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THE INCENTIVE TO ACQUIRE INFORMATION AND FINANCIAL MARKET EFFICIENCY
by
Clas Wihlborg

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THE INCENTIVE TO ACQUIRE INFORMATION AND
FINANCIAL MARKET EFFICIENCY

by

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I. Introduction

In efficient financial markets by a common definition, asset price variability depends solely on variability of so-called fundamental factors influencing the actual cash flows to the owner of the asset. It seems reasonable to define excess volatility in asset markets as fluctuation in price relative to an efficient market benchmark. In finance theory this benchmark price reflects "all available information" and accordingly, prices change only as a result of news about fundamentals.

The normative aspect of efficiency is that asset prices reveal all available information and, therefore, investment decisions by firms using financial market returns to measure the cost of capital would incorporate fully this information. Under the strongest definition of efficiency, available information includes private information while under weaker definitions only "publicly available" information is reflected in price.

Grossman and Stiglitz (1980) showed that if information of relevance for asset pricing is costly, then it is impossible for asset prices to be "informationally efficient" in the sense that prices perfectly reveal all information held by agents. If they did, nobody would incur the costs of acquiring the relevant information. Agents' incentive to acquire information is influenced by an externality which may cause a free-rider problem.

The starting point in this paper is that important information is costly to acquire. The costs may take different forms to be discussed below. Much information about management capability, technological skills and development cannot be described by simple disturbances.
to the stream of earnings or the rate of return. I will therefore distinguish between shocks to fundamentals and to structural parameters such as time series properties of cost or demand variables. The common definition of market efficiency in terms of "available" information becomes ambiguous in this framework.

If most relevant information is costly to acquire or process, then equilibrium price must be determined by simultaneous equilibria in the markets for financial assets and for information. We must ask what information set is consistent with a zero incentive to acquire as well as to provide information taking potential externalities of information acquisition into account. Price fluctuations may depend on changes in the information set as well as in fundamental factors and even if the information set is constant the price response to disturbances depends on the information set of agents. Efficiency of markets becomes a somewhat elusive concept determined by efficiency in information acquisition and dissemination rather than in markets for securities. The efficiency of information markets and information reflected in market prices depend on the degree to which externalities and other market imperfections inhibit information acquisition.

In this paper I discuss how costs of information about fundamental factors as well as parameters influence price adjustment in a market for a risky security. Structural parameters are, for example, the time series characteristics and the probability distributions of fundamental factors. The latter factors may be demand and cost conditions for a firm.

An interesting issue arises with respect to information acquisition about management capability and technological progress. Due to
asymmetric information and principal agent problems, these factors may not be truly exogenous relative to the information contained in the market's valuation of a firm. Information dissemination for such factors may in fact not be the most efficient in a decentralized securities market.

The larger issue for financial market efficiency which is raised by this discussion is what financial institutional structure is most efficient for dissemination of information about firm and project value to suppliers of financial resources? Some types of information may be disseminated efficiently in decentralized securities markets of the type discussed in most of this paper but other types of information may be better disseminated within more centralized structures like industrial groups around a bank with large financial and personal investments in firms.

The paper proceeds as follows: In Section II a simple model for pricing of a multiperiod risky security is developed and sources of price fluctuations are described. The incentive to acquire information and potential free rider problems are described in Section III. Equilibrium adjustment and equilibrium information acquisition are studied in Section IV. In one case structural parameters are known but fundamental disturbances are not, and in another case, current disturbances are observable but structural parameters are not known with certainty. Conditions under which free rider problems arise and conditions for all agents to choose to become informed in spite of information externalities are derived. These conditions are studied further in Section V where "degree of efficiency" is discussed in more detail. In Section VI it is argued that signalling, monitoring,
control, and insider trading are substitutes for information acquisition in the determination of the equilibrium information set. Finally, Section VII contains conclusion about price adjustment and appropriate definitions of market efficiency. Empirical evidence on stock-price behavior is also interpreted in this section.

II. Asset Price Adjustment, Information Acquisition and the Free Rider Problem; A Framework

This section develops a framework for analyzing the determinants of a multiperiod asset's price and for analyzing the incentive to acquire information.

Following Grossman (1976), it is assumed that each agent choosing between one risky asset and one risk-less asset maximizes a utility function characterized by constant absolute risk-aversion:

$$U(W_t) = -e^{-cW_t}$$ (1)

where $W_t$ is wealth in period $t$ and $c$ is a risk-aversion parameter.

There are two types of individuals, informed and uninformed denoted by superscripts $I$ and $U$. Each agent starts in period $t$ with wealth equal to 1. Uninformed agents invest $\theta^U_t$ in a risky asset with the one-period return $r_t$ and $(1-\theta^U_t)$ in the risk-less asset with the one period return $r_F$. Informed individuals spend an amount $z$ on information and invest $(1-z)\theta^I_t$ in the risky asset and $(1-z)(1-\theta^I_t)$ in the risk-free asset. Wealth of the two agents in period $t+1$ are:

$$W^U_{t+1} = \theta^U_t (1+r_{t+1}) + (1-\theta^U_t) (1+r_F)$$ (2a)

$$W^I_{t+1} = (1-z)[\theta^I_t (1+r_{t+1}) + (1-\theta^I_t)(1+r_F)]$$ (2b)
Each agent maximizes expected utility of period t+1 wealth. This is equivalent to maximizing $E_t[W_{t+1}] - \frac{c}{2} \sigma_w^2$, where $\sigma_w^2$ is the variance of wealth in period t+1 and $E_t$ is the expectations operator for U- or I-agents. The first order conditions for the maximization problems, simplifying notation so that $E_t(1+r_{t+1}) = (1+r)^I$ and $E_t(1+r_{t+1}) = 1+r^U$, are:

\[ (1+r^I) - (1+r_F) = c(1-\zeta) r^I \sigma_r^2 \]  \hfill (3a)

\[ (1+r^U) - (1+r_F) = c(1-\zeta) r^U \sigma_r^2 \]  \hfill (3b)

In (3a) and (3b), $\sigma^2_I$ and $\sigma^2_U$ refer to the variances of the rate of return on the risky asset for informed and uninformed, respectively.

Initially it is assumed that a share $\lambda$ of all agents have acquired a certain kind of information. Equilibrium in the market for the risky asset is then described by:

\[ \lambda(1-\zeta) r^I + (1-\lambda) r^U = \theta \]  \hfill (4)

where $\theta$ is the exogenous supply of the risky asset per agent.

