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INDUSTRY EVOLUTION AND R&D EXTERNALITIES

by
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Abstract: R & D externalities can imply ranges of aggregate increasing returns to scale in R & D. A consequence of increasing returns to scale is that several equilibria may exist involving different numbers of firms and R & D investment levels. Analysing adjustment dynamics yields important policy conclusions. One is that subsidies aimed at increasing the number of firms and R & D investment levels sometimes can have the opposite effect, shifting the industry onto an adjustment path toward an equilibrium with fewer firms and less R & D. The model is tested empirically using a unique database comprising competing firms in various R & D races.
1. Introduction

It is commonly accepted that research and development yields externalities in the sense that knowledge acquired in one firm spills over to other firms. Often knowledge spread in this way finds new applications or stimulates further innovative activity in other firms. When these externalities are sufficiently strong an industry can exhibit aggregate increasing returns to scale to R & D.

A typical feature of models with increasing returns to scale is that they give rise to more than one equilibrium. Yet most models in this field have been designed, by use of various assumptions, to yield a single equilibrium. This paper shows that analyzing the adjustment paths toward different equilibria has important policy implications.

Alfred Marshall argued in his "Principles of Economics" that an industry with competitive firms could exhibit a decreasing long-run cost curve due to the fact that one firm's production engenders external economies in terms of educating a skilled work force and spreading knowledge gained by "learning-by-doing". Later the literature on imperfect competition disposed of this view. It was argued that a monopolist would usurp such an industry in order to internalize the external economies.

More recently several papers have followed Arrow's (1962) lead in showing that a decentralized competitive equilibrium can exist with increasing returns to scale and externalities. For example, Romer (1986) constructs a model in which there are increasing returns to knowledge, but the growth of knowledge is limited by decreasing returns to the production of new knowledge. These models are generally designed to yield a single competitive equilibrium.

Fölster (1990) considers a situation with ranges of increasing returns to scale in the production of knowledge. In these situations several competitive equilibria can arise involving different levels of
research. This opens the possibility that a profitable technology may be neglected merely due to a coordination problem: If all firms invested simultaneously they might all find it profitable. Yet none is willing to risk investing too early and losing out at the expense of other firms that can enter later and draw on a pool of skilled researchers and an established knowledge base. The coordination problem is related to that analyzed in the literature on network externalities (e.g. Katz & Shapiro, 1985). However network externalities are generally assumed on the demand side, arising, for example, by the adoption of common standards. In our case the network externality arises on the supply side and determines how many firms find it profitable to enter an R & D race.

It is shown in Fölster (1990) that various coordination mechanisms such as communication between firms or the existence of investors that can buy several firms do not, in general, resolve the coordination problem.\(^1\) Therefore we do not consider these coordination mechanisms further in the present paper.

Here industry evolution toward various equilibria is analyzed. A model is developed in which entries and exits change the number of firms in the industry. Each incumbent firm decides on its R & D investment level based on expected profits. Profits generally decrease with the number of competitors, but there is some range where R & D externalities imply increased profits as the number of firms increases.

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\(^1\)While costless communication has been found to solve some coordination problems (e.g. Crawford & Sobel, 1982; Farrel, 1987) these results rely heavily on the assumption of common knowledge. In the absence of common knowledge Fölster (1990) shows experimentally that communication loses much of its coordination ability.

Coordination could theoretically occur via an investor who internalizes externalities by purchasing all firms. In the absence of perfect information and given some transaction costs, however, coordination may not occur because as soon as the coordinator is expected to succeed in purchasing a number of firms he thereby ensures profitability for the remaining firms. The remaining firms will then not sell out to the coordinator at a price that makes it worthwhile to complete the buy-out of all firms.
(the S-shaped function). This is shown to lead to complicated adjustment paths that in many cases can imply a cyclical development of the number of firms towards long-run equilibria with either high or low numbers of firms.

R&D competition is modelled in two different ways, both supporting the hypothesis of the S-shape profit as a function of the number of incumbent firms. The first approach is a quality ladders model. The second approach is a more traditional set-up where firms minimize unit production costs. The intensity of R&D turns out to be the same for both long-run equilibria in the case of the quality ladders model. However, R&D is more intensive in the high-competition equilibrium for the cost-economizing model.

R&D subsidies can sometimes move the industry toward the high R&D intensity equilibrium. However subsidies may result in a breakdown of the high-competition equilibrium. Subsidizing then increases short-run profits but lowers long-run profits and therefore induces exits from the industry.

The theory is applied empirically to panel data of 45 R&D races. Estimation supports the idea of aggregate increasing returns to scale in R&D within some range of the number of competitors (the S-shaped profit function). Further the data allow an analysis of factors that determine how sensitive entry of firms is to various factors that differ between firms and countries such as subsidies, supply of researchers and macroeconomic variables.
2. The model

Consider an industry consisting of homogeneous incumbent firms engaged in R&D competition. All firms are owned by stockholders who decide whether the firm performs better in this particular industry rather than in other industries. Stockholders can decide to let a firm switch from one industry to another thus giving rise to entry and exit. At each moment in time they choose a combination of entries and exits by comparing net present values that the firm can achieve within the particular industry with outside opportunity costs. The evolution of the number of firms in the industry is thus endogenously determined as a result of entry and exit decisions by stockholders.

