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TAX PROGRESSION: INTERNATIONAL AND INTERTEMPORAL COMPARISONS USING LIS DATA

by Kirill Pogorelskiy*, Christian Seidl, and Stefan Traub‡**

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Abstract

The conventional approach to comparing tax progression (using local measures, global measures or dominance relations for first moment distribution functions) often lacks applicability to the real world: local measures of tax progression have the disadvantage of ignoring the income distribution entirely. Global measures are affected by the drawback of all aggregation, viz. ignoring structural differences between the objects to be compared. Dominance relations of comparing tax progression depend heavily on the assumption that the same income distribution holds for both situations to be compared, which renders this approach impossible for international and intertemporal comparisons.

Based on the earlier work of one of the authors, this paper develops a unified methodology to compare tax progression for dominance relations under different income distributions. We address it as *uniform tax progression for different income distributions* and present the respective approach for both continuous and discrete cases, the latter also being employed for empirical investigations.

Using dominance relations, we define tax progression under different income distributions as a class of natural extensions of uniform tax progression in terms of taxes, net incomes, and differences of first moment distribution functions. To cope with different monetary units and different supports of the income distributions involved, we utilized their transformations to population and income quantiles. Altogether, we applied six methods of comparing tax progression, three in terms of taxes and three in terms of net incomes, which we utilized for empirical analyses of comparisons of tax progression using data from the Luxembourg Income Study. This is the first paper that performs international and intertemporal comparisons of *uniform* tax progression with actual data.

For our analysis we chose those countries for which LIS disposes of data on gross incomes, taxes, payroll taxes and net incomes. This pertains to 15 countries, out of which we selected 13. This gave rise to 78 international comparisons, which we carried out for household data, equivalized data, direct taxes and direct taxes inclusive of payroll taxes. In total we investigated 312 international comparisons for each of the six methods of comparing tax progression.

In two thirds of all cases we observed uniformly greater tax progression for international comparisons. In a bit more than one fifth of all cases we observed bifurcate tax progression, that is, progression is higher for one country up to some population or income quantile threshold, beyond which the situation is the opposite, i.e., progression is higher for the second country. No clear-cut findings can be reported for just one tenth of all cases. But even in these cases some curve differences are so small that they may well be ignored.

We also test consistency of our results with regard to the six methods of comparing tax progression and present here twelve (Germany, the UK and the US) plus four comparing Germany and Sweden out of the total of 312 graphs, each containing six differences of first moment distribution functions. These differences can be interpreted as intensity of greater tax progression. We demonstrate the overall picture of uniform tax progression for international comparisons using Hasse diagrams.

Concerning intertemporal comparisons of tax progression, we present the results for the US, the UK, and Germany for several time periods. We align our findings with respect to major political eras in these countries, e.g., G. Bush senior, W. Clinton, and G. Bush junior for the United States; M. Thatcher, J. Major, and A. Blair for the United Kingdom, and for Germany, the last year before German re-unification (1989), the beginning of H. Kohl's last term as chancellor (1994), and G. Schröder (2000). In addition, we study sensitivity of our results to the equivalence scale parameter.

JEL Classification: H23, H24.
Keywords: income tax progression, measurement of uniform tax progression, comparisons of tax progression, tax progression with different income distributions.

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1 Introduction

Tax progression has ever been of concern, not only to the profession, but also to politicians, let alone to the man on the street. Hence, measurement of tax progression and international as well as intertemporal comparisons of tax progression are of utmost importance. Intuitively, by tax progression we mean a situation when, as income increases, so does the average tax rate, i.e., the higher income strata pay a relatively larger share of their gross income than the lower income strata do. If we take the tax system as a whole, then in addition to the tax schedule we should also take into account the influence of the existing income distribution in order to be able to draw sound conclusions about its real progression. Thus, we agree with Suits (1977, p. 725): "There is nothing inherently regressive about a sales tax or even a poll tax. They are regressive because income is unequally distributed, and the more unequally income is distributed, the more regressive they become."

In contrast to that, the existing methodology of measuring and comparing tax progression allows only answers to problems which are outside central interest. There are three main routes of research, viz. local, global, and uniform measures of tax progression. Local measures of tax progression, in particular its main representatives, tax revenue elasticity and residual income elasticity, concentrate on the tax schedule only and neglect the important role of the income distribution for tax progression. If a certain tax schedule happens to be rather progressive but hits very few people only, then the respective tax system should not be viewed as highly progressive. Global measures of tax progression weigh taxation or the net incomes by the income distribution in addition to some other weights, but this very aggregation procedure is its main drawback. A tax schedule which is regressive over some income intervals may be categorized to be more progressive than another tax schedule which is progressive throughout just because of compensation due to the aggregation procedure. Uniform measures of tax progression which work by way of single-crossing conditions or by relative concentration curves require the same income distributions for all cases to be compared. This means that questions such as "Is the tax schedule of the USA associated with the American income distribution more or less progressive than the German tax schedule associated with the German income distribution?" cannot be answered by using this approach.

To handle these realistic problems, Seidl (1994) developed an approach in which comparisons are based on population quantiles or income quantiles with respect to taxes or net incomes rather than on tax schedules directly in terms of income. This method allows to substitute the income distributions with different supports by quantiles with the unit interval as the common support of different distributions. The idea of this approach is the following: if a tax schedule for one country collects relatively less tax revenue from the lower income strata than does a tax schedule of another country, then the first one is considered as more progressive. Alternatively, if the first tax schedule leaves the lower income strata relatively more net income than does the tax schedule of another country,

then the first tax schedule is considered as more progressive. The comparison of these relative positions is carried out in terms of the tax quantiles for the population quantiles or for the income quantiles.[1] In his theoretical work, Seidl (1994) made use of concavity or convexity conditions of relative concentration curves, which, however, yielded only sufficient conditions in terms of elasticities, but not necessary conditions of uniformly more or less tax progression. It seems that no general analytic solution to this problem exists.[2]

The main purpose of this paper is an empirical investigation of international and intertemporal comparisons of tax progression utilizing this approach. We used data from the Luxembourg Income Study, LIS (2010), for 13 out of 15 countries for which data for gross incomes, direct taxes, payroll taxes and net incomes are available (see Table 1 in Section 4.1). We made separate comparisons for household incomes and for equivalized incomes (using the Luxembourg equivalence scale) and for progression of direct taxes and direct taxes plus payroll taxes (mainly comprising the employees' share of social security contributions). This gave us four times 78 international comparisons. Moreover, we applied six measurement devices for comparisons of tax progression, four in terms of population quantiles and two in terms of income quantiles. In addition to that, we also investigated intertemporal comparisons of tax progression for some selected countries and studied the influence of the scale parameter of the Luxembourg equivalence scales on the results of comparisons of tax progression.

Section 2 reviews local and global measures of tax progression, Section 3 deals with uniform tax progression, first for identical income distributions (or, more generally, for income distributions with the same support), which represents the customary theory, second for different income distributions in the continuous version, and third for different income distributions in the discrete version in preparation for empirical investigations. Section 4 displays the results of our research. This section section starts with a description of LIS data and the procedures we applied to them; it continues by providing some intuition about working with grouped data, and then proceeds with an elaborate discussion of the results of our research. Except for one table and two figures, all tables and figures are placed at the end of the paper.

[1] Dardanoni and Lambert (2002) took a different route: rather than replacing incomes by quantiles, they transplanted the income distributions by *deformation functions* (p. 102), thereby remaining in the domain of incomes. For instance, when comparing the United States with Germany, they propose deforming the German income distribution to the American income distribution and look whether the American tax system, when applied to the deformed compound distribution, is more or less progressive than the American tax system as applied to the American income distribution. Alternatively, the American income distribution can be deformed to the German income distribution, or both distributions can be deformed to a fictive third income distribution.

[2] Note that by a "general analytic solution" we mean the one that yields non-trivial necessary and sufficient conditions. For instance, it is immediate that there is uniformly greater progression if and only if the curve obtained by taking the differences of the transformed first moment curves of taxes or net incomes does not change its sign within the unit interval. This condition, however, is a mere reformulation of the definition of uniform tax progression (see Section 3.3).

2 Local and Global Measures of Tax Progression

This paper employs the following notation: Y denotes income, $[Y_*, Y^*]$ denotes the support of the income distribution, $f(Y) \geq 0$ denotes the density function and $F(Y)$ the distribution function of the income distribution, q denotes the population quantiles and p the income quantiles of gross incomes, $\mu := \int_{Y_*}^{Y^*} Y f(Y) dY$ denotes mean gross income, $T(Y)$ denotes the income tax schedule, $\frac{T(Y)}{Y}$ denotes the average income tax schedule, $\frac{dT(Y)}{dY}$ denotes the marginal income tax schedule,[3] and $\tau := \int_{Y_*}^{Y^*} T(Y) f(Y) dY$ denotes mean tax.

Local measures of income tax progression just focus on the tax schedule. The more primitive ones are the first derivative of the average tax schedule and the difference of the marginal and the average tax schedules. They are positive for progressive and negative for regressive tax schedules. More refined measures are the *tax elasticity*

$$\varepsilon(Y) := \frac{dT(Y)/dY}{T(Y)/Y}$$

and the *residual income elasticity*

$$\eta(Y) := \frac{d[Y - T(Y)]/dY}{[Y - T(Y)]/Y}.$$

Verbally expressed, the tax elasticity is the ratio of marginal and average tax rates, and the residual income elasticity is the ratio of the marginal and the average retention rates. $\varepsilon(y)$ measures *liability progression*, $\eta(y)$ *residual income progression*. According to liability progression, a tax schedule is progressive at $\tilde{Y}$ if $\varepsilon(\tilde{Y}) > 1$; according to residual income progression a tax schedule is progressive at $\tilde{Y}$ if $0 < \eta(\tilde{Y}) < 1$. The meaning of these two local measures of tax progression is simple: $\varepsilon(\tilde{Y}) > 1$ means that the tax on an extra monetary unit for a taxpayer with income $\tilde{Y}$ exceeds his or her average tax burden; $0 < \eta(\tilde{Y}) < 1$ means that an extra monetary unit leaves a taxpayer less net income than under his or her average retention rate. Note that both measures are equivalent for the general diagnosis of tax progression, that is, we have $\varepsilon(\tilde{Y}) > 1 \Leftrightarrow 0 < \eta(\tilde{Y}) < 1$, but this equivalence does not apply to comparisons of tax progression. This means that for two tax schedules $T^1(Y)$ and $T^2(Y)$ it does *not follow* that $\varepsilon^1(\tilde{Y}) > \varepsilon^2(\tilde{Y})$ holds if and only if $\eta^1(\tilde{Y}) < \eta^2(\tilde{Y})$ holds. For a numerical illustration on a former German income tax reform see Seidl and Kaletha (1987).

Local measures of tax progression have a crucial drawback: they are completely separated from income distributions. Hence, the fractions of people affected by the various parts of a tax schedule are neglected by local measures of tax progression. Yet for comparing two situations with respect to tax progression on the whole, the fractions of the population affected by the various parts of a tax schedule are important. Suppose that a

[3] For the sake of mathematical convenience we assume that all tax schedules are continuously differentiable.

tax schedule is very progressive, yet nobody in a society is affected by the very high rates of this tax schedule. Then this tax schedule will be perceived as less progressive than a tax schedule with more moderate rates which, however, cut in broad strata of taxpayers.

For an arbitrary income distribution (but which is assumed to be the same for both schedules under comparison) we can employ local measures of tax progression for purposes of progression comparison if we have dominance relationships throughout, e.g., $\varepsilon^1(Y) > \varepsilon^2(Y)$ or $\eta^1(Y) < \eta^2(Y)$ for all $Y \in [Y_*, Y^*]$. If these relationships apply, then greater tax progression of tax schedules holds trivially for any such income distribution on which both tax schedules operate, and is therefore independent of the choice of the income distribution. We will see below that similar relations represent sufficient conditions for greater tax progression for uniform tax progression. Note that they may, although more complicated, also be expressed in terms of q and p; we will come to these expressions below.

The introduction of income distributions into comparisons of tax progression can be done in two ways: the first one takes the route of aggregate measures which map taxes and incomes into the real numbers—these are *global measures of tax progression*; the second one uses dominance relations—these are *measures of uniform tax progression*.

Global measures of tax progression are based on income distribution measures of gross incomes, net incomes, and taxes.[4] In the simplest cases the Gini coefficient is used. Examples include, inter alia, the measure proposed by Dalton (1922/1954, pp. 107-8) as the mean deviation of average tax rates (which is just the Gini coefficient of the average tax rates), or the measure proposed by Reynolds and Smolensky (1977), which is simply the difference of the Gini coefficients of net and gross incomes. Pechman and Okner (1974) and Okner (1975) proposed to normalize the Reynolds-Smolensky measure by the Gini coefficient of gross incomes. The Musgrave and Thin (1948) measure of effective progression is the ratio of the areas under the Lorenz curves for gross and net incomes.[5] Many more global measures of tax progression were developed, e.g., by Hainsworth (1984), Khetan and Poddar (1976), Suits (1977), Kakwani (1977b, 1984, 1987), Formby et al. (1981, 1984), Pfähler (1982, 1983, 1987), Liu (1984), and Lambert (1988). Blackorby and Donaldson (1984) and Kiefer (1984, pp. 500-1) chose another way: they proposed global measures of tax progression based on the equally distributed equivalent income.

Pfähler (1987, p. 7), suggested a general framework for global measures of tax progression. He showed that most of the measures based on distributional measures can

[4]For short surveys see Kiefer (1984, pp. 498-9), Seidl (1994, pp. 343-6), and Peichl and Schäfer (2008, pp. 3-5). Kiefer (1984, p. 497) distinguished two groups of global measures of tax progression, viz. *structural indices*, which are functions of incomes and their respective taxes, and *distributional indices*, which are functions of the tax structure and the distribution of post-tax incomes. If this classification is extended to measures of uniform tax progression, then our Definitions 1, 2, and 5 in Section 3.3 are structural measures of tax progression, and Definitions 3, 4, and 6 are distributional measures of tax progression.

[5]The area under the Lorenz curve is one minus the Gini coefficient divided by two.

be expressed as the weighted sum of local relative deviations $[T(Y) - \frac{\tau}{\mu}Y]/\tau$ of the actual tax schedule from a revenue-neutral proportional tax schedule. For the expression of the global progression measures in terms of differences between the distributions of net incomes and gross incomes, Pfähler (1987, p. 12) showed that they can be defined using the very same weights for the weighted sum of local relative deviations $[Y - T(Y) - (1 - \frac{\tau}{\mu})Y]/(\mu - \tau)$ of actual net incomes from revenue-neutral net incomes under a proportional tax.[6]

Global measures of tax progression not only serve to categorize tax schedules into progressive, proportional and regressive, but also to derive an *ordering* of tax progression. If progression is measured in terms of positive (negative) values, then tax schedule $T^1(\cdot)$ is *more progressive* than $T^2(\cdot)$ if the global measure applied shows a higher (lower) value for $T^1(\cdot)$ than for $T^2(\cdot)$.

Global measures of tax progression have several advantages. First, they work for different tax schedules and different income distributions. This means that international and intertemporal comparisons of tax progression can be effectuated. Second, they feature a double weighting, both by some weights particular to the specific global measure, and by the income distribution. That is, particular characteristics of a tax schedule gain more (less) weight if more (less) taxpayers are affected. Third, global measures of tax progression are able to compensate income subintervals with opposite properties of tax schedules by appropriate weighting and subsequent aggregation.

However, at the same time this last advantage turns out as a major handicap of global measures of tax progression. Aggregating the effects of tax schedules over the whole support of the income distribution may lead to the result that $T^1(\cdot)$ is categorized as being more progressive than $T^2(\cdot)$, although $T^1(\cdot)$ has a decreasing average tax rate throughout some subinterval of the income support, while $T^2(\cdot)$'s average tax schedule is increasing throughout the whole income support. Alternatively, suppose that a tax schedule is progressive for the lower incomes and regressive for the upper incomes. This may lead to a Lorenz curve of gross incomes which intersects the Lorenz curve of net incomes. In this case we cannot exclude that the two Gini coefficients have the same value (or that their difference is very small), which would indicate a proportional (or close to proportional) tax schedule under some measures of tax progression, although this tax schedule is far from being proportional (see also Suits (1977, p. 752) for a critique of global measures of tax progression). The second handicap of global measures of tax progression is rooted in their aggregation procedure, which presupposes comparability of the tax burden across all income strata. This is much related to the assumption of interpersonal comparability of utility. These handicaps led to the development of uniform measures of tax progression which we consider in the next section.

[6]Using appropriate weighting functions, this approach encompasses many global measures of tax progression, e.g., as proposed by Musgrave and Thin (1948), Hainsworth (1984), Khetan and Poddar (1976), Suits (1977), Kakwani (1977b, 1984, 1987), Pfähler (1987), and Lambert (1988).

3 Uniform Tax Progression

Uniform tax progression adopts yet another concept of progression comparisons. Again, it can be formulated in terms of taxes or in terms of net incomes. For the presentation in this paper we shall stick to their original formulations.

3.1 Uniform Tax Progression for Income Distributions with the Same Support

The main work on uniform tax progression was done under the assumption of identical income distributions for the two tax schedules to be compared. It is immediate that the respective analyses extend to income distributions with the same support.

Jakobsson (1976, p. 165) used elasticity properties of tax schedules to characterize more progressive tax schedules. His theorem purports that if $T^1(\cdot)$ is more progressive than $T^2(\cdot)$ for all possible income distributions with the same support $[Y_*, Y^*]$, then $\eta^1(Y) \leq \eta^2(Y)$ for all $Y \in [Y_*, Y^*]$ and $\eta^1(Y) < \eta^2(Y)$ for a nonempty subinterval of $[Y_*, Y^*]$ (the necessary condition). On the other hand, if for a particular pair of income distributions with the same support $[Y_*, Y^*]$, $\eta^1(Y) \leq \eta^2(Y)$ for all $Y \in [Y_*, Y^*]$ and $\eta^1(Y) < \eta^2(Y)$ for a nonempty subinterval of $[Y_*, Y^*]$, then $T^1(\cdot)$ is more progressive than $T^2(\cdot)$ (the sufficient condition). The key difference between the necessary and the sufficient conditions of greater tax progression lies in the fact that the latter can be applied to some given pair of income distributions, while the former is applicable only if all possible income distributions are considered.

As concerns the proof, the sufficiency part of this theorem is obvious; it follows immediately from the properties of residual income progression (see its definition in Section 2). For the necessity part of this theorem, Jakobsson (1976, p. 165) considered the case that $\eta^1(Y) \leq \eta^2(Y)$ holds generally, except for an income subinterval for which $\eta^1(Y) > \eta^2(Y)$ holds. Then $T^1(\cdot)$ cannot be more progressive than $T^2(\cdot)$ because "we could always choose an income distribution before tax that lies completely within the latter interval." For this latter interval, the tax schedule $T^2(\cdot)$ is more progressive than $T^1(\cdot)$.

Notice that if $\eta^1(Y) < \eta^2(Y)$ holds for all $Y \in [Y_*, Y^*]$, then $T^1(\cdot)$ generates systematically lower net incomes than $T^2(\cdot)$ if the same income distribution holds for both tax schedules. This means that $T^1(\cdot)$ raises more revenue than $T^2(\cdot)$. In other words, Jakobsson's theorem is inconsistent with the assumption that $T^1(\cdot)$ and $T^2(\cdot)$ can raise the same revenue. $T^1(\cdot)$, it is true, causes a more equal distribution of net incomes than $T^2(\cdot)$, however bought at the price of a higher tax burden for all. The lower income strata are only left with the satisfaction that the upper income strata are pinched relatively more under the tax schedule $T^1(\cdot)$.

Kakwani (1977a), too, relied on elasticities for progression comparisons, but took another route. His point of departure are the first-moment distribution functions of

taxes and net incomes:

$$F_T(Y) := \frac{1}{\tau}\int_{Y_*}^{Y} T(y)f(y)dy; \tag{1}$$

$$F_{Y-T}(Y) := \frac{1}{\mu-\tau}\int_{Y_*}^{Y} [y-T(y)]f(y)dy. \tag{2}$$

These functions indicate the share of total tax revenue (total net income) paid (received) by the income recipients with gross incomes less or equal to Y.

For two tax schedules Kakwani (1977a) employed the concentration curve of $F_{T^1}(Y)$ relative to $F_{T^2}(Y)$, and analogously for the net incomes. This relative concentration curve is defined on the unit square [note that the range of both $F_T(Y)$ and $F_{Y-T}(Y)$ is the unit interval], where $F_{T^1}(Y)$ is depicted on the ordinate and $F_{T^2}(Y)$ on the abscissa. If this relative concentration curve lies below the diagonal, then, except at the endpoints Y_* and Y^*, $F_{T^1}(Y)$ collects for all income levels Y a lower share of tax revenue than does $F_{T^2}(Y)$. Hence, $F_{T^1}(\cdot)$ is more progressive than $F_{T^2}(\cdot)$.

The second derivative of a strictly convex (concave) function is positive (negative). For a positive second derivative of the relative concentration curve of taxes $\varepsilon^1(Y) \geq \varepsilon^2(Y)$ holds, for a negative second derivative of the relative concentration curve of net incomes $\eta^1(Y) \leq \eta^2(Y)$ holds. Alas, these conditions are *sufficient* conditions for greater tax progression, but *not necessary* conditions. This results from Kakwani's confinement to a particular income distribution rather than to the universe of all income distributions. A particular income distribution and two tax schedules may yield a relative concentration curve lying wholly below the diagonal, although $\varepsilon^1(Y) < \varepsilon^2(Y)$ holds for some subinterval of the support of the income distribution. This applies *mutatis mutandis* also to net incomes. After all, this case seems to us the more realistic one because one wants to compare tax schedules not with respect to the universe of income distributions, but for an empirically given pair of income distributions.

A third approach developed by Hemming and Keen (1983) relies on *single crossing conditions*. According to their findings, $T^1(\cdot)$ is more progressive than $T^2(\cdot)$ if the net income function $Y - T^1(Y)$ crosses the net income function resulting from $T^2(\cdot)$, viz. $Y - T^2(Y)$, once from below, say, at $\tilde{Y}$. In their proof, they start demonstrating that this holds for tax schedules raising the same revenue. The intuition behind this proposition is clear: tax progression means that the lower income strata pay relatively less than the upper income strata under $T^1(\cdot)$ than under $T^2(\cdot)$. In other words, the lower income strata have relatively more net income than the upper income strata under $T^1(\cdot)$ than under $T^2(\cdot)$. By the assumption of revenue-neutral tax schedules, this translates into absolute figures. Due to revenue neutrality, the upper income strata have to pay exactly the same amount of tax more under $T^1(\cdot)$ than under $T^2(\cdot)$, as the lower income strata pay more under $T^2(\cdot)$ than under $T^1(\cdot)$. If $T^1(\cdot)$ and $T^2(\cdot)$ are not revenue neutral, then

the two cases have to be normalized and the argument translates into relative figures. This is Hemming and Keen's (1983) second proposition.

Obviously, this constitutes a sufficient condition of greater tax progression. The necessary part of the proof comes from Hemming and Keen's (1983) requirement that it should hold for *all* income distributions. Suppose, for instance, that there are two crossings: let $Y - T^1(Y)$ cross $Y - T^2(Y)$ at $\tilde{Y} > Y_*$ from below and at $\bar{Y}$, $Y^* > \bar{Y} > \tilde{Y}$, from above. Consider now $\hat{Y}$, $\bar{Y} > \hat{Y} > \tilde{Y}$. Then there exist income distributions such that $T^1(\cdot)$ and $T^2(\cdot)$ raise the same revenue for $Y \in [Y_*, \hat{Y})$ and raise the same revenue (possibly different from the first interval) for $Y \in [\hat{Y}, Y^*]$. Then $T^1(\cdot)$ is more progressive than $T^2(\cdot)$ on the income interval $[Y_*, \hat{Y})$ and less progressive on $[\hat{Y}, Y^*]$. Hence, the necessary part of the proof requires that the single-crossing condition should hold for the universe of income distributions.

The relationship between Kakwani's (1977a) elasticity condition and Hemming and Keen's (1983) single-crossing condition is the following: obviously

$$\ln \frac{Y - T^1(Y)}{Y - T^2(Y)} = \ln[Y - T^1(Y)] - \ln[Y - T^2(Y)]. \tag{3}$$

Differentiating this with respect to Y, multiplying the right-hand side by $\frac{Y}{Y}$, re-arranging and using the definition for $\eta(Y)$ yields

$$\frac{d}{dY}\{\ln[Y - T^1(Y)] - \ln[Y - T^2(Y)]\} = \frac{\eta^1(Y) - \eta^2(Y)}{Y}. \tag{4}$$

Applying an exponential transformation on Equation (3) and substituting the integral of Equation (4) yields

$$\frac{Y - T^1(Y)}{Y - T^2(Y)} = e^{\int_{\tilde{Y}}^{Y} \frac{\eta^1(y) - \eta^2(y)}{y} dy} \tag{5}$$

for $\tilde{Y} \leq Y \leq Y^*$, where $\tilde{Y}$ denotes the income at which the net income curves cross.

When $\eta^1(Y) \leq \eta^2(Y)$ for all $Y > \tilde{Y}$ and the inequality sign is strict for some nonempty interval of $(\tilde{Y}, Y^*]$, then $[Y - T^1(Y)] < [Y - T^2(Y)]$ for all $y \in (\tilde{Y}, Y^*]$. In other words, the single-crossing condition holds. Hence, the condition $\eta^1(Y) \leq \eta^2(Y)$ for all $Y \in [Y_*, Y^*]$ is sufficient for the single-crossing condition to hold. Conversely, when the single-crossing condition holds, this does not imply that $\eta^1(Y) \leq \eta^2(Y)$ for all $Y \in [Y_*, Y^*]$. $\eta^1(Y) > \eta^2(Y)$ may well hold for a subinterval of $[Y_*, \tilde{Y})$, or for a subinterval of $(\tilde{Y}, Y^*]$, while leaving the integral term in equation (5) negative.

Uniform measures of tax progression for identical income distributions, or, more generally, for income distributions with the same support, have several drawbacks. First, by definition, they are only applicable for comparing tax schedules in situations with the same income distributions or at least the same support of the income distributions. Hence, they cannot be used for international or intertemporal comparisons of tax progression which are typically associated with different income distributions and different

supports. Second, uniform measures of tax progression establish just sufficient conditions of greater tax progression if considered not for all possible income distributions, but for particular ones.[7] It is, in particular, the first drawback which suggests an extension to comparisons of progression for tax schedules associated with different income distributions having different supports.

3.2 Uniform Tax Progression for Different Income Distributions: Continuous Version

The analysis of comparisons of uniform tax progression with different income distributions can be performed in terms of relative concentration curves of first moment distribution functions or in terms of first- or second-order differences of first moment distribution functions. Note, at the outset, that first moment distribution function of the shape (1) or (2) are inappropriate for comparisons of tax progression of different tax schedules associated with different income distributions. The reason is that this analysis holds only if both income distributions have equal support, which is extremely unlikely. For two functions of type (1) unequal support means that $1 = F_{T^1}(\tilde{Y}) > F_{T^2}(\tilde{Y})$, where $\tilde{Y}$ is equal to the maximum income Y^{1*} for the first income distribution, but smaller than the maximum income Y^{2*} for the second income distribution. This means that a relative concentration curve starts at the point (0,0), but does not reach the point (1,1). Thus, it is a degenerate relative concentration curve which cannot be used for comparisons of tax progression.

Instead, we have to apply transformations from the income distributions on $[Y_{1*}, Y^{1*}]$ and $[Y_{2*}, Y^{2*}]$, respectively, onto the unit interval. Several methods are available. Two of them stand out, viz. the expression in terms of population quantiles $q = F(Y)$, and in terms of income quantiles $p = F_Y(Y) = \frac{1}{\mu}\int_{Y_*}^{Y} yf(y)dy$. q indicates the fraction of the persons in the lower income strata with maximum income Y; p indicates the fraction of the aggregate income of the lower income strata with maximum income Y.[8] Obviously $F(Y) > F_Y(Y)$ because every person with income less or equal to Y is counted by $F(Y)$ with the same population weight, whereas $F_Y(Y)$ counts the smallest incomes up to Y and expresses their aggregate as a fraction of total income, because smaller incomes contribute less weight. Conversely, $F^{-1}(q) < F_Y^{-1}(p)$ for $q = p$ because the $(q \times 100)$ percent lowest income earners have a lower maximum income than the maximum income of the $(q \times 100) = (p \times 100)$ percent of aggregate income.

A simple transformation of variables $Y = F^{-1}(q)$ applied to the first moment distri-

[7] On the other hand, Kakwani's (1977a) sufficiency conditions extend to necessary conditions if they should apply to the universe of income distributions.

[8] Note the this approach has also the advantage that the effects of inflation for intertemporal comparisons of tax progression are normalized by population or income shares. At the same time, different currencies are also calibrated and unified by this approach, which renders international comparisons of tax progression viable.

bution function of incomes, $F_Y(Y) = \frac{1}{\mu}\int_{Y_*}^{Y} yf(y)dy$, and to the first moment distribution functions (1) and (2) gives us

$$F_Y(q) = \frac{1}{\mu}\int_0^q F^{-1}(\tilde{q})d\tilde{q}; \tag{6}$$

$$F_T(q) = \frac{1}{\tau}\int_0^q T[F^{-1}(\tilde{q})]d\tilde{q}; \tag{7}$$

$$F_{Y-T}(q) = \frac{1}{\mu-\tau}\int_0^q \{F^{-1}(\tilde{q}) - T[F^{-1}(\tilde{q})]\}d\tilde{q}. \tag{8}$$

A transformation of variables $Y = F_Y^{-1}(p)$ for (1) and (2) gives us[9]

$$F_T^Y(p) = \frac{\mu}{\tau}\int_0^p \frac{T[F_Y^{-1}(\tilde{p})]}{F_Y^{-1}(\tilde{p})}d\tilde{p}, \tag{9}$$

$$F_{Y-T}^Y(p) = \frac{\mu}{\mu-\tau}\int_0^p \frac{\{F_Y^{-1}(\tilde{p}) - T[F_Y^{-1}(\tilde{p})]\}}{F_Y^{-1}(\tilde{p})}d\tilde{p}, \tag{10}$$

where the expression under the integral in equation (9) is the average tax schedule, and in equation (10) it is the average retention schedule.

Note that $F_Y(q)$ is the Lorenz curve of the gross income distribution, $F_T(q)$ is the Lorenz curve of the tax distribution, and $F_{Y-T}(q)$ is the Lorenz curve of the net income distribution.[10] $F_T^Y(p)$ denotes the fraction of total tax revenue paid by all taxpayers whose aggregate gross income amounts to the fraction p of total gross income. $F_{Y-T}^Y(p)$ denotes the fraction of aggregate net income of all taxpayers whose aggregate gross income amounts to the fraction p of total gross income. Notice the difference between $F_T(q)$ and $F_T^Y(p)$: $F_T(q)$ denotes the share of total tax revenue paid by the fraction q of the poorest taxpayers, whereas $F_T^Y(p)$ denotes the share of total tax revenue paid by the poorest taxpayers whose compound gross income is a fraction p of total gross income. This means that we have $F_T(q) < F_T^Y(p)$ for any given unequal income distribution and for any strictly increasing average tax schedule $T(Y)/Y$, if $1 > q = p > 0$, because the fraction q of the poorest taxpayers holds only a fraction of total gross income, $F_Y(q)$, which is smaller than $F_Y^Y(p) = p$ for $q = p$.

Next, we define uniformly greater progression of tax schedules associated with their respective income distributions. This can be done in at least two ways. Firstly, a tax

[9]Equations (9) and (14) (the latter for the discrete case) are the so-called Suits (1977) curves. Suits used the tax curve to construct a global measure of tax progression similarly to the Gini coefficient for measuring income inequality. Note that $F_Y^Y(p)$ is just the diagonal through the unit square. Therefore, this formulation is omitted.

[10]This means that we assume away re-ranking in our theoretical analysis. We will come back to this point in the next section.

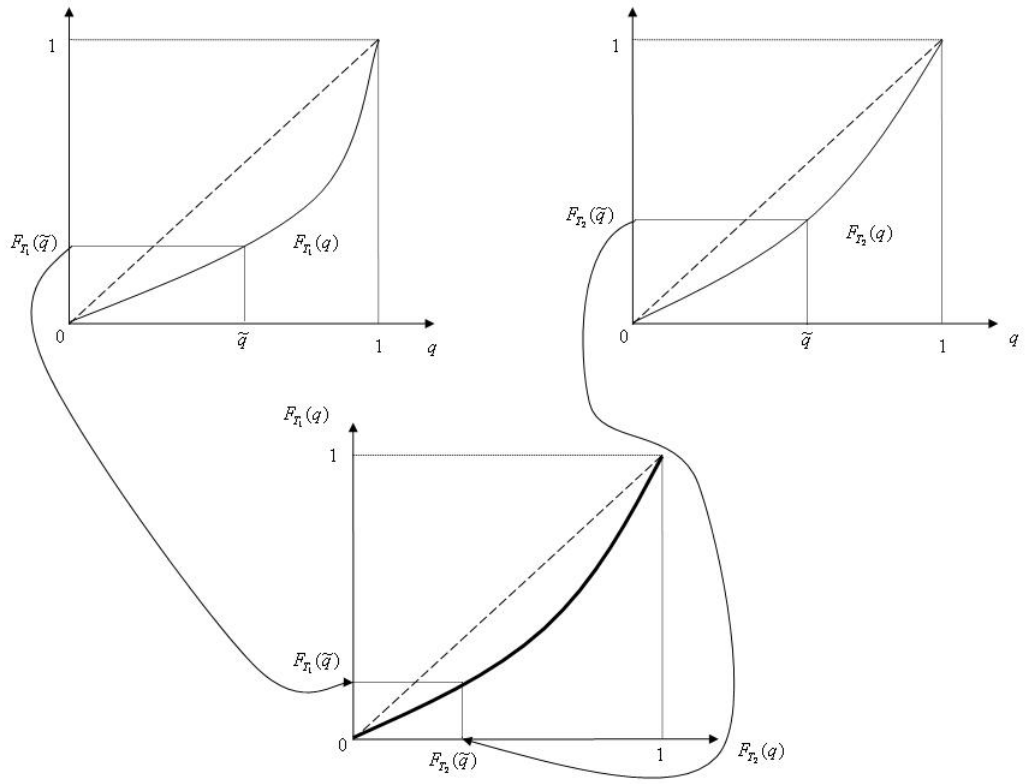

Figure 1: Construction of a relative concentration curve from Lorenz curves for taxes

schedule T^1 can be defined to be uniformly more progressive than T^2 whenever the concentration curve of F_{T^1} relative to F_{T^2} does not cross the diagonal of the unit square except at the endpoints (0,0) and (1,1). To illustrate, we focus on the case of entering F_{T^1} on the ordinate and F_{T^2} on the abscissa, and on the half-space of the unit square below the diagonal. Other arrangements are immediate. This means that for the same fractions q or p as applied to the two income distributions,[11] T^1 collects a smaller fraction of aggregate taxes from smaller incomes than does T^2. A sufficient condition for the concentration curve of F_{T^1} relative to F_{T^2} to lie wholly below the diagonal of the unit square is that it is strictly convex.[12] Figure 1 illustrates this case in terms of q.

Alas, the convexity (or concavity) condition is not a necessary condition for greater tax progression. When comparing two situations, then there may well occur cases such that convexity or concavity of the relative concentration curve is violated without its crossing the diagonal within the unit square. Figure 2 illustrates.

[11] It can also be performed in terms of Y but this would require that both income distributions involved have equal support; see Seidl (1994, pp. 347-9).

[12] Equivalently, one can require that the slope of a relative concentration curve be less than one below a unique value of its argument and greater than one thereafter. Whereas this is equivalent to strict convexity for the case of relative concentration curves, strict convexity produces the more intuitive and precise formulations of the sufficient conditions.

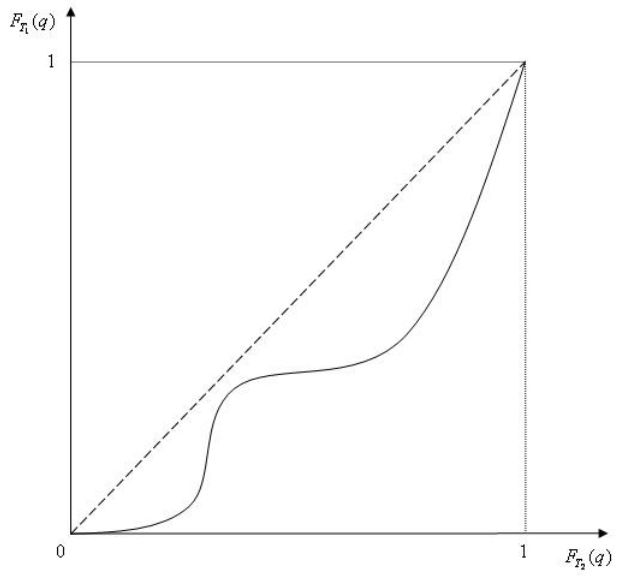

Figure 2: (Y_1, T_1) more progressive than (Y_2, T_2) with nonconvex relative concentration curve

Alternatively, we may define a tax schedule T^1 to be uniformly more progressive than T^2 whenever the concentration curve of $F_{Y^1-T^1}$ (ordinate) relative to $F_{Y^2-T^2}$ (abscissa) lies wholly above the diagonal of the unit square except at the endpoints of the support of the income distribution. This means that T^1 leaves the taxpayers a larger fraction of aggregate net incomes for lower incomes than does T^2. Again the analysis can be performed in terms of q or p. A sufficient condition for the concentration curve of $F_{Y^1-T^1}$ relative to $F_{Y^2-T^2}$ to lie wholly above the diagonal of the unit square is its strict concavity. Negative taxes are excluded from this analysis.

It is readily seen from Figure 1 that, instead of working with relative concentration curves, we can use the differences of the respective first moment distribution functions. To illustrate, we consider the tax case only; the extension to the net income case is immediate. Suppose the relative concentration curve (with F_{T^1} on the ordinate and F_{T^2} on the abscissa) is strictly convex. Note that a concentration curve of $F_{T^1}(\cdot)$ relative to $F_{T^2}(\cdot)$ does not cross the diagonal iff $F_{T^1}(\cdot) - F_{T^2}(\cdot)$ has the same sign for all $q, p \in (0, 1)$. Then the difference $F_{T^1} - F_{T^2}$ is negative with a unique minimum. If the relative concentration curve is not convex, but, as in Figure 2, below the diagonal, then the difference of the curves is negative with multiple minima. If the relative concentration curve crosses the diagonal, the curve differences will be partly negative, partly positive. For a concave

relative concentration curve, the curve differences are positive, single peaked for a strictly concave relative concentration curve, and multiple peaked for a non-concave relative concentration curve which is above the diagonal. Hence, the equivalence between relative concentration curves and first-order curve differences is obvious. We shall see that working with curve differences is the more appropriate method for analyzing empirical data.

