t}(j)$ , the probability of filling a vacancy  $q_{i,t}^l$ , and the current and expected path of nominal wages  $W_{i,t}(j)$ . Define the real wage as  $w_{i,t} = \frac{W_{i,t}(j)}{p_t}$ . The value to the employment agency of adding another worker at time  $t$ ,  $J_{i,t}(j)$ , can be obtained by differentiating  $F_{i,t}(j)$  with respect to  $n_{i,t}(j)$ :

$$J_{i,t}(j) = p_{i,t}^l - w_{i,t} - \frac{\kappa_i}{2} x_{i,t}(j)^2 + (\rho_i + x_{i,t}(j)) \beta E_t \Lambda_{t,t+1} J_{i,t+1}(j).$$

The first-order condition for vacancy posting equates the marginal cost of adding a worker with the

discounted marginal benefit:

$$\kappa_i x_{i,t}(j) = \beta E_t \Lambda_{t,t+1} J_{i,t+1}(j).$$

For workers, the value to a worker of employment at agency  $i$ ,  $V_t(j)$ , is

$$V_{i,t}(j) = w_{i,t}(j) + \beta E_t \Lambda_{t,t+1} [\rho V_{i,t+1}(j) + (1 - \rho) U_{i,t+1}],$$

The value of unemployment,  $U_{i,t}$ , is

$$U_{i,t} = ub_t + \beta E_t \Lambda_{t,t+1} [s_{i,t+1}^l V_{i,t+1} + (1 - s_{i,t+1}^l) U_{i,t+1}],$$

where  $V_{i,t}$  is the average value of employment for a new worker at time  $t$ .<sup>5</sup> The workers' surplus for having a job at employment agency  $j$ ,  $H_{i,t}(j)$ , is

$$H_{i,t}(j) = V_{i,t}(j) - U_{i,t}.$$

Given that

$$U_{T,t} = U_{N,t},$$

it follows that

$$\beta E_t \Lambda_{t,t+1} s_{T,t+1}^l H_{T,t}(j) = \beta E_t \Lambda_{t,t+1} s_{N,t+1}^l H_{N,t}(j).$$

For an unemployed worker, the expected payoff of searching for jobs in either sector must be equal. In equilibrium, a lower job-finding rate in one sector must be compensated by a relatively higher surplus of having a job in that sector.

Employment agencies and workers negotiate a nominal wage  $W_{i,t}(j)$  to maximize the joint product of the workers' surplus  $H_{i,t}(j)$  and the employment agencies' surplus  $J_{i,t}(j)$ . However, every period, each employment agency has only a fixed probability  $1 - \lambda_i$  to negotiate with workers. The Nash bargaining problem between employment agencies and workers is

$$\max H_{i,t}(j)^\eta J_{i,t}(j)^{1-\eta},$$

s.t.

$$\begin{aligned} W_{i,t}(j) &= W_{i,t}^* \text{ with probability } 1 - \lambda_i \\ &= W_{i,t-1}\pi \text{ with probability } \lambda_i, \end{aligned}$$

where  $\pi$  is the steady-state inflation rate. The equation for the real wage  $w_{i,t}^*$  derived from this staggered contracting is

$$\begin{aligned} \Delta_t w_{i,t}^* &= \eta(p_{i,t}^l + \frac{\kappa_i}{2} x_{i,t}^2(i)) + (1 - \eta)(ub_t + s_{i,t+1} \beta \Lambda_{t,t+1} H_{i,t+1}) \\ &\quad + \lambda_i \rho_i \beta E_t \Lambda_{t,t+1} \Delta_{t+1} w_{i,t+1}^*. \end{aligned} \tag{4}$$

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<sup>5</sup>See Gertler and Trigari (2009) for details on the average value of employment.

The first term of equation (4) is the worker's contribution to the match, and the second is the worker's opportunity cost. These are conventional components for Nash bargaining solutions for wages. The third term is from the staggered multi-period contracting. Finally, the aggregate real wage  $w_{i,t}$  can be expressed as<sup>6</sup>

$$w_{i,t} = (1 - \lambda)w_{i,t}^* + \lambda\pi \frac{1}{\pi_{i,t}}w_{i,t-1}.$$

### 3.3 Entrepreneurs

There are entrepreneurs in both the tradable and non-tradable sectors. Following Bernanke, Gertler and Gilchrist (1999), entrepreneurs are risk-neutral and have a finite life. Using a Cobb-Douglas technology, in each sector  $i$ , at each period  $t$ , entrepreneur  $j$  uses capital  $k_{i,t}(j)$  and labour services  $l_{i,t}(j)$  to produce wholesale goods  $y_{i,t}(j)$ :

$$y_{i,t}(j) = a_{i,t}(k_{i,t}(j))^\alpha (l_{i,t}(j))^{1-\alpha},$$

where  $a_{i,t}$  is the technology shock that follows

$$\log a_{i,t} = \rho_{a,i} \log a_{i,t-1} + \epsilon_t^{a_i}, \epsilon_{i,t}^{a_i} \sim i.i.d. N(0, \sigma_{\epsilon^{a_i}}^2).$$

Entrepreneurs purchase capital at price  $q_{i,t}$  from capital producers, using both their own net worth  $N_{i,t}$  and bank loans.

Given that a significant portion of the financing of Canadian corporations is raised in the United States, I assume that Bank loans can originate from both the domestic market  $B_{i,t}$  and the international market  $B_{i,t}^*$ .<sup>7</sup> Entrepreneurs can default due to idiosyncratic shocks, and since only they observe the realization of those shocks, the optimal loan contract in Bernanke, Gertler and Gilchrist (1999) is such that the entrepreneurs pay a risk premium on loans. Thus for the domestic loans, the external finance premium,  $rp(\cdot)$ , depends on the entrepreneur's balance-sheet position. At the aggregate level it can be characterized by

$$rp_{i,t} = rp\left(\frac{q_{i,t}k_{i,t+1}}{N_{i,t+1}}\right), \quad (5)$$

where  $rp'(\cdot) > 0$  and  $rp(1) = 1$ . Equation (5) expresses that, in each sector, the external finance premium increases with leverage.

For the loans from the international market, I assume that entrepreneurs need to pay  $R_t^* \kappa_{i,t}^{rp}$ , where  $R_t^*$  is the foreign interest rate and  $\kappa_{i,t}^{rp}$  is a risk premium related to industry  $i$  when borrowing

<sup>6</sup>For details of the derivations of staggered wage contracts, see Zhang (2011a).

<sup>7</sup>The International Monetary Fund (2008) suggests that a quarter of the financing of Canadian corporations is raised in the United States. Statistics Canada's Survey of Suppliers of Business Financing also suggests that, in 2008, almost 20 per cent of the total debt was foreign. For manufacturing, the fraction is even higher – close to 30 per cent. Although there is a slight decrease after 2008, foreign debt remains a significant portion of total debt financing for Canadian corporations, especially for the manufacturing industry.

from abroad. It is assumed that this industry-specific risk premium  $\kappa_{i,t}^{rp}$  is an increasing function of the fraction of foreign loans in total debt,

$$\kappa_{i,t}^{rp} = \exp \left( \varsigma_i \frac{\frac{e_t B_{i,t}^*}{P_t}}{q_{i,t} k_{i,t+1} - N_{i,t}} \right), \quad (6)$$

where  $\varsigma_i > 0$  is a parameter determining the relative size of  $B_{i,t}(j)$  and  $B_{i,t}^*(j)$  for each sector. The reason for having  $\kappa_{i,t}^{rp}$  is mainly due to technical concerns. Without it, the foreign debt at the sectoral level may be non-stationary, complicating the analysis of local dynamics. The one-period profit function for entrepreneur  $j$  is

$$\begin{aligned} \pi_{i,t}(j) = & \frac{B_{i,t}(j)}{P_t} + \frac{e_t B_{i,t}^*(j)}{P_t} + p_{i,t}^w y_{i,t}^j + q_{i,t}(1 - \delta)k_{i,t}(j) \\ & - p_{i,t}^l l_{i,t}(j) - R_{t-1} r p_{i,t-1} \frac{B_{i,t-1}(j)}{P_t} - R_{t-1}^* \kappa_{i,t-1}^{rp} \frac{e_t B_{i,t-1}^*(j)}{P_t} \\ & - q_{i,t} k_{i,t+1}(j), \end{aligned}$$

where  $p_{i,t}^w$  is the relative price for the wholesale goods in sector  $i$ , and  $p_{i,t}^l$  is the labour service price. Let  $b_{i,t}(j)$  and  $b_{i,t}^*(j)$  denote the real debts in the economy; i.e.,  $b_{i,t}(j) = \frac{B_{i,t}(j)}{P_t}$  and  $b_{i,t}^*(j) = \frac{B_{i,t}^*(j)}{P_t}$ .<sup>8</sup> Entrepreneur  $j$  chooses  $l_{i,t}(j)$ ,  $k_{i,t+1}(j)$ ,  $b_{i,t}(j)$  and  $b_{i,t}^*(j)$  to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \pi_{i,t}(j).$$

The first-order condition yields:

$$l_{i,t}(j) : p_{i,t}^w \frac{\partial y_{i,t}(j)}{\partial l_{i,t}(j)} = p_{i,t}^l,$$

$$k_{i,t+1}(j) : -q_{i,t} + E_t \beta [p_{i,t+1}^w \frac{\partial y_{i,t+1}(j)}{\partial k_{i,t+1}(j)} + q_{i,t+1}(1 - \delta)] = 0,$$

$$b_{i,t}(j) : 1 - E_t \beta \left[ \frac{R_t r p_{i,t}}{\pi_{t+1}} \right] = 0,$$

and

$$b_{i,t}^*(j) : 1 - E_t \beta \left[ \frac{s_{t+1} R_t^* \kappa_{i,t}^{rp}}{s_t \pi_{t+1}^*} \right] = 0.$$

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<sup>8</sup>Dib, Mendicino and Zhang (2008) also allow both domestic and international financing in their model; however, they assume that the tradable sector can obtain outside financing only from the international market and that the non-tradable sector can obtain it only from the domestic market.