The instantaneous expected net profit $\pi$ of a representative incumbent firm is a function of the number of firms $n$ currently populating the industry and of an R&D parameter $u$, so that $\pi = \pi(n,u)$. The time argument is omitted here and henceforth where it does not lead to confusion. At each moment in time $t$ a firm expects to obtain a surplus $\pi(n,u)dt$ during the period $(t, t+dt)$. The number of firms $n$ is a real variable. The R&D parameter $u$ is either the intensity of product innovation or unit production costs. In what follows we examine models of R&D behavior for both cases.
Each incumbent firm chooses the time path of R&D that maximizes the initial net present value: 
\[ v_0 = \int_0^T e^{-rt} \pi(n, u) dt \],
the sum of future profits discounted by the real interest rate \( r \).

Stockholders choose at each moment in time \( \hat{n} \), the change in the number of firms, through their decisions on entries into (and exits from) the industry. The net present value of each entry is the difference between the market value of the incumbent firm and the opportunity cost - the firm's value in the best alternative industry. The cost of creating a new firm in the industry is also assumed to equal the opportunity cost.

The firm's market value at time \( t \) is 
\[ v(t) = \int_t^T e^{-r(t-\tau)} \pi(n, u) d\tau, \]
and the opportunity cost of new entry is \( z = \text{constant} \). Conversely, the net value of each exit from the industry is \( z - v \).

Stockholders are assumed to incur adjustment costs related to turbulence in the industry, as measured by fluctuations in the number of firms. One can think of these e.g. as the investors' increasing costs of analyzing the market or of higher risk premia in a more turbulent market. Further there may be temporarily higher costs for capital goods and specialized labor in a market in which rapid growth increases demand faster than supply can be expanded. In a rapidly shrinking market there may be corresponding capital losses if many firms simultaneously try to exit and sell of their equipment. For the sake of simplicity we assume that the instantaneous adjustment cost is
a squared function of the net change of the number of firms \((2\alpha)^{-1}n^2\), where \(\alpha\) is a positive parameter. Stockholders behavior can be modelled by assuming that a representative stockholder maximizes, at date \(t\), the net entry profit: \((v-z)\dot{n} - (2\alpha)^{-1}n^2\). Under these assumptions the number of firms changes as:

\[
\dot{n} = \alpha(v - z). \tag{1}
\]

Differentiating \(v(t)\) implies

\[
\dot{v} = rv - \pi(n, u), \tag{2}
\]

which means that the rate of return on equity \(\dot{v}/v + \pi(n, u)/v\) equals the real interest rate. Equation (2) implies absence of arbitrage or capital market efficiency.

The net profit function \(\pi(n,u)\) is defined for \(n > 0\). Suppose that for each \(n\) the function \(\pi(n,u)\) has an interior maximum in the interval \(u \geq 0\):

\[
u^*(n) = \arg\max_{u \geq 0} \pi(n, u). \tag{3}\n\]

This assumption reflects decreasing returns to R&D at the firm level. Since R&D expenditures are calculated based on net profits, the firm incurs too high R & D costs if the intensity of innovation is too high or unit production costs are close to the minimum level. In subsections
2.2 and 2.3 we consider examples of net profit functions satisfying this property.

The dynamic problem of the incumbent firm is to choose the time path \( u(t) \) providing a maximum to the initial value \( v_0 \) subject to the equations (1)-(2). The current value Hamiltonian is

\[
H = \pi(n, u) + \Theta_1(v-z) + \Theta_2(rv - \pi(n, u))
\]  

(4)

where \( \Theta_1 \) and \( \Theta_2 \) are costate variables related to the equations (1), (2).

The optimal control function maximizes \( (1 - \Theta_2)\pi(n, u) \), which is equivalent to the static profit maximization provided that \( \Theta_2 < 1 \) for all \( t \). The optimal control is \( u^*(n) \) which by assumption is an interior solution to (3).

To demonstrate that \( \Theta_2 < 1 \) consider the costate equations:

\[
\dot{\Theta}_1 = r\Theta_1 - (1-\Theta_2)\pi'(n),
\]

(5)

\[
\dot{\Theta}_2 = -\alpha\Theta_1,
\]

(6)

where \( \pi(n) = \pi(n, u^*(n)) \) is the optimal static profit. Integrating (5) forward and inserting \( \Theta_1 \) in (6) implies that

\[
\dot{\Theta}_2(t) = -\alpha \int_t^\infty e^{-\alpha(\tau-t)}(1-\Theta_2(\tau))\pi'(n(\tau))d\tau.
\]

If at moment \( t \) it is true that \( \Theta_2(t) < 1 \), but \( \Theta_2(\tau) \) approaches 1, the time derivative \( \dot{\Theta}_2(t) \) tends to 0 \((|\pi'(n)| < \infty \) for all positive \( n \) and \( \Theta_2(\tau) \) remains below 1.

Since the optimal control function \( u^*(n) \) does not depend on the costate variables, we can ignore the costate equations and focus on the dynamics of the state variables \( n \) and \( v \). This is described by the following system:
The industry evolution depends crucially on the shape of the net profit function $\pi(n)$ which is defined and continuously differentiable for all positive $n$. The interval $(0, \infty)$ is assumed to belong to the range of this function. Consider two cases when $\pi(n)$ is either monotonously decreasing for all $n$ (Fig.1), or has an $S$-shape as depicted in Fig.2. Intuitively, the positive effect of competition on incumbents' profits is explained by R&D externalities such as technological spillovers and cluster effects. In what follows we derive the profit function $\pi(n)$ and demonstrate the positive external effect of R&D competition.

