

Chauvel, Louis

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## Intensity and shape of inequalities: The ABG method for the analysis of distributions

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# LIS

## Working Paper Series

No. 609

### **Intensity and shape of inequalities: the ABG method for the analysis of distributions**

Louis Chauvel

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# Intensity and shape of inequalities: the ABG method for the analysis of distributions<sup>1</sup>

Louis Chauvel louis.chauvel@uni.lu

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## Abstract

Inequality is anisotropic: its intensity is variable along the income scale. Therefore, to focus on local inequalities, a new representation, the isograph, is developed to figure their variations. This leads to the expression of three coefficients able to summarize the shape of inequalities: a main coefficient, alpha, measures inequality at the median, and two correction coefficients, beta and gamma, take into account the curvatures at the top and at the bottom of the distribution. A set of 212 samples of microdata surveys from 40 different countries of the LIS datacenter archive provides a systematic view on the properties of these ABG (alpha beta gamma) coefficients that are compared to a set of standard indices (Atkinson indexes, generalized entropy, Wolfson polarization, etc.). This method also provides a smoothing tool that is able to show the differences in shape of distributions (stobiloid) and their change over time.

Income distribution analysis is central in the understanding of inequality structures and social transformations. In his seminal works on distributions, Pareto (1896:99, 1897: v2.305-24) proposed a leptokurtic distribution which approximates the top income hierarchy, and representations of the general architecture of societies based on incomes (Pareto, 1909:380-8). Improvements are to be noticed since the Gini index (Gini, 1914) but the still in use old tools have promoted a general conception where inequality is singular even though the results of these tools can be diverse.<sup>2</sup> On the contrary, this contribution intends to show how local

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<sup>1</sup> This paper received the financial support of the Luxembourg FNR Fonds National pour la Recherche project PEARL-IRSEI.

<sup>2</sup> Improvements are obvious in the understanding of the socioeconomic processes able to generate these Pareto distributions (Gabaix, 2009), and even of double Pareto (Reed, 2001) since the lower tail has this specific shape as well. In this field, general overviews (Kleiber and Kotz, 2003) show the diversity of approaches. Over time, more appropriate and more general statistical distributions have been developed, from the Champnowne-I (1937) and Fisk (1961) distributions to the Generalized Beta of the second kind (GB2) that are becoming standard tools (Jenkins, 2009; Graf and Nedyalkova, 2013). In parallel, many indexes of inequality have been developed (Champnowne and Cowell, 1998:151-3) and a mass of harmonized data has been accumulated (Brandolini and Atkinson, 2001; Cowell 2000, 2003). On top of that, graphical innovation used to represent

inequality can be diverse along the income scale and propose a distinction between inequality at the middle, the top and the bottom. A first empirical example shows why this question is meaningful for incomes.

In this paper, income is defined as the disposable (after tax and transfers) income per consumption unit (square root of the household population), divided by the median income of the population.<sup>3</sup> Therefore we speak of “medianized equivalized disposable income” (medi). On table 1 are figured quantiles of the income distribution in Israel-2007 (il07) and the U.S.-2010 (us10), two countries with close level of general inequality with Gini indexes of 0.365 and 0.371, respectively.<sup>4</sup> The comparison (table 1) shows that the richer quartile p75 is lower in the U.S. than in Israel and the American poorer quartile p25 is better off than its Israeli equivalent. Near the median, there is less inequality in the U.S. than in Israel. Anyway, on the lower quantiles, the poorer Israeli residents are better off than in the U.S., and the richest centile p99 is closer to the median than the American one. Therefore, Israel has more inequality at the middle and less at the extremities of the distribution. In terms of “general inequality” as it is conventionally measured by Gini index for instance, Israel 2007 is slightly less unequal than U.S. 2010. In terms of “local” inequality, a notion that we can intuitively define as local stretching away from the median, the Israel/U.S. comparison is obviously more complicated, with stronger and weaker inequality on various segments of the income scale. This kind of ambiguous situation relates to well-known stochastic dominance problems in the analysis of Gini coefficients, characterized by intercrossing Lorenz curves, impairing the comparison of Gini indices. But this diagnosis is not a solution to the problem of comparison.

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distributions have been reviewed by Dombos (1982) who listed dozens of models of graphs in addition to the still in use log-log Pareto diagram (Nirei and Souma, 2007:444), the Lorenz curve, the Pen’s Parade (1971), as well as the standard density, cumulative distribution function or quantile function graphs. Therefore, the field of inequality analysis could seem like a mature technology.

<sup>3</sup> Zero or negative medi are excluded from the analyze. This method can be adapted to wealth inequality analysis with available datasets of the LIS (Jäntti et al., 2013).

<sup>4</sup> The code for countries is based on the International Organization for Standardization two-characters code [http://en.wikipedia.org/wiki/ISO\\_3166-1](http://en.wikipedia.org/wiki/ISO_3166-1), followed by the two digits of the year of the survey.

## **PLACE TABLE 1 HERE**

To break this deadlock, this paper generalizes the idea of diversity in “local inequality”, along the income scale.<sup>5</sup> It proposes an analysis in terms of the shape of inequalities that has been generally neglected.<sup>6</sup> Champernowne I – Fisk (CF) distribution, which can be related to a relation of power theory of incomes, is a correct first order approximation delivering a “general inequality measure” that the Gini index can measure. Then, the “Isograph”, a tool I introduce to represent the diversity of “local inequality” along the income scale, is able to figure out how the empirical distribution deviates from the CF hypothesis at the median, the top and the bottom. Therefore, alpha, beta and gamma (ABG) method proposes an estimation of three parameters of inequalities, compatible with the Pareto properties of the tails. These coefficients are directly interpretable in terms of level-specific measures of inequality at the median (alpha), the top (beta) and the bottom (gamma). An empirical analysis of 212 datasets from 40 countries gives a characterization of the ABG coefficients. One of the added values of this method is its ability to help researchers understand not only the intensity of inequalities but the shapes of the distributions (strobiloids) in relation to interpretable coefficients.

### **A balance of power theory of stratification and the CF-distribution**

Scholars consider income distributions backed by theory.<sup>7</sup> This is precisely the case of the CF distribution we make use of as a benchmark of empirical observations. I introduce here a balance of power theory of incomes. Developed societies are socially hierarchized on the base of rank (of education, prestige, access to political power, “value” of any kind) expressible as a quantile rank  $p$  in  $]0,1[$ . Each individual  $i$  ( $i=1, \dots, n$ ) with income  $y_i$  is above a proportion of

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<sup>5</sup> Gabaix (2009) pays attention to this local degree of inequality, but his topics (mainly size of cities, firms, and largest actors on the stock market) lead to a focus on the top of the distribution and not on the whole scale: with city sizes, alpha is close to 1 (Zipf law), and so the description “median size city” is a perplexing notion.

<sup>6</sup> Weeden and Grusky, (2012) recently proposed a focus on forms of inequality but in terms of categorical groupings more than on distribution of economic resources.

$p_i$  individuals and has a proportion of  $(q_i = 1 - p_i)$  individuals above her. Champernowne (1937) defined “power of income” as  $Y_i = \ln(y_i)$ , the logged income. Similarly, we define “power of social rank”  $X_i$  as the log of the ratio of the odds of  $p_i$ :  $X_i = \ln(p_i/q_i)$ . Thus,  $X_i$  is the logit of the rank quantile  $p$  in the distribution:  $X_i = \text{logit}(p_i)$ .<sup>8</sup> The higher  $p_i$ , the higher the power of social rank; when  $p_i$  tends to 1, social power tends to  $+\infty$ ; this could explain why, at the top of the distribution of prestige, it is strategic to increase one’s rank since the rewards in terms of  $\text{logit}(\text{quantile})$  tends to be infinite, and is obviously much higher than in the neighborhood of the median. Conversely, close to the bottom, losing several ranks may have immense consequences in terms of power of rank. This could explain why Aristotle sees the top of the distribution as dangerously arrogant and the bottom prone to brutality, when the middle of the scale expresses stability and moderated political attitudes (Aristotle, *Politics* 1295b15).

