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The Structure of the ISAC Model

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THE STRUCTURE OF THE ISAC MODEL

by

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May, 1981

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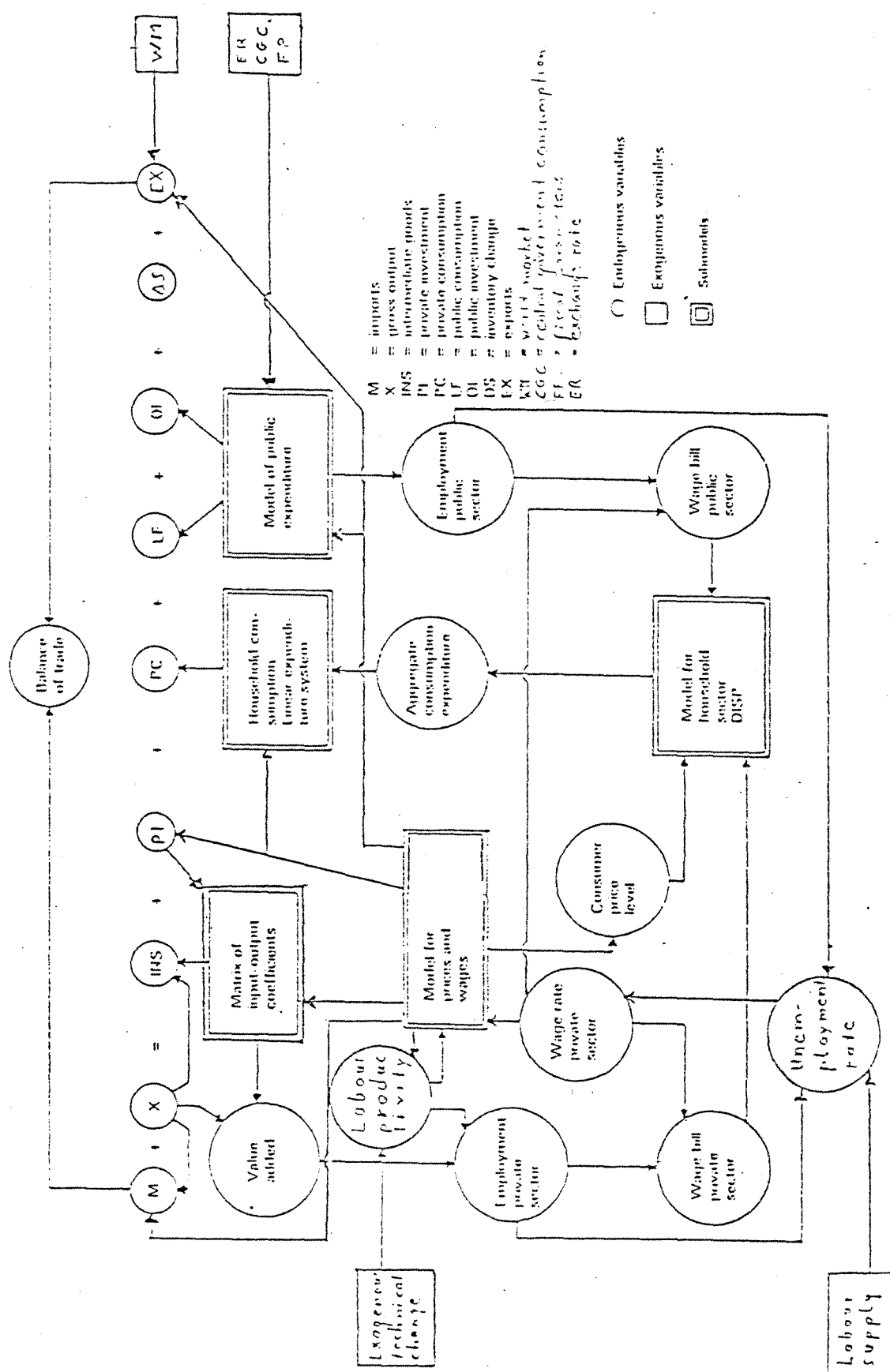
A compact presentation of the formal structure of the ISAC model is given in this paper¹. The model is presented block by block with special emphasis laid on those features which distinguishes ISAC from other growth models hitherto used in Sweden viz. the vintage approach to the formation of industrial capital, the "Phillips curve" - like way of dealing with wage determination, the explicit representation of energy substitution possibilities and the endogenous treatment of local government.