Given these two cases, there may be one, two or three stationary states of the system (7)-(8). From (7) the long-run value of the firm is $v_\infty = z$ in all cases. When $v = v_\infty$ there is no new entry into (or exits from) the industry. The steady-state number of firms in the industry $n_\infty$ is the solution to the equation:

$$\pi(n) = rz.$$  \hspace{1cm} (9)

If $n = n_\infty$ the firm's value does not change. Thus, the steady states differ only in the limit number of firms. In this section we do not deal
with the case of two steady states. If the system (7)-(8) has three stationary states, one can relate them with varying degrees of competition in the market: low, medium, or high.

Proposition. The unique long-run equilibrium is always a saddle point of the system (7)-(8). In the case of three stationary states the long-run equilibria with minimal and maximal number of firms are saddle points, while the steady state with the medium-level number is either the unstable node or the unstable focus.

The characteristic equation for the system (7)-(8) is

$$\xi^2 - r\xi + \alpha \pi'(n) = 0,$$

and the eigenvalues are $\xi = [(r \pm (r^2 - 4\alpha \pi'(n))^{1/2})]/2$. When $\pi'(n_o) < 0$, there are a couple of real eigenvalues with different signs, and the stationary state in this case is a saddle point. When $\pi'(n_o) > 0$, and $\alpha$ is sufficiently small, both characteristic roots are positive and the medium-level equilibrium is an unstable node. If $\pi'(n_o) > 0$, but $\alpha$ is large, the roots are complex-valued with positive real parts. Thus, the medium-level stationary state is an unstable focus.

Define the equilibrium path as a trajectory of the system (7)-(8) converging to one of the long-run equilibria. Given the initial number of firms $n_o$, the equilibrium path is determined by the initial evaluation
of the firm $v_0$. Equilibrium paths for the above cases of the model behavior are drawn as solid lines in figures (3)-(5).

There is one saddle point when incumbent's profit is decreasing with competition, $\pi'(n) < 0$ for all $n$ (fig.3). For each initial number of firms $n_0$ there exists a unique initial value $v_0$ that determines the equilibrium path. The industry converges to the long-run equilibrium $n_\ast$ regardless of the initial number of firms.

When there are three stationary states and the "middle" state is an unstable node, the industry evolution depends on the initial number of firms (fig.4). If it is below the medium-level number $n_\ast = n_2$, then competition will be low in the long-run, since $n$ tends to $n_\ast = n_1$, the minimal limit number. If $n_0$ is in the interval $(n_2, n_3)$, entry dominates exits and the number of firms converges to $n_\ast = n_3$, the maximal equilibrium number. In this case the equilibrium path is unique: for each initial number of firms $n_0$ there exists a unique initial value $v_0$ that specifies the industry evolution to one of the long-run equilibria.

The industry dynamics are more interesting when the medium stationary state is an unstable focus. This is the case when the parameter $\alpha$ is large, which means that entrants' adjustment costs are negligible. The system (7)-(8) describes non-explosive cyclical fluctuations that are not dampened but transform to the regime of monotonous growth.
Cyclical movements are explained by the S-shaped profit function $\pi(n)$. When firm's value $v$ exceeds opportunity costs $z$, the number of firms increases because of (7). As profit goes up, the value begins to decline, according to (8). When it becomes lower than entry costs $z$, the number of firms decreases.

An equilibrium path, departing from the medium-level equilibrium, fluctuates around it with growing amplitude and then converges to one of the saddles. There may be three cases portrayed in figures 5a-5c. In the first case there are two different equilibrium paths with cyclical behavior. One of them converges to the high-competition long-run equilibrium $n_3$, and another - to the low-competition equilibrium $n_1$ (fig.5a).

In the second case there is a unique cyclical equilibrium trajectory that comes to the maximal-level equilibrium $n_3$ (fig.5b) and the non-cyclical equilibrium path going to the minimal-level equilibrium $n_1$. The value of firms is notably below the cost of entry $z$ when the industry moves along the non-cyclical path. The third case is the reverse of the second: the cyclical equilibrium trajectory converges to the low-competition equilibrium $n_1$, while the non-cyclical path overestimates firms' value (as compared with entry costs $z$) and goes to the high-competition saddle point $n_3$ (fig.5c).

Similar cyclical behavior of equilibrium trajectories is demonstrated in a recent paper [Gali, 1994] reconsidering the
neoclassical model of economic growth in the presence of price markups. However, that paper mentions only the first case with two different cyclical equilibrium trajectories departing from the unstable focus.

The cyclical dynamics imply a multiplicity of equilibrium paths. For a given initial number of firms $n_0$ there may be many initial values $v_0$ specifying the movement towards one of the steady states $n_1$ and $n_3$. There are infinitely many initial values $v_0$ if the initial number $n_0$ coincides with the medium equilibrium $n_2$.

The long-run industry evolution is thus determined by the initial evaluation of incumbent firms, $v_0$, given the initial number of firms $n_0$. The institution for this evaluation is the stock market. If it believes that the industry will move to the steady state with a low or high degree of competition, it estimates the value of firms according to these expectations.

