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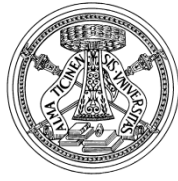
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**Quaderni di Dipartimento**

**Endogenous Market Structures and Labor  
Market Dynamics  
(New version)**

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# Endogenous Market Structures and Labor Market Dynamics

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## **Abstract**

We propose a model characterized by strategic interactions among an endogenous number of producers and search and matching frictions in the labor market. In line with U.S. data: (i) new firms account for a relatively small share of overall employment, but they create a relevant fraction of new jobs; (ii) firms' entry is procyclical; (iii) price mark ups are countercyclical, while aggregate profits are procyclical. In response to a technology shock the labor share decreases on impact and overshoots its long run level. Also the propagation on labor market variables is stronger than in the standard search model. We argue that the countercyclicity of the price mark up is the key mechanism for our results.

*JEL classification: E24, E32, L11.*

*Keywords: Endogenous Market Structures, Job Creation, Firms' Entry, Search and Matching Frictions.*

## 1 *Introduction*

We present a framework where search and matching frictions in the labor market interact with the dynamics of the number of firms and their strategic behavior. Market structures are said to be endogenous since the number of producers and the price mark ups are determined both in the short and in the long run.

The paper is motivated by three stylized facts: the large fraction of job creation (destruction) in the U.S. economy due to the birth (death) of firms; the procyclical variation in the number of market competitors and in aggregate profits; the countercyclical variation of price mark ups.<sup>1</sup>

To account for these facts we consider an economy with distinct sectors, each one characterized by many firms supplying goods that can be imperfectly substitutable to a different extent. As in Jaimovich and Floetotto (2008) and Colciago and Etro (2010 a and b), we take strategic interactions into account and allow firms within a sector to compete either in prices (Bertrand competition) or in quantities (Cournot competition). Following Ghironi and Melitz (2005) and Bilbiie, Ghironi and Melitz (2007) (BGM 2007 henceforth), entry is subject to sunk entry costs and a time-to-build lag. The free entry condition equates the expected present discounted value of profits to the sunk cost to endogenize the number of firms in each sector. As a result the degree of market power, measured as the mark up that firms can impose over marginal costs, depends endogenously on the form of competition, on the degree of substitutability between goods and on the number of firms in the sector.

Firms are large, since they employ multiple-workers and the labor market is characterized by Mortensen and Pissarides (1999)-style search and matching frictions. Workers may separate from a job for two, exogenous, reasons: either because the firm where the job is located exits from the market or because the match is destroyed. The endogeneity of the number of producers together with the large firms assumption allows to characterize both the intensive and the extensive margin of job creation, and to realistically distinguish between the dynamics of the number of producers and that of employment.

Beside addressing the empirical facts mentioned above we use this setup to show two further results. First the model reproduces the comovement of labor share with technology shocks at business cycle frequencies. Rios-Rull and Santaella-Llopis (2010) estimate the response of the labor share, i.e. the ratio of the labor compensation to output, to

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<sup>1</sup>We discuss empirical evidence supporting these facts below. Notice that, as in Jaimovich and Floetotto (2008), we use the terms new firms and new competitors in a broad sense. They refer to both start ups and to new establishments.

a technology shocks in the US economy, and find that it is characterized by two main features: *countercyclical* and *overshooting*. Explaining both these facts is a notorious difficulty for conventional business cycle models.<sup>2</sup> Second we show that under a conservative and standard calibration, our framework outperforms the basic search and matching model at replicating the observed variability of the unemployment rate, vacancies and market tightness.

The mechanism at the basis of both results is the markup countercyclical due to oligopolistic competition between firms. A positive technology shock promotes entry of new firms. The resulting stronger competition leads to a persistent decrease in the price mark up. In particular, when productivity increases the markup is muted on impact and gradually decreases as the goods market gets more crowded. A lower price markup leads to a persistent increase in the value of the marginal product of labor, thereby affecting the dynamic of the real wage. In particular following a technology shock the latter peaks after some periods, when the mark up reaches its minimum, and then reverts to the steady state with a hump shape dynamics. On the contrary, output jumps on impact and then monotonically reverts to the steady state. As a result of the hump shaped response of the real wage, labor income peaks while output is decreasing, leading to the overshooting of the labor share.

Turning to the amplification result, a persistently lower mark up, by increasing the value of the current and future marginal product of labor, boosts hiring both at the intensive and the extensive margin. The effect of technology shocks on unemployment is thus amplified with respect to what happens in a model with constant mark ups and a constant number of producers.

Notice that a similar propagation mechanism based on markup countercyclical holds in sticky-prices environment, as emphasized by Monacelli et al. (2010). However, in our framework although markups are countercyclical, aggregate profits remain strongly procyclical as in the data. BGM (2010) point out that it is notoriously difficult to generate both countercyclical markups and procyclical aggregate profits in models with sticky prices, or, more generally, in any model economy with a constant number of firms.

In the long run stronger competition in the goods market leads to lower unemployment and to higher real wages. The endogenous steady state share of gross job creation due to new firms is 25 percent and the share of overall employment due to startups equals 2.5 percent. These

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<sup>2</sup>Choi and Rios-Rull (2009) analyze real business cycles models where wages are not set competitively and show that none of these models is able to reproduce the labor share overshooting.

figures are in line with U.S. averages. Haltiwanger et al. (2009) consider U.S. annual data between 1992 and 2005. They find that business startups account for roughly 3 percent of U.S. total employment in any given year. While this is a reasonably small share of the stock, it is large relative to net job creation which averages around 2.2 percent of total employment per year.<sup>3</sup> Also, Davis and Haltiwanger (1990) on the basis of U.S. manufacturing data between 1972 and 1986 estimate that 25 percent of annual gross job creation is due to new establishments births. Similarly, Jaimovich and Floetotto (2008) focus on employment data at the establishment level. They estimate that the average fraction of quarterly job-gain (losses) that can be explained by the opening (closing) of establishments is about 20 percent.

What does the empirical evidence have to say about the procyclicality of firms' entry and the countercyclicality of price markups? An early reference on the procyclicality of firms' entry in the U.S. is Chatterjee and Cooper (1993), while a more recent one is Bergin and Corsetti (2008).

The main challenge when trying to measure the cyclicity of price mark ups is the lack of a direct measure of marginal costs. For this reason authors have to make assumptions concerning the relationship between marginal and average costs. As a consequence, the evidence concerning the cyclicity of price mark ups is more controversial than that on firms' entry. Nekarda and Ramey (2010), using average wages and other measures to proxy marginal costs, support the view that mark ups are acyclical. Their result is challenged by Cheremuklin and Tutino (2011), who, using the same data constructed by Nekarda and Ramey (2010), find support for mark ups countercyclicality. Cheremuklin and Tutino (2011) suggest that the acyclicity result may depend on the specific detrending procedure adopted.<sup>4</sup> Bilts (1987), Rotemberg and Woodford (2000) and Galí et al. (2007) also document price mark up countercyclicality.<sup>5</sup>

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<sup>3</sup>Haltiwanger et al (2009) warn that it would be misleading to conclude that new firms account for more than 100% percent of all net new jobs. Other, mature, firms are creating jobs. However the net growth from new firms alone exceeds the average.

<sup>4</sup>Cheremukhin and Tutino (2011) use the series on private sector mark ups constructed by Nekarda and Ramey (2010) and argue that mark ups are countercyclical. They suggest that cyclicity results may depend on the specific detrending procedure adopted. To avoid this issue Cheremuklin and Tutino (2010) compute mark up cyclicity resorting to the third moment of first differenced data, which is independent of the detrending procedure.

<sup>5</sup>More in detail, Bilts (1987) estimates marginal cost in U.S. manufacturing under several assumptions about overtime and adjustment costs and concludes that markups are countercyclical. Rotemberg and Woodford (2000) discuss several reasons for which mark ups should be countercyclical. Galí *et al.* (2007) use VAR analysis

A relevant empirical support to our model comes from the studies by Campbell and Hopenayn (2005) and Oliveira Martins and Scarpetta (1999). These papers convincingly report evidence suggesting that a variation in the number of competitors affects the degree of competition in the market and through this way the mark up that firms can impose on marginal costs. Finally, BGM (2007) and Etro and Colciago (2010) emphasize the procyclicality of real profits.

Overall, the evidence briefly discussed tends to favour countercyclical theories of mark ups and to support the view that competition between an endogenous number of producers is a relevant dimension to consider in order to understand the connection between labor markets and the goods markets. To conclude, to our knowledge, the model we propose is the first one to address in a unified framework the stylized facts listed at the beginning of this Introduction.

The remainder of the paper is organized as follows. Section 2 briefly reviews the relevant literature. Section 3 spells out the model economy. Section 4 contains the main results and Section 5 concludes. Technical details are left in the Appendix.

## 2 Related Literature

The papers closest related to ours are Blanchard and Giavazzi (2003), Hebel and Haefke (2009), Shao and Silos (2008) and more recently Acemoglu and Hawkins (2010), Hawkins (2011) and Kaas and Kircher (2011).

With respect to Blanchard and Giavazzi (2003) we provide a fully specified DSGE model where the dynamics of the number of firms is explicitly modeled. Hebel and Haefke (2009) consider a labor search model with firms' entry. However, their analysis focuses on the long run effects of deregulation in the goods markets for the level of unemployment and the real wage. Shao and Silos (2008) introduce firms' entry in a Mortensen-Pissarides-style model with monopolistic competition in the goods market. Their framework is characterized by small firms and constant mark ups. They identify the countercyclical value of vacancies as the main propagation channel of technology shocks. Also, the small firms assumption does not allow to address the empirical evidence on job creation by new entrants.

Acemoglu and Hawkins (2010) consider a model with large firms, decreasing returns to labor and an extensive margin of job creation due to firms' entry. They assume convex costs of adjusting the labor force

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to document the countercyclicality of price mark ups in response to an expansionary monetary policy shock. Colciago and Etro (2010) use the same multivariate technique to show mark up countercyclicality in response to a positive technology shock.

at the firm level. Decreasing return to labor together with continuous wage bargaining deliver heterogeneity in the size of firms. They find that size heterogeneity implies responses of unemployment and of the job market tightness to a shock to labor productivity which are significantly more persistent than in the Mortensen-Pissarides model. Hawkins (2011), however, argues that the heterogeneity in the size of firms is not enough to strengthen the propagation of shocks. More precisely, he shows that the assumption at the basis of the amplification in Acemoglu and Hawkins (2010) is the convex cost schedule that firms face when they adjust their labor force. This slows down the job creation process at the firm level, leading to an aggregate sluggish dynamics. Notice that in our framework the slow response of the number of firms does not lead to propagation per se, but due to its effect on the price mark up.

