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Quaderni di Dipartimento

Calvo vs. Rotemberg in a Trend Inflation World: An Empirical Investigation

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Calvo vs. Rotemberg in a Trend Inflation World: An Empirical Investigation*

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Abstract

This paper estimates and compares new-Keynesian DSGE monetary models of the business cycle derived under two different pricing schemes - Calvo, Rotemberg - and a positive trend inflation rate. Our empirical findings (i) support trend inflation-equipped models as better fitting during the U.S. great moderation period, (ii) provide evidence in favor of the statistical superiority of the Calvo setting, and (iii) suggest the absence of price indexation under the Calvo mechanism only. Possibly, the superiority of the Calvo model (against Rotemberg) is due to the restrictions implied by such pricing scheme for the aggregate demand equation. The determinacy regions associated to the two estimated models indicate relevant differences in the implementable simple policies. Our findings call for the development of monetary policy models consistently embedding a positive trend inflation rate and possibly based on a Calvo pricing scheme.

JEL classification:

Keywords: Calvo, Rotemberg, trend inflation, Bayesian estimations.

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1 Introduction

DSGE New Keynesian monetary model always features nominal rigidities. The Calvo (1983) and Rotemberg (1982) sticky pricing models are by far the two most common employed both in theoretical and applied work in this literature. Moreover, most models are typically linearly approximated assuming ‘zero inflation in steady state’. Under such assumption, the Calvo and the Rotemberg pricing schemes lead to the very same macroeconomic dynamics (e.g., Rotemberg, 1987, Roberts, 1995) and to equivalent welfare indications (e.g., Nisticò, 2007).\footnote{Lombardo and Vestin (2008) discuss the conditions under which welfare costs might be different under these two pricing schemes.} Given such a model equivalence (up to a first order degree of approximation), the choice of the Calvo vs. Rotemberg pricing scheme has typically been no more than a matter of taste.

In a recent contribution, however, Ascari and Rossi (2009) compare the two sticky pricing schemes and show that, contrary to conventional wisdom, the Calvo and the Rotemberg models result in a substantial different macroeconomic dynamics even at a first order level, whenever the model is log-linearized around a positive steady state inflation rate.\footnote{As in the literature, a positive steady state level of inflation is also indicated with the term "trend inflation".} The two models, then, have very different policy implications regarding the inflation-output long-run and short-run relationships, the determinacy conditions, the response to technology and monetary policy shocks, and the disinflation dynamics.\footnote{This is just another example of the fact that, while simplifying the derivation of the approximated inflation process, the zero steady state assumption is obviously empirically implausible, but also not theoretically innocuous. Ascari (2004) and Yun (2005) show that first-order effects arise on the Calvo price setting setup under trend inflation. Elaborating further with the Calvo set-up, Ascari and Ropele (2007, 2009) study the implications of different trend inflation levels for the optimal monetary policy and for the Taylor principle.} Given that: (i) the Calvo and the Rotemberg models are the two most popular way of modelling nominal price rigidities; (ii) they result in a different log-linearized dynamic macroeconomic model under positive trend inflation; (iii) positive mean inflation is an undeniable features of macroeconomic data for OECD countries in post-WWII sample; it seems natural to proceed to a comparative quantitative investigation of the Calvo vs. the Rotemberg model of price setting. This is what we do in this paper.

This paper, therefore, fits the Calvo and Rotemberg frameworks derived under positive trend inflation to 1984:I-2008:II U.S. macroeconomic data. We first compare the ‘baseline’ New Keynesian model - allowing for no-impact of trend inflation on the first order approximation - to trend inflation-equipped frameworks. Then we evaluate the
empirical performance of Calvo against Rotemberg conditional on trend inflation.

Several findings arise. First, models acknowledging a positive trend inflation rate display a better (or, at least, no worse) fit than a baseline ‘zero-trend inflation’ framework. Given the different theoretical implications for the monetary policy stemming from a trend inflation-equipped framework (as opposed to the baseline model) in terms of optimal policy (Ascari and Ropele, 2007), determinacy of simple monetary policy rules (Ascari and Ropele, 2009), our results push towards the employment and development of macroeconomic frameworks consistently accounting for a positive steady-state inflation rate. Second, the U.S. data support Calvo (as opposed to Rotemberg) as the better fitting pricing scheme. Third, when comparing the two models under the ‘no price-indexation’ restriction, we verify the rejection of the indexation hypothesis by the Calvo framework. Interestingly, this result emerges in absence of any model for the low frequency of the inflation rate, i.e. without appealing to any exogenous process modeling the possibly time-varying trend inflation as in Ireland (2007) and Cogley and Sbordone (2008). Differently, shutting down indexation in the Rotemberg framework leads to a drop in the model’s empirical fit, suggesting a lack of internal dynamics in comparison to Calvo. Fourth, for a given degree of trend inflation, the determinacy area is strongly dependent on the choice of the price setting model. In particular, the set of implementable Taylor rules in the Rotemberg model is bigger than in the Calvo model and contains the latter as strict subset.

Other papers stress the importance of considering trend inflation in empirical work. Benati (2008) estimates a NKPC for a variety of countries, and shows that price-indexation a la Christiano, Eichenbaum, and Evans (2005) is not stable across different samples in countries that explicitly adopted an inflation targeting scheme. He relates this instability to different policy regimes, so demonstrating that indexation is ‘not structural in the sense of Lucas’. Elaborating on this paper, Benati (2009) estimates different NKPCs derived under alternative pricing schemes. His results corroborate and extend his previous findings, i.e. the degree of price indexation is not invariant across different policy regimes, and it tends to zero under the more recent, stable regimes. Notably, Benati (2009) estimates, among others, Ascari and Ropele’s (2009) derivation of the Calvo model under trend inflation for a variety of countries.\footnote{The list considered by Benati (2009) includes the Euro area, West Germany, Germany, France, Italy, U.K., Canada, Sweden, Australia, New Zealand, and Switzerland.} He considers a step-function to model possible drifts in the inflation target. Differently from Benati (2009), who works with a fully-fledged new-Keynesian DSGE framework, Cogley and
Sbordone (2008) estimate a NKPC embedding a drifting trend inflation coupled with a TVC-VAR model. They find that, once drifts in trend inflation are accounted for, price indexation in the U.S. is zero, i.e. a purely-forward looking NKPC fits the data well without the need for *ad-hoc* backward-looking components. In a very recent contribution, however, Barnes, Gumbau-Brisaz, Liex, and Olivei (2009) criticizes this result on the basis of the estimation methodology. Paciello (2009) estimates a Calvo-based NKPC with positive, constant trend inflation for the post-WWII via indirect inference, and shows that such a model is able to match the dynamic responses of inflation to monetary policy and technology shocks even in absence of indexation, an ability not enjoyed by the standard, zero steady-state inflation framework.

Our investigation departs from the ones above along different dimensions. First and foremost, our paper focuses on the comparison between different frameworks, i.e. Calvo and Rotemberg. To our knowledge, this is the first contribution assessing the relative empirical relevance of these two very widely employed pricing schemes. Second, we focus on two models displaying a *constant* trend inflation rate, i.e. displaying no exogenous random-walk type of process for the Fed’s inflation target. Still, the version of the Calvo model preferred by the data is that with no-price indexation. With respect to Benati (2009), we provide evidence for the U.S. case, therefore complementing his battery of estimates. With respect to Cogley and Sbordone (2008) and Paciello (2009), we consider a structural representation of the demand side of the economy, rather than a reduced-form TVC-VAR. This is obviously important from an econometric standpoint, because the identification of forward and backward looking terms in the NKPC also depends on how the remaining structural equations are modeled (Beyer and Farmer, 2007). When such equations are not specified, as in the NKPC-VAR approach, the meaning of the economic restrictions imposed to the estimation is unclear, as pointed out by Cogley and Sbordone (2008) themselves. Also from a theoretical point of view, our analysis shows the importance of estimating the full model equations, because the assumed pricing scheme may affect not only the supply side of the model, but also the other model equations, as in the case of the Rotemberg model. Moreover, differently from Paciello (2009), we conduct our empirical analysis with Bayesian techniques. Our choice is driven by the possibly superior performance against indirect inference (impulse response matching) as far as this class of DSGE models is concerned (Canova and Sala, 2000).