Initial values of firms $v_0$ related to paths coming to the steady states $n_1$ or $n_3$ may not differ much when there are two cyclical equilibrium trajectories (fig.5a). The closer is the initial number of firms to the focus $n_2$, the less may be the distance between those initial values of firms. However, when there is only one cyclical equilibrium trajectory, the stock market notably overvalues (fig.5b) or undervalues (fig.5c) firms (related to $z$) if it does not believe in the cyclical evolution.
These results demonstrate the role played by the stock market, or more broadly by financial markets, in economic development. Their impact on economic growth is discussed in several recent papers, demonstrating the evidence that financial development correlates with growth (Pagano 1993, Atje and Jovanovic 1993, King and Levine 1993). One interpretation of these empirical results could be that current or expected economic growth to a larger extent stimulates development of financial markets. In our model, however, the stock market is responsible for the initial evaluation of firms and, hence, for the pattern of the long-run industry evolution and its dynamic efficiency.

2.2 R&D Externalities and Quality Ladders

We assumed above that the profit function $\pi(n)$ is monotonously decreasing (fig.1) or has an S-shape (fig.2). Now we suggest an explanation for this hypothesis based on a modified quality ladders model.

Following the existing quality ladders literature [e.g. Grossman and Helpman 1991, Segerstrom et al. 1990, Segerstrom 1994] suppose that the industry product has a countable number of qualities $j = -1,0,1,2,3,...$ with higher quality represented by higher number $j$. At each time interval $(t, t+dt)$ firms are engaged in R&D competition with one
winner or no winners. The winner is a firm that innovates first and for a period wins quality leadership in the industry. There is no winner during the period dt if no firm is successful. The winner enjoys a temporary monopoly position during this period and extracts monopoly rents. Losers upgrade their quality level through imitation, so in the case of successful innovation by one firm all others also move to the higher "quality ladder" at the beginning of the next period. The latter point constitutes a difference to the previous quality ladder models where the leading firm becomes a looser only if some other firm manages to reach a higher quality ladder.

If quality is upgraded during time interval \((t, t + dt)\), the market is supplied with two brands of the product. Consumer expenditures are allocated between the obsolete and the new good. At each moment consumers maximize the instantaneous utility \(\ln(d_1 + \lambda d_2)\) subject to the budget constraint \(p_1d_1 + p_2d_2 = E(t)\), where \(d_1\) and \(d_2\) denote the quantities of the obsolete and the new good, respectively; \(p_1\) and \(p_2\) are the respective prices of these goods; \(E(t)\) is consumer expenditure at the time \(t\); \(\lambda > 1\) represents the extent of quality improvement. As in Segerstrom [1994] consumer expenditures are constant over time, \(E(t) = E\).

Quality adjusted prices must be equal in equilibrium: \(p_1 = p_2/\lambda\). Non-leaders are Bertrand competitors charging prices at the unit cost level normalized to one. Hence, the winner of the R&D race increases
the price exactly to the extent of quality improvement: \( p_2 = \lambda \). Actually
the leading firm becomes a monopoly producer, because infinitesimal
price reductions allow it to take over the market. It means that \( d_2 = E/p_2 \). Thus the monopoly surplus obtained by the winner equals:

\[
\Delta = (p_2 - 1)E/p_2 = (1 - 1/\lambda)E.
\]

The expected monopoly profit from each race is a product of \( \Delta \)
and the probability that the firm \( i \) will become a winner. Successful
innovations arise as a result of a Poisson process with an intensity
depending on R&D efforts and the number of firms in the industry.
The probability that one firm innovates successfully during the period
dt is a function of R&D intensity \( h \) and the number of firms at the
current moment \( n(t) \):

\[
\Pr (\text{one firm is a winner}) = h(1 - ae^{-bn(t)})dt, \quad (10)
\]

where \( a, b \) are positive parameters, \( a < 1 \). All firms have equal
chances at the beginning of time interval \((t, t+dt)\). Hence, the
probability that any firm \( i \) innovates successfully is \( h(1 - ae^{-bn})n^{-1}dt \).

The probability (10) roughly equals the probability that at least
one firm innovates successfully because it is unlikely that two or more
firms innovate successfully in a small interval. The exponential function
(10) with discrete \( n \) is a limit case of Bernoulli trials, modelling
attempts by each firm to promote innovations in the market. Let \( m \) be
the number of trials made by a firm, \( p \) be the probability that the
attempt will be successful. Suppose there is a chance - for exogenous
reasons - of one firm innovating successfully without any trial with probability $p_o$. Then the probability that at least one firm innovates successfully is $1 - (1-p_o)(1-p)^m$. Given that the number of trials $m$ is large and the probability $p$ is small, this is close to the exponential function $1 - ae^{-bn}$, and $a = 1-p_o$, $b \approx mp$.

The expected monopoly surplus from winning R&D races in the time interval $(t,t+dt)$ is $\Pi(n,h)dt = \Delta h(1 - ae^{-bn})n^{-1}dt$. The instantaneous expected net profit is:

$$\pi(n,h) = \Delta h(1 - ae^{-bn})n^{-1} - \sigma^{-1}h^\sigma, \quad (11)$$

where $\sigma^{-1}h^\sigma$ are firm's R&D expenditures as a function of R&D intensity. We suppose that $\sigma > 1$, which means diminishing returns to scale in R&D on the disaggregate level. Segerstrom (1994) proposes R&D technology with constant returns to scale for the individual firm and decreasing industry-wide returns.