So far the model is similar to the one period model in Grossman (1976), Grossman and Stiglitz (1980), and Diamond (1985). In order to distinguish clearly between the role of uncertainty about fundamentals and uncertainty about structural parameters, including time-series characteristics of disturbances, I assume that the risky asset is a multiperiod claim on a firm with earnings $R_t$ in period t. The valuation of the earnings stream for the two individuals are

\[ V^I_t = \sum_{j=1}^{\infty} E_t \frac{R_{t+j}}{(1+r^I)^j} \]  \hfill (5a)

\[ V^U_t = \sum_{j=1}^{\infty} E_t \frac{R_{t+j}}{(1+r^U)^j} \]  \hfill (5b)
In a perfect auctioneer-market price adjusts so that

\[ P_t = V_t^I = V_t^U \] (6)

These specifications imply that an informed (uninformed) individual in period t expects to be informed (uninformed) in every future period as well. In the discussion of the information market below it will be argued that this assumption is reasonable. Given expectations and uncertainty about the earnings-stream, and given total supply, individuals of the two types adjust their holdings \( \theta^I \) and \( \theta^U \), under the constraint in (4) that supply \( \theta \) is willingly held, until discount rates in (3a) and (3b) are such that the earnings-stream is valued identically by both types of individuals. It is clearly not meaningful in this framework to assume that risk-adjusted discount rates are exogenous and constant as is often done in the finance literature. In this framework there is a downward sloping demand curve for the risky asset. In an n-asset economy, the demand curve for each risky asset would be downward slipping as long as each asset does not have a perfect substitute.²

To specify how price adjustment depends on fundamentals and structural parameters, it is assumed that earnings in period t consists of two factors, \( x_t \) and \( y_t \), such that

\[ R_t = x_t + y_t \] (7)

The factor \( x_t \) has a degree of persistence with the following properties:

\[ x_{t+1} = \rho x_t + \varepsilon_x \] (8)

where \( \varepsilon_x \sim N(0, \sigma_x^2) \) and independent of other factors. The structural parameter \( \rho \) describes how the contribution of the fundamental variable \( x \) is expected to develop over time. The fundamental \( x \) may be interpreted
as, for example, cost conditions or demand conditions while the parameter shows how a disturbance in one period may have lasting effects. This structural parameter may be related to technological progress, management skill and the like. The factor $y_t$ is assumed to be period specific factor such that $y_t \sim N(0, \sigma^2_y)$. The important distinction for the purposes here is that the two factors have different time-series characteristics. It is therefore desirable for agents to identify how each factor contributes to earnings in period $t$ in order to forecast future earnings.

In one part of the analysis below, it is assumed that informed agents can observe $x_t$ while uninformed observe only $r_t$ and form an expectation $E_t^U[x_t]$. Using (8), (5) and (6) we derive the following semi-reduced form expressions for the price if $p$ is known:

$$V_t^I = p_t = x_t \frac{\rho}{1+r_I - \rho}$$  \hspace{1cm} (9a)

$$V_t^U = p_t = E_t^U[x_t] \frac{\rho}{1+r_U - \rho}$$  \hspace{1cm} (9b)

9a and 9b are not expressed in terms of fundamentals since $(1+r_I)$ and $(1+r_U)$ are given by (3a) and (3b). Using these expressions $p_t$ will be solved for as a function of fundamentals in Section IV.

Before proceeding to the reduced form solution with heterogeneous information, the importance of structural uncertainty can be illustrated under the assumption that nobody is informed. This case is interesting in order to illustrate how different factors and parameters affect price adjustment. Assuming homogeneous agents and using (3), the price in terms of fundamentals is:

$$p_t = E_t[x_t \frac{\rho}{1 + r_f + c\sigma^2 - \rho}]$$  \hspace{1cm} (10)
The adjustment of price to a change in expectations about $x_t$, under the assumption that the covariance between $x_t$ and other variables and parameters is zero is:

$$\frac{dP_t}{dE_t[x_t]} = E_t[p/(1 + r_F + c\sigma^2 - \rho)]$$ (11)

This expression implies that structural uncertainty i.e. uncertainty about, for example $c$, the risk-aversion coefficient, and $\rho$, the time-series parameter for $x$, influence price adjustment. The reason is that by Jensen's inequality for a stochastic variable, $a$, $E_t[\frac{1}{a}]$ exceeds $\frac{1}{E_t[a]}$ by a term that is proportional to the variance of $a$. Thus, increased structural uncertainty as well as higher uncertainty about the risk-free one period interest rate and $\theta$, the supply-variable, cause price variability to increase at constant variability in fundamentals.

**III. The Free-rider Problem and the Incentive to Acquire Information**

Returning to the case with full knowledge about structural parameters and two kinds of agents as expressed in (9a) and (9b) the free rider problem in the traditional RE-framework with structural certainty and perfect auctioneers markets can be observed. Uninformed individuals who know $r_F$, $\theta$, $\rho$, and $\lambda$ can infer $x_t$ by observation of $P_t$ using expression 9a for $P_t$, (3a) for $(1+r_F^T)$, and (4) for $\theta^T$. The share of informed firms, $\lambda$ must be known as well. Under these assumptions no individual would acquire costly information about $x_t$.

It may seem far-fetched that prices reveal private information without agents being able to trade based on such information before the price adjusts. In the following an error term $\varepsilon_P$ will be added to the market price in order for information acquisition to be possible.
Specifically, it is assumed that

$$P_t = v_t + \varepsilon_{p,t}$$  \hspace{1cm} (12)

It is assumed that the market noise term $\varepsilon_{p,t}$ is a normally distributed random variable with variance $\sigma_{\varepsilon_p}$. This assumption is very strong and perhaps unrealistic. Interpretations of this term are discussed in Section V. The consequence of the noise term is that the price is not a "sufficient statistic" for revelation of $x_t$ to the uninformed. As will be noted below, there is still an information externality in the price, as long as $P_t$ contains some information about $x_t$.

The existence of noise in the price signal implies that uninformed agents observe two signals, $R_t$ and $P_t$. $R_t$ is $(x_t + y_t)$ while $P_t$ as given by (12) is a function of $x_t$ and $\varepsilon_{p,t}$ if $\lambda > 0$. In the next section an explicit formulation for price is derived.