\footnote{Moreover, Schorfheide (2005) and Ireland (2007) also embed a time-varying inflation target in their models, but without consequences for the specification of the NKPC due to the assumption of full-indexation.}
2009). Finally, we concentrate on a stable subsample (great moderation), which renders our assumption of a constant trend inflation more palatable. All in all, we see our contribution as complementary to Benati (2009), Cogley and Sbordone (2008), Paciello (2009) and Barnes et al. (2009).

The paper is structured as follows. Section 2 describes the two frameworks we deal with and highlights the relevant differences. Section 3 presents our main econometric analysis, and discuss the two estimated models and their relative fitting power. Section 4 proposes further investigations corroborating our set of benchmark results. Section 5 assesses the determinacy of the rational expectations equilibrium in the two models. Section 6 draws some directions for further research.

2 The theoretical model

In this section we sketch a small-scale new-Keynesian model in the two versions of the Rotemberg (1982) and the Calvo (1983) price setting scheme. The model economy is composed of a continuum of infinitely-lived consumers, producers of final and intermediate goods. Households have the following instantaneous and separable utility function:

$$ U(C_t, N_t) = \frac{C_t^{\gamma - \sigma}}{1 - \sigma} - d_n \frac{N_t^{1+\varphi}}{1+\varphi}, $$

where $C_t$ is a consumption basket (with elasticity of substitution among goods $\varepsilon$) and $N_t$ are labor hours.

Final good market is competitive and the production function is given by $Y_t = \left[ \int_0^1 Y_{i,t}^{\frac{1}{1-\varepsilon}} \, d\eta \right]^{\frac{1-\varepsilon}{\varepsilon}}$. Final good producers demand for intermediate inputs is therefore equal to $Y_{i,t} = Y_{i,t+J} = \left( \frac{P_{i,t}}{P_{i,t+1}} \right)^{-\varepsilon} Y_{i,t+1}$. The intermediate inputs $Y_{i,t}$ are produced by a continuum of firms indexed by $i \in [0, 1]$ with the following simple constant return to scale technology $Y_{i,t} = A_i N_{i,t}$, where labor is the only input and $\ln A_i = a_t$ is an exogenous productivity shock, which follows an AR(1) process. The intermediate good sector is monopolistically competitive.


The Calvo model

The Calvo price setting scheme assumes that in each period there is a fixed probability $1 - \theta$ that a firm can re-optimize its nominal price, i.e., $P_{i,t}^*$. With probability $\theta$, instead,
the firm automatically and costlessly adjust its price according to an indexation rule. The price setting problem is:

$$\max_{\{P_t\}_{t=0}^{\infty}} \mathbb{E}_t \sum_{j=0}^{\infty} \mathcal{D}_{t,t+j} \beta^j \left[ \frac{P_{i,t}^* (\bar{\pi} \chi_j)^{1-\mu} (\Pi^X_{t,t+j-1})^\mu}{P_{t+j}} Y_{i,t+j} - MC_{i,t+j}^r Y_{i,t+j} \right],$$  \hspace{1cm} (1)

subject to:

$$Y_{i,t+j} = \left[ \frac{P_{i,t}^* (\bar{\pi} \chi_j)^{1-\mu} (\Pi^X_{t,t+j-1})^\mu}{P_{t+j}} \right]^{-\varepsilon} Y_{t+j} \text{ and}$$  \hspace{1cm} (2)

$$\Pi^X_{t,t+j-1} = \begin{cases} 
\left( \frac{P_i}{P_{i-1}} \right) \left( \frac{P_{i+1}}{P_i} \right) \times \cdots \times \frac{P_{i+j-1}}{P_{i+j-2}} & \text{for } j = 1, 2, \ldots \\
1 & \text{for } j = 0.
\end{cases}$$  \hspace{1cm} (3)

where $\mathcal{D}_{t,t+j} \equiv \beta^j \frac{Y_{i,t+j}}{Y_{i,j}}$ represents firms’ stochastic discount factor, $MC^r_{i,t+j} = \frac{W_{i,t+j}}{P_{i,t+j}^r A_{i,t+j}}$ is the real marginal cost function, and $\bar{\pi}$ denotes the central bank’s inflation target and it is equal to the level of trend inflation. The indexation scheme in (1) is very general. In particular: (i) $\chi \in [0, 1]$ allows for any degree of price indexation; (ii) $\mu \in [0, 1]$ allows for any degree of (geometric) combination of the two types of indexation usually employed in the literature: to steady state inflation (e.g., Yun, 1996) and to past inflation rates (e.g., Christiano et al., 2005).\(^6\)

In the Calvo price setting framework, prices are staggered because firms charging prices at different periods will set different prices. Then, in each given period $t$, there will be a distribution of different prices, that is, there will be price dispersion, which results in an inefficiency loss in aggregate production. Formally:

$$N^d_t = Y_t \int_0^1 \left[ \left( \frac{P_{i,t}}{P_t} \right)^{-\varepsilon} \right] di = s_t \frac{Y_t}{A_t}. $$  \hspace{1cm} (4)

Schmitt-Grohé and Uribe (2007) show that $s_t$ is bounded below at one, so that $s_t$ represents the resource costs due to relative price dispersion under the Calvo mechanism. Indeed, the higher $s_t$, the more labor $N^d_t$ is needed to produce a given level of output. Note that price dispersion creates a wedge between aggregate output and aggregate employment. To close the model, the aggregate resource constraint is simply given by:

$$Y_t = C_t.$$  \hspace{1cm} (5)

\(^6\)For a detailed derivation of the optimal price equations under these two pricing schemes, see Ascarì and Ropele (2009).
The Rotemberg model

The Rotemberg model assumes that a monopolistic firm faces a quadratic cost of adjusting nominal prices, that can be measured in terms of the final-good and given by:

$$\frac{\varphi_P}{2} \left( \frac{P_{t,t}^\chi}{(\pi_{t-1}^\chi)^\mu (\bar{\pi}^\chi)^{1-\mu} P_{t-1}^\chi} - 1 \right)^2 Y_t, \quad (6)$$

where $\varphi_P > 0$ determines the degree of nominal price rigidity. As stressed in Rotemberg (1982), the adjustment cost looks to account for the negative effects of price changes on the customer-firm relationship. These negative effects increase in magnitude with the size of the price change and with the overall scale of economic activity, $Y_t$. Likewise for the Calvo model, (6) includes a general specification for the adjustment cost used by, e.g., Ireland (2007), among others. In particular, the adjustment cost will depend on the ratio between the new reset price the last period one, adjusted by a (geometric) combination of steady state inflation and of past inflation. $\mu$ and $\chi$ are playing a parallel role as in the indexation scheme in the Calvo model.