At time $t$ firms choose R&D intensity maximizing (11). It is easy to show that $\arg\max_h \pi(n,h) = h(n) = [\Delta (1 - ae^{-bn})n^{-1}]^{1/(\sigma-1)}$. Inserting it in (11) we derive the instantaneous profit function

$$\pi(n) = (1-1/\sigma)[\Delta (1-ae^{-bn})n^{-1}]^{\sigma/(\sigma-1)}. \quad (12)$$

It is monotonously decreasing or has the S-shape. To simplify formulas let $\sigma = 2$. In this case

$$\pi'(n) = -(2\pi(n))^{1/2}\Delta [n^{-2} - ae^{-bn}(bn^{-1} + n^{-2})].$$

The expression in square brackets is negative if $e^{bn} > 1/a(1+bn)$. This either holds for some interval of $n$ bounded away from 0, or does not
hold for all \( n \), because \( a < 1 \). Consequently, the profit function (12) is S-shaped or monotonously decreasing. The intuition behind the S-shape profit function is that the positive external effect of R&D dominates the negative effect of increasing competition.

The intensity of R&D is a monotonously increasing, concave function of expected profits. Indeed, from (12)

\[
h(n) = \left[ (1 - 1/\sigma) \pi(n) \right]^{1/\sigma}.
\]

Expected instantaneous profits \( \pi(n) \) are, in the industry evolution model, the same in the long-run equilibria with low and high numbers of firms. Hence the intensity of R&D is the same under high and low competition. R&D activity is persistent because the long-run expected profits are positive.

2.3 R&D Externalities and Production Costs Economizing.

A profit function with the above properties can also be derived from another model of R&D competition. Suppose that product quality does not change throughout time and the R&D parameter is unit production costs \( c \). Further we assume that there is cost pressure, e.g. rising real wages, that in the absence of R & D continuously raise costs. Thus some R & D is required merely to keep costs constant.

The instantaneous net profit function can be represented as
\[ \pi(n, c) = \Pi(n, c) - S(n, c) \]  
(13)

where \( \Pi(n, c) \) is defined as firms' expected operating profit and \( S(n, c) \) is R&D expenditures. As above, total consumer expenditures are \( E \) constant. The price is also constant in time and normalized to one. Firms divide the market on equal shares, thus each currently producing \( E/n \) units of output. A representative firm expects to obtain operating profit:

\[ \Pi(n, c) = (1 - c)E/n. \]  
(14)

R&D expenditure is a linear function of R&D intensity \( y(c) \):

\[ S(n, c) = d(n)y(c), \]  
(15)

where \( d(n) \) is the marginal cost of R&D which, because of external effects, depends on the number of firms.

Suppose there is a lower technological limit for the unit costs \( c_0 < 1 \). For an individual firm R&D results in keeping the unit production costs at some level above the technological limit. The closer unit cost \( c \) is to the minimal level \( c_0 \), the higher R&D intensity is and, consequently, the higher are R&D expenditures \( S(n, c) \). This is a close parallel to the assumption of decreasing R&D returns used in Segerström [1994] and in the above quality ladders model. We choose the following function as a measure for R&D intensity:

\[ y(c) = 1/(c - c_0), \]  
(16)

which implies that R&D costs are unboundedly increasing if \( c \) converges to \( c_0 \).
Inserting (14)-(16) into (13) we have

$$\pi(n,c) = (1 - c)E/n + d(n)/(c_0 - c).$$  \hspace{1cm} (17)

Profit-maximizing unit cost is

$$c(n) = c_0 + (d(n)n/E)^{1/2}$$  \hspace{1cm} (18)

and

$$\pi(n) = (1 - c_0)E/n - 2(d(n)E/n)^{1/2}. \quad (19)$$

Consider an example of the marginal R&D cost function:

$$d(n) = ne^{-\gamma n},$$

where $\gamma$ is the positive parameter. The profit function (19) in this case has the shape depicted in figures 1 and 2. Indeed:

$$\pi'(n) = -(1 - c_0)E/n^2 + \gamma Ee^{-\gamma n/2}. \quad (20)$$

The function $(1 - c_0)/n^2$ either intersects $\gamma Ee^{-\gamma n/2}$ in two points or locates above it for all $n$. In the former case we have the S-shape of the profit function (19) as shown in figure 2, and in the latter case it is monotonously decreasing as depicted in figure 1.

One can interpret (20) as a result of two opposite effects of new entry on the incumbent’s profit. On the one hand, profit is decreasing with new entry through market competition, and on the other hand, it can increase because of the positive R&D externality. Note, that the same is true for a more general class of marginal cost functions:

$$d(n) = n^B e^{-\lambda n}, \quad (21)$$

where $B$ exceeds -1.
Consider which of the long-run equilibria of the industry evolution model yields higher R&D intensities. Using (18) the profit function (19) is represented as

\[ \pi(n) = (1 + c_0 - 2c(n))E/n, \]  

that is

\[ c(n) = (1 + c_0 - \pi(n)n/E)/2. \]  

Since instantaneous profit \( \pi(n) \) is positive and equal in the long-run equilibria, (23) implies that \( c(n_1) > c(n_3) \). Thus, unlike the above quality ladders model, R&D activity is more intensive in the high-competition steady state.