The expected return on capital in each sector is defined as

$$E_t r_{i,t+1}^k = \frac{E_t [p_{i,t+1}^w \alpha \frac{y_{i,t+1}}{k_{i,t+1}} + q_{i,t+1} (1 - \delta)]}{q_{i,t}}.$$

In each sector, the expected return on capital must equal the expected costs of external finance:

$$E_t r_{i,t+1}^k = E_t \beta \left[ \frac{R_t r p_{i,t}}{\pi_{t+1}} \right].$$

The demand for foreign debt for each sector is determined by

$$E_t r_{i,t+1}^k = E_t \beta \left[ \frac{s_{t+1} R_t^* \kappa_{i,t}^{rp}}{s_t \pi_{t+1}^*} \right].$$

Finally, the aggregate net worth in each sector is given by

$$N_{i,t+1} = \gamma_{i,t} \eta_i^e (r_{i,t}^k q_{i,t-1} k_{i,t} - \frac{R_{t-1} r p_{i,t-1}}{\pi_t} b_{i,N,t-1} - \frac{s_t R_{t-1}^* \kappa_{i,t-1}^{rp}}{s_{t-1} \pi_t^*} b_{i,t-1}^*),$$

where  $\eta_i^e$  is the survival rate of entrepreneurs for each sector, and  $\gamma_{i,t}$  is an exogenous shock to the survival probability. Following Christiano, Motto and Rostagno (2010), I interpret this shock as a financial wealth shock to the entrepreneurs. This shock affects the aggregate financial wealth of the entrepreneurs as follows: in the model, the number of entrepreneurs exiting is balanced by the number that enter. Since those who exit usually have more net worth than those who enter, when a positive (negative) shock occurs, the aggregate net worth of entrepreneurs increases (decreases). I assume that  $\gamma_{i,t}$  follows an AR(1) process:

$$\log \gamma_{i,t} = \rho_{\gamma,i} \log \gamma_{i,t-1} + \epsilon_t^{\gamma_i}, \quad \epsilon_t^{\gamma_i} \sim i.i.d. N(0, \sigma_{\epsilon^{\gamma_i}}^2).$$

Entrepreneurs that are going out of business consume their residue equity:

$$ce_{i,t+1} = (1 - \gamma_{i,t} \eta_i^e) (r_{i,t}^k q_{i,t-1} k_{i,t} - \frac{R_{t-1} r p_{i,t-1}}{\pi_t} b_{i,t-1} - \frac{s_t R_{t-1}^* \kappa_{i,t-1}^{rp}}{s_{t-1} \pi_t^*} b_{i,t-1}^*).$$

The aggregate demand for labour services is relatively simple. Given that the aggregate production function is constant returns to scale,

$$y_{i,t} = k_{i,t}^\alpha (z_{i,t} l_{i,t})^{1-\alpha},$$

the aggregate labour demand equation can be written as

$$p_{i,t}^w (1 - \alpha) \frac{y_{i,t}}{l_{i,t}} = p_{i,t}^l,$$

where  $l_{i,t}$  is the labour services supplied by employment agencies in sector  $i$ ,  $(1 - \alpha) \frac{y_{i,t}}{l_{i,t}}$  is the

marginal product of labour services,  $p_{i,t}^w$  is the relative price for wholesale goods and  $p_{i,t}^l$  is the relative price for labour services.

### 3.4 Capital producers

Capital producers use investment goods to produce new capital purchased by entrepreneurs. At the end of period  $t$ , they buy investment goods  $I_t$ , at real price  $p_{I,t} = P_{I,t}/P_t$  to produce sector-specific capital that can be used by entrepreneurs at time  $t + 1$ . Capital production in each sector is assumed to be subject to an investment-specific shock,  $\tau_{i,t}$ , which follows an AR(1) process:

$$\log \tau_{i,t} = \rho_{i,x} \log \tau_{i,t-1} + \epsilon_{i,t}^\tau, \epsilon_{i,t}^\tau \sim i.i.d. N(0, \sigma_{\epsilon_i^\tau}^2).$$

Following Christiano, Eichenbaum and Evans (2005), I assume that capital producers in each sector face investment adjustment costs  $S(I_{i,t}, I_{i,t-1})$ , such that in steady state  $S = S' = 0$  and  $S'' > 0$ , and  $\xi_i > 0$  is an investment adjustment cost parameter. The production of each capital stock yields the following time- $t$  profit function:

$$\Pi_t^i = q_{i,t} I_{i,t} \tau_{i,t} - q_{i,t} I_{i,t} S(I_{i,t}, I_{i,t-1}) - p_{I,t} I_{i,t}.$$

The aggregate stock of capital evolves as follows:

$$k_{i,t+1} = I_{i,t} \tau_{i,t} - I_{i,t} S(I_{i,t}, I_{i,t-1}) + (1 - \delta) k_{i,t}.$$

### 3.5 Sectoral goods producers

There are sectoral goods producers in all three sectors: the tradable, non-tradable and imported-goods sectors. The sectoral goods producers in both the tradable and non-tradable sectors buy the corresponding inputs from entrepreneurs; those in the imported-goods sector buy foreign homogeneous intermediate inputs, differentiate them slightly into  $z_{i,t}(j)$ , and sell the product at price  $p_{i,t}(j)$ . The final goods for each sector  $i$ ,  $z_{i,t}$ , are the composite of individual variety,

$$z_{i,t} = \left[ \int_0^1 y_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

The price index that minimizes the sectoral goods producers' cost function is

$$p_{i,t} = \left[ \int_0^1 p_{i,t}(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}.$$

Following Calvo (1983), in each period, only a fraction  $1 - \nu_i$  of retailers reset their prices, while the remaining retailers keep their prices unchanged. The retailer chooses  $p_{i,t}(j)$  to maximize



its expected real total profit over the periods during which its prices remain fixed:

$$E_t \sum_{i=0}^{\infty} \nu \Delta_{i,t+i}^p \left[ \left( \frac{p_{i,t}(j)}{p_{i,t+i}} \right) y_{i,t+i}(j) - mc_{i,t+i} y_{i,t+i}(j) \right],$$

where  $mc_{i,t}$  is the real marginal cost, namely, the price of wholesale goods relative to the price of sectoral final goods ( $p_{i,t}^w/p_{i,t}$ ). The real marginal cost for imported intermediate goods is  $e_t P_t^*$  for a given nominal exchange rate,  $e_t$ , and a foreign price level,  $P_t^*$ .  $\Delta_{t,i}^p \equiv \beta^i c_{t+i}/c_t$  is the stochastic discount factor. Let  $p_t^*$  be the optimal price chosen by all firms adjusting at time  $t$ .

The first-order condition is

$$p_{i,t}^* = \left( \frac{\varepsilon}{\varepsilon - 1} \right) \frac{E_t \sum_{s=0}^{\infty} \nu^s \Delta_{s,t+s}^p mc_{i,t+s} y_{i,t+s} \left( \frac{1}{p_{i,t+s}} \right)^{-\varepsilon}}{E_t \sum_{s=0}^{\infty} \nu^s \Delta_{s,t+s}^p y_{i,t+s} \left( \frac{1}{p_{i,t+s}} \right)^{1-\varepsilon}}.$$

The aggregate price evolves according to

$$p_{i,t} = [\nu_i p_{i,t-1}^{1-\varepsilon} + (1 - \nu_i) (p_{i,t}^*)^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}}.$$

Retailers in the tradable sector produce goods for domestic use,  $z_{T,t}^d$ , and exports,  $z_{T,t}^e$ , so that  $z_{T,t} = z_{T,t}^d + z_{T,t}^e$ . The aggregate foreign demand function for exports of manufactured goods is

$$z_{T,t}^e = \varpi \left( \frac{e_t P_{T,t}}{P_t^*} \right)^{-\nu} Y_t^*,$$

where  $Y_t^*$  is foreign output. The elasticity of demand for domestic manufactured goods among foreigners is  $-\nu$ , and  $\varpi > 0$  is a parameter determining the fraction in foreign spending of domestic manufactured goods exported.

### 3.6 Aggregate final-goods producers

A representative firm acts in a perfectly competitive market and uses sectoral output to produce final consumption and investment goods,  $x_t^j$ , with  $j = \{C, I\}$ , according to the following constant elasticity of substitution technology:

$$x_t^j = \left[ (\omega_T^j)^{\frac{1}{\nu_j}} \left( z_{T,t}^{d,j} \right)^{\frac{\nu_j-1}{\nu_j}} + (\omega_N^j)^{\frac{1}{\nu_j}} \left( z_{N,t}^j \right)^{\frac{\nu_j-1}{\nu_j}} + (\omega_F^j)^{\frac{1}{\nu_j}} \left( z_{F,t}^j \right)^{\frac{\nu_j-1}{\nu_j}} \right]^{\frac{\nu_j}{\nu_j-1}},$$

where  $\omega_T^j$ ,  $\omega_N^j$  and  $\omega_F^j$  denote the shares of domestically used tradable, non-tradable and imported composite sectoral goods in the final goods, respectively, with  $\omega_T^j + \omega_N^j + \omega_F^j = 1$ , and  $\nu_j > 0$  is the elasticity of substitution between sectoral goods:

$$p_t^j = \left[ (\omega_T^j)^{\frac{1}{\nu_j}} (p_{T,t}^j)^{1-\nu_j} + (\omega_N^j)^{\frac{1}{\nu_j}} (p_{N,t}^j)^{1-\nu_j} + (\omega_F^j)^{\frac{1}{\nu_j}} (p_{F,t}^j)^{1-\nu_j} \right]^{\frac{1}{1-\nu_j}}.$$

I further define the model's GDP at constant prices as

$$GDP_t = x_t^C + p^I x_t^I + p^T z_{T,t}^e - s z_{F,t},$$

where  $p^I$  and  $p^T$  are the steady-state prices of investment goods and tradable goods in real terms, and  $s$  is the real exchange rate at the steady state.