Our analysis differs from that in Kaas and Kircher (2011) with respect to both assumptions and focus. For what concerns assumptions we feature strategic interactions among large firms which bargaining the wage on a period-by-period basis with their employees. The aforementioned authors consider an alternative framework to characterize firms' dynamics in a frictional labor market, where large, risk-neutral firms can commit to long-term wage contracts. With respect to the focus, we analyze endogenous market structures both in the long and the short run, and we emphasize their role under different forms of competition for the propagation of exogenous technology shocks on labor market variables. Kaas and Kircher (2011) focus instead on the efficiency of the competitive equilibrium in the presence of multiple-workers firms that can commit to long-term wage contracts.

### 3 *The model*

#### 3.1 *Labor and Goods Markets*

There are two main building blocks in the model: oligopolistic competition with endogenous entry in the goods market and search and matching frictions in the labor market. In this paragraph we outlay their main features.

As in Colciago and Etro (2010 a and b), the economy features a continuum of sectors, or industries, on the unit interval. Sectors are indexed with  $k \in (0, 1)$ . Each sector  $k$  is characterized by different firms  $i = 1, 2, \dots, N_{kt}$  producing the same good in different varieties. At the beginning of each period  $N_{kt}^e$  new firms enter into sector  $k$ , while at the end of the period a fraction  $\delta \in (0, 1)$  of market participants exits from the market for exogenous reasons.<sup>6</sup> Below we describe the entry process

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<sup>6</sup>As discussed in BGM (2007), if macroeconomic shocks are small enough  $N_{kt}^e$  is



and the mode of competition within in each sector in detail.

The labor market is characterized by search and matching frictions, as in Andolfatto (1996) and Mertz (1995). A fraction  $u_t$  of the unit mass population is unemployed at time  $t$  and searches for a job. Firms producing at time  $t$  need to post vacancies in order to hire new workers. Unemployed workers and vacancies combine according to a CRS matching function and deliver  $m_t$  new hires, or matches, in each period. The matching function reads as  $m_t = \gamma_m (v_t^{tot})^{1-\gamma} u_t^\gamma$ , where  $\gamma_m$  reflects the efficiency of the matching process,  $v_t^{tot}$  is the total number of vacancies created at time  $t$  and  $u_t$  is the unemployment rate. The probability that a firm fills a vacancy is given by  $q_t = \frac{m_t}{v_t^{tot}}$ , while the probability to find a job for an unemployed worker reads as  $z_t = \frac{m_t}{u_t}$ . Firms and individuals take both probabilities as given. Matches become productive in the same period in which they are formed. Each firm separates exogenously from a fraction  $1 - \varrho$  of existing workers each period, where  $\varrho$  is the probability that a worker stays with a firm until the next period.

As a result a worker may separate from a job for two reasons: either because the firm where the job is located exits from the market or because the match is destroyed. Since these sources of separation are independent, the evolution of aggregate employment,  $L_t$ , is given by

$$L_t = (1 - \delta) \varrho L_{t-1} + m_t \quad (1)$$

where the number of unemployed workers searching for a job at time  $t$  is  $u_t = 1 - L_{t-1}$ .<sup>7</sup>

### 3.2 *Households and Firms*

Using the family construct of Mertz (1995) we can refer to a representative household consisting of a continuum of individuals of mass one. Members of the household insure each other against the risk of being unemployed. The representative family has lifetime utility:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \int_0^1 \ln C_{kt} dk - \chi L_t \frac{h_t^{1+1/\varphi}}{1+1/\varphi} \right\} \quad \chi, \varphi \geq 0 \quad (2)$$

where  $\beta \in (0, 1)$  is the discount factor and the variable  $h_t$  represents individual hours worked. Note that  $C_{kt}$  is a consumption index for a set

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positive in every period. New entrants finance entry on the stock market.

<sup>7</sup>Given that population is normalized to one, the number of unemployed workers and the unemployment rate are identical.

of goods produced in sectors  $k \in [0, 1]$ , defined as

$$C_{kt} = \left[ \sum_{i=1}^{N_{kt}} C_{kt}(i)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (3)$$

where  $C_{kt}(i)$  is the production of firm  $i$  of this sector, and  $\varepsilon > 1$  is the elasticity of substitution between the goods produced in each sector. The distinction between different sectors and different goods within a sector allows to realistically separate limited substitutability at the aggregated level, and high substitutability at the disaggregated level. Contrary to many macroeconomic models with imperfect competition, our focus will be on the market structure of disaggregated sectors: intrasectoral substitutability (between goods produced by firms of a same sector) is high, while intersectoral substitutability is low.<sup>8</sup> The family receives real labor income  $w_t h_t L_t$  and profits from the ownership of firms. Further, we assume that unemployed individuals receive an unemployment benefit  $b$  in real terms, leading to an overall benefit for the household equal to  $b(1 - L_t)$ . This is financed through lump sum taxation by the government. Notice that the household recognizes that employment is determined by the flows of its members into and out of employment according to

$$L_t = (1 - \delta) \rho L_{t-1} + z_t u_t \quad (4)$$

Households choose how much to save in riskless bonds and in the creation of new firms through the stock market according to standard Euler and asset pricing equations.<sup>9</sup>

The intratemporal optimality conditions for the optimal choices of  $C_{kt}$  requires:

$$P_{kt} C_{kt} = EXP_t \quad \text{for any } k \quad (5)$$

where  $EXP_t$  is total nominal expenditure allocated to the goods produced in each sector in period  $t$  and  $P_{kt}$  is the price index for consumption in sector  $k$ : due to the unitary elasticity of substitution, total expenditure is identical across sectors.

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<sup>8</sup>Our functional form implies unitary elasticity of substitution between goods produced in different sectors. In this case the aggregate consumption bundle enjoyed by the household could be defined as  $C_t^A = \exp\left(\int_0^1 \ln C_{kt} dk\right)$  and associated to the aggregate price index  $P_t^A = \exp\left(\int_0^1 \ln P_{kt} dk\right)$ . The same approach has been proposed by Colciago and Etro (2010 a). Atkeson and Burnstein (2008) consider a trade model with multiple sectors. Even if they allow for general substitutability across sectors, their numerical results are obtained assuming a unitary intersectoral elasticity of substitution.

<sup>9</sup>We report these conditions in Appendix A.

Each firm  $i$  in sector  $k$  produces a good with a linear production function. We abstract from capital accumulation issues and assume that labor is the only input. Output of firm  $i$  in sector  $k$  is then:

$$y_{kt}(i) = A_t n_{kt}(i) h_{kt}(i) \quad (6)$$

where  $A_t$  is the, common to all sectors, total factor productivity at time  $t$ ,  $n_{kt}(i)$  is firm  $i$ 's time  $t$  workforce and  $h_{kt}(i)$  represent hours per employee. Since each sector can be characterized in the same way, in what follows we will drop the index  $k$  and refer to the representative sector.<sup>10</sup>

Finally, the marginal value to the household of having one member employed rather than unemployed,  $\Gamma_t$ , which is a determinant of the wage bargaining problem, is

$$\Gamma_t = \frac{1}{C_t} (w_t h_t - b) - \chi \frac{h_t^{1+1/\varphi}}{1+1/\varphi} + \beta E_t [(1-\delta)\rho - z_{t+1}] \Gamma_{t+1} \quad (7)$$

where  $C_t$  is individual consumption.

### 3.3 Endogenous Market Structures

Following Ghironi and Melitz (2005) and BGM (2007) we assume that new entrants at time  $t$  will only start producing at time  $t+1$ . Given the exogenous exit probability  $\delta$ , the average number of firms per sector,  $N_t$ , follows the equation of motion:

$$N_{t+1} = (1-\delta)(N_t + N_t^e) \quad (8)$$

where  $N_t^e$  is the average number of new entrants at time  $t$ . We assume that entry requires a fixed cost  $\psi$ , which is measured in units of output. In each period, the same nominal expenditure for each sector  $EXP_t$  is allocated across the available goods according to the direct demand function:

$$y_t(i) = Y_t \left( \frac{p_t(i)}{P_t} \right)^{-\varepsilon} = \frac{p_t(i)^{-\varepsilon}}{P_t^{1-\varepsilon}} Y_t P_t = \frac{p_t(i)^{-\varepsilon} EXP_t}{P_t^{1-\varepsilon}} \quad i = 1, 2, \dots, N_t \quad (9)$$

where  $P_t$  is the price index

$$P_t = \left[ \sum_{j=1}^{N_t} p_t(j)^{-(\varepsilon-1)} \right]^{\frac{-1}{\varepsilon-1}} \quad (10)$$

<sup>10</sup>We provide analytical details in Appendix A.

such that total expenditure satisfies  $EXP_t = \sum_{j=1}^{N_t} p_t(j)C_t(j) = C_t P_t$ .<sup>11</sup>

Inverting the direct demand functions, we can derive the system of inverse demand functions:

$$p_t(i) = \frac{y_t(i)^{-\frac{1}{\varepsilon}} EXP_t}{\sum_{j=1}^{N_t} y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}} \quad i = 1, 2, \dots, N_t \quad (11)$$

Period  $t$  real profits of an incumbent producer are defined as

$$\pi_t(i) = \rho_t(i) y_t(i) - w_t(i) n_t(i) h_t(i) - \kappa v_t(i) \quad (12)$$

where  $\rho_t(i)$  ( $= \frac{p_t(i)}{P_t}$ ) is the real price of firm  $i$ 's output,  $v_t(i)$  represents the number of vacancies posted at time  $t$  and  $\kappa$  is the output cost of keeping a vacancy open. The value of a firm is the expected discounted value of its future profits

$$V_t(i) = E_t \sum_{s=t+1}^{\infty} \Lambda_{t,s} \pi_s(i) \quad (13)$$

where  $\Lambda_{t,t+1} = (1 - \delta) \beta \left( \frac{C_{t+1}}{C_t} \right)^{-1}$  is the households' stochastic discount factor which takes into account that firms' survival probability is  $1 - \delta$ . Incumbent firms which do not exit from the market have a time  $t$  individual workforce given by

$$n_t(i) = \varrho n_{t-1}(i) + v_t(i) q_t \quad (14)$$

### 3.3.1 *Bertrand Competition*

Let us consider competition in prices. Contrary to the traditional Dixit-Stiglitz approach which neglects strategic interactions between firms, we take these into consideration and derive the exact Bertrand equilibrium. Each firm  $i$  chooses  $p_t(i)$ ,  $n_t(i)$  and  $v_t(i)$  to maximize  $\pi_t(i) + V_t(i)$ , taking as given the price of the other firms. Maximization is subject to three constraints, namely (6), (9) and (14).

The variable  $\phi_t(i)$  is the Lagrange multiplier of the latter constraint, and represents the time- $t$  value of an additional workers to the firm;  $mc_t(i)$  is the time  $t$  real marginal cost faced by firm  $i$  and represents the Lagrange multiplier of constraint (6).

<sup>11</sup>The demand of the individual good and the price index are the solution to the, usual, consumption expenditure minimization problem.

