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Equity Stakes and Hold-up Problems*

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Abstract

Equity ties between businesses change the division of the firms’ joint profits, thereby affecting incentives for relation-specific investments and other strategic actions. Depending on which side owns the equity and how readily the equity can be resold, we find that the changed incentives can resolve all four types of holdup-related problems: underinvestment, overinvestment, undercooperation, and sabotage. Equity stakes indirectly affect bargaining over the joint profits by making the bargaining positions of the firms dependent on each other. For instance, if one firm is made unprofitable by a breakdown in negotiations, the other firm loses as well. Since bargaining positions are linked, each firm benefits less from attempts to grab a larger share of the joint profits by strengthening its relative bargaining position, and benefits more from actions that increase joint profits. While both firms can gain from increased efficiency due to the equity stake, firms in many cases should only acquire an equity stake if they can bargain for a discounted price.

JEL Classification Categories: L14, L22, G31, G32, G34, C78
Key words: equity blocks, subsidiaries, gainsharing, partial ownership, business groups, ESOPs

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1 Introduction

Equity ties between firms link them financially but allow each firm to maintain separate control and separate interests. It has long been suspected that such partial integration might be driven by the same problem of hold-up costs due to relationship-specific investments that is often used to explain full integration.\(^1\) As evidence for this perspective, Allen and Phillips (2001) show that most instances of corporate equity stakes involve firms with ongoing product market relationships, that equity stakes are associated with asset specificity, and that they lead to both more investment and higher research and development expenditures.

This paper considers the effect of equity stakes on the standard holdup problem of underinvestment (Klein, Crawford and Alchian, 1978; Williamson, 1985) and on three related problems: overinvestment (Grossman and Hart, 1986), undercooperation (Che and Hausch, 1999), and sabotage. Underinvestment occurs when an investment would increase the firm’s profitability but worsens its bargaining position so much that the firm cannot benefit. Conversely, overinvestment arises when an investment is not profitable but strengthens the firm’s bargaining position, e.g. a firm invests in redundant capacity in a different country to be in a stronger position vis-a-vis its domestic suppliers. Undercooperation occurs when an action such as helping a business partner improve quality would increase the partner’s profitability, but is not taken because it strengthens the partner’s bargaining position. Finally, sabotage occurs when an action reduces joint profits but undermines the other firm’s bargaining position so much that the larger share of smaller joint profits makes it worthwhile.

The alignment of interests generated by equity stakes and their ability to reduce hold-up problems may seem straightforward. Consider the typical case where a downstream firm takes an equity stake in a small upstream firm. The equity stake should presumably give the downstream firm a larger share of the joint profits generated by the firms, inducing it to invest more in relationship-specific capital. But the essence of the hold-up problem is that the firms must bargain over the division of the profits so it is not clear that an equity stake gives the downstream firm a larger share. Instead, the literature to date has found that if one party owns an equity stake in another, the stakeholder need not end up with a larger

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\(^1\) Among other possibilities, equity stakes could also help share risk (Aoki, 1984; Khanna and Yafeh, 2000), facilitate collusion among competitors (Flath, 1991), reduce quantity distortions in vertical supply arrangements (Flath, 1989; Riordan, 1991), and encourage information sharing between firms (Filson and Morales, 2001). Equity stakes are highly correlated with board representation (Boone, 2001), so control issues are also likely to be important (Hart and Moore, 1990).
share of joint profits, and investment incentives are strengthened not for the party holding the equity, but for the firm that the equity is held in. In an analysis of union-firm bargaining that closely parallels bargaining between firms, Grout (1988) shows that employee stock ownership reduces the ability of workers to appropriate gains from firm investment in the form of higher wages, and thereby induces the firm to invest more. Dasgupta and Tao (2000) show that if an upstream firm has a choice between general investment and more productive investment which is specific to the downstream firm, it has a greater incentive to make the relationship-specific investment if the downstream firm owns an equity stake in it.²

These models assume that the stakeholder cannot resell the equity to a third party due to contractual stipulations or other factors.³ Since the stakeholder retains the equity stake even when negotiations break down, the stakeholder’s bargaining position is dependent on the target’s bargaining position. For instance, if the target firm loses money when the firms cannot reach agreement, the stakeholder also receives less money. Because the bargaining position of the stakeholder weakens along with that of the target, the stakeholder has less ability to hold-up the target. The target firm is therefore more willing to make investments that are highly-specific to the stakeholder.