The most simple way of describing the determination of $\lambda$, the share of informed firms is to assume, like Grossman and Stiglitz (1980) that in each period information about $x_t$ or $y_t$ can be purchased at a fixed cost ($z$) from some agent. If there is no explicit market for information then information may be gathered and analyzed by each agent at a cost $z$ in every period.

Information processing and the ability to interpret information and gain actual knowledge may also require substantial investment in the capability to gather and analyze information. For example, traders in financial market may have to work a long time in the market in order to acquire a "feel" for how the market reacts to different kinds of information and to sort out useless information from valuable information. The formal analysis will be restricted to period by period acquisition of information at a cost but the analysis applies, as well,
to the decision to invest in capability to gather and analyze information. 6

The incentive to gather information for each agent in period t can be written as:

\[ I_t = E_t^U(E_t[I(U(W_{t+1})] - E_t^U[I(U(W_{t+1})]) \]

(13)

This expression evaluates at the time the agent is uniformed the expected gain of becoming informed in terms of expected utility at time \( t+1 \). Utility functions are given by eq. (1) above. A simplifying transformation of (13) is given by:

\[ I_t = E_t^U(E_t[I(W_{t+1})] - \frac{c}{2} W) - (E_t^U[W_{t+1}] - \frac{c}{2} W) \]

(14)

Equilibrium in the information market occurs when \( I_t = 0 \) which implies that \( I_t = 0 \), as well. Using the definition of \( W_{t+1} \) in (2), and first-order conditions in (3) for \( \theta^I \) and \( \theta^U \), (14) can be developed and written as:

\[ I_t = E_t^U((1+\frac{r}{2}) - \frac{2}{c} \frac{\sigma^2}{\gamma}) - (\frac{(1+\frac{r}{2}) - \frac{2}{c} \frac{\sigma^2}{\gamma}}{2})z(1+r) \]

(15)

In (15) \( (1+r) = E_t^I[P_{t+1}/P_t] \) for \( j = I, U \) and the variances in denominators are the variances of these terms. Thus, in order to solve for the equilibrium share of informed firms \( \lambda \) we must first solve for \( P_t \) and \( E[P_{t+1}] \) as functions of \( \lambda \), taking into account that the price itself reveals the information of the informed to the uninformed. 6 This issue is addressed in the next section.

In a model without endogenous information acquisition price variability is usually derived as a function of structural parameters in a straightforward manner. For example, it was shown in (11) that price adjustment to a perceived change in the fundamental \( x_t \) depends in a
specific way on perceptions about, for example, the risk-aversion coefficient, the time-series parameter $p$, as well as, on the variances of these parameters.

With endogenous information acquisition the adjustment-coefficient influences the incentive to acquire information and the share of informed firms. The dependence of the adjustment coefficient on parameters is therefore more complex. For example, a known increase in the time series parameter $p$ would tend to increase the magnitude of price adjustment to a disturbance in $x_t$. Through (15) the incentive to acquire information would change and, as a result, the adjustment coefficient may increase or decrease. An explicit formulation for $P_t$ with heterogeneous information is required to analyze this issue in more detail. 9

IV. Information Markets and Financial Market Price Adjustment

To determine the equilibrium price in the above model it is necessary to specify the information set of agents. This set depends on what kind of information is publicly available as well as what information can and will be acquired at a cost.

a. Structural parameters known. Uninformed agents cannot observe $x_t$

To begin with, it is assumed, as in most RE-models, that all parameters are known to all market participants. A known share of agents ($\lambda$) have acquired information that enables them to determine $x_t$. Others have to infer $x_t$ from observations of $R_t$ in (7) and $P_t$ in (12). In the latter equation $V_t$ is given by (9a). As noted uninformed agents would infer $x_t$ perfectly by observation of $P_t$ if $\varepsilon_{P,t} = 0$.

The price $P_t$ is derived by inserting (9a) in (16). Expression (3a) is used for $(1+r^1)$ in (9), (4) is used for $\theta_{\text{in}}$ in (3a), (3b) used for $\theta^U_t$
in (4) and (9b) is used for \((1+r^U)\) in (3b). The following expression for \(P_t\) is derived:

\[
P_t = \frac{\lambda x_t \rho \omega + (1-\lambda)E^U_t [x_t] \rho}{(1+r^U - \rho)(\lambda \omega + (1-\lambda)) + \sigma^2 U \theta} + \varepsilon_{p,t}
\]  

(17)

The numerator in (17) can also be written as \(x_{t-1} \rho^2 \lambda \omega + (1-\lambda)E^U_t [x_t] \rho\), since \(x_{t-1}\) is observed in period \(t\).

It can be observed that for \(\Omega = (\sigma^2 U/\sigma^2 T) = 1\), the numerator is simply the average expectation of \(x_t\) multiplied by \(\rho\), and the denominator is the same as for homogeneous expectations in (10). Clearly, the share of agents choosing to become informed influences price adjustment to disturbances.

In order to analyze how price responds to changes in fundamentals it is necessary to solve for \(E^U_t [\varepsilon_x,t]\). The information set of uninformed agents consists of \(R_t\) and \(P_t\) as well as all parameters in (17). These two signals must be used to extract information about the three disturbance terms \(\varepsilon_x, t\), \(\varepsilon_x, t\), and \(\varepsilon_{p,t}\). The expectation of current disturbances in \(x\) can be written in the following simple form:

\[
E^U_t [\varepsilon_x,t] = K_1 \varepsilon_x,t + K_2 \varepsilon_x,t + K_3 \varepsilon_{p,t}
\]  

(18)

where \(0 < K_1, K_2, K_3 < 1\), and \(\delta K_1/\delta \sigma^2 x > 0, \delta K_2/\delta \sigma^2 x > 0\) and \(\delta K_3/\delta \sigma^2 x > 0\). Other derivatives with respect to variances are negative.

In other words, for uninformed agents a positive disturbance of any kind has a positive probability of being a disturbance in \(x_t\), and the larger the variance of \(\varepsilon_x\) relative to a weighted variance of other disturbances the larger is the probability that any disturbance is interpreted as a disturbance in \(x_t\). These results are well-known in the RE-literature.

