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## Trade Specialists and Money in an Ongoing Exchange Economy

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**TRADE SPECIALISTS AND MONEY IN  
AN ONGOING EXCHANGE ECONOMY**

by

Robert W. Clower and Daniel Friedman

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TRADE SPECIALISTS AND MONEY IN AN  
ONGOING EXCHANGE ECONOMY\*

The fact surely is that in modern (capitalist) economies there are, at least, two sorts of markets. There are markets where prices are set by producers; and for those markets, which include a large part of the markets for industrial products, the fixprice assumption makes good sense. But there are other markets, "flexprice" or speculative markets, in which prices are still determined by supply and demand....What we [as macroeconomists] need is a theory in which both fixprice and flexprice markets have a place.

-- Hicks (1974, pp. 23-24)

Even in "flexprice" markets, some sort of institutional structure is necessary to transform amorphous "supply and demand" into publicly announced prices and real-time transactions. The fictitious institution known as the "Walrasian auctioneer" — an external agent who elicits excess demand schedules from all transactors, aggregates them, and computes and announces market-clearing prices, but ignores the assignment<sup>1</sup> of actual exchange partners — is a great convenience to theorists but provides little insight into the performance of actual flexprice markets, such as those for metals, grains, or financial assets. In actuality price announcements and arrangements for transactions in these markets are typically made by trade specialists of some sort, such as brokers, dealers or middlemen.

To what extent do ongoing markets organized by trade specialists yield Walrasian outcomes? In particular, do relative prices tend towards competitive (i.e., Walrasian) equilibrium values? In a decentralized monetary economy with markets organized by trade specialists, does the price level adjust proportionately with the stock of money? If one can't obtain positive answers to these questions, a considerable part of received economic theory becomes doubtful. On the other hand, if positive answers are forthcoming, one is then better equipped to proceed with Hicks' suggestion for reconstructing

macroeconomic theory.

In seeking answers to these questions, we focus in this paper on how the transactions structure (trading specialists and money) affects the existence, efficiency and dynamic stability of equilibrium in a many-goods economy. Consequently, we offer only a cursory treatment of other possibly important aspects of markets. We will take as given the existence and general characteristics of the specialist traders who maintain inventories and adjust prices, and we shall ignore such interesting issues as why specialists exist in some markets but not in others,<sup>2</sup> and how they acquire and process information.

For simplicity our models employ strong but standard assumptions on household preferences (mostly to avoid the nuisance of "corner solutions") and are set in discrete time. Likewise, we omit production from consideration and do not deal explicitly with uncertainty or intertemporal choice. Finally, we omit some formal details and proofs (the interested reader can find these in Friedman (1982)).

After a brief review of the standard discrete time Walrasian model of pure exchange in Section 1, we present our basic conceptual experiment for the exchange and price adjustment process, and indicate how specialists can be introduced into the formal model. We then describe the dynamics of the resulting economy in the neighborhood of equilibrium. This microdynamic barter model (MBM) is reminiscent of earlier treatments of multiple-market dynamic price adjustment processes (Samuelson, 1947, p. 269 ff; Clower and Bushaw, 1954), but it has a more explicit conceptual basis and yields slightly sharper conclusions.

Section 2 outlines a more elaborate model, featuring monetary exchange. We argue that certain undesirable features of the Barter model can be removed if we introduce a medium of exchange, and we then outline how a monetary

microdynamic model (MMM) can be formalized. Section 3 analyzes the adjustment properties of our model following moderate "shocks," both real and nominal. It turns out that fairly mild conditions suffice for the economy to be stable and for money to be neutral. Finally, Section 4 of the paper discusses some shortcuts we have taken, some implications of our model, and directions for further work.

#### 1. The Standard Walrasian Model and a Barter Microdynamic Model

Our benchmark is the standard discrete time Walrasian model. It consists of a finite set of households indexed  $j = 1, \dots, n$ , each characterized by unchanging preferences  $U^j$  over and periodic endowments  $s^j(t)$  in, a finite number of goods  $i = 1, \dots, l$  (none of which is stored). Via the standard<sup>3</sup> constrained optimization problem, we arrive at the vector of net trades  $x^j(t) = d^j(t) - s^j(t)$  desired by household  $j$ , once prices  $p(t)$  are specified. By construction, the budget constraint (or Say's Principle)

$$p(t) \cdot x^j(t) = \sum_{i=1}^l p_i(t) x_i^j(t) = 0,$$

holds for each household  $j$ . Walrasian equilibrium (WE) may then be defined as a price vector  $p(t)$  and a set of desired net trades (i.e., excess demand vectors)  $x^j(t)$  corresponding to  $p(t)$  such that the net trades balance, i.e., sum to the zero vector:  $x^T(t) \equiv \sum_j x^j(t) = 0$ . Well-known theorems assert the existence and Pareto-efficiency of WE under quite general assumptions regarding preferences and endowments. Under more restrictive assumptions, one can establish the uniqueness of WE (up to a scale factor in  $p$ ; see Arrow and Hahn, 1972).

