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The Variability of Inflation and Real Stock Returns*

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Abstract

While a negative correlation between inflation and real stock returns have been well documented, the cause of this relationship has been the subject of considerable controversy. The most plausible causal interpretation is the variability hypothesis which points to a chain from higher inflation to greater variability and uncertainty to depressed economic activity, hence generating a link between inflation and expected returns. The previous studies have not found support for this hypothesis, however, and Fama’s non casual proxy hypothesis has gained considerable currency. We argue that there have been serious methodological problem with the previous tests of the variability hypothesis. When these are corrected, we find strong support for the casual variability hypothesis in the post war data for the United State.

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1. Introduction

While a negative relationship between inflation and real stock market returns has been well documented for the United States and a number of other countries, the explanation for this relationship has not been resolved. ¹ The proxy hypothesis, a possibility suggested by Fama (1981), argues that the relationship is not a true causal one, but only a proxy for the true relationship between expected economic activity and stock returns. The statistical association between inflation and stock returns is posited to operate through the effects of anticipated changes in real activity on the price level via the money demand function. ²

Another possibility, namely the variability hypothesis, is that there is direct, causal relationship based on the well-documented tendency for higher rates of inflation to be more variable and hence promote greater uncertainty. According to this variability hypothesis, a rise in inflation generates greater uncertainty which, in turn, depresses stock returns. ³ This effect could operate through several mechanisms. The increased uncertainty could depress future economic activity à la Friedman (1977), and expectations of this future decline in output would in turn depress current stock returns. This channel is somewhat similar to Fama’s proxy hypothesis except that the causation is assumed to run from higher inflation to expectations of a decline in economic activity rather than from expected declines in economic activity to higher inflation. Greater uncertainty generated by higher inflation could also lead to an increased risk premium being added to the discount rate, lowering the discounted present value of expected future cash flows. ⁴ Again, this would lead to a decline in the current level of stock prices.

Unfortunately, the empirical evidence to date on the proxy hypothesis for the United States has been mixed and the only major studies of the variability hypothesis of which we are aware found negative results. ⁵ We should note that not all of the studies which find some support
for the proxy hypothesis imply evidence against the variability hypothesis. For example, Kaul (1987) finds some support for the proxy hypothesis, but his inclusion of a real activity variable does not eliminate the role of the change in expected inflation in his equation, suggesting that the proxy hypothesis is not a complete explanation. In the a more recent study, Balduzzi (1995) finds strong support for the proposition that more than the proxy hypothesis is at work. Taking a VAR approach he uses covariance analysis to show that production growth accounts for only a small portion of the negative correlation between stock return and inflation in the US between 1954 and 1990. The major cause is found to be innovation in the inflation rate, with interest rate innovation being the next most important. This suggests the plausibility of the variability hypothesis as a behavioral explanation for the negative correlation.

However, in his study of the variability hypothesis, Buono (1989) does not find supporting evidence. He uses an autoregressive conditional heteroscedastic (ARCH) model to generate a measure of inflation variability. He finds further support for a positive link between higher inflation and greater inflation variability, but concludes that “the evidence suggests that the negative relation between (nominal) stock returns and the variability of inflation posited by the variability hypothesis is not consistent with the data.” (p. 337). This interpretation is open to question, however. Given a positive relation between the variability of inflation and the rate of inflation, only a negative relation between real stock returns and the variability of inflation can cause the negative real stock returns-inflation relationship. But, when nominal returns are used, a positive relation between nominal stock returns and the variability of inflation may also cause the negative real stock returns-inflation relationship. Therefore, by using nominal returns and examining whether the coefficient on the inflation variability is less than zero, Buono(1987) biased his test against finding support for the variability hypothesis.
In this paper we reexamine the variability hypothesis using real stock returns and find fairly strong evidence in its favor for the whole post-World War II period and first two sub-periods. However, we are unable to test the variability hypothesis for the post-1981 period because the conventional tests do not detect ARCH disturbances in the inflation equation and hence inflation variability cannot be measured by the ARCH model in this period. This reflects a limitation of the ARCH model in studying the variability of inflation.

In the following section, we discuss the methodology we use to model the variability of inflation and the datasets. We then present our results in section III. Section IV offers a concluding remark.

II. Methodology and Data Sources

As noted in the introduction, the test of the variability hypothesis consists of two parts. The first concerns the well-established positive relationship between the inflation rate and the variability of inflation. The second concerns the posited negative relationship between real stock returns and the variability of inflation.

There is a substantial literature on proxies for inflation uncertainty and their relationship to levels of inflation. For compatibility with the previous study of the variability hypothesis by Buono (1989), we proxy inflation variability by the conditional variance of inflation generated by Engle’s (1982) ARCH model or its alternative, GARCH model, when it is more appropriate. As Holland (1993) notes, studies of the U.S. inflation process, which have been based on the assumption of fixed parameters such as the ARCH or GARCH model, have tended to be substantially less successful in finding a positive relationship between inflation and inflation variability and a negative relationship between inflation variability and economic activity in US
data than have other approaches. Thus by using this approach we are likely biasing our analysis against finding support for the variability hypothesis. Therefore we believe that our positive findings with the ARCH or GARCH model provide strong evidence in favor of the variability hypothesis.

