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## INSIDER TRADING ANOMALIES IN A KYLE-TYPE MODEL OF SEQUENTIAL AUCTIONS

by

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***INSIDER TRADING ANOMALIES IN A KYLE-TYPE  
MODEL OF SEQUENTIAL AUCTIONS<sup>1</sup>***

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*July 1993*

IUI, Stockholm

**ABSTRACT:** The sequential auction model proposed in the paper extends the insider trading model by Kyle [1985]. It is demonstrated that insider trading can lead to market anomalies resulting from the abnormal behavior of participants. When outsiders (non-informed small rational traders) are too risk-tolerant, it becomes beneficial for the insider to destabilize market prices at some trading rounds. Multiple solutions to the insider problem are demonstrated and examined by means of a numerical example. The solution chosen by the insider is time-consistent and providing that prices convey the minimal information to outsiders.

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### **1. Introduction.**

The main argument commonly suggested in favour of insider trading is that it improves the efficiency of the stock market. Insiders are best informed (by definition) about actual asset values and if they are allowed to trade, market prices convey their private information to other participants. As a consequence, total welfare will be improved, since risks of the parties are reduced and the losses of outsiders from trade are compensated by a more efficient allocation of investment.

Some theoretical papers discuss this argument in detail. In Glosten and Milgrom [1985] it is demonstrated, how insiders' information is disseminated to market specialists and uninformed traders in a pure dealership market. In the dynamic model of sequential auctions by Kyle [1985] a risk neutral insider confronts risk neutral competitive market makers, who set efficient (in the semi-strong sense) prices, and irrational noise traders. Although the private information is reflected by prices, the insider makes positive profits by exploiting his monopoly power. In the finite number of trading dates (rounds) information is transmitted by prices only gradually, but as the frequency of auctions tends to infinity, insider information is completely incorporated into prices by the end of the period.

However, the ability of insiders to transmit information to the market is sometimes called into question. For example, in Laffont and Maskin [1990] it is argued, that it may be reasonable for an insider to ensure that market prices do not convey private information. It may happen, if the variability of asset returns is sufficiently small and allows the insider to choose "pooling" equilibria in a two-stage signalling game. In this case the stock market is inefficient, since there are no incentives to reveal the true information and to accept it.

It is argued here that insider trading can lead to market inefficiencies, but these situations are related to some trade anomalies, which in turn result from the abnormal behavior of participants. We propose an extended Kyle-type model of sequential auctions [Kyle 1985] to demonstrate how such anomalies can emerge. Besides the three kinds of participants acting in that model, small risk averse portfolio-based investors also participate. Following Leland [1992] we call them *outsiders*. The insider takes into account the influence of trade on beliefs of market makers and outsiders. Unlike to the former expecting to obtain zero profits from each trade and setting a price equal to their conditional expectations, the latter use price signals to update at each trading date their prior estimates of the asset value in the Bayesian fashion. They do not trust to current market prices completely, but only to a degree which corresponds to the relative accuracy of prices. The value estimate they use is actually a moving average of current and past prices. Therefore market makers and outsiders diverge in their opinions.

Such divergence of beliefs sometimes makes it beneficial for the insider to destabilize market prices. As is shown in the paper it is possible under

*abnormal* asset pricing, when the excess supply (demand) of shares causes market makers to increase (decrease) the price even though it is set in efficient way. Such abnormal behavior of market makers can occur because the behavior of outsiders or insiders is abnormal and market makers take into account the anomalies in their trading and, in turn, set abnormal prices. If, for instance, outsiders are tolerant to risk (the risk-aversion degree is too small), they become too sensitive to the divergence of their beliefs from the current price. In this case an excess current supply of shares will signal to market makers that the asset is actually undervalued rather than overvalued and they will increase the price.

Insiders, in turn, can destabilize trade by making extra profits from the divergence of beliefs of market makers and outsiders. As it is demonstrated by means of a numerical example, it may be beneficial for the insider first to trade in the abnormal way and to incur some losses in order to destabilize the market and then to obtain high speculative profits recouping the losses. In particular, it may be the case under an abnormally low degree of risk aversion by outsiders. The abnormal trading and price setting effects are impossible in the original Kyle model of sequential auctions [Kyle 1985].

Another anomaly which also does not happen in that model [Kyle 1985] is the multiplicity of solutions to the insider problem. The solution to it is obtained by the backward induction in a similar way to [Kyle 1985]. The backward induction provides a set of endogenous parameters that determine the behavior of market participants (quantities and prices for each trading date), given a realization of noise trading. We call this set a *trade technology*. As the numerical analysis shows, the trade technology is well defined as a

sequence of endogenous parameters given the terminal zero-profit condition and the terminal variance of beliefs about the asset value, if the exogenous parameters of the model (the degree of risk-aversion by outsiders, the variance of noise trading and the terminal variance of beliefs) are not too small. In this case the trade technology can be computed before the stock market starts to work in the forward recursion.

If, for example, the risk-aversion elasticity is low, it may happen that there are two *trading date equilibria* at some trading round, i.e. two sets of endogenous parameters related to this round both satisfying the second-order condition. In this case the insider has to choose between these equilibria according to the Maximum Principle. Actually the trade technology is represented by a tree with the root at the terminal date and endpoints at the initial date. Therefore the insider has to reoptimize at each trading date comparing the expected profits on each *path* of the tree. Theoretically, it can lead to a time-inconsistent behavior of the insider. However, the sequential profit maximization turns out to be time-consistent in the case of a particular example considered in the paper. The path of the trading tree initially chosen is preferable to other paths at all subsequent trading dates. As computations show, prices determined by this path convey the minimal information and they are the least sensitive to quantities traded by the participants.

The paper is organized as follows. Section 2 presents the model of sequential trading, Section 3 provides a solution to the model as a *recursive linear equilibrium*. The existence of trading date equilibria is examined in Section 4. The anomalies of pricing and trading behavior of participants are discussed in Section 5. Section 6 demonstrates the possibility of multiple

solutions at the penultimate trading date. A numerical example of the abnormal insider trade is analysed in Section 7.

## 2. The model.

The model is an extended Kyle-type sequential trade model of the non-competitive stock market. In the Kyle model [Kyle 1985], referred here as KM, two infinitely divisible assets - risky and riskless - are exchanged between three kinds of traders: an insider having the access to a private information and influencing asset prices, market makers who set these prices in the efficient way and irrational (liquidity) noise traders completely indifferent to price movements. In our model uninformed, portfolio-based risk averse small investors are also participating. Following Leland [1992] we call them *outsiders*.

Trade occurs over a trading period (trading day in KM) consisting of  $N$  sequential auctions (trading rounds) or trading dates:  $n = 1, \dots, N$ . At each date trading takes place in the following way. The insider and noise traders choose simultaneously and independently quantities of risky asset to be traded and place the market orders to market makers. The latter set an efficient price schedule based on observable current and past trades, spread the orders among outsiders and clear the market.

The insider knows at the beginning of the trading period the realization  $v$  of the *ex post* future value of the risky asset  $\tilde{v}$  normally distributed with mean  $p_0$  and variance  $\Sigma_0$ . (We use tilda to distinguish between stochastic variables and their realizations.) He does not observe current prices and quantities currently supplied by noise traders. At each trading date the insider



is maximizing the expected speculative profit  $E_{n-1}\tilde{\pi}_n = E_{n-1} \sum_{k=n}^N (v - \bar{p}_k) \Delta \bar{x}_k$  conditional on the information available at date  $n-1$ . The following notation is used here:  $E_{n-1}$  is the conditional expectation operator;  $\bar{p}_k$  is the risky asset price at date  $k$  and  $\Delta \bar{x}_k = \bar{x}_k - \bar{x}_{k-1}$  is the insider's net order to market makers,  $\bar{x}_k$  - quantity purchased at date  $k$ . The return from the safe asset is normalized to one. The insider acts as an intertemporal informed monopolist taking into account the effect of his current actions on beliefs and behavior of other participants.

Market makers do not know the realization of the asset value  $v$  and do not observe noise trading. They accept the total market order of the parties and cannot distinguish between order flows coming from noise traders and other participants. They set an efficient price  $p_n$ ,  $n = 1, \dots, N$ , equal to the expected asset value conditional on the information available from current and past trading. Although market makers are not explicit maximizers, they expect to earn zero profits from any auction, where they implicitly compete in the Bertrand fashion [Kyle 1985].