Following the same assumption that equity resales are restricted, we show that an equity stake can also resolve the overinvestment problem. Since any improvement in the target’s bargaining position will also benefit the stakeholder, the target has less incentive to overinvest in redundant capacity that is inefficient but strengthens the target’s bargaining position. Regarding the stakeholder’s behavior, we find that incentives to underinvest and overinvest are unaffected, but the interdependence of bargaining positions can solve the undercooperation and sabotage problems. The stakeholder gains more from cooperative actions that strengthen the target’s bargaining position since the stakeholder’s bargaining position also benefits. Likewise, the stakeholder gains less from sabotage that undermines the target’s bargaining position since the stakeholder’s bargaining position is also harmed.

Relaxing the assumption that the stakeholder cannot resell the equity, we find that the im-

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²Dasgupta and Tao consider two models, one where the upstream firm’s investment costs are private to an owner-manager as in Aghion and Tirole (1994), and one where the costs are fully deducted from the firm’s profits. The latter model is closest to Grout’s analysis and to ours.

³For instance, professional partnerships often require members to resell their equity at a fixed price to the partnership rather than sell it on the open market. And when a large firm owns a fraction of a small, research and development firm, information asymmetries will impede the firm’s ability to resell the equity at a favorable price.
pact of an equity stake is more consistent with intuition in that the stakeholder is guaranteed a larger share of joint profits and all of the impact on investment incentives falls on the stakeholder. Because the stakeholder can sell, or even threaten to sell, the equity before bargaining begins, the stakeholder receives both the normal share of joint profits from bargaining plus a share of the target’s profits corresponding to the size of the equity stake. Actions which raise joint profits but strengthen the target’s relative bargaining position raise the market value of the stakeholder’s equity in the target, so the stakeholder gains more from investment and cooperation. And actions which lower joint profits but strengthen the stakeholder’s relative bargaining position lower the market value of the equity, so the stakeholder gains less from overinvestment and sabotage. An equity stake can therefore solve all four problems for the stakeholder, but has no effect on the behavior of the target firm.

Despite the opportunity for efficiency gains, we find that acquiring an equity stake at market prices need not be a good deal for the stakeholder. If the target’s capital is highly specific to the stakeholder, then without an equity stake the stakeholder can demand a large share of joint profits because the target will lose money if agreement cannot be reached. But if the stakeholder acquires an equity stake that cannot be readily resold, the stakeholder will also lose money in the event of disagreement, weakening the stakeholder’s bargaining position. The stakeholder’s total income may be worse than if it had no equity stake at all, and any gain in income will be less than the market price of the equity unless the efficiency gains are sufficiently large. When equity resales are permitted, the problem is less severe in that the stakeholder’s payoff always increases relative to not owning equity. But in this case the payoff

\footnote{Aghion and Tirole (1994) consider the case where a third party takes an equity stake in an upstream firm. This third party can then finance investment in the upstream firm without the bargaining problems that the downstream firm faces. The ability to resell the equity in our model allows the downstream firm itself to play the same role as the third party.}

\footnote{In this paper we assume that each firm maximizes profits, so the target firm’s behavior is unaffected by the stakeholder’s share of profits. If the target firm is owner-managed the private effort costs of the owner will not be fully accounted for in profit calculations and the owner’s effort level will fall as the stakeholder’s equity stake rises (Aghion and Tirole, 1994).}

\footnote{Allowing for bi-directional stakeholding would allow equity stakes to affect both firms’ incentives. Since bi-directional stakeholding is rare (Gerlach, 1992; Allen and Phillips, 2001) we restrict our analysis to unidirectional stakeholding.}

\footnote{In recognition of this problem in the context of union-firm bargaining, unions have traditionally discouraged workers from owning stock in their firm out of fear that it would undermine the union’s bargaining position and lead to lower wages (McElrath and Rowan, 1992).}
increase is never as much as the market price of the equity no matter how large the efficiency gains, implying the stakeholder should always insist on a discounted price for the equity.

In our analysis we assume no contract is available except the equity stake contract. For instance, rather than assuming that a downstream firm contracts with an upstream firm to purchase a product at a certain price but that under some conditions both sides might prefer to renegotiate (Maskin and Moore, 1999), we assume that a binding price contract cannot be signed. By concentrating on this strong version of the contract problem, we are able to highlight the power of equity stakes, but we also exclude the possibility of other contracts that in some cases might be just as effective as an equity stake.8 The unavailability of other contracts could reflect a weak legal environment or a poor information environment. Note that the use of equity stakes requires verifiable information on aggregate firm profits, but does not require verifiable information on specific firm actions and costs, such as the amount of research and development spending that is directly related to a particular project. Even when the information and legal environment is sufficient to support other contracts, an equity stake has the advantage of being a simple, widely used contract that has a large body of law defining its application in different circumstances and that receives automatic enforcement from the Securities and Exchange Commission.