Following Bollerslev (1986), the inflation process with the GARCH(p,q) disturbances is given by:

\[ \pi_t | \psi_{t-1} \sim N(x_t \beta, h_t) \]  

\[ h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_i h_{t-i} \]  

\[ \epsilon_t = \pi_t - x_t \beta \]  

where \( \pi_t \) is the rate of inflation, \( \psi_{t-1} \) is the information set available to forecasters at time t-1, \( x_t \) is a vector of explanatory variables, and \( h_t \) is the conditional variance of inflation. To generate estimates of the conditional variance of inflation, the inflation rate in equation (1) has to be specified. We use the reduced form specification for inflation:

\[ \pi_t = \alpha_0 + a^{k_1} (L) \pi_t + b^{k_2} (L) MG_t + c^{k_3} (L) WG_t + d^{k_4} (L) SG_t + \epsilon_t \]  

Here MG, WG and SG are the growth rates of money, the cost of labor and the oil price, which is used to proxy supply side effects, \( a^{k_1} (L) = a_1 L + a_2 L^2 + \ldots + a_{k_1} L^{k_1} \), and similarly for the other polynomials; \( \epsilon \) is the error term. We follow the procedure proposed by Hsiao (1981) to choose the optimal values for \( k_1, k_2, k_3, \) and \( k_4 \) base on Akaike’s (1970) FPE criterion.

The variables used in this paper are nominal monthly stock returns (equally weighted index of return including dividend, SR_t) from the CRSP Database, the M_t nominal money supply
(M_t), the consumer price index (P_t), producer price index of oil (S_t), and the cost of labor (W_t) from CITIBASE Database. Real stock returns, rsr_t, are given by rsr_t = SR_t - ln(P_t/P_{t-1}). All series are monthly observations from 1954:01 to 1995:12.

III. Empirical Results

We test the variability hypothesis in two stages. In the first stage, we identify the inflation equation and estimate the GARCH model for the sample period. In the second stage, we regress real stock returns on the variance estimates of inflation. Following the above procedure and setting the maximum lag length at twelve, we find the following specification for the 1954-1995 period:

$$\pi_t = \alpha_0 + a^{12}(L)\pi_t + b^5(L)MG_t + c^3(L)WG_t + d^2(L)SG_t + \epsilon_t$$  \hspace{1cm} (5)

Parameter estimates for equation (5) and specification tests are reported in Table 1. The Breusch-Godfrey test does not indicate serial correlation in the disturbances of equation (5). The Lagrange Multiplier test for ARCH disturbances strongly rejects the null hypothesis of conditional homoscedastic disturbances and suggests a high order (eight or higher) ARCH effect.

Based on these test results, if we can assume that the disturbances in the inflation equation follow a particular ARCH process, more efficient estimates, consistent standard errors, and a series of estimates of $h_t$ will be obtained by reestimating equation (5) using the maximum likelihood estimation method. However, the ARCH(8) process in the disturbances is not an ideal specification because too many parameters need to be estimated. Bollerslev (1987) finds that, for quarterly US inflation rate, a GARCH model exhibits a more reasonable lag structure and
provides a slightly better fit than the ARCH model. We find that a GARCH(2,2) process provided a better fit for the monthly inflation rate used in this paper. Table 1, Part 2 reports the maximum likelihood estimation results for equation (5).

To test the variability hypothesis, we regressed real stock returns on current and twelve lagged values of the conditional variance of inflation, $h_t$, as did Buono, and regressed $h_t$ on current and six lagged values of inflation, $\pi_t$. The results for the whole sample period are presented in Table 2, Part 1. In the variability of inflation equation, the sum of the coefficients on the current and lagged $\pi_t$ is 0.03 with a t-statistic of 3.601 and a significance level of less than 0.001, showing a highly significant positive relationship between inflation and the variability of inflation. In the stock returns equation, the sum of the coefficients on the current and lagged $h_t$ is -4.76 with a t-statistic of -6.943 and a significance level of less than 0.001, indicating that the negative relation between real stock returns and the variability of inflation is highly significant. The F-test also decisively reject the null hypothesis that the coefficients on the current and lagged $h_t$ are jointly equal to zero.

Given the substantial difference in inflation experience over our sample period and previous evidence of nonlinear relationships among inflation, inflation variability, and economic activity, it is important to check for robustness of the inflation variability-stock returns relationship over different sub-periods. We divide our whole sample period into three sub-periods: 1954:01-1969:12, 1970:01-1981:12, and 1982:01-1995:12. The first and third periods are dominated by low and relatively stable inflation, the second period is dominated by rising and highly volatile inflation.

Table 2, Parts 2, 3 and 4 report the regression results for the three sub-periods. The results for the first and third periods are similar. While the relation between inflation and its
variability is positive, the coefficients are small and are not close to being significant for the two periods. The sums of the coefficients on the current and lagged $h_i$ in the both stock returns equations are insignificant, indicating that stock returns are not related to the variability of inflation in these two periods. In the second period, the relation between inflation and the variability of inflation remains positive as in the whole period, and is significant at the 10% level. The sum of the coefficients on the current and lagged $h_i$ in the stock returns equations is -3.902 and is significant at the 1% level.