### 2.4 R&D subsidies and industry evolution.

Incumbent firms may incur losses when competition in the industry reaches some critical level. Instantaneous profit can be negative in the version of the model with unit cost economizing through R&D. According to (22) R&D is unprofitable if \( c(n) > (1 + c_0)/2 \). From (18) this is equivalent to

\[ d(n)n > (E/4)(1 - c_0)^2. \]

Thus if minimum unit costs \( c_0 \) are high or consumer expenditures \( E \) are small, the profit function is negative in the interval of firms' number
bounded away from zero. The local minimum of the function \( \pi(n) \) inside this interval is actually its global minimum.

Expected profit is positive in the case of the quality ladders model (12). However this type of R&D competition may become unprofitable if there is a network externality. As an example, sharing network expenses \( C_N \) adds a new increment to the profit function (12):

\[
\pi(n) = \frac{1}{2}[\Delta(1-ae^{bn})n^{-1}]^2 - C_N/n \tag{24}
\]

(here \( \sigma = 2 \)). In this case instantaneous profit turns negative for large \( n \).

Suppose the government is able to intervene by subsidizing R&D through uniform lump-sum transfers. Formally the constant increment \( S > 0 \) adds to the profit functions (22) and (24). This modification significantly affects the model's dynamics. As was pointed above the system (7)-(8) may have two steady states. This occurs when the S-shape profit function \( \pi(n) \) is positive for all \( n \) and the local interior minimum of this function becomes the global minimum (if it is less than \( \lim_{n \to \infty} \pi(n) = S \)). These steady states correspond to the low and medium degrees of competition in the long-run (fig. 6). The high-competition steady state \( n_{3h} \), thus, does not exist in the case of two stationary states.

The equilibrium path starting from the unstable steady state \( n_2 \) goes to the low competition state \( n_1 \). Those trajectories of the system
(7)-(8) moving the number of firms to infinity do not converge to any long-run equilibrium and do not describe equilibrium dynamics. It means that R&D subsidizing implies a pattern of evolution which is the opposite of that generally intended by policy-makers. Subsidizing benefits all firms in the short-run, but in the long-run it decreases firms' values and induces exits from the industry.

R&D subsidies are useful when two steady states do not appear, i.e. the local interior minimum of the S-shape profit function $\pi(n)$ is higher than the amount of lump-sum transfer $S$. In this case subsidizing leads to disappearance of the low and medium-competition steady states. Thus the dynamic is favourable, since it moves the industry to the unique high-competition state. However, policy-makers must be careful about the influence of R&D policy on the long-run evolution of industries.
3. Empirical analysis

A number of issues have to be resolved to make the theory empirically tractable. The main issue is perhaps that we do not expect R & D externalities to be most significant in the kind of step by step technological improvements that characterizes most production. Rather, significant R & D externalities should be most likely in more advanced and long-term R & D projects. An empirical problem is that there is a considerable lag in these projects between R & D investment and eventual production and profits. As a result it is difficult to apply directly the relationship in the theoretical model between R & D investments on the one hand and output and profits on the other hand.

We solve this dilemma by concentrating on advanced R & D projects in a number of areas, using a database that contains all competitors, worldwide, working on the respective technology. The database is a panel which allows us to follow the number of competitors in each technology and their investments in that particular line of research.

The basic theoretical model is implemented in the following manner. The basis for estimation is equation (1) above
\[ \dot{n}_i = \alpha (v_i - z) \]

Here the index \( i \) denotes each R & D race. \( v \) is the present value of future profits for any firm. Expected profits determine current R & D investments. We observe R & D investments, but not profits. Therefore we assume, following equation (21) above, that the functional form for R & D investments \( d(n) \) as a function of \( n \), and omitting index \( i \), is

\[ d(n) = e^{-\lambda n^b} \quad (25) \]

Further, following (19) profits are a function of \( d(n) \) such that

\[ \pi = A/n - 2(d(n)E/n)^{1/2} - S_0 \quad (26) \]

The complete function to be estimated is then

\[ \Delta n_i = \alpha \sum_i e^{-R_0 t} [A/n - 2(En^{b-1})^{1/2} - S_0] dt - az \quad (27) \]

As an alternative we also estimate the quality ladder model which takes the following functional form.
\[ \Delta n_t = \alpha \Sigma_t e^{-R(t)} c \frac{1}{2} \frac{\Delta/c}{n^{-1} \left(1 - ae^{-bn}\right)^2} C_n / n - S_0 \]  

(28)

This can be simplified to

\[ \Delta n_t = \alpha \Sigma_t e^{-R(t)} B \frac{n^{-1} \left(1 - ae^{-bn}\right)^2}{C_n / n - S_0} \]  

(29)

Initially we treat \( \alpha z \) in the unit cost economizing model as a fixed, constant cost that is the same for all entrants. Estimating this function allows tests of whether there is support for the idea of R & D externalities.

As a next step we examine the composition of changes in \( n \) and test how these depend on various country-specific factors. In those estimates \( \alpha z \) is interpreted as an entry cost that differs between countries as a function of various variables like skill levels relative costs and others.
Data

A large part of the data comes from interview surveys among Swedish industrial firms that were conducted between 1987 and 1992. These surveys are described in Fölster (1991a, 1991b). The surveys were concerned with determining the effects of government subsidies for industrial R & D and firms' technological competitiveness in relation to foreign competitors. The parts of the database concerned with R & D cooperation are described in Fölster (1994).