### 3.7 Government

The government is assumed to balance its budget,

$$G_t = T_t,$$

where  $G_t$  follows an AR(1) process,

$$\log G_t = (1 - \rho_x) \log G_{ss} + \rho_x \log G_{t-1} + \epsilon_t^g, \epsilon_t^g \sim i.i.d. N(0, \sigma_{\epsilon^g}^2).$$

### 3.8 Monetary policy rules

The central bank adjusts the nominal interest rate  $R_t$  according to a simple interest rate rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_r} \left(\frac{\pi_t}{\pi}\right)^{\rho_\pi} \left(\frac{GDP_t}{GDP}\right)^{\rho_y} e^{\epsilon_t^m},$$

where  $R^n$ ,  $\pi$  and  $GDP$  are the steady-state values of  $R_t$ ,  $\pi_t$  and  $GDP_t$ , and  $\epsilon_t^m$  is a monetary policy shock that follows

$$\epsilon_t^m \sim i.i.d. N(0, \sigma_{\epsilon^m}).$$

$\rho_\pi$ ,  $\rho_y$  and  $\rho_r$  are policy coefficients chosen by the central bank.

### 3.9 Rest of the world

Given that Canada is a small open economy, I assume that domestic developments do not affect the rest of the world. However, the foreign economy has an impact on the Canadian economy. Following Dib, Mendicino and Zhang (2008), I assume that the foreign output  $Y_t^*$ , foreign interest rate  $R_t^*$ , and inflation  $\pi_t^*$  are exogenous and follow AR(1) processes:

$$\ln(x_t) = (1 - \rho_x) \ln(x) + \rho_x \ln(x_{t-1}) + \varepsilon_{x_t}, \quad \varepsilon_{x_t} \overset{iid}{\sim} N(0, \sigma_{\varepsilon_x}), \quad 0 < \rho_x < 1,$$

where  $x_t = \{R_t^*, Y_t^*, \pi_t^*\}$ ,  $x > 0$  is a steady-state value of  $x_t$ ,  $\rho_x$  is an autoregressive coefficient vector, and  $\varepsilon_{x_t}$  is a vector of uncorrelated and normally distributed innovations with zero means and standard deviations  $\sigma_{\varepsilon_x}$ .

### 3.10 Aggregation and equilibrium

At the equilibrium, the final consumption goods  $x_t^C$  are divided among households' consumption  $c_t$ , government spending  $g_t$ , and entrepreneurs' consumption  $ce_{T,t} + ce_{N,t}$ . In addition, they are also used to cover the vacancy posting costs  $\frac{\kappa_N}{2}x_{N,t}^2n_{N,t} + \frac{\kappa_T}{2}x_{T,t}^2n_{T,t}$ :

$$x_t^C = c_t + g_t + \frac{\kappa_N}{2}x_{N,t}^2n_{N,t} + \frac{\kappa_T}{2}x_{T,t}^2n_{T,t} + ce_{T,t} + ce_{N,t}.$$

The production of investment goods equals the use of investment in the production of capital goods:

$$x_t^I = I_{N,t} + I_{T,t}.$$

A current account equation is yielded by combining the household's budget constraint, government budget, single-period profit functions of firms that produce tradable and non-tradable goods, and foreign goods importers. Under the producer currency pricing assumption, the current account equation in real terms is given by

$$\frac{b_t^*}{\kappa_t R_t^*} = \frac{b_{t-1}^*}{\pi_t^*} + \frac{p_{T,t}}{s_t} z_{T,t}^e - z_{F,t},$$

where  $b_t^* = B_t^*/P_t^*$  is the stock of real foreign debt in the domestic economy,  $z_{T,t}^e$  is the aggregate foreign demand function for exports of manufactured goods and  $z_{F,t}$  is the final good for the imported goods sector.

## 4 Estimation

### 4.1 Calibrated values

I use Bayesian techniques to estimate the model for the Canadian economy. The data sample spans from 1991Q1 to 2010Q4. Some parameters are calibrated to match the salient features of the Canadian economy, and Table 2 reports these parameters and their calibrated values.

For most parameters that govern the sectoral shares, I use the calibrated values in Dib, Mendicino and Zhang (2008). The calibrated value for the discount factor,  $\beta$ , is set at 0.99, which implies an annual steady-state real interest rate of 4 per cent. The curvature parameter in the utility function,  $\gamma$ , is set at 2, implying an elasticity of intertemporal substitution of 0.5. The capital shares in the production of tradable and non-tradable goods,  $\alpha_T$  and  $\alpha_N$ , are set at 0.35 and 0.3, which are close to the values suggested by Macklem et al. (2000). The capital depreciation rate,  $\delta$ , is assumed to be common to both tradable and non-tradable sectors, and is set at 0.025, a value commonly used in the literature. Following Dib (2008), the shares of tradable, non-tradable and imported goods in the production of consumption goods,  $\omega_T^C$ ,  $\omega_N^C$ , and  $\omega_F^C$ , are set at 0.1, 0.57 and 0.33, respectively. Since the share of imported goods in the production of the investment goods is higher than that in consumption goods production, I set  $\omega_T^I$ ,  $\omega_N^I$  and  $\omega_F^I$  equal at 0.2, 0.4 and 0.4, respectively.

The parameter measuring the degree of monopoly power in the intermediate-goods markets,  $\theta$ ,

is set at 6, which implies a 20 per cent markup in the steady state. Based on Dib (2003), both the elasticity of substitution between tradable, non-tradable and imported goods in the production of final consumption goods,  $\nu^C$ , and the elasticity of demand for domestic manufactured goods among foreigners,  $\nu$ , are set equal at 0.8. The elasticity of substitution between tradable, non-tradable and imported goods in the production of final investment goods,  $\nu^I$ , is set at 0.6, implying that imported goods are less substitutable in producing investment than against the consumption-good production. Following Dib, Mendicino and Zhang (2008), I set the parameters determining the steady-state leverage ratios for tradable and non-tradable sectors,  $k_T$  and  $k_N$ , at 0.7 and 0.6, respectively. In calibration, the following functional form is used for the external finance premium:

$$rp_t = \left( \frac{q_t k_{t+1}}{N_{t+1}} \right)^\chi,$$

where  $\chi$  is the elasticity of the external risk premium with respect to leverage and  $\chi > 0$ .  $\chi$  can be viewed as a “reduced-form” parameter capturing financial market frictions.<sup>9</sup>

The steady-state gross domestic and foreign inflation rates,  $\pi$  and  $\pi^*$ , equal 1.0048 and 1.0052, respectively, which are the historical averages over the estimation sample for Canada and the United States. I calibrate the parameter  $v$ , which determines the country-specific risk premium, to match a ratio of foreign debt to GDP of about 30 per cent, as in the data.

For most labour market parameters, I use values from Zhang (2011a and b). The bargaining power parameter,  $\eta$ , is set at 0.5, which is commonly used in the literature. The elasticity of matches to unemployment,  $\sigma_m$ , is set at 0.5, the midpoint of values typically used. Following the suggestion of Zhang (2008), the aggregate job-separation rate,  $1 - \rho$ , is set at 0.09, matching the average job duration of 2.8 years in Canada; the aggregate job-finding rate  $s^l$  is, accordingly, set at 0.927, matching the fact that one-third of unemployed workers find jobs within one month. I normalize the mean of market tightness to 1, which implies that the value of  $\mu_m$  in the matching function equals the quarterly job-finding rate. Following Gertler, Sala and Trigari (2008), I express  $ub$ , the steady-state flow value of unemployment, as

$$ub = \tilde{b}(p^l + \frac{\kappa}{2}x^2),$$

where  $\tilde{b}$  is the fraction of the worker’s contribution to the job. Following Shimer (2005), I set  $\tilde{b}$  at 0.4.

Several new parameters arise from the fact that the labour market is segmented and entrepreneurs can borrow from both domestic and foreign lenders. The survival rate of jobs in the tradable sector,  $\rho_T$ , is set at 0.94, which is taken from Tapp (2011). The steady-state fraction of employed workers in the tradable sector,  $\theta^e$ , is set at 0.2, to match the data. I use debt allocation data from the Survey

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<sup>9</sup>The elasticity of the external risk premium  $\chi$  is the key parameter of the financial accelerator mechanism. It is determined by the “deep” parameters in Bernanke, Gertler and Gilchrist (1999): the variance of idiosyncratic shocks to the return on capital, the bankruptcy costs and entrepreneurs’ survival rate.

Table 2: Calibration of the Parameters

Param.	Definition	Values
$\beta$	Discount factor	0.99
$\gamma$	Inverse of intertemporal substitution of consumption	2
$\nu^C$	Elasticity of substitution between sectors, consumption	0.8
$\nu^I$	Elasticity of substitution between sectors, investment	0.6
$\nu$	Elasticity of demand for domestic tradable goods among foreigners	0.8
$\theta$	Intermediate-goods elasticity of substitution	6
$\alpha_T$	Capital share, tradable	0.35
$\alpha_N$	Capital share, non-tradable	0.30
$\delta_T$	Capital depreciation rate, tradable	0.025
$\delta_N$	Capital depreciation rate, non-tradable	0.025
$\omega_T^C$	Share of tradable goods, consumption	0.10
$\omega_N^C$	Share of non-tradable goods, consumption	0.57
$\omega_T^I$	Share of tradable goods, investment	0.20
$\omega_N^I$	Share of non-tradable goods, investment	0.40
$k_T$	Steady-state net worth to capital ratio, tradable	0.7
$k_N$	Steady-state net worth to capital ratio, non-tradable	0.6
$\pi$	Steady-state domestic inflation rate	1.0048
$\pi^*$	Steady-state foreign inflation rate	1.0052
$\rho$	Aggregate survival rate of jobs	0.91
$\rho_T$	Survival rate of jobs in tradable sector	0.94
$s^l$	Aggregate job-finding rate	0.927
$\eta$	Bargaining power of workers	0.5
$\tilde{b}$	Parameter for unemployment flow value	0.4
$\sigma_m$	Elasticity in matches to unemployment	0.5
$\theta^e$	Fraction of employed workers in tradable sector	0.2

Table 3: Estimation Results: Foreign Shock Processes

			Prior		Posterior
Coef.	Description	Density	Mean	Std	Mode
Autoregressive parameters					
$\rho_{R^*}$	Foreign interest rate	B	0.60	0.10	0.965
$\rho_{\pi^*}$	Foreign inflation	B	0.60	0.10	0.625
$\rho_{y^*}$	Foreign output	B	0.60	0.10	0.83
Standard deviations					
$\sigma_{R^*}$	Foreign interest rate	I	0.50	2.00	0.13
$\sigma_{\pi^*}$	Foreign inflation	I	0.50	2.00	0.25
$\sigma_{Y^*}$	Foreign output	I	0.50	2.00	0.80

of Suppliers of Business Financing to calibrate  $\varsigma_T$  and  $\varsigma_N$ . This survey suggests that, from 2008 to 2010, the average ratio of foreign debt to total debt is 25 per cent for the tradable sector and 17 per cent for the non-tradable sector. I calibrate  $\varsigma_T$  and  $\varsigma_N$  to match these ratios.