Inserting (18) into (17) we obtain that for a given \(\lambda\):

\[
\delta P_t / \delta \varepsilon_{x,t} = \rho [\lambda \omega + (1-\lambda)K_1] / H > 0
\]  

(19)
where $H$ is the denominator in (17). Similarly:

\[
\frac{\delta P_t}{\delta y_t} = (1-\lambda)\rho K_2 y_t / H > 0 \tag{20}
\]

\[
\frac{\delta P_t}{\delta \varepsilon_{p,t}} = (1-\lambda)\rho K_3 \varepsilon_{p,t} / H > 0 \tag{21}
\]

The latter two expressions show that $y_t$ and $\varepsilon_{p,t}$ influence price only if there is confusion about the fundamental factor $x_t$, which has a persistent effect on cash flows.

What is efficiency in this framework? Clearly, given information sets the adjustment to $\varepsilon_{x,t}$ and $\varepsilon_{t}$ is "efficient." The adjustment to $\varepsilon_{p,t}$ is not efficient within standard RE-models, since within these models the existence of $\varepsilon_{p,t}$ is considered an inefficiency. In the next section I argue that shifts in $\varepsilon_{p,t}$ may be consistent with RE if information about structural parameters and others' information is uncertain. In that case the issue of efficiency may be addressed by asking whether $\lambda$ is the efficient share of informed individuals. We turn to the information market to analyze this issue.

The incentive to acquire information was described in eg. (15) above. In Appendix I this equation is expanded further in the case when structural parameters are known but $x_t$ is not directly observed by the uninformed. The following condition for information market equilibrium is derived:

\[
\frac{1}{p^2_t} \sigma^2 U [\Omega(1-z^2)] + \frac{1}{p^2_t} \sigma^2 r^U [\Omega(1-z^2)] = \frac{1}{p^2_t} \sigma^2 r^U [\Omega(1-z^2)-1][(1+r^U)-(1+r^F)]^2 = \rho(1+r^F)_2 \sigma^2 U \tag{22}
\]

In this equation there is a positive incentive ($I_t$) to acquire information if the left hand side is larger than the right hand side. Information market equilibrium occurs for $\lambda = 0$ ($\lambda=1$) when the left hand
side is smaller (larger) than the right hand side for $0 < \lambda < 1$. For values between 0 and 1 the incentive to acquire information is decreasing in $\lambda$. In Appendix II this proposition is explained in more detail. Briefly, as in Grossman and Stiglitz (1980), the price becomes more informative about $x_t$ as informed agents increase in number. Thus, there is a positive externality from information acquisition.

In (22) $(1 + r^U) = E_t^U [P_{t+1}^U / P_t^U]$. The existence of this term indicates that the larger the expected price change, the larger is the incentive to acquire information. Since, the expected price change for the uninformed is increasing in the absolute magnitude of each disturbance, the following proposition is obvious:

**Proposition 1:** For $0 < \lambda < 1$ the equilibrium share of informed agents increases as the absolute magnitude of each disturbance rises. The incentive to acquire information is therefore time-specific and the adjustment coefficient to each disturbance is time-varying.

By conventional tests for efficiency such variation in the adjustment coefficient is considered evidence of inefficiency.\(^{11}\)

From equation (22) the following proposition can also be derived:

**Proposition 2:** For $0 < \lambda < 1$, the equilibrium share of informed agents

a) decreases when the cost of information, $z$, rises;

b) decreases when the risk-free interest rate, $r_F$, rises;

c) has a maximum for an intermediate level of variance of the fundamental factor $\varepsilon_{x,t}$ while it is small or zero for very small and very large variances of this variables.

d) increases when the variances of the noise term $\varepsilon_p$ increases.

Parts a) and b) of this proposition do not require proof. Part c) is shown in a different context in Glick and Wihlborg (1986).

Intuitively, if $\sigma^2_{\varepsilon_{x}} = 0$, then there is no demand for information since $x_{t-1}$ provides full information. On the other hand, as $\sigma^2_{\varepsilon_{x}}$ grows very large relative to $\sigma^2_{\varepsilon_{y}}$, the variance of the observed signal $R_t = x_t + y_t$
becomes dominated by $\sigma_{xt}^2$. In the limit $R_t$ is perfectly informative. Thus, the incentive to acquire information is at a maximum when there is variation in the variable agents must observe for valuation, $\varepsilon_{x,t}$ as well as in the noise-creating disturbances, $y_t$ and $\varepsilon_{p,t}$. The last part of the proposition is explained by the reduction of informativeness of the signal $p_t$ as $\sigma_{ep}^2$ increases. 12

Finally, relating back to the discussion in the previous section of the impact of shifts in structural parameters on price adjustment, Proposition 2 allows a complete analysis of the impact on price adjustment relative to full information adjustment of changes in, for example, the variances of the fundamental factor $x$ and the terms $y$ and $\varepsilon$. Any change in these variances influence price adjustment directly at a constant share of informed agents by their effect on average misperceptions about the factor $x_t$, in (17). They also influence price adjustment through their effect on the share of informed agents as described in Proposition 2. In general, the two effects would be offsetting since the incentive to acquire information is enhanced through (22) by increased misperceptions of uninformed agents about the fundamental factor $x_t$. 13

b. Structural parameters uncertain while current disturbances are observed

To what extent does the above analysis apply when parameters like the risk-aversion coefficient, $c$, and the serial correlation coefficient, $p$, are not known with certainty? Assume in this case that current disturbances in $x_t$ and $y_t$ are observable while one or more of the structural parameters are unknown. The serial correlation terms in particular are interesting since they may vary over time, or at least

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shift with irregular intervals as a result of, for example, the nature of technological innovations, management capability, and general productivity shocks in $x_t$. There is no reason to believe that these parameters are constant over time. If expectations are homogeneous equation (10) for $P_t$ applies in this case.

Information about, for example, time series characteristics for $x_t$ described by $\rho$, and for $y_t$ may also be observable at a cost by means of time series analysis of historical data, and analysis of firms' product development, the nature of the demand for the firm's product, etc. Assume that there is a share of agents who acquire information this way about a time series parameter like $\rho$. All agents are assumed to observe the current level of variables $x_t$ and $y_t$.

As in the previous case, additional information acquired by one group of agents is revealed by the price, $P_t$, to other agents. This variable reflects the difference between individual expectations and others' expectations. If uninformed agents know that any difference in expectations depend only on a difference in estimates of $\rho$, then the previous analysis holds in principle. Revelation occurs when some agents (\lambda) have acquired the information, if remaining agents know that the first group use a forecast rule based on this acquired information. If the noise term $\varepsilon_{p,t}$ is zero, then nobody will acquire costly information. If the noise term is not zero but uncertain, then the price will not be fully informative about informed agents' knowledge. Thus, agents may choose to acquire costly information as in the previous case.