In what follows we distinguish between incumbent firms according to their period of entry. We define as *first period incumbent firms* those producers which entered the market in period  $t-1$  and at time  $t$  produce for the first time. The term *mature incumbent firms* refers, instead, to producers which entered the market in period  $t-2$  or prior. The distinction is relevant because first period incumbents have no beginning of period workforce. Nevertheless, Proposition 1 and 2 show that in both the Bertrand and Cournot equilibria incumbent producers, no matter the period of entry, have the same size, impose the same mark up over marginal costs and have the same individual level of production.

**Proposition 1 (Bertrand Equilibrium)** *In the Bertrand equilibrium, no matter the period of entry: i) the marginal cost and the value of an additional worker are identical across producers:  $mc_t(i) = mc_t$  and  $\phi_t(i) = \phi_t$ ; ii) firms set the same mark up over the nominal marginal cost, given by*

$$\mu_t^P(\varepsilon, N_t) = \frac{\varepsilon(N_t - 1) + 1}{(\varepsilon - 1)(N_t - 1)} \quad (15)$$

*iii) firms have the same level of production, the same size and demand the same number of hours per employee:  $y_t(i) = y_t$ ,  $n_t(i) = n_t$  and  $h_t(i) = h_t$ .*

**Proof.** See Appendix B1 ■

Since in equilibrium firms set the same prices, it follows from (10) that the relative price is also identical across producers and reads as  $\rho_t(\varepsilon, N_t) = \frac{p_t}{P_t} = N_t^{\frac{1}{\varepsilon-1}}$ . The mark up  $\mu_t^P(\varepsilon, N_t)$  is decreasing in the degree of substitutability between products  $\varepsilon$ , with an elasticity  $\epsilon_\varepsilon^P = \frac{\varepsilon N_t}{(1-\varepsilon+\varepsilon N_t)(\varepsilon-1)}$ . Moreover, the mark up vanishes in case of perfect substitutability:  $\lim_{\varepsilon \rightarrow \infty} \mu^P(\varepsilon, N_t) = 1$ . Finally, the mark up is decreasing in the number of firms, with an elasticity  $\epsilon_N^P = \frac{N_t}{(1+\varepsilon(N_t-1))(N_t-1)}$ . Notice that the elasticity of the mark up to entry under competition in prices is decreasing in the level of substitutability between goods, and it tends to zero when the goods are approximately homogenous. When  $N_t \rightarrow \infty$  the mark up tends to  $\varepsilon/(\varepsilon - 1)$ , the traditional one under monopolistic competition. As well known, strategic interactions between a finite number of firms lead to a higher mark up than under monopolistic competition.

### 3.3.2 Cournot Competition

In this section we consider competition in quantities, which has been largely neglected in general equilibrium macroeconomic models with imperfect competition. In this case firms maximize  $\pi_t + V_t$  choosing their production  $y_t(i)$  beside  $n_t(i)$  and  $v_t(i)$ , taking as given the production

of the other firms. Maximization is subject to the same constraints as above, taking care to replace the direct demand function (9) with the inverse demand function given by equation (11). Most of the considerations drawn in the Bertrand competition case extend to Cournot competition. Proposition 2 fully characterizes the equilibrium under Cournot competition.

**Proposition 2 (Cournot Equilibrium)** *Points i),iii) and iii) of Proposition 1 extend to the Cournot case. The symmetric Cournot equilibrium generates the individual output*

$$y_t = \frac{\varepsilon - 1}{\varepsilon} \frac{N_t - 1}{N_t^2} \frac{EXP_t}{MC_t} \quad (16)$$

where  $MC_t$  is the nominal marginal cost, the associated equilibrium mark up is:

$$\mu^Q(\varepsilon, N_t) = \frac{\varepsilon N_t}{(\varepsilon - 1)(N_t - 1)} \quad (17)$$

**Proof.** See Appendix B2. ■

For a given number of firms, the mark up under competition in quantities is always larger than the one obtained under competition in prices, as well known for models of product differentiation (see for instance Vives 1999). Notice that the mark up is decreasing in the degree of substitutability between products  $\varepsilon$ , with an elasticity  $\epsilon_\varepsilon^Q = 1/(\varepsilon - 1)$ , which is always smaller than  $\epsilon_\varepsilon^P$ : higher substitutability reduces mark ups faster under competition in prices. In the Cournot equilibrium, the mark up remains positive for any degree of substitutability, since even in the case of homogenous goods, we have  $\lim_{\varepsilon \rightarrow \infty} \mu^Q(\varepsilon, N_t) = N_t/(N_t - 1)$ . This allow us to consider the effect of strategic interactions in an otherwise standard setup with perfect substitute goods within sectors (as in the standard RBC setting with search and matching frictions of Andolfatto (1996) and Mertz (1995)).

In the general formulation the mark up is decreasing and convex in the number of firms with elasticity  $\epsilon_N^Q = 1/(N - 1)$ , which is decreasing in the number of firms (the mark up decreases with entry at an increasing rate) and independent from the degree of substitutability between goods. Since  $\epsilon_N^Q > \epsilon_N^P$  for any number of firms or degree of substitutability, we can conclude that entry decreases mark ups faster under competition in quantities compared to competition in prices, a result that will have an impact on the relative behavior of the economy under the two forms of competition. Only when  $N_t \rightarrow \infty$  the mark up tends to  $\varepsilon/(\varepsilon - 1)$ , which is the traditional mark up under monopolistic competition.<sup>12</sup>

<sup>12</sup>In what follows, to lighten the notation, we suppress the dependance of  $\rho_t$  and  $\mu_t$  from  $\varepsilon$  and  $N_t$ .

### 3.3.3 Hiring policy

Let  $\pi_t^{FP}$  and  $v_t^{FP}$  be, respectively, the real profits and the number of vacancies posted by a first period incumbent. Symmetrically,  $\pi_t$  and  $v_t$  define, respectively, the individual profits and vacancies posted by mature incumbent firms.

**Proposition 3 (Profits and hiring policy)** *Under both Bertrand and Cournot competition it follows that: i)  $v_t^{FP} = \frac{n_t}{q_t} = v_t - \varrho \frac{n_{t-1}}{q_t}$  and ii)  $\pi_t^{FP} = \pi_t - \kappa \varrho \frac{n_{t-1}}{q_t}$ .*

**Proof.** See Appendix B3. ■

Since all incumbent firms are characterized by the same size, the optimal hiring policy of first period incumbent firms, which have no initial workforce, consists in posting at time  $t$  as many vacancies as required to reach the size of a mature incumbent producer. Given vacancy posting is costly, they will suffer lower profits.

As a consequence of their hiring policy, first period incumbent producers grow faster and pay fewer dividends with respect to mature incumbent producers. The first result is consistent with the U.S. empirical evidence in Haltiwanger et al. (2009), which suggests that a start-up creates on average more new jobs than an incumbent firm, while the second one is consistent with the evidence on the financial behavior of firms discussed by Cooley and Quadrini (2001).

### 3.3.4 Endogenous Entry

In each period the level of entry is determined endogenously to equate the value of a new entrant,  $V_t^e$ , to the entry cost

$$V_t^e = \psi \quad (18)$$

The next Proposition provides a useful relationship between the value of a new entrant and the value of an incumbent firm, denoted by  $V_t$ .

**Proposition 4 (Value of an Incumbent Firm)** *The value of an incumbent firm is larger than that of a new entrant*

$$V_t = V_t^e + \kappa \varrho E_t \Lambda_{t,t+1} \frac{n_t}{q_{t+1}} \quad (19)$$

**Proof.** See Appendix B4. ■

Perspective new entrants have lower value than incumbent firms because they will have, in case they do not exit from the market before starting production, to set up a workforce in their first period of activity. The difference in the value between an incumbent producer and a new entrant is, in fact, the discounted value of the higher vacancy posting cost that the latter will suffer, with respect to the former, in the first period of activity.

### 3.4 Bargaining over Wages and Hours

We assume Nash wage bargaining, so that the firm and each worker split the joint surplus of their employment relationship. Thus, the real wage is set to maximize the product

$$(\phi_t)^{1-\eta} (\Gamma_t C_t)^\eta \quad (20)$$

Recall that the term in the first bracket is the value to the firm of having an additional worker, the second term is the household's surplus expressed in units of consumption. The parameter  $\eta$  reflects the parties' relative bargaining power. The FOC for Nash bargaining is

$$\eta\phi_t = (1 - \eta) \Gamma_t C_t \quad (21)$$

Using the definitions of  $\phi_t$  and  $\Gamma_t$  gives, after some manipulations, the wage equation

$$w_t = (1 - \eta) \frac{b}{h_t} + \eta m c_t A_t + (1 - \eta) \chi C_t \frac{h_t^{1/\varphi}}{1 + 1/\varphi} + \frac{\eta\beta\kappa}{h_t} E_t \frac{z_{t+1}}{q_{t+1}} \frac{C_t}{C_{t+1}} \quad (22)$$

Since  $\frac{z_t}{q_t} = \theta_t$ ,  $\Lambda_{t,t+1} = (1 - \delta) \beta \left( \frac{C_{t+1}}{C_t} \right)^{-1}$  and, importantly,  $m c_t = \frac{\rho_t}{\mu_t}$  we obtain

$$w_t = (1 - \eta) \frac{b}{h_t} + \eta \frac{\rho_t}{\mu_t} A_t + (1 - \eta) \chi C_t \frac{h_t^{1/\varphi}}{1 + 1/\varphi} + \frac{\eta\kappa}{(1 - \delta) h_t} E_t \Lambda_{t,t+1} \theta_{t+1} \quad (23)$$

Clearly the mark up function,  $\mu_t$ , differs according to the form of competition, whether Bertrand or Cournot. In both cases, however, the direct effect of entry on the real wage is captured through the term  $\eta \frac{\rho_t}{\mu_t} A_t$ . Notice that  $\frac{\rho_t}{\mu_t} A_t$  represents the marginal revenue product (MRP) of labor, while  $\eta$  represents the share of the MRP which goes to workers. As described above, entry leads to an increase in the MRP of labor. Thus, ceteris paribus, stronger competition shifts the wage curve up. This result is similar to that in Blanchard and Giavazzi (2003), who find a positive effect of competition on the real wage.

Hours are set to maximize the joint surplus of the match, given by  $S_t = \phi_t + \Gamma_t C_t$ . The FOC with respect of  $h_t$  is

$$\chi C_t h_t^{1/\varphi} = \frac{\rho_t}{\mu_t} A_t \quad (24)$$

where, as above,  $\mu_t$  depends on the form of competition. Hours worked are such that the the marginal rate of substitution between hours and consumption equals the MRP of labor. Stronger competition leads to an increase in hours bargained between the workers and firms for the same reasons for which competition positively affects the wage schedule.