The problem for the firm $i$ is then:

$$\max_{(P_{i,t})_{t=0}^\infty} \sum_{j=0}^\infty D_{t,t+j} \left\{ \frac{P_{t,t+j}}{P_{t+j}} Y_{t,t+j} - MC_{t,t+j}^r Y_{t,t+j} - \frac{\varphi_P}{2} \left( \frac{P_{t,t+j}}{(\pi_{t+j}^\chi)^\mu (\bar{\pi}^\chi)^{1-\mu} P_{t,j}^\chi} - 1 \right)^2 Y_{t+j} \right\}, \quad (7)$$

s.t. $Y_{i,t+j} = \left[ \frac{P_{i,t+j}}{P_{t+j}} \right]^{-\varepsilon} Y_{t+j}$, \quad (8)

where the notation is as above. Firms can change their price in each period, subject to the payment of the adjustment cost. Therefore, all firms face the same problem, and thus will choose the same price, producing the same quantity. In other words: $P_{i,t} = P_t, Y_{i,t} = Y_t, W_{i,t} = W_t$ and $MC_{t,t}^r = MC_t^r \forall i$. Given the symmetry in this economy, and opposite to the Calvo model, in the Rotemberg model the aggregate production function features no inefficiency due to price dispersion, therefore:

$$Y_t = A_t N_t. \quad (9)$$

In the Rotemberg model, the adjustment cost enters the aggregate resource constraint, that creates an inefficiency wedge between output and consumption:

$$Y_t = \left[ 1 - \frac{\varphi_P}{2} \left( \frac{P_t}{(\pi_{t-1}^\chi)^\mu (\bar{\pi}^\chi)^{1-\mu} P_{t-1}^\chi} - 1 \right)^2 C_t = \Psi_t C_t. \quad (10)$$
This is the main difference between the Calvo and the Rotemberg model. In the Calvo model, the cost of nominal rigidities, i.e., price dispersion, creates a wedge between aggregate employment and aggregate output, making aggregate production less efficient. In the Rotemberg model, instead, the cost of nominal rigidities, i.e., the adjustment cost, creates a wedge between aggregate consumption and aggregate output, because part of the output goes in the price adjustment cost. As shown in Asciari and Rossi (2009), and evident from (4) and (10), both these wedges are non-linear functions of inflation and they increase with trend inflation. However, both wedges take the same unitary value under two particular cases: (i) a net steady state inflation equals zero, and/or (ii) full indexation to past or to trend inflation.

2.2 The Log-linearized frameworks

We now present the log-linearized versions of the two pricing frameworks we deal with. For a full derivation of the for the Calvo and the Rotemberg log-linearized model, we refer the reader to Asciari and Ropele (2007 and 2009) and Asciari and Rossi (2009), respectively. Again, we stress that the derivation allows for a non-zero value for the inflation rate in steady state, which may be interpreted as the target pursued by the Federal Reserve in conducting the U.S. monetary policy.

The Calvo model

The Calvo model is described by the first-order difference equations:

\[
\Delta_t = \left[ \beta \pi_t^{1-\varepsilon} + \eta (\theta - 1) \right] E_t \Delta_t + \kappa \hat{y}_t - \lambda \varphi a_t + \lambda \varphi \hat{s}_t + \eta \hat{E}_t \hat{\phi}_{t+1},  \\
\hat{\phi}_t = (1 - \sigma) \left( 1 - \theta \beta \pi_t^{(e-1)(1-\xi)} \right) \hat{y}_t + \theta \beta \pi_t^{(e-1)(1-\xi)} E_t \left[ \varepsilon - 1 \right] E_t \Delta_t + \hat{\phi}_{t+1},  \\
\hat{s}_t = \xi \Delta_t + \varepsilon \pi_t^{(1-\xi)} s_{t-1},  \\
\hat{y}_t = \phi \hat{E}_t \hat{y}_{t+1} + (1 - \phi) \hat{y}_{t-1} - \sigma^{-1} \left( \hat{i}_t - E_t \hat{\pi}_{t+1} \right) + g_t,  \\
\]

where \( \Delta_t \equiv \hat{\pi}_t - \chi \mu \hat{\pi}_{t-1} \), \( \hat{\pi} \) stands for the inflation rate, \( \hat{y} \) for detrended output, \( a \) is the technological shock, \( g \) is the demand shock. Hatted variables indicate percentage deviations with respect to steady state values or, in case of output, from a trend. Moreover: \( \sigma \) is the relative risk aversion parameter, \( \varphi \) the labor supply elasticity, \( \beta \) the discount factor, \( \varepsilon \) the Dixit-Stiglitz elasticity of substitution among goods, \( \theta \) the Calvo parameter, \( \chi \) the degree of price indexation, \( \mu \) the relative weight of indexation to past inflation vs. trend inflation, \( \hat{\pi} \) and the steady-state, trend inflation rate. Finally, \( \lambda, \eta, \kappa, \)
and \( \xi \) in eqs. (11)-(14) are the following convolutions of parameters:

\[
\lambda \equiv \frac{(1 - \theta \pi^{(e-1)(1-\lambda)}) (1 - \theta \beta \tilde{\pi}^{(1-\lambda)})}{\theta \pi^{(e-1)(1-\lambda)}},
\]

\[
\eta \equiv \beta (\tilde{\pi}^{1-\lambda} - 1) [1 - \theta \pi^{(e-1)(1-\lambda)}],
\]

\[
\kappa \equiv (\lambda_{(e,d)} (\sigma + \varphi) + \eta_{(e,d)} (1 - \sigma)),
\]

\[
\xi \equiv \frac{\varepsilon \theta \pi^{(e-1)(1-\lambda)} (\tilde{\pi}^{1-\lambda} - 1)}{1 - \theta \pi^{(e-1)(1-\lambda)}},
\]

As for the aggregate demand equation (17), it is expressed in hybrid terms \textit{a la} Fuhrer and Rudebusch (2004), with the parameter \( \phi_y \) identifying the relative weight of expected output. This semi-structural, flexible version of the IS curve have recently been employed by e.g. Benati (2008), Benati and Surico (2008), Benati and Surico (2009), and Benati (2009).

Notably, all the coefficients of the log-linearized model are a function of the trend inflation rate \( \tilde{\pi} \), that generally tends to increase the coefficients on the forward-looking variables. Moreover, the log-linearized NKPC is influenced by the price dispersion process \( s_t \). This is so because, under Calvo, just a fraction \((1 - \theta)\) of firms is allowed to reoptimize in each period, then price dispersion arises. Under a strictly positive trend inflation rate, price dispersion assumes a first-order relevance and influences the evolution of the log-linearized inflation rate. Moreover, price dispersion has a backward-looking dynamics. The forward looking auxiliary process \( \phi_t \) also participates to the determination of inflation. The IS equation is, instead, standard, because no inefficiencies influence the relationship between output and consumption.

\textbf{The Rotemberg model}

The Rotemberg model is characterized by the following difference equations:

\[
\hat{\pi}_t = \gamma_p \hat{\pi}_{t-1} + \gamma_f \beta \hat{\pi}_{t+1} + \gamma_{dy} \beta (1 - \sigma) \Delta \hat{y}_{t+1} + \gamma_{mc} \hat{mc}_t, \tag{15}
\]

\[
\hat{mc}_t = (\sigma + \varphi) \hat{y}_t - \varsigma_c \sigma \hat{\pi}_t + \varsigma_c \sigma \mu \chi \hat{\pi}_{t-1} - (1 + \varphi) a_t, \tag{16}
\]

\[
\hat{y}_t = \phi_y \hat{y}_{t+1} + (1 - \phi_y) \hat{y}_{t-1} - \varsigma_c \Delta \hat{\pi}_{t+1} + \varsigma_c \mu \chi \hat{\pi}_t - \sigma^{-1} (\hat{t}_t - \hat{\pi}_{t+1}^e) + y_t, \tag{17}
\]

where \( \hat{mc} \) stands for marginal costs, and the notation has the same interpretation as in the previous Subsection. The coefficients \( \gamma_p, \gamma_f, \gamma_{dy}, \gamma_{mc} \), and \( \varsigma_c \) are complicated convolutions of the structural parameters of the model (see Appendix A1). As often assumed in the literature,\(^7\) the Calvo parameter allows to estimate the value of the

\(^7\)The underlying assumption is that of a production function displaying constant returns to scale. See Ascari and Rossi (2009) or Lombardo and Vestin (2009) for details.
Rotemberg adjustment cost $\varphi_p$ via the constraint $\varphi_p = \frac{(\varepsilon - 1)\theta}{(1 - \theta)(1 - \beta\theta)}$, which implies the same first order dynamics as those of the Calvo model in the case of zero steady state inflation.