One can pose that the difference of power of income of two individuals (i) and (j),  $\Delta Y = Y_j - Y_i$ , is proportional to the difference in their social power,  $\Delta X = X_j - X_i$ . Therefore,  $\Delta Y = \alpha \Delta X$ , where the constant alpha expresses the degree of economic inequality in this society. Therefore income inequality between (i) and (j) derives from a balance of social power of rank:

$$\ln(y_j/y_i) = \alpha \ln [p_j/(1 - p_j) (1 - p_i)/p_i]$$

If we consider medianized incomes (incomes divided by the median so that  $y = 1$  for the median individual, which is a convenient standardisation strategy), the expression is simplified:

$$\ln(y_i) = \alpha \ln [p_i/(1 - p_i)], \quad (1)$$

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<sup>7</sup> Some functional forms “claim attention, not only for their suitability in modeling some features of many empirical income distributions, but also because of their role as equilibrium distributions in economic processes” (Cowell, 2002:25-6).

<sup>8</sup> Among others, Clementi and colleagues (2012) log-transform the value of rank, even if quantile, as a ]0,1[ intervalled variable, requires a symmetric treatment that logit transformation does perform. The concept of “logit rank” is more common in epidemiology than in the social sciences. “Logit rank” (O’Brien, 1978; Copas, 1999)

$$\text{or } Y_i = \alpha X_i, \text{ where } X_i = \text{logit}(p_i) \text{ and } Y_i = \ln(y_i)$$

This is precisely the expression of a CF distribution. The CF is a sub-case of the complete Champernowne-II (1937) four-parameter distribution; Fisk (1961) described more generally this simplified form.<sup>9</sup> He called this the “sech<sup>2</sup> distribution” (square of hyperbolic sequant), and it is also called the loglogistic (Shoukri et al., 1988, Dagum, 2006) distribution.

Compared to Dagum’s parametrization, the difference here is that we consider  $\alpha = \delta^{-1}$ . The parameter  $\alpha$  is a coefficient of inequality, and we know that for  $\alpha < 1$ ,  $\alpha$  is equal to the Gini index (Dagum, 1975, 2006, Kleiber and Kotz, 2003:224).<sup>10</sup> In this CF-distribution hypothesis, a variation of one percentage point in alpha generates an increase of income of one percentage point near the third quartile, two percentage points near the ninth decile, etc.

### **Divergences from the CF-distribution**

The analysis of empirical distributions confirms that expression (1) is a first order approximation that should be improved. I propose the introduction of a function ISO that generalizes (1) in (2) and therefore represent the divergences of the empirical curve from the CF hypothesis:

$$Y_i = \text{ISO}(X_i) X_i \quad (2)$$

If  $\text{ISO}(X_i)$  is a constant (alpha), (2) is simplified to (1) and the distribution is a CF with alpha equal to the Gini index; the higher the value alpha, the higher the inequality.

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or “logistic quantile” (Orsini and Bottai, 2011) or other names for logit rescaling of ]0,1[ proportions exist in the literature but have not received the attention they deserve.

<sup>9</sup> It is still unclear how the CF income distribution is really related to stochastic processes initially developed by the same author, Champernowne (1952), that was reworked by Shorrocks (1975) in his analyses of stochastic models of income attainment, and recently renewed by Reed (2001) and Gabaix (2009) in process models based on geometric Brownian motions. See as well Kleiber and Kotz (2003, 65sq) as well. Osberg (1977) criticized this stream of research on the base of its *ad hoc* way to mimic reality, inexact predictions and implicit belief that hierarchy is the result of random processes.

<sup>10</sup> This requires a continuous distribution (infinite population) and an  $\alpha < 1$ . In case of a discrete population and extreme inequality, alpha can be higher than 1; an example is the distribution of the number of casualties in war events over the last century (Cederman, 2003) since the underlying alpha equals 1.5. In the Zipf distribution (Gabaix, 1999) typical of city size distributions, alpha equals 1.

In general, the CF-distribution hypothesis slightly diverges from reality. Therefore, the isograph that represents  $ISO(X_i)$  is not a constant and expresses the intensity and the shape of inequalities. The empirical isographs present horizontal lines that are often bent at the two extremes in different ways by social policies and other empirical biases in the theoretical balance of power. The poor can either benefit from income support or be the victims of extreme social exclusion. The rich can either organize a system of resource hoarding or accept the development of massive redistributive policies. Then, the hypothesis of strict stability of alpha along the income scale can be falsified, since relations of power can be stronger or smoother at the top and at the bottom of the social ladder.

Then,  $ISO(X_i)$  diverges from a constant value. If we define local inequality at level  $X_i$  as the divergence between the median and log-income  $Y_i$ ,  $ISO(X_i)$  is a reliable measure of this local inequality. The isograph is therefore a representation of local inequality over the income scale: the higher the value  $ISO(X_i)$ , the higher the degree of inequality at level  $X_i$ : the stronger the divergence between log-income  $Y_i$  and its median value ( $Y=0$ ). The isograph is sensitive to transfers through the median, to decrease of incomes of individuals above the median and to increase of incomes below.<sup>11</sup> In these three cases the isograph is lower after the transfer than before. Empirical comparisons show that local inequality is variable along the income scale: local income inequality is anisotropic (figure 1) since its measure depends on the level where it is assessed.

When the isograph is almost a flat line (Finland-2004), alpha equals the Gini index (.24 for fi04). In France, Germany and Brazil, this CF distribution hypothesis is an acceptable approximation, but elsewhere the isograph denotes specific local degrees of inequality, often

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<sup>11</sup> In the general case, a non-through-the-median Pigou-Dalton transfer is ambiguous. For Sen (1973), a transfer from a richer person (a) to a poorer person (b), provided that it does not reverse the positions of the two, generates greater equity. From the point of view of local inequalities, the situation is unclear when persons a and b are not on the same side of the median. When both are above the median with a richer than b, the increase of local inequality due to the decline of income of a, is counterbalanced by increasing income of b who diverges from the median.



with a declining level of inequality at the top of the distribution. An extreme case of variation is Israel-2007 with a  $ISO(0) = .52$  at the middle of the distribution, similar to Brazil. At the bottom 5% of the Israeli distribution,  $ISO(-3) = .40$ , like Spain, and at the top 5% of the distribution, the Israeli  $ISO(3) = .36$ , very similar to the U.S. These findings illustrate the large variations of local inequality that are observed along the hierarchy of incomes. In this respect, the isograph helps in the detection of segments in the income scale where country A is more locally unequal than country B. The crossings of the isographs of Israel and U.S. show extreme inequality near to the median in Israel balanced by more equality at the extremes.

## **PLACE FIGURE 1 HERE**

### **The parametric estimation of the ABG method**

The analysis of the 212 isograph shapes let suppose we can synthetize the empirical distributions with only three parameters that I introduce here. The curvatures of the isographs show that we can define a coefficient pertaining to the level of local inequality near to the median ( $\alpha$ ) and two coefficients pertaining to the curvatures of the isograph at the two extremities. Therefore, we define two coefficients  $\beta$  and  $\gamma$  where  $\alpha + \beta$  is the upper asymptote of ISO and conversely  $\alpha + \gamma$  at the bottom. When  $\gamma$  and  $\beta$  are null, we have a CF distribution of coefficient  $Gini = \alpha$ . The interest of this presentation is to deliver interpretable parameters of inequality pertaining respectively to local inequality at the median, and correction at the top and at the bottom of the income distribution.<sup>12</sup>

The parameterization we propose is compatible with the well settled hypothesis that the upper tail has a power-tailed Pareto type shape (Piketty, 2001), so that the upper asymptote of the  $ISO(X)$  function should be a horizontal flat line  $Y = \alpha + \beta$ . We hypothesize, along with

Reed (2001), that the lower tail is Pareto shaped as well.<sup>13</sup> Thus, the lower asymptote of ISO(X) should be a constant  $Y = \alpha + \gamma$ . Between this, progressive variations are observed.