Figure 1 gives a synoptical view of the whole model. The balance equation at the top of the figure represents a 23 sector model where X (gross output), PI (private investment) etc. are column

¹ The growth model ISAC - Industrial Structure And Capital Growth - was developed on the basis of earlier macro-models used within IUI. The first model of this kind developed at the institute was designed for medium-term forecasting purposes and did indeed resemble the model used by the Swedish Government for its medium-term planning surveys, although including in addition a rather extensive specification of household income and payment. (For a detailed account of this model see IUI:s långtidsbedömning 1976. Bilagor, IUI 1977, in particular Chapt. 1-3 by Ulf Jacobsson, Göran Normann and Lars Dahlberg respectively.

To the next IUI economic survey, in 1979, this model was further developed by including i.a. investment functions and price formation equations. (A description of this model and its solution algorithm is given in Jansson-Nordström-Ysander: "Utvecklingsvägar för svensk ekonomi 1978-1985 - en kalkylredovisning" i Kalkyler för 80-talet, IUI 1979.

Since then a major restructuring of the model has taken place. The model now incorporates adjustment mechanisms for wages prices and industrial capital and i.a. local government actions and part of the industrial productivity development are endogenously explained.



vectors. The model can be characterized as a dynamic Keynes-Leontief model with an endogenously changing input-output matrix, with market rigidities and adjustments and with built-in multiplier mechanisms. The arrows emerging from the sector products indicate roughly the way in which an exogenously initiated change would work itself through the model. Specially marked (by single-line square frames) are the exogenous factors driving the model, i.e., the development of world markets, of labor supply, of the exogenous part of technical change and finally of the central government consumption and fiscal parameters. A number of time-lags in the model - i.a. in foreign trade, in private consumption and investment and related to wage and price formation and to tax yields - makes it dynamic and may produce oscillations of the kind associated with business cycles.

In the text the following definitions and conventions have been used.

Large romans = matrices

Small romans without subscripts = vectors

Greek or small romans with subscripts = scalars

Superscripts = indices of categories etc.

"Roofed" letters = vectors turned into diagonal matrices

(t-n) = n-year lag. When no time indication is given, time t is assumed.

Subscripts t-m = vintage t-m

d = gross change operator

Δ = net change operator.

Dotted variable = growth rate.

THE EQUATIONS OF THE ISAC MODEL

Commodity balances in fixed and current prices

The following two accounting identities merely state that total domestic demand (right side) is equal to total domestic supply (left side).

$$m + x = Ax + inv + pc + puc + e + \Delta s$$

$$\hat{p}^m m + \hat{p}^x x = \hat{p}^h (Ax + inv + pc + puc + \Delta s) + \hat{p}^e e$$

The i/o matrix and employment

The i/o matrix for the industry sector is calculated from vintage model for the 14 branches. The i/o matrix thus becomes a slowly moving function of prices of intermediate goods, electricity, fuels, labor and capital (capital stocks together with investments and depreciations)¹. The relationships are described in following formulas

$$A = DA[\widehat{dxcap/xcap}] + A(t-1)\{I - [\widehat{dxcap/xcap}]\}$$

where

$[\widehat{dxcap/xcap}]$ is a diagonal matrix with elements

$$dxcap_i / xcap_i$$

¹ A detailed account of the vintage approach as applied to the iron and steel industry, is given in L. Jansson: A Vintage Model of the Swedish Iron and Steel Industry, Working Paper No 41, IUI, 1981.

and

$$dxcap_i = inv_i / \varepsilon_{t,k,i}$$

$$xcap_i = dxcap_i - dv_i + xcap_i(t-1)$$

$$dv_i = \left(\sum_{\tau} \sum_{j \neq k} \varepsilon_{\tau,j,i} p_{j,i}(t-1) / p_i^x(t-1) \right) \delta_i \cdot xcap_i(t-1)$$

where

$j = el, fu, in, l, k$

$el = \text{electricity}$

$fu = \text{fuel}$

$in = \text{intermediate goods}$

$l = \text{labor (hours)}$

$k = \text{capital}$

$dxcap_i$ is total new installed capacity at time t in branch i , inv is investments and $\varepsilon_{\tau,k,i}$ is the capital output capacity ratio of vintage τ installed in branch i . $xcap_i$ stands for total capacity and dv_i denotes depreciation. The depreciation rate is assumed proportional to the operating cost ratio in each vintage (the term within large bracket) multiplied by a constant δ_i .

The input shares for the new vintage installed at time t in branch i is calculated as

$$DA_i = \sum_j \varepsilon_{t,j,i} f_{j,i}$$

DA_i thus denotes the i :th column of DA . The input shares $\varepsilon_{t,j,i}$ are calculated from a constant elastic function as

$$\varepsilon_{t,j,i} = b_{j,i} \prod_m p_m(t-1)^{\alpha_{m,i}}$$

where $m, j = el, fu, in, l, k$.