## 4.2 Data and priors

Bayesian techniques are used to estimate the model. Because the dynamics of the key variables for the rest of the world are exogenous to the Canadian economy, I assume that foreign output, inflation and the nominal interest rate all follow an AR(1) process and estimate the parameters governing these processes separately. Following the literature, I assume that foreign shocks' autoregressive coefficients follow a beta distribution with a mean of 0.6, and that the standard deviations of the shocks follow an inverse-gamma distribution with a mean of 0.5 per cent and a standard deviation of 2. I use U.S. quarterly real GDP per capita for the foreign output, the federal funds rate in quarterly terms for the foreign interest rate, and the quarter-to-quarter growth rate of the GDP deflator for foreign inflation. Foreign output is logged and HP-filtered, and both the foreign nominal interest rate and inflation are detrended by their means. Table 3 reports the priors and the modes for the posterior distribution.

Taking the estimated foreign shocks as given and using quarterly Canadian data from 1991Q1 to 2010Q4, I estimate the main model using nine series: output in the tradable sector, output in the non-tradable sector, consumption, investment, government spending, the nominal interest rate, inflation, the risk premium and the real exchange rate. Output is measured by real GDP. Consumption is measured by real expenditures of non-durable goods, semi-durable goods and services. Investment is measured by the sum of business gross fixed capital formation, investment in inventories and real expenditure of durable goods. Data on these real variables are expressed in per capita terms using the civilian population aged 15 and up. The nominal interest rate is measured by the overnight rate in quarterly terms. Inflation is the quarter-to-quarter growth rate of the core CPI. The risk premium

is measured by the difference between business prime lending rates and the nominal interest rate.<sup>10</sup> The real exchange rate is measured by multiplying the nominal US\$/Can\$ exchange rate by the ratio of U.S. to Canadian prices. The series of tradable output, non-tradable output, consumption, investment, government spending and real exchange rate are logged and detrended using the HP filter with smoothing parameter 1600. Next, the series of domestic nominal interest rate, inflation and risk premium are detrended by their means.

There are twelve behavioural parameters to estimate: the elasticity of the external risk premium for both tradable and non-tradable sectors  $\chi^T$  and  $\chi^N$ ; the investment adjustment cost parameter for both tradable and non-tradable sectors  $\xi^T$  and  $\xi^N$ ; the Calvo price parameters for all three tradable, non-tradable and imported-goods sectors  $\nu^T$ ,  $\nu^N$  and  $\nu^F$ ; the Calvo wage parameters for both tradable and non-tradable sectors  $\lambda^T$  and  $\lambda^N$ ; and the Taylor rule parameters  $\rho_\pi$ ,  $\rho_y$  and  $\rho_r$ . I also estimate the first-order autocorrelations of all the exogenous shocks and their respective standard deviations.

For most of the priors, I follow the literature. I use beta distributions for all parameters bounded in the [0,1] range. This applies to the shocks' autoregressive coefficient, whose mean I set at 0.6. The parameters of nominal rigidities for prices and wages are also assumed to follow a beta distribution with a mean of 0.75, which corresponds to changing prices and wages every 4 quarters on average. Gamma and inverse-gamma distributions are assumed for parameters that are supposed to be positive. The priors on the investment adjustment cost and risk-premium elasticity are in line with previous literature. For the standard deviation of the shocks, I assume an inverse-gamma distribution with a mean of 0.5 per cent and a standard deviation of 2. The prior assumptions on the monetary policy parameters allow for a range of interest rate inertia between 0 and 1, and a positive response to inflation. I use a normal distribution for the reaction to output in order to allow for a negative response. The priors are reported in Tables 4 and 5.<sup>11</sup>

## 5 Results

### 5.1 Estimates

Table 4 reports the mode, the mean and the 5th and 95th percentiles of the posterior distribution of the behavioural parameters. The estimates indicate significant heterogeneity across sectors. In this section, I focus on the modes of the estimated parameters that reflect the sectoral differences. The estimates of the risk-premium elasticity parameters,  $\chi^T$  and  $\chi^N$ , are quite different for the two sectors:  $\chi^T$  is estimated to be about 0.02, while  $\chi^N$  is estimated to be 0.2. In the non-tradable

<sup>10</sup>In the model, the risk premia that the tradable and non-tradable sectors face are different. Ideally, separate risk-premia series should be used in the estimation to identify the financial frictions in each sector. However, there are no such data available. In order to match the model to the data, I assume that the aggregate risk premium is a weighted average of the risk premium in each sector,  $rp_t = \frac{y_{T,t}}{y_{T,t}+y_{N,t}}rp_{T,t} + \frac{y_{N,t}}{y_{T,t}+y_{N,t}}rp_{N,t}$ .

<sup>11</sup>I use Dynare to estimate the model and the Metropolis-Hastings algorithm to perform simulations. The total number of draws is 20,000 and the first 20 per cent of the draws are neglected. A step size of 0.36 results in an acceptance rate of 0.23.

Table 4: Estimation Result: Behaviour Parameters

		Prior distribution	Posterior distribution			
			Mode	Mean	5 %	95 %
Risk-premium elas., T	$\chi^T$	gamma (0.05,0.02)	0.019	0.018	0.009	0.033
Risk-premium elas., N	$\chi^N$	gamma (0.05,0.02)	0.200	0.197	0.168	0.239
Calvo wage, T	$\lambda^T$	beta (0.75, 0.1)	0.567	0.619	0.429	0.758
Calvo wage, N	$\lambda^N$	beta (0.75, 0.1)	0.797	0.803	0.722	0.896
Calvo price, T	$\nu^T$	beta (0.75, 0.1)	0.734	0.710	0.583	0.818
Calvo price, N	$\nu^N$	beta (0.75, 0.1)	0.600	0.587	0.546	0.636
Calvo price, F	$\nu^F$	beta (0.75, 0.1)	0.959	0.959	0.945	0.975
Inv. adj. cost, T	$\xi^T$	norm (1, 0.5)	1.023	0.922	0.333	2.076
Inv. adj. cost, N	$\xi^N$	norm (1, 0.5)	0.093	0.194	0.042	0.204
Taylor rule inertia	$\rho_r$	beta (0.5, 0.25)	0.636	0.627	0.532	0.711
Taylor rule inflation	$\rho_\pi$	gamma(0.5, 0.5)	0.536	0.596	0.418	0.690
Taylor rule output gap	$\rho_y$	norm (0.125, 0.15)	0.002	0.004	-0.003	0.012

Table 5: Estimation Results: Shock Processes

		Prior distribution	Posterior distribution			
Autoregressive parameters			Mode	Mean	5%	95%
Technology, T	$\rho_{a,T}$	beta (0.6,0.2)	0.801	0.805	0.707	0.906
Technology, N	$\rho_{a,N}$	beta (0.6,0.2)	0.971	0.952	0.916	0.996
Preference	$\rho_e$	beta (0.6,0.2)	0.895	0.888	0.825	0.962
Investment, T	$\rho_{\tau,T}$	beta (0.6,0.2)	0.667	0.564	0.295	0.934
Investment, N	$\rho_{\tau,N}$	beta (0.6,0.2)	0.887	0.796	0.750	0.958
Government	$\rho_g$	beta (0.6,0.2)	0.779	0.761	0.667	0.874
Financial, T	$\rho_{\gamma,T}$	beta (0.6,0.2)	0.634	0.615	0.472	0.722
Financial, N	$\rho_{\gamma,N}$	beta (0.6,0.2)	0.879	0.868	0.790	0.940
Standard deviations						
Technology, T	$\sigma_{\epsilon^a,T}$	invga (0.005,2)	0.017	0.018	0.015	0.020
Technology, N	$\sigma_{\epsilon^a,N}$	invga (0.005,2)	0.004	0.004	0.004	0.005
Monetary	$\sigma_{\epsilon^m}$	invga (0.005,2)	0.003	0.004	0.003	0.004
Preference	$\sigma_{\epsilon^e}$	invga (0.005,2)	0.010	0.011	0.008	0.014
Investment, T	$\sigma_{\epsilon^{\tau,T}}$	invga (0.005,2)	0.002	0.004	0.001	0.005
Investment, N	$\sigma_{\epsilon^{\tau,N}}$	invga (0.005,2)	0.007	0.008	0.005	0.009
Government	$\sigma_{\epsilon^g}$	invga (0.005,2)	0.006	0.006	0.005	0.007
Financial, T	$\sigma_{\epsilon^{\gamma,T}}$	invga (0.005,2)	0.008	0.008	0.007	0.010
Financial, N	$\sigma_{\epsilon^{\gamma,N}}$	invga (0.005,2)	0.003	0.003	0.003	0.004
Log data density		2506.54				



sector, the external finance costs are 10 times as responsive to firms' balance-sheet positions as those in the tradable sector, suggesting that the degree of financial frictions is higher for the firms in the non-tradable sector.<sup>12</sup> The staggering wage contract parameters,  $\lambda^T$  and  $\lambda^N$ , are estimated at 0.57 and 0.80, respectively, suggesting that wages remain unchanged on average for about 2.3 quarters in the tradable sector and for about 5 quarters in the non-tradable sector. The sectoral differences are also captured by the sticky price parameters. For the tradable sector,  $\nu^T$  is estimated to be 0.73, implying that the expected price duration is 3.7 quarters. For the non-tradable sector,  $\nu^N$  is estimated at 0.60, suggesting that the expected price duration is 2.5 quarters. The estimates for the rest of the parameters are consistent with the existing studies in the literature.