Uniformly greater tax progression can also be defined in terms of second-order differences of first moment distribution functions.[13] T^1 is then defined to be uniformly more progressive than T^2 whenever $F_{Y^1} - F_{T^1} > F_{Y^2} - F_{T^2}$ holds for the whole support. This second notion of uniformly greater tax progression indicates that the difference between the first moment distribution curves, which is due to the influence of taxation, is greater for the income-distribution-cum-tax-schedule (Y^1, T^1) than for (Y^2, T^2). The corresponding condition in terms of net incomes can be written as $F_{Y^1-T^1} - F_{Y^1} > F_{Y^2-T^2} - F_{Y^2}$, which means that the difference between the distribution of net incomes and gross incomes is greater for (Y^1, T^1) than for (Y^2, T^2). Therefore, taxation has caused greater equality of net incomes for (Y^1, T^1) as compared with (Y^2, T^2), which is taken as a proxy for greater uniform progression of taxation according to this definition.

Theorem 1 *Assume* $0 < T^i[F_i^{-1}(q)] < F_i^{-1}(q)\ \forall q\ \in (0,1), i = 1,2$, *where* $F_{T^1}(q)$ *is placed on the ordinate and* $F_{T^2}(q)$ *on the abscissa of the relative concentration curve. Then* (Y^1, T^1) *is more progressive than* (Y^2, T^2) *if* $\varepsilon^1(q) > \varepsilon^2(q)\ \forall q \in (0,1)$, *where*

$$\varepsilon^i(q) := \frac{\frac{dT^i[F_i^{-1}(q)]}{dF_i^{-1}(q)} \frac{dF_i^{-1}(q)}{dq}}{T^i[F_i^{-1}(q)]} q,\ i = 1,2.$$

A necessary and sufficient condition is given by the definition of uniformly greater tax progression, applied to (Y^1, T^1) *and* (Y^2, T^2)*:* $F_{T^1}(q) \leq F_{T^2}(q)\ \forall q \in [0,1]$.

In terms of net incomes let $F_{Y^1-T^1}(q)$ *be placed on the ordinate and* $F_{Y^2-T^2}(q)$ *on the abscissa of the relative concentration curve. Then* (Y^1, T^1) *is more progressive than* (Y^2, T^2) *if* $\eta^1(q) < \eta^2(q)\ \forall q \in (0,1)$, *where*

$$\eta^i(q) := \frac{\left[1 - \frac{dT^i[F_i^{-1}(q)]}{dF_i^{-1}(q)}\right] \frac{dF_i^{-1}(q)}{dq}}{F_i^{-1}(q) - T^i[F_i^{-1}(q)]} q,\ i = 1,2.$$

A necessary and sufficient condition is given by the definition of uniformly greater tax progression, applied to (Y^1, T^1) *and* (Y^2, T^2)*:* $F_{Y^1-T^1}(q) \geq F_{Y^2-T^2}(q)\ \forall q\ \in [0,1]$.

The proof of Theorem 1 is easy: for the two sufficient conditions compute the second derivative of the relative concentration curves. Setting the second derivative positive for a strictly convex relative concentration curve and negative for a strictly concave relative concentration curve (see Seidl (1994, p. 349)) yields the respective elasticities.

$\varepsilon(q)$ represents the relative increase in tax revenue collected from the fraction q of the lowest income earners when q is slightly increased. When $\varepsilon^1(q)$ and $\varepsilon^2(q)$ are evaluated at

[13] See Lambert (1989, chap. 7) for similar concepts in the case of identical income distributions.

the same value of q, for different income distributions this means that they are evaluated at different income levels and/or in different monetary units. Assume, for instance, that we have $F_1^{-1}(q) = \hat{Y^1}$ and $F_2^{-1}(q) = \hat{Y^2}$. Then we are actually comparing

$$\varepsilon^1(q) = \frac{dT^1(\hat{Y^1})/dY^1}{T^1(\hat{Y^1})f_1(\hat{Y^1})}q \quad \text{and} \quad \varepsilon^2(q) = \frac{dT^2(\hat{Y^2}))/dY^2}{T^2(\hat{Y^2})f_2(\hat{Y^2})}q,$$

where we have usually $\hat{Y^1} \neq \hat{Y^2}$, even if we are comparing tax schedules defined for the same monetary unit. Notice the tendency of a more unequal income distribution to make the tax system more progressive, because a smaller $\hat{Y}$ is associated with q, which means that there is not much income concentrated in the fraction q of the poorest income earners. Therefore not much tax revenue can be extracted from the lower income strata.

Note that these elasticities are not only cast in terms of tax or net income schedules, but contain elements of both the tax schedules and the income distributions. They are algebraic conditions but, alas, are only sufficient and not necessary conditions. The respective definitions are trivial necessary and sufficient conditions which are helpful for empirical analyses.[14]

Theorem 2 *Assume* $0 < T^i[F_{Y^i}^{-1}(p)] < F_{Y^i}^{-1}(p)\ \forall p \in (0,1)$, $i = 1,2$, *where* $F_{T^1}^{Y}(p)$ *is placed on the ordinate and* $F_{T^2}^{Y}(p)$ *on the abscissa of the relative concentration curve. Then* (Y^1, T^1) *is more progressive than* (Y^2, T^2) *if* $\varepsilon^1(p) - \chi^1(p) > \varepsilon^2(p) - \chi^2(p)\ \forall p\ \in (0,1)$, *where*

$$\varepsilon^i(p) := \frac{\frac{dT^i[F_{Y_i}^{-1}(p)]}{dF_{Y^i}^{-1}(p)}\frac{dF_{Y^i}^{-1}(p)}{dp}}{T^i[F_{Y^i}^{-1}(p)]}p, \text{ and } \chi^i(p) := \frac{dF_{Y^i}^{-1}(p)}{dp}\frac{p}{F_{Y^i}^{-1}}\ i = 1,2.$$

A necessary and sufficient condition is given by the definition of uniformly greater tax progression, applied to (Y^1, T^1) *and* (Y^2, T^2)*:* $F_{T^1}(p) \leq F_{T^2}(p)\ \forall p\ \in [0,1]$.

In terms of net incomes, let $F_{Y^1-T^1}^{Y}(p)$ *be placed on the ordinate and* $F_{Y^2-T^2}^{Y}(p)$ *on the abscissa of the relative concentration curve. Then* (Y^1, T^1) *is more progressive than* (Y^2, T^2) *if* $\eta^1(p) - \chi^1(p) < \eta^2(p) - \chi^2(p)\ \forall p\ \in (0,1)$, *where*

$$\eta^i(p) := \frac{\left[1 - \frac{dT^i[F_{Y_i}^{-1}(p)]}{dF_{Y^i}^{-1}(p)}\right]\frac{dF_{Y_i}^{-1}(p)}{dp}}{F_{Y_i}^{-1}(p) - T_i[F_{Y_i}^{-1}(p)]}p, \text{ and } \chi^i(p) := \frac{dF_{Y^i}^{-1}(p)}{dp}\frac{p}{F_{Y^i}^{-1}}\ i = 1,2.$$

A necessary and sufficient condition is given by the definition of uniformly greater tax progression, applied to (Y^1, T^1) *and* (Y^2, T^2)*:* $F_{Y^1-T^1}(p) \geq F_{Y^2-T^2}(p)\ \forall p\ \in [0,1]$.

Theorem 2 again contains two sufficient and two trivial necessary and sufficient conditions. What we have reasoned after Theorem 1 applies, *mutatis mutandis*, also for

[14] Although having a necessary and sufficient condition for their analysis in terms of deformed income distributions, viz. isoelasticity of the deformation functions to warrant independence of the baseline distribution, Dardanoni and Lambert (2002, p. 106) had to concede that isoelasticity is a rather demanding

Theorem 2. The only difference is that now the comparison of progression is made in terms of shares of aggregate income instead of in income shares of population strata.

As concerns the sufficient conditions, $\varepsilon(p)$ $[\eta(p)]$ denotes the tax [residual income] elasticity with respect to p, and $\chi(p)$ denotes the elasticity of the inverse first moment distribution function with respect to p, which captures the influence of the income distribution evaluated at p. The proof of Theorem 2 is a complete analogue of that of Theorem 1, if applied to relative concentration curves in terms of income shares. When $\varepsilon^1(p) - \chi^1(p)$ and $\varepsilon^2(p) - \chi^2(p)$ are evaluated at the same value of p, this means that different $\hat{Y}_i$'s are involved.

The next theorem analyzes greater progression in terms of second-order differences of first moment distribution functions.

Theorem 3 *Assume* $0 < T^i[F_i^{-1}(q)] < F_i^{-1}(q)\ \forall\ q \in (0,1),\ i = 1,2.$ *Then* $F_{Y^1}(q) - F_{T^1}(q) > F_{Y^2}(q) - F_{T^2}(q) > 0\ \forall\ q \in (0,1)$, *if both* $\varepsilon^1(q) \geq \varepsilon^2(q)$ *and* $\Psi^1(q) \leq \Psi^2(q)\ \forall q \in (0,1)$, *where at least one of the two inequality signs has to be strict. Notice that* $\varepsilon^i(q)$, $i = 1,2$, *is defined as in Theorem 1, and* $\Psi^i(q) := \frac{\frac{dF_i^{-1}(q)}{dq}}{F_i^{-1}(q)}q$, $i = 1,2$, *denotes the elasticity of the inverse distribution function.*
A necessary and sufficient condition is given by the definition of uniformly greater tax progression, applied to (Y^1,T^1) *and* (Y^2,T^2)*:* $F_{Y^1}(q) - F_{T^1}(q) \geq F_{Y^2}(q) - F_{T^2}(q) > 0\ \forall\ q \in (0,1)$.

Moreover, $F_{Y^1}(q) - F_{Y^1-T^1}(q) < F_{Y^2}(q) - F_{Y^2-T^2}(q) < 0\ \forall\ q \in (0,1)$ *if both* $\eta^1(q) \leq \eta^2(q)$ *and* $\Psi^1(q) \geq \Psi^2(q)\ \forall\ q \in (0,1)$, *where at least one of the two inequality signs has to be strict. Notice that* $\eta^i(q), i = 1,2$, *is defined as in Theorem 1.*
A necessary and sufficient condition is given by the definition of uniformly greater tax progression, applied to (Y^1,T^1) *and* (Y^2,T^2)*:* $F_{Y^1}(q) - F_{Y^1-T^1}(q) \leq F_{Y^2}(q) - F_{Y^2-T^2}(q) < 0\ \forall\ q \in (0,1)$.

The proof of Theorem 3 is more involved than the proof of Theorem 1 (see Seidl (1994, pp. 352-3)). Note that the algebraic conditions are only sufficient conditions. They contain in their first components elements of both the tax schedule and the income distribution, while their second components refer to the income distributions only. These latter components serve the role of a calibration device to warrant that the tax distributions are not triggered by great discrepancies in the distributions of gross incomes. The necessary and sufficient conditions are again elementary; they are very helpful for empirical analyses.

Theorem 3 concerns comparisons between differences of cumulative curves of gross incomes and taxes and comparisons between differences of cumulative curves of net incomes and gross incomes. If $F_{Y^1}(q)$ is more diminished by subtraction of the tax curve, viz. $F_{T^1}(q)$, than $F_{Y^2}(q)$ is by $F_{T^2}(q)$ for all q, $0 \leq q \leq 1$, then (Y^1,T^1) is more progressive

condition which will hardly be met in the real world. (For this approach see also Footnote 1.) Hence, com-

than (Y^2, T^2). If $F_{Y^1-T^1}(q)$ is more diminished by subtraction of $F_{Y^1}(q)$ than $F_{Y^2-T^2}(q)$ is by $F_{Y^2}(q)$ for all q, $0 \leq q \leq 1$, then (Y^1, T^1) is more progressive than (Y^2, T^2).

Note that dominance relations of concentrations curves are subrelations of appropriate global inequality measures; of course, the converse does not hold. In this paper, we do not dwell on that; for more details see Seidl (1994, pp. 359-60).

3.3 Uniform Tax Progression for Different Income Distributions: Discrete Version

So far empirical comparisons of tax progression have not been made for the uniform measures of tax progression in terms of q or p that allow to compare progression for different income distributions in different countries or different time periods in the same countries. We investigate the theory developed in the preceding section using the data from the Luxembourg Income Study (LIS). As the LIS data are micro data, we have to re-state all definitions and curves in discrete terms.

As we analyze comparisons of progression of direct taxes on the one hand, and direct taxes plus payroll taxes (mainly employees' share of social security contributions) on the other, we have to introduce the respective notation. We use $Y = [Y_1, Y_2, \ldots, Y_n]$ to denote a distribution of pre-tax or gross incomes arranged in nondecreasing order, $T = [T_1, T_2, \ldots, T_n]$ to denote the distribution of the associated direct taxes, and $S = [S_1, S_2, \ldots, S_n]$ to denote the distribution of direct taxes plus payroll taxes. μ, τ, and ζ denote mean pre-tax or gross income, mean direct taxes, and mean direct taxes plus payroll taxes, respectively. For generic references we will continue to use the terms gross and net incomes. When respective discriminations are needed, the terms pre-tax and post-tax incomes will be used for direct taxes only, and gross and net incomes for analyses of direct taxes plus payroll taxes.

Let (Y, T) denote the income-distribution-cum-tax-schedule for some country or some time period within a country. Let us also define the discrete equivalents of the first moment distribution functions in terms of point coordinates with the first entry being the ordinate, and the second entry being the abscissa of the respective points. For ease of demonstration, T and τ are used as proxies also representing $(T + S)$ and $(\tau + \zeta)$:

$$F_Y(q_k) := \frac{\sum_{i=0}^{k} Y_i}{n\mu}, \quad q_k = \frac{k}{n}, \quad k = 0, \ldots, n; \tag{11}$$

$$F_T(q_k) := \frac{\sum_{i=0}^{k} T_i}{n\tau}, \quad q_k = \frac{k}{n}, \quad k = 0, \ldots, n; \tag{12}$$

$$F_{Y-T}(q_k) := \frac{\sum_{i=0}^{k} (Y_i - T_i)}{n(\mu - \tau)}, \quad q_k = \frac{k}{n}, \quad k = 0, \ldots, n; \tag{13}$$

parison of tax progression "becomes an empirical question", which is another impetus for our present work.

$$F_T^Y(p_k) := \frac{\sum_{i=0}^{k} T_i}{n\tau}, \quad p_k = \frac{\sum_{i=0}^{k} Y_i}{n\mu} \quad k = 0, \ldots, n; \tag{14}$$

$$F_{Y-T}^Y(p_k) := \frac{\sum_{i=0}^{k} (Y_i - T_i)}{n(\mu - \tau)}, \quad p_k = \frac{\sum_{i=0}^{k} Y_i}{n\mu} \quad k = 0, \ldots, n. \tag{15}$$

In formulae (11) to (15), we set Y_0 and T_0 equal to zero, which allows us to include the origin into our curves[15]. Strictly speaking, both the right-hand side and the left-hand side of (11) to (15) are functions of k. Hence we consider the range of the right-hand-side function as the domain of the left-hand-side function, which gives us the discrete versions of the first-moment distribution functions. For all curves in terms of q we use the ranking according to gross incomes, as we have to apply that necessarily also for the curves in terms of p. Here, (11) denotes a discrete equivalent of the Lorenz income curve, (12), and (13) denote the discrete equivalents of the concentration curves of taxes and net incomes, respectively. (14) and (15) denote the discrete equivalents of the concentration curves of taxes and net incomes for the income shares p_k^j. These mappings are just generic; of course, different situations (Y^1, T^1) and (Y^2, T^2) have different components and different numbers of income recipients.[16]

Concentration curves may be different from Lorenz curves because of re-ranking phenomena. Hence, before proceeding further, we have to dwell on the *re-ranking problem* which we assumed away for our theoretical analyses, but which haunts all empirical analyses of distributional problems. For theoretical analyses, the assumption of *co-monotonicity* of gross incomes, taxes, payroll taxes, and net incomes is self-evident: higher gross incomes should imply higher taxes, higher payroll taxes, and higher net incomes. This means that the ordering of gross incomes coincides with the orderings of taxes, payroll taxes, and net incomes. Hence, only one ordering applies to all designs and we can work with Lorenz curves (or their discrete-case equivalents) throughout. However, in the world of empirical data, re-ranking is ubiquitous. Due to different family structures, different tax allowances, different income compositions, and different transfer incomes, households with higher incomes may end up with smaller taxes or, else, with smaller net incomes than households with smaller gross incomes.

Hence, re-ranking opens up Pandora's box of possible other orderings. For instance, for expression (12) we can arrange the entries in nondecreasing order of the taxes instead of arranging them according to the order of their associated gross incomes. In (13), net incomes can be arranged in nondecreasing order instead of arranging them according to their associated gross incomes. Then we could work with Lorenz curves throughout instead of using concentration curves. Indeed, in the presence of re-ranking, concentration curves would be closer to the diagonal than Lorenz curves, or may even cross the diagonal.

[15] Recall that we exclude negative incomes and taxes in our empirical analysis.

[16] Note that for the discrete version we continue to use superscripts to refer to the two different vectors of taxes and incomes, each representing the situation as a whole, while subscripts are used for vector

For the sake of a uniform methodology we decided in favor of ordering all entries according to gross incomes for our empirical analyses. To illustrate, consider the analysis in terms of the aggregate shares p_k. Observe that the mapping for gross incomes boils down to $F_Y^Y(p_k) = p_k$, i.e., it consists just of points on the diagonal of the unit square. Had we ordered the entries in formula (14) not in terms of gross incomes, but in terms of taxes, i.e., had we set $p_k = \frac{\sum_{i=1}^{k} T_i}{n\tau}$, then we would again have only gotten points on the diagonal of the unit square. The same would apply if we had ordered the p_k's according to the net incomes in formula (15). On the other hand, analyses in terms of aggregate shares are sensible from an economic point of view. If from the lowest income recipients whose aggregate income amounts to 20 percent of total income, 5 percent of total tax revenue is collected, then at $p_k^1 = 0.2$, $F_{T^1}^{Y^1}(p_k^1) = 0.05$, and (Y^1, T^1) is more progressive than (Y^2, T^2) if, e.g., $p_k^2 = 0.2$ (or its respective interpolation point on the second moment distribution curve) and $F_{T^2}^{Y^2} = 0.10$. If the same pattern holds for all $0 < p_k^j < 1$ (or for the respective interpolation points of the curves constructed by connecting neighboring points by straight lines), then (Y^1, T^1) is uniformly more progressive than (Y^2, T^2). The same applies *mutatis mutandis* to the net incomes. Hence, the request for comparability of analyses of tax progression in terms of the p_k's on the one hand and the q_k's on the other suggests that we should also use the gross-income rankings for our analyses in terms of q.

Beyond that, there is still another reason for using gross-income rankings throughout: neither taxes nor net incomes exist in isolation. Instead, they derive their existence from their association with gross incomes. Hence, this serves as an additional argument to treat them according to the ordering of gross incomes for our analyses in terms of the q_k's. Finally, using EUROMOD data, Peichl and Schäfer (2008, pp. 9-12) have shown that the difference between the Gini and the concentration coefficients of net incomes are rather small and re-ranking as measured by the Reynolds-Smolensky progression measure is far from dramatic. Hence, the possible error from ignoring re-ranking is quite small.

Another aspect of the re-ranking problem concerns equivalized incomes. Our analyses are carried out in terms of both household and equivalized incomes. The latter are derived from the former by applying equivalence scales to take into account economies of scale in large households.[17] For our analyses we applied the equivalence scales as proposed by LIS: to derive equivalized incomes, we divided the household incomes by m^α, where m denotes the number of household members, and α, $0 \leq \alpha \leq 1$, denotes a scale parameter; mainly we followed the LIS practice of taking $\alpha = 0.5$, but in order to investigate the influence of the scale parameter, we also tried $\alpha = 0.25$ and $\alpha = 0.75$ for selected cases.[18]

components.

[17] See, e.g., Blackorby and Donaldson (1983), Klein (1986), Buhmann et al. (1988), Glewwe (1991), Coulter et al. (1992), Banks and Johnson (1994), Jenkins and Cowell (1994), Faik (1995), Aaberge and Melby (1998), Cowell and Mercader-Prats (1999), Ebert and Moyes (2003), and Schröder (2004).

[18] Buhmann et al. (1988, pp. 119-122) investigated 34 equivalence scales which were proposed by various researchers, and found that the Luxembourg equivalence formula fits them well for various values of α for

To illustrate, consider an income distribution comprising two households: [1000,2500]. Suppose that the first one is a single-person household, while the second one is a four-person household. Then, if we take $\alpha = 0.5$, the equivalized income distribution becomes [1000,1250,1250,1250,1250]; however, taking $\alpha = 0.75$ gives us the equivalized income distribution [884,884,884,884,1000]. Hence, both the household structure[19] and the choice of the scale parameter determine the shape of the equivalized income distribution. In our analysis, equivalized incomes were always arranged in nondecreasing order of equivalized gross incomes.

After the digression on re-ranking, let us return to the concepts of uniformly more progressive tax schedules. Recall that we defined (Y^1, T^1) to be more progressive than (Y^2, T^2) if the tax schedule T^1 associated with the income distribution Y^1 collects for all values of q or p no greater fraction of taxes than does tax schedule T^2 associated with the income distribution Y^2. Alternatively, we defined greater progression of (Y^1, T^1) than (Y^2, T^2) if (Y^1, T^1) leaves the taxpayers for all q or p no less a fraction of post-tax net incomes than does (Y^2, T^2). Finally, we defined greater progression if the difference of the cumulative curves of gross incomes and taxes for (Y^1, T^1) is not smaller than the difference of the cumulative curves of gross incomes and taxes for (Y^2, T^2) for all q, or when the difference of the cumulative curves of gross and net incomes of (Y^1, T^1) is not greater than the difference of the cumulative curves of gross incomes and taxes of (Y^2, T^2) for all q.[20]

For the continuous analyses we expressed the first two concepts in terms of relative concentration curves, and the second two in terms of second-order curve differences. More progression is present if the relative concentration curve does not cut the diagonal within the unit square, or if the second-order curve differences do not cut the abscissa within the unit interval. For the discrete analysis, it is more convenient to use curve differences quite generally.[21] As the respective "curves" in the discrete case consist of finitely many

four representative groups of proposed equivalence scales. Buhmann et al. (1988, p. 128) also observed that income inequality first decreases and then increases as α increases, viz. inequality is an U-shaped function of α; poverty decreases as α increases (p. 132). For more elaborate work see Coulter et al. (1992), Banks and Johnson (1994), Jenkins and Cowell (1994), Faik (1995), and Cowell and Mercader-Prats (1999).

[19] Buhmann et al. (1988, p. 127) argue that equivalence scales have greater effect in case of different household structures associated with the actual income distributions to be compared; greater households, in particular, influence the results. Peichl et al. (2009a,b) observed that part of the increase in income inequality in Germany in terms of equivalized incomes is due to the trend in the direction of smaller households in the last decades.

[20] See also p. 12. Concerning the definitions in terms of p. see Footnote 9, which applies to the discrete case as well.

[21] The case of a relative concentration curve being below (or above) the diagonal in the interior of the unit square is equivalent to a positive (negative) difference of the generating curves within the unit interval. Recall that a concentration curve of $F_{T^1}(\cdot)$ relative to $F_{T^2}(\cdot)$ does not cross the diagonal iff $F_{T^1}(\cdot) - F_{T^2}(\cdot)$ has the same sign for all $q, p \in (0, 1)$. The proof is trivial and therefore omitted. Note that this applies analogously also to net incomes.

points, we have to confine ourselves to the comparison of these points. In the general case, as defined in formulae (11) to (15), we encounter the difficulty that the q_k's and the p_k's need not coincide for (Y^1, T^1) and (Y^2, T^2), so that we may have k^1's for which there are no equal k^2's, and vice versa. This can be handled in a more tedious way by comparing q_k^1 with the equivalent point on the interpolation segment on the second curve. For reasons to be explained in Sections 4.1 and 4.2 we used grouped data with the same number of quantiles. Before that, we explain our measures of comparisons of progression in terms of individual data.

For the sake of unified representation we have arranged all definitions of greater progression in such a way that progression dominance is expressed as nonnegative curve differences and being progression dominated as nonpositive curve differences. Hence, we have:

Definition 1 (Y^1, T^1) *is more [less] progressive than* (Y^2, T^2) *iff* $F_{T^2}(q_k) - F_{T^1}(q_k)$ *is nonnegative [nonpositive] for all* q_k, $0 \leq q_k \leq 1$.

Definition 2 (Y^1, T^1) *is more [less] progressive than* (Y^2, T^2) *iff* $F^Y_{T^2}(p_k) - F^Y_{T^1}(p_k)$ *is nonnegative [nonpositive] for all* p_k, $0 \leq p_k \leq 1$.

Definition 3 (Y^1, T^1) *is more [less] progressive than* (Y^2, T^2) *iff* $F_{Y^1-T^1}(q_k) - F_{Y^2-T^2}(q_k)$ *is nonnegative [nonpositive] for all* q_k, $0 \leq q_k \leq 1$.

Definition 4 (Y^1, T^1) *is more [less] progressive than* (Y^2, T^2) *iff* $F^Y_{Y^1-T^1}(p_k) - F^Y_{Y^2-T^2}(p_k)$ *is nonnegative [nonpositive] for all* p_k, $0 \leq p_k \leq 1$.

Definition 5 (Y^1, T^1) *is more [less] progressive than* (Y^2, T^2) *iff* $[F_{Y^1}(q_k) - F_{Y^2}(q_k)] - [F_{T^1}(q_k) - F_{T^2}(q_k)]$ *is nonnegative [nonpositive] for all* q_k, $0 \leq q_k \leq 1$.

Definition 6 (Y^1, T^1) *is more [less] progressive than* (Y^2, T^2) *iff* $[F_{Y^1-T^1}(q_k) - F_{Y^2-T^2}(q_k)] - [F_{Y^1}(q_k) - F_{Y^2}(q_k)]$ *is nonnegative [nonpositive] for all* q_k, $0 \leq q_k \leq 1$.

Obviously, Definition 1 matches the necessary and sufficient conditions of the first part of Theorem 1. Definition 2 matches the necessary and sufficient condition of the first part of Theorem 2. Definition 3 matches the necessary and sufficient condition of the second part of Theorem 1. Definition 4 matches the necessary and sufficient condition of the second part of Theorem 2. Definition 5 matches the necessary and sufficient condition of the first part of Theorem 3. Note that Definition 5 comes from the formulation that (Y^1, T^1) is more [less] progressive than (Y^2, T^2) iff $[F_{Y^1}(q_k) - F_{T^1}(q_k)] \geq [F_{Y^2}(q_k) - F_{T^2}(q_k)]$ [$\leq$ for less progressive] for all q_k, $0 \leq q_k \leq 1$. Definition 6 matches the necessary and sufficient condition of the second part of Theorem 3. Note that Definition 6 comes from the formulation that (Y^1, T^1) is more [less] progressive than (Y^2, T^2) iff $[F_{Y^1-T^1}(q_k) - F_{Y^1}(q_k)] \geq [F_{Y^2-T^2}(q_k) - F_{Y^2}(q_k)]$ [$\leq$ for less progressive] for all q_k, $0 \leq q_k \leq 1$.

3.4 Heuristics of Progression Comparisons

3.4.1 Heuristics of the First Moment Distribution Functions

To provide some intuition of the proposed method of progression comparisons, we assume in this section that we have the same number of taxpayers n in both situations to be compared and deal with the individual data of the taxpayers. Hence, in formulae (11) to (15) k runs from 0 to n. Then, for the same k the left-hand sides of (12) and (14) have the same value, and the left-hand sides of (13) and (15) have the same value. What makes $F_T(q_k)$ and $F_T^Y(p_k)$ as well as $F_{Y-T}(q_k)$ and $F_{Y-T}^Y(p_k)$ different are the second components of their respective points. For $F_T(q_k)$ and $F_{Y-T}(q_k)$ they are $\frac{k}{n} = q_k$, whereas for $F_T^Y(p_k)$ and $F_{Y-T}^Y(p_k)$ they are $\frac{\sum_{i=1}^k Y_i}{n\mu} = p_k$, i.e. $F_Y(q_k)$. Now $F_Y(q_k) \leq q_k$, since $F_Y(q_k)$ is the Lorenz curve of gross incomes. Hence, the p-curves lie North-West of the respective q-curves.

For proportional taxes with rate t, $1 > t > 0$, we have

$$\frac{\sum_{i=1}^k T_i}{n\tau} = \frac{t\sum_{i=1}^k Y_i}{tn\mu} = \frac{(1-t)\sum_{i=1}^k Y_i}{(1-t)n\mu} = F_Y(q_k).$$

Hence, both $F_T(q_k)$ and $F_{Y-T}(q_k)$ are equal to the Lorenz curve of gross incomes $F_Y(q_k)$, and both $F_T(p_k)$ and $F_{Y-T}(p_k)$ are equal to the diagonal of the unit square.

Suppose co-monotonicity of gross incomes, taxes, and net incomes holds. Then we have for progressive taxes (that is $\frac{T_i}{Y_i}$ is nondecreasing for all i)

$$q_k \geq F_{Y-T}(q_k) \geq F_Y(q_k) \geq F_T(q_k) \tag{16}$$

and

$$F_{Y-T}^Y(p_k) \geq p_k \geq F_T^Y(p_k). \tag{17}$$

Inequality (16) is obvious. To show inequality (17), we consider whether

$$\frac{\sum_{i=1}^k (Y_i - T_i)}{n(\mu - \tau)} \geq \frac{\sum_{i=1}^k Y_i}{n\mu}$$

holds. After some re-arrangement this gives us

$$\frac{\sum_{i=1}^k T_i}{\sum_{i=1}^k Y_i} \leq \frac{\tau}{\mu}. \tag{18}$$

Because of a progressive tax schedule, $\frac{\sum_{i=1}^k T_i}{\sum_{i=1}^k Y_i}$ is also a nondecreasing series of k.[22] Hence it is bounded above by $\frac{\tau}{\mu}$. This establishes the first inequality in (17).

[22]Proof: suppose by contradiction that $\frac{\sum_{i=1}^k T_i}{\sum_{i=1}^k Y_i}$ is decreasing for some k. Then we have $\frac{\sum_{i=1}^k T_i}{\sum_{i=1}^k Y_i} > \frac{\sum_{i=1}^{k+1} T_i}{\sum_{i=1}^{k+1} Y_i}$. Re-arrangement gives us $\frac{T_k}{Y_k} \leq \frac{T_{k+1}}{Y_{k+1}} < \frac{\sum_{i=1}^k T_i}{\sum_{i=1}^k Y_i}$, where the first inequality follows from the nondecreasing average tax rate.

The second part of inequality (17) comes from checking whether

$$\frac{\sum_{i=1}^{k} Y_i}{n\mu} \geq \frac{\sum_{i=1}^{k} T_i}{n\tau}$$

holds. It is immediately seen that this reduces to (18) and, thus, establishes the second part of inequality (17).

Hence, for co-monotonicity and progressive taxation $F^Y_{Y-T}(p_k)$ lies above and $F^Y_T(p_k)$ below the diagonal of the unit square. For co-monotonicity and increasing, but regressive, taxation the opposite inequality signs hold in the inequalities (16) and (17).

When re-ranking occurs, co-monotonicity is violated, and the resulting concentration curves below the diagonal exhibit less curvature, and the concentration curves above the diagonal [this is $F^Y_{Y-T}(p_k)$] more curvature, i.e., the first group moves closer to the diagonal and the second further away from the diagonal. Although cases such that the inequalities (16) and (17) are violated may be constructed, they hold in most cases for empirical data. Recall that Peichl and Schäfer (2008, pp. 9-12) found that the re-ranking effects are not spectacular.

3.4.2 Heuristics of Uniformly Greater Progression

Concerning Definitions 1 to 6, for empirical data the net incomes are more equally distributed than the gross incomes, and gross incomes are more equally distributed than taxes. This means that the q-curves for net incomes exhibit the least curvature, followed by the q-curves for gross incomes, with the q-curves for taxes having the most curvature. As to the p-curves, they become the diagonal for gross incomes, a convex curve for taxes, and a concave curve for net incomes.

Uniformly Greater Tax Progression: Formally Stated Definition 1 states that (Y^1, T^1) is more progressive than (Y^2, T^2), if the first moment distribution function of T^1 with respect to q lies below that of T^2. The degree of higher progression can be measured by taking the difference between these curves, which in turn can be captured by the area under the curve $F_{T^2}(q_k) - F_{T^1}(q_k)$ keeping in mind the sign of the difference. Definition 2 does the same for the first moment distribution functions of taxes with respect to p. Because of our above observation we would expect a smaller difference on average for the p-curves than for the q-curves.

According to Definition 3, (Y^1, T^1) is more progressive than (Y^2, T^2), if the first moment distribution function with respect to q_k of the net incomes $(Y^1 - T^1)$ lies above that of $(Y^2 - T^2)$. That is, for each quantile q_k (except at the end points) the quantile's fraction of the total net income is higher under (Y^1, T^1) than under (Y^2, T^2). A similar

Now, $\frac{T_k}{Y_k} < \frac{\sum_{i=1}^{k} T_i}{\sum_{i=1}^{k} Y_i}$ is equivalent to $T_k \sum_{i=1}^{k-1} Y_i < Y_k \sum_{i=1}^{k-1} T_i$, and, hence, $\frac{T_{k-1}}{Y_{k-1}} \leq \frac{T_k}{Y_k} < \frac{\sum_{i=1}^{k-1} T_i}{\sum_{i=1}^{k-1} Y_i}$, where the first inequality follows from the nondecreasing average tax rate. Backward induction shows for $k = 2$ that $\frac{T_2}{Y_2} < \frac{T_1}{Y_1}$, which contradicts the condition of nondecreasing average tax rates. Q.E.D.

pattern holds for Definition 4, which defines (Y^1, T^1) as more progressive than (Y^2, T^2), if the first moment distribution function with respect to p_k of the net incomes $(Y^1 - T^1)$ lies above that of $(Y^2 - T^2)$. This means that for each gross income quantile p_k (except at the end points) the quantile's fraction of the total net income is higher under (Y^1, T^1) than under (Y^2, T^2). Note that, in contrast to the q-curves, this means that, because of inequality (17), $F^Y_{Y^1-T^1}(p_k)$ lies further apart from the diagonal than does $F^Y_{Y^2-T^2}(p_k)$ for all p_k's if (Y^1, T^1) is more progressive.

Definition 5 uses the difference between $F_Y(q_k)$ and $F_T(q_k)$ as the basis for comparisons of progression. If for (Y^1, T^1) this difference exceeds the one for (Y^2, T^2) for all q's (except at the end points), then (Y^1, T^1) is considered more progressive than (Y^2, T^2). Definition 6 takes an analogous approach using $F_{Y-T}(q_k)$ and $F_Y(q_k)$.

Uniformly Greater Tax Progression: Interaction of Tax Schedules and Income Distributions We employed Definitions 1 to 6 to make comparisons of tax progression. Our approach differs from the conventional one by considering the case of different tax schedules *and* different income distributions for the situations to be compared. This means that both components of tax progression interact.

Starting with Definition 1, suppose $F_{Y^2}(q_k) \geq F_{Y^1}(q_k)\ \forall q_k$ with at least one strict inequality sign, and suppose that we have proportional taxes in both situations. Then the first moment distribution functions of incomes translate immediately to $F_{T^2}(q_k) \geq F_{T^1}(q_k)\ \forall q_k$ with at least one inequality sign strict. This implies that (Y^1, T^1) is, according to Definition 1, considered as being more progressive than (Y^2, T^2), although both taxes are proportional. Hence, the more unequal distribution of gross incomes Y^1 causes (Y^1, T^1) to be more progressive than (Y^2, T^2). This is simply the consequence of allowing the income distribution an equal influence as the tax schedule on the determination of the progression of (Y, T). In other words, the distribution of gross incomes may reinforce or attenuate the effects of progression of the pure tax schedule. For instance, if a slightly progressive tax schedule is associated with a rather unequal distribution of gross incomes, the concentration curve of the taxes may well be dominated by the concentration curves of rather progressive taxes associated with a more equal distribution of gross incomes.[23]

Definition 2 precludes a proportional tax from becoming more progressive than another proportional tax, since $F^Y_Y(p_k)$ happens to be the diagonal. Hence, $F^Y_T(p_k)$ lies below the diagonal for progressive tax schedules. But the distribution of gross incomes interferes also for Definition 2 with the tax schedule and may reinforce or attenuate the progression of the pure tax schedule.

The influence of the distribution of gross incomes is even more pronounced for Definition 3 than for Definition 1 because gross incomes usually have a higher impact on net incomes than on the associated taxes. A more equal distribution of net incomes may

[23]Note that this is merely a possibility. Taxes are levied on absolute rather than relative incomes. Therefore, considerations beyond proportional taxation are subject to speculation.

result from a progressive tax schedule and/or from a more equal distribution of gross incomes. Only if the income distribution is the same can we attribute greater progression to the tax schedule alone. The other end of the gamut is established by the case of a proportional tax for which greater progression is wholly determined by the distribution of gross incomes. In effect, the influence of the distribution of gross incomes is at most pronounced for the net incomes, which, in turn governs the behavior of Definition 3.

Definition 4 precludes a proportional tax from becoming more progressive than another proportional tax, since $F^Y_{Y-T}(p_k)$ becomes equal to $F^Y_Y(p_k)$, which is the diagonal. For progressive tax schedules, $F^Y_{Y-T}(p_k)$ lies, according to (17), above the diagonal. Comparisons of tax progression are again heavily influenced by the distribution of gross incomes. This effect is even more pronounced for Definition 4 than for Definition 2.