Outsiders do not know exactly the asset value realization, but they estimate it from price information. Their initial beliefs about  $v$  are normal with mean  $p_0$  and variance  $\Sigma_0$ . Outsiders have mean-variance preferences over the terminal wealth  $\bar{W}_N = W_0 + \sum_{n=1}^N (\bar{v}_n - \bar{p}_n) \Delta \bar{z}_n$ , where  $W_0$  is the initial wealth,  $\bar{v}_n$  is the current mean estimate of the asset value,  $\Delta \bar{z}_n = \bar{z}_n - \bar{z}_{n-1}$  is net purchases of the risky asset at date  $n$ . Outsiders behave as pure price-takers; unlike the insider and noise traders they observe a current market price. They accept prices  $p_n$  as signals about the asset value updating prior estimates  $\bar{v}_{n-1}$  in the Bayesian fashion (naturally,  $v_0 = p_0$ ).

Noise traders effect the stock market by random net purchases  $\Delta \tilde{u}_n$ ,  $n = 1, \dots, N$ , that are normal and i.i.d. with zero mean and variance  $\sigma^2$ , and independent of the asset value. The parameters of noise trading are common knowledge. The noise traders' activity is usually justified by some exogenous reasons, for example, by the so called "liquidity preference motive" (Glosten and Milgrom [1985], Kyle [1985], Leland [1992]). It is also conventional to treat them as symmetric traders buying and selling the asset with equal probability. The asymmetric behaviour of noisy liquidity traders is discussed in Allen and Gorton [1992].

In line with KM consider trading and pricing rules as functions of relevant observations. When choosing the trade volume  $x_n$ , the insider observes a price history  $p^{(n-1)} = \{p_1, \dots, p_{n-1}\}$ . Hence, his position at the  $n$ -th auction is given by a function  $x_n = X_n(p^{(n-1)}, v)$ . Outsiders base their trade on the current value estimate  $v_n$  and the price history  $p^{(n)}$ :  $z_n = Z_n(p^{(n)}, v_n)$ . Market makers observe the history of trade flows  $h^{(n-1)} = \{x_1 + u_1, \dots, x_{n-1} + u_{n-1}\}$  and the current order flow  $\Delta x_n + \Delta u_n$ . The efficient price schedule  $p_n$  is determined as  $p_n = P_n(h^{(n)})$ .

Denote trading strategies of the insider and outsiders as  $X = \langle X_1, \dots, X_N \rangle$  and  $Z = \langle Z_1, \dots, Z_N \rangle$ , respectively, and the pricing rule applied by market makers as  $P = \langle P_1, \dots, P_N \rangle$ . Let  $\tilde{\pi}_n = \pi_n(X, Z, P) = \sum_{k=n}^N (v - \tilde{p}_k) \Delta \tilde{x}_k$  stands for the profit acquired by the insider at auctions  $n, \dots, N$  as a function of trading strategies and a pricing rule. The certainty equivalent of the end-of-period wealth of outsiders expected at date  $n$  is  $U_n(X, Z, P) = E[\tilde{W}_N | p^{(n)}, v_n] - (a/2) \text{Var}[\tilde{W}_N | p^{(n)}, v_n]$ , where  $a$  is the absolute elasticity of risk-aversion.

### 3. Sequential auction equilibrium.

A *Sequential Auction Equilibrium* is defined for our model as: 1) a pair of trading strategies  $X$  and  $Z$ , 2) a pricing rule  $P$  and 3) a family of conditional beliefs  $g_n(\tilde{v}|p^{(n)})$  that provide for  $n = 1, \dots, N$ :

i) *Sequential profit maximization*: for all  $X'$  such that  $X_1' = X_1, \dots, X_{n-1}' = X_{n-1}$  it is fulfilled:  $E[\pi_n(X, Z, P)|p^{(n-1)}, v] \geq E[\pi_n(X', Z, P)|p^{(n-1)}, v]$ .

ii) *Expected utility maximization*: for all  $Z'$  it is fulfilled:  $U_n(X, Z, P) \geq U_n(X, Z', P)$ .

iii) *Rationality of expectations*: The conditional probability  $g_n(\tilde{v}|p^{(n)})$  is updated by outsiders in the Bayesian fashion.

iv) *Market efficiency*: Asset prices are set by market makers as:  $p_n = E(\tilde{v}|h^{(n)})$ .

Define, following KM, a *recursive linear equilibrium* as a sequential auction equilibrium in which the parties use linear trading strategies and the pricing rule is given recursively as:

$$\Delta \tilde{p}_n = \tilde{p}_n - \tilde{p}_{n-1} = \lambda_n(\Delta \tilde{x}_n + \Delta \tilde{z}_n + \Delta \tilde{u}_n), \quad (1)$$

where  $\lambda_1, \dots, \lambda_N$  are parameters determined from the market efficiency condition. They characterize the depth of the market: the smaller  $\lambda_n$ , the larger trade volume is required for the percentage change of the asset price. The market at date  $n$  is deeper when  $\lambda_n$  is lower. The recursive linearity of equilibrium follows from the pricing rule (1) that, in turn, results from the assumption that both the asset value and noise trading are normally and independently distributed.

The sequential profit maximization (i) implies maximization of the expected profit from trade conditional on the values  $v, p_0, \Sigma_0$ : for all trading strategies  $X'$  it is fulfilled:

$$i') \quad E_0\{\pi_1(X, Z, P)\} \geq E_0\{\pi_1(X', Z, P)\}.$$

However, the reverse is not true. Define a *consistent recursive linear equilibrium* as a triple  $(X, Z, P)$  satisfying i') and ii)-iv) given that trading and pricing rules are linear in corresponding observations.

The following theorem characterizes the linear recursive equilibrium as a solution to a system of difference equations subject to some terminal conditions.

**Theorem.** *Suppose there exists a recursive linear equilibrium. Then in this equilibrium for all trading dates  $n = 1, \dots, N$  it is fulfilled:*

$$\Delta \tilde{x}_n = \beta_{1n}(\tilde{v} - \tilde{p}_{n-1}) + \beta_{2n}(\tilde{v}_{n-1} - \tilde{p}_{n-1}), \quad (2)$$

$$\Delta \tilde{z}_n = \gamma_n(\tilde{v}_{n-1} - \tilde{p}_{n-1} - \lambda_n(\Delta \tilde{x}_n + \Delta \tilde{u}_n)), \quad (3)$$

$$\Delta \tilde{p}_n = C_n(\Delta \tilde{x}_n + \Delta \tilde{u}_n) + \lambda_n \gamma_n(\tilde{v}_{n-1} - \tilde{p}_{n-1}), \quad (4)$$

$$E\{\tilde{\pi}_n | p^{(n-1)}, v\} = \alpha_{1n-1}(v - p_{n-1})^2 + \alpha_{2n-1}(v - p_{n-1})(v_{n-1} - p_{n-1}) + \alpha_{3n-1}(v_{n-1} - p_{n-1})^2 + \delta_{n-1}, \quad (5)$$

$$\tilde{v}_n = \xi_n \tilde{p}_n + (1 - \xi_n) \tilde{v}_{n-1}, \quad (6)$$

where  $\xi_n$  is the relative accuracy of price signal,  $0 < \xi_n < 1$ .