Table 5 reports the mode, the mean and the 5th and 95th percentiles of the posterior distribution of the shock processes. The estimates also suggest significant heterogeneity across sectors. For example, the technology shock in the tradable sector is about 4 times as volatile as that in the non-tradable sector (0.017 versus 0.004). The financial wealth shocks are quite different for the two sectors as well: compared to the non-tradable sector, the shock in the tradable sector is less persistent (0.63 versus 0.88), but more volatile (0.008 versus 0.003). Among all the shocks, the technology shock in the non-tradable sector is the most persistent, with an autoregressive coefficient of 0.97, and the preference shock is the most volatile, with a coefficient of standard deviation of 0.01.

## 5.2 Fit of the model

I first examine how well the model economy is able to account for the overall volatility in the data. Table 6 reports the standard deviations (normalized relative to output) for the nine key variables. On the whole, the model appears to capture well the basic features of the data. The model comes quite close in terms of matching the volatility in aggregate investment  $i$ , consumption  $c$ , and risk premium  $rp$ . Additionally, it captures more than half of the relative volatility in the real exchange rate  $s$ , and inflation  $\pi$ . The model also captures about 40 per cent of the relative volatility in the nominal interest rate  $r$ . For the key labour market variables, the model is able to capture the fact that unemployment is much more volatile than output, although it slightly overestimates the relative volatilities for both employment and unemployment.

Next, I study whether the model economy is able to capture the sectoral differences in output, employment and wages observed in the data. For each variable, I normalize the standard deviations in the tradable sector to those in the non-tradable sector and report the relative volatilities in the data and model in Table 7. In the data, output in the tradable sector is about 4 times as volatile compared

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<sup>12</sup>In the literature,  $\chi$  is typically calibrated at 0.05, although some work based on estimation suggests that it can be larger. For example, De Graeve (2008) estimates  $\chi$  to be 0.10; Zhang (2011 a and b) suggests that  $\chi$  can be as large as 0.20. The result here —  $\chi^N$  is about 10 times larger than  $\chi^T$  — is quite different from the estimates in Dib, Mendicino and Zhang (2008), which also estimate these two elasticities using the Canadian data. In Dib, Mendicino and Zhang (2008),  $\chi^N = 0.028$  and  $\chi^T = 0.033$ . However, their results are based on an unrealistic assumption that the tradable sector can obtain outside financing only from the international financial market, and the non-tradable sector can obtain financing only from the domestic market. Nonetheless, more work is necessary to ensure the robust identification of these two parameters. I leave this to future research.

Table 6: Standard Deviations for the Key Macro Variables: Model vs. Data

	$y$	$c$	$i$	$r$	$\pi$	$rp$	$s$	$u$	$n$
Data	1	0.56	4.04	0.45	0.52	0.09	3.29	5.51	0.63
Model	1	0.42	5.56	0.18	0.28	0.09	2.75	7.76	0.81

to the non-tradable sector. The model is able to capture almost half of the volatility (1.80 vs. 3.95). The model also captures about 40 per cent of the relative volatility in employment, while for real wages it matches the data quite well – the estimated relative volatility is only slightly higher than that in the data (1.82 vs. 1.71).

Table 7: Relative Volatilities: Model vs. Data

	Output	Employment	Wages
Data	3.95	2.81	1.71
Model	1.80	1.07	1.82

### 5.3 Sources of labour market fluctuations

Table 8 reports the forecast error variance decomposition at the infinite horizon based on the mode of the model’s posterior distribution for the key macroeconomic variables, including unemployment, output for both the tradable and non-tradable sectors, employment for the tradable and non-tradable sectors, consumption, investment, the nominal interest rate, inflation and the real exchange rate. Fluctuations in unemployment are primarily driven by financial wealth shocks in the two sectors. Financial wealth shocks in the non-tradable sector explain most of the unemployment variations (27 per cent), while the financial wealth shock in the tradable sector accounts for about 10 per cent. Technology shocks in the two sectors come second, explaining about 22 per cent of the variations in unemployment. Overall, a sizable (60 per cent) fraction of the unemployment variations at the aggregate level can be accounted for by the shocks at the sectoral level, with 20 per cent from the tradable sector and 40 per cent from the non-tradable sector. The rest of the variations in unemployment are explained by foreign shocks (26 per cent), monetary policy shocks (9 per cent) and preference shocks (5 per cent).

Technology and financial wealth shocks turn out to be the most important shocks explaining output and employment fluctuations at the sectoral level. For the tradable sector, sector-specific technology shocks explain about 80 per cent of the variations in output and 35 per cent in employment. For the non-tradable sector, the financial wealth shock explains the majority of variations in output and employment.

Financial wealth shocks in the two sectors also contribute significantly to the other key macro variables. In the tradable sector, they account for about 22 per cent of the volatility in consumption,

46 per cent in the nominal interest rate and 36 per cent in inflation; financial wealth shocks in the non-tradable sector explain about 79 per cent of the variations in investment and 20 per cent in consumption. These shocks seem to “crowd out” the investment-specific shocks, which have limited importance for all the variables. For the rest of the shocks, technology shocks in the non-tradable sector contribute significantly to consumption (25 per cent), monetary policy shocks contribute significantly to inflation (37 per cent) and foreign interest rate shocks are important for explaining the variations in the exchange rate (74 per cent).

Given the focus in this paper on unemployment fluctuations, I also report the forecast error variance decomposition at selected finite horizons (Table 9). It is interesting to note that, within a year, the financial wealth shock in the tradable sector is the main driving force of the unemployment dynamics, explaining more than half of the variations.

## 5.4 Model dynamics

Given the importance of the sector-specific financial wealth and technology shocks for explaining the movement in aggregate unemployment, in this section I use these two shocks in the tradable sector to illustrate how the key variables of the model economy respond to them.

Figure 3 illustrates the responses of the key variables in the model to a one-standard-deviation increase in financial wealth in the tradable sector. The solid line in each panel illustrates the response of the respective variable in the sector where the shocks occur (tradable sector). The dotted line reports the responses of the same variables in the other (non-tradable) sector. The last two panels report the responses of aggregate employment and unemployment.

A positive financial wealth shock increases entrepreneurs’ net worth in the tradable sector. On the one hand, in the tradable sector the rise in net worth causes the external finance premium to decline, leading entrepreneurs to increase their demand for capital. However, this rise in demand is not accompanied by a rise in demand for labour (albeit labour demand rises slightly in the initial periods). Instead, entrepreneurs in the tradable sector substitute labour services with capital, and the demand for labour services declines. The hiring rate in the tradable sector declines after the initial rise, causing the decline in employment. On the other hand, the hiring rate and employment both rise in the non-tradable sector, because the rise in capital demand from the tradable sector pushes up the capital prices, forcing entrepreneurs in the non-tradable sector to face a higher leverage. The risk premium rises significantly in the non-tradable sector given that the value of elasticity of external finance is very high  $\chi^N = 0.2$ . The demand for capital decreases: entrepreneurs in the non-tradable sector substitute capital with labour, and thus the demand for labour rises in this sector. The rise in the hiring rate in the non-tradable sector not only absorbs its unemployed workers, but also attracts unemployed workers from the tradable sector. Overall, both unemployment rates in the tradable and non-tradable sectors decrease, leading to a decline in the aggregate unemployment rate.

Figure 4 shows the response of the model to a positive technology shock in the tradable sector. Initially, output in the tradable sector rises after the shock. In contrast to the case with the financial

Table 8: Variance Decomposition of the Key Variables - Long Run

	Sector-specific						Aggregate			Foreign		
	Tech. (T)	Tech. (N)	Fin. (T)	Fin. (N)	Invest (T)	Invest (N)	Money	Govern.	Prefer.	Int. rate	Inflation	Output
U rate	2.62	19.35	9.23	26.93	0.02	3.18	8.86	0.17	4.42	25.09	0.03	0.09
Output, T	79.99	0.35	4.02	1.99	0.02	0.77	6.41	0.14	2.61	3.5	0.05	0.14
Output, N	4.8	27.35	7.57	40.45	0.01	0.65	4.29	0.09	2.12	12.59	0.02	0.07
Empl., T	35.27	3.37	17.86	8.15	0.03	1.83	10.97	0.3	5	16.04	0.27	0.9
Empl., N	8.61	16.45	11.85	25.54	0.02	3.08	8.26	0.15	3.25	22.62	0.03	0.15
Cons.	1.54	25.28	22.33	20.23	0.01	1.07	2.7	0.76	20.7	5.32	0.01	0.05
Inv.	5.2	2.32	3.87	79.23	0.02	3.51	3.16	0.11	1.02	1.39	0.02	0.14
Int. rate	4.29	13.63	46.04	9.18	0.02	4.52	2.45	0.97	14.58	4.16	0.02	0.14
Inf.	2.05	4.49	35.95	10.03	0.01	2.18	37.02	0.52	6.77	0.91	0.01	0.06
Exch. rate	0.86	1.36	4.95	5.49	0	0.4	7.16	0.25	5.54	73.65	0.32	0.02

Table 9: Variance Decomposition for the Unemployment Rate – Short Run

	Horizon		
	On impact	1 year	4 year
Technology, T	3.1	8.1	6.1
Technology, N	0.4	3.9	21.0
Financial, T	52.8	56.8	12.8
Financial, N	23.0	6.8	12.0
Investment, T	0.1	0.0	0.0
Investment, N	17.3	0.7	6.9
Government	0.0	0.1	0.0
Monetary	0.9	13.8	15.0
Preference	0.0	7.6	2.4
Foreign	2.5	2.2	23.7

wealth shock, the rise in output from a technology shock demands more labour services instead of capital. Thus, the hiring rate rises and the tradable sector employs more workers. However, the rise in labour demand drives up wages, leading to a decline in labour demand in the non-tradable sector. More unemployed workers in the non-tradable sector move to the tradable sector to look for jobs. Overall, the movement in the aggregate unemployment rate is mitigated by the fact that the rise in unemployment in the non-tradable sector partly cancels out the decline in the tradable sector.