Re-arranging Definitions 5 and 6, we have the terms $[F_Y(q_k) - F_T(q_k)]$ and $[F_{Y-T}(q_k) - F_Y(q_k)]$, respectively. Recall that these terms are zero for proportional taxes. Hence, Definitions 5 and 6 become zero for proportional tax schedules. In a way, Definitions 5 and 6 calibrate for the gross income distributions, as they just consider the deviations of the first moment distribution of the gross incomes from the first moment distributions of the taxes or net incomes, respectively. Hence, the influence of the distributions of gross incomes is partly neutralized. Moreover, at first sight Definitions 5 and 6, as they were formulated above, may invoke the wrong conclusion that they provide a separation between the influence of the income distribution on the one hand, and the tax schedule on the other. But this impression is not correct, since the terms $F_T(q_k)$ and $F_{Y-T}(q_k)$ are by themselves influenced by the respective gross income distributions. This is also evidenced from Theorems 1 to 3, which show us that the tax schedules and the income distributions are intrinsically amalgamated so that a straightforward separation of their influence is not at hand.[24] Hence, Definitions 5 and 6 may be considered a second-best approach at separating the influence of the distributions of gross incomes and tax schedules. Here also the tax terms and the net income terms depend on the income distribution, which prevents a clear-cut separation between these influences.

4 Data and Results

Our empirical investigation addresses several problems:

[24] For the *sufficient* conditions, Theorems 2 and 3 may at first sight impart the impression of separate influences of the elasticities of the tax schedules or net incomes on the one hand, and the elasticities $\chi(p)$ and $\Psi(q)$ on the other. However, this impression obscures that the elasticities $\varepsilon(\cdot)$ and $\eta(\cdot)$ themselves depend in intricate ways on the income distributions (which applies also to Theorem 1). [Note that things are different for the analysis in terms of income. For the *sufficient* conditions in terms of taxes and net incomes we observe the sum of the elasticity of the density function of the income distribution on the one hand, and the tax elasticity or the residual income elasticity on the other; see Seidl (1994, pp. 347-8). However, this analysis applies only to cases of identical monetary units and identical support of the income distributions involved.] The work of Dardanoni and Lambert (2002) may also be viewed under the aspect of separating tax schedules and income distributions. These authors employ deformation functions to mimic the income distribution of the other country to be compared. However, this possible

Firstly, recall that the method of comparing tax progression which we proposed determines not a complete, but only a partial ordering. This provokes the question whether it is of major relevance because it might be that the tax schedules associated with their respective income distributions are so involved that only few clear-cut dominance relations emerge. In other words: is our method of comparison of tax progression in the real world only a will-o'-the-wisp, or can it command major occurrence?

Secondly, if our first question is responded in the affirmative, what is the relative performance of the six proposed measures of the comparison of tax progression? Are there interrelationships? What are the economic message and content of these methods?

Thirdly, what is the relative importance of sufficient and necessary conditions of comparisons of tax progression? For which fraction of greater tax progression the relative concentration curves would be strictly convex or concave, and for which they would just not cross the diagonal of the unit square without being strictly convex or concave? If the respective elasticity conditions do not hold, can we safely assume that greater tax progression is unlikely or can we expect it to be rather common?

Fourthly, what is the pattern of comparisons of tax progression when dominance relations do not hold? Do we mainly encounter bifurcate or more intricate progression patterns? Is there a change of the progression pattern at a unique threshold, or do we have a whole series of changes of the progression relations?

Our analyses are carried out for both household data and equivalized data. We used the respective data from the Luxembourg Income Study database (see LIS (2010)). We mainly focus on international comparisons of tax progression, but also carry out intertemporal comparisons of tax progression for selected countries. Furthermore, we perform a sensitivity analysis with respect to the LIS equivalence scale parameter. To address these and related questions, we need household data for both gross and net incomes. In addition to that, for intertemporal comparisons for each country we required the data from at least three recent survey periods ("waves" in LIS terminology; in particular, we used waves III up to VI if the respective data allowed). Data sets showing gross incomes, direct taxes, payroll taxes, and net incomes were available just for 15 countries out of which we took 13, representing in our view the most salient ones. As a shorthand terminology we will use "taxes" to refer either to direct taxes or to direct taxes plus payroll taxes.

We will start with a short description of handling the LIS data, then will report six tables with pairwise progression comparisons for Definitions 1 to 6, followed by comments on six tables with summary results. Then we will analyze sixteen selected graphs for progression comparison related to comparing Germany, the United Kingdom, and the United States, as well as comparing Germany and Sweden, in terms of household incomes with direct taxes and direct taxes plus payroll taxes, and in terms of equivalized data for direct taxes and direct taxes plus payroll taxes. To provide an overview of our findings

way of decomposition works only for isoelastic deformation functions to secure independence of the baseline distribution (see also Footnotes 1 and 14).

for international progression comparisons, we will present Hasse diagrams. Finally, we provide examples of intertemporal progression comparisons and sensitivity results with respect to the parameter of the Luxembourg equivalence scale.

Note that this paper is entirely devoted to *numerical* analyses. We defer the statistical analysis to subsequent work due to much higher computation requirements for data processing on the LIS side (see also Footnote 30).

4.1 Handling LIS Data

Our empirical analysis resorts to micro data drawn from the LIS (2010) database. It is a cross-national data archive located in Luxembourg.[25] Currently it includes micro data from more than 30 countries, most of which are OECD member states. The data sets are organized into "waves" of about five years each, starting with Wave I in 1980 and the most recent being Wave VI (around 2004). The micro data from the different surveys is harmonized and standardized in order to facilitate comparative research. For many countries, however, only a limited number of waves is available, and even if a data set is available, not all income variables are included. In particular, gross incomes, direct taxes and payroll taxes are available only for 15 countries, of which we used 13 in our study.

Table 1 gives a summary of the countries and waves used. The countries are listed according to the country codes which were used to access the data sets. Columns III to VI list the years to which the data sets, included in the respective waves, refer to. For international comparisons, we took Wave V (around 2000) because of its relative recency and as maximizing the number of available country data. We also included France, but due to data availability, we had to rely on wave 1994. Poland's Wave III data set was not included as we deem it being too close to the fall of the Iron Curtain.

Within the data sets, at the household level LIS reports, inter alia, gross income (GI), disposable net income (DPI), income taxes (V11) and mandatory payroll taxes (PAYROLL). In what follows, we employ two different net income definitions. Net income is defined either in accordance with LIS as DPI=GI−(V11+PAYROLL), or it is redefined by us as GI−V11, i.e., the analysis is based on taxes only.[26] It should be noted, however, that V11 in some cases already includes social security contributions if these were lumped together in the original data set before adding it to LIS. Furthermore, some countries like

[25]For more information about the LIS database see Smeeding et al. (1985), Smeeding (2004), and Atkinson (2004). For illustrative examples of applications of the LIS database see, e.g., Allegrezza et al. (2004), Bardasi (2004), Bronchetti and Sullivan (2003), Gornick (2004), Förster and Vleminckx (2004), and Mahler and Jesuit (2006).

[26]This means that taxes include so-called *clawbacks*. i.e., taxes which return to government part of the transfer. Prasad and Deng (2009, p. 439) remark: "This means that where transfers are high, taxes on transfers may be high, but this is not 'true' tax, simply a reduction in the amount of transfer given. To achieve a measure of true tax paid, then, the amount of clawbacks should be subtracted from the total tax paid." However, we opine that clawbacks are an integral part of the tax system. In contrast to Prasad and Deng (2009) we also do not consider indirect taxes in our investigations.

Table 1: LIS Data Sets Used for International and Intertemporal Comparisons

		LIS Wave			
Country	Code	III	IV	V	VI
Australia	au	1989	1995	**2001**	2003
Canada	ca	1991	1994	**2000**	2004
Switzerland	ch	1992	–	**2000**	2004
Denmark	dk	1992	1995	**2000**	2004
Germany	de	1989	1994	**2000**	–
Finland	fi	1991	1995	**2000**	2004
France	fr	./.	**1994**	–	–
Netherlands	nl	1991	1994	**1999**	–
Norway	no	1991	1995	**2000**	2004
Poland	pl	./.	1995	**1999**	2004
Sweden	se	1992	1995	**2000**	–
United Kingdom	uk	1991	1995	**1999**	2004
United States	us	1991	1994	**2000**	2004

Table notes. Boldfaced years were used for international comparisons. A dash means that a gross income data set is not available. ./. means that the respective wave was not used though available.

Denmark do not have separate mandatory social security contributions for most of the population. In these cases PAYROLL stays empty and we cannot distinguish between taxes and payroll.

The analysis was carried out at the household level as well as the level of equivalized data. The former analysis rests upon the original data provided by LIS, weighed by household weights (HWEIGHT). These weights are intended to secure representativeness of the results for the whole population of a country. However, the representativeness of the current study is somewhat reduced by the fact that up to 7 percent of the households listed in each data set had to be truncated to warrant nonnegative gross and net incomes,[27] nonnegative taxes and payroll taxes. For the individual-based analysis, equivalized data was used. First, all monetary variables were multiplied by the Luxembourg equivalence scale which is $m^{-\alpha}$, where m represents the number of household members (D4). If not otherwise stated, α was set equal to 0.5. We also performed sensitivity analyses for some countries with $\alpha = \{0.25, 0.5, 0.75\}$. Second, household weights had to be replaced by person weights which were computed as D4×HWEIGHT.

In order to access the LIS data, we wrote a program in SPSS that computed the

[27]Some data sets are censored in the way that negative incomes are reported as zeros. Hence, we decided to leave out all entries which are nonpositive with respect to either GI or DPI or both.

values of $F_Y(q)$, $F_T(q)$, and so on, at 20 equally spaced quantiles of the unit interval and printed back these results for every data set (country and year), for taxes and taxes plus payroll taxes for both household and equivalized data. We had to use this approach because direct access to LIS data is not permitted. We then wrote a Visual Basic macro to facilitate processing of the LIS output offline.

4.2 Heuristics of Progression Comparisons for Grouped Data

Our formulae and definitions presented in Section 3.3 and discussed in Section 3.4 were cast in terms of individual data. Although this approach appears *prima facie* as the proper one, we could not apply it to our empirical work because, first, LIS does not allow direct access to its data (hence we had to compute all the aggregates for our analysis online, which presents a challenge since a resource-demanding program code is not welcomed by LIS), and, second, the numbers of taxpayers in the populations to be compared are typically different. This last feature would have necessitated tedious ad hoc interpolation procedures for all pairwise comparisons. To deal with both issues in a more expedient way, we employed the following approach using grouped data.

We divided the unit interval into 20 shares, i.e., taking five-percent steps[28] as equally spaced quantiles for q in terms of population shares and for p in terms of income shares. This assumption implies that the respective transformed indices j coincide for all respective situations to be compared in formulae (11) to (15), that is $j = 0, 1, \ldots, 20$. For our work with LIS data this approach has the decisive advantage that we are able to quickly process the data for all 13 countries at the LIS server (the critical time consuming part of our computations) and then analyze the results offline.

We started our procedure by arranging the raw data (taxes and associated net incomes) in our sample in the increasing order of the associated gross incomes; for equivalized incomes the data were re-arranged in the increasing order of equivalized gross incomes. Next, we divided the unit intervals, which correspond to the ranges of the cumulative distribution function for gross incomes, into 20 equally spaced five-percent groups for the analysis in terms of population quantiles q, and in terms of income quantiles p, respectively. Then we took the respective shares of aggregate taxes, gross and net incomes associated with these five-percent quantiles. Thus we obtained the values for q_j, p_j, $F_Y(q_j)$, $F_T(q_j)$, $F_{Y-T}(q_j)$, $F_T^Y(p_j)$, and $F_{Y-T}^Y(p_j)$ for our grouped data, where $j = 0, 1, \ldots, 20$.

But the curves for grouped data deviate from the curves for individual data in several respects. For individual data, as we observed in Section 3.4.1, if we have the same

[28] As compared to other empirical work, this is a rather fine grid. Sala-i-Martin (2006, pp. 355 and 357), for instance, had to work with quintiles and had to resort to widespread data interpolations for carrying out his ambitious study. Bishop et al. (1991a, p. 464) worked with deciles for their construction of Lorenz curves, arguing that "increasing the number of quantiles does not necessarily improve the quality of the overall test" (p. 476, Footnote 5).

number of income recipients and (for the same value of k) the same partial sums for both the q-curve and the p-curve, then we have the same corresponding values on either curve's ordinate; hence, by $p_k \leq q_k$, the curves differ only by their corresponding values on the abscissa. For grouped data, we apparently have the same equally distanced quantiles on the abscissa; therefore, the q-curve and the p-curve differ with respect to their corresponding values on the ordinate.

Consider now the q and p curves for taxes for grouped data. For the lower income strata, the five-percent quantiles contain more taxpayers in terms of p than in terms of q. In turn, this implies also more relative tax revenue for the p-curve as compared with the q-curve. Hence, for the lower income strata we have $F_T^Y(p_j) \geq F_T(q_j)$ for $p_j = q_j$. This means that the tax revenue associated with upper quantiles is smaller in terms of p than in terms of q. Hence, the p-curve for the taxes has a steeper slope than the q-curve for the lower quantiles and a flatter slope for the upper quantiles. But the q-curve can never cut the p-curve from below in the interior of the unit square. This is easily seen: obviously we have for the gross incomes $F_Y(q) \leq p = F_Y^Y(p)$ for $q = p$. Hence, for $q = p$ the gross income shares according to q can never exceed the gross income share according to p. For a progressive tax schedule this implies that $F_T(q) \leq F_T^Y(p)$ for $q = p$. Moreover, note that neither curve crosses the diagonal.

Consider the q and p curves of net incomes for grouped data. For the q-curves, basically the same reasoning applies as to Lorenz curves[29] because the net income shares increase for increasing quantiles. Hence, $F_{Y-T}(q_j)$ lies below the diagonal. For the p-curves, basically the same reasoning applies as for the analysis of inequality (18). For the individual data we can define an increasing series $k_1, k_2, \ldots, k_{20}$, such that

$$\frac{\sum_{i=1}^{k_j} Y_i}{n\mu} - \frac{\sum_{i=1}^{k_{j-1}} Y_i}{n\mu} \simeq 0.05, \; j = 1, \ldots, 20,$$

due to the fineness of the LIS data. Replace now k_j by j and

$$\left[\frac{\sum_{i=1}^{k_j}(Y_i - T_i)}{n(\mu - \tau)} - \frac{\sum_{i=1}^{k_{j-1}}(Y_i - T_i)}{n(\mu - \tau)}\right] \text{ by } F_{Y-T}^Y(p_j),$$

then (17) translates into our grouped data and, hence, inequality (18) translates into our grouped data as well.

This means that our analyses are carried out for Definitions 1 to 6 in terms of grouped data for q_j and p_j, where $j = 0, 1, \ldots, 20$.

Separation of tax progression with respect to the influence of the income distribution on the one hand and the tax schedule on the other was discussed in section 3.4.2. As concerns empirical data, we have to add another problem. Even for the case of identical monetary units, identical support of the income distribution, and confinement to the

[29] See Gastwirth and Glauberman (1976) for errors in estimating Lorenz curves when grouped date are used instead of individual data.

sufficient conditions, we would need the exact tax schedule to disentangle the influence of the income distribution and the tax schedule. However, the nominal tax schedule would not suffice for this purpose. We would have to capture also the administration of a going tax schedule. All tax codes contain many tax benefits, itemized deductions from the tax base, etc. While these instances are captured by the real micro data of taxation, they are not captured by the nominal tax schedule. Yet they constitute a major element of real tax progression. This was also one of our reasons to use the LIS data instead of simulated data, such as, e.g., the EUROMOD data. Hence, even under favorable theoretical conditions, decomposition of tax progression into a term dealing with the income distribution and a term dealing with taxation is limited by empirical data problems.

4.3 Numerical Progression Comparisons

Categorical data of progression relationships just inform on progression dominance, bifurcate progression dominance, or multiple crossings of the respective curves. Curve differences inform also on the intensities of progression dominance. We present first our categorical results and thereafter progression intensities for selected countries.

The categorical results of our international comparisons of tax progression are presented in Tables 2 to 7. For these tables we use the following notation:

D and d denote *progression dominance*: D means that the country in the row dominates the country in the column with respect to progression; d means that the country in the row is dominated by the country in the column with respect to progression. Note that the entries D and d are asymmetric: a D in cell (i, j) implies a d in cell (j, i), and vice versa. A superscript C indicates convexity or concavity of the associated relative concentration curve, i.e, that the respective sufficient conditions (elasticities) of Theorems 1 to 3 hold.

R and r denote *bifurcate progression*: R means that the country in the row is more progressive than the country in the column for the lower income strata and less progressive for the upper income strata; r means that the country in the row is less progressive than the country in the column for the lower income strata and more progressive in the upper income strata. Note again that the entries R and r are asymmetric.

$\#$ denotes multiple changes of the progression pattern.

4.3.1 International Progression Comparisons: Categorical Summary Results

Tables 2 to 7 present the results corresponding to Definitions 1 to 6. Each cell in these tables contains four entries: the entries in the first line refer to household data, the entries in the second line to equivalized data using the Luxembourg equivalence scales $m^{-\alpha}$ with $\alpha = 0.5$. The left-hand side entries in a cell refer to direct taxes only, while the right-hand

side entries to direct taxes plus payroll taxes (which consist mainly of employees' share in social security contributions).

Inspection of the cells of Tables 2 to 7 shows that only few left-hand side and right-hand side columns differ within each cell. Disregarding the C-superscripts, which just indicate that the respective curve differences have a unique extremum, still increases the similarity of the columns within each cell. This means that comparative progression is not changed much if we extend the direct taxes by payroll taxes. This is remarkable since payroll taxes are by and large proportional to the incomes for the lower and middle income strata. For incomes exceeding some benchmark payroll taxes on incomes beyond this benchmark expire altogether. Moreover, for the lower income strata they are usually considerably higher than the direct taxes. This implies that they have a regressive effect on overall tax progression (for the effect of payroll taxes in isolation see Peichl and Schäfer (2008, pp. 13-4)). On top of that, their structure may be different for different countries. An exceptional case is Denmark, where social security contributions are negligible, because most social expenditures are paid out of the tax revenue. Nevertheless, our results show few differences in comparative progression for direct taxes and direct taxes plus payroll taxes.

Basically the same observation applies to the rows within each cell. Although being less similar than the columns, most of them have similar appearance. This means that replacing taxes and net incomes for household data by equivalent taxes and equivalent net incomes does not cause dramatic changes in the comparative progression pattern, although the changes are more pronounced than comparisons between direct taxes only and direct taxes plus payroll taxes. Note that equivalized incomes and taxes tend to be more equally distributed than household incomes and taxes (cf. Peichl et al., 2009a,b). Countries with a higher proportion of large households are more likely to experience changes between the first and the second rows in the cells of the tables.

Tables 8 to 11 survey the results of the comparisons across Definitions 1 to 6. They show us that for international comparisons of uniform tax progression, progression dominance is the rule rather than the exception. In about to two thirds of all 312 international comparisons, uniform progression dominance holds.[30] Among these cases, for between

[30]Note that these are the results of our numerical calculations. As our curves do not draw on the data universe, but on samples only, the question arises whether curve crossings are statistically significant. Whereas Atkinson (1970, p. 258) had asserted that for comparisons among twelve countries "in only 16 out of 66 cases do the Lorenz curves not intersect", Bishop et al. (1991a, p. 462) found statistically significant intersections of Lorenz curves only in three percent of all cases, whereas 97 percent of the Lorenz curves were ranked; in contrast to that, simple numerical comparisons would have ranked only some 75 percent of the comparisons of Lorenz curves. This result holds under the assumption that the differences between the population Lorenz ordinate and the sample Lorenz ordinate are normally distributed (see also Bishop et al. (1991b), who showed impressive results also for the double criterion of Lorenz dominance and higher mean income). This means for our results that we might end up with even more dominance relations if we required curve intersections to be statistically significant. On the other hand, Dardanoni and Forcina (1999) showed using Monte-Carlo simulations that the statistical power

60.78 and 83.05 percent of progression dominance the respective curve differences have a single extremum, which means that the sufficient conditions of convexity or concavity of the associated relative concentration curves hold. However, in between 16.95 and 39.22 percent of cases do we observe that the sufficient conditions of Theorems 1 to 3 do not hold, although the associated relative concentration curves do not cross the diagonal in the unit square.

Uniform progression dominance does not hold for about one third (exactly 32.69 percent) of all cases only. Among these cases we observe between 65.38 and 84.21 percent for which bifurcate progression holds. This means that in these situations there is a unique benchmark such that uniform progression dominance for one country holds up to this benchmark and reverses for the quantiles exceeding this benchmark for the other country. With respect to all cases this means that only for between 3.85 and 11.54 percent do we not have clear-cut patterns of progression dominance: the associated relative concentration curves have in these cases multiple crossings with the diagonal of the unit square or, alternatively, the respective curve differences change their sign more than once. This is negligible as compared with the two thirds of all cases in which uniform progression dominance holds and with more than one fifth of all cases in which bifurcate progression holds.

Progression dominance is a transitive relation. Hence, we can arrange the strict progression dominance relationship of Tables 2 to 7 in terms of Hasse diagrams, which are presented in Figures 3 to 26. We have Hasse diagrams for each of our six definitions and each of our four data sets, which produces 24 figures. At the top of these figures we find the countries with the highest progression dominance, at the bottom those countries which are progression dominated by most other countries, but do not progression dominate other countries. Except for Figures 5, 11, 17, and 23, we find Switzerland and Poland at the bottom, and Denmark, Sweden, Norway, Finland, and the United Kingdom in the lower parts of the figures. At the top we find Germany, the United States, Australia, Canada, and the Netherlands. Only Figures 5, 11, 17, and 23 put everything upside down. All these figures concern Definition 3, which turns out as an outlier among our six definitions. This phenomenon will subsequently crop up also in our other findings.

4.3.2 International Progression Comparisons: Consistency among Progression Concepts

Tables 12 and 13 report the robustness of progression comparisons across our six definitions. These tables show us that full consistency is rather rare: it varies between 2.56 and 8.97 percent for progression dominance[31] and between 5.13 and 15.38 percent for

of the Lorenz curve as a test device is low. However, in the present paper our focus is on the numerical analysis. The respective statistical tests will be conducted in our future research.

[31] A 100 percent full consistency for a set of definitions would mean that any one definition out of this set can represent the rest, while the rest would become superfluous because all measure the same

bifurcate progression. Note that full consistency is highest for taxes plus payroll taxes for equivalized data. However, when we look for at least four consistencies, the consistency rates increase to numbers between 34.64 and 48.71 percent for progression dominance and to numbers between 62.22 and 70.52 percent for bifurcate progression.

Table 14 contains rough indications of similarities and dissimilarities among the progression comparison concepts.[32] This table is based on pairwise comparisons of the entries in Tables 2 to 7 counting the dominance and bifurcate relationships which are identical between pairs of tables. The first entries in the cells of Table 14 contain the percentages (as averages of all four datasets) of congruence of the respective D's and R's in the cells of the pairs of the compared tables, the second entries contain the percentages (as averages of all four datasets) of cases in which a D or R is not matched by the respective symbol in the other table.[33] Multiple crossings were ignored. We observe strong similarity of 83.66 percent between Definitions 1 and 2, although Definition 1 is expressed in terms of q and Definition 2 in terms of p; note that both concern definitions in terms of taxes. Strong similarities are also observed between Definitions 1 and 4 (63.46 percent), Definitions 1 and 5 (65.06 percent), Definitions 2 and 4 (60.90 percent), Definitions 2 and 5 (67.62 percent), Definitions 4 and 6 (69.23 percent), and Definitions 5 and 6 (58.97 percent). The similarities between Definitions 1 and 2 on the one hand, and Definitions 5 on the other, can be explained by their common casting in terms of taxes. The similarities between Definitions 4 and 6 can be explained because they are cast in terms of net incomes. However, the similarities between Definitions 1 and 2 on the one hand, and Definition 4 on the other would not have been expected a priori because Definitions 1 and 2 are cast in terms of taxes and Definition 4 is cast in terms of net incomes. Moreover, Definition 1 is based on q, whereas Definitions 4 is based on p. Hence, Table 14 suggests that we have two (overlapping) similarity clusters: Definitions 1, 2, 4, and 5 on the one hand, and Definitions 4, 5, and 6 on the other. This is also confirmed upon checking higher-order consistencies (not reported in the present text).

Table 14 demonstrates also the particularity of Definition 3, which has very high rates of dissimilarity with the other definitions. It behaves as a complete outlier, which indicates that it measures phenomena which are dissimilar as compared with the other definitions. Interestingly enough, even Definition 4, which is, on theoretical grounds, the one most related to Definition 3, exhibits lower dissimilarity rates with respect to all other definitions than does Definition 3.

Let us for a moment set aside Definition 3 and concentrate on the other definitions. When looking at Tables 2-3 and 6-7 as well as at Figures 3-26 (except Figures 5, 11,

phenomenon. Hence, less than full consistency justifies a multiple of measures for progression comparisons because all measure different traits.

[32]Note that consistency between our definitions depends on the data used. Hence, the present analysis reflects the data used for this study.

[33]Note that we consider a pair of definitions similar if it is either D or R [d or r], but not the opposite ($\sharp, d, r$) [$\sharp, D, R$], respectively.

17, and 23), it is striking to see that "high tax" (as conventionally perceived) countries like Sweden and Denmark, medium tax countries like the United Kingdom, and low tax countries like Switzerland and Poland are all classified as less progressive than most other countries. Furthermore, high tax countries like Germany, medium tax countries like France, and low tax countries like the United States are classified as more progressive than most other countries. This is because the measures in this paper are developed for comparing *uniform tax progression*, not the *level of taxation*. Sweden and Denmark have taxes that reach a high percentage of income rather fast and remain there, which is more akin to proportional taxation; the same pattern applies to the United Kingdom for a medium tax burden, and to Switzerland and Poland for a low tax burden. In contrast to that, the income interval for which taxation is steadily increasing as a percentage of income is comparatively extensive in Germany, in France, and in the United States. This explains their dominance with respect to comparisons of tax progression. For similar results using another approach see Peichl and Schäfer (2008, pp. 8-12).

Let us now focus on the discrepancy between Definition 3 and the other definitions. To illustrate, we single out the case of Definitions 3 and 4 which have a high inconsistency rate of 71.47 percent according to Table 14, although both definitions should be closely related on theoretical grounds.

First of all we check whether both definitions can command plausibility. They can: if $F_{Y^1-T^1}(q_k) > F_{Y^2-T^2}(q_k)$ for all $q_k,\ 0 < q_k < 1$, then the $q_k \times 100$ percent of the lowest income recipients have relatively higher aggregate net income under (Y^1, T^1) than under (Y^1, T^2); if $F^Y_{Y^1-T^1}(p_k) > F^Y_{Y^2-T^2}(p_k)$ for all $p_k,\ 0 < p_k < 1$, then the $p_k \times 100$ percent of the recipients of the lowest aggregate gross income have relatively higher aggregate net income under (Y^1, T^1) than under (Y^1, T^2). Hence, both definitions stand a plausibility test.

Consider now the cells US/CH in Tables 4 and 5. They show for all four entries that Switzerland has higher uniform progression than the United States according to Definition 3, whereas the United States has higher uniform progression than Switzerland according to Definition 4. Let us start with the fictitious assumption that, for whatever reasons, Switzerland had an equal distribution of gross incomes and a proportional tax, whereas gross and net incomes were unequally distributed in the United States. Then we know from Section 3.4.1 that both $F_{Y^{CH}-T^{CH}}(q_k)$ and $F^Y_{Y^{CH}-T^{CH}}(p_k)$ are equal to the diagonal of the unit square, the first by the assumptions of an equal distribution of gross incomes and a proportional tax, and the second by the assumption of a proportional tax. However, this case implies higher uniform progression of the income-distribution-cum-tax system in Switzerland than in the United States according to Definition 3 and higher uniform progression in the United States than in Switzerland according to Definition 4. Hence, in this case Definition 4 is accurate, while Definition 3 is misleading. For moderate tax progression in one country (in this case: Switzerland) and higher tax progression in another country (in this case: the United States), this misleading statement of Definition

3 seems to translate to cases for which the distribution of gross incomes in one country (Switzerland) is sufficiently more equally distributed than in another country (United States).

Suppose now that the population principle applies and that the same distribution of gross incomes holds in both countries.[34] Then

$$F_{Y-T^1}(q_k) \geq F_{Y-T^2}(q_k) \Leftrightarrow F^Y_{Y-T^1}(p_k) \geq F^Y_{Y-T^2}(p_k), \tag{19}$$

if $T^1(\cdot)$ is more progressive than $T^2(\cdot)$. Note that expression (19) follows immediately from Section 3.4.1.[35] Hence, Definitions 3 and 4 are equivalent for identical gross income distributions and greater progression depends only on the tax schedules involved.

Let us now consider another extreme case, viz. total progression. A totally progressive tax schedule consists of just two marginal tax rates, $+100\%$ and -100%.[36] All incomes above mean income are taxed away and all incomes below mean income are replenished by transfers up to mean income. Hence, the net income distribution is equal. Suppose, Switzerland had a totally progressive tax schedule, while the United States had some more common progressive tax schedule. Then Definition 3 correctly indicates higher progression for Switzerland than for the United States. So does Definition 4, since $\tau^{CH} = 0$ for total progression and by assumption $\tau^{US} > 0$:

$$\frac{k\mu^{CH}}{n\mu^{CH}} = \frac{k}{n} \geq \frac{\sum_{i=1}^{k}(Y_i^{US} - T_i^{US})}{n(\mu^{US} - \tau^{US})};\ Y_i^{US} \geq T_i^{US}.$$

Therefore, it seems that the ostensibly deviant behavior of Definition 3 results from its high sensitivity to the influence of the distribution of gross incomes, which may easily overcompensate the influence of the tax schedule and lead to implausible results. In contrast to Definition 3, Definition 4 is less susceptible to the preponderance of the distribution of gross incomes vis-á-vis the tax schedule. Hence, Definition 3 has to be taken with some caution. It is too sensitive with respect to more equal distributions of net incomes irrespective of how they came about.

4.3.3 International Progression Comparisons: Progression Intensities for Selected Countries

Tables 2 to 7 contain categorical data only; they just report whether we have uniform progression dominance, bifurcate progression or multiple crossings of the associated rela-

[34] Switzerland's population is about two percent of the population of the United States. An equal income distribution in both countries in discrete terms requires therefore 50 income clones in the United States for each Swiss income recipient. Take an income distribution consisting of one representative out of the 50 US clones, then both income distributions are identical under the population principle.

[35] The second part of expression (19) may not seem immediate. Consider $F^Y_{Y-T^1}(p_k) \geq F^Y_{Y-T^2}(p_k)$. Then there corresponds a unique q_k to each p_k (recall that $q_k \geq p_k$). Since the same set of taxpayers has higher relative aggregate net income under p_k for T^1, it must also have higher relative aggregate net income under q_k for T^1, as the distribution of gross incomes is the same.

[36] Note that for several reasons we exclude negative taxes for most of our analysis. We use this example

tive concentration curves. They are silent about the intensity of progression comparisons. Information about this aspect is provided by the 312 graphs of international progression comparisons, where each graph corresponds to one out of the four entries in each cell of Tables 2 to 7 and contains six curves corresponding to the six concepts of comparison of progression as embodied in these six tables. From this set we single out sixteen for presentation in this paper, of which four cover the comparisons between the United Kingdom and the United States, four the comparisons between Germany and the United Kingdom, four the comparisons between Germany and the United States, and four the comparison between Germany and Sweden.

United Kingdom versus United States Figure 27 depicts the progression comparisons for UK 1999 versus US 2000 for the direct taxes and household data. In accordance with Tables 2 to 7 we see that the US progression dominates the UK for all definitions except Definition 3. For Definition 3 tax progression is at first higher in the UK than in the US and switches at $q = 0.55$ to higher progression for the US than for the UK. Moreover, note that the curves for Definitions 3, 4, and 6 show much weaker intensity than progression dominance according to the other definitions.

Figure 28 depicts the progression comparisons for UK 1999 versus US 2000 for the direct taxes plus payroll taxes and household data. The pattern is the same as for Figure 27 except that the inclusion of payroll taxes now makes the intensity of progression somewhat less pronounced, as the entries on the ordinate show. Figure 29 depicts the progression comparisons for UK 1999 versus US 2000 for the equivalized direct taxes and equivalized incomes. In accordance with Tables 2 to 7 we see that the US progression dominates the UK for all definitions except Definition 3. For Definition 3 tax progression is uniformly higher in the UK than in the US. Figure 30 repeats this pattern for equivalized direct taxes plus payroll taxes and equivalized incomes.

Germany versus United Kingdom Figure 31 depicts the progression comparisons for Germany 2000 versus UK 1999 for direct taxes and household data. In accordance with Tables 2 to 7 we see that German progression dominates the progression in the United Kingdom for all definitions.

Figure 32 depicts the progression comparisons for Germany 2000 versus UK 1999 for the direct taxes plus payroll taxes and household data. Uniformly greater progression for Germany vis-á-vis the UK is again observed for all definitions except Definition 1. For Definition 1, Germany is more progressive except for the highest 15 percentiles, for which progression switches to the UK. Note that the ordering of progression dominance changes when going from direct taxes to direct and payroll taxes: progression dominance according to Definition 3 is moderate for direct taxes, but becomes far more pronounced for direct and payroll taxes, possibly because payroll taxes render the German distribution

here to compare equally distributed net incomes resulting from unequally distributed gross incomes.

of net incomes more unequal. Progression dominance according to Definitions 1, 2, and 5 becomes less pronounced for direct and payroll taxes (Definition 1 even acts partly in the direction of less progression for Germany). Figures 33 and 34 repeat this pattern also for the equivalized data.

Germany versus United States Figure 35 shows higher progression of the German income-distribution-cum-tax system for Definitions 2, 3, and 5 (except the 95^{th} percentile) for direct taxes. Definitions 4 and 6 do not bear a pronounced message. Definition 1 indicates higher progression for Germany up to $q \in (0.6, 0.65)$ and higher progression for the United States for the upper income strata.

With respect to direct and payroll taxes, Figure 36 shows higher progression for Germany only for Definition 3, whereas Definitions 1 and 2 indicate higher progression for the United States. Definition 5 indicates higher progression for Germany up to $q \in (0.55, 0.6)$ and higher progression for the United States for the upper income strata. These shifts reflect the influence of the higher importance of payroll taxes in Germany. Again Definitions 4 and 6 do not bear a pronounced message.

The equivalized data in Figures 37 and 38 completely repeat the pattern of Figures 35 and 36, respectively.

Germany versus Sweden It is interesting to compare Germany and Sweden, as Sweden is notorious for her high tax burden. Figure 39 shows progression comparison for direct taxes and household data. We see that Germany is uniformly more progressive for Definitions 1, 2, and 5. The other definitions are of no major importance; they also indicate slightly more progression for Germany. This pattern is repeated for direct taxes and payroll taxes in Figure 40, but now with only about half the intensity as for Figure 39 (compare the entries on the ordinates).

This pattern is repeated also for the equivalized data in Figures 41 and 42, except that Sweden appears now slightly more progressive according to Definition 3, which might be a reflection of larger family sizes in Sweden.

Hence, the general picture exhibits greater progression in Germany as compared to Sweden, although Sweden's relative tax burden is higher than Germany's.[37]

4.3.4 Intertemporal Progression Comparisons

We have data for intertemporal progression comparisons for all selected countries except France. However, because of space limitations we restrict ourselves here to just three

[37] Eurostat, Tables tec00018 and tec00019, tell us the percentages of taxes from income and wealth and social security contributions (in square brackets; employees' and employers' contributions taken together) of GDP for 2008 (selected figures): EU 27 mean 13.1 [13.7]; EU 15 mean 13.5 [13.9]; Belgium 16.6 [16.1]; United Kingdom 16.7 [8.4]; Sweden 17.4 [12.00]; Finland 17.5 [12.2]; Iceland 18.3 [2.8]; Norway 22.0 [8.9]; Denmark 29.8 [1.8]. The figures for Germany are 11.3 [16.4]. This shows that Germany is a low-tax country with respect to income and wealth taxes and a high-tax country with respect to social security

countries, viz. the United States, the United Kingdom, and Germany. Of course, we could cover only periods for which full sets of comparable data are available. The confines of data availability precludes too long time intervals.[38] Interestingly enough, the curve differences for intertemporal progression comparisons have much lower intensity than the curve differences for international progression comparisons. They are in the order of between one fifth and one tenth of the curve differences for international progression comparisons. This fact is somewhat obscured by curve calibration, but may readily be seen from the ordinates of the respective figures.

United States For the United States we have data of waves 1991, 1994, 2000, and 2004. Wave 1991 was in the mid-term of the Bush Senior Administration (1989-93). In his election campaign, Bush Senior had promised: "Read my lips: no new taxes." However, he broke his election pledge, it seems for financing the first Iraq War, and was not re-elected. Waves 1994 and 2000 concern the Clinton Administration (1993-2001). Clinton took over office on January 20, 1993, which ended on January 20, 2001. Clinton was very successful in reducing budget deficits, promoting NAFTA, and getting on good terms with China and Russia. On August 5, 1997, Clinton signed the Tax Relief Act, which meant a significant tax cut effective as of January 1, 1998. Moreover, his administration was a peaceful period. The very end of his administration was overshadowed by the dotcom crisis. Wave 2004 concerns the fourth year of the Bush Junior administration. Bush Junior and Cheney were supposed of favoring the upper income strata of the American society. Moreover, this period was overshadowed by the second Iraq War, which was largely financed by high budget deficits. Hence, the progression changes 1991/94 (Bush Senior to Clinton), 1994/2000 (beginning and end of the Clinton Administration), and 2000/04 (Clinton to Bush Junior) are of interest.

Table 15 provides a concise summary on the categorical data of comparisons of tax progression. We use the following notation: Bsen means "Bush Senior", C* means "Clinton 1994", C** means "Clinton 2000", and Bjun means "Bush Junior". An arrow means that the first entry is taken as a starting point and the second entry as the end point of the comparison. For instance, the entry d [D] under 91→94 and Bsen → C*, respectively, means that the tax system in 1991 under the Bush Senior Administration was less progressive [more progressive] than in 1994 under the Clinton Administration. The first three double columns in Table 15 concern the comparisons over adjoining periods, the second three double columns concern comparisons over longer periods. The twelve Figures 43 to 54 are the associated graphs for the first three double columns of Table 15. The remaining graphs are available from the authors upon request.

contributions; as concerns social security contributions, it is topped only by France [17.9] among all 25 member countries of the EU and the two associated countries Iceland and Norway.

[38] Piketty and Saez (2003) and Piketty (2003) endeavored to study income inequality for extremely long periods, relying on income tax data. As income taxes affected in their first decades only the top income strata, they had to confine their investigations to the top income decile of the respective countries.