We are now ready to introduce trade specialists — agents who maintain stocks of inventory from which they can accommodate households' desired net

trades, and who adjust prices so as to maintain control over their inventory. In our underlying conceptual experiment, each period  $t$  represents a market day. In the morning of day  $t$ , the specialists post prices  $p(t)$  computed on the previous evening as described below. Households observe these prices, receive their endowments, and compute their desired net trades  $x^j(t)$  as in the Walrasian model. In the afternoon, the households show up in no particular order at the market place with the goods they wish to "sell" (those for which the desired net trade is negative) and shopping lists of goods they wish to "buy". After checking that the value of goods to be bought is no greater than the value of goods to be sold, the specialists allow a household to enter the market. In the market, there are  $l$  storage bins, one for each good. Each household places its "sold" goods in the appropriate bins and withdraws its "bought" goods from their bins, in the desired quantities (we assume away possible outages for the moment). When these transactions are complete, the household leaves the market place. In the evening, households consume their goods and enjoy their leisure. Specialists note how the ending level in each bin  $S_i(t+1)$  differs from (a) the beginning or previous level  $S_i(t)$ , and (b) the desired level for the next day  $D_i(t+1)$ . They then decide on the price adjustment for the next day,  $\Delta p_i(t) = p_i(t+1) - p_i(t)$ , according to some given rule based on these differences.

To formalize this story, we can retain the Walrasian specification of households, and reinterpret the goods as non-perishable, the specialists having unique access to a storage technology (assumed costless in this section). Collectively, specialists observe the aggregate excess (flow) demand vector<sup>4</sup>  $x(t) \equiv x^T(p(t)) = S(t) - S(t+1)$  and know their own excess (stock) demand vector  $X(t) \equiv D(t+1) - D(t)$ . Other information available to specialists can be incorporated in specifying desired stock holdings  $D$ .

Therefore it is reasonable and quite general to specify the price adjustment rule for the  $i^{\text{th}}$  good as  $p_i(t+1) = p_i(t) + f_i(x(t), X(t))$ . If  $f_i$  depends only on the  $i^{\text{th}}$  components of  $x$  and  $X$  (i.e., if the specialist for good  $i$  looks only at his own bin, and not at excess demand for goods  $j \neq i$ ), then the pricing rule  $f_i$  is called simple; otherwise we call  $f_i$  sophisticated. In any case, a specification of specialist behavior via  $f_i$  and  $D_i$  completes the formal model, which we shall refer to as a Microdynamic Barter Model (MBM). We now turn to dynamics.

In general, a discrete-time dynamical process is a rule that assigns the next period's state as a function of this period's state. In the present instance, the state on day  $t$  of our Barter model consists of a pair of positive vectors  $p(t)$  and  $S(t)$ , specifying prices and initial inventories. Given today's state  $(p(t), S(t))$ , we obtain tomorrow's state  $(p(t+1), S(t+1)) = F(p(t), S(t))$  from the household sector's (Walrasian) excess demand vector  $x(t) \equiv x^T(p(t))$  and the pricing rules  $f$ ; viz.,  $p(t+1) = p(t) + f(x(t), X(t))$  and  $S(t+1) = S(t) - x(t)$ . For the rest of this paper, we will assume that  $D$  is positive and constant,<sup>5</sup> that  $f(0,0) = 0$  (i.e., if purchases equal sales and actual inventories equal desired inventories, the prices don't change), and that each  $f_i$  is twice differentiable and non-degenerate at  $(0,0)$ ; thus one obtains a tractable closed model.

The appropriate notion of equilibrium for such a MBM is that of a rest state, i.e., a state  $(\bar{p}, \bar{S})$  such that if  $(p(0), S(0)) = (\bar{p}, \bar{S})$ , then  $(p(t), S(t)) = (\bar{p}, \bar{S})$  for all  $t > 0$ . It is easy to see that a necessary and sufficient condition is that  $(\bar{p}, \bar{S})$  is invariant under  $F$ , i.e.,  $F(\bar{p}, \bar{S}) = (\bar{p}, \bar{S})$ ; we refer to such states as (Barter) Steady-state Equilibria, BSE. A fixed point theorem could be invoked to prove existence of BSE, but we think it more edifying to establish a "correspondence principle". Each MBM contains



within it a Walrasian model (viz., its household sector) and it is not hard to see that the WE of this Walrasian model stand in 1:1 correspondence with BSE of the MBM. The argument is simply that if in the MBM inventories are all at desired levels, and prices at WE values so desired net trades aggregate to zero, then prices will remain steady and so will inventories; hence we have a BSE. On the other hand, if we are at a BSE, then inventories remain constant, so (outages aside) we must have desired net trades which aggregate to zero; hence we have a WE. Thus existence and efficiency<sup>6</sup> of BSE are inherited from the Walrasian model.