Given the terminal conditional variance  $\Sigma_N$  and the terminal zero-profit condition on the parameters:  $\alpha_{1N} = \alpha_{2N} = \alpha_{3N} = \delta_N = 0$ , the set of endogeneous parameters  $T = \{\beta_{1n}, \beta_{2n}, \alpha_{1n}, \alpha_{2n}, \alpha_{3n}, \lambda_n, \gamma_n, \Sigma_n, \xi_n, \delta_n\}_{n=1}^N$  is a solution to the backward induction system:

$$\beta_{1n} = \frac{1 - C_n(2\alpha_{1n} + \alpha'_{2n})}{2C_n(1 - C_n\alpha_n)} \quad (7)$$

$$\beta_{2n} = -\frac{\gamma_n \lambda_n}{2C_n} + \frac{\lambda_n \gamma_n \alpha_n - (2\alpha'_{3n} + \alpha'_{2n})}{2(1 - C_n \alpha_n)} \quad (8)$$

$$\alpha_{1n-1} = \alpha_{1n} A_n^2 + \alpha_{3n}'(1-A_n)^2 + \beta_{1n} A_n - \alpha_{2n}' A_n(1-A_n), \quad (9)$$

$$\alpha_{2n-1} = -2\alpha_{1n} A_n B_n - 2\alpha_{3n}'(1-A_n)(1-B_n) + \beta_{2n} A_n - \beta_{1n} B_n + \alpha_{2n}' B_n(1-A_n) + \alpha_{2n}' A_n(1-B_n), \quad (10)$$

$$\alpha_{3n-1} = \alpha_{1n} B_n^2 + \alpha_{3n}'(1-B_n)^2 - \beta_{2n} B_n - \alpha_{2n}' B_n(1-B_n), \quad (11)$$

$$C_n = \beta_{1n} \Sigma_n / \sigma^2, \quad (12)$$

$$\gamma_n = 1/(a\Sigma_{n-1} + \lambda_n), \quad (13)$$

$$\xi_n = \beta_{1n} C_n, \quad (14)$$

$$\Sigma_n = (1 - \xi_n) \Sigma_{n-1}, \quad (15)$$

$$\delta_{n-1} = \delta_n + \alpha_n \lambda_n^2 \sigma^2, \quad (16)$$

subject to the second order condition:

$$C_n(1 - C_n \alpha_n) > 0, \quad (17)$$

where  $A_n = 1 - \xi_n$ ,  $B_n = C_n \beta_{2n} + \lambda_n \gamma_n$ ,  $C_n = \lambda_n(1 - \lambda_n \gamma_n)$ ;

$$\alpha_{2n}' = A_n \alpha_{2n}, \quad \alpha_{3n}' = A_n^2 \alpha_{3n}, \quad \alpha_n = \alpha_{1n} + \alpha_{2n}' + \alpha_{3n}'.$$

**Proof.** In the essence it replicates the proof of Theorem 2 in [Kyle 1985].

Let (5) be a solution to the insider problem. Then it must satisfy:

$$\begin{aligned} E\{\tilde{\pi}_n | p^{(n-1)}, v\} &= \max_{\Delta x} E\{(v - \tilde{p}_n) \Delta x + \alpha_{1n}(v - \tilde{p}_n)^2 + \alpha_{2n}(v - \tilde{p}_n)(\tilde{v}_n - \tilde{p}_n) + \\ &\alpha_{3n}(\tilde{v}_n - \tilde{p}_n)^2 + \delta_n | p^{(n-1)}, v\}. \end{aligned} \quad (18)$$

In a linear recursive equilibrium prices change according to (1). Outsiders' trade at date  $n$  is a linear function of price:

$$\Delta \tilde{z}_n = (\tilde{v}_n - \tilde{p}_n) / a \Sigma_n.$$

This and the Bayesian updating rule (6) with the price equation (1) imply:

$$\Delta \tilde{z}_n = \gamma_n (\tilde{v}_{n-1} - \tilde{p}_{n-1} - \lambda_n (\Delta \tilde{x}_n + \Delta \tilde{u}_n)), \quad (19)$$

where  $\gamma_n$  is given by (13).

Inserting (19) into (1) and this into (18) yields a profit-maximizing insider trade:

$$\Delta \tilde{x}_n = \beta_{1n}(v - \bar{p}_{n-1}) + \beta_{2n}(\tilde{v}_{n-1} - \bar{p}_{n-1}), \quad (20)$$

where  $\beta_{1n}$  and  $\beta_{2n}$  are given by (7) and (8).

From (18) and (20) it is easy to get the difference equations on the parameters  $\alpha_{in}$ ,  $i=1,2,3$ ,  $\delta_n$  that ensure the optimality of trade. Summing up the items related to  $(v - p_{n-1})^2$ ,  $(v - p_{n-1})(v_{n-1} - p_{n-1})$ ,  $(v_{n-1} - p_{n-1})^2$  and the residual item in the both sides of (18) implies these equations as (9)-(11), (16). Thus, given  $\lambda_n$ ,  $\Sigma_n$  and  $\xi_n \in (0,1)$  the trade (20) satisfying the second order condition (17) is the solution to the insider problem.

Parameters  $\lambda_n$ ,  $\Sigma_n$  and  $\xi_n$  are obtained from the market efficiency condition which takes the form (details are found in [Kyle 1985]):

$$\bar{p}_n - \bar{p}_{n-1} = E\{\tilde{v} - \bar{p}_{n-1} \mid \Delta x_n + \Delta u_n\}.$$

Application of the projection theorem for normally distributed random variables confirms the linearity of the pricing rule (1) and yields that

$$C_n = \frac{\beta_{1n}\Sigma_{n-1}}{\beta_{1n}^2\Sigma_{n-1} + \sigma^2} \quad (21)$$

$$\Sigma_n = (1 - \xi_n)\Sigma_{n-1} = \frac{\sigma^2\Sigma_{n-1}}{\beta_{1n}^2\Sigma_{n-1} + \sigma^2} \quad (22)$$

which imply (12), (14), (15). □

The dynamics of trades, prices and value estimates is specified in the forward recursion as (2)-(6) given the initial price  $p_0$ , the noise trading

realization and the set of endogenous parameters  $T$ . Equations (7) through (16) determine this set in the backward fashion.

Parameters  $\beta_{1n}, \beta_{2n}$  ( $n = 1, \dots, N$ ) specify the insider's behavior at the auction  $n$ : they measure the intensity of his trade subject to the *ex post* error of market makers ( $v - p_{n-1}$ ) and the deviation of the outsiders' asset value estimate from the market price ( $\bar{v}_{n-1} - v$ ). One can represent (2) as the weighted sum of *ex post* errors of market makers and outsiders:

$$\Delta \bar{x}_n = \beta_n(v - \bar{p}_{n-1}) + \beta_{2n}(\bar{v}_{n-1} - v), \quad (23)$$

where

$$\beta_n = \beta_{1n} + \beta_{2n} = \frac{1 - 2C_n \alpha_n}{2\lambda_n(1 - C_n \alpha_n)} \quad (24)$$

is the parameter similar to the constant  $\beta_n$  given by eq.(3.15) in [Kyle 1985].

Parameters  $\alpha_{1n}, \alpha_{2n}, \alpha_{3n}$  determine the profit expected by the insider at date  $n$  as a square form of date  $n-1$  mistakes of market makers and outsiders. Parameters  $\alpha_{1n}$  and  $\alpha_{3n}$  relate the expected profit to the absolute errors of both parties, while  $\alpha_{2n}$  indicates how much the insider will gain from the divergence of their beliefs. Parameter  $\delta_n$  ascertains the value of future trading opportunities for the insider.

Parameter  $\gamma_n$  indicates the intensity of outsider trade. The product of parameters  $\lambda_n \gamma_n$  shows, if the outsider trade is stabilizing or destabilizing the stock market at date  $n$ . As it follows from (3), outsiders are less flexible to the insider trading if  $\lambda_n \gamma_n < 1$ , and more flexible if  $\lambda_n \gamma_n > 1$ . Parameter  $C_n$

determines the sensitivity of market prices to the insider trading. It is positive if  $\lambda_n > 0$  and the outsider trading is stabilizing prices.

As it was mentioned, market makers currently observe total order flows of market participants. Market makers, however, can determine the volumes traded by outsiders as (3). Therefore the price equation (1) is written as (4). It says that the price movement is determined by the current insider trade plus noise trade and by a deviation of outsiders' asset value estimate from the market price. The price schedule (4) is actually a convex linear combination of increments  $\lambda_n(\Delta\tilde{x}_n + \Delta\tilde{u}_n)$  and  $(\tilde{v}_{n-1} - \tilde{p}_{n-1})$ . The more sensitive are outsiders to prices, the more significant for market makers the deviation of their beliefs from market prices. Note, that according to (6),  $\tilde{v}_{n-1}$  represents a moving average of past prices  $\tilde{p}^{(n-1)}$  and, hence, the difference  $\tilde{v}_{n-1} - \tilde{p}_{n-1}$  means the deviation of the current price from the tendency of prices.