The $f_{j,i}$'s are distribution vectors which convert the aggregate input shares to commodity input shares. Thus all elements in a distribution vector sum to unity. For instance in $f_{el,i}$ all elements are zero except the element for the electricity generating branch which is equal to unity.

All vintages are assumed to be used at the same intensity level. Since input-output ratios are supposed to be independent of the degree of utilization, this implies that the A matrix, defined above, describes the input-output relations not only at full capacity production but also at any other actual production level.

Employment l in branch i is determined as

$$l = [\varepsilon_{t,l,i} \cdot dxcap_i / xcap_i + \varepsilon_{l,i}(t-1) \cdot (1 - dxcap_i / xcap_i)] \cdot x_i$$

Investments, input shares and depreciations are all functions of only predetermined variables which means that they are not dependent on the model solution for year t and accordingly neither is A.

Fuel in the industry is mostly used to produce heat. Heat can be produced by different types of fuel. Ex post substitution between major types of fuels are allowed. Three fuels used for heating are distinguished: oil, coal and domestically produced fuel(dfu). The aggregate use of heat (fuel) in industry is simply the sum of heat produced by oil, coal and domestic fuel. Thus

$$x_{fu,i} = x_{oil,i} + x_{coal,i} + x_{dfu,i}$$

or

$$l = q_{oil,i} + q_{coal,i} + q_{dfu,i}$$

The amount of oil etc required to produce one unit of heat is assumed constant. The shares for coal and domestic fuel are dependent on prices and time according to following formula

$$q_j = a_j \cdot (b_j - p_j^{c_j})$$

where j = coal, domestic fuel.

The p 's are up to five year lagged energy prices, including capital cost, relative to oil produced energy. The share of oil is determined as a residual.

For heat to be separable in the production function of the various industrial branches choice of fuel must have negligible effect on the rest of the installed technique, i.e., the other input shares should be independent of the choice of fuel.

The switching between fuels can be dampened by an adjustment mechanism.

$$q_j = (1-c) q_j(t-1) + c q_j \quad 0 < c < 1$$

Private investments

The investments in branch i of the industry depends on past profits.

$$inv_i = k_i(t-1) \left(\sum_{j=1}^4 \gamma_{j,i} \cdot ep(t-j)_i + d_i \right)$$

where the parameters $\gamma_{j,i}$ are all > 0 and

$$ep_i = vafa_i / (p_i^k k_i + w_i l_i)$$

ep_i is an "excess profit" ratio fluctuating around unity.

Disposable income in the household sector

The model of household income distinguishes between two kinds of individuals. "Pensioners" are people with most of their income from the social security system. The remainder is simply called "wage-earners" although it includes entrepreneurs as well as unemployed persons.

Individuals receive factor-income, capital-income and transfers from other sectors. After deduction of income- and payroll-taxes, and transfers to other sectors they are left with disposable income.

Factor income

The main part of factor income is gross wages and salaries, including payroll taxes and other collective fees, but factor income also includes part of the net surplus in producing sectors.

Gross wages and salaries, bill, is the product of wage/hour and the number of hours worked in different sectors plus the public sector:

$$bill = w' \cdot l + obill$$

$$\text{enet} = \text{he}' \cdot \hat{1} \cdot \text{vafa}$$

where he = entrepreneurs' share of total hours worked in each branch and vafa = value added (i.e. their productivity is assumed equal to the employers).

According to accounting conventions enet also includes imputed income from owner-occupied houses. Thus:

$$\text{fink} = \text{bill} + \text{enet}.$$

Capital income

Capital income consist of interest-payments calculated as a constant fraction of entrepreneurs' income:

$$\text{ip} = \alpha_{id} \text{enet}.$$

Other net capital income is calculated from financial assets which in turn are accumulated from the total financial surpluses and deficits of the household sector.

$$\text{ii} = \gamma_{if} \text{fa}$$

Thus:

$$\text{cinc} = \text{ip} + \text{ii}.$$

Transfer income

This part of the model is fairly disaggregated and to a large extent exogenous except for inflation. There are six different types of transfer incomes:

- etra 1: National general pension (old age plus others)
 2: Dito local
 3: National supplementary pensions (old age plus others)
 4: Private (collective) pensions (old age)
 5: Other transfer income (non-taxable)
 6: Other transfer income (taxable)

The first four items are calculated as number of persons times real income per capita with figures taken from various official sources

$$\text{etra}_i = n p_i \cdot r p_i \cdot \text{cpi}; \quad i=1, \dots, 4.$$

Other transfer income, which mainly goes to wage-earners, is divided according to whether it is taxable or not.