Since Table 15 contains categorical data only, we draw on Figures 43 to 46. They exhibit a higher progression for 1991 than for 1994 for Definition 3 and a higher progression for 1994 than for 1991 for Definition 1. Definitions 2 and 5 (this one for the upper income strata) also tend to higher progression for 1994 than for 1991. The other definitions show no major progression intensity. The deviant behavior of Definition 3 may be a reflection of a slightly more equal distribution of gross incomes in 1991 than in 1994.

The picture of the comparison C* → C** (see Figures 47 to 50) is less clear-cut. The graphical picture is rather disparate. Whereas Definitions 2 (for the upper income strata) and 3 signal higher progression for 1994 than for 2000, Definition 1 signals for the upper income strata a higher progression for 2000 than for 1994. The lower progression for 2000 according to Definition 3 signals a more equal distribution of gross incomes in 2000 than in 1994. Definition 1 signals a more progressive tax schedule in 2000 than in 1994, which seems to have overcompensated the more equal distribution of gross incomes. The relative shift of aggregate income from the upper to the lower income strata due to a more equal gross income distribution seems to have overcompensated the more progressive tax schedule in 2000 according to Definition 2. But, taken as a whole, the Clinton 1997 tax reform did not dramatically change the pattern of tax progression. It was a more a general tax cut than a change of tax progression. Thus, the Clinton 1997 tax reform did not benefit the lower income strata in relative terms.

The picture of the comparison C** → Bjun (see Figures 51 to 54) shows uniformly less progression for 2004 as compared to 2000 for Definitions 3, 4, and 6. Note that these three definitions are cast in terms of net incomes. As concerns the definitions in terms of taxes, the picture is disparate. For Definitions 1, 2, and 5 there are crossings for direct taxes and household data, but by and large greater progression for 2000 than for 2004 for household data and direct and payroll taxes. For equivalized data and direct taxes Definitions 1, 2, and 5 indicate less progression in 2000 than in 2004, whereas they do not provide a clear-cur picture for equivalized data and direct taxes and payroll taxes. Remeber, however, the small intensity of progression for intertemporal comparisons.

Concerning other comparisons, we refer to the last three columns of Table 15. The progression between the two Bush Administrations show either more progression under Bush Senior (Definition 3), or for Definitions 1 (direct taxes and payroll taxes), 5, and 6 lower progression for the upper income strata under Bush Junior. Concerning early Clinton to Bush Junior, we find either generally less progression under Bush Junior, or less progression for the upper income strata. The comparison between Bush Senior and late Clinton does not provide an explicit picture.

United Kingdom For the United Kingdom we have data of the waves 1991, 1995, 1999, and 2004. With respect to fiscal policy, the last two years before 1991 witnessed Margaret Thatcher's unfortunate replacement of local community taxes by a poll tax, called "community charge". It was introduced in Scotland in 1989 and in England and Wales in 1990.

It led to serious unrest among the population: in March 31, 1990, there was a demonstration on Trafalgar Square with more than 100,000 protesters. Margaret Thatcher, who became Prime Minister on May 4, 1979, resigned on November 28, 1990. Her successor became John Major, whose terms of office ended on May 2, 1997. The United Kingdom slid into an economic recession in the period 1990-3. In addition the Black Wednesday on September 16, 1992, occurred during the Major Government: by short selling sterling, George Soros succeeded in a sweeping speculation against the British Pound, which caused a loss of £3.4 billion for Britain. On January 27, 1991, Britain participated in the Gulf War joining the UNO forces with 53,462 soldiers. Major succeeded in overcoming Britain's economic recession: the number of unemployed had decreased from 3 million in 1993 to 2 million in 1996. However, Major had problems with the "Euro-Rebels" in his party. After a catastrophic loss of the Conservatives in the 1997 elections, Major resigned. His follower was Tony Blair of the Labour Party. His office extended from May 2, 1997, to June 27, 2007. He was Prime Minister for the three periods 1997-2001, 2001-5, 2005-7. Blair increased public expenditure for education and health-care; at the same time he planned to reduce budget deficits. After his victory in the 2001 elections he increased taxes. In 2003 he sided the Bush Junior Administration in the Iraq War with 46,000 soldiers, about one third of Britain's military force. Blair introduced minimum wages and reforms to strengthen the private sector of the British economy. Because of incorrect information of the public concerning the reasons for participating in the Iraq War, Blair had to resign.

Table 16 shows rather clear-cut results: for most three double columns, viz. 95→99, 99→04, and 91→04, we observe less progresion of the tax system for the later year, with the notorious exception of Definition 3 for the two last periods. As Table 16 is explicit enough for these cases, we can dispense with presenting the respective graphs. The situation is more intricate for the change from Thatcher to Major, viz. 91→95: Figures 55 to 58 impart the impression of less progression for the 1995 tax system than for 1991. This appears for household data and for Definitions 1, 2, and 5, but not for equivalized data. Definition 3 shows lower progression in 1991 for the lower income strata for household data; it shows higher progression in 1991 throughout for equivalized data.

Germany For Germany we have data of the waves 1989, 1994, and 2000. In Germany, there was a big tax reform extending from 1986 to 1990 (see Seidl and Kaletha (1987)). Hence, 1989 was the last year before the end of the tax reform. November 9, 1989, was the date of the fall of the Berlin Wall. Thereafter the German Democratic Republic existed until October 3, 1990, when the German unification took place. Hence, 1989 was the last year before the 1986-90 tax reform was completed, and the last year of West Germany as a self-contained state. Kohl had become Chancellor of Germany on October 1, 1982, and resigned on October 27, 1998, after an electoral defeat. Then Schröder succeeded him in office. Kohl had survived the 1994 elections as chancellor only after severe electoral losses.

For the purpose of financing the German unification, a solidarity surcharge on top of the income tax was introduced on July 1, 1991, and expired on June 30, 1992. It amounted to 7.5 percent of the income tax. As the German unification proved more expensive than originally anticipated, it was reintroduced in 1995 amounting again to 7.5 percent of the income tax; in 1997 it was reduced to 5.5 percent. The top tax rate for business income was reduced to 47 percent as of January 1, 1994. In 1997, another tax reform was enacted (see Seidl and Traub (1997)). On July 6, 2000, the German Parliament enacted another tax reform extending from January 1, 2001 to January 1, 2005.

Germany shows us a development opposite to that of the United Kingdom. Whereas in the United Kingdom the tax system became less progressive in later time, the German tax system became more progressive in the lapse of time. Except some negligible intersections and except Definition 3, the Figures 59 to 70 show that the German tax system became between 1989 and 2000 more progressive for the lower income strata and less progressive for the upper income strata. Definition 3 makes here again for its notorious exception. Thus, in spite of all reforms to cut minimum and top marginal tax rates, the German tax system had become more progressive in the period 1989 to 2000, at least for the lower income strata; interestingly enough, it had also partly become less progressive for the upper income strata for taxes plus payroll taxes. This reflects that the cuts in top marginal tax rates for the high income strata overcompensated the relative increase in social security contributions which had taken place in this time period.

4.4 Equivalence Scales Matter

Finally we checked the influence of equivalence scales on uniform tax progression. For this purpose we started with choosing $\alpha = 0.25$ for the LIS equivalence scale $m^{-\alpha}$ and let α increase to 0.5 and further to 0.75. It is interesting to see that equivalence scales do indeed matter for comparisons of greater tax progression. Interestingly enough, the curve differences for progression comparisons for equivalence scales have, akin to the curve differences for intertemporal progression comparisons, much lower intensity than for international progression comparisons. They are in the order of between one fifth and one tenth of the curve differences for international progression comparisons. This fact is somewhat obscured by curve calibration, but may readily be seen from the ordinates of the respective figures.

We investigated the influence of the effects of different values of the scale parameter α on progression comparisons. We selected the United States, the United Kingdom, and Germany, and checked the effects of choosing 0.25, 0.5, and 0.75 for α. We present selected graphs in Figures 71 to 96. The graphs differ both from country to country and for different comparisons within a country. Interestingly enough, we observe particular patterns of progression comparisons for certain parameter combinations which are rather robust for different time periods within a country.

Concerning the United States, Figures 71 to 78 show characteristic patterns of pro-

gression comparisons for different parameter pairs. Figures 71 to 74 depict progression changes of $\alpha = 0.25$ versus $\alpha = 0.50$. Whereas the graphs for 1991 and 1994 show a trapezoidal shape with the curves of Definitions 1 and 3 largely at the frontiers, the graphs for 2000 and 2004 show the shape of a bath tub with all curves decreasing for the lower quantiles and increasing for the higher quantiles. The curves for $\alpha = 0.50$ versus $\alpha = 0.75$ and $\alpha = 0.25$ versus $\alpha = 0.75$ are pear-shaped with the graph of Definition 3 at the upper and the graph of Definition 1 at the lower frontier (see Figures 75 to 78).

For the United Kingdom we observe again bath-tub-shaped curve families for $\alpha = 0.25$ versus $\alpha = 0.50$ (see Figures 79 to 81). The curve families for $\alpha = 0.50$ versus $\alpha = 0.75$ are basically also pear-shaped with the curve of Definition 3 largely at the top and the curve of Definition 1 largely at the bottom, but the pears are more clumsy than the ones for the United States; in particular they have a thicker end with reversed roles of Definitions 1 and 3. The curve families for $\alpha = 0.25$ versus $\alpha = 0.75$ are butterfly-shaped with Definition 1 representing the upper frontier for the lower quantiles and the lower frontier for the upper quantiles. Definition 3 has exactly the opposite pattern (see Figures 86 to 88).

For Germany we observe again bath-tub-shaped curve families for $\alpha = 0.25$ versus $\alpha = 0.50$, the curve of Definition 1 making largely for the top and the curve for Definition 3 by and large for the bottom (see Figures 89 to 91). The curve families for $\alpha = 0.50$ versus $\alpha = 0.75$ are basket-cap-shaped rather than pear-shaped. They show several curve intersections with most curves decreasing except the curve of Definition 3 (see Figures 92 and 93). The curve families for $\alpha = 0.25$ versus $\alpha = 0.75$ also look a bit like being butterfly-shaped, although this picture is disturbed by the curve of Definition 2 (see Figures 94 to 96).

Thus, we can draw the conclusion that equivalence scales matter for progression comparisons, although it is not easy to discover a general pattern of their mode of operation. Their influence changes between countries and between the pairs of scale parameter values which are juxtaposed. It is in particular this last influence which is perhaps the most robust behavioral pattern of the influence of equivalence scales on progression comparisons.

5 Conclusion

This paper starts with a concise review of methods of measuring and comparing tax progression. Local measures of tax progression suffer from their neglect of the income distribution, global measures of tax progression suffer from the disadvantage of all aggregation, viz. that much information is lost in the aggregation procedure, and uniform comparisons of tax progression suffer from their assumption that the same income distribution has to hold for all situations to be compared. Based on uniform comparisons of tax progression, Seidl (1994) proposed that, instead of comparing tax schedules and income

distributions in terms of incomes, they should be compared in terms of population and income quantiles. This approach replaces the different supports of income distributions by the distributions of population or income quantiles, whose support is the unit interval. This allows constructing relative concentration curves and curve differences by using the same values of population or income quantiles for the situations to be compared.

We use this approach in empirical research. We investigate uniform tax progression comparing 13 countries from the LIS database, which have, of course, different tax and payroll tax schedules, as well as different income distributions. We employ six different measurement devices of progression comparison, three in terms of taxes and three in terms of net incomes. Our analyses are carried out for household data and for equivalized data using the Luxembourg equivalence scale with parameter value $\alpha = 0.5$. Although we expected more intricate patterns to be the case rather than dominance relations, we observe uniformly greater tax progression in about two thirds of all cases. Out of these cases about two thirds can be thought of (in continuous terms) as having convex or concave relative concentration curves, which means that for those cases the sufficient conditions of Theorems 1 to 3 hold (see Figure 1). In terms of first-order or second-order curve differences this case is represented by a single extremum. For about one third of those cases we observe progression dominance without concavity or convexity of the associated relative concentration curves. In terms of first-order or second-order curve differences, this case is represented by non-intersections , with the abscissa and multiple extrema of the respective curve differences. For about one fifth of all cases we observe bifurcate progression, i.e., higher progression for one country up to a certain threshold and higher progression for the other country beyond this threshold. Only for about one tenth of all international comparisons do we observe interlaced progression patterns.

Note that the proposed methodology compares tax progression, not the level of the tax burden. This means that not all high-tax countries dominate with respect to progression. This is in particular the case for Scandinavian countries which reach a high tax level already at comparatively modest incomes; hence, their tax schedules do not emerge as notedly progressive. On the other hand, low-tax countries like the United States and high-tax countries like Germany emerge as rather progressive because the increase in taxation extends over longer intervals. Based on actual data, some of the progression concepts show consistencies. This is in particular the case for Definitions 1 and 2, but also, and to a lesser extent, for Definitions 1 on the one hand, and Definitions 4 and 5 on the other. Consistency is also observed for Definition 2 on the one hand, and Definitions 4 and 5 on the other. Other consistencies are registered between Definitions 4 and 6. We observed two clusters of high consistency, one comprising Definitions 1, 2, 4, and 5, and another one comprising Definitions 4, 5, and 6 (see Section 3.3). Definition 3 jars with the other definitions; it has high rates of dissimilarity. It is remarkable that Definition 3 is at variance even with Definition 4, although both are defined in terms of net incomes. Formulations in terms of income quantiles seem to be more compliant

both among themselves and among the other progression concepts except Definition 3. Tables 2 to 7 show the categorical data of international comparisons of tax progression. The strict progression dominance relationships are arranged in terms of Hasse diagrams in the 24 Figures 3 to 26.

This is followed by sixteen graphs for comparisons among the United States, the United Kingdom, Germany, and among Germany and Sweden. We see that, except for Definition 3, both Germany and the United States have more progressive tax systems than the United Kingdom. For progression comparisons between Germany and the United States, we find clear tendencies indicating higher progression for Germany than for the United States. Comparing Germany and Sweden shows by and large greater progression for Germany, although Sweden has the higher tax level. In several cases we observe bifurcate progression, or multiple crossings with the abscissa. Progression comparison in this case depends very much on the data set and on the progression concept applied.

Then we conduct intertemporal comparisons of tax progression for the United States, the United Kingdom, and Germany. We did not find a clear-cut picture for the period 1991 to 2004 for the United states except a weak tendency towards less progression for the upper income strata under the Bush Junior Administration. The developments in the United Kingdom and in Germany are more clear-cut: while the tax system became less progressive in the United Kingdom in the lapse of time, it became more progressive in Germany except the highest income stratum.

Finally, we investigate the influence of the equivalence scales on comparisons of tax progression. We observe rather different effects of changes of the parameter values of the equivalence scale depending on the country and, within a country, on the parameter values of the equivalence scales considered. There seems to be an antagonistic influence of Definitions 1 and 3 with respect to the scale parameter. The curve families of progression comparisons are by and large bath-tub-shaped or trapezoidal-shaped for $\alpha = 0.25$ versus $\alpha = 0.50$, pear-shaped or basket-cap-shaped for $\alpha = 0.50$ versus $\alpha = 0.75$, and pear-shaped or butterfly-shaped for $\alpha = 0.25$ versus $\alpha = 0.75$. This seems to be a rather robust pattern which can be observed for different countries and, even more so, for different time periods within the same country. More general patterns are difficult to discover.

Hence, this paper shows that different tax schedules and different income distributions are no obstacle to international and intertemporal comparisons of tax progression in terms of dominance relations. This approach enables more detailed judgments than, for instance, global measures of tax progression. Uniform comparisons of tax progression inform about the structure of tax progression, e.g., whether the tax system of a country or a time period is more progressive at the lower or at the upper end of the income strata, or whether it dominates the tax system of another country or period throughout. On top of this categorical information, our graphs of pairwise country or time period comparisons also provide cardinal information about the intensity of greater or smaller tax progression in terms of the shape of the respective curves and the areas below these curves.

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Table 2: Progression (non)dominance according to Definition 1
Compares (T^1, Y^1) for the country in the row with (T^2, Y^2) for the country in the column for Definition 1 (taxes, parameter: q) and vice versa

	au01		ca00		dk00		fi00		fr94		de00		nl99		no00		pl99		se00		ch00		uk99		us00	
au01	-		D	D	D^C	D^C	D^C	D	R	R	d^C	D^C	R	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D	D	$\#$	R
			$\#$	D^C	D^C	D^C	D^C	D^C	R	R	d^C	D^C	R	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D	D^C	$\#$	R
ca00	d	d			D^C	D^C	D	D	R	R	d^C	$\#$	R	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D	D	d^C	$\#$
	$\#$	d^C	-		D^C	D^C	D^C	D^C	R	R	d^C	D	R	D	D^C	D^C	D^C	D	D^C	D^C	D	D^C	D	D	d^C	R
dk00	d^C	d^C	d^C	d^C			d^C	d^C	d^C	d^C	d^C	d^C	d^C	d	d	d	D^C	D^C	$\#$	$\#$	R	D^C	d^C	d^C	d^C	d^C
	d^C	d^C	d^C	d^C	-		d^C	d^C	d^C	d^C	d^C	d^C	d^C	d	d^C	d	r	r	d	d	R	D	d^C	d^C	d^C	d^C
fi00	d^C	d	d	d	D^C	D^C			d^C	d^C	d^C	d^C	d^C	r	r	r	D^C	D^C	D	D	D^C	D^C	R	R	d^C	d^C
	d^C	d^C	d^C	d^C	D^C	D^C	-		d^C	d^C	d^C	d^C	d^C	D	r	D^C	D^C	D	$\#$	D^C	D	D	R	R	d^C	d^C
fr94	r	r	r	r	D^C	D^C	D^C	D^C			r	D	$\#$	D^C	r	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D	D	r	r
	r	r	r	r	D^C	D^C	D^C	D^C	-		r	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	r	r
de00	D^C	d^C	D^C	$\#$	D^C	D^C	D^C	D^C	R	d			R	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	R	R	d^C
	D^C	d^C	D^C	d	D^C	D^C	D^C	D^C	R	d^C	-		R	D	D^C	D^C	D^C	D	D^C	D^C	D	D^C	D^C	R	R	d^C
nl99	r	d^C	r	d^C	D^C	D	D^C	R	$\#$	d^C	r	d^C			r	$\#$	D^C	D	D^C	$\#$	D^C	D^C	D^C	R	d	d^C
	r	d^C	r	d	D^C	D	D^C	d	d^C	d^C	r	d	-		D^C	$\#$	D^C	D	D^C	$\#$	D	D	D^C	R	d	d^C
no00	d^C	d^C	d^C	d^C	D	D	R	R	R	d^C	d^C	d^C	R	$\#$			D^C	D^C	D	$\#$	D^C	D^C	R	R	d^C	d^C
	d^C	d^C	d^C	d^C	D^C	D	R	d^C	d^C	d^C	d^C	d^C	d^C	$\#$	-		D^C	D	R	$\#$	D	D	R	R	d^C	d^C
pl99	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d	d^C	d^C			d^C	d	R	R	d^C	d^C	d^C	d^C
	d^C	d^C	d^C	d	R	R	d^C	d	d^C	d^C	d^C	d	d^C	d	d^C	d	-		$\#$	$\#$	R	D^C	R	R	d^C	d^C
se00	d^C	d^C	d^C	d^C	$\#$	$\#$	d	d	d^C	d^C	d^C	d^C	d^C	$\#$	d	$\#$	D^C	D			D	D	R	R	d^C	d^C
	d^C	d^C	d^C	d^C	D	D	$\#$	d^C	d^C	d^C	d^C	d^C	d^C	$\#$	r	$\#$	$\#$	$\#$	-		$\#$	D	R	R	d^C	d^C
ch00	d^C	d^C	d^C	d^C	r	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	r	r	d	d			d^C	d^C	d^C	d^C
	d^C	d^C	d	d^C	r	d	d	d	d^C	d^C	d	d^C	d	d	d	d	r	d^C	$\#$	d	-		d^C	d^C	d^C	d^C
uk99	d	d	d	d	D^C	D^C	r	r	d	d	d^C	r	d^C	r	r	r	D^C	D^C	r	r	D^C	D^C			d^C	d^C
	d	d^C	d	d	D^C	D^C	r	r	d^C	d^C	d^C	r	d^C	r	r	r	r	r	r	r	D^C	D^C	-		d^C	d^C
us00	$\#$	r	D^C	$\#$	D^C	D^C	D^C	D^C	R	R	r	D^C	D	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C		
	$\#$	r	D^C	r	D^C	D^C	D^C	D^C	R	R	r	D^C	D	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	-	

Legend. D (d) means that country *row* (*column*) dominates country *column* (*row*) in terms of concentration curves for taxes (Definition1). ^{C} indicates convexity (concavity) of the relative concentration curve (sufficient conditions from Theorems 1-3 hold). R (r) means that country *row* (*column*) dominates country *column* (*row*) for a lower quantile and is dominated by country *column* (*row*) for an upper quantile (just 1 intersection). # indicates more than 1 intersection. For each cell: upper-left and upper-right corners correspond to household-based analysis (taxes and taxes + other mandatory contributions, respectively); bottom-left and bottom-right corners correspond to equivalized-income-based analysis (taxes and taxes + other mandatory contributions, respectively).

Table 3: Progression (non)dominance according to Definition 2
Compares (T^1, Y^1) for the country in the row with (T^2, Y^2) for the country in the column for Definition 2 (taxes, parameter: p) and vice versa

	au01		ca00		dk00		fi00		fr94		de00		nl99		no00		pl99		se00		ch00		uk99		us00	
au01	-		R	D	D^C	D^C	D^C	D	R	R	d^C	D	R	D	D^C	D	D	D	D^C	D^C	D	D	D	D^C	$\#$	D
			R	D^C	D	D	D	D	R	R	d^C	D	R	D	D	D	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	$\#$	D
ca00	r	d			D^C	D^C	D	D	d^C	d^C	d^C	$\#$	R	D	D	D	D	D	D^C	D^C	D	D	D	D	r	$\#$
	r	d^C	-		D^C	D^C	D	D	d^C	d^C	d^C	$\#$	R	D	D	D	D	D	D^C	D^C	D	D^C	D	D	r	$\#$
dk00	d^C	d^C	d^C	d^C			d^C	r	d^C	d^C	d^C	d^C	d^C	d	r	r	D	D	$\#$	r	R	D	r	r	d^C	d^C
	d	d	d^C	d^C	-		d^C	r	d^C	d^C	d^C	r	d^C	d	r	r	r	r	d	r	R	D	r	r	d^C	d^C
fi00	d^C	d	d	d	D^C	R			d^C	d^C	d^C	d	d^C	d	D	D	D	D	R	R	R	D^C	$\#$	$\#$	d^C	d^C
	d	d	d	d	D^C	R	-		d	d^C	d^C	d	d^C	$\#$	D	D	D^C	D^C	R	R	R	D^C	D	$\#$	d^C	d^C
fr94	r	r	D^C	D^C	D^C	D^C	D^C	D^C			r	D^C	$\#$	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	$\#$	D
	r	r	D^C	D^C	D^C	D^C	D	D^C	-		r	D^C	D	D	D	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C
de00	D^C	d	D^C	$\#$	D^C	D^C	D^C	D	R	d^C			R	R	D^C	D^C	D^C	D^C	D^C	D^C	D	D	D^C	D^C	D	$\#$
	D^C	d	D^C	$\#$	D^C	R	D^C	D	R	d^C	-		R	R	D^C	D	D^C	D	D^C	R	D^C	D^C	D^C	D^C	D	R
nl99	r	d	r	d	D^C	D	D^C	D	$\#$	d^C	r	r			D^C	D	D	D	D^C	D	D^C	D^C	D^C	$\#$	r	r
	r	d	r	d	D^C	D	D^C	$\#$	d	d	r	r	-		D^C	D^C	D^C	D	D^C	D	D^C	D^C	D^C	$\#$	r	r
no00	d^C	d	d	d	R	R	d	d	d^C	d^C	d^C	d^C	d^C	d			D^C	D	R	$\#$	R	R	R	R	d^C	d^C
	d	d	d	d	R	R	d	d	d	d^C	d^C	d	d^C	d^C	-		D^C	D^C	R	R	R	R	$\#$	R	d^C	d^C
pl99	d	d	d	d	d	d	d	d	d^C	d^C	d^C	d^C	d	d	d^C	d			d	d	$\#$	$\#$	d	d	d^C	d^C
	d^C	d^C	d	d	R	R	d^C	d^C	d^C	d^C	d^C	d	d^C	d	d^C	d^C	-		d	$\#$	R	R	d^C	d	d^C	d^C
se00	d^C	d^C	d^C	d^C	$\#$	R	r	r	d^C	d^C	d^C	d^C	d^C	d	r	$\#$	D	D			$\#$	D	$\#$	$\#$	d^C	d^C
	d^C	d^C	d^C	d^C	D	R	r	r	d^C	d^C	d^C	r	d^C	d	r	r	D	$\#$	-		$\#$	D	$\#$	r	d^C	d^C
ch00	d	d	d	d	r	d	r	d^C	d^C	d^C	d	d	d^C	d^C	r	r	$\#$	$\#$	$\#$	d			r	d^C	d^C	d^C
	d^C	d^C	d	d^C	r	d	r	d^C	d^C	d^C	d^C	d^C	d^C	d^C	r	r	r	r	$\#$	d	-		r	d^C	d^C	d^C
uk99	d	d^C	d	d	R	R	$\#$	$\#$	d^C	d^C	d^C	d^C	d^C	$\#$	r	r	D	D	$\#$	$\#$	R	D^C			d^C	d^C
	d^C	d^C	d	d	R	R	d	$\#$	d^C	d^C	d^C	d^C	d^C	$\#$	$\#$	r	D^C	D	$\#$	R	R	D^C	-		d^C	d^C
us00	$\#$	d	R	$\#$	D^C	D^C	D^C	D^C	$\#$	d	d	$\#$	R	R	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C		
	$\#$	d	R	$\#$	D^C	D^C	D^C	D^C	d^C	d^C	d	r	R	R	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	-	

Legend. D (d) means that country *row* (*column*) dominates country *column* (*row*) in terms of concentration curves for taxes, based on gross income share cumulants (Definition2). ^{C} indicates convexity (concavity) of the relative concentration curve (sufficient conditions from Theorems 1-3 hold). R (r) means that country *row* (*column*) dominates country *column* (*row*) for a lower quantile and is dominated by country *column* (*row*) for an upper quantile (just 1 intersection). # indicates more than 1 intersection. For each cell: upper-left and upper-right corners correspond to household-based analysis (taxes and taxes + other mandatory contributions, respectively); bottom-left and bottom-right corners correspond to equivalized-income-based analysis (taxes and taxes + other mandatory contributions, respectively).

Table 4: Progression (non)dominance according to Definition 3

Compares (T^1, Y^1) for the country in the row with (T^2, Y^2) for the country in the column for Definition 3 (net incomes, parameter: q) and vice versa

	au01		ca00		dk00		fi00		fr94		de00		nl99		no00		pl99		se00		ch00		uk99		us00	
au01	-		D	$\#$	d	d	d	d	r	r	R	d	d^C	d^C	$\#$	$\#$	d^C	d^C	R	R	d^C	d^C	D	$\#$	D^C	D^C
			$\#$	$\#$	d^C	d^C	d^C	d^C	r	r	R	d^C	d^C	d^C	r	r	r	r	d^C	d^C	d^C	r	D^C	D^C	D^C	D^C
ca00	d	$\#$	-		d^C	d^C	d^C	d^C	d	d^C	d^C	d	d^C	d^C	d^C	d^C	d^C	d^C	d	d	d^C	d^C	r	r	D	D
	$\#$	$\#$			d^C	d^C	d^C	d^C	d^C	d^C	d	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d	r	r	D^C	D^C
dk00	D	D	D^C	D^C	-		D	D	D	D	D	D	R	d^C	D	D	$\#$	$\#$	D^C	D^C	$\#$	$\#$	D^C	D^C	D^C	D^C
	D^C	D^C	D^C	D^C			D^C	D^C	D^C	D^C	D^C	D^C	R	$\#$	D	D	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C
fi00	D	D	D^C	D^C	d	d	-		$\#$	$\#$	R	$\#$	d^C	d^C	$\#$	$\#$	r	$\#$	D	D	d^C	r	D^C	D^C	D^C	D^C
	D^C	D^C	D^C	D^C	d^C	d^C			D^C	D^C	R	D^C	R	R	$\#$	$\#$	D^C	D^C	$\#$	R	D	D^C	D^C	D^C	D^C	D^C
fr94	R	R	D	D^C	d	d	$\#$	$\#$	-		R	R	d^C	d^C	$\#$	$\#$	d^C	d^C	R	$\#$	d^C	d^C	D^C	D^C	D^C	D^C
	R	R	D^C	D^C	d^C	d^C	d^C	d^C			R	R	d^C	d^C	d^C	r	$\#$	$\#$	R	d^C	d	d	D^C	D^C	D^C	D^C
de00	r	D	D^C	D	d	d	r	$\#$	r	r	-		d^C	d^C	$\#$	$\#$	r	r	r	$\#$	r	r	r	D^C	D^C	D^C
	r	D^C	D	D^C	d^C	d^C	r	d^C	r	r			d^C	d^C	r	r	r	D	d^C	d^C	r	r	D^C	D^C	D^C	D
nl99	D^C	D^C	D^C	D^C	r	D^C	D^C	D^C	D^C	D^C	D^C	D^C	-		D^C	D^C	r	D^C	D^C	D^C	r	D^C	D^C	D^C	D^C	D^C
	D^C	D^C	D^C	D^C	r	$\#$	r	r	D^C	D^C	D^C	D^C			$\#$	D	D^C	D^C	D^C	D^C	r	D^C	D^C	D^C	D^C	D^C
no00	$\#$	$\#$	D^C	D^C	d	d	$\#$	$\#$	$\#$	$\#$	$\#$	$\#$	d^C	d^C	-		d	d	$\#$	$\#$	d^C	d^C	r	r	D^C	D^C
	R	R	D^C	D^C	d	d	$\#$	$\#$	D^C	R	R	R	$\#$	d			D^C	R	R	R	$\#$	R	D^C	D^C	D^C	D^C
pl99	D^C	D^C	D^C	D^C	$\#$	$\#$	R	$\#$	D^C	D^C	R	R	R	d^C	D	D	-		R	R	d	d^C	D^C	D^C	D^C	D^C
	R	R	D^C	D^C	d^C	d^C	d^C	d^C	$\#$	$\#$	R	d	d^C	d^C	d^C	r			d^C	d^C	d	d	D^C	D^C	D^C	D^C
se00	r	r	D	D	d^C	d^C	d	d	r	$\#$	R	$\#$	d^C	d^C	$\#$	$\#$	r	r	-		d^C	r	D^C	r	D	D^C
	D^C	D^C	D^C	D^C	d^C	d^C	$\#$	r	r	D^C	D^C	D^C	d^C	d^C	r	r	D^C	D^C			r	r	D^C	D^C	D^C	D^C
ch00	D^C	D^C	D^C	D^C	$\#$	$\#$	D^C	R	D^C	D^C	R	R	R	d^C	D^C	D^C	D	D^C	D^C	R	-		D^C	D^C	D^C	D^C
	D^C	R	D^C	D	d^C	d^C	d	d^C	D	D	R	R	R	d^C	$\#$	r	D	D	R	R			D^C	D^C	D^C	D^C
uk99	d	$\#$	R	R	d^C	d^C	d^C	d^C	d^C	d^C	R	d^C	d^C	d^C	R	R	d^C	d^C	d^C	R	d^C	d^C	-		$\#$	$\#$
	d^C	d^C	R	R	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C			D	D
us00	d^C	d^C	d	d	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d	d^C	d^C	d^C	d^C	d^C	d^C	d	d^C	d^C	d^C	$\#$	$\#$	-	
	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d	d		

Legend. D (d) means that country *row* (*column*) dominates country *column* (*row*) in terms of concentration curves for net incomes (Definition3). ^{C} indicates convexity (concavity) of the relative concentration curve (sufficient conditions from Theorems 1-3 hold). R (r) means that country *row* (*column*) dominates country *column* (*row*) for a lower quantile and is dominated by country *column* (*row*) for an upper quantile (just 1 intersection). # indicates more than 1 intersection. For each cell: upper-left and upper-right corners correspond to household-based analysis (taxes and taxes + other mandatory contributions, respectively); bottom-left and bottom-right corners correspond to equivalized-income-based analysis (taxes and taxes + other mandatory contributions, respectively).

Table 5: Progression (non)dominance according to Definition 4

Compares (T^1, Y^1) for the country in the row with (T^2, Y^2) for the country in the column for Definition 4 (net incomes, parameter: p) and vice versa

	au01	ca00	dk00	fi00	fr94	de00	nl99	no00	pl99	se00	ch00	uk99	us00
au01	-	R $\#$ R r	D D R $\#$	D^C r D $\#$	D^C D^C D^C D^C	R r $\#$ r	R D R D	D^C D D D	D D D D	D^C D^C R r	D^C D D D	D^C D^C D D	$\#$ $\#$ $\#$ $\#$
ca00	r $\#$ r R	-	D $\#$ $\#$ R	D $\#$ D r	D^C D^C D^C D^C	$\#$ r $\#$ r	R $\#$ R $\#$	D D D D	D^C D^C D^C D^C	D^C D D^C D	D^C D^C D D	D D D^C D	$\#$ $\#$ $\#$ $\#$
dk00	d d r $\#$	d $\#$ $\#$ r	-	r r r r	D D D^C D^C	d r R r	R $\#$ R $\#$	D^C r D D^C	D^C D^C D^C D^C	r r r r	D^C D D D	D^C D D^C D^C	r r r r
fi00	d^C R d $\#$	d $\#$ d R	R R R R	-	D^C R D^C R	R d R d	R R R R	D D D D	D D^C D^C D	R R R R	D^C D^C D D^C	D^C D D^C D^C	R R R R
fr94	d^C d^C d^C d^C	d^C d^C d^C d^C	d d d^C d^C	d^C r d^C r	-	d d^C d^C d^C	d^C d^C d^C d^C	r r r r	D^C D^C D^C D^C	d d d^C d^C	D r D D	r r r r	d^C d^C d^C d^C
de00	r R $\#$ R	$\#$ R $\#$ R	D R r R	r D r D	D D^C D^C D^C	-	R R R R	D D^C D^C D^C	D^C D D^C D^C	D^C R r R	D D D D^C	D^C D D^C D^C	$\#$ R r R
nl99	r d r d	r $\#$ r $\#$	r $\#$ r $\#$	r r r r	D^C D^C D^C D^C	r r r r	-	r D r D	D^C D^C D^C D^C	r r r r	D^C D^C D^C D	D D $\#$ D^C	r r r r
no00	d^C d d d	d d d d	d^C R d d^C	d d d d	R R R R	d d^C d^C d^C	R d R d	-	D^C D^C D^C R	d d d $\#$	R R R R	$\#$ $\#$ R R	d^C d^C d^C d^C
pl99	d d d d	d^C d^C d^C d^C	d^C d^C d^C d^C	d d^C d^C d	d^C d^C d^C d^C	d^C d d^C d^C	d^C d^C d^C d^C	d^C d^C d^C r	-	d^C d^C d^C d^C	$\#$ $\#$ R R	d d^C d^C d^C	d^C d^C d^C d^C
se00	d^C d^C r R	d^C d d^C d	R R R R	r r r r	D D D^C D^C	d^C r R r	R R R R	D D D $\#$	D^C D^C D^C D^C	-	D D D^C D	$\#$ $\#$ D D	r r $\#$ $\#$
ch00	d^C d d d	d^C d^C d d	d^C d d d	d^C d^C d d^C	d R d d	d d d d^C	d^C d^C d^C d	r r r r	$\#$ $\#$ r r	d d d^C d	-	d d d d	d^C d^C d^C d^C
uk99	d^C d^C d d	d d d^C d	d^C d d^C d^C	d^C d d^C d^C	R R R R	d^C d d^C d^C	d d $\#$ d^C	$\#$ $\#$ r r	D D^C D^C D^C	$\#$ $\#$ d d	D D D D	-	d^C d^C d^C d^C
us00	$\#$ $\#$ $\#$ $\#$	$\#$ $\#$ $\#$ $\#$	R R R R	r r r r	D^C D^C D^C D^C	$\#$ r R r	R R R R	D^C D^C D^C D^C	D^C D^C D^C D^C	R R $\#$ $\#$	D^C D^C D^C D^C	D^C D^C D^C D^C	-

Legend. D (d) means that country *row* (*column*) dominates country *column* (*row*) in terms of concentration curves for net incomes, based on gross income share cumulants (Definition4). ^{C} indicates convexity (concavity) of the relative concentration curve (sufficient conditions from Theorems 1-3 hold). R (r) means that country *row* (*column*) dominates country *column* (*row*) for a lower quantile and is dominated by country *column* (*row*) for an upper quantile (just 1 intersection). # indicates more than 1 intersection. For each cell: upper-left and upper-right corners correspond to household-based analysis (taxes and taxes + other mandatory contributions, respectively); bottom-left and bottom-right corners correspond to equivalized-income-based analysis (taxes and taxes + other mandatory contributions, respectively).