It appears that only the results for the second period are consistent with the results for the whole sample period. However, we should not jump to the conclusion that the variability hypothesis is rejected in the first and third periods. Because the inflation dynamics in the three periods are quite different, the inflation variability series used to test the hypothesis for the three periods is likely to be misspecified when we use a single equation to model the inflation process for the whole period. Therefore, the results in Parts 2 to 4 do not reveal the true relations between real stock returns and the inflation variability.

To formally check the robustness of the variability hypothesis, we model the inflation processes and estimate the variability of inflation separately for the three sub-periods following the procedure used in the whole sample period. The results reported in Tables 3, 4, and 5 show that the inflation processes in the three sub-periods are indeed quite different. The sum of the coefficients on the lagged oil prices is about 0.02 in the first and second periods compared to a sum of 0.004 in the third period, suggesting that oil price shocks are more important driving forces of inflation in the first and second periods than in the third period. The most striking difference is that the disturbances in the inflation equation follow a ARCH(2) and ARCH(1) processes in the first and second periods respectively, while in the third period the conditional
variance of the disturbances appears to be constant according to the ARCH and White’s tests. As a result, we are unable to estimate the inflation variability using the ARCH models and to examine the relation between stock returns and the variability of inflation in the third period.

The maximum likelihood estimation results for the inflation equations with ARCH(2) and ARCH(1) innovations for the first and second periods are reported in Parts 2 in Table 3 and Table 4 respectively. They show little difference from the OLS estimations in the both cases.

Next, we reexamine the relation between stock returns and the variability of inflation for the first and second periods using the series on the conditional variances generated from the maximum likelihood estimations in the two periods. The results are reported in Tables 6 and 7. As in the whole sample period, we found a positive and significant relation between the inflation variability and inflation and a negative and significant relation between stock returns and the inflation variability in both periods. In particular, the sum of the coefficients on the current and lagged $h_t$ in the stock returns equation is -80.08 with a t-statistic of 3.4 and a significance level of less than 0.001 for the first period, and is -2.767 with a t-statistic of 4.085 and a significance of less than 0.001 for the second period. The absolute values of the coefficients on the inflation variability and the sum of these coefficients in the stock return equations are smaller for the second period than for the first period, suggesting that the negative relation between real stock returns and the variability of inflation is sensitive to the inflation dynamic.

IV. Concluding Remarks

Our empirical results present strong evidence that higher inflation is more volatile and the increased inflation uncertainty does tend to reduce real stock returns. Thus the well known negative correlation between inflation and stock returns does not appear to be entirely spurious.
as was suggested by Fama’s proxy hypothesis. However, we cannot conclude that the variability hypothesis that inflation uncertainty explains the negative inflation-stock returns relationship presents a full explanation. While the negative correlation between inflation and the real stock returns in the third period remains as strong as it does in the first two periods, we are unable to test directly the variability hypothesis because the conventional tests do not detect the ARCH disturbances in the inflation equation and hence the inflation variability cannot be measured by the ARCH model in this period.

One possible interpretation is that economic agents had come to expect inflation to be associated with greater uncertainty and did not revise these views in the third period even though the ARCH structure of the inflation process had changed. While we used the ARCH methodology for consistency with previous studies, our results for the third period highlight a major limitation of this methodology and reinforce the suggestion of Holland (1993) that ARCH and GARCH models do not adequately capture the relationship between rates of inflation and the uncertainty generated by variation in the rate of inflation. While this problem increases the weight that we would put on our positive findings for the first two periods, it also points to the need to develop better measure of inflation variability that is sensitive to differences across inflation regimes. Following a long period of stable and low inflation rates, we would not be surprised to see the negative correlation between real stock returns and the minor innovations in inflation rate disappear. The results from our first and third periods suggest, however, that it could take quite a long period of stable inflation for this to occur.
Reference


Footnotes

1. See Bodie (1976), Nelson (1976), Fama and Schwert (1977), and Gultekin (1983).

2. Geske and Roll (1983) present another possible mechanism for spurious correlation operating through the monetization of government debt.


6. Fama’s original contribution also finds a continued negative relationship between expected inflation and stock returns until he includes the growth rate of a monetary variable in his regression. As McCarthy et al. (1990) note, the monetary growth variable should be highly correlated with expected inflation.

7. As part of a larger study, Vanderhoff and Vanderhoff (1986) consider Malkiel’s (1979) risk hypothesis version of the variability hypothesis. They do not consider the variability hypothesis directly, but argue that the risk hypothesis implies that the effects of changes in expected inflation on stock return should vary across industries. Using data for seven US industries from 1968 to 1982, their results are mixed. While coefficients do tend to vary across industries, when changes in expected income are added to the model, significant negative coefficients are found for only few industries. They conclude that the spurious correlation (proxy) hypothesis works best in their study.

8. This of course depends upon the sensitivity of the inflation variability to the level of inflation.
9. See, for example, the analysis and references in Evans and Wachtel (1993) and Holland (1993).

10. The ARCH(q) process is the special case of GARCH(p, q) when p=0.

11. For simplicity, we do not attempt to estimate both stages simultaneously. Pagan (1984) shows that the estimates of the parameters of interest produced from the two-stages procedure are consistent.

12. We assume that inflation causes the variability of inflation. The values of inflation after sixth lag are not included in the variability of inflation regression because they are insignificant.

13. For example, Graham (1996) found that the relation between real stock returns and inflation is unstable. In particular, he found that the relation is negative before 1976 and after 1982, but positive between these years. Also see the analysis and references in Burdekin, Salamun, and Willett (1995) and Holland (1984, 1993).