A parametric expression of such curvatures is based on two hyperbolic functions  $\theta_1$  and  $\theta_2$  related to hyperbolic tangent functions:  $\theta_1(X) = \tanh(X/2)$  and  $\theta_2(X) = \tanh^2(X/2)$  (see figure 2).

$\theta_1$  stylizes a progressive increase of local inequality along the income scale: lower-class targeted policy of redistribution based on progressive taxation at the top can generate such a  $\theta_1$ . South Africa 2010 shows a positive  $\theta_1$  slope and Japan 2008 a negative one.  $\theta_2$  expresses the relatively lower degree of inequality (more homogeneity) at the center than at the extreme of the distribution. This could happen when middle class policies for the included central part of society create a middle-class oriented welfare ignoring both the excluded classes at the bottom and the very rich at the top. Israel 2007 shows a negative  $\theta_2$ .

### PLACE FIGURE 2 HERE

It is easier for the interpretability of coefficients to make use of two simple linear combinations of these theta functions, B and G. We consider the decomposition

$$\text{ISO}(X) = \alpha + \beta B(X) + \gamma G(X) \quad (3)$$

$$\text{Where } B(X) = (\theta_1(X) + \theta_2(X))/2$$

$$\text{and } G(X) = (-\theta_1(X) + \theta_2(X))/2$$

$$\text{and } \theta_1(X) = \tanh(X/2) \text{ and } \theta_2(X) = \tanh^2(X/2)$$

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<sup>12</sup> This aspect is important: the GB2 distribution offers in general correct fits of empirical distributions (Jenkins, 2009), but the interpretability of its p and q shape coefficients is unclear.

<sup>13</sup> In this respect, at some point, this hypothesis will have to be tested along with the Milanovic et al. (2011) hypothesis that the vital subsistence minimum is at \$PPP 300 per year (1990 prices).

$$\text{Then, } Y = X (\alpha + \beta B(X) + \gamma G(X)) \quad (4)$$

where  $X = \text{logit}(\text{quantile})$  and  $Y = \ln(i)$ ,

This decomposition can be obtained via linear regression of (3), and provides these estimates:

- the coefficient  $\alpha$  of inequality near to the median is the constant,
- $\beta$  characterizes the additional inequality at the top, and  $\beta$  is positive when the rich are richer than in the CF distribution of coefficient  $\alpha$ ,
- $\gamma$  characterizes the additional inequality at the bottom, and  $\gamma$  is positive when the poor are poorer than in the CF distribution of coefficient  $\alpha$ .

When  $\beta$  and  $\gamma$  are equal to zero, the distribution is a CF of coefficient  $\alpha = \text{Gini index}$ . Since we observe empirically that the absolute values of  $\beta$  and  $\gamma$  are always much smaller than  $\alpha$ , the CF distribution is an acceptable simplified first-order hypothesis, and  $\beta$  and  $\gamma$  are corrective coefficients. When  $\beta$  (respectively,  $\gamma$ ) is 1% higher, the  $\text{ISO}(X)$  function increases by 1% at the upper (respectively, lower) asymptote. Thus, we have a three interpretable parameter distribution deriving from the  $\text{ISO}(X)$  function estimation (3).<sup>14</sup>

In this decomposition,  $\alpha$ ,  $\alpha + \beta$  and  $\alpha + \gamma$  are measures of inequality at the median, the top and the bottom of the distribution, respectively, and are compatible with Gini index measures. There is no formula for these measures since they are coefficients obtained via ordinary least squares (OLS) regression of  $\text{ISO}(X)$ . The higher the values calculated using these measures, the higher the inequality at the respective levels of the distribution. They satisfy the criteria of appropriate inequality measures (mean independence, population size independence, symmetry, through the median transfer sensitivity, statistical testability and decomposability (see Jenkins ,1995; Cowell and Jenkins,1995). For the two

latest criteria, the demonstration is less obvious than for the others, but since the ABG estimation is based on linear regressions, it is possible to make use of control variables and of confidence intervals to decompose and statistically test the results of ABG.

### **Comparative analysis of 212 datasets**

The added value of the ABG method over other measures comes from a comparison with more usual indices of inequality on a set of 212 harmonized microdata files covering 40 countries is provided by the LIS datacenter project.<sup>15</sup> This source is very frequently used in the analysis of socioeconomic inequality (Brandolini and Atkinson, 2001, Gornick and Jäntti, 2013) and the set can be used as a large sensitivity test for the three indicators and as a rather general sample of contemporary countries. The first result is that the absolute values of beta and gamma are small compared to alpha: In other words, inequality levels at the extremes,  $\alpha + \beta$  and  $\alpha + \gamma$  are always in the interval  $[0,1]$ . The signs of beta and gamma can be positive or negative and thus generate a four category typology (table 2).

### **PLACE FIGURE 3 HERE**

The mean of the betas is negative: inequality is more often lower at the upper tail than at the median, relative to the CF distribution. The average gamma is close to zero. Even if type 3 and 4 are more usual, all the other configurations exist in the set of 212 samples. Thus, the CF distribution approximation is somewhere in the middle of the datasets (figure 3).

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<sup>14</sup> In the conventional literature, we could speak of a 4 parameter curve, but since we consider the medianized income, the traditional b coefficient is automatically set to 1.

<sup>15</sup> This international consortium archives and harmonizes income relevant datasets in the Western developed world and elsewhere and is devoted to the microdata based analysis of inequalities of disposable incomes after taxes and social transfers. If some datasets are questionable either because sources of bias impairing the possibilities of comparison are documented or because the comparison shows that some cases are unexplainable outliers, the 212 samples available today (23/11/13) are of specific interest for the empirical diversity of case they provide. The codes of the samples in LIS data center are based on the standard ISO 2 digits codes of countries followed by the 2 final digits of the year.

## PLACE TABLE 2 HERE

To test the accuracy and added value of this proposition, the three ABG indexes are compared to other relatively standardized and easily accessible measures of inequality (Jenkins, 1999/2010 Abdelkrim and Duclos, 2013). These indicators are either standard or based on income ratios, and four simple indicators are added: the value of ISO function measured at 4 levels ( $X = -3, -1, +1$  and  $+3$ ). On top of that, the size (proportion in the total population) of five income classes: the poor (po), lower middle class (mcl), middle class (mc), upper middle class (mcu), and the rich (ri). Overall, we generate a table of 212 samples and 30 variables:

- ABG class: alpha, beta, gamma, three coefficients resulting from the ABG method
- Atkinson class: a2, a1, ahalf = Atkinson class of indexes, with parameters 2, 1,  $\frac{1}{2}$  (Atkinson, 1970, also see Yitzhaki, 1983), the higher parameter (2) overweighs the focus on the bottom of the distribution.
- Generalized entropy class: ge2, ge1, ge0, gem1 = Generalized entropy class of indexes, with parameters 2, 1, 0, -1 (Berry et al., 1983). With lower parameter (-1), the focus is put on the top of the distribution.
- Gini inequality index = Gini index (Gini, 1914)
- Wolfson polarization index = Wolfson index (Wolfson, 1986) of polarization.<sup>16</sup>
- Foster-Greer-Thorbecke poverty class: fgt0 1 2 3 Foster-Greer-Thorbecke (Foster et al., 1984) poverty index, with parameters 0, 1, 2, 3, and poverty threshold at 60%. The higher the parameter, the stronger the focus on extreme low income.
- Income ratios: r90v50 = ratio of the last decile by the median (top inequality), r50v10 = ratio of the median by the first decile (bottom inequality), r90v75 = ratio of (the last decile by the last quartile) by (the last quartile by the median): this measures the degree to which the top decile is even higher than expected when the top quartile is known = acceleration of inequality at the top, r25v10 = ratio of (the first quartile by the first decile) by (the median by the first quartile): this measures the degree to which the bottom decile is even lower than expected when the lower quartile is known = acceleration of inequality at the bottom
- ISO(X) class of measure of inequality along the income scale: v2, v6, v10, v14 = the ISO(X) values for  $X = -3, -1, +1$  and  $+3$  (derived from the isograph)
- Income class proportions: po, mcl, mc, mcu, ri: respectively, the proportion of poor (medi < .5), lower middle class (.5 <= medi < .75), middle class (.75 <= medi < 1.25), upper middle class (1.25 <= medi < 2) and rich (2 <= medi) in the total population.<sup>17</sup>

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<sup>16</sup> We consider the Wolfson index since it might be the most standard indicator but more reliable propositions exist (Alderson et al., 2005; Chakravarty and D'Ambrosio, 2010).