This discussion has bearing on the literature on convergence to RE equilibrium as e.g., in Frydman (1982), and Bray and Savin (1986) and
Marcet and Sargent (1988). In this literature agents run in every period a regression of price on fundamental factors in previous periods estimating reduced form coefficients as in (10) or (17). Under reasonable economic assumptions agents' information about adjustment coefficients would converge towards RE-coefficients if the process is costless. However, it can be expected that the ability to perform this kind of analysis requires investment in information gathering capability and the ability to analyze results is required. Thus, the analysis of information acquisition in the previous sub-section applies in principle.

Specifically, the incentive to invest in capability to gather and analyse relevant information would depend on the degree of uncertainty about time series parameter instead of $\sigma^2_x$ as in (22). With modifications the analysis would proceed as in the previous case. If information costs are sufficiently low, while the variance of the price-noise term $\sigma^2_{xp}$ and the uncertainty about the structural parameter $\rho$ are sufficiently high all agents may choose to become informed by investing in capability to gather and analyze data. However, in the normal case we would expect information investments to halted before all agents acquire capability to analyze $\rho$ or other structural parameters, since the larger the share of informed agents the more information about these parameters is revealed in the price.

V. Degree of Market Efficiency and Sources of Noise in the Market Price

If the concept of efficiency is restricted to the financial market without regard to the information market the informativeness of the price signal to those not acquiring information would seem to be a reasonable definition of efficiency. If information acquisition is
disregarded, efficiency in this sense is reduced by an increasing variance of the noise terms in the earnings and the price signal, $\sigma_{\epsilon_p}^2$. Grossman and Stiglitz (1980) show how increasing noise in a price signal increases the incentive to acquire information. The larger share of informed agents offsets the effect of higher price noise on informational efficiency.

Another way to look at efficiency is to evaluate how financial and information markets work simultaneously in order to optimize the information available to agents. In this sense, inefficiency would be increasing with the difference between an optimal share of informed agents and the actual share. The optimal share is the share of firms choosing to become informed when the marginal cost of information equals the social benefit of information acquisition. Social and private benefits are equal in the absence of externalities from information acquisition. In the framework presented above there is a positive externality due to the revelation of informed agents' information through the financial asset price. The effect on total information availability in the market of this externality could formally be measured by calculating the difference between equilibrium $\lambda$ when the incentive to acquire information is evaluated conditional on $R_t$ and $P_t$ as in (22) and equilibrium $\lambda$ when the incentive is evaluated conditional only on $R_t$. This difference measures the effect of the externality caused by revelation through the price. As noted in Section IV the externality is relatively high when the noise in the price signal, $\sigma_{\epsilon_p}^2$, is relatively low. The following Proposition can be formulated based on Proposition 2d:

Proposition 3: Simultaneous financial and information market efficiency, reflecting the share of agents being informed relative to an
optimum share based on costs of acquiring information, is increased by an increase in the variance of the noise in the price signal.

\[ \sigma_{\xi P}^2 \]

Intuitively, when \( \sigma_{\xi P} \) is zero, the externality implies that no agent will acquire costly information. On the other hand, when this variance grows toward infinity, \( P \) becomes a worthless signal to the uninformed and the externality in the determination of \( \lambda \) becomes negligible. Bray and Savin (1986) show that all agents may choose to become informed if noise is sufficiently large and information costs sufficiently low.\(^{14}\)

Welfare consequences of this analysis are not obvious unless the source of noise in the price signal is specified and alternative arrangements under which information can be acquired are evaluated. For example, if the noise is interpreted as a result of pure financial market inefficiency, such as laziness of traders in response to incentives, then information acquisition serves to offset the welfare effect of inefficient trading. On the other hand, if the noise is due to transactions costs in efficient organizations or other factors which cannot be improved by financial market institutions, then the concept of efficiency developed here is more appropriate for welfare analysis. I argue below that in industrialized countries' financial markets, most sources of noise in price signals cannot be ascribed to inefficiencies.

Before turning to the sources of noise, it can be noted that externalities of information acquisition occur for other reasons than simply through the price. For example, a speculator or a trader, who is known to have invested substantial resources in capability to gather and analyze information will be closely watched and imitated. Similarly, a trader who has developed a profitable trading rule will be noticed and imitated.
As noted above, a high variance of the noise term $\epsilon_{p,t}$ in the previous analysis would be interpreted as an inefficiency in most financial market analysis although its existence is a requirement for information acquisition. The existence of the noise term within the framework of the above RE model can be explained by several factors, the existence of which are contrary to the strong assumptions of traditional RE models, but not to efficiency in financial markets per se.

The following list contains some alternative reasons for lack of informativeness in the asset price:

a) Transaction costs

b) Simultaneous uncertainty about both fundamental factors and structural parameters

c) Imperfect knowledge of what information is acquired

d) Imperfections in the information market

e) Imperfect market clearing

a) Transactions costs

Explicit costs of buying and selling assets in the form of commissions and bid-ask spreads represent an obvious reason why the asset price need not reflect average expectations perfectly. However, in well developed and thick financial markets these costs are very small and perhaps negligible for a large share of market participants.

b) Simultaneous uncertainty about both fundamental factors and structural parameters

The price $P_t$ can be interpreted generally as a signal which constitutes uninformed agents' observation of the expectations of the informed. When $\epsilon_{p,t} = 0$ the price $P_t = V_t^I$ and it reflects informed agents' perception about structural fundamentals and parameters. Assume $P_t$ can be written as:
where $A$ is an expression containing parameters such as time series parameters, costs of information, the risk-aversion coefficient, etc., as well as the share of informed firms $\lambda$.

In the previous section it was assumed that uninformed agents know that informed agents can observe either $x_t$ or the structural parameters in $A$. In general, uninformed may not know exactly what the information set of the informed is. To extract information about $x_t$ uninformed must form expectations $E^{U}[E^{I}[x_t]|P_t]$. The market noise term $\varepsilon_{P,t}$ can be viewed as a simplified way of capturing uncertainty about $E^{I}[A]$.