The non-taxable is set by an exogenous trend plus inflation:

$$\text{etra}_5 = \text{ec}_{51} \cdot \exp(\text{ec}_{52} \cdot t) \cdot \text{cpi}.$$

Taxable transfers are divided into sickness benefits, unemployment benefits and others:

$$\begin{aligned} \text{etra}_6 = & \text{ec}_{61} \cdot \text{bill} + \text{ec}_{62} \cdot \bar{w} \cdot \text{ue} + \text{ec}_{63} \\ & \cdot \exp(\text{ec}_{64} \cdot t) \cdot \text{cpi} \end{aligned}$$

Thus total transfer income is

$$\text{etra} = \sum_{i=1}^6 \text{etra}_i$$

Transfer payments

Five types of transfer payments are distinguished:

- btr 1: National income tax
- 2: Local income tax
- 3: Social security contributions
- 4: Other payroll fees
- 5: Other transfer payments.

National income tax is calculated from an aggregated progressive tax-function:

$$btr_1 = bc_1 \cdot skind \cdot n(besk/skind/n)^{bc_2}$$

Local tax is:

$$btr_2^1 = utd \cdot besk$$

Payroll taxes and fees are shares of total gross wage:

$$btr_3 = bc_3 \cdot bill$$

$$btr_4 = bc_4 \cdot bill$$

Other transfer payments are calculated as:

$$btr_5 = bc_5 \cdot \exp(bc_6 \cdot t) \cdot cpi.$$

¹ btr is the local tax payed in year t. As spelled out in the local government model below it, however, only reaches the local authorities in the year (t-2).

Disposable income

Summing up incomes and payments we get disposable income for the household sector

$$\text{disp} = \text{fink} + \text{cink} + \text{etra} - \text{btr}$$

Private consumption

Private consumption expenditure per capita equals disposable income less savings. Savings are set as a constant fraction of disposable income.

$$\text{pcons} = (1-s) \cdot \text{disp}/\eta$$

The total consumption per capita is distributed on 14 consumer goods by the following linear expenditure system

$$p_i^c \cdot c_i = \gamma_i p_i^c \cdot c_i(t-1) + \beta_i \left(\text{pcons} - \sum_{k=1}^{14} \gamma_k p_k^c c_k(t-1) \right)$$

$$\text{and } \sum_{j=1}^{14} \beta_j = 1 \quad i=1, \dots, 14$$

The price vector p^c represents the prices of domestic absorption converted to the consumer goods level

$$p^c = \text{PKONV}' \cdot p^h$$

Private consumption per capita is then transformed to total private consumption of commodities

$$pc = PKONC \cdot c \cdot \eta.$$

Export

The general form of the export function for all branches except textile and paper and pulp industries is:

$$e_i = \beta_{i1} \cdot \exp(\beta_{i2} \cdot t) \cdot (p_i^{ew}/p_i^w)^{\beta_{i3}} \cdot (p_i^{ew}(t-1)/p_i^w(t-1))^{\beta_{i4}} w_{mi}^{\beta_{i5}}$$

for $i = 1, \dots, 6, 9, \dots, 23$.

where w_{mi} stands for the volume of the world market and $p_i^{ew} = p_i^e/\theta$

For the textile and paper and pulp industries export is given by

$$e_i = \beta_{i1} \cdot (p_i^{ew}/p_i^w)^{\beta_{i2}} (p_i^{ew}(t-1)/p_i^w(t-1))^{\beta_{i3}} \cdot w_{mi}^1 + \\ + \beta_{i4} \cdot (p_i^{ew}/p_i^w)^{\beta_{i5}} (p_i^{ew}(t-1)/p_i^w(t-1))^{\beta_{i6}} w_{mi}^2$$

The export functions for textile and paper and pulp industries have been achieved from econometric studies, where the world market has been divided into two regions, wm^1 and wm^2 .

The rest of the export functions are based on econometric studies for only chemicals and engineering. The four branches thus covered in the studies were, however, responsible for more than 70 % of the total export of traded goods in 1977.

Import

The import functions have the following general form

$$m_i = \gamma_{i1} \cdot h_i^{\gamma_{i2}} \cdot \prod_{j=0}^2 [p_i^w(t-j) \cdot \theta / p_i^{xh}(t-j)]^{\gamma_{i,3+j}}$$

The import function for the mining industry merely restates of the fact that all the coal used and all other mining products used outside the iron and steel industry are imported.

$$m_3 = \sum_j a_{3j} x_j + \sum_j \varepsilon_{\text{coal},j} x_j$$

where the first term excludes deliveries of iron ore to the iron and steel industry and where $\varepsilon_{\text{coal},j}$ is the average input share of coal in sector j . Thus all imports in this sector is treated as complementary imports.