First, the impact of trend inflation is evident when looking at eqs. (15)-(17) and their convolution parameters. As in the Calvo model, trend inflation alters the inflation dynamics compared to the usual Rotemberg model by directly affecting the NKPC coefficients. Higher trend inflation increases the coefficient relative to expected and past inflation as well as the coefficient of real marginal costs. As in the Calvo model, the presence of past inflation in (15) is due to indexation to past inflation. With no indexation to past inflation, i.e. with $\mu = 0$, however, the coefficient $\gamma_p$ equals zero and the NKPC becomes completely forward looking. Recall that this is not the case in the Calvo model, because even if past inflation disappears when $\mu = 0$, price dispersion dynamics introduces a backward-looking component. Furthermore, in the NKPC eq. (15) the expected difference of detrended output shows up, because of the influence that trend inflation exerts over firms’ discount factor.

Second, because of the presence of the price adjustment cost, in the Rotemberg model the log-linearized resource constraint can be written

$$\hat{c}_t = \hat{y}_t - \frac{\varphi (\bar{\pi}^{1-x} - 1) \bar{\pi}^{1-x}}{1 - \frac{\theta}{2} (\bar{\pi}^{1-x} - 1)^2} \hat{\pi}_t + \frac{\varphi (\bar{\pi}^{1-x} - 1) \bar{\pi}^{1-x} \mu \chi}{1 - \frac{\theta}{2} (\bar{\pi}^{1-x} - 1)^2} \hat{\pi}_{t-1}. \quad (18)$$

This equation shows that to a first order approximation the Rotemberg model: (i) implies a wedge between output and consumption; (ii) this wedge depends positively on the current and past inflation level; (iii) the elasticity of the wedge with respect to inflation (i.e., the term in the square bracket) increases with trend inflation; (iv) the wedge disappears with zero steady state inflation rate or with full indexation, i.e. with $\chi = 1$. Such a wedge affects also the amount of resources produced in the economy and this is the reason why the IS eq. (17) features the first-order difference inflation rates. The price adjustment cost causes the real marginal cost to depend also on actual inflation and past inflation (see the additional term $\zeta_c \sigma \hat{\pi}_t$ and $\zeta_c \sigma \mu \chi \hat{\pi}_{t-1}$ in (16)).

Notably, under the peculiar case of zero trend inflation, i.e., $\bar{\pi} = 1$, both the Rotemberg and the Calvo frameworks lines up with the standard hybrid new-Keynesian formulation allowing for price indexation to past/steady state inflation. The same holds true in a full indexation scenario, i.e. when $\chi = 1$, regardless to the value assumed by the relative weight $\mu$.

To sum up, because of the different wedges which characterize the Calvo and the Rotemberg model the two log-linearized systems present three main differences. First
of all, in the Calvo model the presence of a price dispersion wedge creates an endoge-
nous predetermined variable in the NKPC, which is absent in the Rotemberg model. 
Secondly, in the Rotemberg model, the presence of the price adjustment cost causes 
the real marginal cost to depend also on actual and past inflation. Finally, the price 
adjustment cost generates a wedge between output and consumption in the resource 
constraint, (10), that appears in the IS curve. As shown by Ascari and Rossi (2009), 
these differences are relevant from a policy standpoint, because of their impact on the 
definition of the determinacy territory associated to simple, implementable Taylor-type 
rules.

2.3 Closing the models

The two models are closed by a common set of equations, i.e.

\begin{align}
\hat{\iota}_t &= \alpha_\iota \hat{\iota}_{t-1} + (1 - \alpha_\iota) \left( \alpha_\pi \hat{\pi}_t + \alpha_g \hat{\gamma}_t \right) + m_t, \\
\hat{z}_t &= \rho_z \hat{z}_{t-1} + \varepsilon_{zt}, \varepsilon_{zt} \sim N(0, \sigma_z^2), \ z \in \{a, g, m\}. 
\end{align}

Eq. (19) is a standard policy rule postulating a smoothed reaction of the policy 
rate \( \hat{\iota}_t \) to fluctuations in inflation and output, with stochastic deviations driven by the 
monetary policy shock \( m_t \). Eq. (20) defines the stochastic properties of the shocks 
hitting the system.

3 Econometric exercise

Our investigation focuses on U.S. data. We employ three 'observables', i.e. the quarterly 
net growth rate of the GDP deflator \( \pi_t^{\text{obs}} \), the log-deviation of real GDP with respect to 
its Hodrick-Prescott trend (relative weight for the smoothing component: 1,600) \( y_t^{\text{obs}} \),

and the net Federal Funds Rate \( \iota_t^{\text{obs}} \). Our measure of detrended output, being mainly 
statistical, is robust to model misspecification, and it is also justified by the absence 
in this model of physical capital, which would probably return a severely misspecified 
model-consistent measure of natural output. Output pre-filtering for the estimation

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8Further discussions on the filtering strategy are proposed in Section 4.

9The source of the data is the Federal Reserve Bank of St. Louis’ website, i.e. 
http://research.st louisfed.org/fred2/. Quarterly observations of the federal funds rate were constructed 
by taking averages of monthly observations. The detrended output and the policy rate were demeaned 
prior to estimation in a model-consistent manner. The observables employed in the estimation are not 
percentualized.
of this small-scale model has recently been performed, among others, by Lubik and Schorfheide (2004), Boivin and Giannone (2006), Benati (2008), Benati and Surico (2008), Benati and Surico (2009), and Benati (2009).¹⁰

Several authors (Clarida, Gali, and Gertler (2000), Lubik and Schorfheide (2004), Boivin and Giannone (2006), Benati and Surico (2009), and Mavroeidis (2009)) documented a break in the U.S. monetary policy conduct corresponding to the advent of Paul Volcker as chairman of the Federal Reserve. Changes in the U.S. macro-dynamics possibly consequential to such a monetary policy shift have also been investigated by D’Agostino, Giannone, and Surico (2006), Benati and Surico (2008) and Cogley, Primiceri, and Sargent (2009), who document a variation in inflation predictability when entering the 1980s, and by Castelnuovo and Surico (2009), who show how VAR impulse response functions may be affected by a drift towards a more hawkish monetary policy. To control for such breaks, we focus on the ‘great moderation’ period 1984:I-2008:II. Our end-of-sample choice enables us to avoid dealing with the acceleration of the financial crises began with the bankruptcy of Lehman Brothers in September 2008, which triggered non-standard policy moves by the Fed (Brunnermeier (2009)).

3.1 Bayesian inference and priors

We estimate the Rotemberg (15)-(17), (19)-(20) and Calvo (11)-(14), (19)-(20) models with Bayesian techniques (An and Schorfheide (2007)). Canova and Sala (2009) show that this technique is less prone to identification issues with respect to alternatives in the context of DSGE models. The Technical Appendix describe the details of our estimation strategy.

The following measurement equations link our observables to the latent factors of our models:

\[
\begin{bmatrix}
    \bar{\pi}_t^{o_{bs}} \\
    \bar{y}_t^{o_{bs}} \\
    \bar{t}_t^{o_{bs}}
\end{bmatrix} = \begin{bmatrix}
    \bar{\pi} - 1 \\
    \bar{y} \\
    \bar{t}
\end{bmatrix} + \begin{bmatrix}
    \hat{\pi}_t \\
    \hat{y}_t \\
    \hat{t}_t
\end{bmatrix}
\]

(21)

where \(\bar{y}\) and \(\bar{t}\) are the sample means of, respectively, detrended output and the federal funds rate, and \(\pi_t^{o_{bs}}\) is the observed net inflation rate.