Table 6: Progression (non)dominance according to Definition 5
Compares (T^1, Y^1) for the country in the row with (T^2, Y^2) for the country in the column for Definition 5 (differences of tax concentration curves, parameter: q) and vice versa

	au01		ca00		dk00		fi00		fr94		de00		nl99		no00		pl99		se00		ch00		uk99		us00	
au01	-		D	D^C	D^C	D^C	D^C	D	d	d	R	D^C	d	r	D^C	D	D^C	D^C	D^C	D^C	D^C	D	D^C	D^C	R	D
			D	D^C	D^C	D^C	r	r	d^C	d^C	d^C	D^C	d^C	D	r	D^C	D^C	D^C	D^C	D^C	D	D^C	D^C	D^C	R	#
ca00	d	d^C	-		D^C	D^C	r	r	d^C	d^C	d^C	r	d^C	r	r	r	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D	R	R
	d	d^C			D^C	D	r	r	d^C	d^C	d^C	r	d^C	r	r	r	D^C	D^C	r	r	D	D	D^C	D	R	R
dk00	d^C	d^C	d^C	d^C	-		d	d	d^C	d^C	d^C	d^C	d^C	d	d^C	r	D	D	#	#	R	D^C	R	R	d^C	d^C
	d^C	d^C	d^C	d			d^C	d^C	d^C	d^C	d^C	d^C	d^C	d	d^C	r	r	D^C	#	#	R	D	R	R	R	R
fi00	d^C	d	R	R	D	D	-		d^C	d^C	R	R	d^C	d	D	D	D^C	D^C	D^C	D	D^C	D^C	R	#	R	R
	R	R	R	R	D^C	D^C			d^C	d^C	R	R	d^C	R	D	D	D^C	D	R	D^C	D	D^C	R	D	R	R
fr94	D	D	D^C	D^C	D^C	D^C	D^C	D^C	-		#	D^C	#	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D	D^C
	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C			D	D^C	D	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C	D^C
de00	r	d^C	D^C	R	D^C	D^C	r	r	#	d^C	-		#	#	D^C	D^C	D^C	D	D^C	D^C	D^C	D	D^C	D^C	R	R
	D^C	d^C	D^C	R	D^C	D^C	r	r	d	d^C			#	#	r	r	D^C	D	D^C	D^C	D	D^C	D^C	D	R	R
nl99	D	R	D^C	R	D^C	D	D^C	D	#	d^C	#	#	-		D^C	D	D^C	D^C	D^C	D	D^C	D^C	D^C	D	D	R
	D^C	d	D^C	R	D^C	D	D^C	r	d	d^C	#	#			D^C	D	D^C	D	D^C	D^C	D^C	D	D^C	D	D	R
no00	d^C	d	R	R	D^C	R	d	d	d^C	d^C	d^C	d^C	d^C	d	-		D^C	D	R	#	D^C	D^C	R	R	R	R
	R	d^C	R	R	D^C	R	d	d	d^C	d^C	R	R	d^C	d			D^C	D	R	R	R	D	R	R	R	R
pl99	d^C	d^C	d^C	d^C	d	d	d^C	d^C	d^C	d^C	d^C	d	d^C	d^C	d^C	d	-		d	d	#	#	R	R	d^C	d^C
	d^C	d^C	d^C	d^C	R	d^C	d^C	d	d^C	d^C	d^C	d	d^C	d	d^C	d			d	#	#	D^C	R	R	R	R
se00	d^C	d^C	d^C	d^C	#	#	d^C	d	d^C	d^C	d^C	d^C	d^C	d	r	#	D	D	-		D	D^C	R	R	R	R
	d^C	d^C	R	R	#	#	r	d^C	d^C	d^C	d^C	d^C	d^C	d^C	r	r	D	#			#	D	R	#	R	R
ch00	d^C	d	d^C	d^C	r	d^C	d^C	d^C	d^C	d^C	d^C	d	d^C	d^C	d^C	d^C	#	#	d	d^C	-		R	d	d^C	d^C
	d	d^C	d	d	r	d	d	d^C	d^C	d^C	d	d^C	d^C	d	r	d	#	d^C	#	d			d	d^C	d^C	d^C
uk99	d^C	d^C	d^C	d	r	r	r	#	d^C	d^C	d^C	d	d^C	d	r	r	r	r	r	r	r	D	-		d^C	d^C
	d^C	d^C	d^C	d	r	r	r	d	d^C	d^C	d^C	d	d^C	d	r	r	r	r	r	#	D	D^C			d^C	d^C
us00	r	d	r	r	D^C	D^C	r	r	d	d^C	r	r	d	r	r	r	D^C	D^C	r	r	D^C	D^C	D^C	D^C	-	
	r	#	r	r	r	r	r	r	d^C	d^C	r	r	d	r	r	r	r	r	r	r	D^C	D^C	D^C	D^C		

Legend. D (d) means that country *row* (*column*) dominates country *column* (*row*) in terms of second-order differences of concentration curves for gross incomes and taxes (Definition5). ^{C} indicates convexity (concavity) of the relative concentration curve (sufficient conditions from Theorems 1-3 hold). R (r) means that country *row* (*column*) dominates country *column* (*row*) for a lower quantile and is dominated by country *column* (*row*) for an upper quantile (just 1 intersection). # indicates more than 1 intersection. For each cell: upper-left and upper-right corners correspond to household-based analysis (taxes and taxes + other mandatory contributions, respectively); bottom-left and bottom-right corners correspond to equivalized-income-based analysis (taxes and taxes + other mandatory contributions, respectively).

Table 7: Progression (non)dominance according to Definition 6

Compares (T^1, Y^1) for the country in the row with (T^2, Y^2) for the country in the column for Definition 6 (differences of net income concentration curves, parameter: q) and vice versa

	au01		ca00		dk00		fi00		fr94		de00		nl99		no00		pl99		se00		ch00		uk99		us00	
au01	-		D	$\#$	r	r	r	r	D^C	D^C	$\#$	r	D	r	D^C	r	D^C	D^C	r	r	D^C	D^C	D^C	D^C	R	R
			D	r	r	r	r	r	D^C	D^C	$\#$	r	D	r	r	r	D^C	D^C	r	r	D	D	D^C	D^C	R	R
ca00	d	$\#$		-	r	$\#$	r	$\#$	D	D^C	$\#$	r	$\#$	$\#$	r	r	D^C	D^C	r	r	D^C	D	D^C	D^C	R	R
	d	R			r	r	r	r	D^C	D^C	$\#$	r	D	r	r	r	D^C	D^C	r	r	D^C	D	D^C	D^C	R	R
dk00	R	R	R	$\#$		-	r	r	D^C	D^C	R	r	R	r	D^C	$\#$	D^C	D^C	$\#$	r	D^C	D^C	D^C	D	R	R
	R	R	R	R			r	r	D^C	D^C	R	$\#$	R	r	D^C	D^C	D^C	D^C	R	r	D^C	D^C	D^C	D^C	R	R
fi00	R	R	R	$\#$	R	R		-	D	D^C	R	R	R	$\#$	D	D	D^C	D^C	R	D	D^C	D^C	D	D	R	R
	R	R	R	R	R	R			D^C	D^C	R	R	R	R	D^C	D	D^C	D^C	R	R	D^C	D^C	D	D	R	R
fr94	d^C	d^C	d	d^C	d^C	d^C	d	d^C		-	d^C	d^C	d^C	d	d	r	D^C	D^C	d	d	D	r	R	r	d^C	d^C
	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C			d^C	d^C	d^C	d^C	r	r	r	D^C	d^C	d^C	D	D^C	R	r	d^C	d^C
de00	$\#$	R	$\#$	R	r	R	r	r	D^C	D^C		-	D^C	r	r	D^C	D^C	D^C	r	$\#$	D^C	D	D^C	D^C	R	R
	$\#$	R	$\#$	R	r	$\#$	r	r	D^C	D^C			D^C	r	r	D	D^C	D	r	r	D^C	D^C	D^C	D	R	R
nl99	d	R	$\#$	$\#$	r	R	r	$\#$	D^C	D	d^C	R		-	r	D	D^C	D^C	r	D	D^C	D^C	D^C	D	R	R
	d	R	d	R	r	R	r	r	D^C	D^C	d^C	R			r	D	D^C	D	r	r	D^C	D	D^C	D	R	R
no00	d^C	R	R	R	d^C	$\#$	d	d	D	R	R	d^C	R	d		-	D^C	D^C	d	d	D^C	D^C	$\#$	$\#$	R	R
	R	R	R	R	d^C	d^C	d^C	d	R	R	R	d	R	d			D^C	D^C	d	$\#$	D	D	R	R	R	R
pl99	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C		-	d^C	d^C	$\#$	$\#$	R	d^C	d^C	d^C
	d^C	d^C	d^C	d^C	d^C	d^C	d^C	d^C	R	d^C	d^C	d	d^C	d	d^C	d^C			d^C	d^C	R	$\#$	R	d^C	d^C	d^C
se00	R	R	R	R	$\#$	R	r	d	D	D	R	$\#$	R	d	D	D	D^C	D^C		-	D^C	D^C	D	$\#$	R	R
	R	R	R	R	r	R	r	r	D^C	D^C	R	R	R	R	D	$\#$	D^C	D^C			D^C	D	D	D	R	R
ch00	d^C	d^C	d^C	d	d^C	d^C	d^C	d^C	d	R	d^C	d	d^C	d^C	d^C	d^C	$\#$	$\#$	d^C	d^C		-	d^C	d	d^C	d^C
	d	d	d^C	d	d^C	d^C	d^C	d^C	d	d^C	d^C	d^C	d^C	d	d	d	r	$\#$	d^C	d			d	d^C	d^C	d^C
uk99	d^C	d^C	d^C	d^C	d^C	d	d	d	r	R	d^C	d^C	d^C	d	$\#$	$\#$	r	D^C	d	$\#$	D^C	D		-	d^C	d^C
	d^C	d^C	d^C	d^C	d^C	d^C	d	d	r	R	d^C	d	d^C	d	r	r	r	D^C	d	d	D	D^C			d^C	d^C
us00	r	r	r	r	r	r	r	r	D^C	D^C	r	r	r	r	r	r	D^C	D^C	r	r	D^C	D^C	D^C	D^C		-
	r	r	r	r	r	r	r	r	D^C	D^C	r	r	r	r	r	r	D^C	D^C	r	r	D^C	D^C	D^C	D^C		

Legend. D (d) means that country *row* (*column*) dominates country *column* (*row*) in terms of second-order differences of concentration curves for gross and net incomes (Definition6). ^{C} indicates convexity (concavity) of the relative concentration curve (sufficient conditions from Theorems 1-3 hold). R (r) means that country *row* (*column*) dominates country *column* (*row*) for a lower quantile and is dominated by country *column* (*row*) for an upper quantile (just 1 intersection). # indicates more than 1 intersection. For each cell: upper-left and upper-right corners correspond to household-based analysis (taxes and taxes + other mandatory contributions, respectively); bottom-left and bottom-right corners correspond to equivalized-income-based analysis (taxes and taxes + other mandatory contributions, respectively).

Table 8: Summary tax progression statistics for household data, taxes

	Progression, # of cases	Progression, % of total cases	Single extr.	Single extr., % of possible	No progression, # of cases	No progression, % of total cases	One intersection, # of cases	One intersection, % of possible
Definition 1	59	75.64	49	83.05	19	24.36	16	84.21
Definition 2	53	67.95	37	69.81	25	32.05	17	68.00
Definition 3	52	66.67	37	71.15	26	33.33	17	65.38
Definition 4	51	65.38	31	60.78	27	34.62	20	74.07
Definition 5	55	70.51	45	81.82	23	29.49	18	78.26
Definition 6	45	57.69	34	75.56	33	42.31	27	81.82
Average	52.50	67.31	38.83	73.70	25.50	32.69	19.17	75.29
Total cases	78							

Table 9: Summary tax progression statistics for household data, taxes+payroll

	Progression, # of cases	Progression, % of total cases	Single extr.	Single extr., % of possible	No progression, # of cases	No progression, % of total cases	One intersection, # of cases	One intersection, % of possible
Definition 1	60	76.92	47	78.33	18	23.08	12	66.67
Definition 2	60	76.92	30	50.00	18	23.08	10	55.56
Definition 3	52	66.67	40	76.92	26	33.33	11	42.31
Definition 4	46	58.97	23	50.00	32	41.03	22	68.75
Definition 5	56	71.79	36	64.29	22	28.21	17	77.27
Definition 6	42	53.85	29	69.05	36	46.15	26	72.22
Average	52.67	67.52	34.17	64.76	25.33	32.48	16.33	63.80
Total cases	78							

Table 10: Summary tax progression statistics for equivalized data, taxes

	Progression, # of cases	Progression, % of total cases	Single extr.	Single extr., % of possible	No progression, # of cases	No progression, % of total cases	One intersection, # of cases	One intersection, % of possible
Definition 1	56	71.79	47	83.93	22	28.21	17	77.27
Definition 2	56	71.79	40	71.43	22	28.21	18	81.82
Definition 3	56	71.79	50	89.29	22	28.21	16	72.73
Definition 4	46	58.97	29	63.04	32	41.03	25	78.13
Definition 5	49	62.82	38	77.55	29	37.18	25	86.21
Definition 6	43	55.13	33	76.74	35	44.87	33	94.29
Average	51.00	65.38	39.50	77.00	27.00	34.62	22.33	81.74
Total cases	78							

Table 11: Summary tax progression statistics for equivalized data, taxes+payroll

	Progression, # of cases	Progression, % of total cases	Single extr.	Single extr., % of possible	No progression, # of cases	No progression, % of total cases	One intersection, # of cases	One intersection, % of possible
Definition 1	62	79.49	44	70.97	16	20.51	12	75.00
Definition 2	55	70.51	33	60.00	23	29.49	17	73.91
Definition 3	59	75.64	51	86.44	19	24.36	15	78.95
Definition 4	45	57.69	26	57.78	33	42.31	25	75.76
Definition 5	51	65.38	33	64.71	27	34.62	22	81.48
Definition 6	42	53.85	28	66.67	36	46.15	33	91.67
Average	52.33	67.09	35.83	67.76	25.67	32.91	20.67	79.46
Total cases	78							

Table 12: Summary statistics: consistency for strict dominance

Incidence of strict dominance (out of 6)	Household data, taxes		Household data, taxes+payroll		Equivalized data, taxes		Equivalized data, taxes+payroll	
	# of cases	% of total cases	# of cases	% of total cases	# of cases	% of total cases	# of cases	% of total cases
6	2	2.56	2	2.56	5	6.41	7	8.97
5	20	25.64	23	29.49	14	17.95	15	19.23
4	16	20.51	11	14.10	8	10.26	9	11.54
3	19	24.36	25	32.05	29	37.18	25	32.05
2	15	19.23	7	8.97	16	20.51	13	16.67
1	5	6.41	8	10.26	5	6.41	8	10.26

Table 13: Summary statistics: consistency for bifurcate dominance (one consistent intersection is allowed)

Incidence of bifurcate dominance (out of 6)	Household data, taxes		Household data, taxes+payroll		Equivalized data, taxes		Equivalized data, taxes+payroll	
	# of cases	% of total cases	# of cases	% of total cases	# of cases	% of total cases	# of cases	% of total cases
6	4	5.13	4	5.13	8	10.26	12	15.38
5	28	35.90	29	37.18	20	25.64	19	24.36
4	23	29.49	18	23.08	21	26.92	21	26.92
3	21	26.92	22	28.21	28	35.90	26	33.33
2	0	0.00	4	5.13	1	1.28	0	0.00
1	2	2.56	1	1.28	0	0.00	0	0.00

Table 14: (Dis)Similarities among progression comparison concepts

	Definition 2		Definition 3		Definition 4		Definition 5		Definition 6	
Definition 1	83.66	13.46	20.51	78.52	63.46	35.26	65.06	33.02	49.36	50.00
Definition 2	-		21.16	78.52	60.90	36.54	67.62	29.81	45.83	52.88
Definition 3	.		-		27.88	71.47	46.16	53.52	48.72	50.64
Definition 4	.		.		-		51.92	47.12	69.23	26.60
Definition 5	.		.		.		-		58.97	40.06

Legend. The first [second] entry in a cell denotes the average percentage of similarity [dissimilarity] between a pair of definitions. Symmetric entries are indicated by the dot sign (.)

Table 15: US progression dominance for Bush sen., Clinton, and Bush jun.

	91→94 Bsen → C*		94→00 C* → C**		00→04 C** → Bjun		91→04 Bsen → Bjun		91→00 Bsen → C**		94→04 C* → Bjun	
Def. 1	d^C	d^C	R	#	#	#	d	d	R	d	R	#
	R	d^C	R	R	d^C	r	R	R	R	R	R	R
Def. 2	d	d	#	D	#	#	#	#	#	#	#	D
	r	d	#	#	r	r	#	#	#	#	#	#
Def. 3	D^C	D^C	D^C	D^C	D	D	D^C	D^C	D^C	D^C	D^C	D^C
	D^C	D^C	r	r	D	D	D^C	D^C	#	D	D	D^C
Def. 4	d	d	r	r	D^C	D	#	D	r	r	D	D
	d	d	r	r	D^C	D	#	#	r	r	#	D
Def. 5	R	R	D	D	R	#	R	R	R	R	#	D
	R	R	#	#	R	#	R	R	R	R	#	#
Def. 6	d^C	d^C	d^C	d^C	D^C	D^C	R	R	d	d^C	D	D
	d^C	d^C	d^C	d^C	D	D	R	R	d^C	d^C	#	D

Table 16: UK progression dominance for Thatcher, Major, and Blair

	91→95 Th→Ma		95→99 Ma→ Bl*		99→04 Bl* → Bl**		91→04 Th → Bl**	
Def. 1	R	$\#$	D^C	R	D	D	D^C	D^C
	$\#$	r	D^C	R	D	D	D	D
Def. 2	r	r	D^C	D^C	D	D	D	D
	r	r	D	D	D	D	D^C	D^C
Def. 3	$\#$	r	D^C	D^C	$\#$	$\#$	$\#$	$\#$
	D	$\#$	r	r	$\#$	$\#$	$\#$	$\#$
Def. 4	$\#$	r	D^C	R	D	D	D	D
	$\#$	r	D	$\#$	D	D	D	D
Def. 5	$\#$	r	D^C	D^C	D^C	D^C	D^C	D
	$\#$	$\#$	D^C	D	D^C	D	D^C	D
Def. 6	$\#$	d	D^C	D^C	D	D	D^C	D
	$\#$	$\#$	D	R	D^C	D	D^C	D

Table 17: German progression dominance for Kohl and Schröder

	89→94 K*→ K**		94→00 K** → S		89→00 K* → S	
Def. 1	r	$\#$	d	d	d^C	d^C
	r	r	d	d	d^C	r
Def. 2	r	r	r	r	r	r
	r	$\#$	r	r	r	r
Def. 3	R	$\#$	r	r	r	r
	D^C	D	$\#$	$\#$	D	D
Def. 4	D	$\#$	r	r	$\#$	r
	D	$\#$	r	r	$\#$	r
Def. 5	$\#$	$\#$	d	$\#$	d	$\#$
	$\#$	$\#$	$\#$	d	$\#$	$\#$
Def. 6	D	$\#$	d	d	$\#$	$\#$
	D	$\#$	d	d	$\#$	$\#$

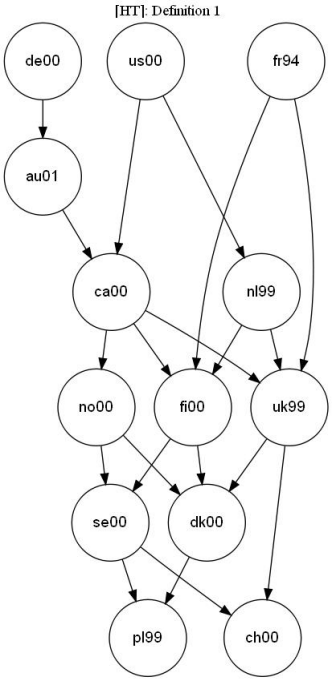

Figure 3: Progression dominance for direct taxes and household data: Definition 1

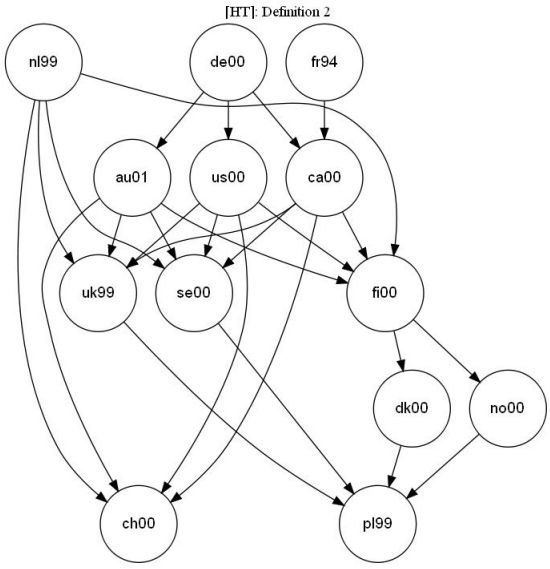

Figure 4: Progression dominance for direct taxes and household data: Definition 2

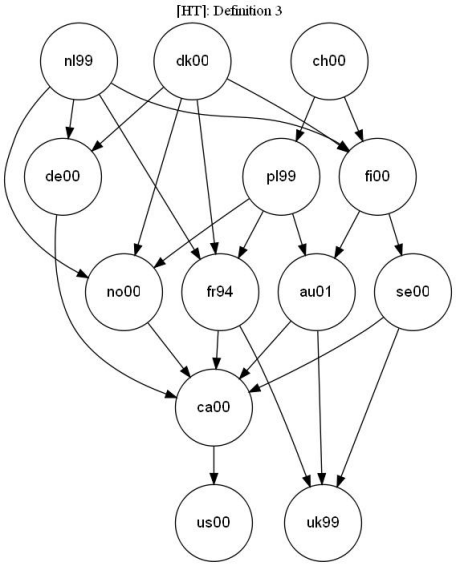

Figure 5: Progression dominance for direct taxes and household data: Definition 3

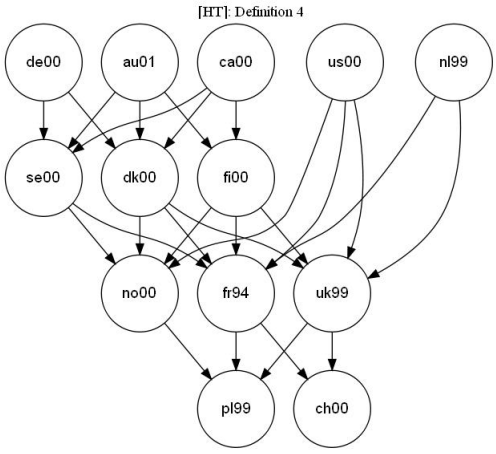

Figure 6: Progression dominance for direct taxes and household data: Definition 4

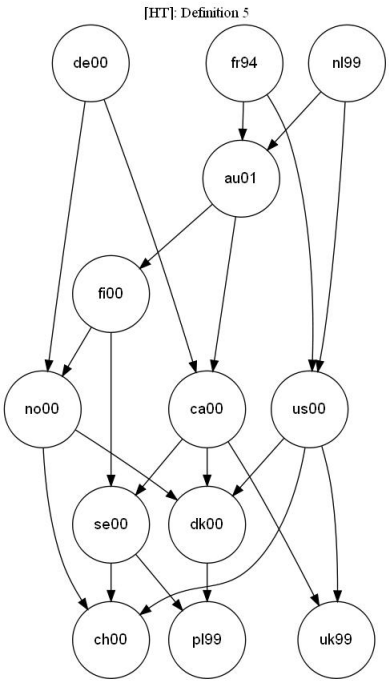

Figure 7: Progression dominance for direct taxes and household data: Definition 5

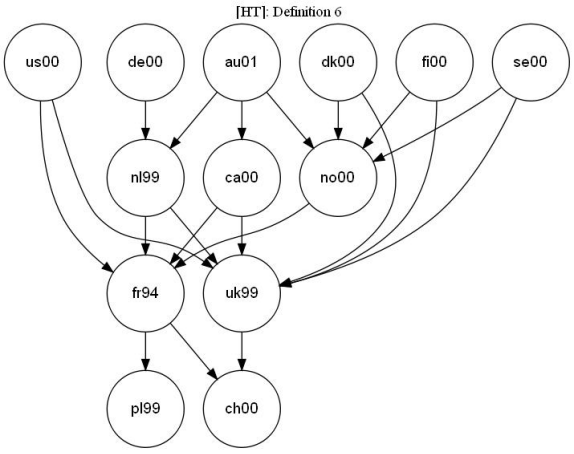

Figure 8: Progression dominance for direct taxes and household data: Definition 6

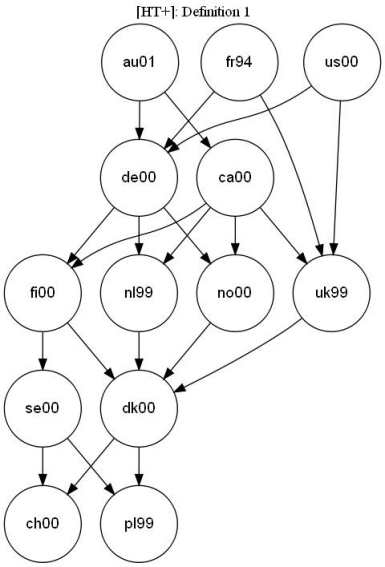

Figure 9: Progression dominance for direct and payroll taxes and household data: Definition 1

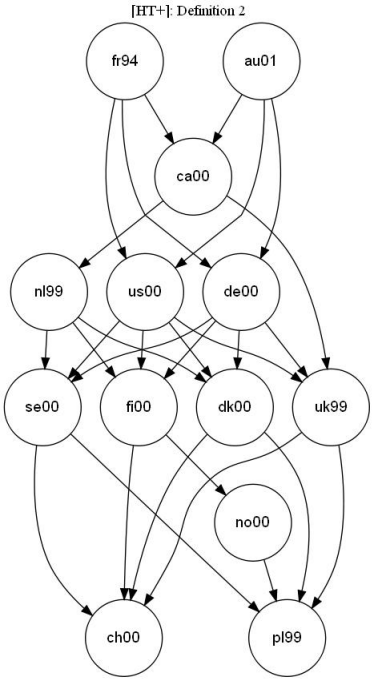

Figure 10: Progression dominance for direct and payroll taxes and household data: Definition 2

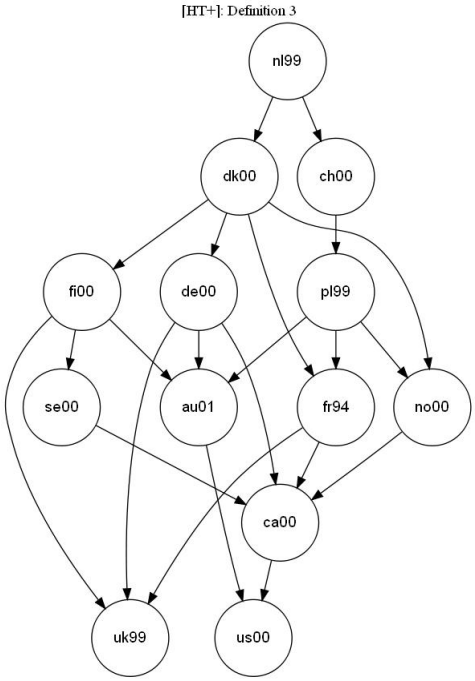

Figure 11: Progression dominance for direct and payroll taxes and household data: Definition 3

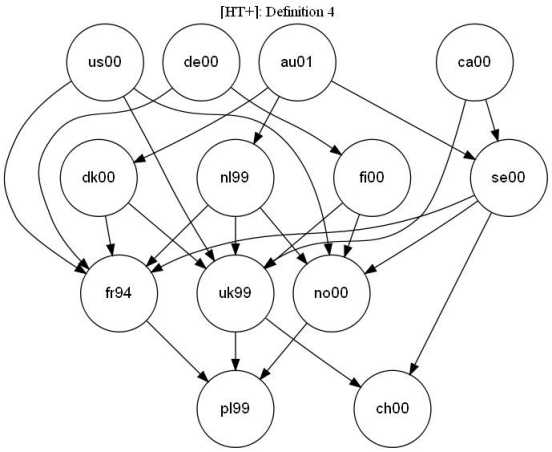

Figure 12: Progression dominance for direct and payroll taxes and household data: Definition 4

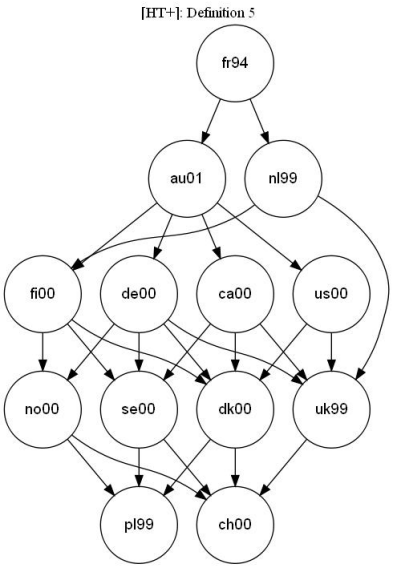

Figure 13: Progression dominance for direct and payroll taxes and household data: Definition 5

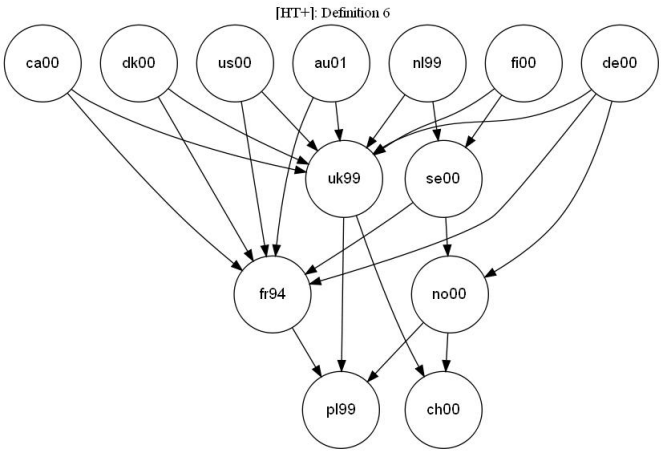

Figure 14: Progression dominance for direct and payroll taxes and household data: Definition 6

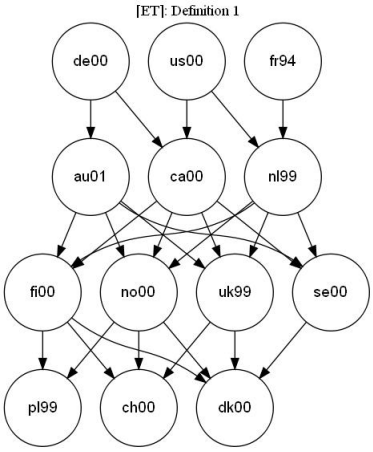

Figure 15: Progression dominance for direct taxes and equivalized data: Definition 1

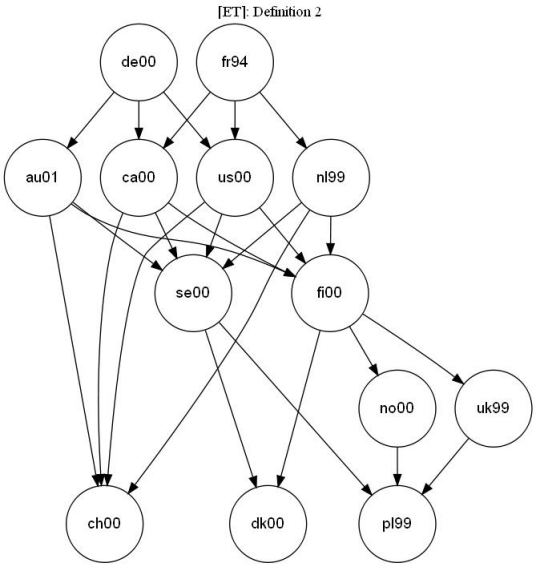

Figure 16: Progression dominance for direct taxes and equivalized data: Definition 2

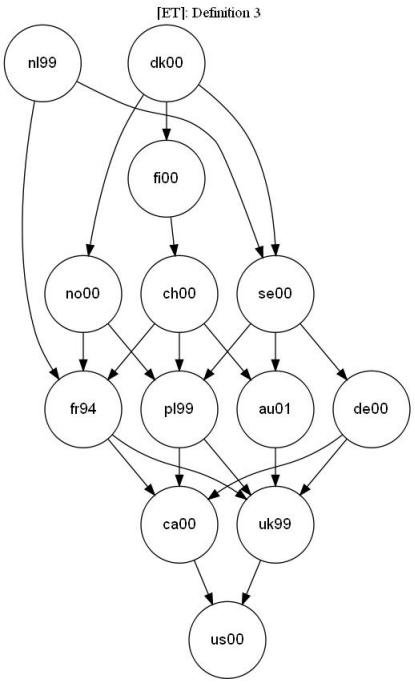

Figure 17: Progression dominance for direct taxes and equivalized data: Definition 3

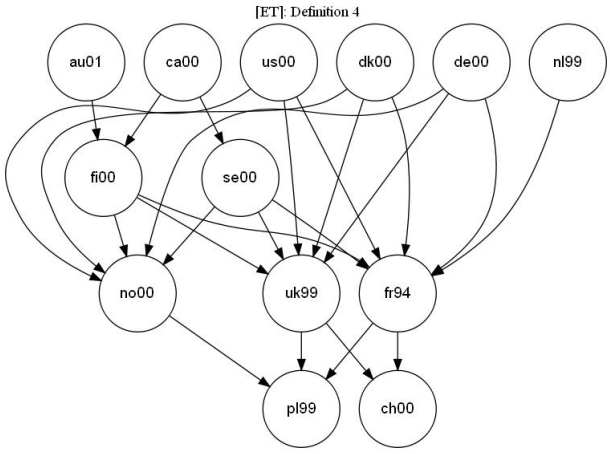

Figure 18: Progression dominance for direct taxes and equivalized data: Definition 4

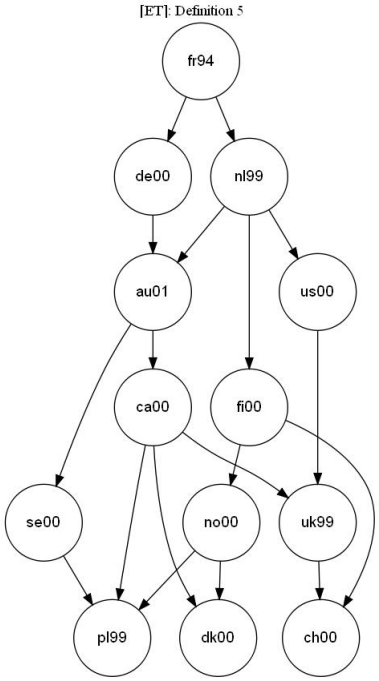

Figure 19: Progression dominance for direct taxes and equivalized data: Definition 5

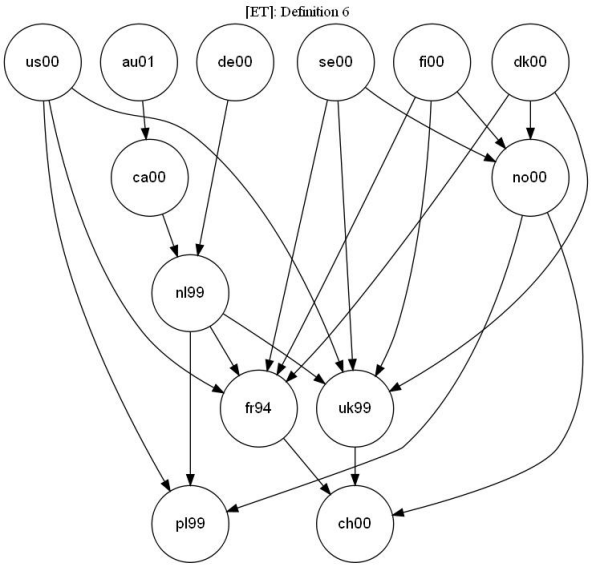

Figure 20: Progression dominance for direct taxes and equivalized data: Definition 6

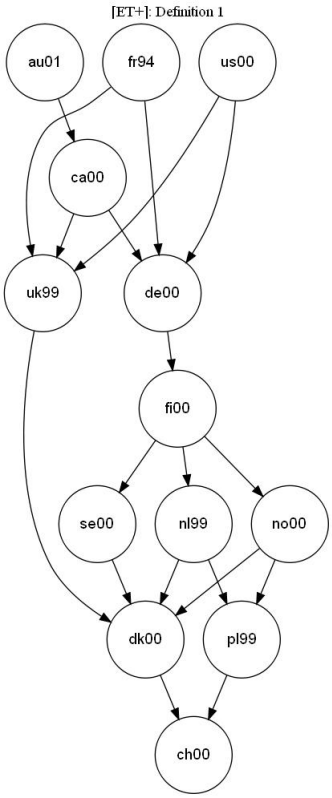

Figure 21: Progression dominance for direct plus payroll taxes and equivalized data: Definition 1

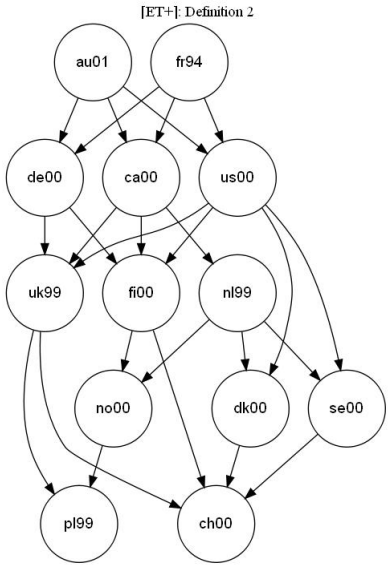

Figure 22: Progression dominance for direct plus payroll taxes and equivalized data: Definition 2

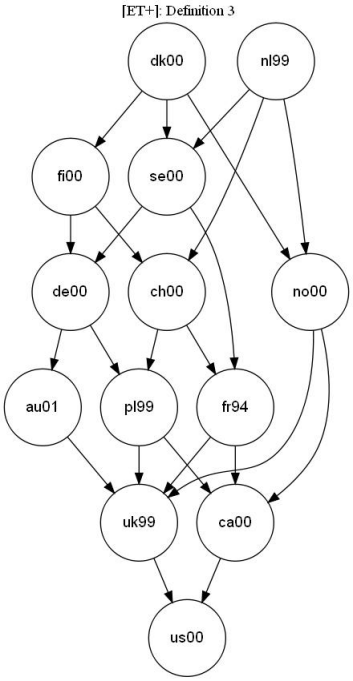

Figure 23: Progression dominance for direct plus payroll taxes and equivalized data: Definition 3

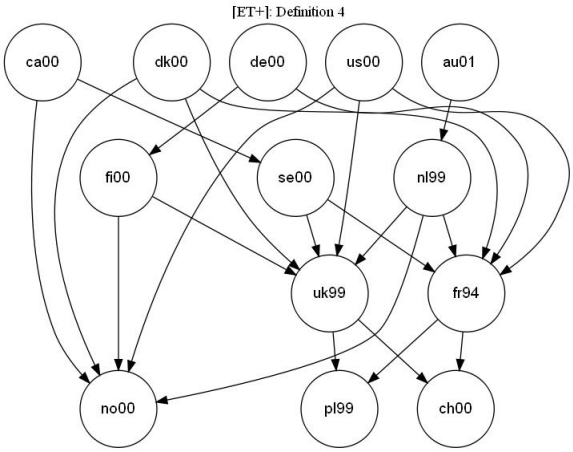

Figure 24: Progression dominance for direct plus payroll taxes and equivalized data: Definition 4