<sup>17</sup> We would prefer a log-symmetric definition such as .75 to 1.33, but the .75 to 1.25 of the median definition is far more common in the literature (Pressman, 2007). We can notice that working on quintile dynamics, Dallinger (2013) found similar variations in the changes of the different sub-strata of the middle classes.

- Income class based indicator of polarization:  $r_{pol} = (mcl + mcu)/mc$  when the upper and lower middle classes are large compared to the middle class, we have a polarization in the sense of a shrinking middle class.

First, there is a very strong relation between alpha and Gini index (table 6) ( $R = +.95$ ) that confirms the relation of the two measures of inequality when the CF approximation is acceptable. Second, the beta and gamma coefficients provide information complementary to alpha: the degree to which inequality increases or not at the top and at the bottom of the distribution.

Third, a principal component analysis (PCA) of the complete table allows a multidimensional analysis of the relation between these measures (table 3). The first axis of the PCA (68% of the total variance) is of a similar nature as many of the other measures of inequality. Alpha is representative of the first axis of the PCA, with the Atkinson parameter 1 ( $a_1$ ) and the Atkinson parameter  $\frac{1}{2}$  ( $a_{half}$ ), generalized entropy parameter 1 ( $ge_1$ ) and parameter 0 ( $ge_0$ ), Gini index and several quantile ratios. In this respect, alpha is a supplementary measure, more specifically related to the median of the distribution, and having distributional backgrounds in the Champernowne's tradition and related to the Gini index.

### **PLACE TABLE 3 HERE**

On axes 2 and 3 (12 and 7% of the variance, respectively), the role of beta and gamma is clarified:

- On the second axis, beta and gamma are well correlated along with  $r_{25v10}$  and  $r_{90v75}$  that express the degree to which the extreme deciles are farther than the quartiles would have let us expect. Thus, this second axis is a measure of distribution tails over-elongation. Here, the generalized entropy index with parameter 2 has a relatively stronger correlation on axis 2 than the other traditional measures. On the negative part of axis 2 are the indicators of polarization: the larger beta or gamma, the smaller the

upper middle and the lower middle class, respectively. So beta (respectively, gamma) measures the lack of density of the upper (respectively, lower) middle class: when beta (respectively, gamma) is large, the upper (respectively, lower) middle class shrinks. Then, at a given level of overall inequality, when both beta and gamma are small, polarization occurs because the median is relatively less dense than the upper and lower middle classes.

- Axis 3 reveals the difference between gamma and beta, along with the opposition between  $r_{25v10}$  and  $r_{90v25}$ . On this axis, the generalized entropy index with parameter -1 and the Atkinson index with parameter 2 are located on the same side as gamma: Here are measures sensitive to inequality at the bottom. Conversely, the generalized entropy index with parameter 2, located on the same side as beta, is sensitive to inequality at the top. Therefore, beta and gamma are sensitive to important features that others measures partially detect but with less sensitivity.

**PLACE FIGURE 4 HERE**

**PLACE TABLE 4 HERE**

These samples show a relation between the Gini index, Atkinson 2, generalized entropy 2 and the ABG coefficients (table 4):

- Gini index is very similar to alpha and is very sensitive to the values of beta as well, but is almost independent of gamma: As a measure of inequality, Gini index is first median oriented, and secondly rich oriented.
- The Atkinson 2 index is more sensitive to lower tail inequality. The significance of beta on Atkinson 2 is very low, but high with gamma and alpha: then, Atkinson 2 is both sensitive to poverty and general inequality (Gini index).

- Conversely, the generalized entropy 2 combines beta and alpha.

It is possible to test the superiority of the ABG coefficients and of the GA2GE2 triple coefficients (Gini index, Atkinson 2, generalized entropy 2) by a nested model: In our data we compare the explanation (in terms of delta  $r^2$ ) of income class proportions (po, mcl, mc, mcu, ri) by ABG first and the GA2GE2 second, and vice versa. The ABG coefficients are always superior to the GA2GE2 triple, and the improvement proposed by ABG is particularly strong for the explanation of mcl and mcu, respectively, the lower and upper middle class.

Additionally, the ABG could improve the measurement polarization over that of the Wolfson index (Wolfson, 1986). The Wolfson index has been developed on the basis of the Gini index; therefore, it improves its sensitivity to median stretches when the others remain almost unchanged. Consequently, the ratio  $r_{pol} = (mcl + mcu)/mc$  as defined earlier should increase in a configuration of polarization. The linear correlation matrix (table 6) shows that the Wolfson index is an improvement of the Gini index in terms of  $r_{pol}$  ratio detection, but alpha is even more efficient in terms of  $r^2$ .

**PLACE TABLE 5 HERE**

**PLACE TABLE 6 HERE**

Nested model comparison on the 212 datasets shows that when  $r_{pol}$  is explained by alpha first and the Wolfson index second, Wolfson index adds 2.5% to the  $R^2$ ; on the contrary, alpha adds 13.3% to the  $R^2$  when it is added after the Wolfson index. Hence, the Wolfson index works as a measure of polarization although in this purpose it is weaker than alpha.

### **Representing the shapes of income distribution: the strobiloid**

The ABG decomposition provides a method for smoothing the empirical quantile function of the distribution. If, for instance, one is interested in the architecture of societies deriving from the density curve of distributions, like in the seminal works of Pareto (1897: 315), we can



represent incomes as a vertical hierarchical axis and the density value horizontally (see figure 5). A convenient way of standardizing the representations, so that comparisons are possible, is to normalize the income curve: With both medianization of income and the normalization of the surface to 1 (so that it defines the density of the distribution), one can superpose the shapes of two periods or two nations to compare the differences. This is the way of processing in the strobiloid representation (Chauvel, 1995, Lipietz, 1996, Chauvel, 2013).<sup>18</sup>

### **PLACE FIGURE 5 HERE**

The empirical representations are suggestive, reveal the diversity of the shapes of income distribution in the world and show how some countries experienced complete change in their internal socioeconomic architecture. There, the shape of society appears in a representation where the larger a curve, the more individuals are in this level of the graph: a middle class society will appear with a large belly (Sweden) in contrast to the contemporary American distribution where a large proportion of the population is close to the bottom. Kernel smoothing can produce similar curves, but our method relies on a Pareto power-tail compatible methodology producing interpretable parameters.<sup>19</sup> This tool allows country and time comparison of the large transformations in the intensity and shape of inequalities.

### **PLACE FIGURE 6 HERE**

The incomes in Sweden-1987 are generally “more equal” than elsewhere, even if the Swedish shape is more specific by its lack of rich than its lack of poor; there has been a slight trend of

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<sup>18</sup> The strobiloid comes from Pareto’s idea (1897:313) that the shape is the one of an arrow or of a spinning top. This representation, close to Pareto’s first representations of the income pyramid, allows 2 by 2 comparisons of countries, time, etc. Nielsen (2009) gives an overview of Pareto’s legacy and analyses why it is generally neglected in social science.

decline in the density at the median level. The bottom part of the curve in Germany-1983 shows the same level of inequality as in Sweden-1987 (on the isograph), even though the other income levels above are more unequal in Germany-1983. In terms of public policy, this German-1983 structure is a specific model of homogeneity below the median with a high implicit minimum income.

The French distribution is rather commonplace in Europe and stable over time with no major change during this time span. On the contrary, the U.K. shows a strong trend of polarization, with a trend of convergence to the onion-shaped strobiloid of the U.S. The U.S. itself has an ever more typical onion shape with increasing inequality. An important aspect of this shape is found less in the extreme values at the top but rather in the lower values with very a strong gamma. Israel, the last case, may be the most symbolic one in terms of shift from rather equal to strongly unequal distribution with a very specific feature: a steadily declining median class of incomes with a relatively strong minimum income scheme, leading to the creation of an unprecedented arrow-head-shaped curve. Israel appears then as an extreme case of rapid polarization in the most recent decades (García-Fernández et al., 2013), and this is confirmed by the isograph in figure 6.