## 5.5 Importance of market segmentation

In this section I restrict certain features to be the same across the two sectors and re-estimate the model. I then compare the log-likelihoods of these estimations with the baseline model to examine how crucial it is to allow the two sectors to be different. In the first exercise, I restrict the elasticity of external finance so that it is the same across the two sectors by assuming  $\chi^N = \chi^T = 0.02$ . In the second exercise, I restrict the Calvo wage rigidity parameter in the non-tradable sector so that it has the same value as that in the tradable sector,  $\lambda^N = \lambda^T = 0.57$ .

Table 10 provides the estimates of the modes of the parameters and the log data densities. In both cases, the restrictions lead to a deterioration of the marginal likelihood. When the nominal wage contract lengths in the two sectors are restricted to be the same,  $\lambda^N = \lambda^T = 0.57$ , the log data density falls by 65 points. When the two sectors are restricted to have the same degree of financial market frictions,  $\chi^N = \chi^T = 0.02$ , the likelihood falls even more, by 90 points.

## 5.6 Subsample estimates

The estimation results can be sensitive to the sample period. In particular, the important role of the financial wealth shocks might be directly linked to the inclusion of the recent financial crisis data in the estimation. In this section I re-estimate the model using a pre-financial crisis subsample, 1991Q1

Table 10: Testing the Importance of Market Segmentation

		Baseline	$\chi^N = \chi^T = 0.02$	$\lambda^N = \lambda^T = 0.57$
Log data density		2506.5	2416.1	2441.2
Mode of the structural parameters				
Taylor rule inertia	$\rho_r$	0.6359	0.3147	0.6100
Taylor rule inflation	$\rho_\pi$	0.5359	2.6078	0.4940
Taylor rule output gap	$\rho_y$	0.0014	0.0205	-0.0039
Calvo price, T	$\nu^T$	0.734	0.7844	0.6271
Calvo price, N	$\nu^N$	0.6004	0.8018	0.6067
Calvo price, F	$\nu^F$	0.9596	0.9621	0.9585
Calvo wage, T	$\lambda^T$	0.5672	0.4841	0.5700
Calvo wage, N	$\lambda^N$	0.7973	0.9154	0.5700
Inv. adj. cost, T	$\xi^T$	1.0226	0.013	0.7858
Inv. adj. cost, N	$\xi^N$	0.0939	2.3045	0.0691
Risk-premium elas., T	$\chi^T$	0.0193	0.020	0.0209
Risk-premium elas., N	$\chi^N$	0.2003	0.020	0.2051
Mode of the autoregressive parameters of the exogenous shock processes				
Technology, T	$\rho_{z,T}$	0.8013	0.9163	0.8106
Technology, N	$\rho_{z,N}$	0.9705	0.8088	0.9517
Government	$\rho_g$	0.7785	0.792	0.7790
Preference	$\rho_e$	0.8949	0.9157	0.8636
Financial, T	$\rho_{\gamma,T}$	0.6337	0.968	0.5631
Financial, N	$\rho_{\gamma,N}$	0.8785	0.751	0.8407
Investment, T	$\rho_{\tau,T}$	0.6679	0.8099	0.5718
Investment, N	$\rho_{\tau,N}$	0.887	0.9981	0.8932

Table 11: Comparison of the Subsample and Baseline Sample – Parameter Estimates

	Structural parameters			Shock process	
	Subsample	Baseline		Subsample	Baseline
	1991-2007	1991-2010		1991-2007	1991-2010
$\chi^T$	0.009	0.019	$\rho_{a,T}$	0.867	0.801
$\chi^N$	0.154	0.200	$\rho_{a,N}$	0.968	0.971
$\lambda^T$	0.760	0.567	$\rho_e$	0.871	0.895
$\lambda^N$	0.729	0.797	$\rho_{\tau,T}$	0.417	0.667
$\nu^T$	0.596	0.734	$\rho_{\tau,N}$	0.580	0.887
$\nu^N$	0.621	0.600	$\rho_g$	0.794	0.779
$\nu^F$	0.964	0.959	$\rho_{\gamma,T}$	0.842	0.634
$\xi^T$	0.047	1.023	$\rho_{\gamma,N}$	0.896	0.879
$\xi^N$	1.972	0.093	$\sigma_{\epsilon^a,T}$	0.014	0.017
$\rho_r$	0.607	0.636	$\sigma_{\epsilon^a,N}$	0.004	0.004
$\rho_\pi$	0.613	0.536	$\sigma_{\epsilon^m}$	0.003	0.003
$\rho_y$	-0.005	0.002	$\sigma_{\epsilon^e}$	0.009	0.010
			$\sigma_{\epsilon^{\tau,T}}$	0.006	- 0.002
			$\sigma_{\epsilon^{\tau,N}}$	0.009	0.007
			$\sigma_{\epsilon^g}$	0.042	0.006
			$\sigma_{\epsilon^{\gamma,T}}$	0.005	0.008
			$\sigma_{\epsilon^{\gamma,N}}$	0.001	0.003
Log data density				2087.59	2506.54

to 2007Q4.<sup>13</sup> Table 11 compares the modes of the posterior distribution of the model parameters over the subsample and our baseline sample. The noticeable changes regarding the financial frictions and shocks are as follows: the estimates of the elasticity of the external risk premium in the two sectors  $\chi^T$  and  $\chi^N$  are reduced from 0.019 to 0.009 and from 0.200 to 0.154, respectively, suggesting that, before the financial crisis, firms' borrowing costs are less sensitive to their balance-sheet conditions. Moreover, financial wealth shocks are less volatile in the subsample compared to the baseline sample (0.005 versus 0.008 for the tradable sector, and 0.001 versus 0.003 for the non-tradable sector). Due to these differences, a smaller fraction of the labour market fluctuations are explained by the financial shocks for the pre-crisis subsample (Table 12). Overall, about 13 per cent of the unemployment fluctuations are explained by the financial wealth shocks, with each sector contributing about 6 per cent. These results suggest that the data from the recent financial crisis period play an essential role in determining the importance of the financial wealth shocks.

<sup>13</sup> According to the C. D. Howe Institute's Business Cycle Council, Canada's last recession began in November 2008 and ended in May 2009. The U.S. recession lasted 18 months, beginning in December 2007 and ending a month after the Canadian downturn in June 2009. I choose 2007Q4 to be the last data point in the sample in order to exclude the periods during which the recent financial crisis would have the most impact on the Canadian economy.

Table 12: Comparison of the Subsample and Baseline Sample – Variance Decomposition

	Sector-specific						Aggregate		Foreign	
	Tech. T	Tech. N	Fin. T	Fin. N	Invest T	Invest N	Money	Govern.	Prefer.	
1991-2007	1.37	27.35	6.46	6.74	4.64	16.62	18.68	0.52	7.92	9.80
1991-2010	2.62	19.35	9.23	26.93	0.02	3.18	8.86	0.17	4.42	25.21

## 6 Extensions

### 6.1 Introducing a commodity sector

Given that Canada is a net exporter of commodities, commodity price fluctuations in the world market can lead to fluctuations in the terms of trade, and this can have a notable impact on the tradable and non-tradable sectors. As a result, commodity price shocks can potentially be an important source of unemployment fluctuations. In order to assess whether the financial wealth and technology shocks will remain the main driving forces for unemployment fluctuations in the presence of commodity price shocks, in this section I introduce a commodity sector to the model and describe the modifications.

#### 6.1.1 Commodity sector

In order to capture the importance of natural resources for production in the commodity sector, the production inputs include capital, labour and land. The commodity output is divided between domestic uses (as direct inputs in the tradable and non-tradable sectors) and exports.

The commodity sector is indexed by  $X$ . At each period  $t$ , entrepreneur  $j$  in the commodity sector uses capital,  $k_{X,t}$ , labour services,  $l_{X,t}$ , and land,  $L_{X,t}$ , to produce commodity output  $y_{X,t}$  using a Cobb-Douglas technology,

$$y_{X,t}(j) = a_{X,t}(k_{X,t}(j))^{\alpha_x}(l_{X,t}(j))^{\beta_x}(L_{X,t}(j))^{1-\alpha_x-\beta_x},$$

where  $\alpha_x, \beta_x$  are shares of capital, and labour services in the production of commodities. The supply of land  $L_{X,t}$  is assumed to evolve exogenously according to the following AR(1) process:

$$\log L_{X,t} = (1 - \rho_L) \log L_{X,ss} + \rho_L \log L_{X,t-1} + \epsilon_t^L, \epsilon_t^L \sim i.i.d.N(0, \sigma_{\epsilon^L}^2).$$

It is assumed that the nominal commodity price  $P_{X,t}^*$  is determined exogenously in world markets



and denominated in the foreign currency.<sup>14</sup> It follows an AR(1) process:

$$\log P_{X,t}^* = (1 - \rho_{p_{X,t}}) \log P_{X,ss}^* + \rho_{p_{X,t}} \log P_{X,t-1}^* + \epsilon_t^L, \epsilon_t^{P_X^*} \sim i.i.d.N(0, \sigma_{\epsilon_{P_X^*}}^2).$$

The one-period profit function for entrepreneur  $j$  is

$$\begin{aligned} \pi_{X,t}(j) = & s_t p_{X,t}^* y_{i,t}^j + q_{X,t}(1 - \delta)k_{X,t}(j) \\ & - p_{X,t}^l l_{X,t}(j) - q_{X,t}k_{X,t+1}(j) - L_{X,t}(j)p_{X,t}^L, \end{aligned}$$

where  $s_t p_{X,t}^*$  is the relative price for the commodity output that the entrepreneur produces and  $p_{X,t}^l$  is the labour service price. The entrepreneur purchases capital at price  $q_{X,t}$  from capital producers. For simplicity, I assume that the entrepreneur in the commodity sector does not face financial frictions. The entrepreneur in the commodity sector chooses  $l_{X,t}(j)$ ,  $k_{X,t+1}(j)$  and  $L_{X,t}$  to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \pi_{X,t}(j).$$