Eq. (21) identifies the quintessence of a trend inflation model, i.e., its ability to shape the steady-state inflation rate. Clearly, different trend inflation values will lead to dif-

¹⁰For an alternative approach, based on a model-consistent treatment of the real GDP trend, see Smets and Wouters (2007), Justiniano and Primiceri (2008), and Castelnuovo and Nisticò (2009).
ferent empirical performances of the different models we will investigate. Our empirical investigation exactly wants to discriminate such models on the basis of their ability to replicate inflation’s long-run value on top of its dynamics. Microfounded models, as the ones above, log-linearized around a general trend inflation level can treat trend inflation in a model consistent way. This consideration is important when searching for the encompassed ‘baseline new-Keynesian model’. Indeed, an obvious way to collapse to such model would be that of setting the gross trend inflation rate $\bar{\pi} = 1$, so reconstructing the ‘zero-steady state’ assumption typically employed in the literature when deriving such model. However, eq. (21) makes it clear that, while being clearly logically grounded, this choice would force us to leave the mean of observed inflation unmodeled, so condemning the standard new-Keynesian model to a poor empirical performance. To circumvent this issue, one could demean observed inflation prior to estimation. However, this would probably penalize, in relative terms, the trend inflation models, one of their edges being their ability to ‘naturally’ model the first moment of observed inflation. To estimate the encompassed baseline, ‘zero trend inflation’ framework, we will then set the indexation parameter $\chi = 1$, as in Christiano et al. (2005), therefore switching off the trend inflation-related ‘extra terms’, as well as muting the impact of trend inflation on the relative weights of inflation expectations and marginal costs in the NKPC and IS schedules (when present). We can then assign a positive trend inflation rate (with which to model inflation mean) to the baseline new-Keynesian model in a theoretically-consistent manner.

Our dogmatic priors and prior densities read as follows. We assume standard values for a sub-set of parameters, i.e. we set the discount factor $\beta$ to 0.99, the elasticity of substitution among goods $\varepsilon = 6$, and the inverse of the labor elasticity $\varphi$ to 1. To favor a smooth convergence towards the ergodic distribution, we fix the relative indexation weight and set it to $\mu = 1$, i.e., we concentrate on indexation to past inflation, in line with Benati (2009). We calibrate the steady state inflation rate by appealing to inflation’s sample mean, i.e., $\bar{\pi} = 1.0063$, which translates to a net yearly percentualized inflation target of about 2.5%. We set $\bar{\pi} = 1.0063$, with $\pi = 0.0012$ and $\tau = 0.0131$. Table 1 reports the standard prior densities for the estimated parameters. Notice that such densities are common across models. The standard assumption $\varphi_p = \frac{(\varepsilon - 1)\theta}{(1 - \theta)(1 - \beta\theta)}$ allows us to impose

\footnote{We conducted econometric exercises in which we estimated also the trend inflation rate. Our results turned out to be virtually unchanged.}

\footnote{For a different strategy, based on the calibration of the priors of the auxiliary parameters via pre-sampling or the exploitation of information coming from different datasets, see Del Negro and Schorfheide (2008).}
the very same prior on the parameter $\theta$ so to estimate, in the Calvo model, the Calvo-lottery parameter, and in the Rotemberg model the amount of adjustment costs.\footnote{We also estimated a version of the Rotemberg model in which the adjustment cost is a free parameter. We recorded a small improvement in the marginal likelihood, i.e. 34.95. The remaining results remain unaffected.}

3.2 Posteriors and model comparison

Figure 1 displays the posterior densities of the structural parameters across the three models we focus on, i.e. the ‘Baseline’ model (featuring full indexation to past inflation), the Calvo model, and the Rotemberg model.\footnote{The convergence towards the target distribution was checked \textit{via} (and confirmed by) the univariate and multivariate statistics proposed by Brooks and Gelman (1998).} First, the data appear to be quite informative as regards two key parameters in the pricing context, i.e. the degree of indexation in Calvo and Rotemberg, and the degree of price stickiness in our three models. Indeed, different frameworks suggest different indications as regards these key parameters, with Calvo pointing towards a lower indexation and a higher stickiness than Rotemberg, a result we will scrutinize further. In general, the likelihood function turns out to be informative for most of the structural parameters of interest, the only exception being the reaction to output in the Taylor rule.

Table 2 collects our posterior estimates. The posterior means of the Calvo parameter and the degree of relative risk aversion is very close to that estimated by other authors, as, e.g., Rabanal and Rubio-Ramírez (2005), Christiano et al. (2005), Smets and Wouters (2007), Rabanal (2007). Interestingly, the IS curve turns out to be (almost) fully forward looking. The estimated Taylor rule parameters suggest a strong long-run systematic reaction to the inflation gap fluctuations - in line with recent estimates provided by Blanchard and Riggi (2009) - and a more moderate reactivity to output oscillations, both tempered in the short run by a fairly large amount of policy gradualism. As in previous studies, (e.g., Smets and Wouters (2007)), the persistence of the technological shock is large.

In terms of model comparison, the marginal likelihood (computed with the modified harmonic mean estimator developed by Geweke (1998)) clearly points towards the superiority of trend inflation-equipped frameworks.\footnote{Recall that, to assess the standard new-Keynesian model, we set $\chi = 1$ and allowed for a positive trend inflation so to model the first moment of observed inflation. An alternative strategy, often followed by researchers when estimating zero steady state inflation models, would have been that of demeaning the observed inflation rate prior to estimation and let the indexation parameter free. Admittedly, when doing so, we obtained a marginal likelihood equal to $-33.24$, i.e., very close to our estimated trend inflation models. But demeaning inflation in an a-priori fashion is logically inconsistent} The Bayes factor involving the
baseline and the Calvo models (unrestricted) reads $exp(4.57) \approx 96.54$, which suggests a "strong" support for the trend inflation model.\footnote{According to Kass and Raftery (1995), a Bayes factor between 1 and 3 is "not worth more than a bare mention", between 3 and 20 suggests a "positive" evidence in favor of one of the two models, between 20 and 150 suggests a "strong" evidence against it, and larger than 150 "very strong" evidence.} Interestingly, the Rotemberg model is also supported by the marginal likelihood comparison, even if the wedge with the baseline NK model is much smaller. In the light of the different normative indications coming from models with zero vs. positive trend inflation, this result appears to be very relevant.

Conditional on a positive trend inflation rate, one may also detect two important differences when contrasting Calvo and Rotemberg. First, a comparison based on their power of fit speaks in favor of the Calvo model, with a log-difference that translates into a Bayes factor of about 13.46. Second, while both models point towards a degree of indexation clearly lower than 1 (the calibration suggested by e.g., Christiano et al., 2005), there is a clear difference in the estimated degree of indexation $\chi$, an object whose microfoundations is theoretically scant. The estimated posterior mean associated to the Calvo model clearly points towards a negligible value for price indexation (0.15), and the 5th percentile is virtually zero. By contrast, the Rotemberg model calls for a more than double posterior mean, 0.38, the zero value does not belong to the standard 90% credible set, and it calls for a very high 95th percentile reading 0.72.

As already stressed, the theoretical justification for the introduction of indexation in a macroeconomic model is somewhat questionable. Moreover, as shown by Benati (2008 and 2009) and Cogley and Sbordone (2008), such a parameter is hardly structural in the sense of Lucas, so that policy exercises conducted with models appealing to indexation may very well be misleading. Then, our posterior estimates point to the Calvo model as the more appealing from a 'structural' standpoint. To gauge the statistical relevance of the difference in the estimated indexation parameters, Figure 3 displays the distribution obtained by plotting 10,000 pairwise differences between the draws sampled from the posterior of the $\chi$ parameter under Rotemberg and those sampled from the posterior under Calvo. Notably, the larger part of the mass is clearly associated to positive realizations, with a share of about 82%. While the standard [5th pct, 95th pct] credible set includes the zero value, the stricter [25th pct, 75th pct] credible set -
in our context. In fact, with partial indexation the coefficients of the log-linearized model would depend on the level of trend inflation. A priori-demeaning, thus, just 'kills' one of the implications of the microfounded restrictions imposed by positive trend inflation on the framework, i.e., that of jointly modeling inflation's first moment and its dynamics. Consequently, we intentionally stick to our theoretically-consistent strategy when conducting our model comparison.
recently employed by e.g. Cogley, Primiceri, and Sargent (2009) - does not. Thus, the data support a lower indexation parameter called for by the Calvo model.