The same of course holds for oil. It is also assumed that a constant fraction of total supply of oil products is refined domestically. It thus becomes:

$$m_{12} = \gamma_{12,1} \cdot x_{12}$$

Stock building

For goods producing branches the total change of inventories is assumed constant. The addition to inventories in each branch is proportional to its share of total production.

$$\Delta s_i = \Delta s \cdot Y_i / \sum_{j=1}^{18} Y_j \quad i=1, \dots, 18$$

where Y_i is gross production in branch i and index i covers the goods producing part of the business sector.

There also exists a simple stock building model which can be included when necessary. Only one aggregate inventory good is distinguished but demand for inventories is generated by four stock-holding branches which are simple subsums of the 18 goods producing branches. The four branches are foresting, agriculture and fishing, producers of intermediate goods, producers of finished goods and the merchandise trade. The stocks are proportional to branch production. The proportion might change over time.

$$s_i = sr_i Y_i \quad i=1, \dots, 4.$$

$$\Delta s = \sum_{i=1}^4 [s_i - s_i(t-1)]$$

The additions of inventory are distributed according to the formula above.

Prices

Swedish producers' prices on the export and home markets are determined by:

$$p_i^e = b_{i1}(p_i^w)^{b_{i2}} \text{ucost}_i^{b_{i3}} \text{ur}_i(t-1)^{b_{i4}}$$

$$p_i^{xh} = c_{i1}(p_i^w)^{c_{i2}} \text{ucost}_i^{c_{i3}} \text{ur}_i(t-1)^{c_{i4}}$$

where

$$\text{ucost} = A'p^h + \text{vac}$$

and where

$$\text{vac} = [\hat{l} \cdot w + (\hat{r} + \delta) \cdot \hat{k} \cdot p^{\text{inv}}] \cdot \hat{x}^{-1}$$

that is, value added at normal profits and

$$\text{ur}_i = (x_i/x\text{cap}_i)/(x_i/x\text{cap}_i)_{1980}$$

The value of domestic absorption is

$$\hat{h}p^h = (\hat{x} - \hat{e})p^{xh} + \hat{m} + p^m$$

with

$$\hat{n} = \hat{x} - \hat{e} + \hat{m}.$$

Since p^h is appear in ucost the two blocks of equations for export prices and domestic producer prices on the home market are not in reduced. But the model is solved in such a way that equality is established between the two sides.

The value of domestic production is

$$\hat{x}p^x = \hat{e} \cdot p^e + (\hat{x} - \hat{e}) \cdot p^{xh}$$

For all business sectors the gross profit q_i is residually determined since a pure mark up pricing is not used.

$$q_i = (p_i^x - p^h A_i) x_i - w_i l_i$$

where A_i denotes column i of the matrix A .

Wages

The changes in the wage rate is the same for all sectors though the level differs.

$$w_i = w_{\text{conv}_i} \cdot w_0 \quad i=1, \dots, 37$$

where index i includes both the 23 branches of the business sector and the 14 subbranches of the public sector. w_0 is either exogenous or the percentage change of wage rate $\dot{w}_0$ is endogenously determined by the following function:

$$\dot{w}_0 = d_0 + d_2(u - u_0) + cpi(t-1) + d_3[\Pi(t-1) - \Pi_0] + d_4\dot{\lambda}(t-1)$$

Thus the change in the wage rate is a function of current deviation of the unemployment rate from a "normal" unemployment rate $(u - u_0)$, past inflation, $cpi(t-1)$, past deviation from a "normal" level of aggregate gross profit margin in industry, $(\Pi(t-1) - \Pi_0)$ and finally past change of labor productivity $\lambda(t-1)$. When w_0 is determined endogeneously it is assumed that wage changes in the public sector is lagging one year compared to the business sector.

Central government

The developoment of central government consumption is exogenously determined. Seven different consumption purposes are distinguished.

1. National defense.
2. Public order and safety.
3. Education.
4. Health.
5. Social security and welfare services.
6. Roads.
7. Other services.

The rate of growth of production and consumption for the various purposes is a constant proportion of an exogenously given common growth factor g_0 .

$$sp = g_0 \cdot gr \cdot sp(t-1)$$

$$sc = (1-sale) \cdot sp.$$

The need for intermediate goods is related to production in each central government sector and then converted to demand from the 23 business branches;

$$sf = asf \cdot sp$$

$$lf = SGA \cdot sf.$$

Employment is derived from productivity assumption for each sector. Investment is exogeneously determined.

Local government

Real current expenditure of local governments (consumption + market sales according to national accounting conventions) are split into five categories.