A more difficult and perhaps more important task is to find conditions that guarantee the stability of BSE. Stability is clearly a crucial issue: if a BSE isn't stable, then it has little economic significance in an economy subject to even the mildest of shocks. We say that a BSE  $(\bar{p}, \bar{S})$  is globally stable if, for an arbitrary initial state  $(p(0), S(0))$ , we have  $p(t) \rightarrow \bar{p}$  and  $S(t) \rightarrow \bar{S}$  as  $t \rightarrow \infty$ . If this convergence holds only for initial states in some neighborhood of the BSE, it is locally stable.

Global stability is too much to hope for in our barter model for two reasons. The first has to do with the indeterminacy of the price level. Since  $p$  is never unique in the Walrasian model, there can be no unique BSE; but it is easy to see that the definition of global stability entails uniqueness. This problem can be eliminated at the cost of minor technical complications by an appropriate normalization of  $p$ . The second difficulty is more fundamental: our dynamical process is not well defined if outages occur, i.e., if  $x_i^T(t) > S_i(t)$  for some good  $i$ . Such states can't always be avoided; indeed, one would expect an outage in good  $i$  if the initial price  $p_i(0)$  is sufficiently low. Hence our process is not even globally defined, much less globally stable. On the other hand, it is clear that outages won't occur if

we begin in a sufficiently small neighborhood of a locally stable BSE.

Local stability of a BSE  $(\bar{p}, \bar{S})$  evidently depends on the price elasticities of aggregate demand at  $\bar{p}$  as well as on the price adjustment rules; the former can be represented by the matrix  $A$  of partial derivatives of aggregate excess (flow) demand with respect to prices, evaluated at  $\bar{p}$ :

$$A = \left( \begin{array}{c} \frac{\partial x_l^T}{\partial p_k} \Big|_{p=\bar{p}} \end{array} \right)$$

We have been able to prove two stability results for our Microdynamic Barter model. Apart from some technical qualifications having to do with price level indeterminacy, the first says that if we allow sophisticated pricing rules, then specialists can (locally) stabilize any BSE at which the  $A$ -matrix is non-singular. The second result says that if the Hicksian matrix  $A$  is symmetric (or nearly so) and negative definite at a BSE then simple pricing rules (in fact, wide families of simple pricing rules) can ensure the local stability of the BSE. Negative definiteness of  $A$  has been a well-known condition for stability results in economics since Samuelson (1947). It may be interpreted as the requirement that own price effects are normal and not counteracted by cross-price effects. Near symmetry may be thought of as small income effects. Non-singularity is a much weaker condition that may be interpreted as saying no bundle of other goods is a perfect substitute for any given good.

To summarize, our MBM has several attractive features. By introducing only the single institution of trade specialists, we are able to come up with an ongoing process in which notional trading plans can be realized through logistically plausible transactions and in which prices can be adjusted over time in a simple fashion. The steady states of our process correspond precisely to the equilibria of our Walrasian benchmark, and these steady states

are locally stable under a wide range of intuitively plausible conditions.

The MBM with sophisticated price adjustment rules can be viewed as a concrete version of the "central supermarket" model of Clower-Leijonhufvud (1975), and provides strong verification of the conjecture: "Except in circumstances where trader reactions to price variation are both erratic and violent...it should be possible for the trade coordinator to devise some strategy of price adjustment that would ensure stability." (p. 186). Indeed, we are able to prove that our specialists can manage even "erratic and violent" reactions; all we require (given our standard assumptions on preferences, etc.) is that traders regard the goods as distinct.<sup>7</sup>

However, our MBM still has several major defects. We have already noted that outages can be expected if the economy is perturbed too far from equilibrium, but the logic of our model precludes any simple way of dealing with outages. The usual household constrained maximization problem (whose solution yields desired net trades) is inappropriate if desired trades might not be realized. Hence there is no direct way to define  $F$  globally. Another defect is that our barter exchange process is centralized, in that we require the services of some specialist to check that each household takes away goods from the marketplace whose value does not exceed the value of goods brought to the marketplace. Also, the maintenance of some fixed price level would seem to require the efforts of a specialist who looks at all prices and renormalizes them. Thus even with simple pricing rules, the MBM can't really be decentralized.

## 2. A Monetized Microdynamic Model (MMM)

We are hardly the first to find that money can solve many of our problems (at best we can claim that our problems are novel). The introduction of a

commodity called "money" (or "cash") as the medium of exchange and as a store of value allows us to define an exchange process that is more decentralized than our barter process and that can be defined globally. The key point is that in monetized exchange, each transaction is quid pro quo; that is, each component of a household's net trade is accompanied by an offsetting cash flow. Given quid pro quo, we need no longer postulate a centralized check-point for verifying households' budget constraints. Likewise outages need not upset household plans to any great extent, so we can hope to define a dynamical process globally. There is also reason to believe the price level will take care of itself, at least in the long run.