The estimation of a constrained version of the two models, i.e. that with the degree of indexation \( \chi \) set to zero, also confirms the superiority of the Calvo model. As shown by Table 2, all the structural parameters display an appreciable stability across the different model versions. Interestingly, the marginal likelihood gives an even more clear indication: the fit of the Calvo framework improves (suggesting that indexation is just unwarranted), while the one of the Rotemberg set up deteriorates (suggesting this model needs the indexation assumption to fit the data at hand). Consequently, the Bayes factor, which in this case reads 188.67, leads to a more solid preference in favor of the Calvo model, i.e., a "very strong" evidence in the language of Kass and Raftery (1995).

4 Further investigations

In comparing Calvo and Rotemberg, our empirical exercises support (i) trend inflation equipped models, (ii) the empirical superiority of the Calvo model, and (iii) the low (or zero) degree of indexation to past inflation called for by the Calvo model. These conclusions have been drawn by relying on some assumptions whose relevance for our findings deserves further scrutiny. Therefore, we perform some robustness checks along different relevant dimensions.

4.1 Robustness checks

- **Calibration of the trend inflation rate.** In our baseline exercises, we calibrate the trend inflation rate to the inflation sample mean, that is, 2.5\% in annualized and percentualized net terms. However, given that the magnitude of the trend inflation rate drives the relevance of the ‘extra-components’ showing up in the NKPC (Calvo, Rotemberg) and the IS schedule (Rotemberg), as well as it exerts a non-linear impact on most of the parameters of the system, a sensitivity analysis along this dimension is warranted. We then re-estimate the Calvo and Rotemberg models under two alternative trend inflation calibrations, i.e. 2\% and 3\%. Table 3 collects in columns second to fifth the results concerning our unrestricted estimates. Our main results are by and large robust to these perturbations. In particular, the Calvo model still fits the data better, and with a call for indexation lower than that by Rotemberg - notably, zero indexation belongs to the 90\%
credible set just in the Calvo cases. As regards the calibration of trend inflation, perhaps not surprisingly the marginal likelihoods tend to favor 2.5%, i.e. the annualized and percentualized inflation sample mean.

- **Indexation to trend inflation.** Following Benati (2009), in our baseline exercise we set the relative indexation weight $\mu = 1$, i.e., we assume that firms index their price to past inflation, so ruling out the possibility for firms to index prices to trend inflation. This strategy allows current inflation to have lagged inflation among its determinants, and it contributes to the creation of ‘model-consistent inflation persistence’. In fact, the unconstrained estimates put forward by Ireland (2007) suggest that the calibration preferred by the data may very well be the opposite: U.S. firms may be more prone to index their prices to trend inflation. We then re-estimate our Calvo and Rotemberg model under $\mu = 0$. Our posteriors, collected in Table 3 (sixth and seventh column), still show support for our three main results above. The marginal likelihoods of the two models is clearly higher than that of the model estimated under zero trend inflation, which reads $-66.49$ (estimates not shown for the sake of brevity but available upon request). The Bayes factor still favors the Calvo model, even if this preference is mild. Interestingly, the estimated degree of indexation-to-trend-inflation is higher than in the previously commented versions of the model, with a posterior mean for Calvo reading 0.44 vs. Rotemberg’s 0.77. However, the realizations within the [5th, 95th] percentiles suggest a very imprecise estimate for Calvo, and a large mass in favor of a positive realization for Rotemberg. However, when looking at the marginal likelihoods, the ‘no indexation’ constraint (Table 2, columns four and five) still returns a better likelihood for the Calvo model than that suggested by the $\mu = 0$ plus free indexation-to-trend-inflation scenario.\textsuperscript{17} In contrast, and in line with our previous findings, the fit of the Rotemberg model clearly deteriorates.

- **Informativeness of the prior on the indexation parameter.** Model comparison of nested models performed on the basis of improper priors (e.g. priors having infinite variance) may lead to biased results bases on an improper Bayes factor (Gelfand (1996)). In fact, our model comparison is based on diffuse but proper priors, which makes our model comparisons sensible. Of course, different priors may lead to different results because of their influence on the marginal likelihood. To verify

\textsuperscript{17}Notice that, under ‘no indexation’, the relative weight $\mu$ does not exert any influence on the dynamics of the system, and consequently does not affect our marginal likelihoods.
the robustness of our results, we then re-estimate the baseline model by employing a different prior for our ‘key’ indexation parameter. In particular, we assume \( \chi \sim Beta(0.25, 0.10) \), a density with much more mass on indexation values in line with the literature (e.g. Smets and Wouters (2007)). Table 3 (last two columns) exhibits the results of this further check. The estimated indexation degrees are in this case somewhat closer, with Calvo suggesting 0.19 and Rotemberg 0.26. Still, the Calvo model is again favored by the data.

- **Piecewise quadratic trend.** An important check concerns the robustness of our results to a different filtering strategy. It is well known that different filters may induce dramatically heterogeneous representations of the economic cycle (Canova (1998)). We then re-estimate our models with an alternative business cycle representation, which is obtained by detrending the log-real GDP with a quadratic trend. In detrending the series, as in the case of the Hodrick-Prescott filtering, we employ the extended sample 1954:IV-2008:II. In so doing, we account for the 1973:1 break in the deterministic trend identified by Perron and Wada (2009), who show that differing filtering methods (Beveridge-Nelson, Unobserved Component) return the same picture of detrended output conditional on such a break.\(^{18}\) Interestingly, our point estimates are similar to those obtained under Hodrick-Prescott filtering, thus confirming our benchmark results.

- **Frisch labor supply elasticity.** Our benchmark calibration is \( \varphi = 1 \). We experimented with a variety of different values belonging to the set \([0.5, 1.5]\), and verified that our results are clearly robust to these variations.\(^{19}\)

Overall, our checks confirm our main results, i.e. trend inflation leads to a superior fit, and Calvo calls for a superior fit and a lower indexation degree with respect to Rotemberg.

### 4.2 Why does Calvo ‘make it better’?

Why does Calvo ‘make it better’? The differences between Calvo and Rotemberg are fundamentally three: (i) the different order of the dynamics because of the presence of price dispersion \( \hat{s}_t \) and the auxiliary process \( \hat{\phi}_t \) in Calvo but not in Rotemberg; (ii) the

\(^{18}\)We allow for both a break in the constant and in the slope coefficients.

\(^{19}\)We do not present the results of the last two robustness checks for the sake of brevity. However, these results are available upon request.
different non-linear impact of trend inflation on the convolutions of the two systems; (iii) the different ‘regressors’ in the NKPC and IS schedules of the two models. We discuss these elements in turn.

Price dispersion is an autoregressive process that might in principle explain the lower ‘request of price indexation’ by Calvo. The auxiliary process, even if purely forward looking, might in principle be important in shaping the dynamics of the system. Figure 3 contrasts observed inflation with these two latent processes. When looking at the two top panels, which display raw processes, one may easily realize that such latent processes are hardly responsible of the superiority of the Calvo framework. Indeed, the price dispersion volatility (left column) is way lower than that of raw inflation. In contrast, the auxiliary process (right column) is extremely volatile. Of course, this does not imply that these processes are uncorrelated with raw inflation. The two bottom panels, which show standardized processes, make us appreciate the correlations between price dispersion and raw inflation (0.80) and the auxiliary process and inflation (0.60). Nevertheless, given the very different volatilities characterizing these processes, the explanatory power of these two processes is likely to be very low.20

However, further investigations conducted over these latent processes to isolate their contribution for the description of the U.S. inflation rate turn out to be inconclusive. In particular, when switching these latent processes off and re-estimating our models, we did not observe a clear impact on the estimated parameters or a deterioration of the marginal likelihoods. However, one should take the results coming from this attempt with a grain of salt. Indeed, given the structure of the Calvo-model at hand, it is not possible to mute the price dispersion process in a theoretically coherent manner. We then leave the attempt to identify the role played by price dispersion for the description of raw inflation to future research.