### **Conclusion: Added value and further extension of the ABG method**

This ABG methodology represents a progress in terms of measurement and graphical representation (CF curve, isograph, strobiloid) of the diversity of inequalities at different levels of the income scale, since in many cases inequality is an anisotropic dimension along the income scale. In terms of public policies, it can offer interesting interpretations about the diversities of dynamics of inequalities, where median dynamics proposed by alpha is to be analyzed in parallel with the behavior of the extremes described by beta and gamma.

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<sup>19</sup> Kernel density analysis is generally unable to give correct assessment of the extremities of the curves.

The significance of this proposition lies in its easy-to-use family of distributions that enable the modeling of income distributions; it could be useful, for instance, for modeling extremely unequal distributions such as Zipf laws (Gabaix, 1999) that are extreme Pareto distributions with alpha close to 1, and ABG could be useful in the understanding of why the Gini index could be a problematic indicator of inequality when the isograph is far from a constant (when beta and gamma differ greatly from 0).

The ABG methodology explicitly claims that magnitudes of ranks and of incomes defined by  $\text{logit}(\text{quantile})$  and  $\text{log}(\text{income})$  are almost linearly related. Therefore, the  $\text{logit}(\text{quantile})$  could be an important tool in the measurement of inequalities, and it could be possible to make use of it in other fields such as income mobility. Later developments of the ABG should include an examination of statistical significance issues and aspects of group decomposability: Since the ABG coefficients result from a linear regression, control variables can be added to understand how gaps between groups explain overall inequality.

The results here can be confirmed with more traditional tools, but the alpha-beta-gamma ABG method, the CF and the isograph, and the associated strobiloid offer more systematic and easier tools for the detection of these specificities, propose better measures of the income distribution and help us to understand the anisotropy of inequalities.

## References

- Alderson, A. S., J. Beckfield and F. Nielsen, "Exactly How Has Income Inequality Changed? Patterns of Distributional Change in Core Societies." *International Journal of Comparative Sociology*, 46, 405-423, 2005.
- Araar, A. and J.-Y. Duclos, *DASP: Distributive Analysis Stata Package*, PEP, World Bank, UNDP and Université Laval, 2007.
- Atkinson, A. B., "On the Measurement of Inequality," *Journal of Economic Theory*, 2, 244–63, 1970.
- Berry, A., F. Bourguignon, and C. Morrisson, "Changes in the World Distributions of Income Between 1950 and 1977," *The Economic Journal*, 93, 331–50, 1983.

- Brandolini, A. and A. B. Atkinson, "Promise and Pitfalls in the Use of "Secondary" Data-Sets: Income Inequality in OECD Countries as a Case Study," *Journal of Economic Literature, American Economic Association*, 39, 771–99, 2001.
- Cederman, L.-E., "Modeling the Size of Wars: From Billiard Balls to Sandpiles," *American Political Science Review*, 97, 135–50, 2003.
- Champernowne, D. G. "The Theory of Income Distribution," *Econometrica*, 5, 379–81, 1937.
- Champernowne, D. G. and F. A. Cowell, *Economic Inequality and Income Distribution*, Cambridge University Press, Cambridge, UK, 1998.
- Chakravarty, S.R. and C. D'Ambrosio, "Polarization Orderings Of Income Distributions," *Review of Income and Wealth*, 56, 47–64, 2010.
- Chauvel, L., "Inégalités singulières et plurielles : l'évolution de la courbe de répartition des revenus," *Revue de l'OFCE*, 55, 211–40, 1995 <http://louis.chauvel.free.fr/STROFCED.pdf>
- , "Welfare Regimes, Cohorts and the Middle Classes," in J. Gornick and M. Jäntti (eds), *Economic Inequality in Cross-National Perspective*, Stanford, CA, Stanford University Press, 115–41, 2013.
- Clementi, F., M. Gallegati, and G. Kaniadakis, "A New Model of Income Distribution: The  $\kappa$ -Generalized Distribution," *Journal of Economics*, 105, 63–91, 2012.
- Copas, J. "The Effectiveness of Risk Scores: The Logit Rank Plot Quick View," *Journal of the Royal Statistical Society. Series C (Applied Statistics)*, 48, 165–83, 1999.
- Cowell, F. A., *Measuring Inequality*, 2nd edition, Harvester Wheatsheaf, Hemel Hempstead, 1995.
- , "Measurement of Inequality," in A. B. Atkinson and F. Bourguignon (eds.), *Handbook of Income Distribution Volume 1*, Elsevier Science, Amsterdam, 59–85, 2000.
- , *The Economics of Poverty and Inequality: Introduction*, London School of Economics, mimeo, 2002.
- , (ed.), *The Economics of Poverty and Inequality, Volume I Inequality*. Edward Elgar, Cheltenham, UK, 2003.
- Cowell, F. A. and S. P. Jenkins, "How Much Inequality Can We Explain? A Methodology and an Application to the USA," *Economic Journal*, 105, 421–30, 1995.
- Dagum, C., "A model of income distribution and the conditions of existence of moments of finite order", *Proceedings of the 40th session of the International Statistical Institute*, Vol. XLVI, Book 3, Warsaw, 196-202, 1975.
- , "A New Model of Personal Income Distribution: Specification and Estimation," *Economie Appliquée*, 30, 413–36, 1977.
- , "Wealth distribution models: analysis and applications," *Statistica*, 6, 235–68, 2006.
- Dallinger, U., "The Endangered Middle Class? A Comparative Analysis of the Role Public Redistribution Plays", *Journal of European Social Policy*, 23(1), 83–101, 2013.
- Dombos, P., "Some Ways of Examining Dimensional Distributions," *Acta Sociologica*, 25, 49–64, 1982.
- Fisk, P. R., "The Graduation of Income Distributions," *Econometrica*, 29, 171–85, 1961.
- Foster, J., J. Greer, and E. Thorbecke, "A Class of Decomposable Poverty Measures," *Econometrica*, 52, 761–66, 1984.

- Gabaix, X., "Zipf's Law for Cities: An Explanation," *Quarterly Journal of Economics*, 114, 739–67, 1999.
- , "Power Laws in Economics and Finance," *Annual Review of Economics*, 1, 255–94, 2009.
- García-Fernández, R. M., D. Gottlieb, and F. Palacios-González, "Polarization, Growth and Social Policy in the Case of Israel, 1997–2008," *Economics Discussion Papers*, No 2012-55, Kiel Institute for the World Economy, 2013.
- Gini C., Sulla misura della concentrazione e della variabilità dei caratteri. Atti del Reale Istituto Veneto di Scienze, Lettere ed Arti, LXXIII, parte II, 73, 1203–48, 1914.
- Gornick J. C. and M. Jäntti (eds.), *Income Inequality: Economic Disparities and the Middle Class in Affluent Countries*, Stanford University Press, Stanford, CA, 2013.
- Graf, M. and D. Nedyalkova, "Modeling of Income and Indicators of Poverty and Social Exclusion using the Generalized Beta Distribution of the Second Kind," *Review of Income and Wealth*, 1, 1–20, 2013.
- Jäntti M., E. Sierminska and P. Van Kerm, "The Joint Distribution of Income and Wealth" in J. Gornick and M. Jäntti (eds), *Economic Inequality in Cross-National Perspective*, Stanford, CA, Stanford University Press, 312–33, 2013.
- Jenkins, S. P., "The Measurement of Income Inequality," in L. Osberg (ed.), *Economic Inequality and Poverty: International Perspectives*, M. E. Sharpe Inc., Armonk, NY, 1991.
- , "Accounting for Inequality Trends: Decomposition Analyses for the UK, 1971-86," *Economica*, 62, 29–63, 1995.
- , "Distributionally-Sensitive Inequality Indices and the GB2 Income Distribution," *Review of Income and Wealth*, 55, 392–98, 2009.
- , "INEQDECO: Stata Module to Calculate Inequality Indices with Decomposition by Subgroup," Statistical Software Components S366007, Boston College Department of Economics, revised 19 Apr 2001, 1999/2010.
- Kleiber, C. and S. Kotz, *Statistical Size Distributions in Economics and Actuarial Sciences*, John Wiley, Hoboken, NJ, 2003.
- Lipietz, A., *La Société en sablier. Le partage du travail contre la déchirure sociale*. Paris, La Découverte, 1996
- Luxembourg Income Study (LIS) Database. 2014. <http://www.lisdatacenter.org> (multiple countries; waves for 1985-2005, microdata runs completed in jan 2014). Luxembourg: LIS.
- Lorenz, M. O., "Methods of Measuring the Concentration of Wealth," *Publications of the American Statistical Association*, 9, 209–19, 1905.
- Milanovic, B., P.H. Lindert, J.G. Williamson, "Pre-industrial inequality", *Economic Journal*, 121, 255–272, 2011.
- Nielsen, F., "Economic Inequality, Pareto, and Sociology: The Route Not Taken.", *American Behavioral Scientist* 50, 619–38, 2007.
- Nirei, M. and W. Souma, "A Two Factor Model of Income Distribution Dynamics," *Review of Income and Wealth*, 53, 440–59, 2007.
- O'Brien, P. C., "A Nonparametric Test for Association with Censored Data," *Biometrics*, 34, 243–50, 1978.