### 3.5 Job Creation and Amplification

Combining the first conditions for profits maximization we get the Job Creation Condition (JCC), which, under both forms of competition, reads as

$$\frac{k}{q_t} = \left( \frac{\rho_t}{\mu_t} - \frac{w_t}{A_t} \right) A_t h_t + \varrho E_t \Lambda_{t,t+1} \frac{k}{q_{t+1}} \quad (25)$$

The JCC equates the real marginal cost of hiring a worker, the left hand side, with the marginal benefit, the right hand side. Note that we assumed that firms take individual wages as given when choosing employment.<sup>13</sup> Importantly, the marginal benefit depends positively on the ratio  $\frac{\rho_t}{\mu_t}$ , which is a positive function of the number of firms in the market,  $N_t$ .<sup>14</sup> As the number of competitors increases, agents consume more goods and enjoy higher welfare for any given level of nominal expenditure. For this reason the welfare based price level must decrease and the relative price of variety  $i$  increases. This increases the profitability of the marginal worker. At the same time, stronger competition leads to a lower mark up, stimulating demand by consumers and thereby increasing output.

As we show below, a positive technology shock leads to entry of new firms and thus to an increase in  $\frac{\rho_t}{\mu_t}$ . In equilibrium, since hiring depends on the current and expected future values of the marginal product of labor, this boosts hiring and employment.<sup>15</sup>

Notice that a similar propagation mechanism based on markup countercyclicality holds in sticky-prices environment, as emphasized by Monacelli et al. (2010). However, in our framework although markups are countercyclical, aggregate profits remain strongly procyclical as in the data. BGM (2010) point out that it is notoriously difficult to generate both countercyclical markups and procyclical aggregate profits in models with sticky prices, or, more generally, in any model economy with a constant number of producers.

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<sup>13</sup>A similar assumption can be found, *inter alia*, in chapter 3 of Pissarides (2000) and Krause and Lubik (2007). This assumption rules out the hiring externality emphasized by Ebell and Haefke (2009). However, the same authors show that the over-hiring effect on unemployment and wages is quantitatively very small.

<sup>14</sup>Of course,  $\mu_t$  differs according to the mode of competition.

<sup>15</sup>The love for variety dimension is not essential for our results. Normalizing the size of the product space leads to minor qualitative and quantitative changes to our results. The essential dimension is that of the countercyclical mark up.

### 3.6 Aggregation and Market Clearing

Considering that the individual workforce,  $n_t$ , is identical across producers leads to

$$L_t = n_t N_t \quad (26)$$

To obtain aggregate output notice that  $P_t Y_t = \sum_{i=1}^{N_t} p_t y_t = N_t p_t y_t$ , further given  $\rho_t = \frac{p_t}{P_t}$  and the individual production function it follows that

$$Y_t = \rho_t N_t y_t = \rho_t A_t L_t h_t \quad (27)$$

Aggregating the budget constraints of households we obtain the aggregate resource constraint of the economy

$$C_t + \psi N_t^e = W_t h_t L_t + \Pi_t \quad (28)$$

which states that the sum of consumption and investment in new entrants must equal the sum between labor income and aggregate profits,  $\Pi_t$ , distributed to households at time  $t$ . Aggregate profits are defined as

$$\Pi_t = (1 - \delta) N_{t-1} \pi_t + [N_t - (1 - \delta) N_{t-1}] \pi_t^{NP} \quad (29)$$

where  $(1 - \delta) N_{t-1}$  is the number of mature incumbent producers, and  $N_t - (1 - \delta) N_{t-1}^e$  is the number of time- $t$  first period incumbent firms. Goods' market clearing requires

$$Y_t = C_t + N_t^E \psi + \kappa v_t^{tot} \quad (30)$$

Finally, the dynamics of aggregate employment reads as

$$L_t = (1 - \delta) \varrho L_{t-1} + q_t v_t^{tot} \quad (31)$$

which shows that workers employed to a firm which exits the market join the mass of unemployed. Appendix C lists the full set of equilibrium conditions for the economy.

### 3.7 Steady State and Calibration

In order to obtain values for the steady state levels of variables and for the deep structural parameters, we need to impose 14 restrictions. Calibration is conducted on a quarterly basis. The discount factor,  $\beta$ , is set to the standard value of 0.99 for quarterly data, while the rate of business destruction,  $\delta$ , equals 0.025 to match the U.S. empirical level of 10 percent business destruction a year reported by BGM (2007). The baseline value for the entry cost is set to match the U.S. ratio of investment equal

to 15 per cent as in BGM (2007). This allows both models to hit the same steady state markup, equal to 27 percent. Notice that this value is within the range estimated by Oliveira Martins and Scarpetta (1999) for a large number of U.S. manufacturing sectors. The implied values of the entry cost are  $\psi = 1.69$  under Bertrand competition and  $\psi = 0.48$  under Cournot competition.<sup>16</sup> With no loss of generality, the value of  $\chi$  is such that steady state labor supply equals one. In this case the Frisch elasticity of labor supply reduces to  $\varphi$ , to which we assign a value of one as in Monacelli et al. (2010). We take as the baseline value for the intersectoral elasticity of substitution  $\varepsilon = 6$ , as estimated by Christiano, Eichenbaum and Evans (2005) using U.S. quarterly data between 1965 and 1995. Technology is assumed to follow a first order autoregressive process given by  $\hat{A}_t = \rho_A \hat{A}_{t-1} + \varepsilon_{At}$ , where  $\hat{A}_t = \ln(A_t/A)$  and  $\rho_A \in (0, 1)$  and  $\varepsilon_{At}$  is a white noise disturbance, with zero expected value and standard deviation  $\sigma_A$ . As standard in the literature we set the steady state marginal productivity of labor,  $A$ , to 1. As in BGM (2007) and King and Rebelo (2000) we set  $\rho_a = 0.979$  and  $\sigma_a = 0.0072$ .

Next we turn to parameters that are specific to the search and matching framework. We adopt a conventional parameterization. The aggregate separation rate is  $1 - (1 - \delta) \varrho$ . We set  $\varrho$  such that the latter equals 0.1, as suggested by estimates provided by Hall (1995) and Davis et al. (1996). The elasticity of matches to unemployment is  $\gamma = \frac{1}{2}$ , which is within the range of the plausible values of 0.5 to 0.7 reported by Petrongolo and Pissarides (2001) in their survey of the literature on the estimation of the matching function. In the baseline parameterization we impose symmetry in bargaining and set  $\eta = \frac{1}{2}$ , as in the bulk of the literature. We then set the efficiency parameter in matching,  $\gamma_m$ , and the steady state job market tightness to target an average job finding rate,  $z$ , equal to 0.7 and a vacancy filling rate,  $q$ , equal to 0.9. We draw the latter value from Andolfatto (1996) and Dee Haan et al. (2000), while the former from Blanchard and Galì (2010). Notice that a job finding rate equal to 0.7 corresponds, approximately, to a monthly rate of 0.3, consistent with US evidence. Finally, we set the unemployment benefit in real terms,  $b$ , such that the replacement ratio  $\frac{b}{w}$  equals

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<sup>16</sup>We also experimented with an alternative calibration strategy of this parameter. We fixed  $\psi = 1$  and held it constant across market structures. In this case different market structures lead to different values of the steady state great ratios and markups.

Given a common entry cost, the Bertrand equilibrium is characterized by a lower number of goods compared to the Cournot equilibrium.

Not surprisingly a lower entry cost would be, in both frameworks, associated to a higher number of producers and thus to stronger competition and, a lower markup and a higher real wage. However, the impulse response functions below are not qualitatively affected by the value of  $\psi$ .

0.48. This value is consistent with the US replacement ratio reported in the OECD Economic Outlook of 1996 for the US.<sup>17</sup> Given these parameters we can recover the cost of posting a vacancy  $\kappa$  by equating the steady state version of the JCC and the steady state wage setting equation.

The steady state rate of unemployment is equal to

$$u = \frac{1 - (1 - \delta) \varrho}{q\theta + (1 - (1 - \delta) \varrho)} = 0.125$$

which is increasing in the rate,  $\delta$ , of business destruction and in the exogenous, firm-level job separation rate,  $\varrho$ . As expected the unemployment rate is decreasing in the job filling probability  $q$ . While the endogenous steady state rate of unemployment is larger than the average quarterly rate for the U.S., it is in line with the value used by Krause and Lubik (2007) and much lower than those in Andolfatto (1996) and Trigari (2009).<sup>18</sup> Notice that the steady state ratio between jobs created by first period incumbent firms ( $JC^{FP}$ ) and total job creation ( $JC$ ) is given by

$$\frac{JC^{FP}}{JC} = \frac{(1 - \delta) N^e v^{NP} q}{v^{tot} q} = \frac{\delta (1 - u)}{\theta q u} = 0.25$$

which implies that job creation by new producers account for about 25 per cent of total (gross) job creation, close to the quarterly U.S. average of 20 per cent reported by Jaimovich and Floetotto (2008). Also notice that the ratio between workers employed by first period incumbent firms ( $L^{FP}$ ) and total employment ( $L$ ) is

$$\frac{L^{FP}}{L} = \frac{(1 - \delta) N^e \frac{L}{N}}{L} = \delta = 0.025$$

New producers account for about 2.5 percent of total employment, slightly lower than the 3 percent reported by Haltiwanger et al. (2010) as the average value for the U.S. between 1976 and 2005. Notice that the shares considered are independent of both the entry cost and the competitive framework.

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<sup>17</sup>See Chapter 3 table 3.2. The OECD computes two distinct replacement ratio concepts. The first one is the Replacement Ratio at the beginning of the unemployment spell (54% for the U.S.). The second one is the average replacement ratio over a 60 months unemployment spell (36% for the U.S.).

<sup>18</sup>The computation of the steady state is in Appendix D.

## 4 *Business Cycle Analysis*

In what follows we will first study the impulse response functions to a technology shock, and finally we will evaluate the second order moments. To assess the role of endogenous market structures, we compare the performance of the Bertrand and Cournot model to that of a standard search model à la Shimer (2005), augmented with monopolistic competition in the goods market. Thus, we consider a search model featuring a Cobb-Douglas production function of the form  $y_t = A_t k_{t-1}^a (L_t h_t)^{1-\alpha}$  and a dynamics of physical capital given by  $k_t = (1 - \delta) k_{t-1} + I_t^k$ , where  $I_t^k$  represents investment in physical capital. As in the bulk of the literature we set  $\delta = 0.025$  and  $\alpha = \frac{1}{3}$ .

The calibration strategy of remaining parameters is identical across the models. Importantly, monopolistic competition implies that the price mark up is exogenous. Firms do not interact strategically but set a constant mark up over marginal costs equal to  $\mu = \frac{\varepsilon}{\varepsilon-1}$ .<sup>19</sup>

### 4.1 *IRFs to a technology Shock*

In this section we show the qualitative reactions of the economy to a persistent technology shock. Figure 1 and Figure 2 depict percentage deviations from the steady state of key variables in response to a one standard deviation technology shock. Time on the horizontal axis is in quarters. Solid lines refer to the case with competition in prices, dashed lines to that with competition in quantities, dotted lines refer to the benchmark search model.