In these surveys individual R & D projects are identified. For each project aimed at developing a specific technology firms have supplied information on R & D investment levels as well as who potential international competitors are and which of those actually are engaged in similar R & D. Also patterns of R & D cooperation were identified. Firms were asked to reconstruct this information for the period since 1980, thus creating a panel.

Often firms are reluctant to reveal such detailed information. Since the surveys were conducted by an institute (The Industrial Institute of Economic and Social Research) with close ties to industry, and the surveys were carried out as personal telephone calls to managers or research managers, firms were quite open.

The firms were randomly selected among industrial firms that spend at least 5% of revenue on R & D. Of the contacted firms 9%
refused to answer at all. The remaining firms were asked to pick a representative sample of 3 R & D projects. Only in 13 cases did firms say that they could not pick a representative sample because they did not want to reveal details of a particular R & D project. In sum, the data should represent a reasonably unbiased sample of research-intensive Swedish industrial firms' R & D projects.

The Swedish firms could often supply sufficiently detailed information even about competitors' projects. To be sure, however, the competitors were contacted as well and asked to confirm the information. Among the foreign competitors the reply rate was considerably lower. Of those contacted 46% responded. Not all of those supplied detailed information. The response of these firms was sufficient however to ascertain that the Swedish firms' information about their competitors was generally accurate.

All in all 45 technologies are included in the sample. A "technology" is narrowly defined based on the Swedish firm's project definition. Table 1 shows various characteristics of the technologies, the market structure in each of these "races", and what forms of cooperation existed.
Table 1. Description of the data. Number of technologies in each field, and average number of competitors and cooperative agreements within each technology.

<table>
<thead>
<tr>
<th>Technology</th>
<th># of technologies</th>
<th>Average # of competitors</th>
<th>Average # of cooperative agreements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medical-and biotech</td>
<td>7</td>
<td>12.8</td>
<td>1.4</td>
</tr>
<tr>
<td>Communication</td>
<td>4</td>
<td>9.6</td>
<td>1.6</td>
</tr>
<tr>
<td>Energy</td>
<td>5</td>
<td>8.2</td>
<td>2.1</td>
</tr>
<tr>
<td>Environment</td>
<td>4</td>
<td>16.3</td>
<td>3.1</td>
</tr>
<tr>
<td>Information</td>
<td>8</td>
<td>8.1</td>
<td>1.7</td>
</tr>
<tr>
<td>Lasers</td>
<td>2</td>
<td>14.6</td>
<td>2.0</td>
</tr>
<tr>
<td>New materials</td>
<td>4</td>
<td>5.7</td>
<td>1.3</td>
</tr>
<tr>
<td>Robotics</td>
<td>6</td>
<td>21.2</td>
<td>4.1</td>
</tr>
<tr>
<td>Transport</td>
<td>5</td>
<td>12.0</td>
<td>3.6</td>
</tr>
</tbody>
</table>

Table 2 lists the variables used in the estimations. The first estimation only uses the number of firms in each race and R & D expenditures, while subsequent decomposition of the number of competitors in each race also makes use of the country variables.
Table 2. The variables.

<table>
<thead>
<tr>
<th>n</th>
<th>Number of firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>R &amp; D in US dollars</td>
</tr>
</tbody>
</table>

Country data:

<table>
<thead>
<tr>
<th>NC</th>
<th>number of firms in the country that are competing in the R &amp; D race.</th>
</tr>
</thead>
<tbody>
<tr>
<td>SALES</td>
<td>The firm's total sales in related technology, in US dollars.</td>
</tr>
<tr>
<td>TECHCOMP</td>
<td>The firms' technological competence in the technology at the inception of the project, as judged by competitors, on a scale of 1 - 10.</td>
</tr>
<tr>
<td>SUB</td>
<td>Size of average subsidies to a research project, in percent of RD.</td>
</tr>
<tr>
<td>ED</td>
<td>University graduates in each respective category in 1000's</td>
</tr>
<tr>
<td>SIZE</td>
<td>Size of the country in millions of inhabitants</td>
</tr>
</tbody>
</table>

R & D race estimations

The dependent variable consists of integer values between -4 and +5 seen over all time periods and can be approximated by a continuous variable. We use a standard maximum likelihood method. The coefficients are estimated as shown in Table 3. The overall fit of the model is quite good.

Table 3 also shows estimation of the quality ladder model with and without the network externality coefficient $C_N$. 
Table 3. Maximum likelihood estimates of equation (24).

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Basic Model</th>
<th>Quality ladder with $C_N$</th>
<th>Quality ladder without $C_N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CONSTANT</td>
<td>1.24</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.43)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A$</td>
<td>4.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.95)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.91</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.298)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>2.21</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.96)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_0$</td>
<td>0.18</td>
<td>0.08</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
<td>(0.02)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>$E$</td>
<td>0.73</td>
<td>35.31</td>
<td>34.81</td>
</tr>
<tr>
<td></td>
<td>(0.218)</td>
<td>(11.2)</td>
<td>(11.9)</td>
</tr>
<tr>
<td>$a$</td>
<td>7.29</td>
<td>7.21</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.99)</td>
<td>(2.69)</td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>0.3</td>
<td>0.31</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.10)</td>
<td></td>
</tr>
<tr>
<td>$C_N$</td>
<td>0.1</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In likelihood 916.4399 1012.87 915.01

450 Observations. Asymptotic standard errors in parentheses.

We interpret these coefficients in figure 7 where the profit function is drawn for basic model and the quality ladder model with $C_N$. We do not show the quality ladder model without $C_N$ since the coefficients and the shape of the profit function is so similar to the version of the model including $C_N$.