The impact of trend inflation on the convolutions is actually unlikely to be responsible of the different between Calvo and Rotemberg. Cogley and Sbordone (2008) perform an exercise in which they switch off the impact of trend inflation on the convolutions of a NKPC estimated with U.S. data. When estimating the so constrained model, they obtain an estimated NKPC virtually equivalent to that estimated in a theoretically-consistent manner. Then, the edge of the Calvo model over Rotemberg is likely not to be driven by the impact of trend inflation on the convolutions of the NKPC and the IS

20Of course, a more volatile price dispersion process, possibly stochastic, could very well become a determinant of raw inflation. We leave the development of a model with a stochastic price dispersion process to future research.
curve.

We are then left with the distinct structure of the two models. Being two fundamentally different models, they propose under trend inflation two fundamentally different structures. In particular, they differ in the IS curves, because of the diverse implications of the pricing mechanisms on the relationship between consumption and output. We then implement the following exercise to investigate if the difference between the two IS curves influences the fit of the overall frameworks. We estimate the ‘Calvo NKPC - Rotemberg IS’ model, set up by considering eqs. (11)-(13), (17), (19), and (20), and the ‘Rotemberg NKPC - Calvo IS’ model, which consists by eqs. (15)-(16), (14), (19), and (20). The idea is that of ‘swapping’ the different, theoretically based IS structures between the two models to check the consequences over the price indexation estimate and the model fit.

This ‘swap’ leads to two interesting findings. First, the estimated indexation parameter for the ‘Calvo NKPC - Rotemberg IS’ turns out to be $\chi = 0.36 \ [0.03, 0.68]$ (posterior mean and 90\% credible set), i.e., the indexation parameter more than double with respect to the trend inflation Calvo model. Moreover, the empirical fit deteriorates, with the marginal likelihood reading $-35.52$. On top of that, we detect a deterioration of about one log-point in the marginal likelihood when imposing the no-indexation constraint $\chi = 0.36$. Contrasting results emerge when moving to the estimation of the ‘Rotemberg NKPC - Calvo IS’ set up, which returns $\chi = 0.17 \ [0, 0.35]$, with a marginal likelihood equal to $-33.12$, higher than the ‘Calvo NKPC - Rotemberg IS’ framework. The imposition of the $\chi = 0$ constraint on this latter framework leaves the marginal likelihood basically unchanged.\(^{21}\)

These findings suggest that the assessment of the empirical abilities of the Calvo vs. Rotemberg frameworks must involve all the model equations, i.e. the study on the NKPCs per se is not exhaustive. It is important to recall that, when considering the microfounded Calvo and Rotemberg set ups, Calvo does require neither a counterfactual zero net inflation rate in steady state nor an unappealing full indexation to steady state/past inflation. The standard model, instead, needs these two features to square up with the data. In comparison with Rotemberg, Calvo maintains a more standard Euler equation for consumption and inflation. Ex-post, it is perhaps not surprising that the Calvo model under trend inflation turns out to be the best-fitting model. What it appears as a striking fact is that just a small number of empirical applications have

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\(^{21}\)We omit the presentation of the whole set of ‘Calvo NKPC - Rotemberg IS’ and ‘Rotemberg NKPC - Calvo IS’ estimates for the sake of brevity, but these results are available upon request.
been conducted so far with this trend inflation equipped framework.

5 The estimated role of trend inflation: An application

As shown in Ascari and Rossi (2009), whenever the degree of indexation is only partial, trend inflation affects the determinacy regions in the Calvo and in the Rotemberg model in a different way. It enlarges the determinacy region in the Rotemberg model, while it shrinks it in the Calvo model. Since in both models indexation counteracts the effects of trend inflation, then, indexation will have opposite effects in the two models. Moreover, with full indexation (both to trend and to past inflation) the two models exhibit the same area of determinacy. Indeed, in this case, or likewise in the case of zero steady state inflation, the two wedges in equations (10) and (4) disappear and the two models are equivalent.

Figure 4 shows the determinacy regions in the Calvo model under the three estimated model: (i) the baseline NK with $\chi = \mu = 1$, (ii) the Calvo model with $\mu = 1$ and $\chi = 0$; (iii) the Rotemberg model with $\mu = 1$ and $\chi = 0.38$. When $\chi = \mu = 1$, i.e., full backward-looking indexation, both models collapses to the baseline NK model (with an hybrid NKPC), and the condition for a rational expectation equilibrium to be determinate coincides with the Taylor principle: $\alpha_\pi > 1$. Our estimates, however, suggests that the degree of indexation is far from full in both the Calvo and the Rotemberg model. Indeed, in the Calvo model the best fit is obtained with zero indexation, while the Rotemberg model requires a degree of backward-looking indexation equal to 0.38. The determinacy regions under these two estimated frameworks look quite different. Indeed, under zero indexation in the Calvo model the boundary of the determinacy region rotates clockwise. In other words, the system becomes more prone to indeterminacy in presence of a systematic reaction to output oscillations. This prediction is in stark contrast with the one coming from the Rotemberg model, which suggests a counter-clockwise rotation conditional on the estimated degree of indexation (as well as the remaining structural parameters).

To sum up, our estimates show that for a given degree of trend inflation, the determinacy area is strongly dependent on the choice of the price setting model. It follows that the two models may imply different normative policy prescriptions, so that, the need to assess the relative performance of the two models is reinforced. In fact, when seeking for optimal and implementable rules, e.g., Schmitt-Grohe and Uribe (2007), dif-
ferent indications may come from the two frameworks, simply because some rules can be optimal and implementable under Rotemberg pricing, but could be not implementable under Calvo. It is important to realize that the model that offers a more likely description of the data, i.e., Calvo, it also generates a more restrictive determinacy region, casting some shadows on the results in the optimal policy rules literature based on the Rotemberg model.

6 Conclusions

This paper compares two new-Keynesian DSGE monetary models of the business cycle derived under different pricing schemes - Calvo and Rotemberg - and a positive trend inflation rate. We exploit the structural differences in the first-order presentation of the two models, derived by Ascani and Rossi (2009), to assess their relative empirical fit for the 1984:I-2008:II U.S. data. Our main results are the following.

First, we find evidence supporting trend inflation-endowed models as superior on a positive ground. In the light of the recent literature that showed that normative conclusions (Ascani and Ropele, 2007) may be importantly influenced by trend inflation, our empirical support call for an increase in the use and development of trend inflation-equipped frameworks.

Second, our empirical exercises support the Calvo model as the better fitting pricing scheme. This finding is relevant, given the different policy implications stemming from these two frameworks (Ascani and Rossi, 2009). Section 5, for example, shows how the indeterminacy regions implied by the two models for a standard Taylor rule are very different. A policy rules implementable under Rotemberg, thus, may lead to indeterminacy under Calvo, which should then be taken as the referenced model to rule out sunspot-driven inefficient fluctuations.

Third, the estimated degree of indexation in the Calvo model is statistically zero. In line with the results in Cogley and Sbordone (2008), it seems there is no need to hardwire a persistence term in the Calvo model by assuming backward-looking indexation as in Christiano et al. (2005), an assumption theoretically questionable and empirically not observed in the micro data on price setting. We are aware that this result may be driven for the particular sample we concentrate upon, and in future work we are planning to undertake a similar analysis on a longer sample, assuming time-varying trend inflation (see Cogley and Sbordone, 2008, Benati, 2009, Barnes et al., 2009).