1. OLS Estimation

\[
\pi_t = 0.008 + 0.225\pi_{t-1} + 0.188\pi_{t-2} + 0.039\pi_{t-3} + 0.058\pi_{t-4} + 0.120\pi_{t-5} + 0.067\pi_{t-6} + 0.026\pi_{t-7} + (0.486) (4.400) (3.623) (0.766) (2.420) (1.462) (0.553) \\
+ 0.042\pi_{t-8} + 0.146\pi_{t-9} + 0.067\pi_{t-10} + 0.042\pi_{t-11} + 0.099\pi_{t-12} + 0.012SG_{t-1} + 0.004SG_{t-2} + 1.351WG_{t-1} + (0.919) (3.212) (1.463) (0.920) (2.282) (3.905) (1.241) (1.071) \\
+ 0.510WG_{t-2} + 2.281WG_{t-3} + 0.063MG_{t-1} - 0.687MG_{t-2} + 4.383MG_{t-3} + 2.237MG_{t-4} + 3.015MG_{t-5} + (0.400) (1.806) (0.030) (0.311) (2.037) (1.011) (1.415) (1.071)
\]

\[R^2 = 0.586, \quad SEE = 0.203\]

Breusch-Godfrey LM Test for Serial Correlation

<table>
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<tr>
<th>Order of serial correlation</th>
<th>T*R²</th>
<th>Prob&gt;(\chi^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.205</td>
<td>0.651</td>
</tr>
<tr>
<td>4</td>
<td>2.080</td>
<td>0.721</td>
</tr>
<tr>
<td>8</td>
<td>5.751</td>
<td>0.675</td>
</tr>
<tr>
<td>12</td>
<td>15.12</td>
<td>0.235</td>
</tr>
</tbody>
</table>

ARCH Test

<table>
<thead>
<tr>
<th>Order of ARCH</th>
<th>T*R²</th>
<th>Prob&gt;(\chi^2)</th>
</tr>
</thead>
<tbody>
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<td>0.000</td>
</tr>
<tr>
<td>4</td>
<td>12.15</td>
<td>0.016</td>
</tr>
<tr>
<td>8</td>
<td>16.74</td>
<td>0.033</td>
</tr>
<tr>
<td>10</td>
<td>16.62</td>
<td>0.083</td>
</tr>
</tbody>
</table>

2. Maximum Likelihood Estimation: assuming \(h_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\varepsilon_{t-2}^2 + \beta_1h_{t-1} + \beta_2h_{t-2}\)

\[
\pi_t = 0.012 + 0.217\pi_{t-1} + 0.109\pi_{t-2} + 0.049\pi_{t-3} + 0.107\pi_{t-4} + 0.129\pi_{t-5} + 0.060\pi_{t-6} + 0.021\pi_{t-7} + (0.761) (3.631) (2.376) (1.096) (2.548) (3.202) (1.426) (0.474) \\
+ 0.085\pi_{t-8} + 0.151\pi_{t-9} + 0.077\pi_{t-10} + 0.024\pi_{t-11} - 0.090\pi_{t-12} + 0.008SG_{t-1} - 0.004SG_{t-2} + 2.203WG_{t-1} + (2.055) (4.071) (1.782) (0.676) (2.095) (3.002) (1.502) (1.827) \\
+ 1.129WG_{t-2} + 3.297WG_{t-3} - 0.047MG_{t-1} - 3.893MG_{t-2} + 3.902MG_{t-3} + 4.886MG_{t-4} + 1.077MG_{t-5} + (1.084) (2.879) (0.027) (2.426) (2.012) (2.878) (0.553) \\
+ h_t = 0.018 + 0.471\varepsilon_{t-1}^2 - 0.007\varepsilon_{t-2}^2 - 0.054h_{t-1} + 0.200h_{t-2} + (3.617) (6.030) (0.071) (0.310) (2.146) \\
\]

Note: T-Statistics are given in parentheses below the coefficients estimates
Table 2: Tests of the Variability Hypothesis


**Variability of Inflation Equation:**
\[ h_t = \beta_0 + \beta_2 \pi_t + \beta_3 \pi_{t-1} + \ldots + \beta_6 \pi_{t-6} \]

\[ h_t = 0.033 - 0.013\pi_t + 0.049\pi_{t-1} - 0.040\pi_{t-2} + 0.014\pi_{t-3} - 0.020\pi_{t-4} + 0.016\pi_{t-5} + 0.025\pi_{t-6} \]

\( (9.168) (1.276) (4.570) (3.708) (1.270) (1.802) (1.463) (2.416) \)

\[ \bar{R}^2 = 0.085, \quad SEE = 0.048, \quad \sum_{i=1}^{7} b_i = 0.030 \]

\[ H_0: \sum_{i=1}^{7} \beta_i = 0, \quad t-Statistic: 3.601, \quad Significance Level: 0.000 \]

\[ H_0: \beta_1 = \ldots = \beta_7 = 0, \quad F-Statistic: 7.680, \quad Significance Level: 0.000 \]