Let us first see what happens to our conceptual experiment when we introduce money. As before, specialists post prices for their goods each morning. Households receive their daily endowments of goods, check their cash balances carried forward from the previous day, and plan today's purchases and sales. To ensure that their plans are robust with respect to possible disappointments due to outages, we impose a finance constraint that the value of each household's purchases does not exceed its cash balances.<sup>8</sup> Consequently we need not and do not require the budget constraint that the value of planned purchases not exceed the value of planned sales of goods.

In the afternoon, householders travel around to various specialists, buying and selling goods for money at the posted prices. Each specialist accommodates his customers if possible, but turns away buyers if and when his bin of goods is empty, and sellers if and when his cash balances are exhausted. In the evening, households consume their purchases and any unsold endowments, and update their cash balances. Specialists check their bin levels and compute price adjustments as in the previous model.

It turns out to be convenient for modeling purposes to complicate this conceptual experiment a bit. Specialists may charge a "spread" between buying and selling prices; specifically, the price  $p_1^+$  at which a household can purchase a good may exceed the price  $p_1^-$  at which it can sell that good by some fixed percentage  $\sigma_1 > 0$ . We also allow for the possibility that specialists may return "excess" cash at the end of the day to "shareholder" households.

We now sketch how this story can be formalized. Each household (index  $j$  suppressed for the moment) is characterized by an endowment of goods  $s = (s_1, \dots, s_n)$ , and by preferences described by a (Patinkinesque) utility function  $U$  defined over consumption  $d = (d_1, \dots, d_n)$  and net cash income  $y$ , with current prices  $p$  and cash balances  $M$  possibly serving as shift parameters. The commodity  $M$  differs from the  $n$  "goods" commodities in that it can be stored by households, and is neither produced nor consumed.  $U$  is assumed homogeneous of degree 0 in the "nominal" variables  $y$ ,  $p$  and  $M$ . An example, which we call the Modified Cobb-Douglas (MCD)<sup>9</sup> is

$$U(d, y; M) = \sum_{i=1}^l a_i \log d_i + y/M;$$

where  $\sum_1 a_i < 1$ ,  $a_i > 0$ .

The household chooses its desired consumption  $d^*(t)$  by maximizing  $U$  subject to the finance constraint  $m^+ \leq M(t)$ , where

$$m^+ = p^+ \cdot (d-s)^+ = \sum_{i=1}^l p_i^+ \max\{0, d_i - s_i\}$$

is the planned gross expenditure required to obtain  $d$ , given bid prices  $p^+(t)$  and endowment  $s$ . The household's desired net trade is then  $x^*(t) = d^*(t) - s$ . Actual net trade  $x(t)$  may differ from  $x^*(t)$  if outages occur; in this case some given rationing rule determines  $x(t)$ .<sup>10</sup> Realized net

income for a household then consists of dividends received from specialists less net expenditures; i.e.,  $y(t) = \pi(t) - m_+(t) - m_-(t)$ , where dividends  $\pi(t)$  are described below,

$$m_+(t) = p^+ \cdot x^+(t) = \sum_{i=1}^l p_i^+ \max\{0, x_i\}$$

is realized gross expenditure, and the absolute value of

$$m_-(t) = p^- \cdot x^-(t) = \sum_{i=1}^l p_i^- \min\{0, x_i\}$$

is realized gross sales revenue. End of period cash balances for the household then are (reverting now to the use of the household index  $j$ )  $M^j(t+1) = M^j(t) + y^j(t)$ .

We also need to characterize specialists' behavior with respect to cash balances,  $M_i(t)$ . The specialist's actual net cash revenue in period  $t$  is

$$NCR_i(t) = p_i^+(t) \cdot x_i^{+T}(t) + p_i^-(t) \cdot x_i^{-T}(t);$$

occasional rationing ensures that  $M_i(t) + NCR_i(t) > 0$  for all  $t$ . Presumably, NCR will ordinarily be positive, since  $p_1^+ > p_1^-$ , while  $x_1^{+T} = \left| x_1^{-T} \right|$  near equilibrium. Hence we need some rule for distributing excess cash accumulated by specialists. We hypothesize that there is some desired cash balance  $M^*(t)$  for each specialist  $i$ , possibly depending on prices and purchases and sales volume, and that balances in excess of  $M^*(t)$  are paid out to shareholders (households):

$$\pi_i(t) = (M_i(t) + NCR_i(t) - M_i^*(t))^+$$

so next period's cash balance

$$M_i(t+1) = M_i(t) + NCR_i(t) - \pi_i(t)$$

is bounded between 0 and  $M^*(t+1)$ . Finally, we close the MMM by assuming some fixed share distribution

$$\theta_i = (\theta_i^1, \dots, \theta_i^n), \quad \theta_i^j > 0, \quad \sum_{j=1}^n \theta_i^j = 1, \quad i = 1, \dots, \ell.$$

Hence

$$\pi^j(t) = \sum_{i=1}^{\ell} \theta_i^j \pi_i(t)$$

is the dividend payment received by the  $j^{\text{th}}$  household.