- Orsini, N. and M. Bottai, "Logistic Quantile Regression in Stata," *The Stata Journal*, 11, 327–44, 2011.
- Osberg, L. S., "Stochastic Process Models and the Distribution of Earnings," *Review of Income and Wealth*, 23, 205–15, 1977.
- Pareto, V., "La courbe des revenus", *Le Monde économique*, juillet, 99–100, 1896.
- , *Cours d'économie politique*, Rouge, Lausanne, 1897.
- Pen, J., *Income Distribution: Facts, Theories, Policies*, Praeger Publishers, New York, 1971.
- Piketty, T., *Les hauts revenus en France au XX<sup>ème</sup> siècle. Inégalités et redistributions, 1901-1998*, Grasset, Paris, 2001.
- Pressman, S., "The Decline of the Middle Class: An International Perspective," *Journal of Economic Issues*, 41, 181–200, 2007.
- Reed, W. J., "The Pareto, Zipf and Other Power Laws," *Economics Letters*, 74, 15–19, 2001.
- Shoukri, M. M., I. U. M. Mian, and D. S. Tracy, "Sampling Properties of Estimators of the Log-Logistic Distribution with Application to Canadian Precipitation Data," *The Canadian Journal of Statistics/La Revue Canadienne de Statistique*, 16, 223–36, 1988.
- Shorrocks, A.F., "On Stochastic Models of Size Distributions", *The Review of Economic Studies*, 42, 631–41, 1975.
- Weeden, K. A., and D.B. Grusky, "The Three Worlds of Inequality", *American Journal of Sociology*, 117, 1723–85, 2012.
- Wolfson, M. C., "Stasis amid Change: Income Inequality in Canada, 1965-1983," *Review of Income and Wealth*, 32, 337–69, 1986.
- Yitzhaki, S., "On an Extension of the Gini Inequality Index," *International Economic Review*, 24, 617–28, 1983.

**Table 1: Percentiles of incomes in Israel and the U.S., and differences**

	p1	p5	p10	p25	p50	p75	p90	p95	p99
il07	0.188	0.306	0.389	0.586	1.000	1.595	2.237	2.751	4.467
us10	0.072	0.236	0.361	0.608	1.000	1.532	2.169	2.731	4.511
Diff.	-0.115	-0.070	-0.028	0.022	0.000	-0.063	-0.068	-0.020	0.044

Note: in Israel 2007, the fifth percentile level (p5) is equal to 30.6% of the median (p50) and percentile 95 is 2.75 times the median. The line Diff. presents the simple difference between the U.S. and Israel.

**Table 2: Typology of income shapes**

	Gamma negative	Gamma positive
Beta positive	Type 1: Rich are richer and the poor richer than in the CF hypothesis. The isograph has a positive slope. 13 cases. Typical country: za08	Type 2: Rich are richer and the poor poorer, but the middle class is relatively homogeneous. The isograph has a U shape. 22 cases. Typical country: de04
Beta negative	Type 3: Rich are poorer and the poor are richer than in the CF. The isograph has an inversed U shape. 90 cases. Typical country: il07	Type 4: Rich are poorer and the poor are poorer. The isograph has a negative slope. 87 cases. Typical country: us10

**Table 3: Correlation between the principal components and 30 indicators of inequality**

Indic	v1	v2	v3	indic	v1	v2	v3
Alp	0.2149	-0.1103	-0.0251	r90v50	0.2075	0.0614	-0.1851
Bet	-0.0416	0.3938	-0.2423	r50v10	0.2144	0.0492	0.0830
gam	0.0020	0.3767	0.3663	r90v75	-0.0187	0.3364	-0.3846
a2	0.1453	0.0240	0.2757	r25v10	-0.0063	0.3765	0.3046
a1	0.2184	0.0591	-0.0690	v2	0.2095	0.0589	0.1579
ahalf	0.2140	0.0770	-0.1290	v6	0.2158	-0.0817	0.0331
ge2	0.1332	0.1500	-0.2466	v10	0.2162	-0.0639	-0.0648
ge1	0.2074	0.0969	-0.1768	v14	0.2117	0.0369	-0.1728
ge0	0.2155	0.0817	-0.0938	po	0.2139	-0.0184	0.1253
gem1	0.0606	0.0345	0.3022	mclo	-0.0807	-0.3585	-0.1896
gini	0.2182	0.0146	-0.0918	mc	-0.2086	0.1450	-0.0046
wolfson	0.2187	-0.0146	-0.0831	mcu	0.0633	-0.3605	0.1706
fgt0	0.2062	0.0963	0.1411	ri	0.2148	-0.0232	-0.1084
fgt1	0.2117	0.0736	0.1334	rpol	0.1889	-0.2465	-0.0318
fgt2	0.2157	0.0263	0.1274				
fgt3	0.2102	-0.0860	0.1099				

**Table 4: OLS coefficients: Gini index, Atkinson 2 and Generalized entropy 2 by the ABG coefficients**

Gini index	Coef.	Std. Err.	T	P > t	[95% Conf. Interval]	
Gini index $R^2 = .979$						
Alp	.8742	.0088	98.38	0.000	.8567 .891816	
Bet	.4284	.0187	22.89	0.000	.3915 .4654096	
Gam	.0993	.0198	5.01	0.000	.0602 .1384843	
Cons	.0344	.0028	12.03	0.000	.0288 .040119	
Atkinson 2 $R^2 = .4607$						
Alp	1.3423	.1206	11.12	0.000	1.1044 1.5802	
Bet	-.5571	.2542	-2.19	0.030	-1.0583 -.0558	
Gam	1.8344	.2694	6.81	0.000	1.3031 2.3657	
_cons	-.0420	.0389	-1.08	0.281	-.1187 .0346	
Generalized entropy 2 $R^2 = .3952$						
alp	3.1272	.2755	11.35	0.000	2.5840 3.6704	
bet	3.5504	.5804	6.12	0.000	2.4060 4.6948	
gam	-.1475	.6153	-0.24	0.811	-1.3606 1.0654	
_cons	-.5299	.0888	-5.96	0.000	-.7051 -.3548	

Note: Vif < 1.35; N = 212

**Table 5:  $R^2$  added value in nested models of income class proportions of ABG coefficients and of GA2GE2 triple coefficients (Gini index, Atkinson 2, generalized entropy 2)**

	ABG first	GA2GE2 delta $r^2$	GA2GE2 first	ABG delta $r^2$
po	0.9709	0.0036	0.8919	0.0827
mcl	0.423	0.0224	0.1329	0.3125
mc	0.9343	0.0091	0.8953	0.0481
mcu	0.3029	0.0679	0.2065	0.1643
Ri	0.9822	0.0014	0.9687	0.0149