The set of endogeneous parameters  $T$  is computed in backward induction given a terminal value of the conditional variance  $\Sigma_N$  and the zero-profit terminal condition  $\alpha_{1N} = \alpha_{2N} = \alpha_{3N} = \delta_N = 0$ . The set  $T$  specifies the *trade technology* used by the insider, i.e. trading and pricing rules that determine the behavior of participants. Since endogenous parameters including conditional variances  $\Sigma_n$  are computed in the backward fashion, it is more convenient to condition the family of trade technologies  $T$  by the terminal error variance  $\Sigma_N$  instead of the initial variance  $\Sigma_0$ . If the backward induction system (7)-(17) gives a unique solution for all dates  $n = 1, \dots, N$ , there is one-to-one correspondence between the initial and the terminal error variances. Hence, the trade technology is determined by three exogenous parameters: the degree of risk aversion  $\alpha$ , the variance of noise trade  $\sigma^2$ , and



the terminal error variance  $\Sigma_N$  as  $T = T(a, \sigma^2, \Sigma_N)$  and computed in backward fashion before the stock market starts to work in the forward recursion (2)-(6). However, the system (7)-(17) does not generally provide a unique solution: it is possible that at some trading rounds the insider has to choose among different sets of endogenous parameters. We discuss this problem in what follows.

#### 4. Existence of trading date equilibria.

Theorem from the previous section provides *interior* solutions to the insider problem: it must hold that the relative accuracy of a price signal  $\xi_n$  strictly belongs to the unit interval at each trading date. Consider how these solutions are found iteratively.

The parameters of trade technology are considered as recursive or non-recursive. Given date  $n$  parameters of the backward system (7)-(17), parameters  $\alpha_{1n-1}, \alpha_{2n-1}, \alpha_{3n-1}, \Sigma_{n-1}, \delta_{n-1}$  of the preceding trading date  $n-1$  are determined recursively from the equations (9)-(11), (15)-(16). The non-recursive parameters  $\beta_{1n}, \beta_{2n}, \lambda_n, \gamma_n, \xi_n$  form a non-linear algebraic system (7)-(8), (12)-(14), given current recursive parameters  $\alpha_{1n}, \alpha_{2n}, \alpha_{3n}, \Sigma_n$ . At each date  $n$  the backward induction determines current parameters of trade technology in two steps. First, date  $n$  recursive parameters are computed from the difference equations (9)-(11), (15)-(16), given date  $n+1$  parameters. Second, date  $n$  non-recursive parameters are found from the system (7)-(8), (12)-(14), given date  $n$  recursive parameters.

Define a *trading date equilibrium* as a solution to the non-recursive system (7)-(8), (12)-(14) satisfying the second-order condition (17) and the

constraint  $0 < \xi_n < 1$ . This equilibrium specifies the trade expected to take place at date  $n$ : the intensity of insider trade and outsiders' response, the sensitivity of market price (the depth of the stock market) and the relative accuracy of price signal. The non-recursive system (7)-(8), (12)-(14) can be reduced to a three-dimensional system (7), (12), (14) that determines parameters  $\beta_{1n}$ ,  $C_n$  and  $\xi_n$ . The other non-recursive parameters  $\lambda_n$ ,  $\gamma_n$  and  $\beta_{2n}$  are then found directly from  $C_n$ , (13) and (8).

**Proposition 1.** *Suppose at some trading date  $n$  it is fulfilled:  $\alpha_{1n} > \alpha_{3n} > 0$ . Then there exists a trading date equilibrium such that  $0 < C_n < C_n^* = \Sigma_n^{1/2}/\sigma$ .*

**Proof.** From (12) and (14)  $\beta_{1n} = C_n \sigma^2 / \Sigma_n$  and the relative accuracy of a price signal is:

$$\xi_n = C_n^2 \sigma^2 / \Sigma_n. \quad (25)$$

Inserting this into  $\alpha_{2n}'$ ,  $\alpha_{3n}'$  and into (7) we have an equation on  $C_n$ :

$$C_n = F_n(C_n) \quad (26)$$

where  $F_n(C_n)$  is the right-hand side of (7) multiplied by  $\Sigma_n/\sigma^2$ .

From (7)  $F_n(C_n) \rightarrow +\infty$  as  $C_n \rightarrow 0$ . Since  $\alpha_{1n} > 0$ , we have:  $F_n(C_n^*) < 1/2(\sigma/\Sigma_n^{1/2}) < C_n^* = \Sigma_n^{1/2}/\sigma$ . Hence, the equation (26) has a solution located between 0 and  $C_n^*$ , if the second order condition  $C_n \alpha_n < 1$  is fulfilled for all  $C_n$  in the interval  $(0, C_n^*)$ .

If some  $C_n$  in this interval is equal to  $1/\alpha_n$ , then  $F_n(C_n)$  has a vertical asymptote inside  $(0, C_n^*)$  and tends to  $-\infty$  as  $C_n$  goes to  $1/\alpha_n$  from the left by the assumption that  $\alpha_{1n} > \alpha_{3n}$ . Consequently, in this case there also exists a solution to (26).  $\square$

The condition of Proposition 1 means, that the insider expects to receive gains from the errors of market makers and outsiders at date  $n-1$ , and the mistake of the former is more valuable. If trading date equilibria exist at all trading rounds, then the recursive linear equilibrium exists and is given by (2)-(17).

The proof provides an iterative procedure solving the system (7), (12), (14) that can be applied for numerical computations of the backward system (7)-(17). This iterative procedure is interpreted as a search of the relative accuracy of prices  $\xi_n = C_n^2 \sigma^2 / \Sigma_n$  brought about by the insider. He chooses some initial value  $\xi_n^0$  from the admissible interval  $(0, 1)$ , computes the responses of market makers and outsiders, compares  $C_n^0 = \xi_n^0 \Sigma_n / \sigma^2$  to  $F_n(C_n^0)$ , then chooses a new value  $\xi_n = \xi_n^1$  and so forth.

Most likely there exists a unique trading date equilibrium in the case of Proposition 1. This conjecture has not been proved yet, but numerical results further discussed support it. If the condition of the Proposition does not hold, it may happen, that at some date the system of non-recursive parameters (7), (12), (14) provides an even number of interior solutions (in fact zero or a couple).

Suppose there are two interior solutions to the non-recursive system (7), (12), (14) at some trading date  $n$ , both satisfying the second order condition (17), i.e. two local maximum points. Let the first solution provides a set of recursive parameters  $\alpha_{1n-1}$ ,  $\alpha_{2n-1}$ ,  $\alpha_{3n-1}$ ,  $\Sigma_{n-1}$ ,  $\delta_{n-1}$  for the preceding date according to the recursive equations (9)-(11), (15)-(16) and the second one - another set of the same parameters. In line with the optimality principle the insider has to choose between these two sets comparing corresponding

expected profits given by the square form (5). But he is unable to do this, unless the current asset price  $p_{n-1}$  and the asset value expectation  $v_{n-1}$  are unknown. These variables are to be computed by the forward recursion (2)-(6) given the history of noise trading and the trade technology  $T$  (the solution to the backward system (7)-(17)). It means, that the insider is not able to choose the trade technology before the stock market starts to work in the forward fashion in line with (2)-(6).

In the case of multiple solutions the trade technology is represented as a tree with the root at date  $N$  and the top at the initial date  $0$ . The nodes of this tree correspond to the trading dates in which there are two or more trading date equilibria. Then at each date  $n = 1, \dots, N$  the insider chooses the *best branch* of the tree given date  $n$  parameters of the expected profit (5):  $\alpha_{in}$ ,  $\delta_{ni}$ ,  $i=1,2,3$  and date  $n-1$  asset price  $p_{n-1}$  and value expectation  $v_{n-1}$ . He actually has to reoptimize at each trading date that may lead to time-inconsistent solutions. Clearly, the time-inconsistency problem does not arise in the case when trading date equilibria exist and unique for all trading rounds, since there is no need for the insider to compare expected profits in the computation of trade technology. This problem also does not arise if the insider acts in line with the consistent recursive linear equilibrium (i'), (ii)-(iv), i.e. he precommits to a particular path of the trading tree selected before the market starts to work. We examine an example of trading tree in Section 7.

## 5. Abnormal price setting.

Define trade at some date  $n$  as *abnormal*, if  $\lambda_n < 0$ . It means that the efficient price responds to the current flow of market orders in abnormal way. For example, the inference of market makers from the observation of excess demand at the trading round  $n$ :  $\Delta x_n + \Delta z_n + \Delta u_n > 0$  will be that the asset value is overestimated rather than underestimated by the price at date  $n-1$ . Consequently, they will set a new price  $p_n$  at a lower level than  $p_{n-1}$ .