We shall retain the assumptions on the price-adjustment rules  $f_i$  from the previous model, and also postulate (for decentralization) that each is simple.

We noted at the beginning of this section that certain problems regarding household behavior can be solved by the introduction of money. On the other hand, money can create new problems, in this case for specialists. The quid pro quo nature of monetary exchange allows us to regard each specialist as an autonomous agent, attempting in some sense to maximize profits. (Thus, for instance,  $D_i$  might best be thought of as arising from balancing marginal inventory storage costs against the marginal convenience yield of inventories in facilitating trade.) To properly pose an optimization problem for specialists, however, requires a lot of structure — inventory storage costs, outage penalties, information structures, stochastic specification of household arrivals and desired trades, etc. — that is extraneous to our present purposes. Hence we content ourselves here with the plausible but general rules listed above, and refer the interested reader to Friedman (1982b) for further discussion and a derivation of rules conforming to present assumptions from a simplified (but still fairly messy) optimal control problem.

### 3. Local Dynamics

The monetary model laid out in the previous section gives rise to a dynamical system that differs in crucial ways from that of our MBM. First, a state of our MMM is now a trio of non-negative vectors  $p$ ,  $S$  and  $M$ , where  $p$  and  $S$  are as in the MBM and  $M = (M_1, \dots, M_\ell, M^1, \dots, M^n)$  represents the distribution of the economy's stock of money among specialists and households. Our specifications evidently yield a globally defined dynamical system  $G$ , with  $(p(t+1), S(t+1), M(t+1)) = G(p(t), S(t), M(t))$ . Note that  $G$  conserves the total money stock  $M_T^T(t) = \sum_{i=1}^{\ell} M_i(t) + \sum_{j=1}^n M^j(t)$ , i.e.,  $M_T^T(t) = M_T^T(0)$  for all  $t \geq 0$ . A steady-state equilibrium for our Monetized model (MSE) is a fixed point  $(\bar{p}, \bar{S}, \bar{M})$  of  $G$ .

One can again use a "correspondence principle" to establish the existence of such equilibria, although the matter is more delicate than before. From  $E_0$ , a given MMM with  $\sigma = 0$ , one can extract a Walrasian model by suitably restricting the domain of the household utility functions, while retaining the household endowments. For each WE price vector  $\bar{p}$  of that Walrasian model, one can find vectors  $\bar{M}$  and  $\bar{S}$  such that  $(\bar{p}, \bar{S}, \bar{M})$  is a no-rationing MSE of  $E_0$ , and employs the same net trades as the WE. Let  $E_\sigma$  be a MMM exactly like  $E_0$  except that  $\sigma > 0$ . One then can find a no-rationing MSE of  $E_\sigma$  corresponding to  $(\bar{p}, \bar{S}, \bar{M})$ . For  $\sigma = 0$ , these MSE are "efficient" in the sense that all households have the same marginal rates of substitution of goods for income and these coincide with prices, i.e.,  $MRS_{iy}^j = \bar{p}_i$  for all  $j$  and  $i$ . For  $\sigma > 0$  we only have approximate efficiency in the sense that  $\bar{p}_i \equiv \bar{p}_i^- < MRS_{iy}^j < \bar{p}_i^+ = (1 + \sigma_i) \bar{p}_i$ .

Taking these existence and efficiency results as established, we shall be concerned for the rest of this section with the dynamics of an MMM,  $E$ , in a neighborhood of  $(\bar{p}, \bar{S}, \bar{M})$ , a no-rationing<sup>11</sup> MSE. In particular, we will



investigate conditions under which the economy returns to our MSE following various types of "shocks". By a temporary real shock, we mean an exogenous shift at  $t=0$  in the inventory stocks; i.e., initial conditions for the dynamics are  $p(0) = \bar{p}$ ,  $M(0) = \bar{M}$ , but  $S(0) \neq \bar{S}$ . A permanent real shock would consist of a change in household endowments or preferences (or perhaps a change in specialists' markup). We can describe such a shock by means of initial conditions that may have constituted an MSE for the original pre-shock economy, but are not an MSE of our given (post shock) economy,  $E$ . That is, for a permanent real shock, we generally have  $p(0) \neq \bar{p}$ ,  $S(0) \neq \bar{S}$  (perhaps), and  $M(0) \neq \bar{M}$ , although  $M_T^T(0) = \bar{M}_T^T$ . The MMM economy  $E$  is locally stable at  $(\bar{p}, \bar{S}, \bar{M})$  if  $(p(t), S(t), M(t)) \rightarrow (\bar{p}, \bar{S}, \bar{M})$  as  $t \rightarrow \infty$  for all sufficiently mild<sup>12</sup> real shocks.