The estimations clearly support the notion of aggregate increasing returns to scale for a range of number of firms. The shape
of the profit functions shown in figure 8 is by no means forced onto the data. Both functions can exhibit monotonically decreasing forms for wide ranges of coefficients.

In order to illustrate the implications of these results further a number of simulations are performed.

Simulations

Numerical analysis focuses on the influence of the real interest rate $r$ on the industry evolution. The system (7)-(8) dynamics is simulated for the estimated parameters of the cost economizing model (Table 3). The behavior of trajectories for the quality ladders model does not differ much, so we present simulation results concerning only that model.

Simulations demonstrate that all above cases of industry evolution are possible for a quite narrow domain of the real interest rate: $r \in (0.005, 0.07)$. The result is shown in Table 4 where "focus-a, b, c" relates to the cases depicted in figures 5 a, b, c, respectively, and "un. saddle p." means that there is a unique long-run equilibrium which is a low-competition state.

Table 4

<table>
<thead>
<tr>
<th>$r \times 100%$</th>
<th>0.5 - 2.8</th>
<th>2.85 - 3.99</th>
<th>4.0 - 5.74</th>
<th>5.75 - ...</th>
</tr>
</thead>
</table>

Dynamics: "focus-b" "focus-a" "focus-c" un. saddle p.

The case of unstable node (fig.4) is obtained for a higher value of entry cost parameter $z$ than we used in the simulation. The favourable zone for the real interest rate is 2.85 - 5.75 %%, when the high-
competition long-run equilibrium is attainable. However in the interval 4.0 - 5.75% (case "focus-c") the stock market can undervalue firms and move the industry to the low-competition state.

Figures 8-10 demonstrate the trajectories of the system (7)-(8) for \( r = 0.01, 0.03 \) and 0.05. The long-run equilibria \( n_1 \) and \( n_3 \) are characterized in Table 5.

Table 5

\[
\begin{array}{ccc}
 r & 0.01 & 0.03 & 0.05 \\
\hline
 n_1 & 3.05 & 2.75 & 2.62 \\
n_3 & 20.15 & 16.05 & 12.25 \\
\end{array}
\]

Country comparisons

A final step is to disaggregate equation (24) into where changes in the number of firms occur. We therefore replace the dependent variable with the change in \( NC \), the number of competitors in each country. The countries included are the US, Japan, Germany, France, England, Italy and Sweden. These countries account for 85% of competitors in the R & D races.

The entry cost \( z \) is replaced by a vector of country-specific explanatory variables weighted by coefficients to be estimated.

One problem with disaggregating the dependent variable is that \( NC \) never is larger than one or smaller than -1 and thus only takes three values: -1, 0, 1. To solve this problem the dependent variable is adjusted by the relative size of each country. This creates variety.
among the values of the dependent variable and at the same time controls for country size.

Table 6. Maximum likelihood estimations of country-specific changes in number of competing firms.

<table>
<thead>
<tr>
<th>Explanatory variables</th>
<th>Basic model</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5.46</td>
</tr>
<tr>
<td></td>
<td>(2.533)</td>
</tr>
<tr>
<td>γ</td>
<td>0.61</td>
</tr>
<tr>
<td></td>
<td>(0.243)</td>
</tr>
<tr>
<td>λ</td>
<td>0.896</td>
</tr>
<tr>
<td></td>
<td>(0.242)</td>
</tr>
<tr>
<td>β</td>
<td>1.90</td>
</tr>
<tr>
<td></td>
<td>(0.62)</td>
</tr>
<tr>
<td>S₀</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>(0.06)</td>
</tr>
<tr>
<td>SALES</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>(0.0002)</td>
</tr>
<tr>
<td>TECHCOMP</td>
<td>0.501</td>
</tr>
<tr>
<td></td>
<td>(0.086)</td>
</tr>
<tr>
<td>SUB</td>
<td>-0.031</td>
</tr>
<tr>
<td></td>
<td>(0.027)</td>
</tr>
<tr>
<td>ED</td>
<td>0.0023</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
</tr>
<tr>
<td>SIZE</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
</tr>
</tbody>
</table>

Log likelihood 1267.9

Standard error in parentheses.

In table 6 the coefficients for the profit equation are similar as in the previous estimation. Of the country specific variables SALES, TECHCOMP and ED have the expected sign and appear significant. The degree of subsidization appears insignificant and actually has a negative sign. We hesitate, however to interpret this as support for the
theoretical result that subsidies can push firms on to the adjustment path toward the low-level equilibrium. An alternative explanation could be that countries with few entrants more often subsidize research.

5. Conclusion

The analysis supports the notion that there can be ranges of aggregate increasing returns to research in which an industry can converge to different equilibria. The empirical estimations yield results consistent with the hypothesis that subsidies can reduce chances of reaching the high-number-of-firms equilibrium, while technological competence and education seem to increase chances.
References


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Gali, Jordi, 1994, Monopolistic competition, endogenous markups and growth, European Economic Review 38, 748-756.


Segerstrom, Paul S., 1994, Reexamining the quality ladders growth model, Unpublished manuscript, Michigan State University.
Fig. 6
FOCUS A

\[ r = 0.03 \]
Fig. 10

FOCUS C

$r = 0.05$

value

number of firms