This can occur by the following reason: either outsider or insider trade is abnormal and market makers take it into account and, in turn, set abnormal prices. If outsiders are too sensitive to the divergence of their beliefs from current prices, then  $\lambda_n \gamma_n > 1$  and  $C_n > 0$  if  $\lambda_n < 0$ . The second order condition (17) is then fulfilled if  $\alpha_n > 0$  and  $C_n$  is not too large. In this case market makers place too much weight on the increment  $(\bar{v}_{n-1} - \bar{p}_{n-1})$  in the price schedule (4) and outsiders are responsible for such abnormal price movement.

Insiders can destabilize trade by making extra profits from the divergence of beliefs of market makers and outsiders. According to (5), the insider expects to gain from the *ex-post* mistakes of market makers and outsiders if  $\alpha_{1n} > 0$  and  $\alpha_{3n} > 0$ , and from a divergence of their beliefs depending on the sign of  $\alpha_{2n}$ . When  $\alpha_{2n}$  is negative, it is profitable for the insider if both parties are making mistakes in the same direction, but the error of outsiders is higher (i.e. either  $v < p_{n-1} < v_{n-1}$  or  $v > p_{n-1} > v_{n-1}$ ). We can say, that the insider is exploiting the relative mistake of outsiders if  $\alpha_{2n} < 0$  and  $\alpha_{3n} > 0$ . It may happen, that at some date  $\alpha_n$  becomes negative because  $\alpha_{2n} < 0$  and its absolute value is large. In this case the insider exploits the mistake of outsiders at the expense of the reduction of future profit

opportunities  $\delta_{n-1}$  in (16). Since  $\alpha_n$  can be negative, the second order condition can hold under negative  $C_n$  if  $\lambda_n < 0$  but  $\gamma_n > 0$ . In other words, market makers overweight the increament  $\lambda_n(\Delta\bar{x}_n + \Delta\bar{u}_n)$  in the price schedule (4) and the price is set in abnormal way despite the normal behavior of outsiders.

As the numerical analysis discussed in what follows shows, the behavior of the both parties becomes abnormal under low values of  $a$ ,  $\sigma^2$  or  $\Sigma_N$ . It is easy to show, that the trade at the terminal date is abnormal, if and only if the degree of risk aversion, the variance of noise trade or the terminal error variance is small.

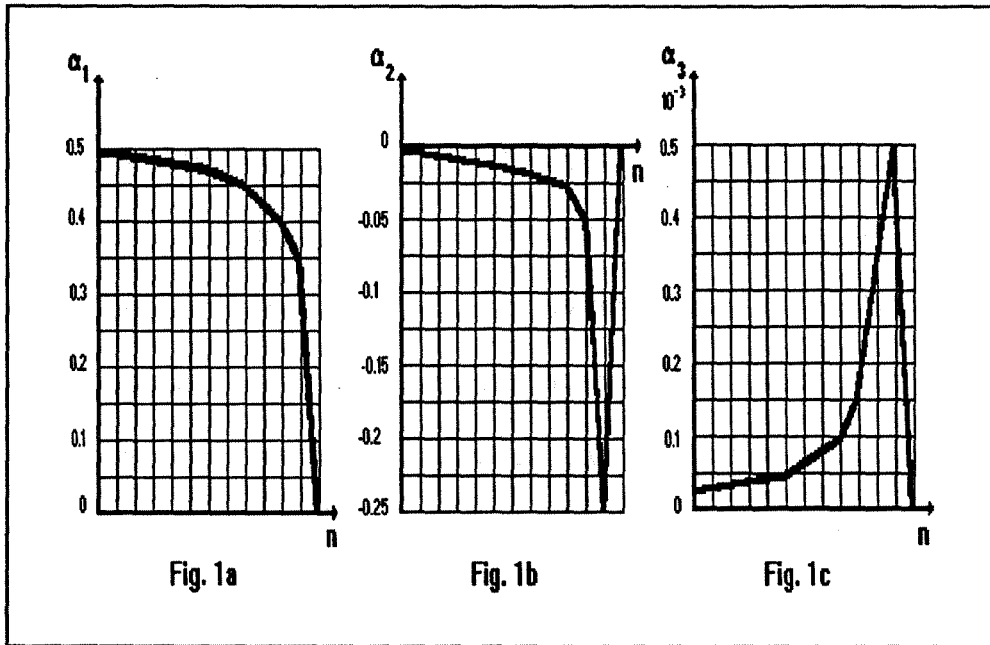
**Proposition 2.** *The terminal trade is abnormal, if and only if  $a\Sigma_N^{1/2}\sigma < 1/2\sqrt{2}$ .*

**Proof.** Equations (7), (12) and the zero-profit condition:  $\alpha_{iN} = 0, i=1,2,3$ , imply that  $C_N = \Sigma_N^{1/2}/\sigma\sqrt{2}$ . From (7)  $\beta_{iN} = \sigma/\Sigma_N^{1/2}\sqrt{2}$  and from (14)  $\xi_N = 1/2$ . Hence, (13) and (15) imply that  $\lambda_N = a\Sigma_N / \sqrt{2}(a\Sigma_N^{1/2}\sigma - 1/2\sqrt{2})$ . Consequently,  $\lambda_N < 0$ , if and only if  $a\Sigma_N^{1/2}\sigma < 1/2\sqrt{2}$ .  $\square$

The numerical analysis of the penultimate trading round presented in the next section shows, that conditions of the same kind ensure uniqueness and existence of a pair of trading date equilibria. Note, that the larger is the number of trading rounds, the smaller is the terminal error variance  $\Sigma_N$  given the asset value variance  $\Sigma_0$ . It means that the condition of Proposition 2 will not hold for sufficiently long trading periods and the terminal date trade will be abnormal if trading occurs throughout many rounds.

Proposition 2 suggests the necessary and sufficient condition for the normal behavior of outsiders ( $\gamma_N > 0$ ,  $\lambda_N\gamma_N < 1$ ). Similarly, the behavior of the insider at some trading date presumably is normal (in the sense that  $\alpha_n >$

0) if the values of exogenous parameters are not too small. The numerical analysis supports this idea. Figure 1 shows the dynamics of endogenous parameters  $\alpha_{1n}$ ,  $\alpha_{2n}$ ,  $\alpha_{3n}$  in the case of 12 trading dates, given the exogenous parameters:  $a = 1$ ,  $\sigma^2 = 100$ ,  $\Sigma_N = 100$ . Numerical simulations demonstrate, that this picture is typical and  $\alpha_n > 0$ , when the degree of risk aversion, the variance of noise trade and the terminal variance of beliefs are not too low. These computations are summarized in the following way.



**Observation 1.** *If the exogenous parameters  $a$ ,  $\sigma^2$  and  $\Sigma_N$  are not too low, then at each date  $n = 1, \dots, N-1$  the trade is normal and it is fulfilled:  $\alpha_{1n} > 0$ ,  $\alpha_{2n} < 0$ ,  $\alpha_{3n} > 0$ ,  $\alpha_n > 0$ ,  $\beta_{1n} > 0$ ,  $\beta_{2n} < 0$ .*

As the numerical example in Section 7 shows, it is possible that  $\alpha_n$ ,  $C_n$  and  $\lambda_n$  are negative at some trading round given at least one of the exogenous parameters is small.

The effects of abnormal trading are impossible in the original Kyle model of sequential auctions [Kyle 1985]. They are simply forbidden by the

second order condition that is similar to (17):  $\lambda_n(1 - \alpha_n \lambda_n) > 0$ , where  $\lambda_n$ ,  $\alpha_n$  are parameters of KM similar to  $\lambda_n$  and  $\alpha_n$  in our model. Since  $\alpha_n > 0$  at all trading dates in KM, the second order condition does not hold for negative  $\lambda_n$ . It means that it is not beneficial for the insider first to destabilize the market at the expense of some losses and then to recoup the losses and to obtain high speculative profits. However, it may be beneficial in the case of our model under abnormally low values of exogenous parameters.