Again, the local stability analysis for  $E$  is similar to that for a corresponding MBM but a bit more delicate. In general, stability of a MSE requires that the "real sector" of the economy be stable; i.e., the pricing rule stabilizes demand in the sense of Section 3 above, given the matrix<sup>13</sup>

$$A_{\sigma} = \left( \begin{array}{c|c} \frac{\partial x^*}{\partial p} & \frac{\partial x^*}{\partial k} \\ \hline \frac{\partial x^*}{\partial p} & \frac{\partial x^*}{\partial k} \end{array} \right)_{p=\bar{p}}$$

Restricting our attention to simple pricing rules, we must therefore rule out aggregate demand functions that yield eigenvalues of  $A_{\sigma}$  with positive real part — roughly speaking, we rule out the possibility that there is a basket of goods with the Giffen property.

Stability of the real sector does not suffice, however. If the composition of aggregate demand responds sensitively to the distribution of  $M_T^T$ , we could have a self-reinforcing process wherein an initial shock causes a shift in demand, which induces a shift in income that intensifies the demand

shift as cash balances adjust. The simplest way to eliminate this possibility is to assume that distributional effects are small, i.e.,  $x^T$  is relatively insensitive to small changes in  $M$ , for  $M_T^T$  fixed.<sup>14</sup> Finally, to avoid price level stickiness due to reallocations of  $M_T^T$  from households to specialists (or vice versa), we make the convenient assumption<sup>15</sup> that specialists' desired cash balances are proportional to their prices, ceteris paribus. We refer to this as homogeneous desired balances. This assumption will serve its purpose if specialists' have positive payout in equilibrium, i.e., if  $\sigma \gg 0$ .

Given these additional conditions, an extension of the local stability proposition for the MBM may be established. Specifically<sup>16</sup>:

Proposition 1 (Local Stability). A no-rationing steady-state equilibrium  $(\bar{p}, \bar{S}, \bar{M})$  of an MMM economy  $E_\sigma$  with  $\sigma \gg 0$ , is locally stable if the following conditions are satisfied:

- (a) The pricing rule  $f$  is stabilizing given  $A_\sigma$ ;
- (b) distributional effects are sufficiently small; and
- (c) specialists have homogeneous desired balances.

A similar analysis applies to nominal shocks. An exogenous shift at  $t=0$  in the distribution of cash balances or in prices is a temporary nominal shock, i.e.,  $p(0) \neq \bar{p}$ ,  $M(0) \neq \bar{M}$  but  $S(0) = S$  and  $M_T^T(0) = M_T^T$ . A nominal shock is permanent if  $M_T^T(0) \neq \bar{M}_T^T$ . The local stability result above establishes that mild temporary nominal shocks have only transient effects, but the case of permanent nominal shocks requires a little further analysis.

Given that specialists and households have homogeneous cash balances (the latter being a consequence of the maintained assumption of degree zero homogeneity of preferences), it is easy to see that we have money neutrality in

the sense that  $(c\bar{p}, \bar{S}, c\bar{M})$  is a MSE if  $(\bar{p}, \bar{S}, \bar{M})$  is, for any  $c > 0$ . However, even an equiproportionate change in cash balances does not result in an immediate shift to the new MSE; unless we suspend our price adjustment rules,<sup>17</sup> the best we can hope for is that  $p(t) \rightarrow c\bar{p}$  as  $t \rightarrow \infty$  following an exogenous shift of  $\bar{M}$  to  $c\bar{M}$  at  $t=0$ . The more interesting case of a permanent nominal shock which is not equiproportionate must a fortiori be analyzed in asymptotic terms.

Let  $(\bar{p}, \bar{S}, \bar{M})$  now refer to an MSE of  $E$  before the shock, and for some  $c > 0$  suppose the shock consists of an increase in the total money stock of  $(c-1)100\%$ , i.e.,  $M_T^T(0) = c\bar{M}_T^T$ , with distribution arbitrary. We say that the asymptotic neutrality property (ANP) holds if  $(p(t), S(t), M(t)) \rightarrow (c\bar{p}, \bar{S}, c\bar{M})$  as  $t \rightarrow \infty$ ; i.e., if all nominal quantities respond in proportion to the change in the total money stock. Hence our neutrality property is an equilibrium relationship whose validity depends on the stability properties of the economy. Recall that even to obtain stability with respect to real shocks, we required mechanisms that remove "distortions" in the distribution of the money stock. Hence it is not surprising that these same mechanisms also ensure ANP:

Proposition 2 (Local Asymptotic Neutrality). Conditions (a)-(c) of Proposition 1 guarantee that ANP also holds for mild permanent nominal shocks at a no-rationing steady-state equilibrium  $(\bar{p}, \bar{S}, \bar{M})$  of an MMM economy  $E_\sigma$  with  $\sigma \gg 0$ .