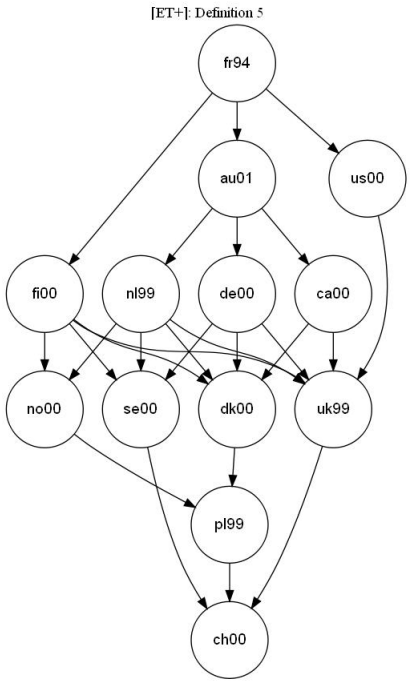

Figure 25: Progression dominance for direct plus payroll taxes and equivalized data: Definition 5

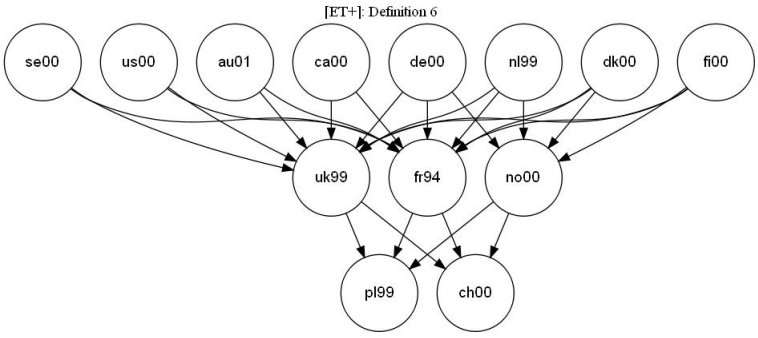

Figure 26: Progression dominance for direct plus payroll taxes and equivalized data: Definition 6

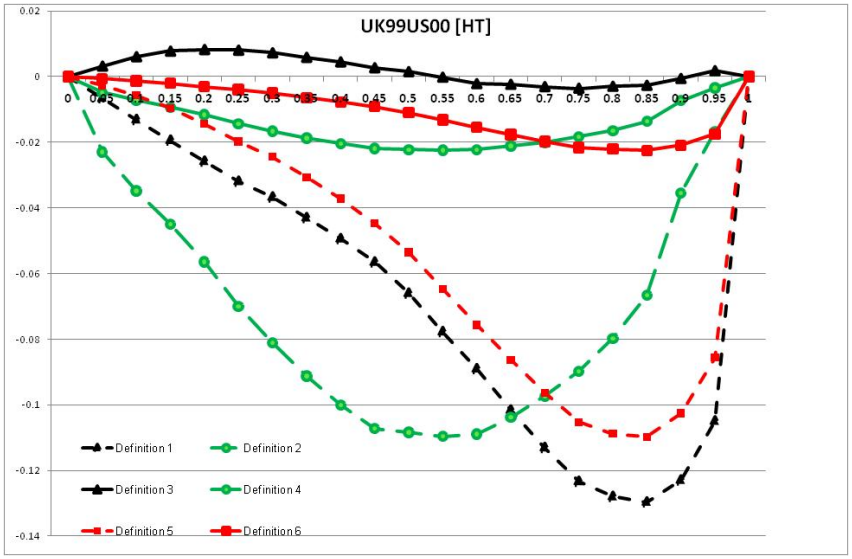

Figure 27: Progression comparison for direct taxes and household data: UK 1999 vs. US 2000

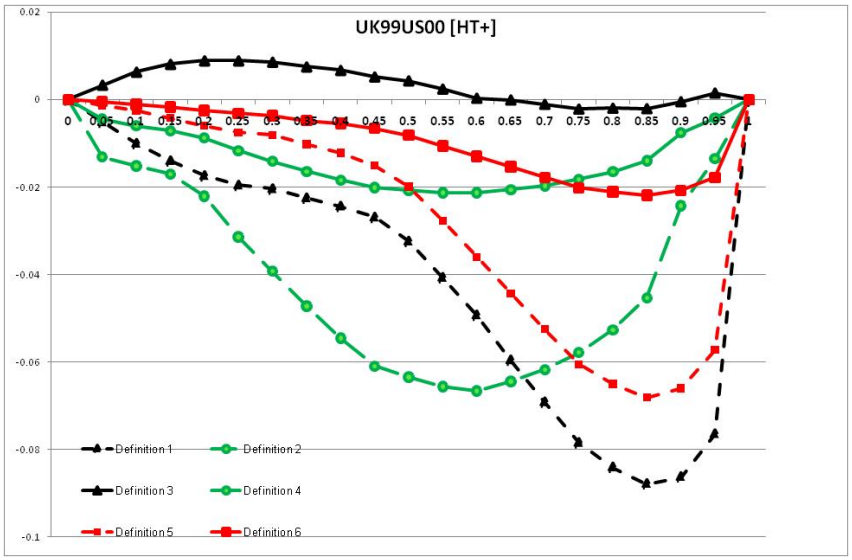

Figure 28: Progression comparison for direct plus payroll taxes and household data: UK 1999 vs. US 2000

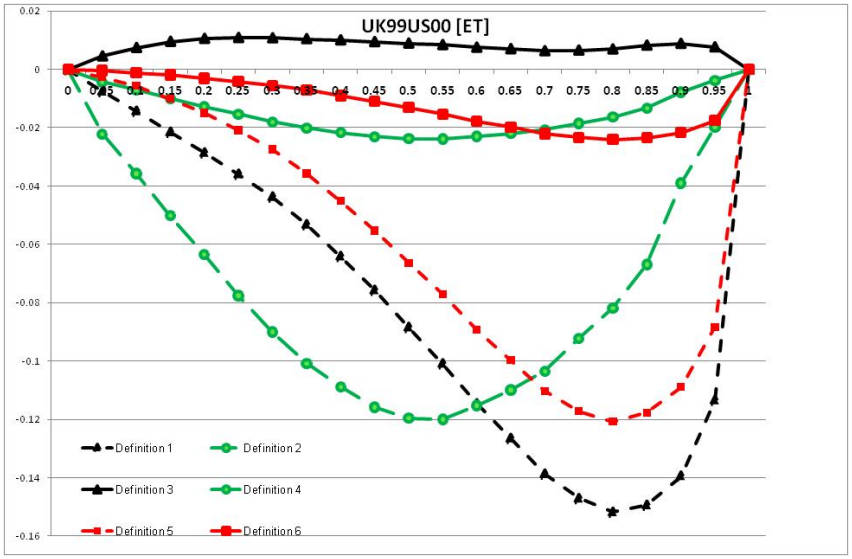

Figure 29: Progression comparison for direct taxes and equivalized data: UK 1999 vs. US 2000

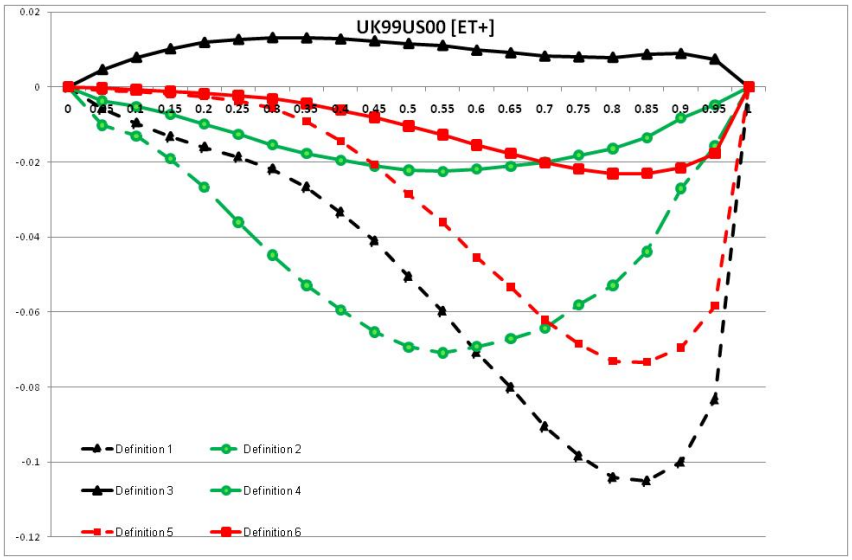

Figure 30: Progression comparison for direct plus payroll taxes and equivalized data: UK 1999 vs. US 2000

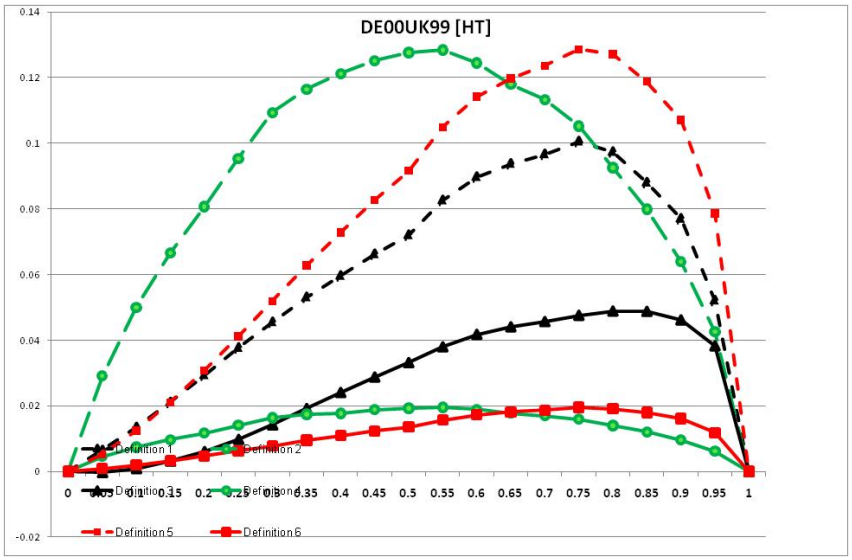

Figure 31: Progression comparison for direct taxes and household data: Germany 2000 vs. UK 1999

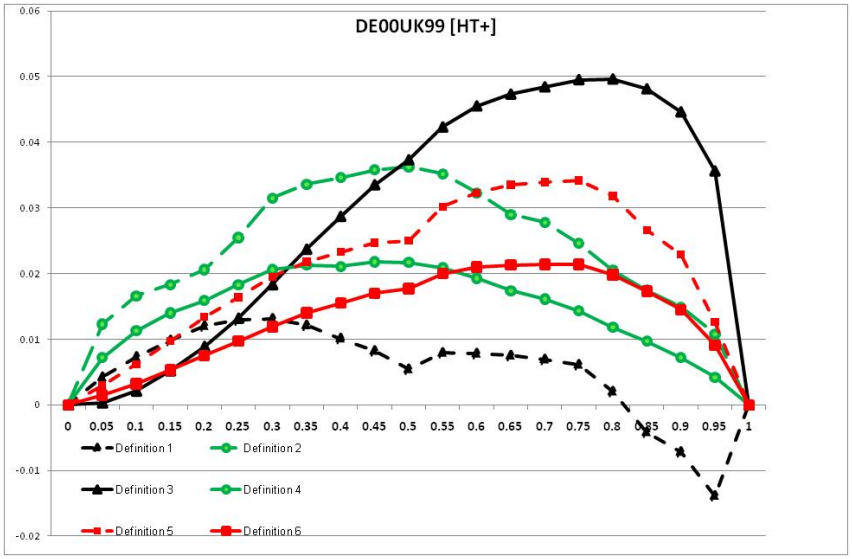

Figure 32: Progression comparison for direct plus payroll taxes and household data: Germany 2000 vs. UK 1999

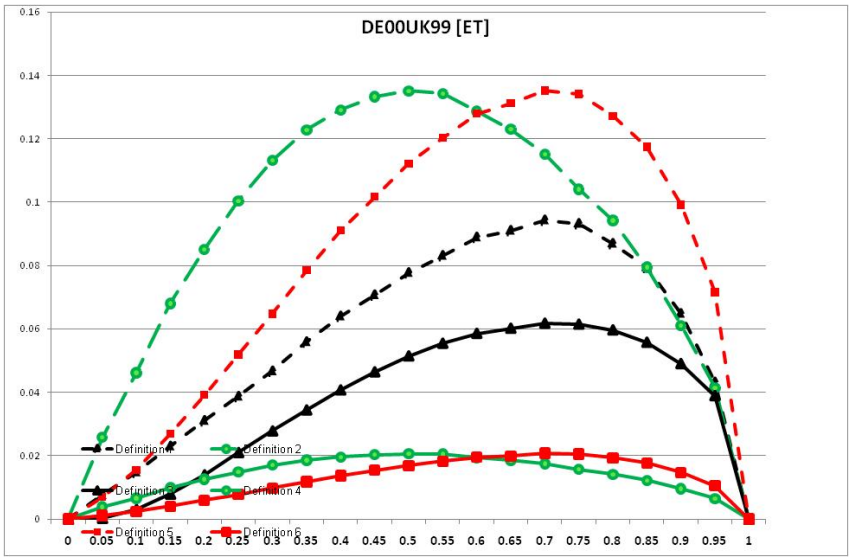

Figure 33: Progression comparison for direct taxes and equivalized data: Germany 2000 vs. UK 1999

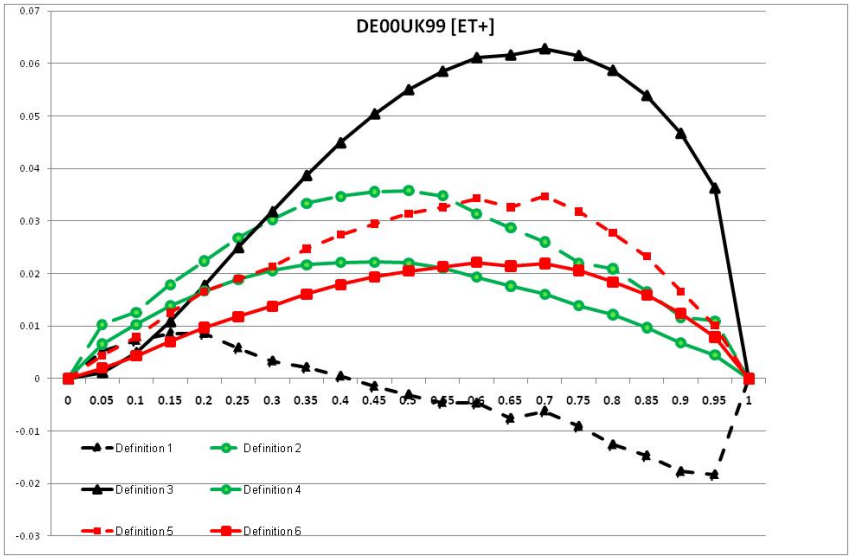

Figure 34: Progression comparison for direct plus payroll taxes and equivalized data: Germany 2000 vs. UK 1999

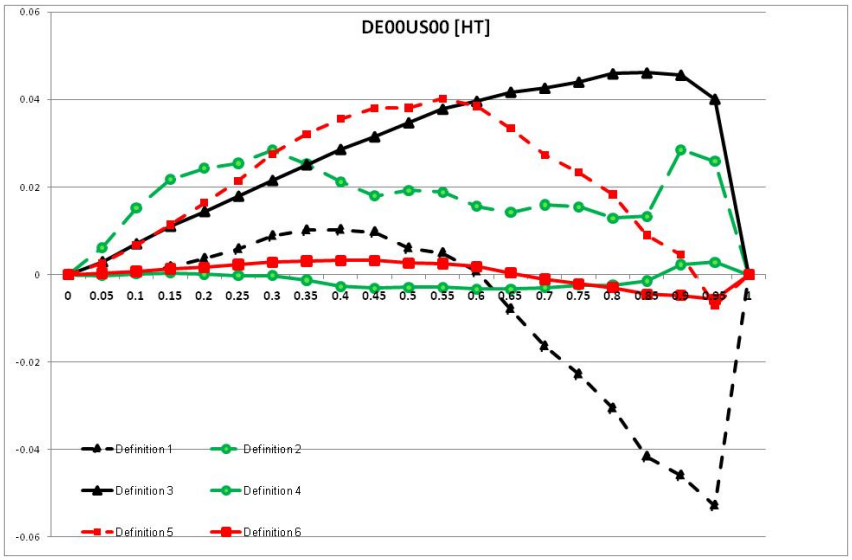

Figure 35: Progression comparison for direct taxes and household data: Germany 2000 vs. US 2000

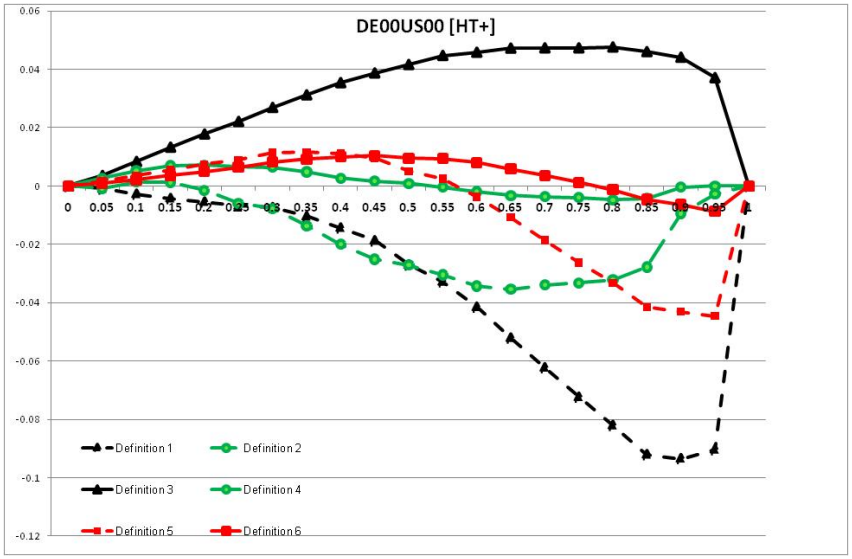

Figure 36: Progression comparison for direct plus payroll taxes and household data: Germany 2000 vs. US 2000

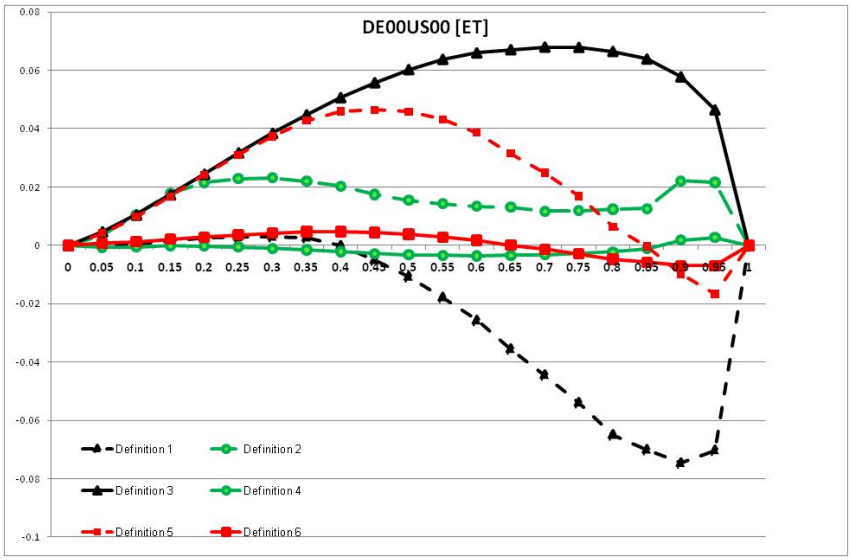

Figure 37: Progression comparison for direct taxes and equivalized data: Germany 2000 vs. US 2000

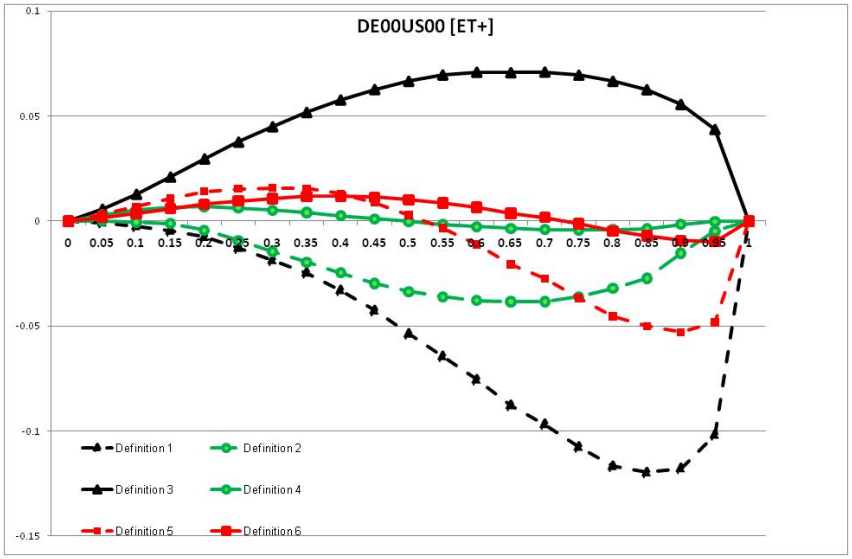

Figure 38: Progression comparison for direct plus payroll taxes and equivalized data: Germany 2000 vs. US 2000

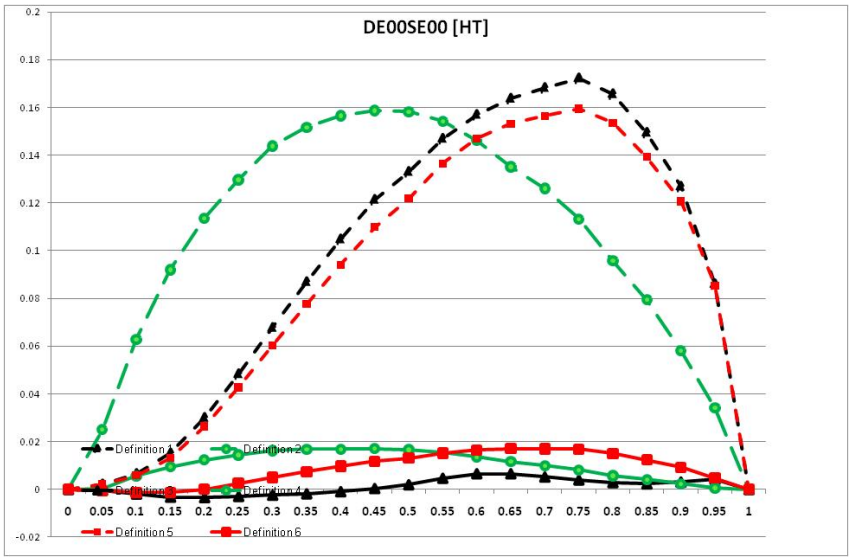

Figure 39: Progression comparison for direct taxes and household data: Germany 2000 vs. Sweden 2000

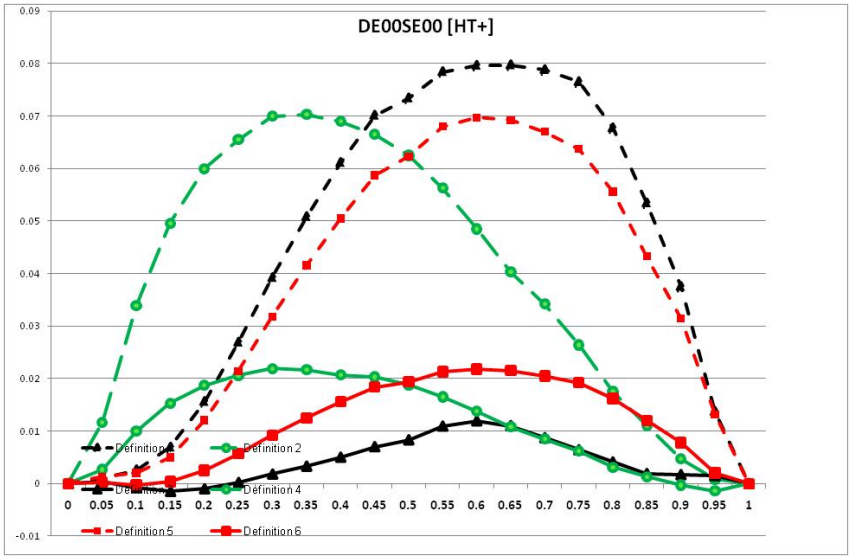

Figure 40: Progression comparison for direct plus payroll taxes and household data: Germany 2000 vs. Sweden 2000

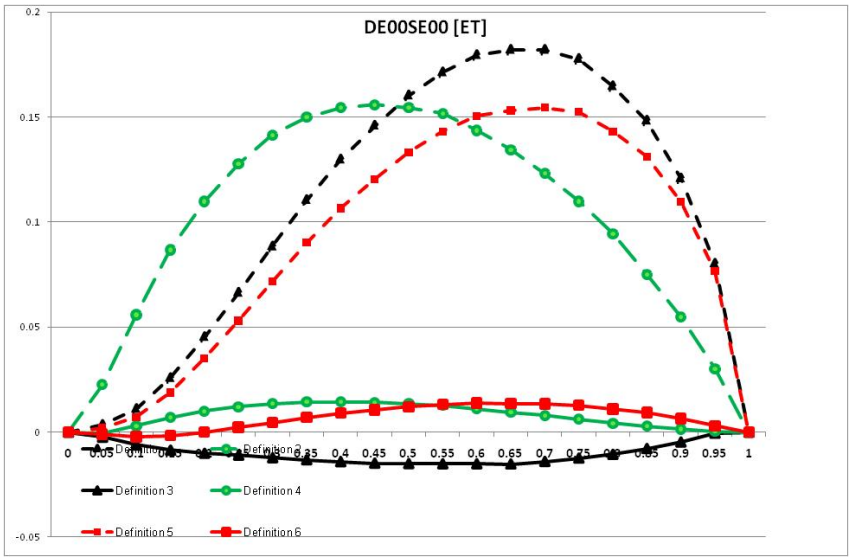

Figure 41: Progression comparison for direct taxes and equivalized data: Germany 2000 vs. Sweden 2000

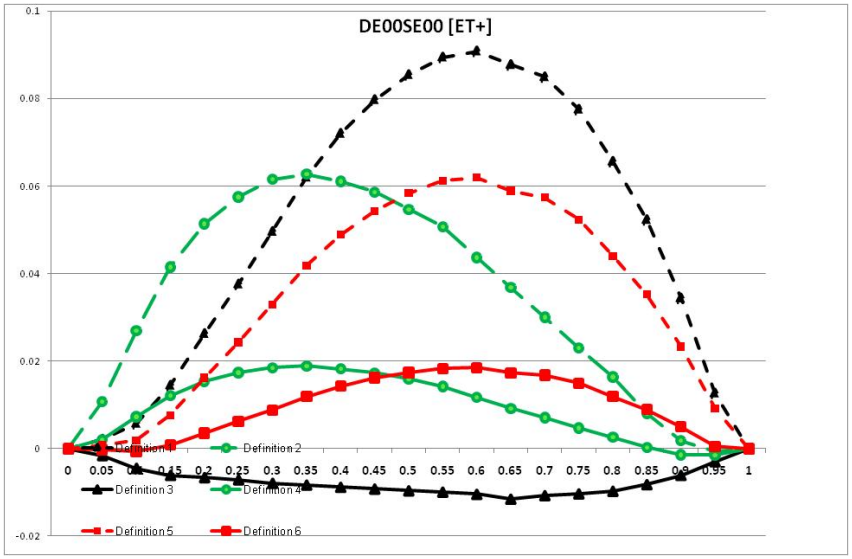

Figure 42: Progression comparison for direct plus payroll taxes and equivalized data: Germany 2000 vs. Sweden 2000

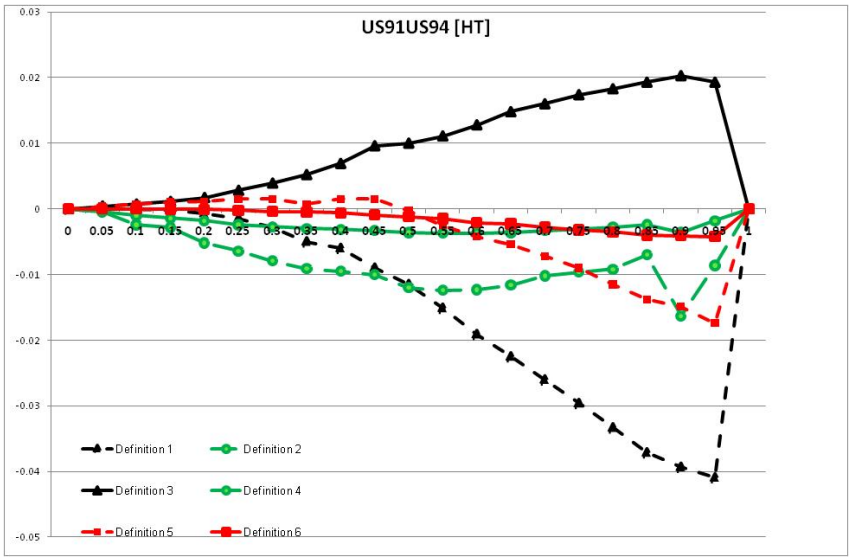

Figure 43: Progression comparison for direct taxes and household data: US 1991 vs. 1994

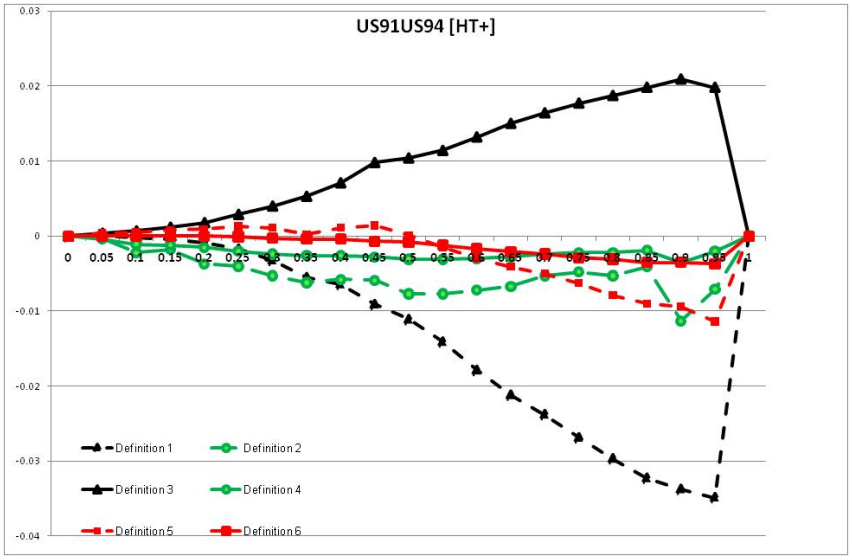

Figure 44: Progression comparison for direct plus payroll taxes and household data: US 1991 vs. 1994

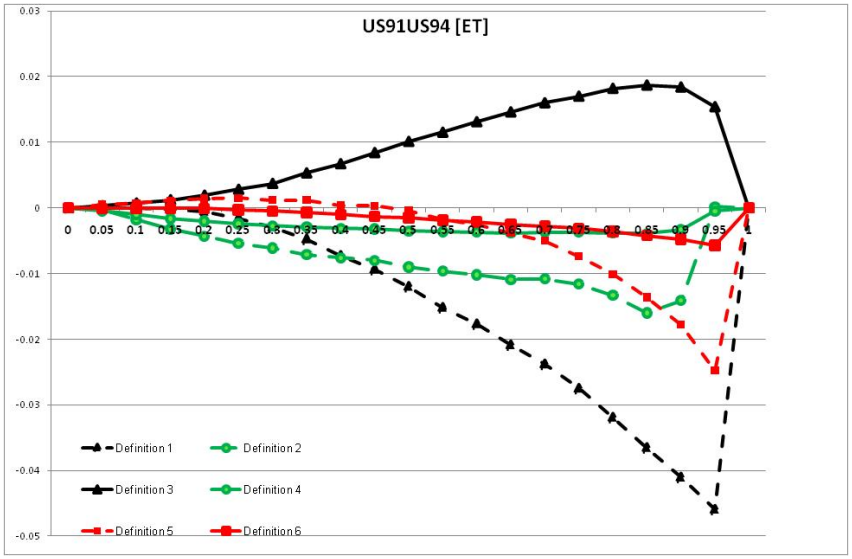

Figure 45: Progression comparison for direct taxes and equivalized data: US 1991 vs. 1994

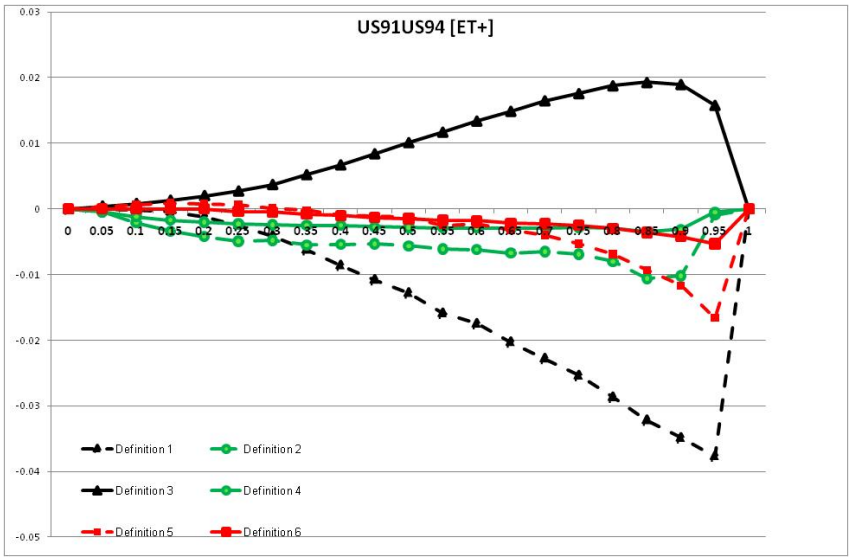

Figure 46: Progression comparison for direct plus payroll taxes and equivalized data: US 1991 vs. 1994

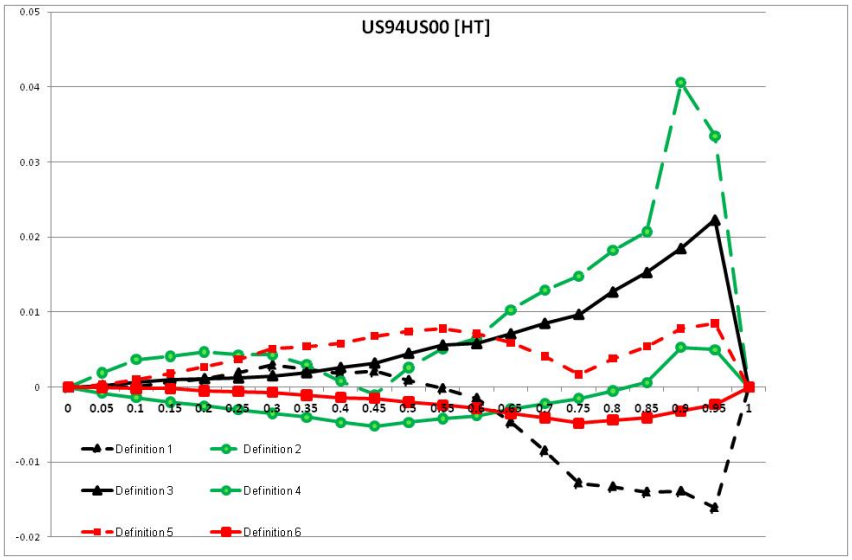

Figure 47: Progression comparison for direct taxes and household data: US 1994 vs. 2000

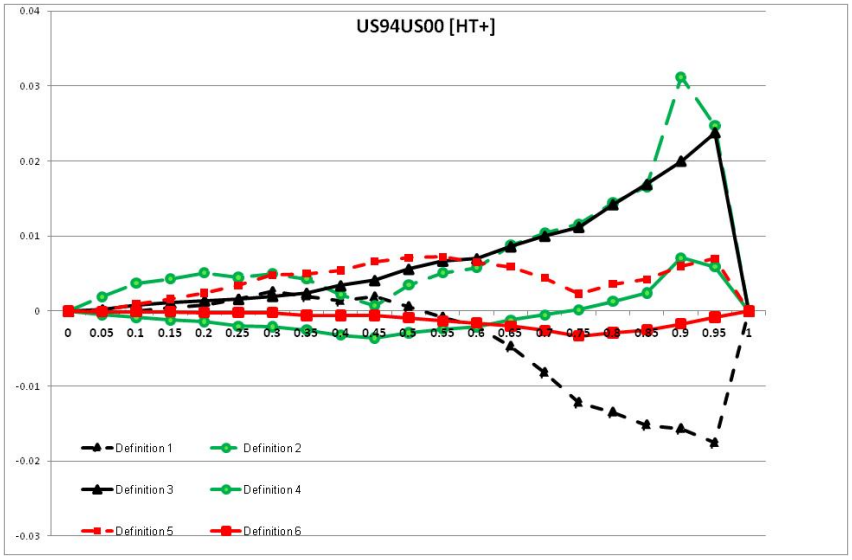

Figure 48: Progression comparison for direct plus payroll taxes and household data: US 1994 vs. 2000

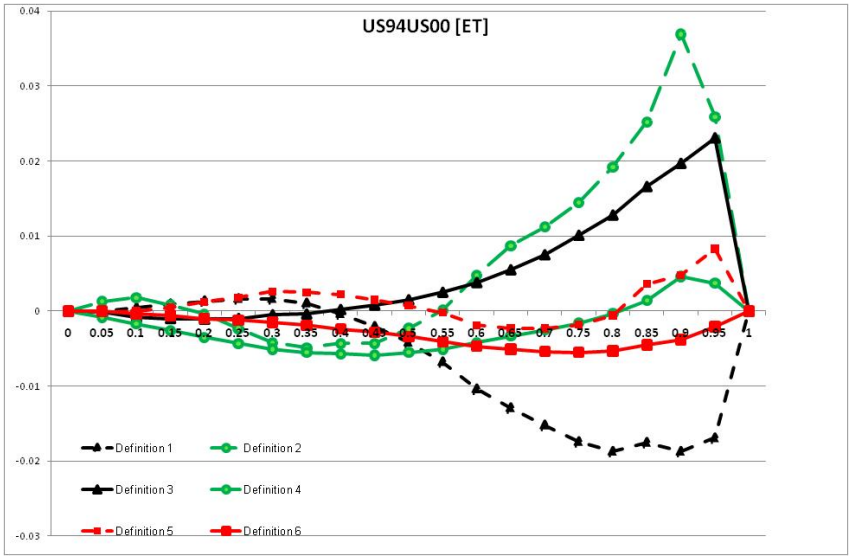

Figure 49: Progression comparison for direct taxes and equivalized data: US 1994 vs. 2000