#### 6. The case of multiple solutions.

A high degree of non-linearity of the backward induction system (7)-(17) makes it difficult for the formal analysis. The non-recursive part of this system (7)-(8), (12)-(14) is reduced to the equation (26), that determines trading date equilibria. This equation can be represented as a 7th order polynomial. Therefore it is reasonable to analyze the system (7)-(17) numerically. By means of computer we are able, first, to estimate exactly the number of solutions for different values of the exogenous parameters  $a$ ,  $\sigma^2$ ,  $\Sigma_N$  and, second, to examine directly trade technologies in the case of multiple trading date equilibria.

As would be expected, the system performs well if the exogenous parameters are "normal", i.e. they are not too low. There is a unique trading date equilibrium at each trading round, i.e. a unique  $C_n$  solving (26), satisfying the second-order condition (17) and belonging to the admissible interval  $(-\Sigma_n^{1/2}/\sigma, \Sigma_n^{1/2}/\sigma)$ , for which the accuracy of prices  $\xi_n$  is strictly between zero and unity. (Besides that, the trading date equilibrium is normal, i.e. providing  $\lambda_n > 0$  at each date.) Hence, there is a unique linear recursive equilibrium



and the trade technology  $T = T(a, \sigma^2, \Sigma_N)$  is time-consistent and determined before the market begins to operate.

It is easy to show, that there is always a unique trading date equilibrium at the terminal date  $N$ , and  $C_N = \Sigma_N^{1/2}/\sigma\sqrt{2}$ . However, as it follows from Proposition 2, it may happen, that the trade is abnormal at this date. The numerical example in the next section illustrates this case.

Consider trading date equilibria for the penultimate trading round  $N-1$ . We have examined the equation (26) numerically for this date under different values of exogenous parameters  $a$ ,  $\sigma^2$  and  $\Sigma_N$ . Computations revealed the following opportunities: either a unique or a coupled trading date equilibria exist, or they do not exist.

Our numerical results are summarized in figure 2. It demonstrates the regions of one, two and no trading date equilibria for the round  $N-1$  in the plane of exogenous parameters and for three levels of the terminal error variance  $\Sigma_N = 0.01, 1, 100$ . There are three shaded regions in the figure corresponding to the different levels of  $\Sigma_N$ . Parameters  $\sigma^2$  and  $a$  belonging to these regions provide a couple of trading date equilibria, those lying above the shaded zones correspond to the case of a unique equilibrium; if they are located below the shaded domains, there is no solution. One can see, that the higher the terminal error variance, the larger the domain of a unique solution and the smaller other regions. The two interesting facts have been established numerically.

**Observation 2.** 1) *An interval of risk-aversion degree  $a$  providing the existence of a unique, a coupled trading date equilibria or non-existence is identical for the pairs of variances: a)  $\Sigma_N = 100$ ,  $\sigma^2 = 0.01$ ; b)  $\Sigma_N = 1$ ,  $\sigma^2 = 1$ ;*

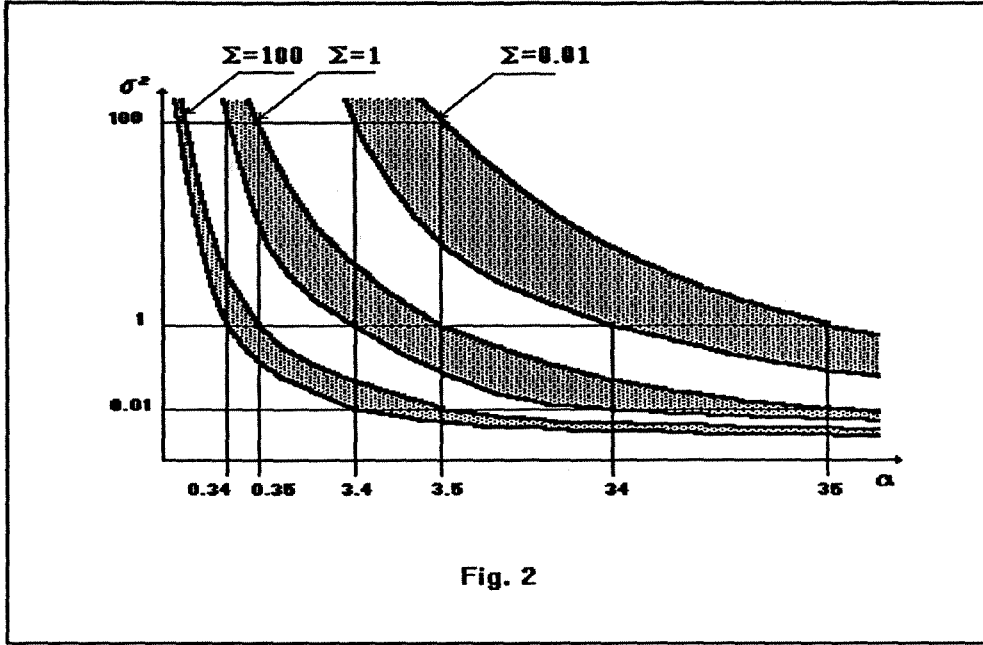


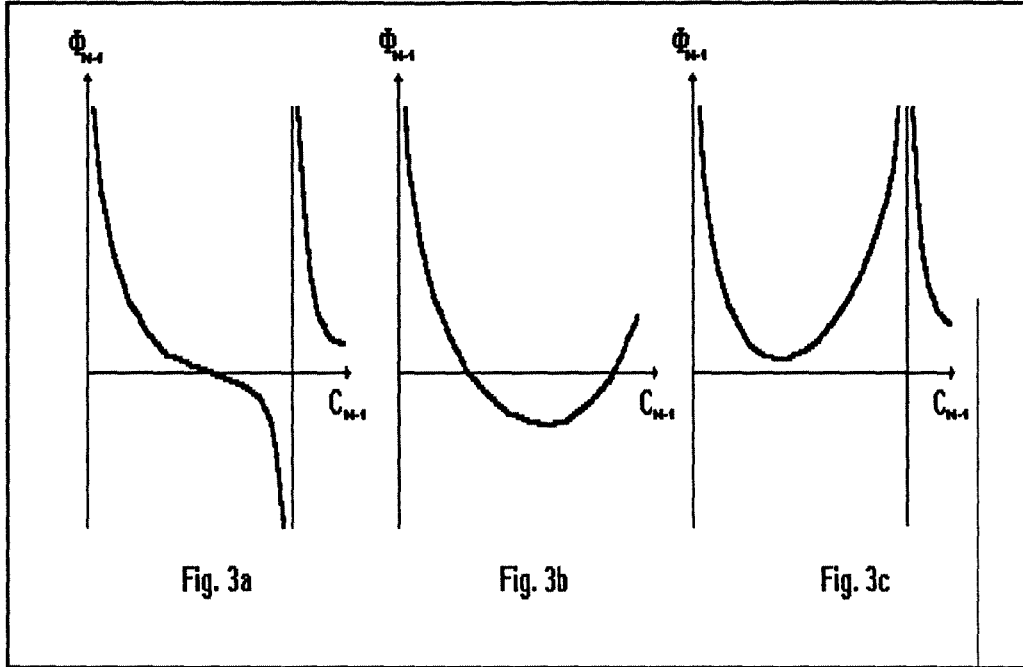
Fig. 2

c)  $\Sigma_N = 0.01$ ,  $\sigma^2 = 100$ . 2) For a given value of  $\sigma^2$  the ratio of lengths of these intervals is reciprocal to the square root from the ratio of correspondig variances  $\Sigma_N$ .

Figure 2 illustrates this Observation. The values of the risk-aversion degree  $\alpha$  are given approximately. For example, there is a couple of solutions if  $\alpha$  belongs to the interval  $[3.4, 3.5]$  and it is fulfilled:  $\sigma^2 \Sigma_N = 1$ . If, on the other hand,  $\sigma^2 = 1$ , then the interval of  $\alpha$  providing a couple of solutions is:  $[0.34, 0.35]$ ,  $[3.4, 3.5]$  and  $[34, 35]$  for  $\Sigma_N = 100$ ,  $\Sigma_N = 1$  and  $\Sigma_N = 0.01$ , correspondingly.

Figure 3 shows the graphical solutions to (26) in the cases of one, two and zero trading date equilibria at the trading round  $N-1$ . The function  $\Phi_{N-1}(C_{N-1}) = F_{N-1}(C_{N-1}) - C_{N-1}$  intersects the absciss inside an interval satisfying the second order condition and the condition  $0 < \xi_{N-1} < 1$  in one point (fig.3a), in two points (fig.3b) or in no points (fig.3c). The vertical asymptotes in the

figure correspond to the bounds of the second order condition:  $\alpha_{N-1}C_{N-1} = 1$ .