Corollary: The Equation of Exchange  $M_T^T V = PT$  holds asymptotically with constant  $V$  and  $T$  under the conditions of the Propositions.

The argument for the corollary is as follows. Let  $\bar{x}$  be the vector of transactions occurring at  $(\bar{p}, \bar{S}, \bar{M})$ ; for arbitrary  $x$  and  $p$ , define the index numbers  $P = p^+ \cdot \bar{x}^{+T} / \bar{p}^+ \cdot \bar{x}^{+T}$ ,  $T = \bar{p}^+ \cdot x^{+T} / p^+ \cdot x^{+T}$  for prices and transactions, and let  $V = (M_T^T)^{-1}$ . Following a nominal shock, if ANP holds, we asymptotically reach a new equilibrium  $(\bar{p}, \bar{S}, \bar{M}) = (c\bar{p}, \bar{S}, c\bar{M})$ . But  $\bar{M}_T^T V = \bar{M}_T^T / \bar{M}_T^T = c$ ,  $\bar{T} = 1$  since  $\bar{x} = \bar{x}$  (due to homogeneity of demands in  $M$  and  $p$ ), and  $P = (c\bar{p}^+) \cdot \bar{x}^{+T} / (\bar{p}^+ \cdot \bar{x}^{+T}) = c$ . Hence we have  $\bar{M}_T^T V = \bar{T} = c$  for constant  $V$  and  $T$  at the new equilibrium as long as ANP holds.

Our neutrality result is reminiscent of that of Howitt (1974). He posited a price adjustment rule that depended only on excess flow demands in a model without explicit inventories, and hence had rationing except at equilibrium. Under the strong assumption of Gross Substitutes (see Arrow, Block and Hurwicz, 1959; Howitt (1974) extended this assumption to include a real balance effect) and an assumption on the rationing scheme (or alternatively that there were no distribution effects), he used a system of differential equations approximating his model's discrete time dynamics and demonstrated global asymptotic neutrality. Our Proposition 2 can be viewed as an extension of Howitt's result to an economy with a more general price adjustment process and weaker assumptions on excess demand. We obtain only local neutrality because our assumptions allow for multiple equilibria.

#### 4. Discussion

We have shown that it is possible to model a logistically plausible exchange process in which trading plans can normally be realized even when the plans are not mutually consistent. We have also specified decentralized real-time adjustment processes for prices and stocks of goods (and money), and explained some simple stability and neutrality results. Before presenting

some final perspectives, a brief discussion of some of our modeling short-cuts may be in order.

In our models we employed only one specialist for each good. We really have in mind a situation in which many specialists compete in selling each good (or closely related goods), so arbitrage would enforce essentially unified prices. In that case, any spread  $\sigma > 0$  should reflect specialists' costs of storing and transacting. It is not easy to model these activities explicitly, especially if the economy as a whole is adjusting, but it would be desirable to derive specialists' desired inventory levels, cash balances, and pricing rules from optimizing behavior in a continuous time stochastic setting. Friedman (1982b) begins this task.

As a second short cut, we have followed Patinkin in putting money (cash income in our case) directly into the utility function. Of course, we really believe that households value current income only to the extent that it enhances future consumption opportunities. It does so in our model for two logically distinct reasons: cash is the only store of value for households and, given the finance constraint and updating rules for  $M(t+1)$ , it should be regarded as the sole means of payment. Again it would be desirable to pose an appropriate intertemporal stochastic optimization problem that incorporates these roles, whose solution would yield indirect single period utility functions of the sort we have postulated.<sup>18</sup>

We do not believe, however, that the local dynamics we have discussed here are at all sensitive to our short cuts. Our existence results are obtained from quite general "correspondence" principles, and the local stability and neutrality results arise from a study of the linearized total excess demand, pricing rules, etc. at a steady-state equilibrium. Given the quite general specification of these functions, it seems clear that they will

include the linearized versions of functions derived from more fully articulated optimization problems.