**Stock Returns Equation:**
\[ rsr_t = \beta_0 + \beta_1 h_t + \beta_2 h_{t-1} + \ldots + \beta_{13} h_{t-12} \]

\[ rsr_t = -0.129 - 0.111h_t - 0.479h_{t-1} - 0.299h_{t-2} - 0.210h_{t-3} - 0.476h_{t-4} - 0.333h_{t-5} - 0.352h_{t-6} \]

\( (3.903) (0.389) (1.674) (1.024) (0.721) (1.631) (1.139) (1.205) \)

\[ - 0.022h_{t-7} - 0.436h_{t-8} - 0.401h_{t-9} - 0.136h_{t-10} - 0.672h_{t-11} - 0.833h_{t-12} \]

\( (0.075) (1.495) (1.377) (0.466) (2.350) (2.923) \)

\[ \bar{R}^2 = 0.075, \quad SEE = 0.311, \quad \sum_{i=1}^{13} b_i = -4.760 \]

\[ H_0: \sum_{i=1}^{13} \beta_i = 0, \quad t-Statistic: -6.943, \quad Significance Level: 0.000 \]

\[ H_0: \beta_1 = \ldots = \beta_{13} = 0, \quad F-Statistic: 4.155, \quad Significance Level: 0.000 \]


**Variability of Inflation Equation:**
\[ h_t = \beta_0 + \beta_2 \pi_t + \beta_3 \pi_{t-1} + \ldots + \beta_6 \pi_{t-6} \]

\[ h_t = 0.037 - 0.009\pi_t + 0.016\pi_{t-1} - 0.007\pi_{t-2} + 0.012\pi_{t-3} - 0.008\pi_{t-4} - 0.009\pi_{t-5} + 0.009\pi_{t-6} \]

\( (17.06) (1.219) (2.144) (0.998) (1.638) (1.139) (1.175) (1.179) \)

\[ \bar{R}^2 = 0.034, \quad SEE = 0.019, \quad \sum_{i=1}^{7} b_i = 0.003 \]

\[ H_0: \sum_{i=1}^{7} \beta_i = 0, \quad t-Statistic: 0.326, \quad Significance Level: 0.745 \]

\[ H_0: \beta_1 = \ldots = \beta_7 = 0, \quad F-Statistic: 1.955, \quad Significance Level: 0.063 \]
Table 2: Tests of the Variability Hypothesis (continued)

**Stock Returns Equation:**
\[ r_{sr_t} = \beta_0 + \beta_1 h_t + \beta_2 h_{t-1} + \ldots + \beta_{13} h_{t-12} \]

\[ r_{sr_t} = -0.284 + 0.701 h_t + 0.235 h_{t-1} + 0.380 h_{t-2} + 1.418 h_{t-3} + 1.621 h_{t-4} - 1.389 h_{t-5} - 0.449 h_{t-6} \]
\[ (3.151) \quad (0.786) \quad (0.286) \quad (0.452) \quad (1.694) \quad (1.928) \quad (0.165) \quad (0.536) \]
- 0.105 h_{t-7} - 0.845 h_{t-8} + 0.088 h_{t-9} + 0.380 h_{t-12} \]
\[ (0.126) \quad (1.009) \quad (0.104) \quad (0.452) \]
\[ \bar{R}^2 = -0.15, \quad SEE = 0.221, \quad \sum_{i=1}^{13} b_i = -3.149 \]

\[ H_0: \sum_{i=1}^{13} \beta_i = 0, \quad t-Statistic: 1.329, \quad Significance Level: 0.186 \]
\[ H_0: \beta_1 = \ldots = \beta_{13} = 0, \quad F-Statistic: 0.787, \quad Significance Level: 0.673 \]

**Variability of Inflation Equation:**
\[ h_t = \beta_0 + \beta_1 \pi_t + \beta_2 \pi_{t-1} + \ldots + \beta_7 \pi_{t-6} \]
\[ h_t = 0.026 - 0.019 \pi_t + 0.092 \pi_{t-1} - 0.076 \pi_{t-2} - 0.002 \pi_{t-3} + 0.034 \pi_{t-4} + 0.041 \pi_{t-5} + 0.039 \pi_{t-6} \]
\[ (1.564) \quad (0.726) \quad (3.335) \quad (2.639) \quad (0.055) \quad (1.167) \quad (1.489) \quad (1.429) \]
\[ \bar{R}^2 = 0.118, \quad SEE = 0.077, \quad \sum_{i=1}^{7} b_i = 0.041 \]

\[ H_0: \sum_{i=1}^{7} \beta_i = 0, \quad t-Statistic: 1.627, \quad Significance Level: 0.100 \]
\[ H_0: \beta_1 = \ldots = \beta_7 = 0, \quad F-Statistic: 3.744, \quad Significance Level: 0.001 \]