The first-order condition yields:

$$l_{X,t}(j) : s_t p_{X,t}^* \frac{\partial y_{X,t}(j)}{\partial l_{X,t}(j)} = p_{X,t}^l,$$

$$k_{X,t+1}(j) : -q_{X,t} + E_t \beta [p_{X,t+1}^* \frac{\partial y_{X,t+1}(j)}{\partial k_{X,t+1}(j)} + q_{X,t+1}(1 - \delta)] = 0,$$

and

$$L_{X,t}(j) : s_t p_{X,t}^* \frac{\partial y_{X,t}(j)}{\partial L_{X,t}(j)} = p_{X,t}^L.$$

### 6.1.2 Modifications in the labour market

As in the benchmark model, the labour market is segmented. Employment agencies in the commodity sector post vacancies for entrepreneurs, and unemployed workers need to decide which sector to search. Those seeking jobs in the commodity sector are denoted as  $u_{X,t}$ . In equilibrium, searching for jobs in any of the sectors gives the same expected payoff for them. The number of unemployed workers at the beginning of period  $t$  is

$$u_t = 1 - n_t = 1 - n_{T,t} - n_{N,t} - n_{X,t},$$

where  $n_{X,t}$  is the number of employed workers in the commodity sector. For an employment agency in the commodity sector, the value of adding another worker at time  $t$  is the price of selling one unit

<sup>14</sup>The sources of the commodity price increase can be different. For example, a rise in commodity prices can be due to either a positive external demand shock or a negative supply shock. Dorich et al. (2013) show that the overall impact of commodity prices on the Canadian economy depends on the underlying sources.

of labour service  $p_{X,t}^l$ , minus the wage cost  $\frac{W_{X,t}(j)}{p_{X,t}}$  and hiring costs  $\frac{\kappa_X}{2}x_{X,t}(j)^2$ , plus the continuation value of the filled vacancy:

$$J_{X,t}(j) = p_{X,t}^l - \frac{W_{X,t}(j)}{p_{X,t}} - \frac{\kappa_X}{2}x_{X,t}(j)^2 + (\rho + x_{X,t}(j))\beta E_t \Lambda_{t,t+1} J_{X,t+1}(j).$$

The first-order condition for vacancy posting equates the marginal cost of adding a worker with the discounted marginal benefit:

$$\kappa_X x_{X,t}(j) = \beta E_t \Lambda_{t,t+1} J_{X,t+1}(j).$$

The workers' surplus for having a job at employment agency  $j$ ,  $H_{i,t}(j)$ , in the commodity sector is

$$H_{X,t}(j) = V_{X,t}(j) - U_{X,t},$$

where  $V_{X,t}(j)$  is the value of employment for a new worker at employment agency  $j$  at time  $t$ , and  $U_{X,t}$  is the value of unemployment. Since

$$U_{T,t} = U_{N,t} = U_{X,t},$$

it follows that

$$\beta E_t \Lambda_{t,t+1} s_{T,t+1}^l H_{T,t}(j) = \beta E_t \Lambda_{t,t+1} s_{N,t+1}^l H_{N,t}(j) = \beta E_t \Lambda_{t,t+1} s_{X,t+1}^l H_{X,t}(j).$$

That is, for an unemployed worker, the expected payoff of searching for jobs in the three sectors must be equal.

As in the tradable and non-tradable sectors, employment agencies and workers in the commodity sector negotiate a nominal wage  $W_{X,t}(j)$  to maximize the joint product of the workers' surplus  $H_{X,t}(j)$  and the employment agencies' surplus  $J_{X,t}(j)$ . However, every period, each employment agency has only a fixed probability  $1 - \lambda_X$  to negotiate with workers. Thus, the Nash bargaining problem between employment agencies and workers is

$$\max H_{X,t}(j)^\eta J_{X,t}(j)^{1-\eta},$$

s.t.

$$\begin{aligned} W_{X,t}(j) &= W_{X,t}^* \text{ with probability } 1 - \lambda_X \\ &= W_{X,t-1}\pi \text{ with probability } \lambda_X, \end{aligned}$$

where  $\pi$  is the steady-state inflation rate.

### 6.1.3 Modifications in the tradable and non-tradable sectors

It is assumed that a commodity is used as an input in the production of tradable and non-tradable goods, and the production functions for tradable and non-tradable sectors are modified as follows:

$$y_{i,t}(j) = a_{i,t}(k_{i,t}(j))^{\alpha_i}(l_{i,t}(j))^{\beta_i}(X_{i,t}(j))^{1-\alpha_i-\beta_i},$$

where  $i = T, N$ . The one-period profit function for entrepreneur  $j$  in the tradable and non-tradable sectors is modified as follows:

$$\begin{aligned}\pi_{i,t}(j) = & \frac{B_{i,t}(j)}{P_t} + \frac{e_t B_{i,t}^*(j)}{P_t} + p_{i,t}^w y_{i,t}^j + q_{i,t}(1 - \delta)k_{i,t}(j) \\ & - p_{i,t}^l l_{i,t}(j) - R_{t-1} r p_{i,t-1} \frac{B_{i,t-1}(j)}{P_t} - R_{t-1}^* \kappa_{i,t-1}^{rp} \frac{e_t B_{i,t-1}^*(j)}{P_t} \\ & - q_{i,t} k_{i,t+1}(j) - s_t p_{X,t}^* X_{i,t},\end{aligned}$$

and the demand for a commodity is given by

$$X_{i,t}(j) : p_{i,t}^w \frac{\partial y_{i,t}(j)}{\partial X_{i,t}(j)} = s_t p_{X,t}^*.$$

### 6.1.4 Variance decomposition with the presence of a commodity price shock

In this section, I simulate the modified model and compute the variance decomposition for unemployment, based on that model. For most of the parameter values and shock processes, I use the estimated modes from the baseline model. For the parameter values related to the commodity sector, however, I rely on the calibrated values in Dib (2008). Table 13 reports the calibrated values.

I introduce three new shocks in the modified model: a commodity price shock, a natural resource shock and an investment-specific shock in the commodity sector. For the first two shocks, I use the estimates in Dib (2008). I set the persistence and standard deviation of the commodity price shock at 0.86 and 0.04, respectively, and the persistence and standard deviation of the natural resource shock at 0.64 and 0.06. For the investment-specific shock in the commodity sector, I set its autoregressive coefficient and standard deviation at 0.66 and 0.0023, the same values for the tradable sector in the baseline model.

Table 14 reports the unconditional forecast error variance decomposition of the key macro variables for the modified model. It is interesting to note that the commodity price and foreign interest rate shocks seem to replace the role of technology shocks – the contribution of the technology shocks to the key variables is very limited. The role of the financial wealth shocks in the tradable sector is also significantly reduced. For unemployment fluctuations, in the modified model, foreign interest rate and commodity price shocks explain almost 60 per cent of the variations. Financial shocks in the non-tradable sector still remain relatively important, explaining about 16 per cent of the variations in unemployment.

Table 13: Calibration of the Parameters

Param.	Definition	Values
$\beta$	Discount factor	0.99
$\gamma$	Inverse of intertemporal substitution of consumption	2
$\nu^C$	Elasticity of substitution between sectors, consumption	0.8
$\nu^I$	Elasticity of substitution between sectors, investment	0.6
$\theta$	Intermediate-good elasticity of substitution	6
$\delta$	Capital depreciation rate	0.25
$\alpha_T$	Capital share, tradable	0.26
$\alpha_N$	Capital share, non-tradable	0.28
$\alpha_X$	Capital share, commodity	0.41
$\beta_T$	Labour share, tradable	0.63
$\beta_N$	Labour share, non-tradable	0.66
$\beta_X$	Labour share, commodity	0.39
$\omega_T^C$	Share of tradable good, consumption	0.10
$\omega_N^C$	Share of non-tradable good, consumption	0.57
$\omega_T^I$	Share of tradable good, investment	0.20
$\omega_N^I$	Share of non-tradable good, investment	0.40
$k_T$	Steady-state net worth to capital ratio, tradable	0.7
$k_N$	Steady-state net worth to capital ratio, non-tradable	0.6
$\pi$	Steady-state domestic inflation rate	1.0048
$\pi^*$	Steady-state foreign inflation rate	1.0052
$\rho$	Aggregate survival rate of jobs	0.91
$\rho_T$	Survival rate of jobs in tradable sector	0.94
$s^I$	Aggregate job-finding rate	0.927
$\eta$	Bargaining power of workers	0.5
$\tilde{b}$	Parameter for unemployment flow value	0.4
$\sigma_m$	Elasticity in matches to unemployment	0.5