Finally, our model highlights the importance of estimating the full model equations,
rather than only the NKPC generated from a particular price setting mechanism. Indeed, the way the cost of nominal rigidities enters the model may also affect the demand side of the model, as in the Rotemberg one. Furthermore, this may have important consequences for the overall fit of the model.

All in all, this paper offers solid support to the Calvo model. Such a model has been criticized for the implausibility of the pricing mechanisms proposed. While being clearly misspecified, the Calvo pricing mechanism deserves in our opinion high attention. Moreover, also recent contributions contrasting different pricing mechanisms on the basis of micro data end up supporting Calvo as largely superior with respect to alternatives (e.g. Costain and Nakov (2008)). While welcoming new contributions carrying out more realistic price setting mechanism, our feeling is that the Calvo model will be a difficult competitor to beat in applied macroeconomics.

7 Technical appendix

7.1 Convolution parameters in the log-linearized Rotemberg model

\[
\begin{align*}
\varsigma_c &= \frac{\varphi_p (\pi^{1-\chi} - 1) \pi^{1-\chi}}{[1 - \frac{\varphi_p}{\varphi} (\pi^{1-\chi} - 1)^2]}, \\
\gamma_p &= \frac{(2\pi^{2(1-\chi)} - \pi^{1-\chi}) (1 + \beta \chi \mu) + \beta \chi \mu + \beta [(\pi^{1-\chi} - 1) \pi^{1-\chi}]^2 \sigma \varphi_p (1+\mu \chi)}{(2\pi^{2(1-\chi)} - \pi^{1-\chi}) (1 + \beta \chi \mu) + \beta [(\pi^{1-\chi} - 1) \pi^{1-\chi}]^2 \sigma \varphi_p (1+\mu \chi)}, \\
\gamma_f &= \frac{(2\pi^{2(1-\chi)} - \pi^{1-\chi}) (1 + \beta \chi \mu) + \beta \chi \mu + \beta [(\pi^{1-\chi} - 1) \pi^{1-\chi}]^2 \sigma \varphi_p (1+\mu \chi)}{(2\pi^{2(1-\chi)} - \pi^{1-\chi}) (1 + \beta \chi \mu) + \beta \chi \mu + \beta [(\pi^{1-\chi} - 1) \pi^{1-\chi}]^2 \sigma \varphi_p (1+\mu \chi)}, \\
\gamma_{dy} &= \frac{(2\pi^{2(1-\chi)} - \pi^{1-\chi}) (1 + \beta \chi \mu) + \beta \chi \mu + \beta [(\pi^{1-\chi} - 1) \pi^{1-\chi}]^2 \sigma \varphi_p (1+\mu \chi)}{(2\pi^{2(1-\chi)} - \pi^{1-\chi}) (1 + \beta \chi \mu) + \beta \chi \mu + \beta [(\pi^{1-\chi} - 1) \pi^{1-\chi}]^2 \sigma \varphi_p (1+\mu \chi)}, \\
\gamma_{mc} &= \frac{\varepsilon - 1 + \varphi_p (2\pi^{2(1-\chi)} - \pi^{1-\chi}) (1 - \beta)}{\varphi_p \left( (2\pi^{2(1-\chi)} - \pi^{1-\chi}) (1 + \beta \chi \mu) + \beta [(\pi^{1-\chi} - 1) \pi^{1-\chi}]^2 \sigma \varphi_p (1+\mu \chi) \right)}.
\end{align*}
\]

7.2 Estimation Procedure

To perform our Bayesian estimations we employed DYNARE (release 4.0), a set of algorithms developed by Michel Juillard and collaborators. DYNARE is freely available at the following URL: http://www.cepremap.cnrs.fr/dynare/. The simulation of the target distribution is basically based on two steps.
• First, we initialized the variance-covariance matrix of the proposal distribution and employed a standard random-walk Metropolis-Hastings for the first \( t \leq t_0 = 20,000 \) draws. To do so, we computed the posterior mode by the ‘csminwel’ algorithm developed by Chris Sims. The inverse of the Hessian of the target distribution evaluated at the posterior mode was used to define the variance-covariance matrix \( C_0 \) of the proposal distribution. The initial VCV matrix of the forecast errors in the Kalman filter was set to be equal to the unconditional variance of the state variables. We used the steady-state of the model to initialize the state vector in the Kalman filter.

• Second, we implemented the ‘adaptive Metropolis’ (AM) algorithm developed by Haario, Saksman, and Tamminen (2001) to simulate the target distribution. Haario, Saksman, and Tamminen (2001) show that their AM algorithm is more efficient than the standard Metropolis-Hastings algorithm. In a nutshell, such algorithm employs the history of the states (draws) so to ‘tune’ the proposal distribution suitably. In particular, the previous draws are employed to regulate the VCV of the proposal density. We then exploited the history of the states sampled up to \( t > t_0 \) to continuously update the VCV matrix \( C_t \) of the proposal distribution. While not being a Markovian process, the AM algorithm is shown to possess the correct ergodic properties. For technicalities, refer to Haario, Saksman, and Tamminen (2001).

We simulated two chains of 400,000 draws each, and discarded the first 75\% as burn-in. To scale the variance-covariance matrix of the chain, we used a factor so to achieve an acceptance rate belonging to the [23\%,40\%] range. The stationarity of the chains was assessed via the convergence checks proposed by Brooks and Gelman (1998). The region of acceptable parameter realizations was truncated so to obtain equilibrium uniqueness under rational expectations.

References


<table>
<thead>
<tr>
<th>Param.</th>
<th>Description</th>
<th>Prior Distr.</th>
<th>Prior Mean (St.dev.)</th>
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<td>Beta</td>
<td>0.50 (0.15)</td>
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<td>2.50 (0.25)</td>
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</tr>
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</table>

Table 1: Priors for structural parameters.
Figure 1: Prior and posterior densities. 'Baseline' model: Full indexation. 'Calvo' and 'Rotemberg': See description in the text. Models estimated under a 2.5 per cent trend inflation net rate (yearly rate, percentualized) and indexation to past inflation.
<table>
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<th>Rotemb.</th>
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Table 2: **Posterior means for structural parameters.** Models estimated under a 2.5 percent trend inflation net rate (yearly rate, percentualized) and indexation to past inflation. The Table reports the posterior means and the [5th,95th] percentiles. The Marginal Likelihood are computed with the modified harmonic mean estimator by Geweke (1998). Details on the model estimation are reported in the text.
<table>
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<th>Calvo $\mu=0$</th>
<th>Rotemb. $\mu=0$ $\chi\sim$Beta(1/4,1/10)</th>
<th>Rotemb. $\chi\sim$Beta(1/4,1/10)</th>
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Table 3: Posterior means for structural parameters. 1 / 2 / 3 / 4: Models estimated under a 2 / 3 / 2.5 / 2 per cent trend inflation net rate (yearly rate, percentualized), full indexation to past / past / trend / past inflation, and a Beta(0.50,0.285) / Beta(0.50,0.285) / Beta(0.50,0.285) / Beta(0.25,0.10) prior density on the indexation parameter. The Table reports the posterior means and the [5th,95th] percentiles. The Marginal Likelihood is computed using the modified harmonic mean estimator by Geweke (1998). Details on the model estimation are reported in the text.
Figure 2: **Indexation: Difference between densities.** Green vertical line: zero line. Red dotted vertical lines: 25th and 75th percentiles.
Figure 3: **Calvo model: Inflation vs. latent factors.** Filtered factors estimated under 2.5 per cent trend inflation and zero indexation.
Figure 4: **Determinacy under different frameworks.** Determinacy regions lie at the right of the model-consistent boundaries.