**Stock Returns Equation:**
\[ r_{sr_t} = \beta_0 + \beta_1 h_t + \beta_2 h_{t-1} + \ldots + \beta_{13} h_{t-12} \]
\[ r_{sr_t} = -0.428 + 0.100 h_t - 0.359 h_{t-1} - 0.287 h_{t-2} - 0.259 h_{t-3} - 0.544 h_{t-4} - 0.384 h_{t-5} - 0.258 h_{t-6} \]
\[ (7.243) \quad (0.283) \quad (1.016) \quad (0.798) \quad (0.718) \quad (1.512) \quad (1.065) \quad (0.714) \]
+ 0.178 h_{t-7} - 0.291 h_{t-8} - 0.234 h_{t-9} - 0.043 h_{t-10} - 0.685 h_{t-11} - 0.834 h_{t-12} \]
\[ (0.492) \quad (0.808) \quad (0.652) \quad (0.121) \quad (1.937) \quad (2.357) \]
\[ \bar{R}^2 = 0.058, \quad SEE = 0.335, \quad \sum_{i=1}^{13} b_i = -3.902 \]

\[ H_0: \sum_{i=1}^{13} \beta_i = 0, \quad t-Statistic: 3.887, \quad Significance Level: 0.000 \]
\[ H_0: \beta_1 = \ldots = \beta_{13} = 0, \quad F-Statistic: 1.672, \quad Significance Level: 0.074 \]

**Variability of Inflation Equation**: \( h_t = \beta_0 + \beta_1 \pi_t + \beta_2 \pi_{t-1} + ... + \beta_7 \pi_{t-6} \)

\[
\begin{align*}
\hat{h}_t &= 0.039 - 0.0063\pi_t + 0.023\pi_{t-1} - 0.024\pi_{t-2} + 0.013\pi_{t-3} - 0.016\pi_{t-4} + 0.012\pi_{t-5} + 0.012\pi_{t-6} \\
(4.514) & \hspace{1cm} (0.369) \hspace{1cm} (1.288) \hspace{1cm} (1.334) \hspace{1cm} (0.689) \hspace{1cm} (0.892) \hspace{1cm} (0.683) \hspace{1cm} (0.762)
\end{align*}
\]

\[
\bar{R}^2 = -0.018, \hspace{1cm} SEE = 0.038, \hspace{1cm} \sum_{i=1}^{7} \hat{b}_i = 0.013
\]

\( H_0 : \sum_{i=1}^{7} \beta_i = 0 \), \hspace{1cm} \text{t-Statistic: 0.489, Significance Level: 0.625}

\( H_0 : \beta_1 = ... = \beta_7 = 0 \), \hspace{1cm} \text{F-Statistic: 0.587, Significance Level: 0.766}

**Stock Returns Equation**: \( r_{srt} = \beta_0 + \beta_1 h_t + \beta_2 h_{t-1} + ... + \beta_{13} h_{t-12} \)

\[
\begin{align*}
r_{srt} &= -0.268 + 0.005h_t - 0.381h_{t-1} - 0.002h_{t-2} + 0.313h_{t-3} - 0.058h_{t-4} + 0.612h_{t-5} - 0.192h_{t-6} \\
&\hspace{1cm} (6.482) \hspace{1cm} (0.010) \hspace{1cm} (0.726) \hspace{1cm} (0.004) \hspace{1cm} (0.601) \hspace{1cm} (0.111) \hspace{1cm} (1.173) \hspace{1cm} (0.369)
\end{align*}
\]

\[
\begin{align*}
- 0.290h_{t-7} - 0.255h_{t-8} - 0.422h_{t-9} + 0.129h_{t-10} + 0.216h_{t-11} + 0.018h_{t-12} \\
&\hspace{1cm} (0.556) \hspace{1cm} (0.491) \hspace{1cm} (0.811) \hspace{1cm} (0.245) \hspace{1cm} (0.419) \hspace{1cm} (0.036)
\end{align*}
\]

\[
\bar{R}^2 = -0.057, \hspace{1cm} SEE = 0.224, \hspace{1cm} \sum_{i=1}^{13} \hat{b}_i = -0.305
\]

\( H_0 : \sum_{i=1}^{13} \beta_i = 0 \), \hspace{1cm} \text{t-Statistic: 0.356, Significance Level: 0.722}

\( H_0 : \beta_1 = ... = \beta_{13} = 0 \), \hspace{1cm} \text{F-Statistic: 0.313, Significance Level: 0.989}

Note: T-Statistics are given in parentheses below the coefficients estimates

1. OLS Estimation

\[ \pi_t = 0.018 + 0.115\pi_{t-1} + 0.182\pi_{t-2} - 0.047\pi_{t-3} + 0.122\pi_{t-4} + 0.087\pi_{t-5} + 0.118\pi_{t-6} + 0.077\pi_{t-7} + 0.152\pi_{t-8} - 1.132MG_{t-1} + 4.655MG_{t-2} + 2.327MG_{t-3} + 5.106MG_{t-4} + 0.011SG_{t-1} - 0.001SG_{t-2} + 0.010SG_{t-3} + 2.348WG_{t-1} + 1.206WG_{t-2} + 5.154WG_{t-3} \]

\[(0.770) (1.538) (2.448) (1.608) (1.276) (1.159) \]

\[ R^2 = 0.261, \quad SEE = 0.188 \]

Breusch-Godfrey LM Test for Serial Correlation

<table>
<thead>
<tr>
<th>Order of serial correlation</th>
<th>T*R^2</th>
<th>Prob&gt;\chi^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.852</td>
<td>0.356</td>
</tr>
<tr>
<td>4</td>
<td>1.595</td>
<td>0.810</td>
</tr>
<tr>
<td>8</td>
<td>4.006</td>
<td>0.857</td>
</tr>
<tr>
<td>12</td>
<td>18.05</td>
<td>0.114</td>
</tr>
</tbody>
</table>