1. Education.
2. Health.
3. Social welfare.
4. Roads (total expenditures).
5. Central administration, fire service etc.

These expenditures are explained by linear expressions of the following form:

$$lp_i = a_{1i}z_{1i} + a_{2i}z_{2i} + a_{3i}z_{3i} + a_{4i}z_{4i} + a_{5i}z_{5i}$$

where z_{1i} are shift variables, z_{2i} and z_{3i} stand for investment consequences and capacity restrictions, while z_{4i} and z_{5i} reflect the impact of changes in real income, local tax rates and relative prices¹.

$$z_{2i} = \frac{1}{kp} \left(\frac{\Delta kp}{\Delta lp_i} \right)^*$$

$$z_{3i} = z_{2i} \frac{\Delta kp(t-1) - \Delta kp^*(t-1)}{kp}$$

$$z_{4i} = \phi_i \frac{besk}{besk(t-2)}$$

$$z_{5i} = z_{4i} \text{ besk}(1 - utd - av_0)$$

with ϕ_i defined as $\phi_i = (1 - sb_i - av_i) \frac{pi}{cpi}$

¹ All expressions explaining local authority behavior are derived from maximizing a quadratic goal-function under a budget restriction. A detailed account of the model is given in Ysander, B-C, An Econometric Model of Local Government Budgeting, IUI Working Paper, No 43, 1981.

Aggregate investments by local authorities are explained by a gradual adjustment to desired capital stock levels, with the rate of adjustment depending on capital good prices, interest rates and real income development.

$$\Delta kp = a_{16}z_{16} + a_{26}z_{26} + a_{36}z_{36} + a_{46}z_{46} + a_{56}z_{56}$$

$$z_{16} = kp; \quad z_{26} = \Delta kp^*; \quad z_{36} = \Delta liq(t-1) \cdot z_{16};$$

$$z_{46} = \phi_6 \cdot kp^2 \cdot r \cdot \frac{besk}{besk(t-2)}$$

with ϕ_6 , net real price of capital goods, defined in the same way as ϕ_i .

$$z_{56} = (1-utd-av_0)besk \cdot z_{46} \cdot z_{16}$$

The depreciation is assumed to be a constant fraction of existing capital stock. Gross investments then becomes

$$lp_6 = \Delta kp + a_{66}z_{66}$$

where a_{66} is the depreciation rate and $z_{66} = kp(t-1)$.

Investments may also be computed in a simplified manner as equal to desired capital stock changes plus reinvestments.

Sales made by local authorities of goods and services at market prices are assumed to be a constant fraction of local production. The local governments' final demand of commodities from the business sector can be summarized:

$$lp_i = AL'_i \cdot Z_i$$

$$lc = LGA \cdot lp;$$

where AL_i and Z_i are the i :th column of AL and Z respectively. The matrix LGA converts local governments' final demand to commodities.

Employment is derived from productivity assumption for each sector.

Transfer payments are split into two categories, - subsidies to public utilities and direct transfers to the household sector. In the model the explanation of these payments is derived from the idea that the provision of housing space and of public utilities are arguments in the local governments' goal function, pursued indirectly by way of "price subsidies".

$$t_i = a_{2i}\zeta_i + a_{3i}\mu_i + a_{4i}\gamma_i$$

$i=1,2$ 1 = public utilities subsidies
 2 = direct household (housing) subsidies,

where ζ_i represents the cost for the households relative to disposable income and μ_i and γ_i express the impact of developments in real income, local tax rate and relative prices.

$$\bar{w}_1 = 1; \quad \bar{w}_2 = z_{13}; \quad \varepsilon_i = \frac{\tau_i/cpi}{(1-utd-av_0)(t-1)besk} \quad \zeta_i = \bar{w}_i \varepsilon_i$$

$$\mu_i = \varepsilon_i \frac{\tau_i}{cpi} \frac{besk}{besk(t-2)}; \quad \gamma = \mu_i (1-utd-av_0)besk$$

with τ_i defined as $\tau_i = p_i^c(1-sb_i)$.

The transfer payments can alternatively be treated in a simplified manner. The net amount of subsidies to public utilities are then approximated as a constant fraction of production in sector 18 (electricity, heat etc.). Direct household transfers are set as a linear function of households' housing expenditures (z_{13}).

Local government expenditures are financed by taxes and state grants with liquidity changes acting as a buffer against planning failures. Given state grants the local tax rate is determined residually by way of the budget restriction:

$$\text{utd} = \frac{\text{besk}}{\text{besk}(t-2)} [-\text{utd}(t-2)(\text{besk}(t-2) - \text{besk}(t-4)) - \text{sb}_0 - \Delta \text{dt} + e_0 + \text{rdt}]$$

To simulate possible restrictions or inertia in local government political behavior, the model can alternatively be supplemented with a floor restriction on the tax rate complemented with a rule, prescribing that planned surpluses that increase liquidity above a certain relative level are used to scale up current expenditure.