Consider our baseline economy with endogenous entry. As shown in Figure 1 and Figure 2, the model dynamics shows very similar pattern under both Bertrand and Cournot competition. The shock increases consumption and creates expectations of future profits. This, in turn, leads to entry of new firms.

Recall that entry is subject to a one period time-to-build lag, which implies that the number of producing firms,  $N_t$ , does not change on impact. This translates into an initially muted response of both the love for variety and the mark up. In particular, the price mark up finds its negative peak after few periods and then gradually reverts to its long run value.<sup>20</sup> As pointed out in the discussion of equation (25), an increase in

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<sup>19</sup>We say that the markup is exogenous in the case of monopolistic competition because its value is fully determined once the elasticity of substitution between goods is fixed. In other words the two magnitudes cannot be set independently. Given that our baseline calibration features  $\varepsilon = 6$ , the markup characterizing the benchmark search model is fixed at 20 percent.

<sup>20</sup>This correlation pattern is consistent with the analysis of prices and costs in Rotemberg and Woodford (2000) and with the VAR evidence for the U.S. in Colciago

the ratio  $\frac{\rho_t}{\mu_t}$  raise the current and future value of the marginal product of labor, boosting vacancy creation and hiring.

As a result, under endogenous market structures the response of unemployment, the job finding rate and the vacancy filling rate are amplified with respect to those obtained in the benchmark search model. The peak effect on vacancy creation, however, is on impact because hiring is a forward looking phenomenon.

Hall (2005) argues that the job finding rate is the key variable in understanding the large fluctuation in unemployment over the past 50 years. The strongly procyclical response of the job finding rate delivered by the Bertrand and Cournot models is at the basis of the large swing in unemployment. The endogeneity of the market structures implies a response of the unemployment rate in the period after the shock which is almost six times larger, and more persistent, than that observed in the benchmark search model. Further, fluctuation in output, labor market tightness and aggregate hours remain larger in our baseline model than in the benchmark search model.

While the shock vanishes and entry strengthens competition, output and profits of the firms drop and the incentives to enter disappear. At some point net exit from the market occurs and the rate of unemployment, the mark up and thus the incentive to create vacancy gradually return to the steady state. Importantly, our framework delivers procyclical aggregate profits, which is a notorious difficulty for models featuring countercyclical mark ups. In the next section we argue that the sluggish adjustment in the number of firms, which is at the basis of the dynamic correlation path between aggregate output and the price mark up, is key to address the response of the labor share to a technology shock.

## 4.2 Productivity shocks and the Labor share

In this section we show that our model with endogenous entry in the goods market can address the co-movement of labor share with technology shocks at business cycle frequency. Rios-Rull and Santaeulàlia-Llopis (2010) estimate the response of the labor share, i.e. the ratio of the labor compensation to output, to a technology shocks in the US economy, and find it characterized by two main features: *countercyclicity* and *overshooting*.

Explaining both these facts is a notorious difficulty for conventional business cycle models. Consider the definition of labor share  $\frac{wLh}{Y} = w\frac{H}{Y}$ . In log-deviations

$$\widehat{l}_{s_t} = \widehat{w}_t - \left( \widehat{y}_t - \widehat{H}_t \right), \quad (32)$$

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and Etro (2010a).

equation (32) simply states that the log-deviation of the labor share is the difference between the log-deviation of the real wage and that of labor productivity. To understand the dynamics of the labor share, we need to understand the dynamics of these two variables. In a standard RBC model the real wage is equal to  $(\hat{y}_t - \hat{H}_t)$ , that is, real wages and labor productivity move identically. Hence, whatever the amount of the shock, the deviations of the labor share  $\hat{l}s_t$  from its steady state is always zero. Thus, as a general rule, in order to obtain a non constant labor share the allocative role of the real wage has to be broken.<sup>21</sup>

In the standard search and matching model the allocative role of the real wage is broken through Nash bargaining. In response to a productivity shock real wages jump on impact. Bargaining over wages implies that only a fraction  $\eta$  of the increase in productivity goes to workers. As a result  $\hat{w}_t - \hat{A}_t < 0$  and the labor share is countercyclical on impact. However, as shown in Figure 3, after peaking on impact the real wage returns monotonically to its initial level and the labor share goes back to long run level without overshooting in the benchmark search model.

In our model, while the labor share is countercyclical for the same reasons mentioned above, the labor share overshoots its long run value. This is so under both Bertrand and Cournot competition. Figure 4 shows that the labor share overshoots its long run level after about five quarters, it peaks at about the fifth year at a level much larger than its long-run level and seven years after the shock has hit the economy is still halfway toward its average. This pattern resembles very closely that in the data (see Figure 1 of Rios - Rull and Santaaulàlia-Llopis 2010).

The overshooting is related to the dynamic correlation between output and the markup described above. When productivity increases the markup is muted on impact and gradually decreases as firms enter into the market. This pattern is mirrored in the response of the real wage which peaks after some periods, when the mark up reaches its minimum, and then reverts to the steady state with a hump shape dynamics. On the contrary, output jumps on impact and then monotonically reverts to the steady state. As a result of the hump shaped response of the real wage, labor income peaks while output is decreasing, leading to the overshooting result. The dynamic response of the markup to the technology

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<sup>21</sup>For example, Gomme and Greenwood (1995) break the relationship between the real wage and productivity augmenting the RBC model with long term labor contracts, which insure workers against income fluctuations. In this case workers are paid more than the marginal product of labor in bad times, and viceversa, leading to a countercyclical labor share. Nevertheless, the authors do not tackle the issue of the overshooting of the labor share.



shock is thus key for the overshooting.

### 4.3 *Second Moments*

To further assess the implications of endogenous market structures for the business cycle, we compute second moments of the key macroeconomic variables. In this exercise we follow the RBC literature and assume that the only source of random fluctuations are temporary exogenous technology shocks. We calibrate the productivity process as in King and Rebelo (2000), with persistence  $\rho_A = 0.979$  and standard deviation  $\sigma_A = 0.0072$ . We use the same process as in King and Rebelo (2000) for comparison purposes with the bulk of the literature and to verify the additional impact of our propagation channel for a given shock.

Table 1 reports statistics on US data for the period 1964:1-2005:1. We take from Gertler and Trigari (2009) moments relative to output for consumption  $C$ , investment  $I$ , the real wage,  $w$ , aggregate hours,  $L$ , the unemployment rate,  $u$ , the labor market tightness,  $\theta$  and vacancies  $v$ . Moments for aggregate profits  $\Pi$ , the price mark up  $\mu$ , and job creation by new entrants,  $m^e$ , are based on our own calculations.<sup>22</sup> In the same Table we report the moments produced by the benchmark search model and by the two models with competition in quantities and with competition in prices under the baseline parameterization.<sup>23</sup>

The two models characterized by endogenous market structure deliver a very similar performance at replicating the U.S. business cycle. The endogeneity of market structures implies a higher volatility of aggregate hours with respect to the benchmark models. This is due to both a higher volatility of the intensive and extensive margin of labor.<sup>24</sup> While unemployment is as volatile as output in the benchmark search model, it is 4.32 times as volatile as output in our framework. Vacancies perform even better. They are 8.07 times as volatile as output in both models. Further, both frameworks deliver an extremely volatile

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<sup>22</sup>We consider a labor share-based mark up series as in Rootemberg and Woodford (1992). See Colciago and Etno (2010) for the details concerning the construction of this measure. The source for the data on job creation by new entrants is the Business Employment Dynamics dataset constructed by the Bureau of Labor Statistics. Data are relative to job creation by new establishments. Notice that for this variable data are available from 1992:3. In order to have a longer series, we consider, for this specific variable, data up to 2010:3. For consistency, the relative standard deviation and the contemporaneous correlation with output are computed using real GDP spanning over the same time period.

<sup>23</sup>For the computation of the moments concerning the models with endogenous market structures we consider data-consistent variables. That is, variables have been deflated for the love for variety effect. See BGM (2007) for details.

<sup>24</sup>The standard deviation of individual hours is 0.22 under both Bertrand and Cournot competition, while it is 0.03 under the standard search model.



	$C$	$I$	$u$	$L$	$w$	$ls$	$\theta$	$v$	$m^e$	$\Pi$	$\mu$
US. data											
Rel.St.Dev.	0.41	2.71	5.15	0.6	0.52	0.51	11.28	6.30	4.34	5.12	0.61
Corr.with $Y$	0.81	0.94	-0.86	0.78	0.56	-0.20	0.90	0.91	0.26	0.64	-0.26
Baseline Search											
Rel.St.Dev.	0.86	1.51	1.28	0.16	0.81	0.02	3.11	2.40	0.00	2.46	0.00
Corr.with $Y$	0.99	0.99	-0.72	0.99	0.99	-0.98	0.99	0.91	0.00	0.99	0.00
Bertrand Competition											
Rel.St.Dev.	0.58	3.37	4.32	0.52	0.47	0.03	10.49	8.07	3.83	0.99	0.02
Corr.with $Y$	0.98	0.98	-0.73	0.99	0.99	-0.50	0.99	0.90	0.83	0.99	-0.08
Cournot Competition											
Rel.St.Dev.	0.56	3.48	4.32	0.53	0.46	0.02	10.48	8.07	3.94	0.99	0.02
Corr.with $Y$	0.98	0.98	-0.73	0.99	0.99	-0.60	0.99	0.90	0.83	0.99	-0.07

Table 1: Aggregate Statistics

labor market tightness. Indeed, labor market tightness is almost 10.5 times higher than output. Job creation by new firms displays a relative volatility in line with that in the data, although the contemporaneous correlation with output is too high. The volatility of investment is about three times higher than output in our framework, essentially matching that in the data. Given the monopolistic competitive nature of the market structure, the benchmark search models cannot deliver information on the volatility of the mark up. On the contrary our models do a relatively good job at matching the negative contemporaneous correlation between output and the mark up. The relative volatilities of profits and the mark up are underestimated, however. Finally notice that the real wages are less volatile than in the benchmark search model and closer to the data. The contemporaneous correlation with output are also qualitatively in line with the data. In particular, the correlation of the labor share with output is closer to the data in the models with endogenous entry than in the benchmark search model.

Considering that we adopt a very standard and conservative model calibration we see the performance of both the Bertrand and the Cournot frameworks as a success for three main reasons. First, the model can reproduce the procyclicality of entry and the countercyclicality of the mark up observed in the data. Second, it matches the nonlinear time profile of the correlation between the mark up and the cycle documented by Rotemberg and Woodford (1992) and emphasized by BGM (2007). Finally, without resorting to nominal rigidities in wages or prices, it substantially outperforms a standard model with search in the labor market in terms of variability of labor market variables. For these reasons we claim that endogenous market structures are a relevant amplification channel of technology shocks in an otherwise standard model of search in the labor market.