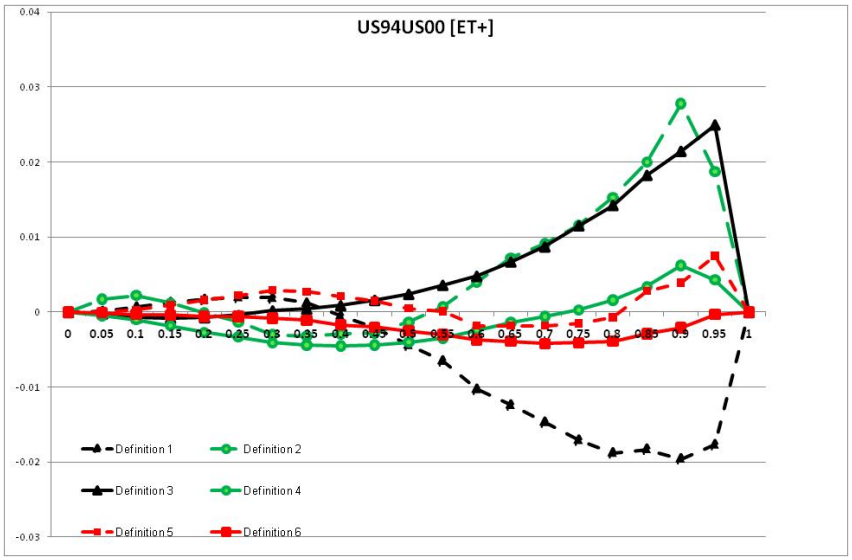

Figure 50: Progression comparison for direct plus payroll taxes and equivalized data: US 2000 vs. 2004

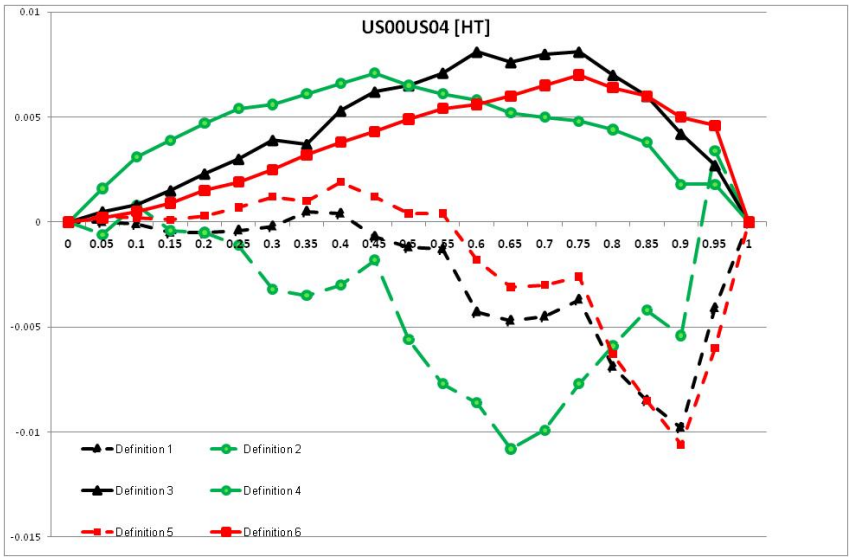

Figure 51: Progression comparison for direct taxes and household data: US 2000 vs. 2004

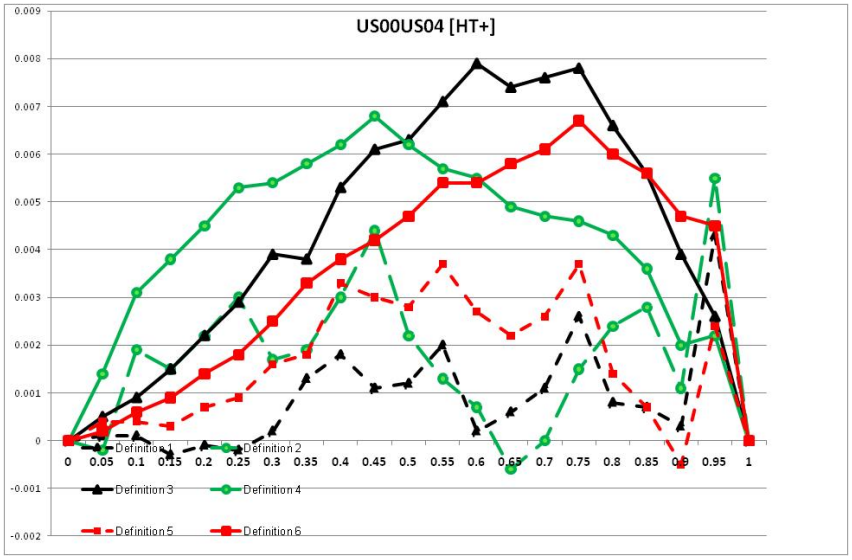

Figure 52: Progression comparison for direct plus payroll taxes and household data: US 2000 vs. 2004

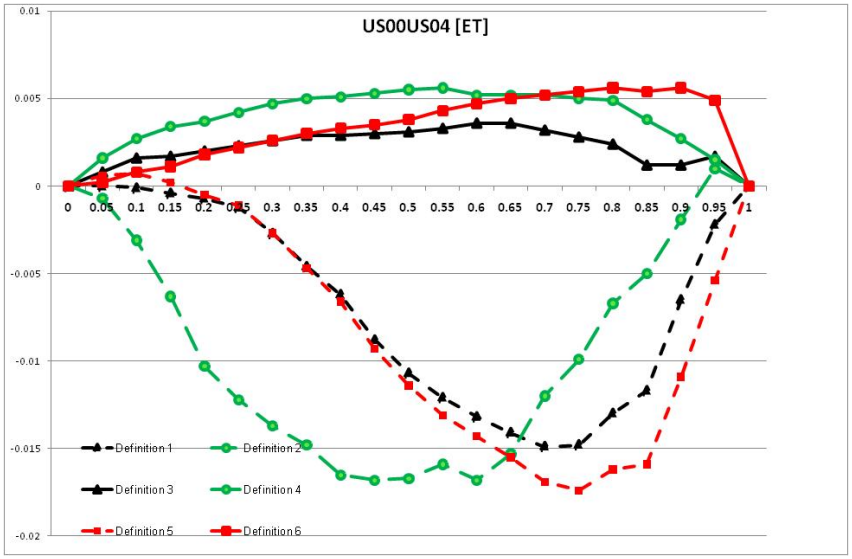

Figure 53: Progression comparison for direct taxes and equivalized data: US 2000 vs. 2004

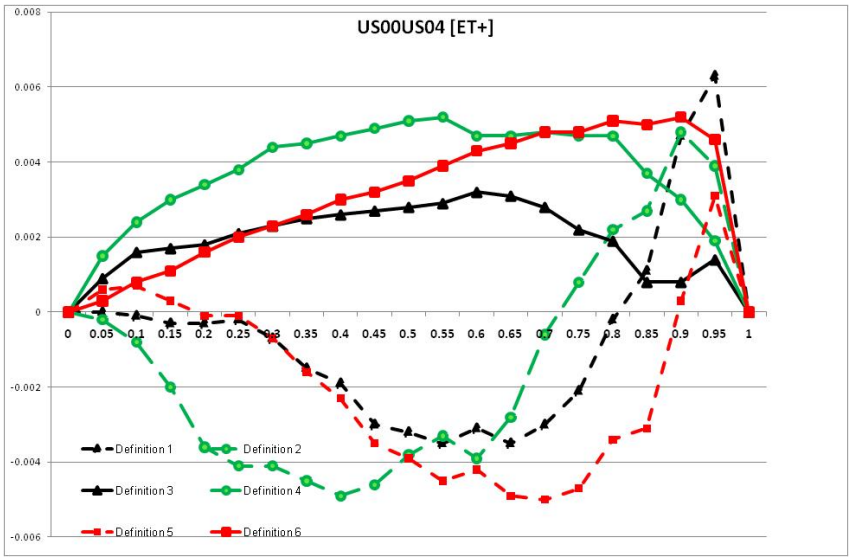

Figure 54: Progression comparison for direct plus payroll taxes and equivalized data: US 2000 vs. 2004

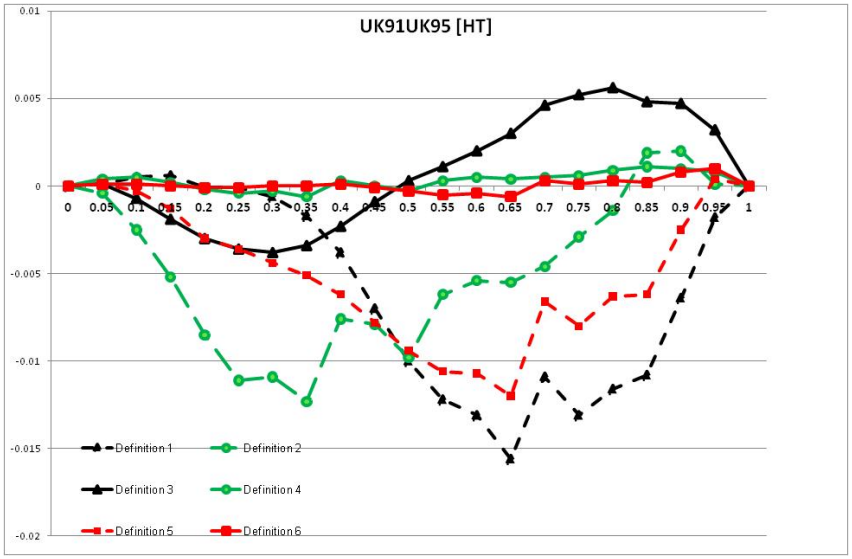

Figure 55: Progression comparison for direct taxes and household data: UK 1991 vs. 1995

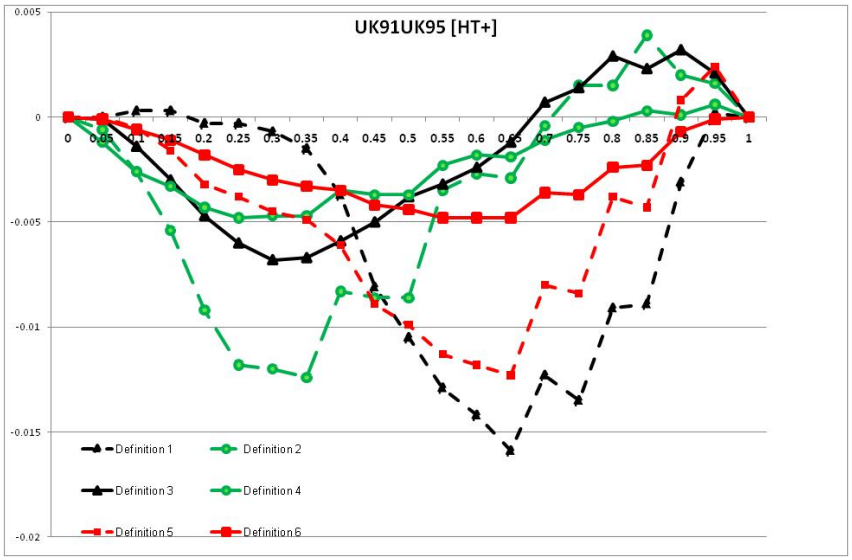

Figure 56: Progression comparison for direct plus payroll taxes and household data: UK 1991 vs. 1995

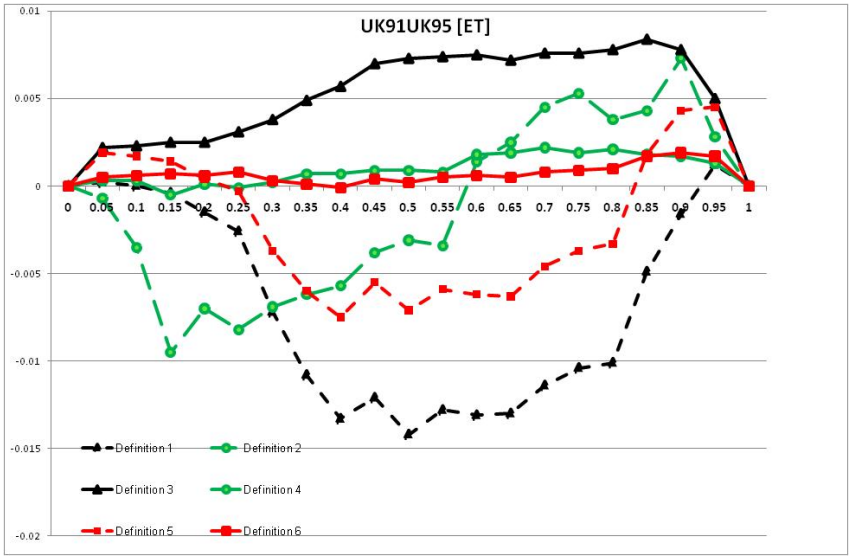

Figure 57: Progression comparison for direct taxes and equivalized data: UK 1991 vs. 1995

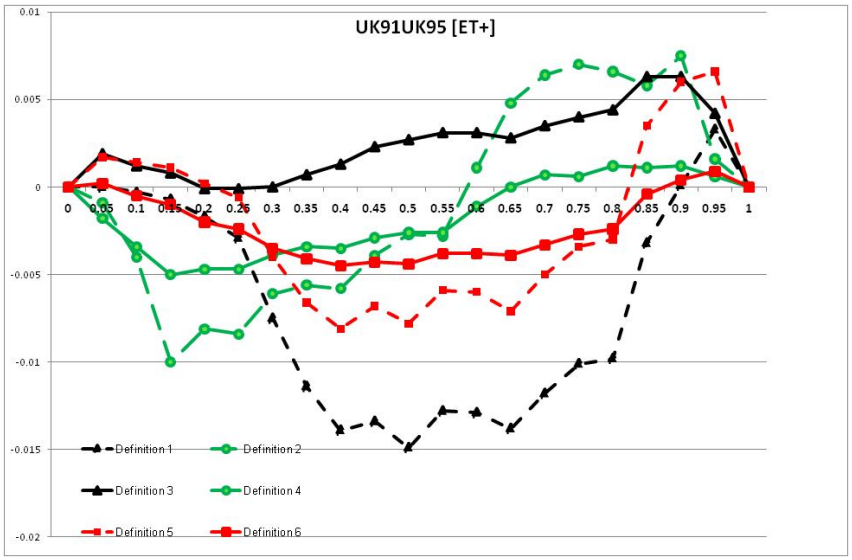

Figure 58: Progression comparison for direct plus payroll taxes and equivalized data: UK 1991 vs. 1995

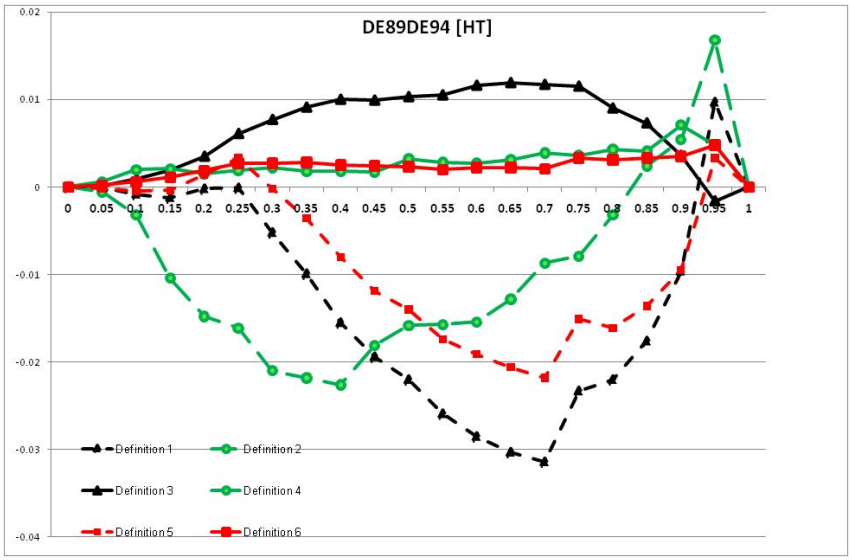

Figure 59: Progression comparison for direct taxes and household data: Germany 1989 vs. 1994

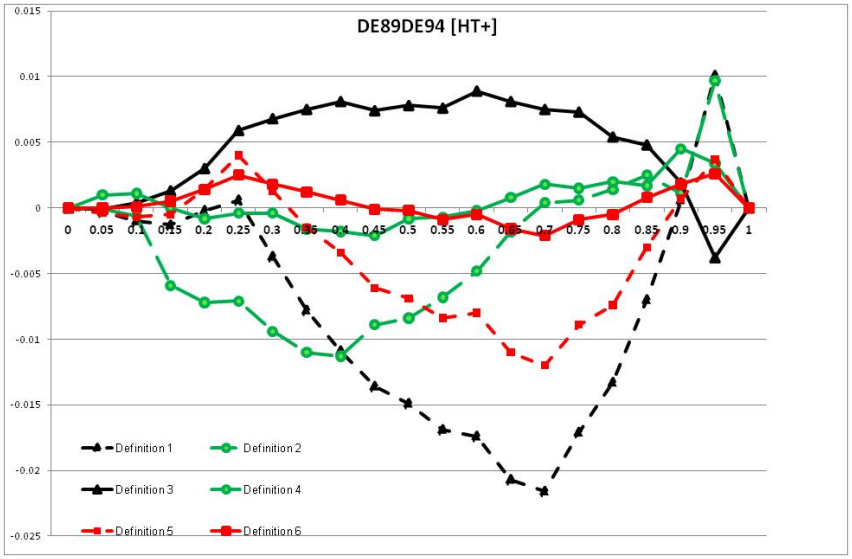

Figure 60: Progression comparison for direct plus payroll taxes and household data: Germany 1989 vs. 1994

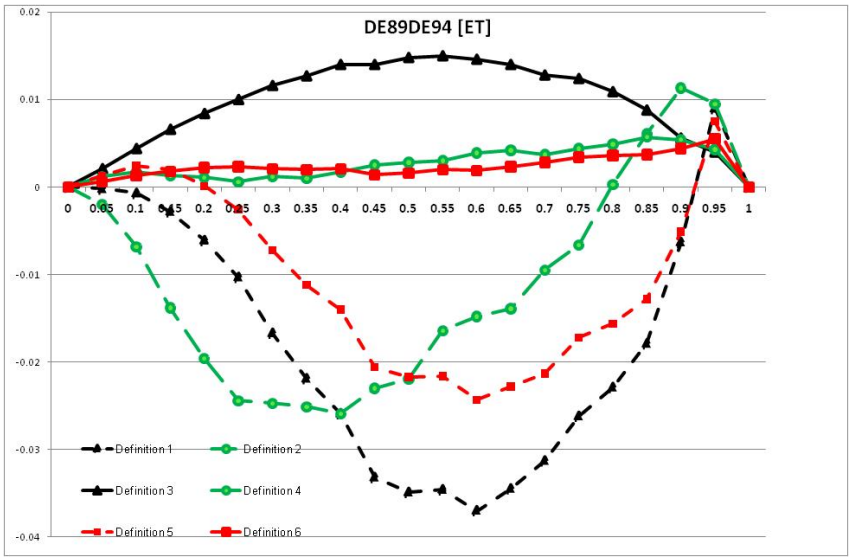

Figure 61: Progression comparison for direct taxes and equivalized data: Germany 1989 vs. 1994

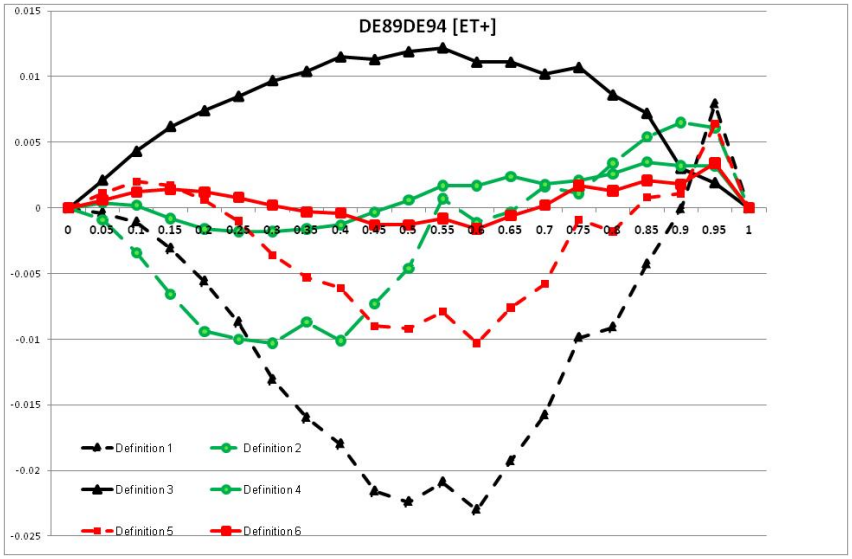

Figure 62: Progression comparison for direct plus payroll taxes and equivalized data: Germany 1989 vs. 1994

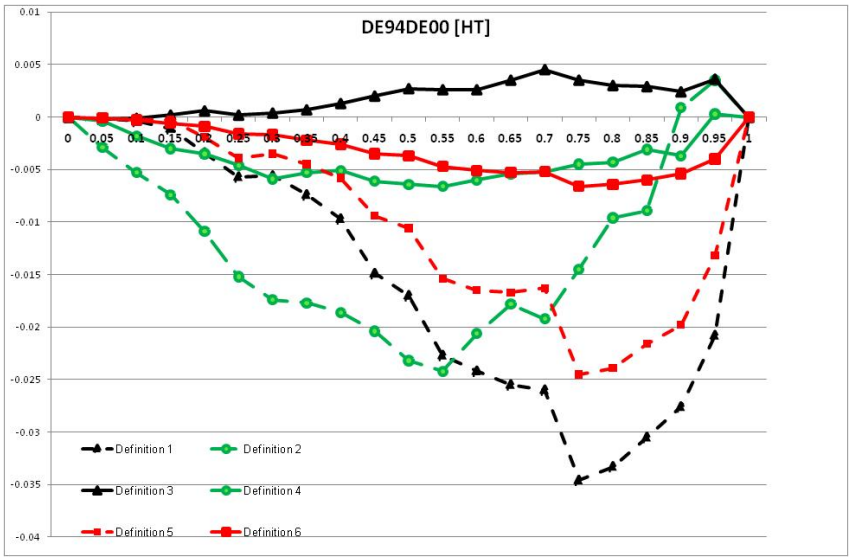

Figure 63: Progression comparison for direct taxes and household data: Germany 1994 vs. 2000

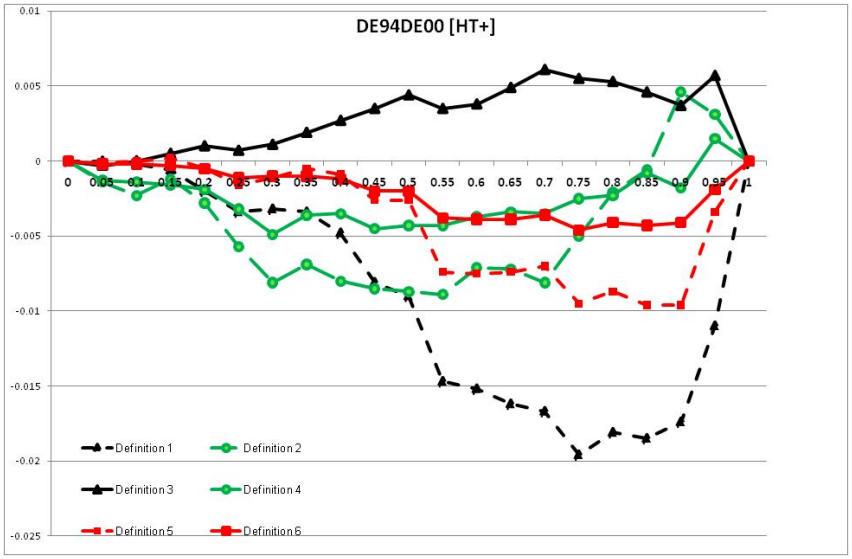

Figure 64: Progression comparison for direct plus payroll taxes and household data: Germany 1994 vs. 2000

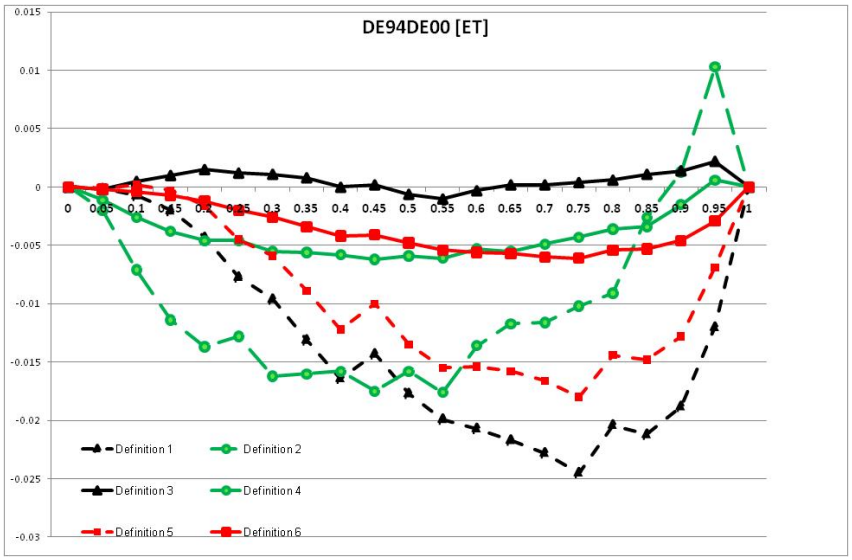

Figure 65: Progression comparison for direct taxes and equivalized data: Germany 1994 vs. 2000

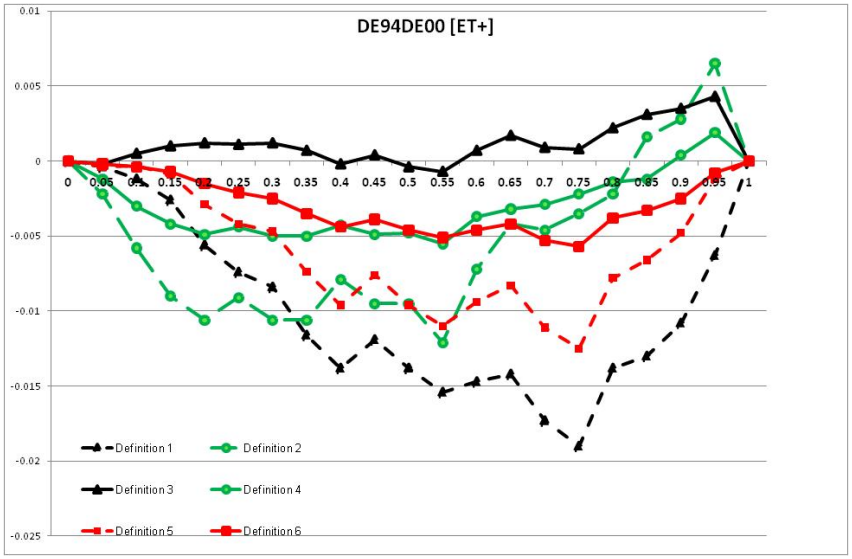

Figure 66: Progression comparison for direct plus payroll taxes and equivalized data: Germany 1994 vs. 2000

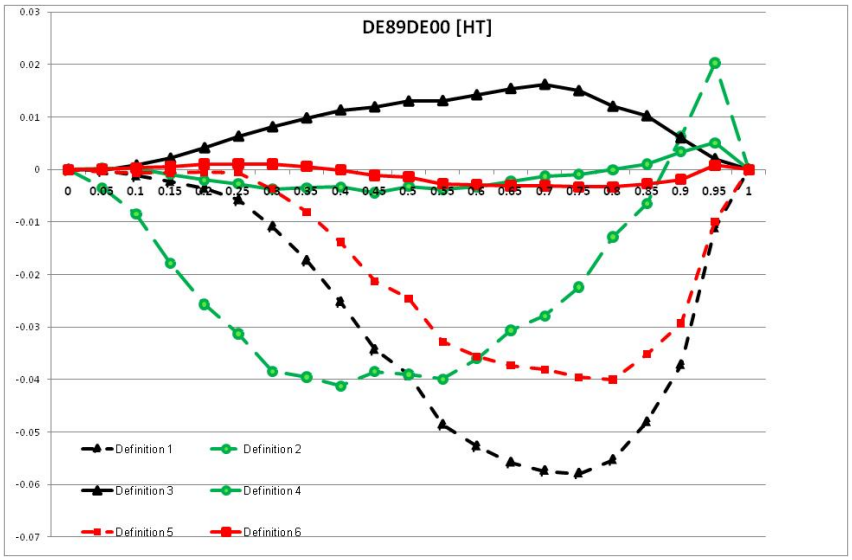

Figure 67: Progression comparison for direct taxes and household data: Germany 1989 vs. 2000

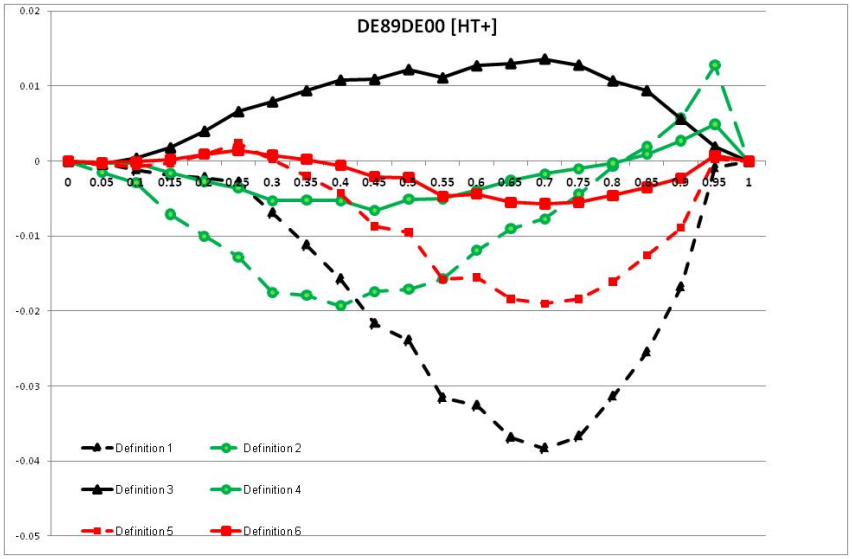

Figure 68: Progression comparison for direct plus payroll taxes and household data: Germany 1989 vs. 2000

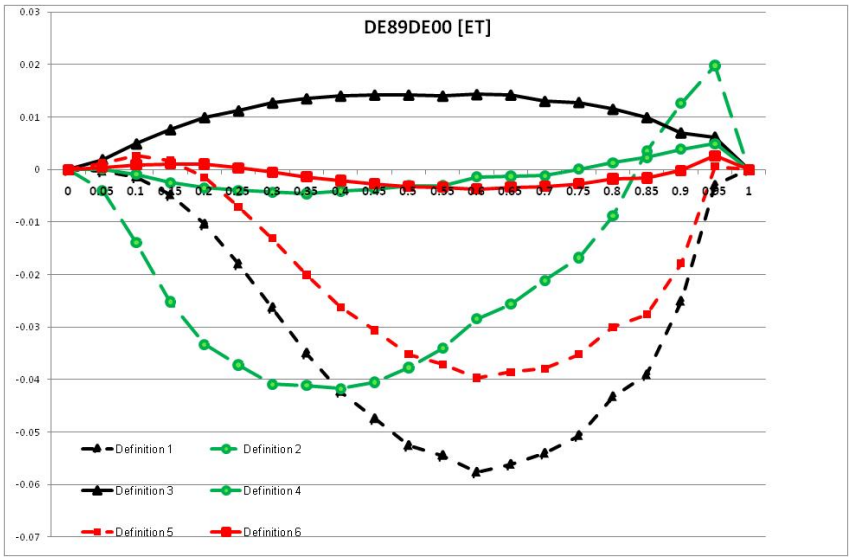

Figure 69: Progression comparison for direct taxes and equivalized data: Germany 1989 vs. 2000

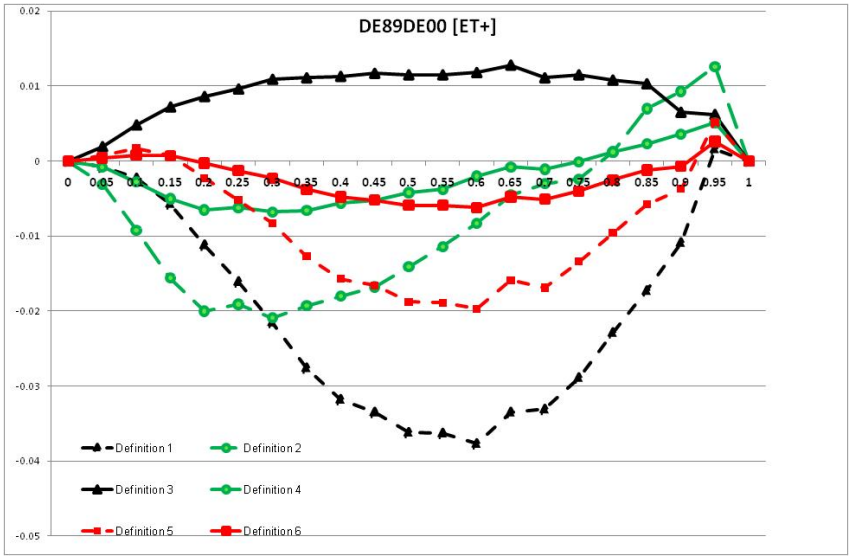

Figure 70: Progression comparison for direct plus payroll taxes and equivalized data: Germany 1989 vs. 2000

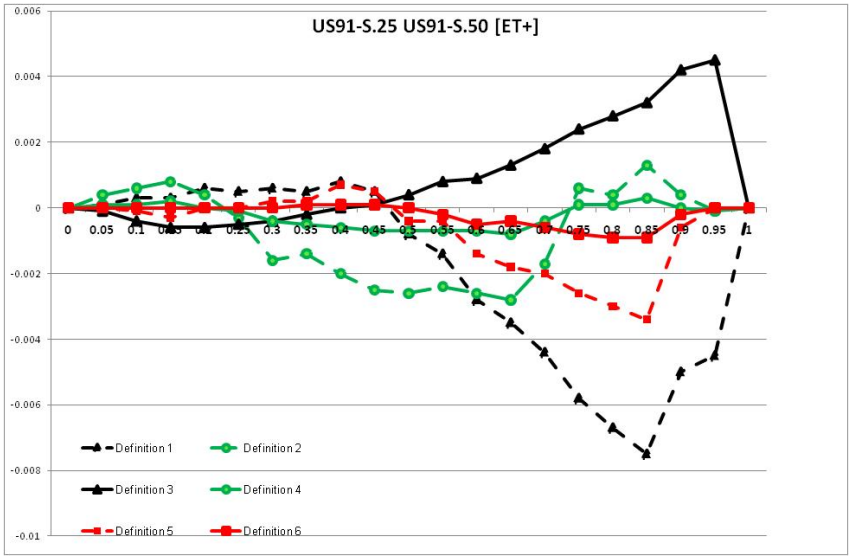

Figure 71: Progression comparison for direct plus payroll taxes and scale parameters 0.25 and 0.50: United States 1991

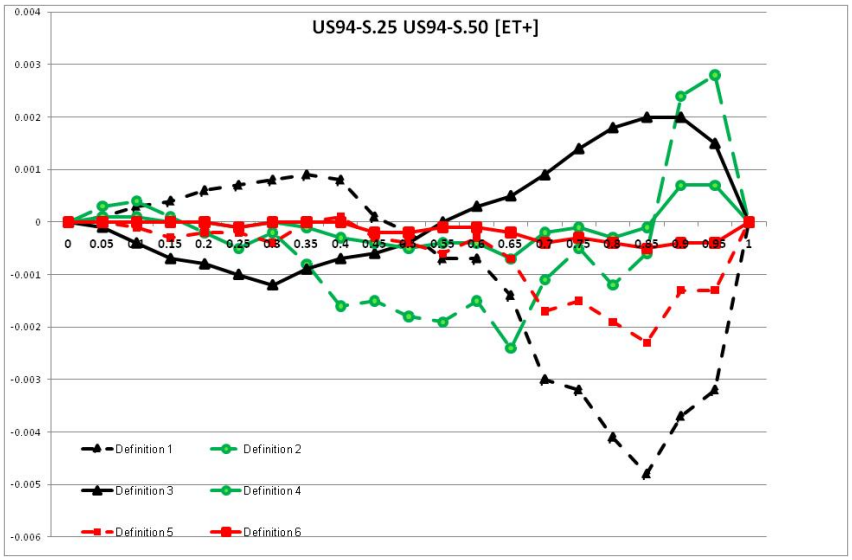

Figure 72: Progression comparison for direct plus payroll taxes and scale parameters 0.25 and 0.50: United States 1994

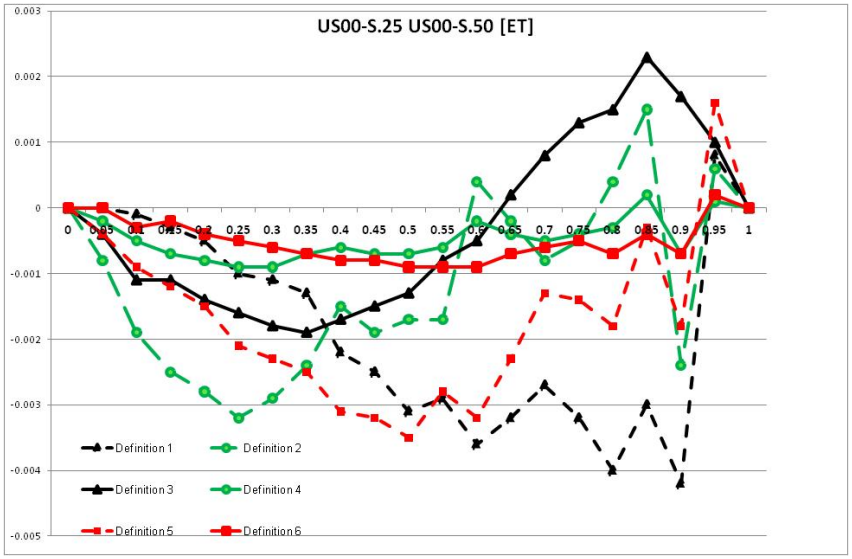

Figure 73: Progression comparison for direct taxes and scale parameters 0.25 and 0.50: United States 2000