We have not examined, what happens with the market, when a trading date equilibrium does not exist, i.e. there are no solutions to (26) lying in the admissible interval of the relative accuracy of prices  $\xi_n \in (0, 1)$  and satisfying the second order condition (17). In this case the insider faces a couple of corner solutions for which  $\xi_n = 1$  or 0. If  $\xi_n = 1$ , the private information is completely revealed at date  $n$  since the error variance  $\Sigma_n$  computed in the forward induction as  $\Sigma_n = (1-\xi_n)\Sigma_{n-1}$  becomes zero (however, in this case we cannot condition a trade technology by the terminal error variance  $\Sigma_N$ ). If  $\xi_n = 0$  the date  $n$  price does not convey any new information about the asset value. There is a similarity between these extreme cases and the notions of completely separating and pooling equilibria in signalling games [Laffont, Maskin 1990].

## 7. An example of abnormal trade: the trading tree and the sequential reoptimization.

Consider a numerical example demonstrating the insider's choice among multiple solutions. Suppose the unconditional asset value mean is  $p_0 = 1000$ , the number of trading dates is  $N = 7$  and both noise trade and terminal error variances are not too small:  $\sigma^2 = 100$ ,  $\Sigma_N = 100$ . But let the degree of risk aversion by outsiders be sufficiently low:  $a = 0.0035$ , so the condition of Proposition 4 is not fulfilled. This example illustrates the insider trading anomalies, that can occur due to a non-cautious behavior of outsiders.

Figure 4 shows, how the trading tree looks like. It consists of *points*, each corresponding to a unique or multiple trading date equilibria and denoted by squares in the graph, and *branches* connecting adjacent points according to the backward induction (7)-(17). A sequence of branches and points connecting the endpoints (i.e. the points at the top of the tree) with the origin is called the *trading path*. A *node* is a point which is an immediate predecessor to more than one points. Given a trading path, a node corresponds to a trading date when there are multiple trading date equilibria. The trading tree depicted in figure 4 consists of 6 trading paths drawn as bold lines and 5 nodes denoted as bold squares.

In the case of our example all trading date equilibria exist at each round and there are only double solutions at each node. (We have not discovered examples of trading trees with more than two trading date equilibria).

Denote a point in the tree as  $P_{in}$ , where  $i$  is a trading path number,  $i = 0, 1, \dots, 5$ ,  $n$  is a number of trading date. In figure 4 the numeration of paths is given at the bottom and at the top, and the numeration of dates is at the left.

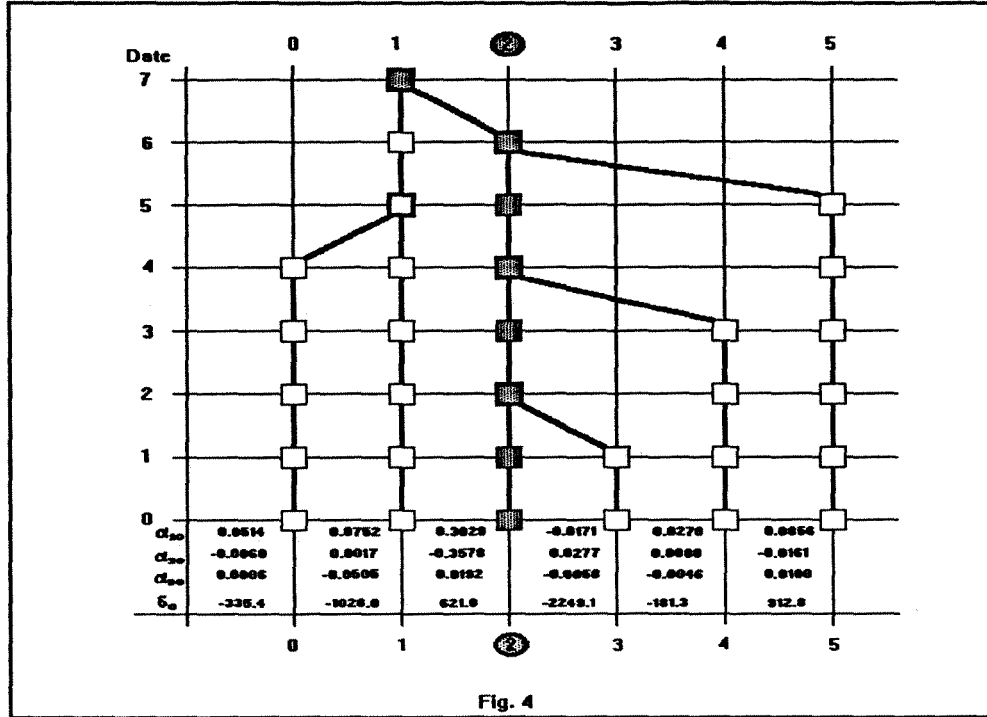


Fig. 4

For instance, the origine node in the graph is  $P_{17}$ . We add a superscript of trading path in the notation of endogenous parameters, for example,  $\alpha_{14}^2$  means  $\alpha_{14}$  in the second path.

The first path goes down from the origin  $P_{17}$  and the second path goes to the right. There are three nodes belonging to the second path:  $P_{26}$ ,  $P_{24}$ ,  $P_{22}$ . Branches growing to the right side from this nodes build up the paths 3, 4, 5. Branches going down from these nodes continue the second path. Each of the paths 2 through 5 is originated by a maximal root of the equation (26). Branches that continue the second path correspond to minimal roots of (26) which are positive. The zero path growing to the left from the node  $P_{15}$  corresponds to the minimal root of (26) which is negative. In this case  $C_5^1 < 0$ , but the second order condition (17) holds, since  $\alpha_5^1 < 0$  and its absolute value is large. The trade on the zero path is abnormal at the 5th date:  $\lambda_5^1 = -2.313$ ,  $C_5^1 = -2.851$ ,  $\gamma_5^1 = 0.1005$ ,  $\lambda_5^1 \gamma_5^1 = -0.231$ .

Another point of abnormal trade is the origin  $P_{17}$ :  $\lambda_7^1 = -69.65$ ,  $C_7^1 = -2.851$ ,  $\gamma_7^1 = -0.0145$ ,  $\lambda_7^1 \gamma_7^1 = 1.01$ . It implies that trade is abnormal at the terminal trading date for all paths of the tree. This effect is opposite to a stabilizing effect of the last date trade, which presents under "normal" conditions when risk aversion is not very small. Numerical experiments show that if the behavior of outsiders is normal and both the insider and outsiders do not destabilize prices, the effect of price stabilization is most pronounced at the last trading date. But it is precisely at this date the price drastically diverges from the asset value, if the trade is abnormal. Figure 5a demonstrates the dynamics of prices in the case, when  $v = p_0 = 1000$  and the trade technology is specified as the second path of the trading tree (the noise trade is modelled as a realization of the pseudo-random variable  $N(0,100)$ ). In our example the price raises significantly at the last trading round when large quantities are sold by the insider, as it is shown in figure 5b. One can interpret it as an emergence of a large speculative asset bubble. The insider obtains large profit at the expense of outsiders as a result of abnormal price movement at the last trading date. Thus, outsiders who are rational in the Bayesian sense are cheated by a "wrong" price signal.

We have chosen the second trading path to demonstrate the effect of abnormal trade at the terminal date, because it is this path, that typically (under various initial conditions) dominates the others in terms of expected trade profits. Consider, using our numerical example ( $v = p_0 = 1000$ ,  $\sigma^2 = 100$ ,  $\Sigma_N = 100$ ,  $a = 0.0035$ ), the selection of the trading path for different asset value realizations:  $v = 500, 750, 1000, 1250, 1500$ .