## 5 *Conclusions*

We provided a DSGE model where firms' dynamics and matching frictions in the labor market interact endogenously. We accounted for strategic interactions in both prices and quantities among producers. As in the data, while new firms account for a relatively small share of overall employment, they create a relevant fraction of new jobs. The model explains the procyclicality of profits together with the countercyclicality of price mark ups and reproduces, at least qualitatively, the behavior of the labor share in response to a labor productivity shock.

The interplay between search and matching frictions, endogenous entry and strategic interactions among producers constitutes a strong amplification channel of technology shocks on labor market variables. Our framework outperforms the standard search models in terms of business cycle statistics of the labor market variables.

Our analysis could be extended in various dimensions. One aspect we neglect is the asymmetry between market competitors in terms of both size and the probability of exit from the market. Davis et al. (2009) document that the distribution of vacancy creation is strongly biased in favor of small firms; Haltiwanger et al. (2009) show that younger firms are more likely to exit from the market than more mature firms. Another important aspect that we do not discuss is, as documented by Davis et al. (2009), that a large fraction of new hires happens without prior vacancy creation.

In ongoing research we extend our framework to a government sector and analyze the transmission of government spending shocks to the labor market. We believe that the strong propagation embodied in the model with endogenous market structures could help resolving the unemployment fiscal multiplier puzzle emphasized in some recent contributions

without departing from a flexible prices approach.

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## Figures

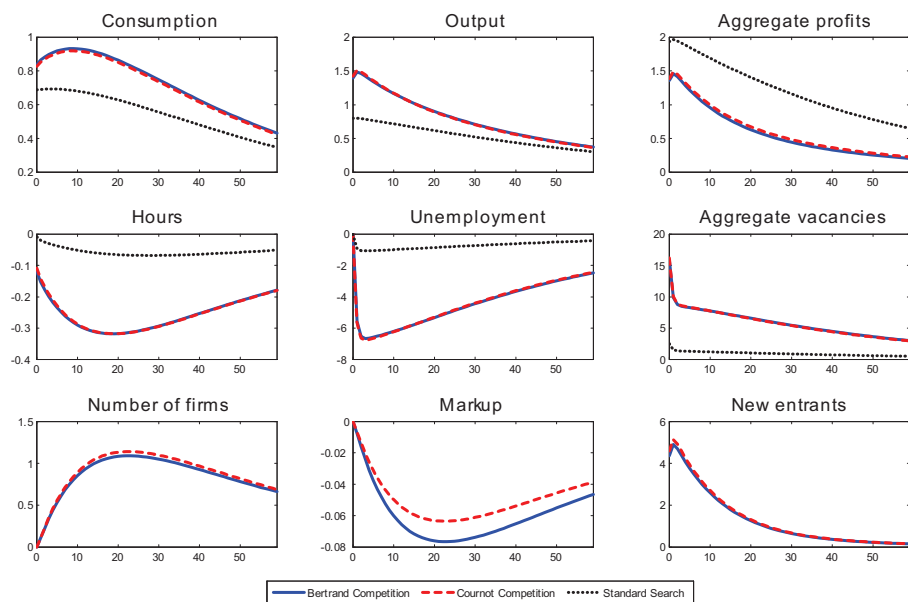


Figure 1: Impulse response functions to a temporary technology shock.

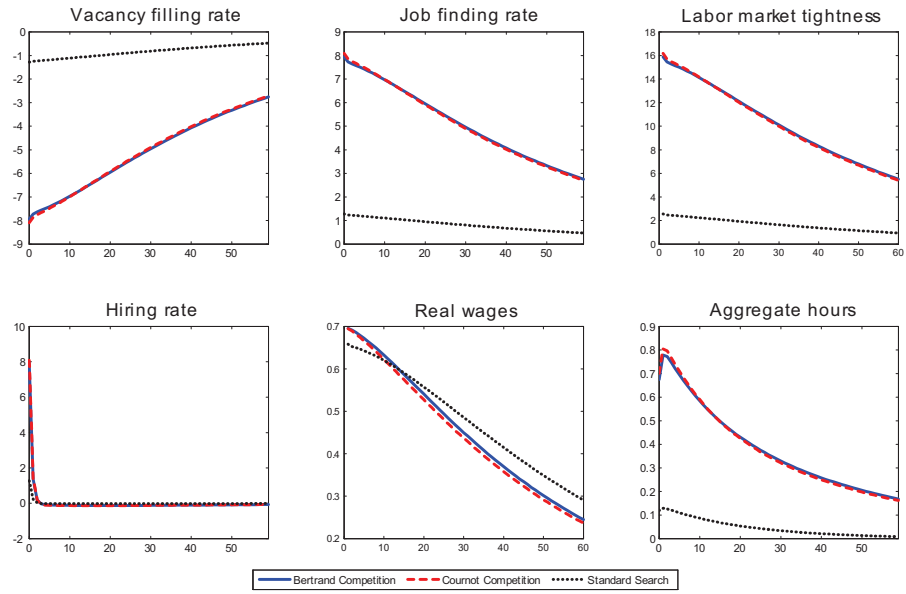


Figure 2: Impulse response functions to a temporary technology shock.

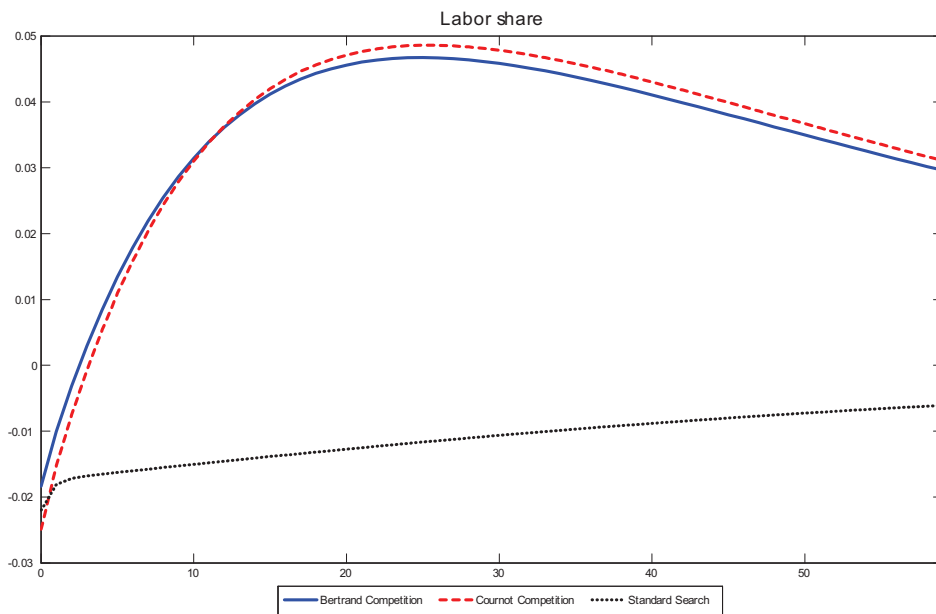


Figure 3. Labor share dynamics in response to a positive technology shock.



## Appendix

### A. Analytical Details

The representative agent maximizes intertemporal utility (2) choosing how much to invest in bonds and risky stocks out of labor and capital income. We assume that household invest in both incumbent firms and new entrants. Bonds and stocks are denominated in terms of an aggregate price index  $P_t^A$ . The budget constraint expressed in nominal terms is

$$\begin{aligned}
& P_t^A B_{t+1} + \int_0^1 P_{kt} C_{kt} dk + \\
& + P_t^A \int_0^1 V_{kt} N_{kt} s_{kt+1} dk + P_t^A \int_0^1 V_{kt}^e N_{kt}^e s_{kt+1}^e dk \\
= & W_t L_t h_t + (1 - L_t) P_t^A b + (1 + r_t) P_t^A B_t + \\
& + (1 - \delta) P_t^A \int_0^1 [\pi_{kt}(\varepsilon, N_{kt}) + V_{kt}] N_{kt-1} s_{kt} dk + \\
& + (1 - \delta) P_t^A \int_0^1 [\pi_{kt}^{FP}(\varepsilon, N_{kt}) + V_{kt}] N_{kt-1}^e s_{kt}^e dk - P_t^A T_t
\end{aligned}$$

where  $B_t$  is net bond holdings with interest rate  $r_t$ ,  $V_{kt}$  is the value of an incumbent firm in sector  $k$  and  $V_{kt}^e$  is the value of a new entrant in sector  $k$ . The variables  $N_{kt}$  and  $N_{kt}^e$  represent the number of active firms in sector  $k$  and the new firms in this sector at the end of the period. The variable  $s_{kt}$  is the share of the stock market value of the incumbent firms of sector  $k$  that are owned by the agent while  $s_{kt}^e$  is the share of portfolio of new entrants held by the household. The term  $(1 - \delta) P_t^A \int_0^1 [\pi_{kt}(\varepsilon, N_{kt}) + V_{kt}] N_{kt-1} s_{kt}$  represents the sum between the value of the portfolio of mature incumbent firms held by the household and the profits distributed by these firms. Notice that in period  $t$  there are  $(1 - \delta) N_{kt-1}$  mature incumbent firms in each sector. The term  $(1 - \delta) P_t^A \int_0^1 [\pi_{kt}^{FP}(\varepsilon, N_{kt}) + V_{kt}] N_{kt-1}^e s_{kt}^e$  denotes the sum between the value of the portfolio of first period incumbent firms held by the household and the profits distributed by these firms, where  $(1 - \delta) N_{kt-1}^e$  is the number of first period producers at time  $t$ . Recall that the superscript FP indicates variables relative to first period incumbent firms. In the budget constraint we have imposed the condition that  $V_{kt}^{FP} = V_{kt}$  i.e. symmetry between incumbents. Finally  $P_t^A T_t$  represent nominal lump sum taxes imposed to finance unemployment benefits. Equations ?? and 4 represents the constraint to the utility maximization problem. We denote with  $\xi_t$  the Lagrangian multiplier of the first constrain, while  $\Gamma_t$  is the one of the second constraint.