The system of equations

The list of equations on the next page summarizes the formal structure of the ISAC model¹. At the bottom the two main disequilibria in the model - in foreign trade and in the aggregate labor market - are listed. Clearance also of these markets requires a "steering" by way of exogenous policy parameters like the exchange rate or state consumption. (Financial entities like tax rates and transfers are excluded from this very compressed list.) To ensure also a "normal" rate of capacity utilization in all branches and periods would obviously require a large and special set of policy parameters. To the left of the equation list in the table the number of variables are given, while the corresponding number of equations are registered in the right-most column. Excluded from the count are exogenous variables not used as instruments, predetermined variables and identities.

¹ The Gauss-Seidel algorithm is used to compute the solution. This algorithm, according to our experience, is easy to program and implement, its speed of convergence is mostly satisfactory and it has rarely failed to converge, though convergence is not theoretically assured.

LIST OF EQUATIONS¹

Number of variables

n=23

Number of equations

n=23

$$4n \quad m + x = Ax + inv + pc + puc + \Delta s + e \quad n$$

Prices

$$2n+1 \quad p^e = p^e[\theta p^w, ucost, ur(t-1)] \quad n$$

$$n \quad p^{xh} = p^{xh}[\theta p^w, ucost, ur(t-1)] \quad n$$

$$p^h = (\hat{x} - \hat{e})p^{xh} + \theta \cdot \hat{m}p^w$$

$$n+1 \quad ucost = p^h A + w \hat{e}_\ell + \quad n$$

$$+ p^K(p^h, r, \delta) \hat{e}_K \cdot \hat{u}r^{-1}$$

Wages

$$w = [1 + \dot{w}(cpi(t-1), u, \Pi(t-1), \varepsilon_\ell(t-1))] \quad 1$$

$$\cdot w(t-1)$$

Production capacity and technique

$$ur_i = x_i / xcap_i \quad n$$

$$A(t) = DA(t) \cdot \widehat{dxcap} \cdot \widehat{xcap}^{-1} +$$

$$A(t-1) \cdot (I - \widehat{dxcap} \cdot \widehat{xcap}^{-1})$$

$$dxcap = inv \cdot \hat{e}_{t,k}^{-1}$$

$$inv = inv[ep \cdot \Lambda(1, 4), \delta, k(t-1)]$$

Privat consumption and labour demand

$$(n+14) \quad pc = pc(c_0 w' \ell - c_1, p^h) \quad n$$

$$\ell = (\varepsilon_\ell \hat{x}, \varepsilon_{puc} \cdot puc)^2 \quad (n+14)$$

¹ Financial streams are excluded from the brief list of equations above.

² ℓ includes employment in both business and public sectors.

Government production and investment

$$2n+1 \quad puc = sc + lc \quad n$$

$$lc = LGA \cdot lp \quad n$$

Foreign trade

$$e = e[(p^e_\theta, p^w)\Lambda(0,1), wm] \quad n$$

$$m = m[(p^w_\theta, p^{xh})\Lambda(0,2), h]; \quad h=x-e+m \quad n$$

Unemployment

$$\Sigma l_i - l_s = \text{disequilibrium 1} \quad 1$$

Balance of trade

$$\Sigma [p^e_e - p^m_m] = \text{disequilibrium 2} \quad 1$$

10n+3

10n+3

¹ In the equations above the lagoperator operates on the left hand variabel in the follwing way

, $x \Lambda(i,j)$, $\rightarrow$, $x(t-i), \dots, x(t-j)$.

LIST OF VARIABLES

Commodity balances

m	= imported goods and services
x	= domestically produced goods and services in the business sector
inv	= private investments
puc	= public use of intermediate goods and public investments
e	= exported goods and services
Δs	= changes in inventory stocks (exogenous)
p^e	= Swedish producers export prices
p^m	= import prices
p^h	= prices of domestic absorption

The i/o matrix and employment

$dxcap$	= total new capacity in branch i
inv_i	= investments in branch i
$\varepsilon_{\tau,k,i}$	= capital output ratio of vintage τ installed in branch i
$xcap_i$	= total capacity in branch i
dv_i	= depreciation in branch i
A	= matrix of i/o coefficients
DA	= i/o matrix for the new vintages
$\varepsilon_{\tau,j,i}$	= the i/o ratio of vintage τ for input j in branch i
$f_{j,i}$	= distribution vector which converts aggregate input shares to commodity input shares
q_j	= share of aggregate use of fuel
l_i	= employment in branch i

Private investments

k_i	= capital stock
ep	= excess profits
$w_i h_i$	= gross wages and salaries in branch i
p_i^k	= $p^i(r+\delta_i)$ user cost of capital
r	= pretax discount rate
δ_i	= depreciation rate (historical average)
A_j	= the j :th column of the i/o matrix A
p_i^{xh}	= domestic producers prices on the home market