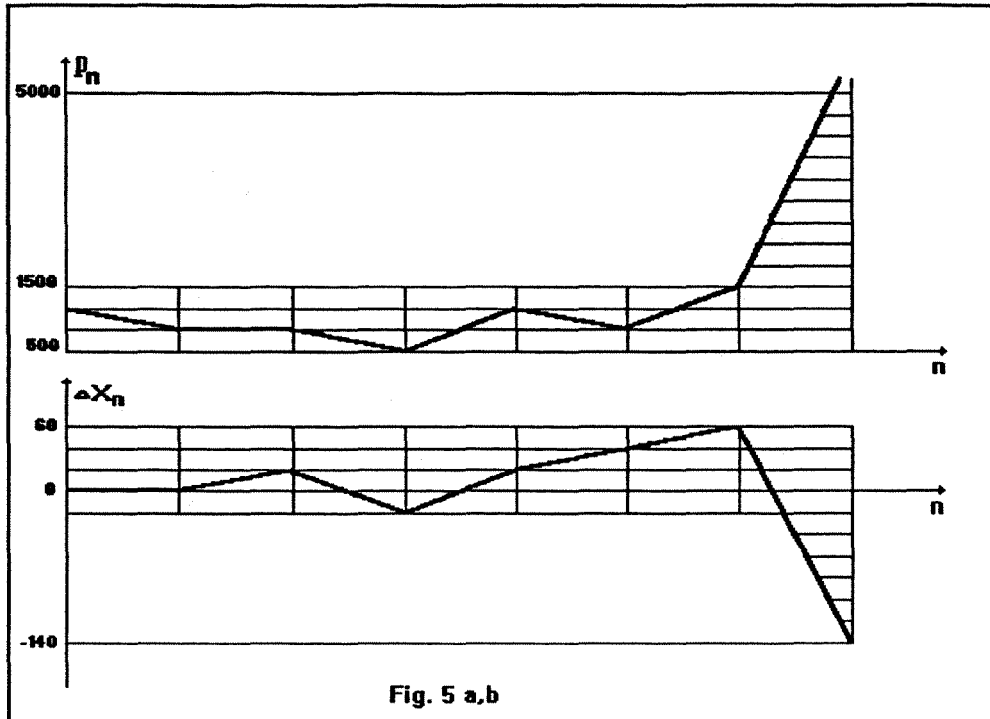


Fig. 5 a,b

The insider compares expected profits (5) at each trading round. The parameters of expected profits are computed in the backward fashion for each trading path. Their values at the initial date 0 are given at the bottom line of the tree graph (fig.4). As it turns out, the second trading path provides the maximal expected profit for all above asset value realizations and for all trading dates  $n = 1, \dots, 6$  at which multiple trading date equilibria exist.

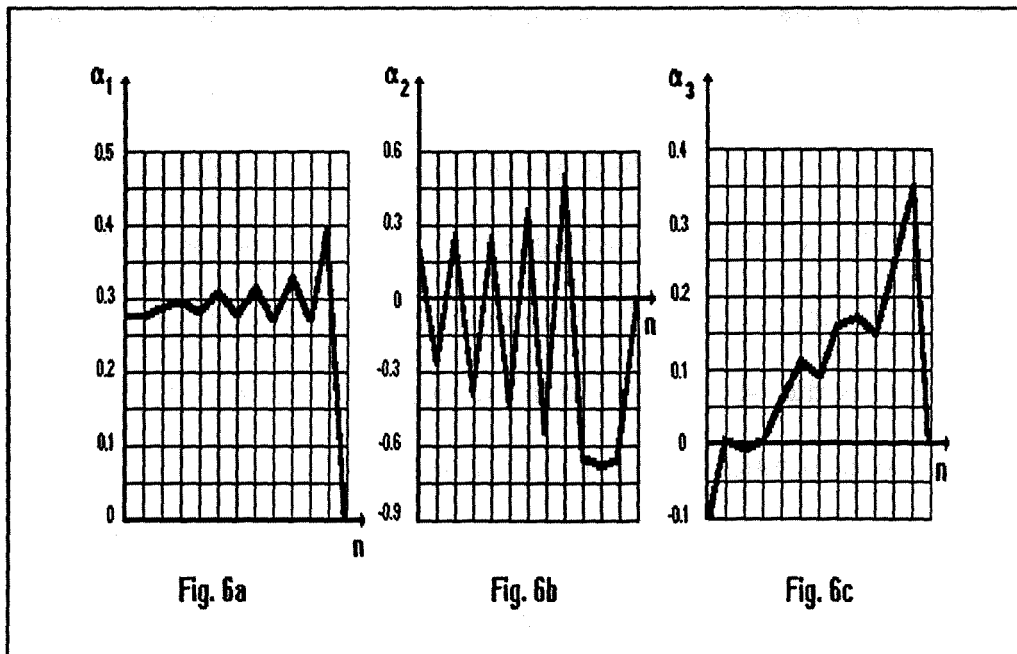
Thus, the insider will pick up the second path of the trading tree, regardless of combinations of mistakes made by the other parties. Since the second path is the best solution at all subsequent dates  $n = 1, \dots, 6$ , it is the time-consistent solution for the particular values of  $v$ .

We have extended the trading period in this example to 12 dates to check the robustness of these numerical result. It turns out, that the additional trading dates preceding the 7 original dates, do not add new nodes and branches to the tree. The extended tree consists of the same 6 (prolonged)

Number of path	0	1	2	3	4	5
$\alpha_{10}$	0.0578	0.0824	0.2871	0.0147	0.0333	0.0873
$\alpha_{20}$	-0.0018	-0.0064	0.1699	-0.0030	-0.0034	-0.0003
$\alpha_{30}$	0.0004	-0.0284	-0.0098	-0.0052	-0.0034	0.0059
$\delta_0$	1333.7	-333.4	950.1	420.2	23256.6	2216.2

Table 1.

trading paths as above. But the set of initial parameters of the expected profit (5) modifies. (See Table 1; Figure 6 demonstrates the trajectories of the parameters  $\alpha_{in}$ ,  $i = 1, 2, 3$ , for the second path in the case of 12 trading dates.



One can compare this picture with the one in figure 1, typical for a "normal" degree of risk aversion.) However the second path is again more



preferable at all dates  $n = 1, \dots, 12$  for the above values of  $v$ . We tried to reveal numerically some features peculiar for the second path. They are summarized in the following observation.

**Observation 3.** *For all 12 trading dates parameters  $\xi_n$ ,  $C_n$ ,  $\lambda_n$  are minimal for the optimal (the second) trading path, while parameters  $\gamma_n$ ,  $\alpha_{1n}$  are maximal for this path.*

As a result the insider in our example will choose the trade technology such that:

- prices convey the minimal information to the market;
- prices are the least sensitive to trade (in the sense of [Kyle, 1985] the market is the most deep);
- outsiders are the most responsive to prices;
- profits obtained from market makers' mistakes are the highest.

Of course, an open question is to what extent one can generalize this observation. We also have not looked for the examples of time-inconsistent solutions, when the insider changes the trading path at some dates. Probably, it may occur under another realization of noise trading, in particular, if the variance  $\sigma^2$  is higher than in the above example. In this case the risk aversion elasticity  $a$  should be taken less than 0.0035 to satisfy the abnormal trade condition from Proposition 2.

## 8. Concluding remarks.

The paper demonstrates that under certain circumstances insider trading in the stock market can lead to abnormal outcomes. In particular, if uninformed traders are too risk-tolerant, it may be beneficial for insiders to

send wrong price signals by destabilizing the trade. The effects of abnormal price setting revealed in our model can explain some anomalies that happen in reality, in particular, because of non-cautious behavior of uninformed traders. However, normally, i.e. when outsiders are sufficiently risk averse and the variance of noise trade is not too low, this does not occur and one should expect prices to converge to the asset value as the number of trading dates increases.

These considerations are related to some abnormal situations and do not justify the legal rules against insider trading. From our view losses from price destabilizing insider trading are in many cases compensated by positive effects of the improved investment allocation. This argument is suggested by means of a simple model of insider trade [Leland 1992] with explicit investment behavior of firms.

The model proposed here was applied by the authors in numerical experiments with the Micro-to-Macro model of Swedish Economy (MOSES) [Albrecht et al, 1989]. As the simulations showed, the stock market with insider trading improves the long-run efficiency of investment, if firms are responsive to stock market prices [Antonov, Trofimov 1992]. As it is argued by Eliasson [1990 p.293-294], the rules that do not allow members of the competent team to hold stock in their business and to trade with it, prevent the efficient allocation of managerial competence. Rather than prohibiting insider trading the legislators should be more concerned about efficient signalling devices quickly detecting the emergence of insiders in the market.

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