The intertemporal optimality conditions with respect to  $s_{kt+1}$ ,  $s_{kt+1}^e$  for each sector, and with respect to  $B_{t+1}$  are:

$$P_t^A V_{kt} = \beta E_t (1 - \delta) \frac{\xi_{t+1}}{\xi_t} P_{t+1}^A [\pi_{kt+1}(\varepsilon, N_{kt+1}) + V_{kt+1}]$$

$$P_t^A V_{kt}^e = \beta E_t (1 - \delta) \frac{\xi_{t+1}}{\xi_t} P_{t+1}^A [\pi_{kt+1}^{FP}(\varepsilon, N_{kt+1}) + V_{kt+1}]$$

$$P_t^A \xi_t = \beta E_t (1 + r_{t+1}) P_{t+1}^A \xi_{t+1}$$

The optimal choice of consumption of the bundle of good produced in sector  $k$ ,  $C_{kt}$ , is instead

$$P_{kt} C_{kt} = P_t^A C_t = EXP_t \text{ for any } k \in [0, 1]$$

the latter implies that nominal expenditure is identical in each sector and, given sectors are atomistic with aggregate unit mass, that sector nominal expenditure equals aggregate nominal expenditure, defined as  $EXP_t$ . Also, it follows that  $\xi_t = \frac{1}{P_t^A C_t}$ . Notice that  $\Gamma_t$  has the meaning of the marginal value to the household of having a member employed rather than unemployed. The latter affects bargaining over the real wage and individual hours and it is given by

$$\Gamma_t = \frac{1}{C_t} (w_t h_t - b) - \chi \frac{h_t^{1+1/\varphi}}{1 + 1/\varphi} + \beta E_t [(1 - \delta) \rho - z_{t+1}] \Gamma_{t+1}$$

where  $w_t = \frac{W_t}{P_t^A}$  is the real wage. Following Ghironi and Melitz (2005), we adopt a probability  $\delta \in [0, 1]$  with which any firm can exit from the market for exogenous reasons in each period. The dynamic equation determining the number of firms in each sector is then:

$$N_{kt+1} = (1 - \delta) (N_{kt} + N_{kt}^e) \quad \forall k$$

which provides the dynamic path for the average number of firms:

$$N_{t+1} = (1 - \delta) \int_0^1 (N_{kt} + N_{kt}^e) dk = (1 - \delta) (N_t + N_t^e)$$

where, of course, we have  $N_t \equiv \int_0^1 N_{kt} dk$  and  $N_t^e \equiv \int_0^1 N_{kt}^e dk$ .

Market clearing in the asset markets requires  $B_t = 0$  for any  $t$  in the bond market, and  $s_{kt} = s_{kt}^e = 1$  for any sector  $k$  in the stock market. In a symmetric equilibrium, the number of firms, the mark up and individual profits are the same in every sector, which leads to the following equilibrium relations:

$$\begin{aligned} P_{kt} &= P_t^A \quad C_{kt} = C_t \quad \forall k \\ V_t &= E_t \Lambda_{t,t+1} [\pi_{t+1}(\varepsilon, N_{t+1}) + V_{t+1}] \end{aligned} \quad (33)$$

$$\begin{aligned}
V_t^e &= E_t \Lambda_{t,t+1} [\pi_{t+1}^{FP}(\varepsilon, N_{t+1}) + V_{t+1}] \\
C_t^{-1} &= \beta(1 + r_{t+1})E(C_{t+1}^{-1})
\end{aligned} \tag{34}$$

The variable  $\Lambda_{t,t+1} = \beta(1 - \delta) \frac{C_t}{C_{t+1}}$  represents the household's stochastic discount factor, which takes into account that a firm exits from the market with probability  $\delta$ .

## B. Proofs of Propositions

### B1. Proposition 1

**Proof.** Notice that  $C_{k,t}(i) = \left(\frac{p_{k,t}(i)}{P_{k,t}}\right)^{-\varepsilon} C_{k,t} = \frac{p_{k,t}^{-\varepsilon}(i)}{(P_{k,t})^{1-\varepsilon}} P_{k,t} C_{k,t} = \frac{p_{k,t}^{-\varepsilon}(i)}{(P_{k,t})^{1-\varepsilon}} EXP_{k,t} = \frac{p_{k,t}^{-\varepsilon}(i)}{(P_{k,t})^{1-\varepsilon}} EXP_t$ . Since

$$P_{k,t} = \left[ \sum_{j=1}^{N_t} p_{k,t}(j)^{-(\varepsilon-1)} \right]^{\frac{-1}{\varepsilon-1}} \tag{35}$$

we can write the demand faced by firm  $i$  as

$$C_{k,t}(i) = \frac{p_{k,t}(i)^{-\varepsilon} EXP_t}{\left[ \sum_{j=1}^{N_{k,t}} p_{k,t}(j)^{-(\varepsilon-1)} \right]}$$

Each sector can be similarly described, so we drop the index referring to sectors and consider a representative sector. Substituting the direct demand for the individual good into period  $t$  real profits, we obtain

$$\pi_t = \frac{p_t(i)^{1-\varepsilon}}{\left[ \sum_{j=1}^{N_t} p_t(j)^{-(\varepsilon-1)} \right]} \frac{EXP_t}{P_t^A} - w_t(i) n_t(i) h_t(i) - kv_t(i)$$

The profit maximization problem of a mature producer reads as

$$\max_{\{p_t(i), n_t(i), v_t(i)\}_t} \pi_t + E_t \sum_{s=t+1}^{\infty} \Lambda_{t,s} \pi_s$$

subject to

$$A_t n_t(i) h_t(i) = \frac{p_t(i)^{-\varepsilon} EXP_t}{\left[ \sum_{j=1}^{N_t} p_t(j)^{-(\varepsilon-1)} \right]} \tag{36}$$

$$n_t(i) = \rho n_{t-1}(i) + v_t(i) q_t \tag{37}$$

Lagrangian multipliers on constraints (36), and (37) are respectively  $mc_t(i)$  and  $\phi_t(i)$ . Setting up the Lagrangian  $\mathbb{L}$ , the FOCs for profit maximization are

$$\frac{\partial \mathbb{L}}{\partial n_t(i)} = 0 : w_t(i) h_t(i) + \phi_t(i) - mc_t(i) A_t h_t(i) = \varrho E_t \Lambda_{t,t+1} \phi_{t+1}(i)$$

$$\frac{\partial \mathbb{L}}{\partial v_t(i)} = 0 : k = \phi_t(i) q_t$$

$$\begin{aligned} \frac{\partial \mathbb{L}}{\partial p_t(i)} = 0 : & \frac{(1 - \varepsilon) \left[ \sum_{j=1}^{N_t} p_t(j)^{-(\varepsilon-1)} \right] - (1 - \varepsilon) p_t(i)^{1-\varepsilon}}{\left[ \sum_{j=1}^{N_t} p_t(j)^{1-\varepsilon} \right]^2} p_t(i)^{-\varepsilon} \frac{EXP_t}{P_t^A} \\ & mc_t(i) \frac{\varepsilon p_t(i)^{-1} \left[ \sum_{j=1}^{N_t} p_t(j)^{-(\varepsilon-1)} \right] + (1 - \varepsilon) p_t(i)^{-\varepsilon}}{\left[ \sum_{j=1}^{N_t} p_t(j)^{1-\varepsilon} \right]^2} p_t(i)^{-\varepsilon} EXP_t \\ & = 0 \end{aligned} \quad (38)$$

Notice we assumed that firms take individual wages as given when choosing employment. Also notice that since there is a continuum of sectors, the individual firm takes the aggregate price level as given. The second condition shows that  $\phi_t(i)$ , the surplus created by a match, is identical across mature incumbent firms. Before providing an explicit formula for the individual price level and the price mark up, we turn to the profit maximization problem of a first period incumbent producer which sets the price for the first time. The relevant difference with respect to the previous case is represented by the form of constraint (37) which reads as  $v_t(i) q_t = n_t(i)$ , since producers in their first period of activity have no stock initial workforce. However, FOCs with respect to  $p_t(i)$ ,  $n_t(i)$  and  $v_t(i)$  are identical to those reported above. Since the surplus  $\phi_t$  created by a match is identical across incumbent firms, they will face the same wage bargaining problem, thus will face the same wage,  $w_t(i) = w_t$ , the same marginal cost,  $mc_t(i) = mc_t$ , and will demand the same amount of hours,  $h_t(i) = h_t$ . As a result the third condition can be written as

$$(1 - \varepsilon) P_t^{1-\varepsilon} - (1 - \varepsilon) p_t(i)^{1-\varepsilon} = MC_t \left[ \varepsilon p_t(i)^{-1} P_t^{1-\varepsilon} + (1 - \varepsilon) p_t(i)^{-\varepsilon} \right]$$

where  $MC_t (= P_t^A mc_t)$  is the nominal marginal cost, which shows that  $p_t(i)$  does not depend on any firm specific variable. In other words all incumbent

firms, no matter the period of entry, choose the same price. Since firms face the same demand function and adopt the same technology, it follows that  $y_t(i) = y_t$  and  $n_t(i) = n_t$ . We are now ready to provide an expression for the common price chosen by firms. Given firms choose the same price

level, it follows that  $P_t^A = P_t = \left[ \sum_{j=1}^{N_t} p_t(j)^{-(\varepsilon-1)} \right]^{\frac{-1}{\varepsilon-1}} = N_t^{\frac{1}{1-\varepsilon}} p_t$ . Imposing symmetry and rearranging, condition 3 can be rewritten as

$$p_t = \mu_t^P MC_t$$

where

$$\mu_t^P = \frac{\varepsilon (N_t - 1) + 1}{(\varepsilon - 1) (N_t - 1)}$$

■

## B2. Proposition 2

**Proof.** The main difference with the proof of proposition 1 is that profit maximization must take the inverse demand function as a constraint. The latter is

$$p_t(i) = \frac{y_t(i)^{-\frac{1}{\varepsilon}} EXP_t}{\sum_{j=1}^{N_t} y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}}$$

which implies that period profits can be written as

$$\pi_t = \frac{y_t(i)^{1-\frac{1}{\varepsilon}}}{\sum_{j=1}^{N_t} y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}} \frac{EXP_t}{P_t^A} - w_t(i) n_t(i) h_t(i) - kv_t(i)$$

and constraint 36 is replaced by  $A_t n_t(i) h_t(i) = y_t(i)$ . We proceed as above and initially consider the problem of a mature incumbent. Setting up a Lagrangian function as in the proof of Proposition 1 and differencing with respect to  $y_t(i), n_t(i), v_t(i)$ , it can be easily verified that the FOCs with respect to  $n_t(i), v_t(i)$  are unchanged with respect to the Bertrand case. Turning to the problem of a first period incumbent firm, it can be verified that the consideration made under Bertrand competition extend to this case. Incumbent firms, independently of the period of entry, face the same marginal cost and assign the same value to the marginal worker. In particular, notice that the

FOC with respect to  $y_t(i)$  reads as

$$\frac{\frac{\varepsilon-1}{\varepsilon}y_t(i)^{-\frac{1}{\varepsilon}}\sum_{j=1}^{N_t}y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} - \frac{\varepsilon-1}{\varepsilon}y_t(i)^{\frac{\varepsilon-2}{\varepsilon}}}{\left[\sum_{j=1}^{N_t}y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}\right]^2}\frac{EXP_t}{P_t^A} = mC_t$$

which shows that individual production is not firms specific. Imposing symmetry and rearranging leads to the individual output

$$y_t = \frac{\varepsilon - 1}{\varepsilon} \frac{N_t - 1}{N_t^2} \frac{EXP_t}{MC_t}$$