Assume, for example, that uninformed agents do not know the perception of informed agents about $\rho$ in $A$. Then the price $P_t$ becomes a noisy signal for extracting information about the disturbances $x_t$. Approximating structural uncertainty with a stationary, normally distributed noise term in a RE model imposes these distributional processes on parameters such as time-series characteristic of disturbances. The processes for such variables may be better described as non-normal and non-stationary. Nevertheless, the approximation offers a simple way of adding realism to the strong assumptions of RE models.\(^{15}\)

The process by which agents become informed about structural parameters and disturbances would depend on relative information costs for the two types of information. If information costs for disturbances in $x$ are relatively high, then all agents may choose to acquire information about structural parameters, since disturbances create noise in the price signal about parameters. As knowledge about structural parameters improves the externality from acquiring information about
disturbances increases and in the final equilibrium agents are well-informed structurally but ill informed about disturbances. The reverse process is also possible leading to an equilibrium in which agents are well-informed about current disturbances but ill informed about structural parameters.

c) Imperfect knowledge of what information is acquired

Even if uninformed agents know structure with certainty, the nature of explicit information that is purchased or acquired in information markets may not be known to those who do not buy it. For example, in foreign exchange markets forecast advisory services sell an exchange rate forecast but this forecast is often accompanied by the service's reasoning about several fundamental factors like $x_t$ which may cause expected exchange rate changes. (Glick and Wihlborg, 1986). In other words, informed agents buy a bundle of information about disturbances with instructions for its decomposition, which is unavailable to the uninformed. Admati and Pfleiderer (1985) argue that an informative supplier actually has an incentive to "contaminate" the information in order to overcome a free-ride problem.

d) Imperfections in the information market

So far, little has been said about the functioning of the information market. It was simply assumed that agents, who have the incentive to acquire information do so and that the equilibrium share is known. There are a number of problems with this assumption. In the absence of knowledge about $\lambda$, the price signal in (23) can be seen as a composite signal for the share of informed ($\lambda$), their expectations about other factors in $A$, and $x_t$. 

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To what extent can $\lambda$ be known to agents observing a market price? The answer to this question depends on what the cost of information actually refers to. In the setting of a market for information with many suppliers it is of course impossible for each supplier to inform each agent about how many others have purchased information. However, the incentive to purchase information in (22) requires knowledge of $\lambda$. Thus, either the market may break down due to the indeterminacy of the incentive to gather information or a monopolistic supplier may be formed. This monopolist would have to announce to each potential customer the value of $\lambda$ at different times. In the absence of a market, knowledge of $\lambda$ for each agent is possible if there is sufficient heterogeneity in the incentive to acquire information due to differential costs of access to information, and if each agent knows the structure of these costs across agents. Some uncertainty about the share of informed agents may remain. In this case, the price signal in the market becomes more noisy and the free rider problem is alleviated.

e) Imperfect market clearing and market organization

Though asset markets are perhaps closer to perfect auction markets than other markets, they cannot be considered as such. We enter here into the rapidly expanding analysis of market micro-structure. In this literature the determinants of bid-ask spreads are analyzed but also the mechanism by which participants' information is revealed.

For the purposes of this paper we are concerned with the extent to which an agent can use superior information without revealing that he or she has superior information before a contract is entered. For example, if each agent is small in the market-place and this agent believes that others do not have the same information, then the expected effect on
price of transacting with the superior information is negligible. The first-comer advantage can be utilized fully. However, if many agents have the same thought then no profit will materialize.

The literature on micro-market structure deals with the specific rules under which dealers, specialists, traders, etc. interact in the market to determine a price. Information revelation depends on these rules. There is no space to enter into this whole literature here but each specific market structure allows different opportunities for an individual to enter a contract without revealing information to the other party to the transaction. For example, in the interbank foreign exchange market each bank offers other agents to buy or sell at a given price. This arrangement allows the bank's customers to take advantage of specific information and only through actual transactions will it be revealed to other market participants.

In this example there is an interval between the time at which an agent enters a contract and the time at which the price includes the information based on which the contract is entered. The longer this time interval the more noisy is $P_t$ about the current information set of other agents.

The conclusion of this section is that market imperfections often exist and/or there is substantial uncertainty about the contents of price signal. It cannot be ruled out, however, that externalities of information acquisition exist through price revelation as well as through imitation in the marketplace. Empirical evidence is discussed in the concluding section.
VI. Alternative Modes for Information Acquisition and Transmission
Under Asymmetric Information

There are many channels through which information is disseminated
only some of which involve outright acquisition. Advertising and public
relations may be informative about the demand and cost conditions faced
by a firm (see e.g. Kotowitz and Mathewson, 1979). Voluntary or
required disclosure related to takeovers has an informational role
(Grossman and Hart, 1980). Management may simply precommit to disclose
information on a continuous basis absorbing the cost of information
dispersal (Diamond, 1985). Such information release serves as a
substitute for information gathering activities by outside market
participants. Signalling of information, monitoring by specialized
agents, insider trading, and outsiders with a stake taking control, may
also serve as substitutes for information acquisition.

In this section, as few aspects of alternative modes for
transmitting firm-specific information from well-informed managers to
market participants are discussed. Of particular interest here is the
potential interdependence between the internal efficiency of a firm and
the information available to market participants.

Consider a situation in which the current performance \( R_t \) based on
levels of ability, effort and technology is known but for valuation it
is necessary to obtain information about \( \rho \) in the model. This parameter
describes how the technology and management will succeed against
competition over time. Two problems arise. Managers are likely to be
better informed than agents outside the firm about specific skills and
technology even if outsiders spend substantial resources on information
acquisition. In this situation, the performance of management,
development of technology and other factors captured by \( \rho \) may be
endogenous relative to both asset pricing and market participants' information sets.

A literature has developed on the possibilities of signalling information to market participants. For example, the debt-equity structure of the firm may be used as a signal about the earnings prospects of the firm (Ross, 1977). Leland and Pyle (1977) propose that an increase in stock-holdings of managers signal a belief in the future earnings relative to the market's valuation. Masulis (1987) contain an overview of the expanding literature on signalling through financial structure. The thrust of this literature is that by incurring a cost of some kind, managers information about the prospects of the firm can be revealed. Only very general information can be revealed this way. More specific information allowing market participants to form their own judgment about the firm's prospects cannot be revealed this way.

In the principal-agent literature current and future values of the firm-specific factor are not completely exogenous. The effort level as well as investment and product development decisions of managers are often based on objectives that differ from those of stock-holders. Therefore, the specification of the contract between the principals (stock-holders) and their agents (managers) are important for creating managerial incentives that are compatible with stock-holders objectives.