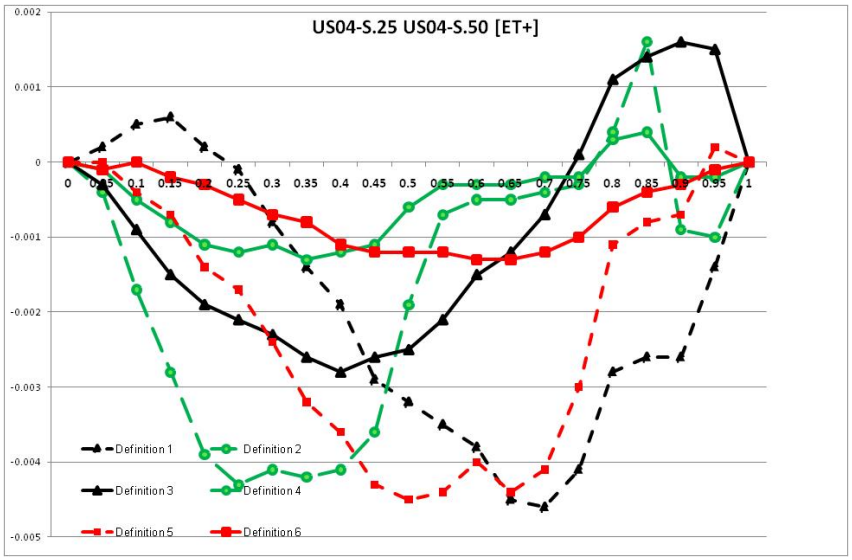

Figure 74: Progression comparison for direct plus payroll taxes and scale parameters 0.25 and 0.50: United States 2004

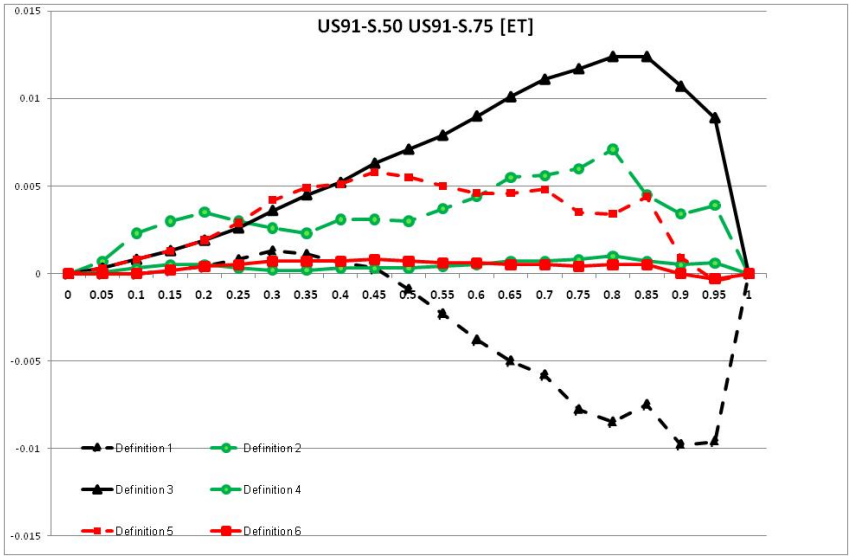

Figure 75: Progression comparison for direct taxes and scale parameters 0.50 and 0.75: United States 1991

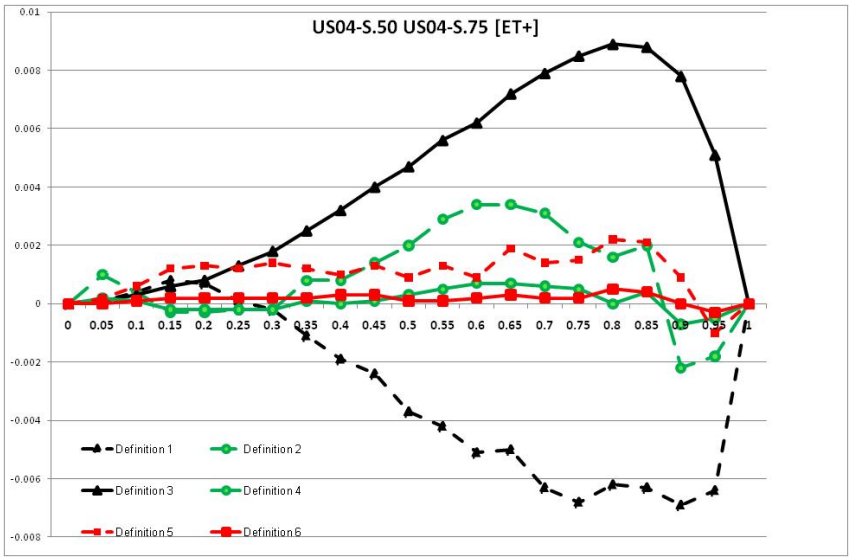

Figure 76: Progression comparison for direct plus payroll taxes and scale parameters 0.50 and 0.75: United States 2004

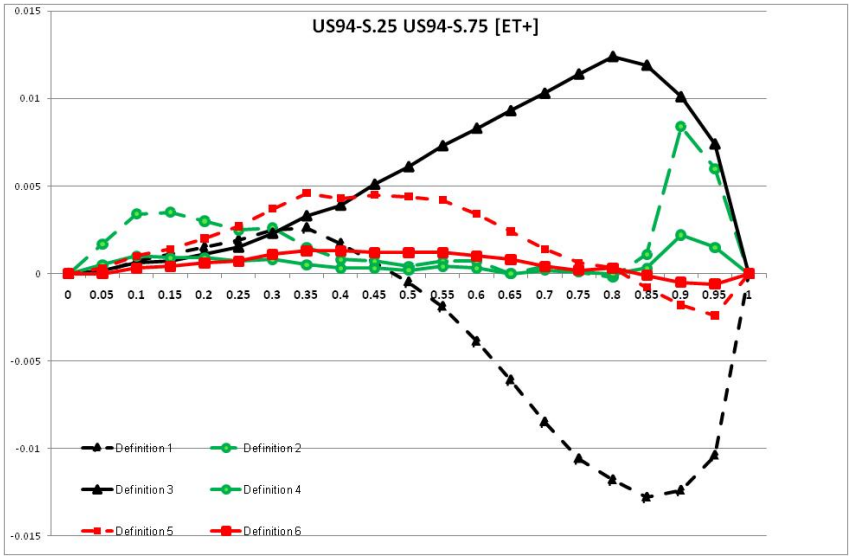

Figure 77: Progression comparison for direct plus payroll taxes and scale parameters 0.25 and 0.75: United States 1994

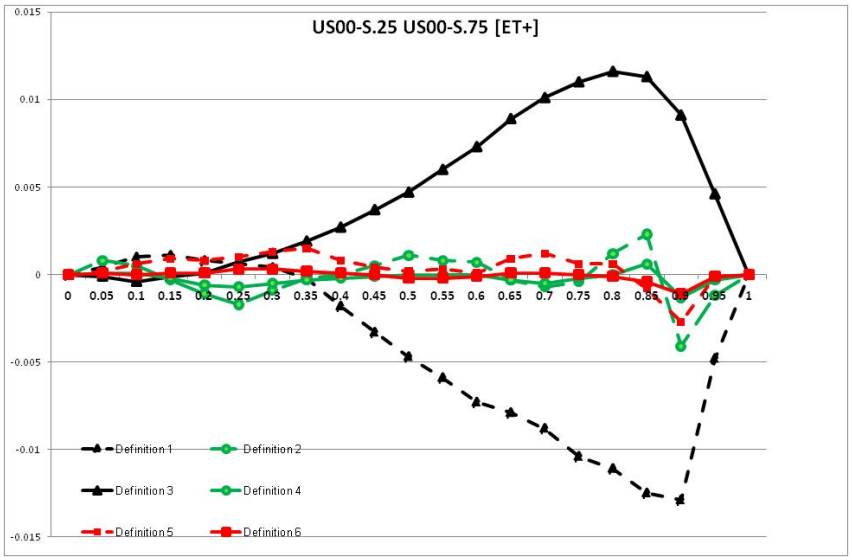

Figure 78: Progression comparison for direct plus payroll taxes and scale parameters 0.25 and 0.75: United States 2000

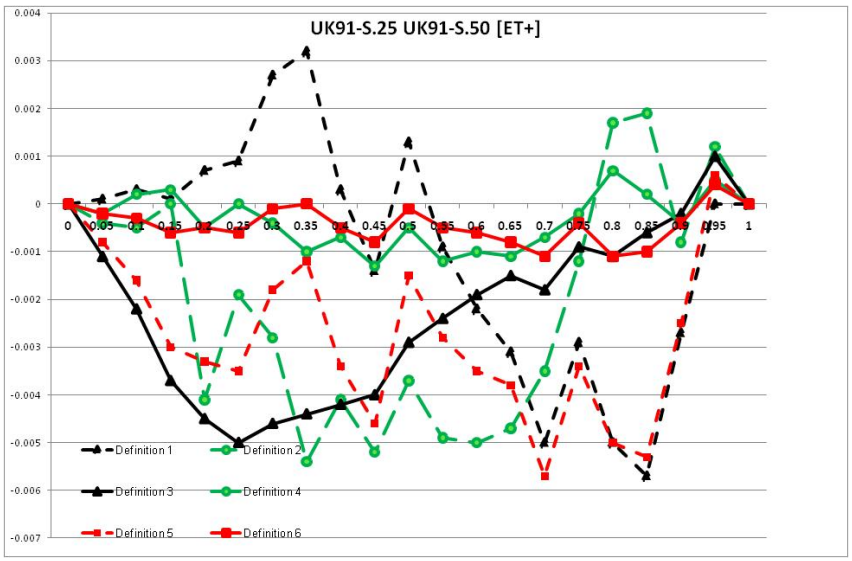

Figure 79: Progression comparison for direct plus payroll taxes and scale parameters 0.25 and 0.50: United Kingdom 1991

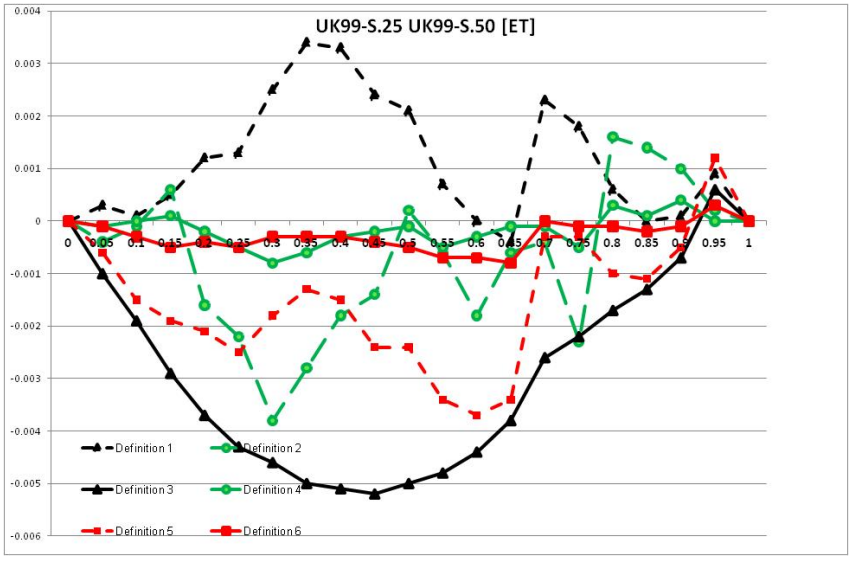

Figure 80: Progression comparison for direct taxes and scale parameters 0.25 and 0.50: United Kingdom 1999

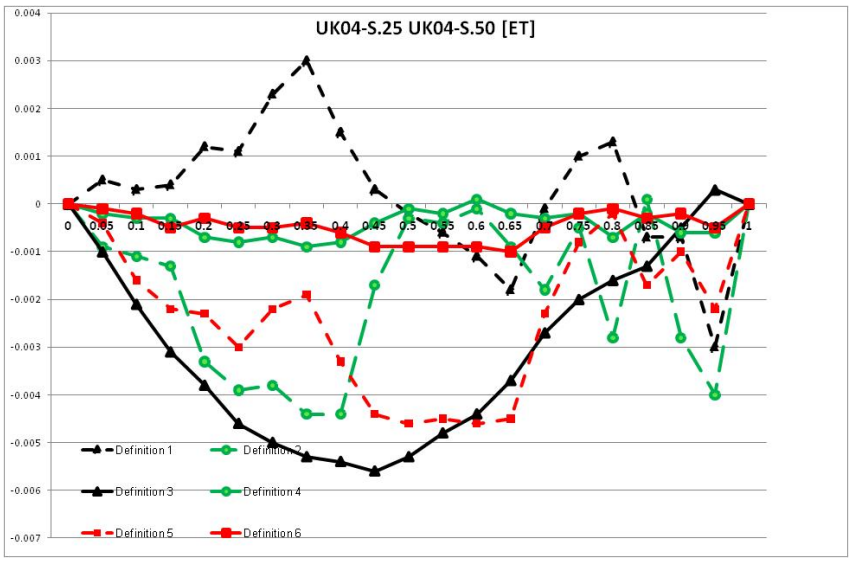

Figure 81: Progression comparison for direct taxes and scale parameters 0.25 and 0.50: United Kingdom 2004

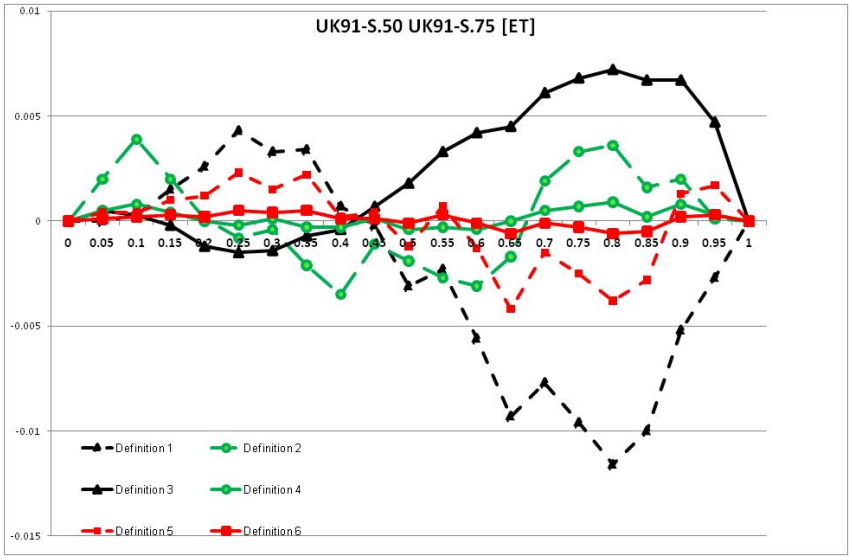

Figure 82: Progression comparison for direct taxes and scale parameters 0.50 and 0.75: United Kingdom 1991

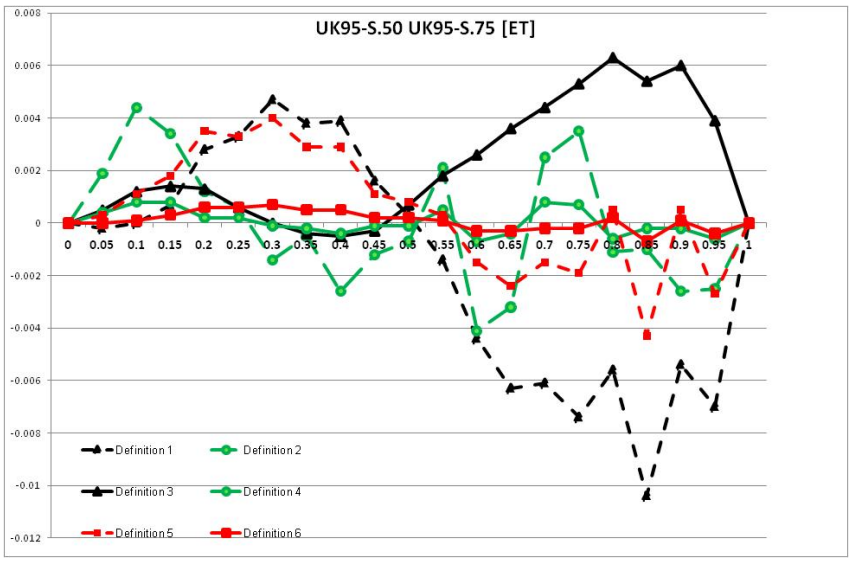

Figure 83: Progression comparison for direct taxes and scale parameters 0.50 and 0.75: United Kingdom 1995

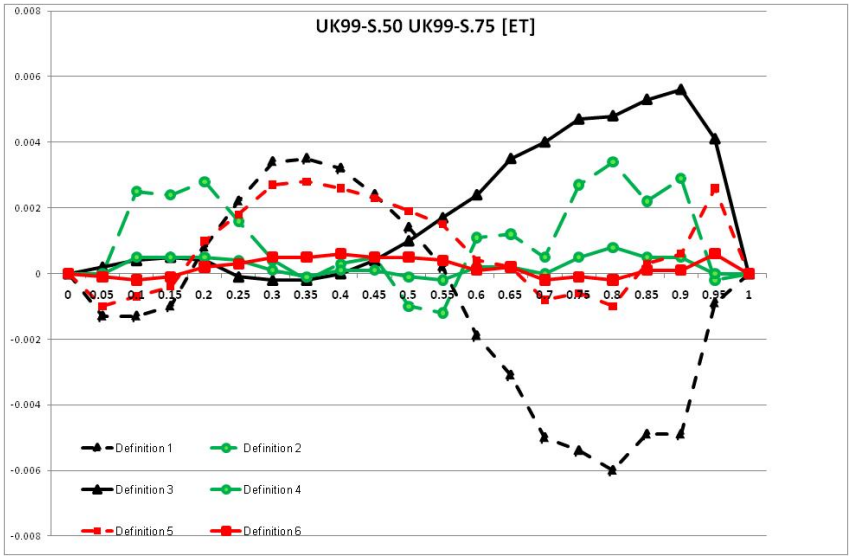

Figure 84: Progression comparison for direct taxes and scale parameters 0.25 and 0.75: United Kingdom 1999

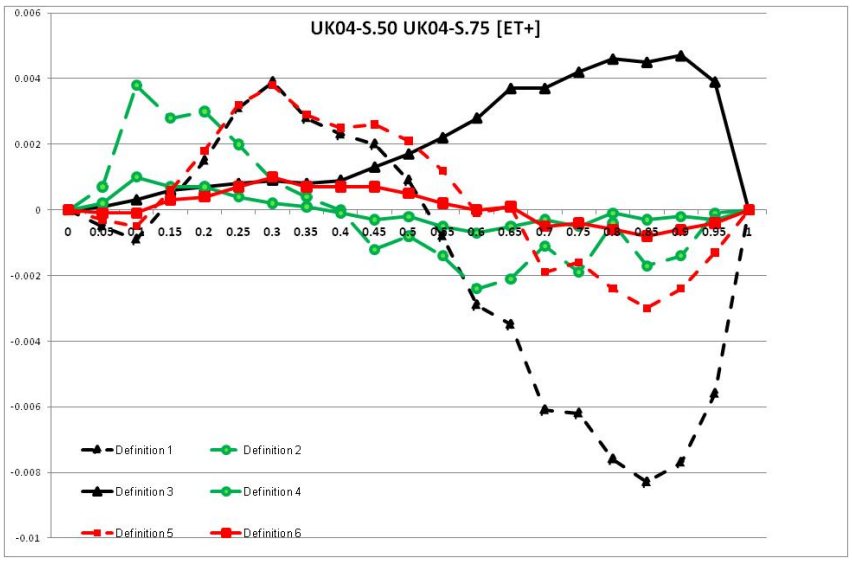

Figure 85: Progression comparison for direct taxes and scale parameters 0.50 and 0.75: United Kingdom 2004

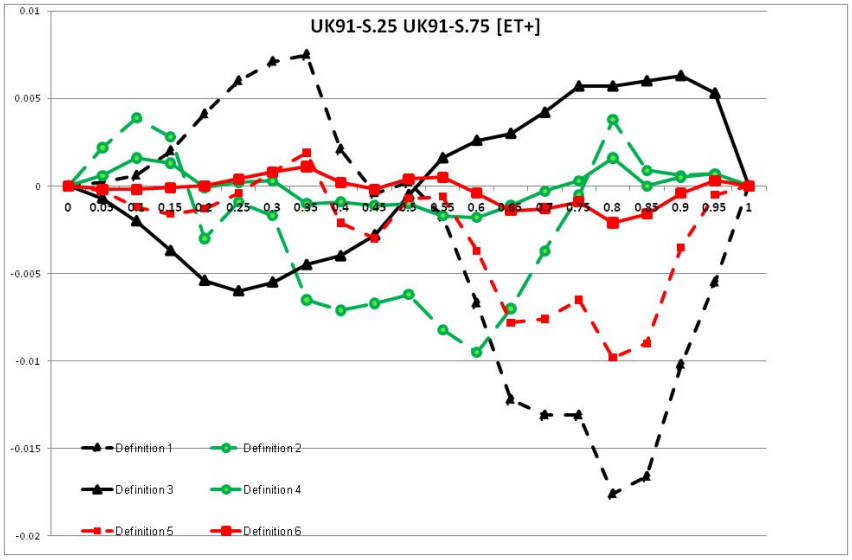

Figure 86: Progression comparison for direct plus payroll taxes and scale parameters 0.25 and 0.75: United Kingdom 1991

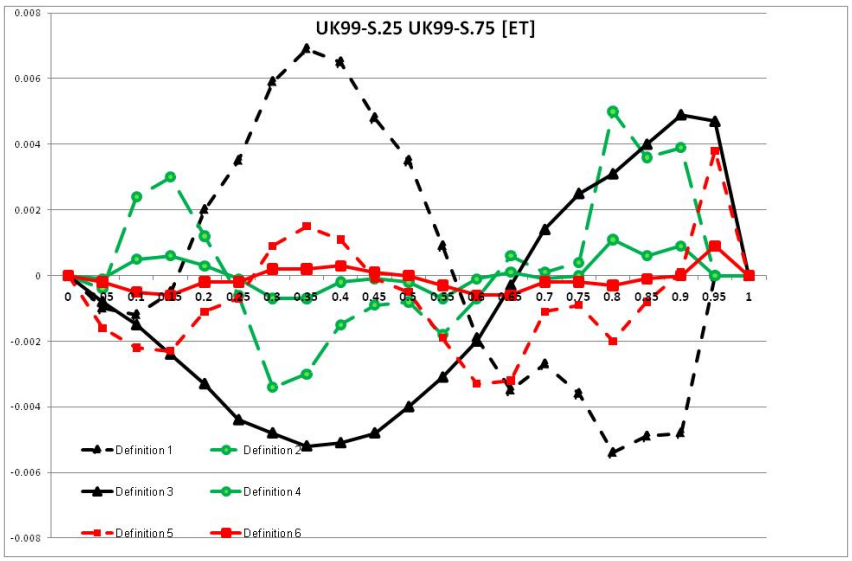

Figure 87: Progression comparison for direct taxes and scale parameters 0.25 and 0.75: United Kingdom 1999

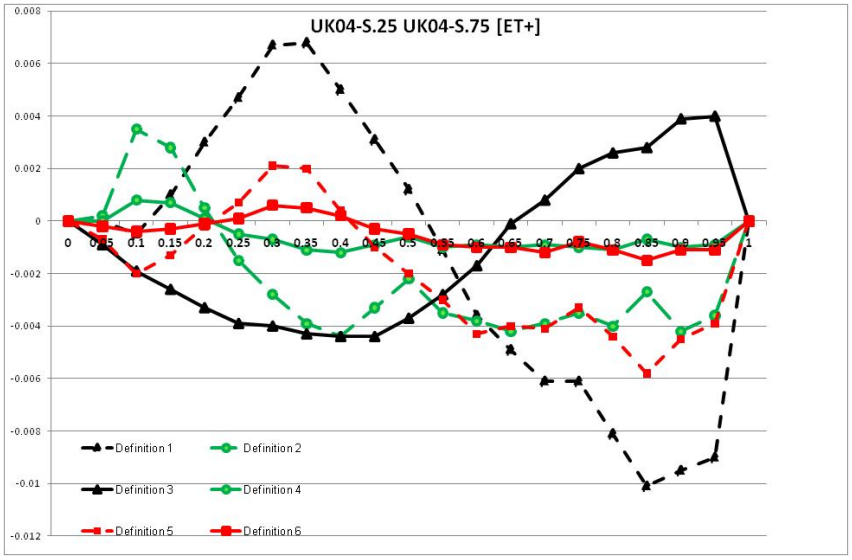

Figure 88: Progression comparison for direct plus payroll taxes and scale parameters 0.25 and 0.75: United Kingdom 2004

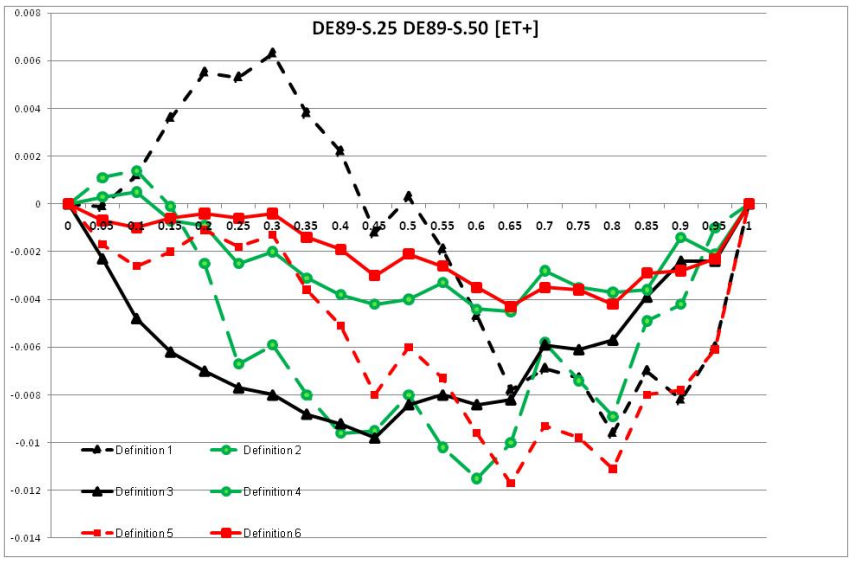

Figure 89: Progression comparison for direct plus payroll taxes and scale parameters 0.25 and 0.50: Germany 1989

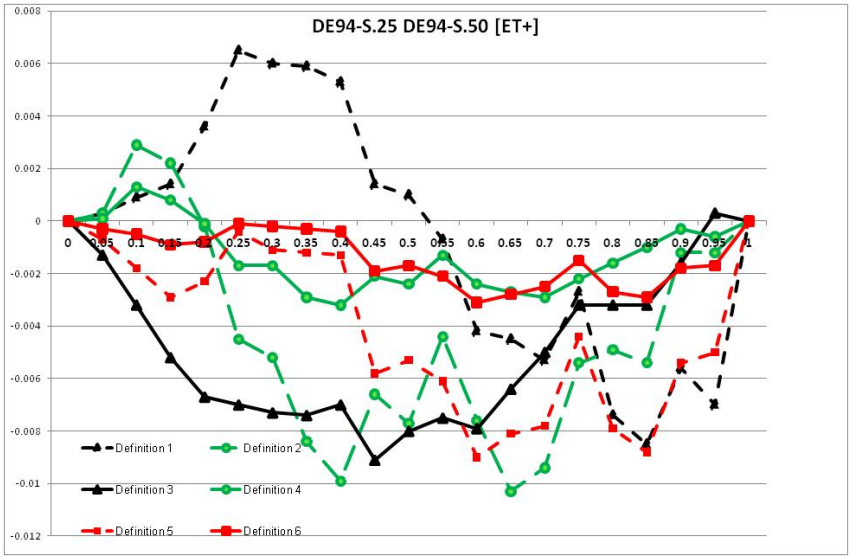

Figure 90: Progression comparison for direct plus payroll taxes and scale parameters 0.25 and 0.50: Germany 1994

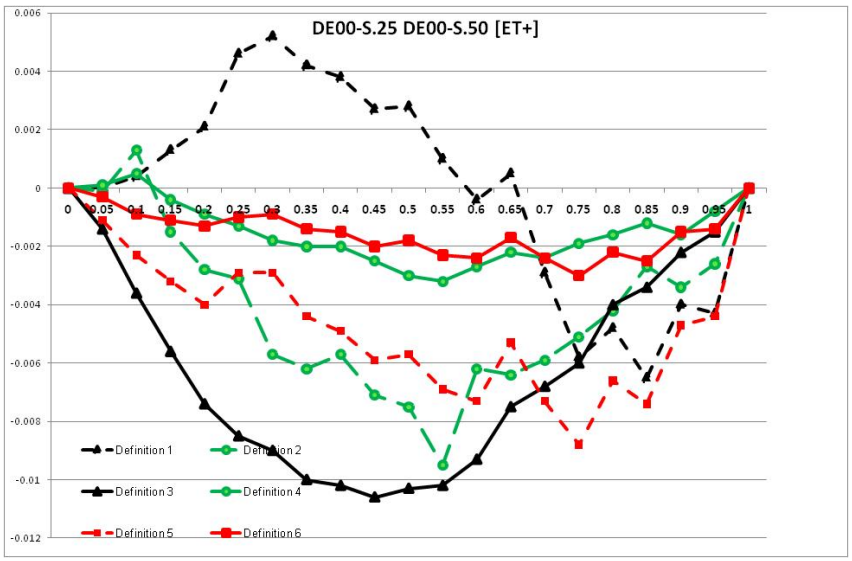

Figure 91: Progression comparison for direct plus payroll taxes and scale parameters 0.25 and 0.50: Germany 2000

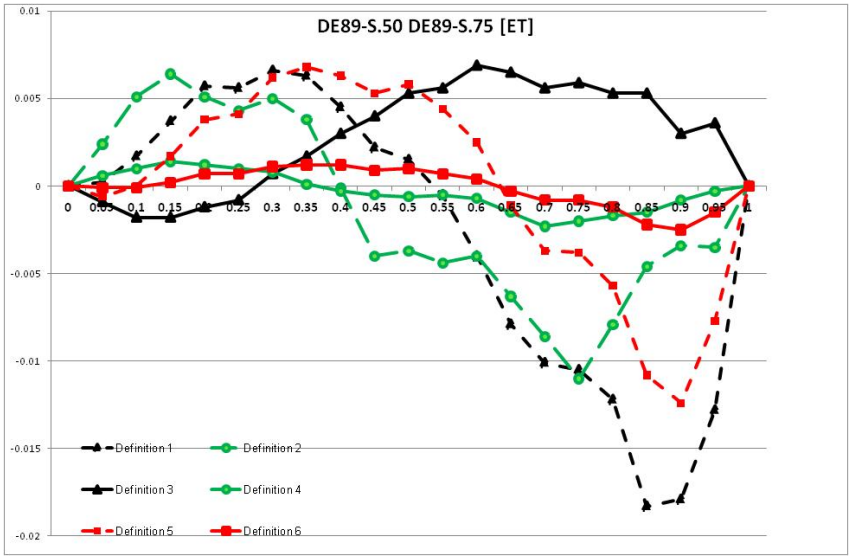

Figure 92: Progression comparison for direct taxes and scale parameters 0.50 and 0.75: Germany 1989

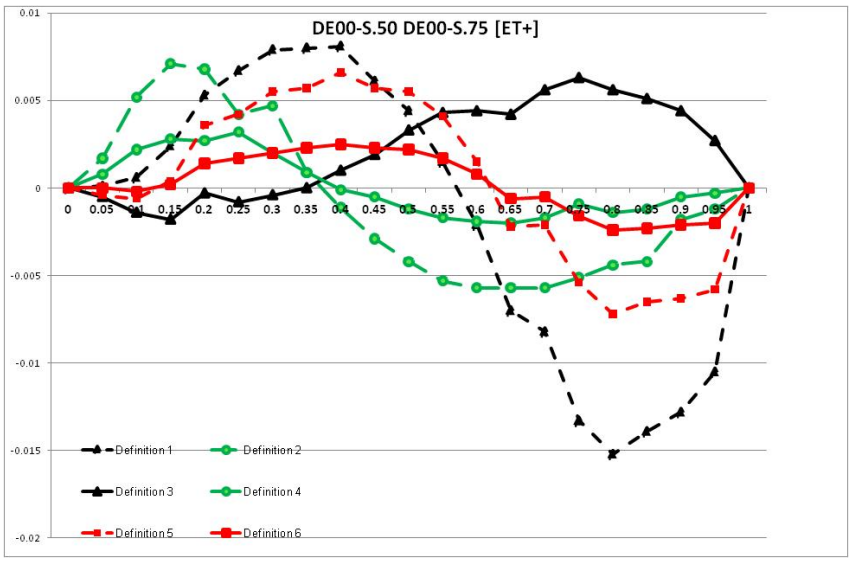

Figure 93: Progression comparison for direct plus payroll taxes and scale parameters 0.50 and 0.75: Germany 2000

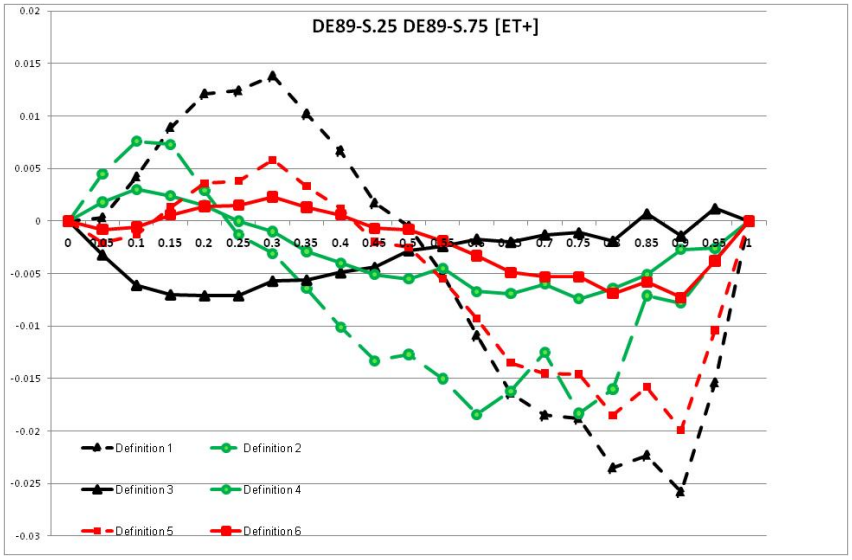

Figure 94: Progression comparison for direct plus payroll taxes and scale parameters 0.25 and 0.75: Germany 1989

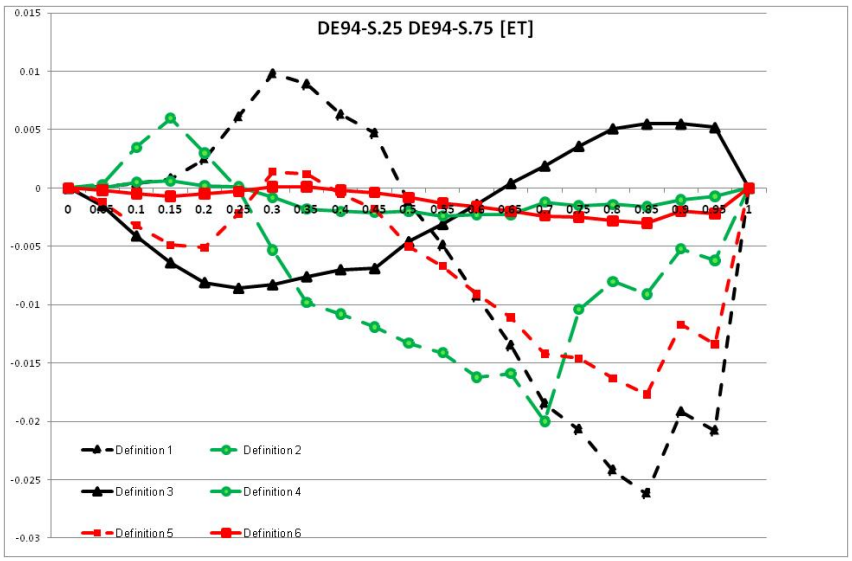

Figure 95: Progression comparison for direct taxes and scale parameters 0.25 and 0.75: Germany 1994

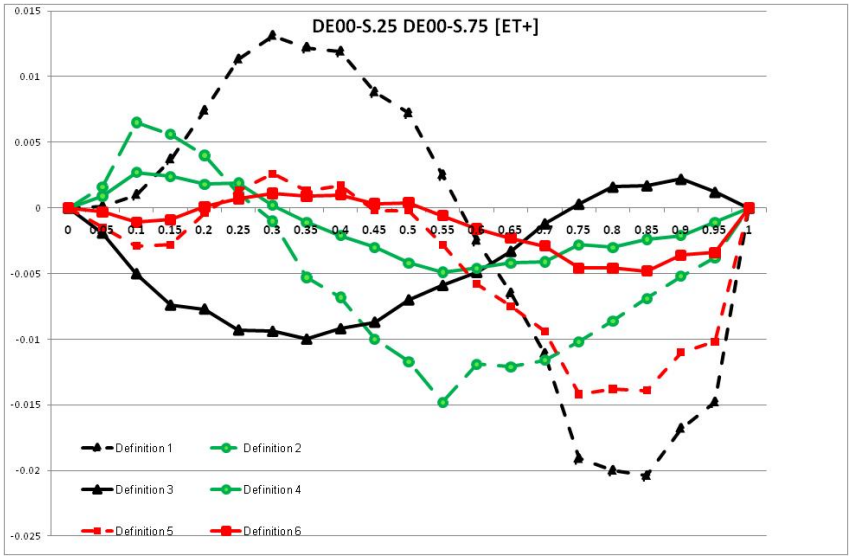

Figure 96: Progression comparison for direct plus payroll taxes and scale parameters 0.25 and 0.75: Germany 2000