Table 14: Variance Decomposition of the Key Variables: With Commodity Sector

	Financial shock		Financial shock		Invest shock		Oil price shock		Tech. shock		Tech. shock		Land shock		Invest shock		Invest shock		Money shock		Govern shock		Prefer. shock		Int.rate shock		Output shock		Inflation shock	
	T	N	X	X	T	N	X	X	T	N	X	X	T	N	X	N	N	X	X	X	X	foreign	shock	foreign	shock	foreign	shock	foreign	shock	
U rate	0.65	15.84	0.22	18.38	0.2	1.94	0.02	0	2.22	2.65	0.08	19.75	38.04	0	2.22	2.65	0.08	19.75	38.04	0	0.01	0	0.01	0	0.01	0	0.01	0	0.01	
Empl, T	0.66	7.85	0.32	32.02	1.8	0.64	0.02	0	5.08	0.82	0.01	12.72	38.04	0	5.08	0.82	0.01	12.72	38.04	0.01	0.01	0	0.01	0	0.01	0	0.01	0	0.01	
Empl, N	1.33	18.94	0.24	14.62	0.7	2.24	0.02	0	2.29	3.12	0.11	28.58	27.81	0	2.29	3.12	0.11	28.58	27.81	0	0	0	0	0	0	0	0	0	0	
Empl, X	0.15	1.37	0.05	7.82	0.04	0.08	0.04	0	0.73	0.14	0.01	1.63	87.78	0	0.73	0.14	0.01	1.63	87.78	0	0.18	0	0.18	0	0.18	0	0.18	0	0.18	
Output, T	3.3	6.96	0.2	25.13	21.7	0.42	0.02	0.01	2.96	1.5	0.02	14.38	23.35	0.04	2.96	1.5	0.02	14.38	23.35	0.04	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	
Output, N	1.19	21.39	0.15	11.91	0.55	6.57	0.01	0	1.46	1.74	0.05	37.72	17.25	0	1.46	1.74	0.05	37.72	17.25	0	0.01	0	0.01	0	0.01	0	0.01	0	0.01	
Output, X	0.07	12.68	0.09	1.5	0.02	0.01	3.8	0	1.19	0.08	0.01	6.62	73.88	0	1.19	0.08	0.01	6.62	73.88	0	0.05	0	0.05	0	0.05	0	0.05	0	0.05	
Cons.	0.07	0.13	0.02	1.6	0.01	0.27	0	0	0.16	0.03	0.02	92.43	5.27	0	0.16	0.03	0.02	92.43	5.27	0	0	0	0	0	0	0	0	0	0	
Inv.	0.6	34.87	0.17	9.39	0.66	0.63	0.01	0	2.09	1.07	0.09	12.8	37.61	0	2.09	1.07	0.09	12.8	37.61	0	0	0	0	0	0	0	0	0	0	
Int. rate	1.9	2.06	0.42	33.06	0.18	1.21	0.04	0	2.77	0.62	0.26	24.16	33.3	0	2.77	0.62	0.26	24.16	33.3	0	0.01	0	0.01	0	0.01	0	0.01	0	0.01	
Inf.	3.87	1.93	0.32	20.29	0.19	1.01	0.02	0	1.41	9.83	0.36	45.07	15.66	0	1.41	9.83	0.36	45.07	15.66	0	0.02	0	0.02	0	0.02	0	0.02	0	0.02	
Exch. rate	0.52	9.31	0.16	25.7	0.07	1.19	0.04	0	1.6	0.41	0.1	21.74	38.56	0	1.6	0.41	0.1	21.74	38.56	0	0.6	0	0.6	0	0.6	0	0.6	0	0.6	

## 7 Conclusions

Are sector-specific shocks important in shaping unemployment fluctuations in the Canadian labour market? The analysis in this paper suggests that they are. In the paper I introduce a segmented labour market structure into a small open-economy model with financial and labour market frictions. Estimation results suggest that there is a lot of heterogeneity across sectors: compared to the non-tradable sector, prices are more rigid but wages are more flexible in the tradable sector. Moreover, the tradable sector observes external financial costs as being less responsive to firms' balance-sheet position. Shocks are different at the sectoral level. Compared to the non-tradable sector, both the technology and financial wealth shocks in the tradable sector are more volatile but less persistent. I find that the financial wealth shocks in the two sectors are the most important shocks for explaining the unemployment fluctuations in the Canadian labour market. In the short run, the financial wealth shock in the tradable sector plays a greater role – explaining about half of the unemployment variations. In the long run, the fluctuations in unemployment are mainly driven by the financial wealth shock in the non-tradable sector.

With the structural estimation, however, the effects of a particular shock depend on the data used to identify the shock, and the other shocks included in the model. It is interesting to note that, if the benchmark model is estimated using the pre-financial crisis subsample, the resulting financial wealth shocks in both sectors are much less volatile and account for much less fluctuation in aggregate unemployment. It is also interesting to note that once the model is extended and a commodity price shock is included, the contributions of the financial wealth shocks are reduced, although they still account for almost 20 per cent of the unemployment volatility. The results of the extended model, however, are based on calibration. A variance decomposition based on estimation would provide a more rigorous analysis. I leave this to future research.

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Figure 1: Output and Employment: Tradable vs. Non-tradable

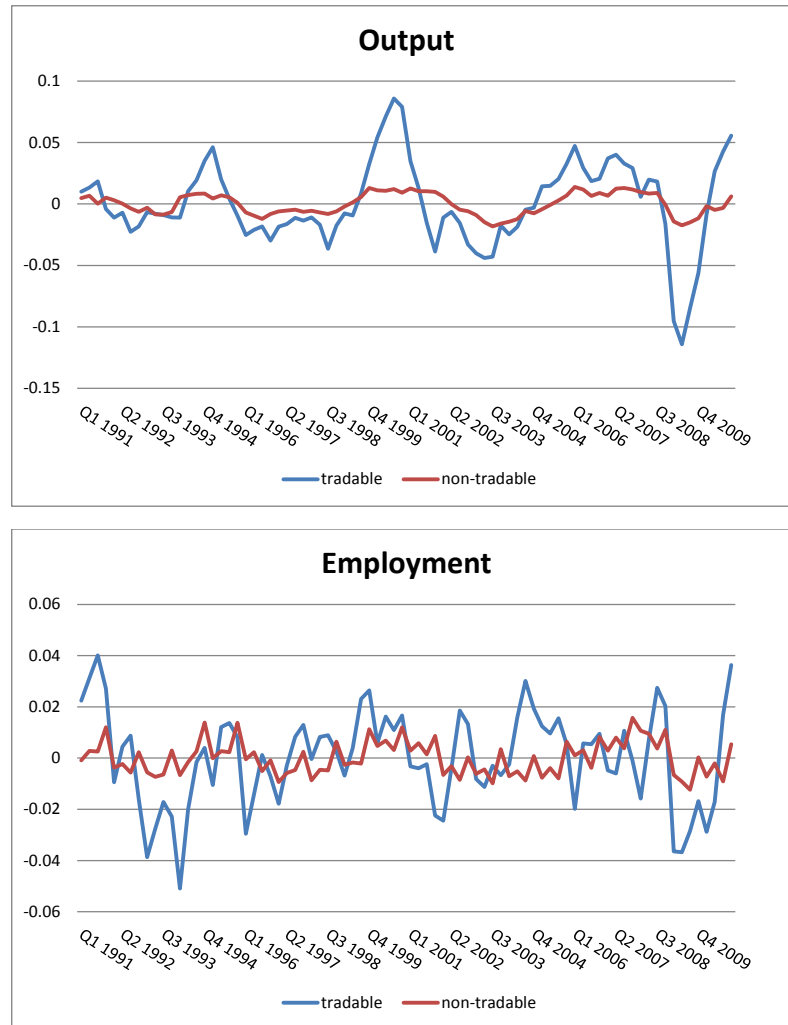


Figure 2: Wages: Tradable vs. Non-tradable

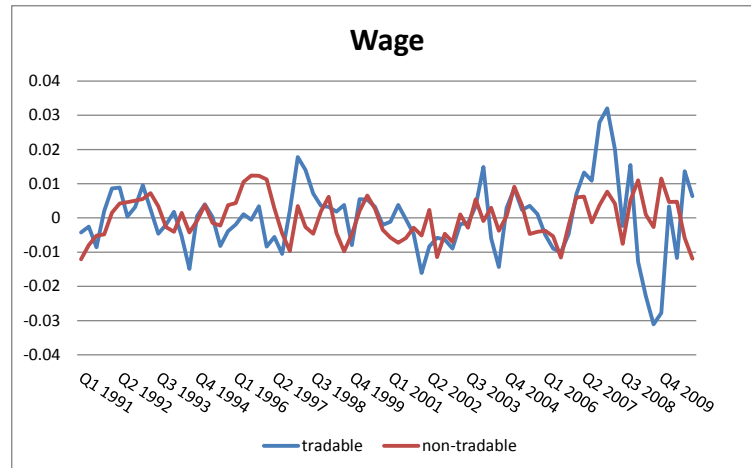


Figure 3: Model Dynamics after a Financial Wealth Shock in the Tradable Sector

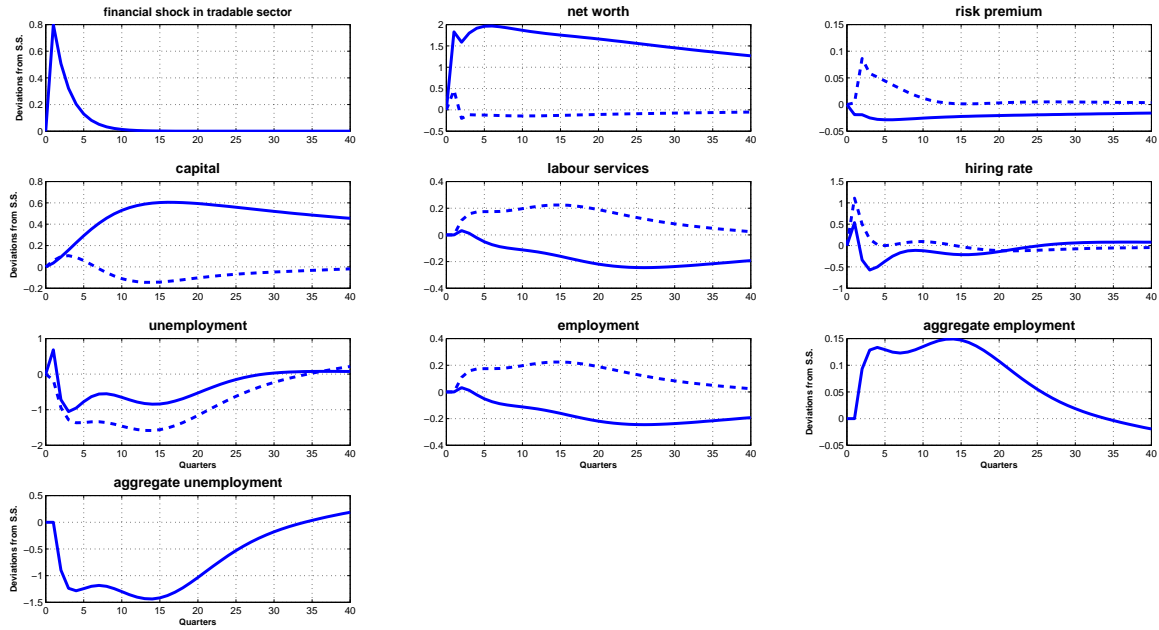


Figure 4: Model Dynamics after a Technology Shock in the Tradable Sector