Our analysis generally supports the view that the basic (static) Walrasian model is a reasonable approximation to the long-run tendencies of an ongoing "flexprice" market organized by trade specialists. Of course, our Propositions apply only to "mild shocks" that do not cause any outages (inventory stock outages or cash outages) and consequent rationing. The MMM admits adjustment paths (following "severe shocks") that exhibit these inefficiencies over a prolonged transitory period,<sup>19</sup> so the model can exhibit something akin to Leijonhufvud's (1973) "effective demand failures" and "corridor effects". We have not emphasized such phenomena here because we believe that a proper understanding of them requires an examination of "fixprice markets" (such as that for labor) and more sophisticated financial arrangements than direct cash-for-goods spot markets. Clearly much work remains to be done before one can construct a theory of the sort called for by Hicks, but in describing the dynamic interactions of individual agents in terms of a simple institutional structure, we believe we have taken a crucial first step.

## FOOTNOTES

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<sup>1</sup>A non-trivial problem even at known market-clearing prices, see Ostroy and Starr (1974).

<sup>2</sup>Also, one notes that specialists play slightly different roles in different markets. For instance, in foreign exchange markets, brokers don't hold inventory ("open positions") but dealers do; brokers are price takers in lumber markets (Balderston and Hoggatt (1962)) but price makers in grain markets. We will employ a fairly general specification of the specialist's role in the present study.

<sup>3</sup>Throughout this paper we assume for convenience that households satisfy standard strong assumptions to guarantee an interior solution  $d^j(t)$  to the optimization problem <sup>so it</sup> that is a differentiable function of its parameters; e.g., the assumptions  $s^j \gg 0$ ;  $U^j$  smooth, strictly convex and monotone, and a strong boundary condition, suffice.

<sup>4</sup>That  $x^T = S(t) - S(t+1)$  assumes no stock outages; presumably  $D(t)$  is chosen largely to make outages extremely unlikely. We will assume away outages for the next few paragraphs.

<sup>5</sup>This is not as severe a restriction as it might seem. One can show that a model with  $D_1$  depending on last period desired purchases and sales (probably the most relevant information, see Friedman (1982b)) yields a model that is essentially equivalent to one with constant  $D_1$ .

<sup>6</sup>That is, households' marginal rates of substitution all coincide with relative prices.

<sup>7</sup>More precisely, that  $A$  has maximum rank. In defining  $A$ , we use strong standard assumptions on preferences to ensure the differentiability of demand functions. If one is willing to tolerate more complicated statements and proofs, there seems to be no obstacle to relaxing these assumptions.

<sup>8</sup>The finance constraint can be rationalized with a Kohn (1981) story that households consist of a wife (who, for a change, let's say, sells labor or other goods) and a husband (who does all the shopping); the assumption that the husband and wife do not meet during the working day yields the finance constraint. One should also note that a finance constraint is more natural than a budget constraint in a continuous-time setting.

<sup>9</sup>Those who don't like to see cash balances  $M$  look "bad" should feel free to use the utility function  $V = (M/P)U$ , where  $P$  is some appropriate price index;  $U$  and  $V$  yield the same demand functions.

<sup>10</sup>We won't need to employ specific rationing rules in this paper, but certain extensions of our analysis (e.g., to effective demand failures) would require them. Basically, any rationing rule will do, as long as (a) it doesn't provide incentives for households to misrepresent their desired trades (thus rationing proportional to  $x^*$  is not acceptable); (b) no component of  $x$  exceeds that of  $x^*$  in absolute value, or differs in sign; and (c)  $x = x^*$  if feasible. An example of an acceptable rule is "rationing by priority": households with lower indices  $j$  are allowed to transact first.

<sup>11</sup>That is,  $x^* = x$  at  $(\bar{p}, \bar{S}, \bar{M})$ . It turns out that MSE with rationing ( $x^* \neq x$ ) are also possible.

<sup>12</sup>Technically, "mild" means there is some given open neighborhood  $N$  of  $(\bar{p}, \bar{S}, \bar{M})$  such that  $(p(0), S(0), M(0)) \in N$ .

<sup>13</sup> $A_\sigma$  is not well defined on a set of measure zero in  $M$  and  $p$ , a fact we can safely ignore here. See Friedman (1982).



<sup>14</sup>Distributional effects will be zero under many common assumptions, e.g., identical homothetic preferences.

<sup>15</sup>It appears that in the absence of this assumption there may be shifts in the MSE, greatly complicating the statement of Propositions 1 and 2.

<sup>16</sup>Propositions 1 and 2 below are proved in Friedman (1982, Proposition 5).

<sup>17</sup>It might make sense to suspend the rules if the precise nature of a forthcoming shock were common knowledge; we prefer to keep the rules and regard the shock as a surprise, the nature of which is only gradually realized.

<sup>18</sup>See Howitt (1974) and Grandmont (1982) for a discussion of these matters.

<sup>19</sup>Preliminary results suggest that these effective demand failure equilibria can be asymptotically stable if  $\sigma = 0$ , but are unstable if  $\sigma \gg 0$ . Hence the presumption is that our economies eventually return to some sort of no-rationing MSE following severe shocks.

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