Disposable income in the household sector

$bill$	= gross wages and salaries
$w' \cdot l$	= gross wages and salaries in the household sector
$obill$	= gross wages and salaries in the public sector
$enet$	= entrepreneurs' income
$vafa$	= value added in current prices (factor values)
$fink$	= factor income

Capital income

ip	= interest payments
α_{id}	= interest payment as ratio of entrepreneurs' income
ii	= other net capital income
γ_{it}	= average yield on financial assets
fa	= net financial assets
$cinc$	= capital income

Transfer income

np = number of pensioners
rp = real pension income per capita
cpi = consumer price index
etra = transfer income
ec_{51...} = constants
ec₆₃
ue = unemployment

Transfer payments

btr_i = transfer payments of type i
bc_{1...} = constants
bc₆
skind = tax index (one year lagged cpi)
besk = assessed income
n = number of taxpayers
utd = local tax rate
disp = disposable income

Private consumption

pcons = private consumption expenditures
per capita
s = net savings ratio
η = total population
c_i = per capita demand for consumption of
goods i
PKONV = matrix of conversion
pc_i = consumption of commodity i
p_i^c = consumer price of commodity i

Export

- e_i = export from branch i
 β_{ij} = parameters
 p^{ew} = export prices of domestic producers in foreign currency
 wm_i = world market demand of commodity i
 θ = the exchange rate

Import

- m_i = import of commodity i
 γ_{ij} = parameter
 h_i = domestic absorption of commodity i

Stock building

- Δs_i = the change of stock in branch i
 Δs = the aggregate change of stock
 y_i = gross production in producers prices in branch i
 s_i = stock in stock holding branch i
 sr_i = stock output ratio
 ys_i = aggregated output in stock holding branch i

Prices

- p_i^{xh} = Swedish producers price of commodities i on the domestic market
 b_{ij} = parameters
 c_{ij}
 p_i^w = average world market prices in foreign currency
 θ = the exchange rate

$ucost$ = unit cost of production
 u = relative utilisation rate
 (index=1 at normal capacity use)
 vac = value added computed with a normalized
 pretax discount rate
 w = wage rate
 l = employment in the business sector
 p^{inv} = price index for investment goods
 r = pretax discount rate
 δ = depreciation rate
 k = capital stock
 q_i = gross profit in branch

Wages

w_i = wage rate in branch i
 w_{conv_i} = branch wage - average wage rate ratio
 w_0 = the average wage rate
 d_j = constants
 u = unemployment rate
 u_0 = "normal" unemployment rate
 Π = average gross profit margin in
 the industrial sector
 Π_0 = the "normal" gross profit margin in
 the industrial sector
 λ = average labor productivity in
 the industrial sector

Central government consumption

- gr = vector of relative growth numbers of
state consumption purposes
- g_o = the common growth factor of state
consumption
- sp = central government production volume
(market prices)
- sc = central government consumption volume
- sale = share of production sold
- sf = demand for intermediate goods for each
state sector
- asf = share of intermediate goods
- gf = demand for intermediate goods from
the business branch
- SGA = conversion matrix (23×7)

Local government

- lp_i = volume of local government production, category i
 z_{11} = index of population change, ages 7-19 years
 z_{12} = index of population change, weighted with average number of hospital days for the various age-brackets
 z_{13} = index of population change, ages over 71 years
 z_{14} = value added of business sector
 z_{15} = total population index

The rest of the z variables are explained in the text.

- Δkp = aggregate net investments
 kp = aggregate capital stock in local government
 av_i = local government fees as share of total production costs for category i
 sb_i = categorical state grants as share of production costs for category i
 p_i = production cost index, category i
 Δkp^x = desired change of capital stock
 Δliq = net change of short-term assets
 r = rate of interest on local government bonds

lp_6 = gross investments

AL = matrix of the parameters al_{ij}

Z = matrix of the variables z_{ij}

LGA = conversion matrix

lp_c = diagonal matrix that transforms local government production to local government consumption

$(\frac{\Delta kp}{\Delta lp_i})^x$ = marginal capital output ratio, category i

av_0 = local government fees as share of total production costs

utd = local tax rate

t_i = subsidies as share of net expenditures for category i by the households

$\phi_i, \phi_I, Y_i, \bar{w}_i, \varepsilon_i, \mu_i, \gamma_i$ and τ_i = cf definitions in the text

sb_0 = general state grant

dt = net financial assets

Δdt = net change of financial assets

e_0 = total expenditure for production investment and transfer payments net of state categorical grants and fees

lc = local government production and investment