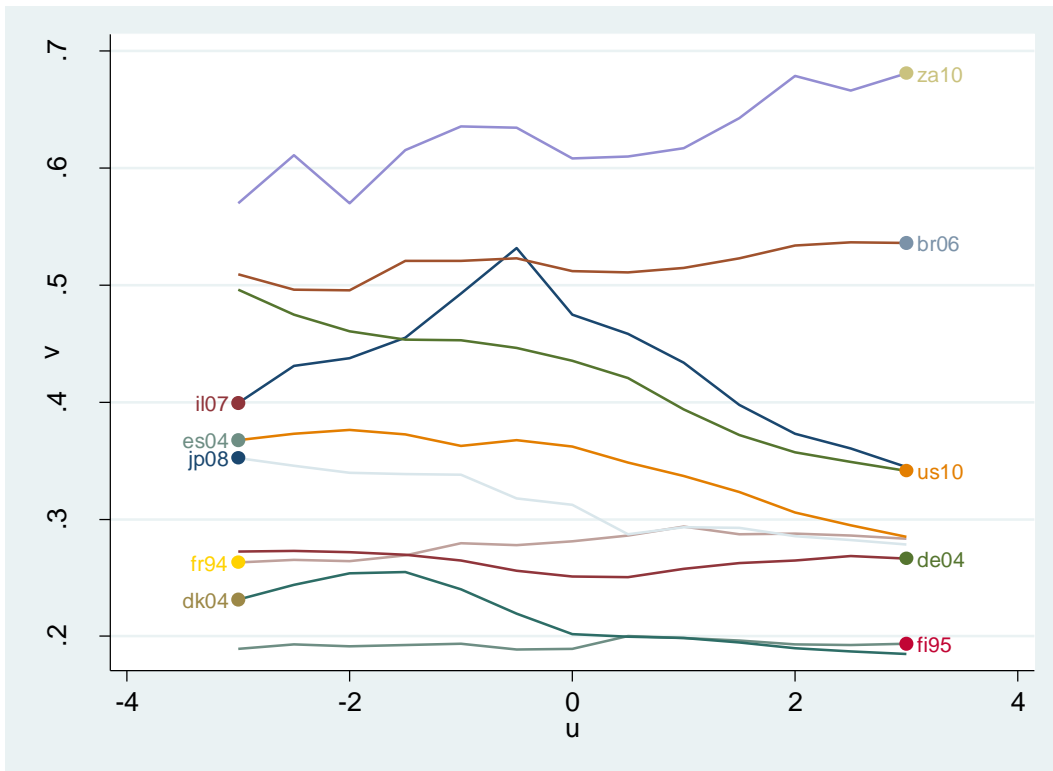
Note: In the explanation of the proportion of po, the ABG coefficients explain 97% and the GA2GE2 triple (Gini index, Atkinson 2, generalized entropy 2) add 0.36% to the  $R^2$ ; when GA2GE2 is first, the  $R^2$  is 89.2% and the added value of ABG is 8.3%.

**Table 6: Correlation between the ratio of polarization, Gini, Wolfson index and ABG**

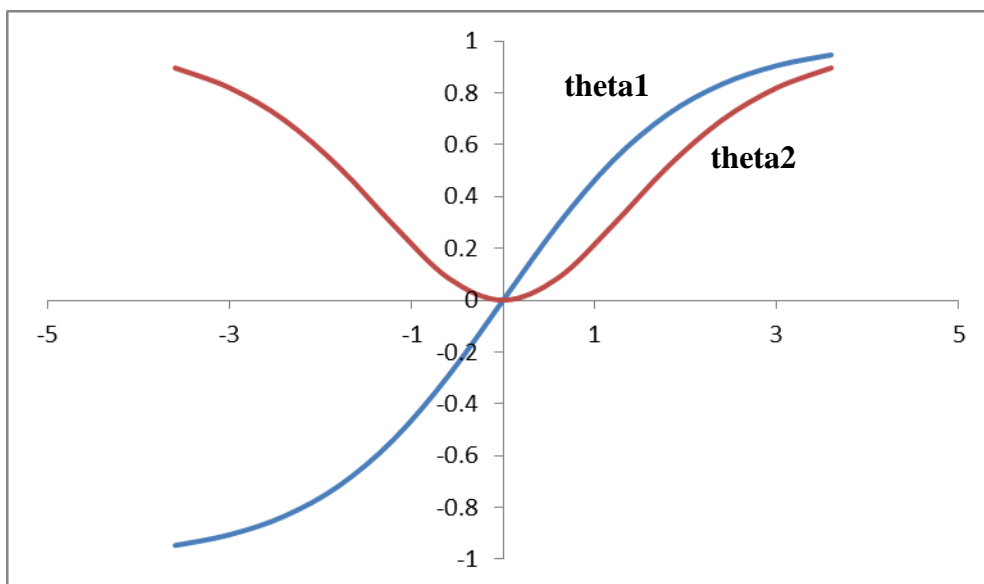
var	rpol	gini	wolfson	alp	bet	gam
rpol	1.0000					
gini	0.8389	1.0000				
wolfson	0.8596	0.9853	1.0000			
alp	0.9200	0.9510	0.9737	1.0000		
bet	-0.4496	-0.0966	-0.1543	-0.3648	1.0000	
gam	-0.3258	-0.0356	-0.0867	-0.1914	0.4197	1.0000



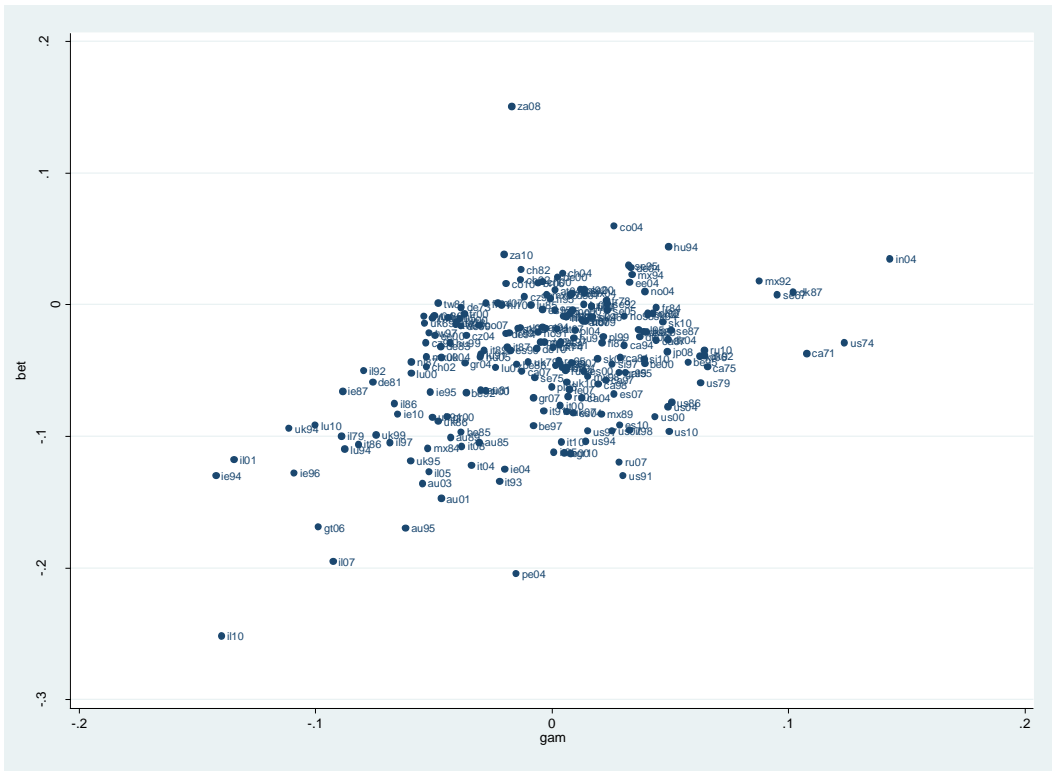
**Figure 1: Isograph in 10 contrasted cases**