Substituting the latter into the inverse demand function, after imposing symmetry, we get

$$p_t = \frac{EXP_t}{N_t} y_t^{-1} = \frac{EXP_t}{N_t} \frac{\varepsilon N_t^2 MC_t}{(\varepsilon - 1)(N_t - 1) EXP_t} = \mu_t^Q MC_t$$

where

$$\mu_t^Q = \frac{\varepsilon}{(\varepsilon - 1)} \frac{N_t}{(N_t - 1)}$$

■

### B3. Proposition 3

**Proof.** Since all incumbent firms are, under both forms of competition, characterized by the same size, first period incumbent firms, which have no initial workforce, must post at time  $t$  as many vacancies as required to reach the size of a mature incumbent producer. Given the time- $t$  workforce of a first period incumbent is  $v_t^{FP} q_t = n_t$ , i) follows. To prove ii) notice that

$$\pi_t^{FP} = \frac{p_t}{P_t} y_t - w_t h_t n_t - k v_t^{FP} = \frac{p_t}{P_t} y_t - w_t h_t n_t - k \frac{n_t}{q_t}$$

Since it also holds that  $n_t = \rho n_{t-1} + v_t q_t$  the latter can be written as

$$\begin{aligned} \pi_t^{FP} &= \frac{p_t}{P_t} y_t - w_t h_t n_t - k \frac{\rho n_{t-1} + v_t q_t}{q_t} \\ &= \underbrace{\frac{p_t}{P_t} y_t - w_t h_t n_t - k v_t}_{\pi_t} - k \rho \frac{n_{t-1}}{q_t} \\ &= \pi_t - k \frac{\rho n_{t-1}}{q_t} \end{aligned}$$

■

#### B4. Proposition 4

**Proof.** The value of a new entrant reads as

$$V_t^e = E_t \Lambda_{t,t+1} \pi_{t+1}^{NP} + E_t \sum_{s=t+2}^{\infty} \Lambda_{t,s} \pi_s = E_t \Lambda_{t,t+1} (\pi_{t+1}^{NP} + V_{t+1}) \quad (39)$$

Proposition 3 implies that

$$\pi_{t+1}^{NP} = \pi_{t+1} - k \frac{\varrho n_t}{q_{t+1}}$$

Using the latter into (39) it follows

$$V_t^e = E_t \Lambda_{t,t+1} \left( \pi_{t+1} - k \frac{\varrho n_t}{q_{t+1}} \right) + E_t \Lambda_{t,t+1} V_{t+1} \quad (40)$$

Notice that the value of an incumbent firm must satisfy the recursive equation

$$V_t = E_t \Lambda_{t,t+1} (\pi_{t+1} + V_{t+1})$$

Substituting the latter into (40) we obtain equation (19). A similar result can be obtained combining equations (34) and (33) and using the result in Proposition 3. ■

#### C. Equilibrium Conditions

In what follows we list the equilibrium conditions of the model. The definition of aggregate employment is

$$L_t = N_t n_t \quad (41)$$

Since  $P_t Y_t = N_t p_t y_t$  and  $\rho_t = \frac{p_t}{P_t}$  it follows that aggregate output reads as

$$Y_t = \rho_t N_t y_t = \rho_t A_t L_t h_t \quad (42)$$

In equilibrium  $B_t = B_{t-1} = 0$  and  $s_t = s_{t+1} = s_{t+1}^e = 1$ . Further since the Government runs a balanced budget it follows that  $G_t = b(1 - L_t) = T_t$  and the aggregate resource constraint reads as

$$C_t + V_t^e N_t^e = W_t L_t h_t + (1 - \delta) N_{t-1} \pi_t + (1 - \delta) N_{t-1}^e \pi_t^{NP} \quad (43)$$

Good's market clearing requires

$$Y_t = C_t + N_t^E \psi + k v_t^{tot} \quad (44)$$

where

$$v_t^{tot} = (1 - \delta) N_{t-1} v_t + (1 - \delta) N_{t-1}^e v_t^{NP} \quad (45)$$

and

$$v_t^{NP} = \frac{n_t(i)}{q_t} \quad (46)$$

The motion of the number of firms reads as

$$N_t = (1 - \delta) (N_{t-1} + N_{t-1}^E) \quad (47)$$

while the dynamic of aggregate employment

$$L_t = (1 - \delta) \rho L_{t-1} + q_t v_t^{tot} \quad (48)$$

The JCC

$$\frac{k}{q_t} = (mc_t A_t - w_t) h_t + \rho E_t \Lambda_{t,t+1} \frac{k}{q_{t+1}} \quad (49)$$

where

$$q_t = \frac{m_t}{v_t^{tot}} \quad (50)$$

The definition of the household's stochastic discount factor is

$$\Lambda_{t,t+1} = (1 - \delta) \beta \left( \frac{C_{t+1}}{C_t} \right)^{-1} \quad (51)$$

The wage schedule reads

$$w_t = (1 - \eta) b + \eta mc_t A_t + (1 - \eta) \chi \frac{C_t h_t^{1/\varphi}}{1 + 1/\varphi} + \eta k E_t \frac{C_t/h_t}{C_{t+1}} \theta_{t+1} \quad (52)$$

where job market tightness is defined as

$$\theta_t = \frac{v_t^{tot}}{u_t} \quad (53)$$

Hours worked satisfy

$$h_t = \left( \frac{1}{\chi} \frac{\rho_t A_t}{\mu_t C_t} \right)^\varphi \quad (54)$$

The mark up function depends of the form of competition; for Bertrand competition we have

$$\mu_t^P = \frac{\varepsilon (N_t - 1) + 1}{(\varepsilon - 1) (N_t - 1)} \quad (55)$$

while under Cournot Competition

$$\mu_t^Q = \frac{\varepsilon N_t}{(\varepsilon - 1) (N_t - 1)} \quad (56)$$



Next we have to consider three Euler equations; the one for bonds

$$\frac{1}{C_t} = \beta(1 + r_t)E_t \left( \frac{1}{C_{t+1}} \right) \quad (57)$$

that for shares of incumbent firms

$$V_t = E_t \Lambda_{t,t+1} (\pi_{t+1} + V_{t+1}) \quad (58)$$

and finally the Euler equation for shares in new entrants

$$V_t^e = E_t \Lambda_{t,t+1} \left( \pi_{t+1} - k \frac{\varrho n_{t+1}}{q_{t+1}} + V_{t+1} \right) \quad (59)$$

Next we consider the pricing equation

$$mc_t = \frac{\rho_t}{\mu_t} = \frac{N^{\frac{1}{\varepsilon-1}}}{\mu_t} \quad (60)$$

and the definition of profits of incumbent firms which have been in the market for more than a period

$$\pi_t = \frac{p_t}{P_t} y_t(i) - w_t n_t L_t - k v_t(i) \quad (61)$$

The total number of matches is

$$m_t = \gamma_m (u_t)^\gamma (v_t^{tot})^{1-\gamma}, \quad (62)$$

where the definition of the unemployment rate is

$$u_t = 1 - L_{t-1} \quad (63)$$

Finally we have to take into account the entry condition

$$V_t^e = \psi \quad (64)$$

and the definition of the job finding rate

$$z_t = \frac{m_t}{u_t} \quad (65)$$

The equilibrium contains 24 equations for 25 variables: 24 endogenous variables  $Y_t, L_t, n_t, h_t, C_t, m_t, q_t, z_t, \theta_t, N_t^e, N_t, v_t^{tot}, v_t^{NP}, v_t^e, mc_t, w_t, \Lambda_{t,t+1}, \mu_t, r_t, V_t, \pi_t, u_t, V_t^e, \rho_t$  and 1 exogenous variable,  $A_t$ . In addition the equilibrium features 13 parameters:  $\gamma, \gamma_m, \kappa, \delta, \varrho, \beta, \varphi, \chi, \eta, \varepsilon, b, A$  and  $\psi$ .

## D. Steady State

Given the restrictions reported in the text, the steady state can be obtained as follows. By definition  $q = \frac{m}{v^{tot}} = \gamma_m \theta^{-\gamma}$ , thus  $\gamma_m = q\theta^\gamma$  and  $z = \frac{m}{u} = \gamma_m \theta^{1-\gamma}$ . To pin down the steady state rate of unemployment notice that  $v^{tot} = \theta u = \theta(1-L)$ . Substituting for total vacancies into the steady state counterpart of equation (48) leads to

$$\begin{aligned} L &= (1-\delta) \varrho L + qv^{tot} = (1-\delta) \varrho L + q\theta(1-L) \\ &= \frac{q\theta}{1 - (1-\delta) \varrho + q\theta} \end{aligned}$$

As a consequence we can determine

$$\begin{aligned} v^{FP} &= (1-\delta) \frac{N^e L}{N} \frac{L}{q} = \delta \frac{L}{q} \\ u &= \frac{1 - (1-\delta) \varrho}{1 - (1-\delta) \varrho + q\theta}. \end{aligned}$$

and

$$v = v^{tot} - v^{FP}$$

Notice that  $b = \frac{b}{w}w$ , where we calibrate the ratio  $\frac{b}{w}$ . Evaluating the wage schedule and the JJC at the steady state leads respectively to

$$wh = \left[ 1 - (1-\eta) \frac{b}{w} \right]^{-1} \left[ \frac{\eta}{\mu} \rho Ah + (1-\eta) \chi C \frac{h^{1+1/\varphi}}{1+1/\varphi} + \eta \kappa \beta \theta \right]$$

and

$$wh = mcAh - (1 - \varrho(1-\delta) \beta) \frac{\kappa}{q}$$

Combining the latter two equations, after substituting for  $\chi = \frac{\rho}{\mu} \frac{A}{C} h^{-1/\varphi}$ , delivers the cost of posting a vacancy,  $k$ , as a function of the number of firms

$$\kappa = \frac{1 - \left( 1 - (1-\eta) \frac{b}{w} \right)^{-1} \left( \frac{\eta + \varphi}{1 + \varphi} \right)}{\frac{(1 - \varrho(1-\delta) \beta)}{q} + \left( 1 - (1-\eta) \frac{b}{w} \right)^{-1} \eta \beta \theta \mu} \frac{\rho}{\mu} Ah \quad (66)$$

The value of  $k$  increasing with the extent of competition since  $\frac{\rho}{\mu}$  is an increasing function of  $N$ . The same holds for the steady state wage, given by

$$w = mcA - (1 - \varrho(1-\delta) \beta) \frac{\kappa}{hq}$$

Combining the steady state counterparts of equations (43) and (44) delivers

$$Y = wLh + (1-\delta) N_{t-1} \pi_t + (1-\delta) N_{t-1}^e \pi_t^{NP} + kv^{tot} \quad (67)$$

where  $\pi = \left(\rho(N) - \frac{w}{A}\right) \frac{AL}{N} - (1 - \rho) \frac{k}{q} \frac{L}{N}$ ,  $V = \frac{1-\Lambda}{\Lambda} \pi$  and  $\pi^{NP} = \frac{\psi}{\Lambda} - V = \frac{\psi}{\Lambda} - \frac{\Lambda}{1-\Lambda} \pi$ . Substituting the definitions of  $\pi$ ,  $V$  and  $\pi^e$  into (67) delivers an equation which can be solved for  $N$ . Our numerical analysis shows that the latter has a unique solution for  $N > 1$ .