ARCH Test

<table>
<thead>
<tr>
<th>Order of ARCH</th>
<th>T*R^2</th>
<th>Prob&gt;\chi^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.836</td>
<td>0.085</td>
</tr>
<tr>
<td>2</td>
<td>3.551</td>
<td>0.077</td>
</tr>
<tr>
<td>3</td>
<td>6.006</td>
<td>0.198</td>
</tr>
<tr>
<td>4</td>
<td>7.126</td>
<td>0.211</td>
</tr>
</tbody>
</table>

2. Maximum Likelihood Estimation: assuming \( h_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\varepsilon_{t-2}^2 \)

\[ \pi_t = -0.004 + 0.164\pi_{t-1} + 0.204\pi_{t-2} - 0.054\pi_{t-3} + 0.158\pi_{t-4} + 0.117\pi_{t-5} + 0.151\pi_{t-6} + 0.026\pi_{t-7} + 0.146\pi_{t-8} + 1.214MG_{t-1} + 6.006MG_{t-2} + 4.176MG_{t-3} + 5.906MG_{t-4} + 0.013SG_{t-1} - 0.002SG_{t-2} + 0.012SG_{t-3} + 2.650WG_{t-1} + 0.231WG_{t-2} + 5.108WG_{t-3} \]

\[(0.186) (2.120) (2.301) (0.791) (2.203) (1.828) (2.514) (0.389) \]

\[ h_t = 0.025 - 0.020\varepsilon_{t-1}^2 + 0.269\varepsilon_{t-2}^2 \]

\[(5.097) (0.255) (1.705) \]

Note: T-Statistics are given in parentheses below the coefficients estimates

1. OLS Estimation

\[ \pi_t = -0.023 + 0.217\pi_{t-1} + 0.290\pi_{t-2} + 0.103\pi_{t-3} + 0.016\pi_{t-4} + 0.141\pi_{t-5} + 0.085\pi_{t-6} + 0.039\pi_{t-7} \]

\[ -0.055\pi_{t-8} + 0.205\pi_{t-9} + 3.039MG_{t-1} + 2.007MG_{t-2} + 11.77MG_{t-3} + 6.123MG_{t-4} \]

\[ + 0.022SG_{t-1} - 0.015SG_{t-2} + 0.014SG_{t-3} + 2.539WG_{t-1} + 0.353WG_{t-2} - 1.540WG_{t-3} - 1.569WG_{t-4} \]

\[ \bar{R}^2 = 0.479, \quad SEE = 0.235 \]

Breusch-Godfrey LM Test for Serial Correlation

<table>
<thead>
<tr>
<th>Order of serial correlation</th>
<th>T*R²</th>
<th>Prob&gt;χ²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.289</td>
<td>0.591</td>
</tr>
<tr>
<td>4</td>
<td>3.341</td>
<td>0.502</td>
</tr>
<tr>
<td>8</td>
<td>6.783</td>
<td>0.560</td>
</tr>
<tr>
<td>12</td>
<td>14.07</td>
<td>0.296</td>
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</tbody>
</table>

ARCH Test

<table>
<thead>
<tr>
<th>Order of ARCH</th>
<th>T*R²</th>
<th>Prob&gt;χ²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.040</td>
<td>0.081</td>
</tr>
<tr>
<td>2</td>
<td>3.554</td>
<td>0.470</td>
</tr>
<tr>
<td>3</td>
<td>3.956</td>
<td>0.861</td>
</tr>
<tr>
<td>4</td>
<td>6.324</td>
<td>0.899</td>
</tr>
</tbody>
</table>

2. Maximum Likelihood Estimation: assuming \( h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 \)

\[ \pi_t = -0.028 + 0.410\pi_{t-1} + 0.335\pi_{t-2} + 0.034\pi_{t-3} + 0.024\pi_{t-4} + 0.091\pi_{t-5} - 0.020\pi_{t-6} - 0.075\pi_{t-7} \]

\[ - 0.031\pi_{t-8} + 0.240\pi_{t-9} + 5.032MG_{t-1} + 5.951MG_{t-2} + 10.55MG_{t-3} + 3.122MG_{t-4} \]

\[ + 0.019SG_{t-1} - 0.021SG_{t-2} + 0.013SG_{t-3} + 3.614WG_{t-1} + 0.362WG_{t-2} + 0.153WG_{t-3} - 2.153WG_{t-4} \]

\[ h_t = 0.017 + 0.823\varepsilon_{t-1}^2 \]

Note: T-Statistics are given in parentheses below the coefficients estimates

OLS Estimation

\[ \pi_t = 0.189 + 0.315 \pi_{t-1} + 9.748 W_{t-1} + 0.009 S_{t-1} - 1.303 M_{t-1} - 0.418 M_{t-2} \]
\[ - 0.351 M_{t-3} - 0.215 M_{t-4} - 0.814 M_{t-5} + 8.733 M_{t-6} - 8.624 M_{t-7} \]
\[ (5.503) \quad (3.263) \quad (3.425) \quad (-1.935) \quad (-0.425) \quad (-0.126) \]
\[ R^2 = 0.267, \quad \text{SEE} = 0.163 \]