Holmstrom (1977) discusses such contract design under moral hazard and the role of monitoring. Managerial remuneration can be based on relatively easily monitored proxies for the quality of management's decision. In this case, information acquisition by stock-holders become an endogenous part of the relationship between financial market participants and the firm. In other models, optimal contracts include
equity participation by managers in which case the information contents of the price of equity becomes a concern of managers and influence their behavior. In terms of the model in Section III an incentive contract may, as noted, influence the growth of earnings, $\rho$, and uncertainty about this parameter. Thus, the contract provides partial information to the market about some insiders' view of the firm's prospects. Such a contract must be monitored, however, a process which is costly. In the model framework, earnings in every period is a relatively easily monitored variable but it provides only period-specific information and this information is noisy. Incentive contracts linked to this period-specific variable may cause welfare losses of the type discussed in Marino and Campbell (1989) in this volume, if the objective of the contract is to influence managers' performance with respect to earnings growth, $\rho$. This parameter would be reflected in the stock market price in an informationally efficient market but not in current earnings.

Contracts linked to the stock market price are obviously easily monitored but the analysis above implies that information about $\rho$ is revealed in the price only if information costs are incurred by some agents. Thus, the internal efficiency properties of incentive contracts and the information reflected in stock market prices are determined simultaneously.

In general, one would expect contractual and institutional configurations that minimize the sum of internal welfare losses and costs of information acquisition and monitoring by stockholders. As suggested by the analysis of Chan, Siegel and Thakor (1987), the optimal contractual arrangement could be that stockholders take control of a
venture and become insiders in order to reduce information costs related to monitoring of performance.

A controversial issue in financial markets is the role of inside information. It may be the only type of information that is available to potential market-participants at a zero cost. Accordingly, only if insider trading is allowed is it possible to obtain market efficiency in the traditional sense.

Is inside information a dominant factor in asset markets? This question cannot be answered here but we may note that if general market participants know the factors about which insiders have superior information, then the noise in the price signal about this information is low (compare with Section V). The model suggests that in this case, the gains from insider trading will also be low while its information value is high. In other situations insiders and the nature of their information may not be clearly identifiable and the price becomes a noisy information signal. Then there are large gains from insider trading but its information value is low. Thus, it can be argued that insider trading would affect income distribution less and contribute more to informational efficiency when markets are characterized by low transactions costs, fast market clearing, and agents are well informed about insiders' activities. A cost related to insider trading even under these circumstances may occur when such trading potentially influences managers' choice of activities.

VII. The Concept of Financial Market Efficiency; Conclusions and Empirical Evidence

The analysis in the previous section suggests a broader view of financial market efficiency than that usually taken in the finance literature. Once information costs are recognized as non-negligible
alternative institutional structures have different costs associated with the dissemination of information about the value of projects and firms to suppliers of financial resources. An efficient financial market minimizes these costs which may vary among sectors and projects. Some factors of relevance for evaluation are easily observed by outsiders and can be valued in decentralized securities markets without great efficiency losses. Information about other factors like the value of intangible assets in a firm and the ability of a firm to remain technologically competitive over time can be costly to obtain and require continuous monitoring. Only few insiders have such information. A decentralized financial market need not be the most efficient organization for dissemination of information in this case.

Most of the formal analysis in the paper has been devoted to the more limited notion of efficiency in a decentralized securities market. It was argued that if important information about fundamental variables and/or structural parameters can be acquired only at a cost, then common finance concepts of market efficiency are misleading since they take the information set of agents as exogenous. An alternative definition of efficiency recognizing the simultaneous determination of financial market and information market equilibrium was suggested.

Financial and information markets can be considered increasingly inefficient the fewer are the agents who become informed relative to those who become informed in the absence of externalities in information acquisition. These externalities occur by revelation of information through the equilibrium price and/or through imitation and observation of well-informed agents in the market.
It was shown that the inefficiency in the markets is decreasing as the noise in the price signal to the uninformed is increasing. This result is seemingly contrary to conventional analysis of efficiency with exogenous information sets. The conventional concept of efficiency presumes that noise in the price signal is caused by some remediable market failure such as the existence of monopolistic institutions protecting inefficient traders and inefficient organizational structures. However, in competitive markets, the noisiness of price with respect to specific types of information is caused by simultaneous uncertainty about disturbances, structural parameters and others' information sets, as well as by the lack of a perfect auctioneer. In this analysis these factors contribute to market efficiency by alleviating a free rider problem in information acquisition.

A welfare oriented analysis of asset pricing is complex. The definition of market efficiency suggested here is welfare oriented but it does not consider a potential governmental role in information dissemination. A topical welfare oriented policy issue is the role of insider trading. It was noted that since insiders are the only agents who are costlessly informed, insider trading may improve informational efficiency. However, permission of such trading could increase transactions costs in the market (King and Roell, 1988) and managers' decisions may be influenced by the ability to profit in the market in the short term.

Finally, considering the empirical evidence on market efficiency in the traditional sense there is an increasing body of evidence in foreign exchange and stock markets indicating inefficiency in the traditional sense.

Schiller's (1981) work on stock-market price variability and his controversial statement that prices are characterized by "excess" variability due to mass-psychological phenomena has stimulated a substantial amount of research on "bubbles" and other sources of excess variability.

Most bubble tests have been performed on inflation and foreign exchange market data (see, for example, Flood and Garber, 1980; Frankel and Froot; 1986, Meese, 1987). The analysis pertaining to bubbles in stock markets is limited to variance bound tests as, for example, in Grossman and Schiller (1981), LeRoy (1984), Mankiw, Romer and Shapiro (1985), and Kleidon (1987). Rational expectations imply certain bounds on the variance of security price relative to the price that would have existed had agents known the ex post development of fundamentals. The variance bound tests for excess variability presumes that agents are not very risk-averse, however, and that fundamentals follow stationary processes over time. Their power to identify excess variability is accordingly quite weak as pointed out by Le Roy and Kleidon.

The evidence of bubbles is at least conceptually easy to reconcile with periods of learning of structural parameters, time-series properties of disturbances and the behavior of other agents, particularly in periods of high uncertainty about fiscal and monetary
policies. Empirical work remains, however, on making the connection between the timing of bubbles and timing of policy shifts and policy uncertainty.

The strongest evidence on inefficiency is provided by Sweeney (1986a and b) who demonstrates that trading rules can be profitable and that the profitability seems persistent. They are not decreasing over time even though the trading rule does not change over time. Even with information costs we would expect that over time, more and more agents discover the rule with the consequence that profits would decrease.