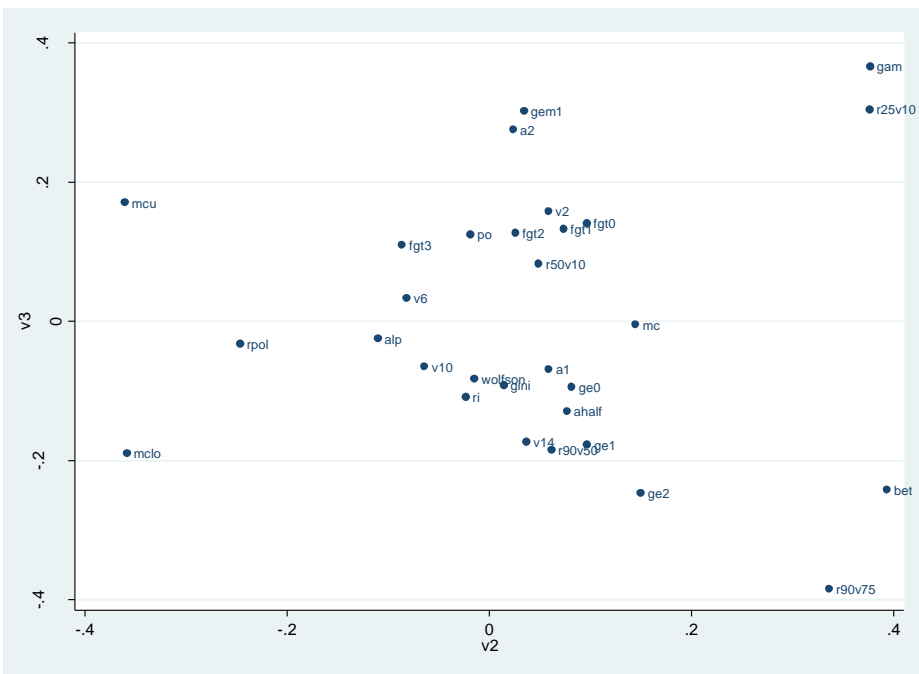
**Figure 2: The theta1 and theta2 functions**



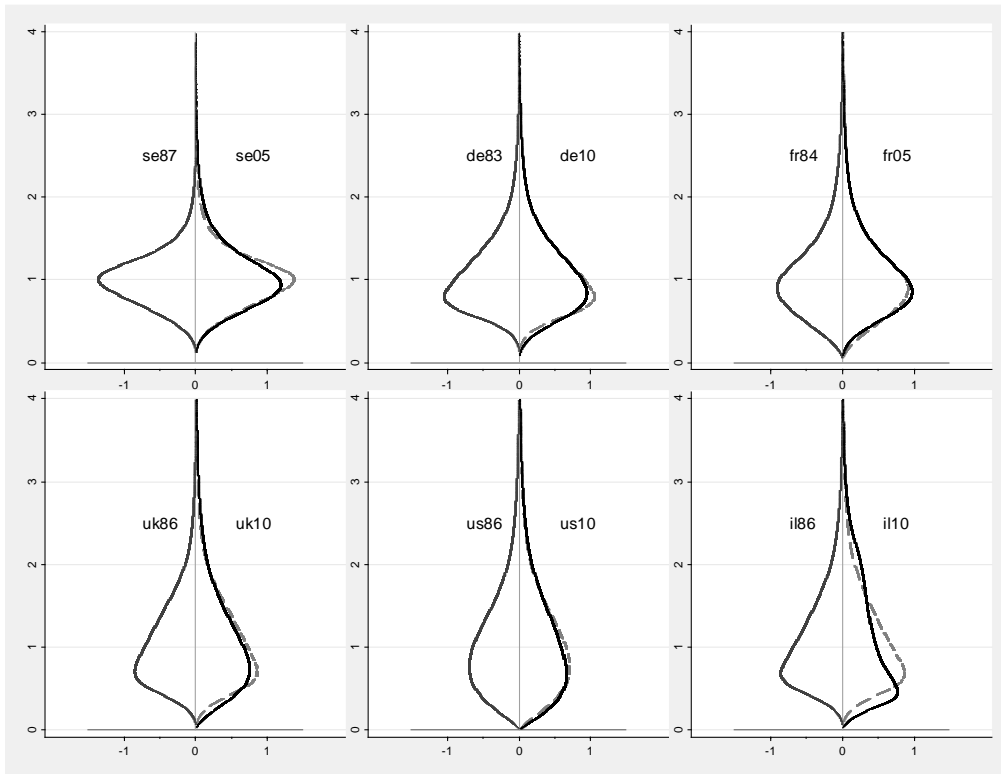
**Figure 3: The relation between beta and gamma**



**Figure 4: Unrotated principal components of the 30 indicators of inequality, poverty and stratification. X = axis 2 Y = axis 3**

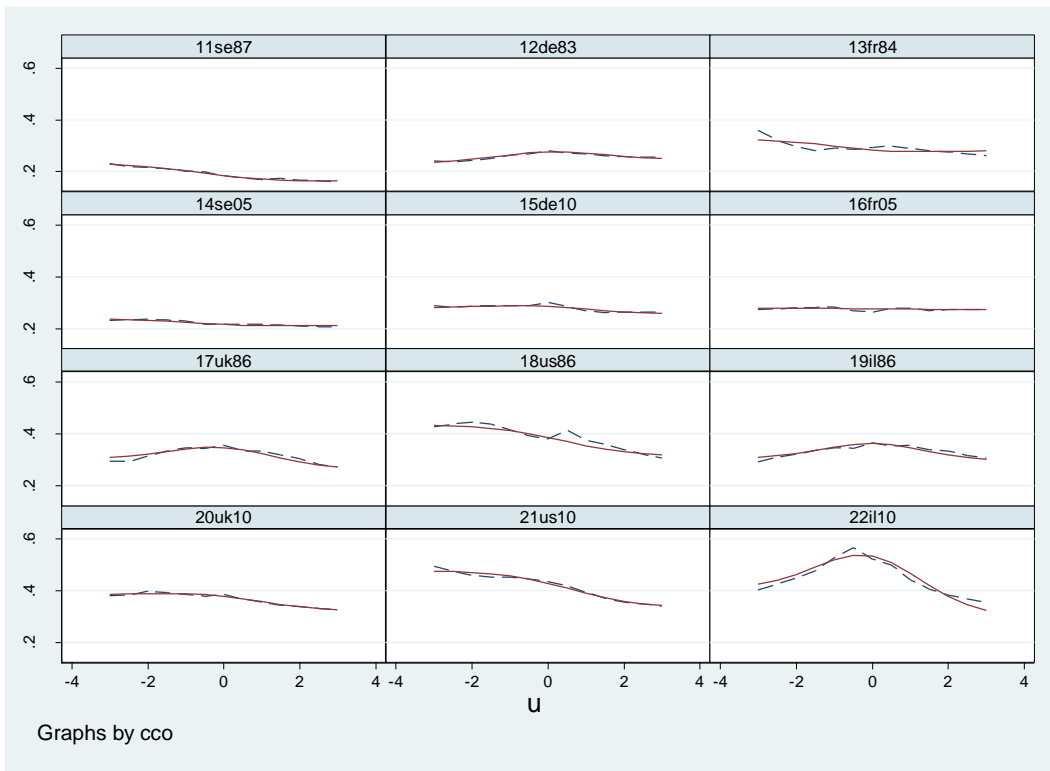


**Figure 5: Six typical strobiloids (Sweden, Germany, France, U.K., U.S., Israel)**



Reading note: The strobiloid presents the income hierarchy (on the Y axis, 1 = median). The curve is larger (X axis) when the density of individuals at this level is stronger: Many individuals are at the intermediate level and their number diminishes at the top and at the bottom. Thus, the intermediate middle class is stronger with increasing belly size of the strobiloid.

**Figure 6: Isographs pertaining to six typical countries**



Reading note: Straight lines indicate the fitted isographs (ABG method) and dashed lines indicate the empirical ones. The higher the curve is at a given level of X (logit rank), the stronger are the income inequalities at this level. Israel 1986-2010 shows an obvious case of extreme polarization.