Breusch-Godfrey LM Test for Serial Correlation

<table>
<thead>
<tr>
<th>Order of serial correlation</th>
<th>T*R^2</th>
<th>Prob&gt;\chi^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.572</td>
<td>0.450</td>
</tr>
<tr>
<td>4</td>
<td>2.172</td>
<td>0.704</td>
</tr>
<tr>
<td>8</td>
<td>9.351</td>
<td>0.313</td>
</tr>
<tr>
<td>12</td>
<td>12.58</td>
<td>0.401</td>
</tr>
</tbody>
</table>

ARCH Test

<table>
<thead>
<tr>
<th>Order of ARCH</th>
<th>T*R^2</th>
<th>Prob&gt;\chi^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.083</td>
<td>0.298</td>
</tr>
<tr>
<td>2</td>
<td>1.104</td>
<td>0.576</td>
</tr>
<tr>
<td>4</td>
<td>2.530</td>
<td>0.639</td>
</tr>
<tr>
<td>8</td>
<td>10.04</td>
<td>0.262</td>
</tr>
</tbody>
</table>

White’s Test

\[ \chi^2 = 9.498, \quad \text{Prob}(\chi^2 > 9.498) = 0.393 \]

Note: T-Statistics are given in parentheses below the coefficients estimates
Table 6: Tests of the Variability Hypothesis: 1954:01-1969:12

**Variability of Inflation Equation:**

\[ h_t = \beta_0 + \beta_1 \pi_t + \beta_2 \pi_{t-1} + ... + \beta_7 \pi_{t-6} \]

\[ \bar{R}^2 = 0.270, \quad SEE = 2.394, \quad \sum_{i=1}^{7} \beta_i = 11.54 \]

- \( H_0: \sum_{i=1}^{7} \beta_i = 0, \quad t\text{-Statistic}: 9.328, \quad Significance Level: 0.000 \)
- \( H_0: \beta_1 = ... = \beta_7 = 0, \quad F\text{-Statistic}: 12.51, \quad Significance Level: 0.000 \)

**Stock Returns Equation:**

\[ rsr_t = \beta_0 + \beta_1 h_t + \beta_2 h_{t-1} + ... + \beta_{13} h_{t-12} \]

\[ \bar{R}^2 = 0.037, \quad SEE = 0.215, \quad \sum_{i=1}^{13} \beta_i = -80.08 \]

- \( H_0: \sum_{i=1}^{13} \beta_i = 0, \quad t\text{-Statistic}: -3.405, \quad Significance Level: 0.001 \)
- \( H_0: \beta_1 = ... = \beta_{13} = 0, \quad F\text{-Statistic}: 1.621, \quad Significance Level: 0.082 \)

Note: 1. \( h_t \) is the conditional variance of inflation estimated from the inflation equation in Part 2 of Table 3. T-Statistics are given in parentheses below the coefficients estimates.
Table 7: Tests of the Variability Hypothesis: 1970:01-1981:12

Variability of Inflation Equation: $h_t = \beta_0 + \beta_1 \pi_t + \beta_2 \pi_{t-1} + \ldots + \beta_7 \pi_{t-6}$

\[
\begin{align*}
\hat{h}_t & = 0.016 - 0.022 \pi_t + 0.244 \pi_{t-1} - 0.117 \pi_{t-2} - 0.073 \pi_{t-3} + 0.035 \pi_{t-4} + 0.030 \pi_{t-5} + 0.043 \pi_{t-6} \\
& (0.612) (0.525) (5.668) (2.612) (1.628) (0.772) (0.698) (1.012)
\end{align*}
\]

$R^2 = 0.177$, $SEE = 0.124$, $\sum_{i=1}^{7} b_i = 0.070$

$H_0: \sum_{i=1}^{7} \beta_i = 0$, \hspace{1cm} $t$-Statistic: 1.748, $Significance Level: 0.082$

$H_0: \beta_i = \ldots = \beta_7 = 0$, $F$-Statistic: 5.765, $Significance Level: 0.000$

Stock Returns Equation: $r_{sr_t} = \beta_0 + \beta_1 h_t + \beta_2 h_{t-1} + \ldots + \beta_{13} h_{t-12}$

\[
\begin{align*}
\hat{r}_{sr_t} & = -0.463 + 0.0002 h_t - 0.276 h_{t-1} - 0.158 h_{t-2} - 0.190 h_{t-3} - 0.296 h_{t-4} - 0.174 h_{t-5} \\
& (9.352) (0.0008) (1.368) (0.782) (0.941) (1.464) (0.862)
\end{align*}
\]

$- 0.151 h_{t-6} + 0.056 h_{t-7} - 0.252 h_{t-8} - 0.229 h_{t-9} - 0.110 h_{t-10} - 0.424 h_{t-11} - 0.564 h_{t-12} \\
(0.750) (0.280) (1.248) (1.136) (0.546) (2.102) (2.796)$

$\overline{R}^2 = 0.078$, $SEE = 0.331$, $\sum_{i=1}^{13} b_i = -2.767$

$H_0: \sum_{i=1}^{13} \beta_i = 0$, \hspace{1cm} $t$-Statistic: -4.085, $Significance Level: 0.000$

$H_0: \beta_i = \ldots = \beta_{13} = 0$, $F$-Statistic: 1.930, $Significance Level: 0.032$

Note: 1. $h_t$ is the conditional variance of inflation estimated from the inflation equation in Part 2 of Table 4. $T$-Statistics are given in parentheses below the coefficients estimates