One explanation for the persistence of profits which implies a degree of inefficiency in the sense suggested in this paper is that, if costs of discovering and developing trading rules are high, and, if there are imitators once good rules are found, then there may be "under-investment" in the search for profit-opportunities.
FOOTNOTES

1. Limiting the analysis to one risky asset is convenient, but it will be argued that most results generalize to the case with many risky assets.

2. Only in a risk-neutral world is the demand curve infinitely elastic. With risk-aversion the demand curve for each risky asset is downward sloping. Since firms are rarely identical, this argument implies that each firm faces a downward sloping demand curve for its financial obligations.

3. More generally, the variable y should have a different time series coefficient applied to it, but this coefficient is set to zero for analytical convenience.

4. If a market price is not a "sufficient statistic" for agents to extract information about a disturbance, then all agents may choose to incur the cost of acquiring information about this disturbance. See, for example, Grossman, 1976, and Bray, 1982 for a further discussion of this concept.

5. Reasons for failures of information markets are discussed in, for example, Admati and Pfleiderer (1986), Demsetz, (1969) and Glick and Wihlborg (1985).

6. See Glick and Wihlborg, 1985, fn. 11 and footnote below.

7. The terms within { } in (13) can be written as:

\[-e^{(E_{t+1}^{Iw} - \frac{c}{2}\sigma^2I)} - e^{(E_{t+1}^{Uw} - \frac{c}{2}\sigma^2U)}\]

8. If each individual acquires information expecting no one else to do the same as in, for example, Diamond (1985), then the incentive to acquire information is independent of the externality.

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9. Glick and Wihlborg (1985) and (1989) contain explicit analyses of this problem in the context of goods market adjustments. They show that the effect of information acquisition is often to offset fully or partially the direct effect on adjustment of a shift in a structural parameter.

10. In recent years an empirical literature has developed on testing for "bubbles" and/or "sunspots" (see e.g., Meese, 1987 and Flood and Garber, 1982). These concepts have the connotation of some irrationality and "excessive" price volatility. Bubbles refer to price changes due to self-fulfilling expectations about future price changes unrelated to fundamentals while sunspots are normally associated with extraneous variables i.e., non-fundamentals which by agents are believed to be fundamentals.

11. This proposition does not hold if information acquisition requires investment in capability to gather and analyze information. In this case, the incentive to invest is obtained by forming expectations in period t of the gains from being informed in future periods. The sum over time of the incentive described in equation (22) on the left hand side would be compared to the investment. The time specific term in (22) would be a variance, since future realizations of variables would be unknown at the time expectations are formed.

12. Analogous results are obtained in Glick and Wihlborg (1985) and (1986).

13. In Grossman and Stiglitz (1980) an increase in noise has no effect on the informativeness of the price system due to the exactly
offsetting effects of increased noise and increased information acquisition.

14. Stein (1987) argues that the informativeness of asset prices could fall as a result of speculative activity when risk-neutral speculators are less informed than other risk-averse market participants. When information is costly this situation seems implausible, since risk-neutral agents acquire information based on any expected improvement in the valuation of the asset while a risk-averse agent would acquire information only if the improvement is expected to exceed a risk-premium.

15. Individuals' assumptions and knowledge about others' expectation formation play an important role in rational expectations models. In macroeconomics, it is often assumed that all agents use simple, identical forecast rules, based on knowledge of structural parameters. Frydman (1982) points out that the assumption about expectation formation is somewhat arbitrary and it may not represent optimizing behavior by agents. He argues that, if agents, in order to forecast future prices, estimate the parameters of a model by running regressions of price on exogenous variables, then convergence to a RE equilibrium may not occur unless each individual's price forecast is a "consensus" forecast of price on exogenous variables alone. However, each agent has an incentive to try to discover how others form expectation and try to profit from this information.

16. Such heterogeneity of information costs provides the reason why it is possible to assume in equations (5a) and (5b) that an informed
(uninformed) agent expects to be an informed (uninformed) agent in all future periods.

REFERENCES


Appendix I. The incentive to acquire information when structural parameters are known.

By adding and subtracting appropriate terms (15) can be rewritten as:
\[ I_t' = \Omega(1-z^2)[E_U((1+r)^I - (1+r)^U)^2] \]
\[ - 2(1+r)^U_E((1+r)^I - (1+r)^U)] + \]
\[ + [\Omega(1-z^2)-1]((1+r)^U - (1+r)^I)^2 - z(1-r)^2 \sigma_{y}^2 \]
\[ \Omega = \frac{\sigma_x^2}{\sigma_y^2}. \] These variances are known to informed as well as uninformed. (9a) and (9b) make it possible to express (1+r)^I and (1+r)^U as functions of x_t and E[x_t], respectively. The observation that (1+r)^j = E_t[t_{t+1}/P_t] for j = I, U is also used. With these substitutions E_U((1+r)^I - (1+r)^U)^2 = 0. The term E_U((1+r)^I - (1+r)^U)^2 can be written as (1/P_t^2 \cdot \rho^2(E_x[x_t]^2 - (E_x[x_t])^2) = (1/P_t^2) \cdot \rho^2 \sigma_x^2. After making these substitutions in (A1), equation (22) in the text is derived.

Appendix II. The incentive to acquire information is decreasing in the share of informed agents.

Equilibrium in the information market requires that the left hand side in (22) is decreasing relative to the right hand side as \lambda increases. First we study \sigma_x^2 as a function of \lambda. Without formal proof we observe in (17) that as \lambda increases a larger proportion of the variance in price P_t is due to the variance in \varepsilon_x relative to variances in y and \varepsilon_p. Thus the informativeness of P_t with respect to \varepsilon_x is increasing in \lambda and \sigma_x^2 is decreasing in \lambda. Consider next \Omega = \sigma_x^2/\sigma_y^2. This term is also decreasing in \lambda since, as \lambda increases, the perception error of the uninformed about \varepsilon_{x,t} decreases and therefore, the forecast of P_{t+1} by the uninformed approaches the forecast of the informed.
Formally, this proposition is proven by comparing the variances of $P_{t+1} - E_t[P_{t+1}]$ and $P_{t+1} - E_t'[P_{t+1}]$.

The right hand side of (22) is also decreasing in $\lambda$ since the information cost $z$ is multiplied by $\sigma^{2U}_r$ but the effect of a change in $\lambda$ on the left hand side is multiplicative through $\sigma^{2U}_x$ and $\Omega$. 