2 Underinvestment, Overinvestment, Undercooperation and Sabotage without an Equity Stake

Before considering the impact of equity stakes, we first show how a standard bargaining model can capture the different inefficiencies that have been identified in the literature. Problems arise if actions that increase joint profits are unrewarded in bargaining over the division of joint profits, or if actions that decrease joint profits are rewarded. Perfect contracts can avoid such inefficiencies by properly rewarding productive actions and penalizing non-productive actions, but for a variety of reasons such contracts are difficult to write and enforce (Williamson, 1985). We assume that no contract is available so the division of joint profits is determined by ex post bargaining.

Let there be two firms, s and t, engaged in business dealings that create joint profits

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8 For example, Edlin and Reichtelstein (1996) show that investment inefficiencies can be solved with a fixed price contract. Che and Hausch (1999) show that the problem of undercooperation is not so readily solved.
$S$. Assuming both firms are risk neutral and a side payment from one firm to the other is possible, the standard Nash axiomatic bargaining model predicts that the two sides just split the difference. The firms do not receive exactly the same profits, but in agreement each side receives the same amount over and above its own disagreement profits or “bargaining position”. Let $A_s$ and $A_t$ represent the firms’ respective agreement profits where $A_s + A_t \leq S$. For instance, if one firm buys a good from the other, these profits incorporate the bargained price of the good and any costs of producing the good. Let $D_s$ and $D_t$ represent the firms’ disagreement profits. For instance, these are the profits that each firm receives if the good is sold instead to a third firm. Assuming $D_s + D_t < S$, the Nash solution is the pair of agreement profits which maximize

$$\left( A_s - D_s \right)^{\frac{1}{2}} \left( A_t - D_t \right)^{\frac{1}{2}}$$

subject to the constraint $A_s + A_t \leq S$. The maximum clearly occurs where $A_s + A_t = S$ and $A_s - D_s = A_t - D_t$, or

$$A_i = D_i + \frac{1}{2} (S - D_t - D_j)$$
$$= \frac{1}{2} S + \frac{1}{2} D_i - \frac{1}{2} D_j,$$  \hspace{1cm} (2)

for $i, j \in \{s, t\}, i \neq j$. Given the standard nature of the problem we will use this result directly rather than derive it separately as the outcome of an alternating offers bargaining game.\(^9\) Note that each side’s agreement profits are its disagreement profits plus half of joint profits net of both sides’ disagreement profits, $S - D_s - D_t$. As a result, each firm’s agreement payoff is a positive function of joint profits and its own disagreement payoff, and a negative function of the other firm’s disagreement payoff. These general properties, rather than the particular Nash solution, are the key to this paper’s results. They are shared by almost all standard bargaining solutions.\(^{10}\)

\(^9\)The interpretation of the disagreement payoffs can differ depending on the exact structure of such an alternating offers game. Binmore, Rubinstein and Wolinsky (1986) show that the disagreement payoffs correspond to the “inside” disagreement point in the Rubinstein bargaining game and the “outside” disagreement point in a game with an exogenous risk of breakdown. This distinction does not affect the qualitative conclusions of our analysis, though the two disagreement points will often differ in practice, affecting the exact solution.

\(^{10}\)See Roth (1979) and Thomson (1994) for surveys. The Nash solution with equal bargaining power ensures the split-the-difference result. If the parties have different bargaining power, or in a non-cooperative framework
Before bargaining starts the two sides might take non-productive actions or fail to take productive actions in order to better position themselves for bargaining (Nash, 1953). For simplicity we consider discrete actions that are either taken or not taken, and assume profits are additive functions of different actions so that each firm has a dominant strategy of taking an action or not. Since each firm $i$ maximizes its profits $A_i = \frac{1}{2}S + \frac{1}{2}D_i - \frac{1}{2}D_j$ rather than joint profits $S$, a wedge is driven between individual and collective interests. Letting the difference operator $\Delta$ indicate the difference of the operand from taking an action or not, firm $i$ will take an action if $\Delta A_i > 0$, which occurs if the gain in joint profits is greater than any loss in relative bargaining position, $\Delta S > -(\Delta D_i - \Delta D_j)$. In Figure 3 the effect on joint profits $\Delta S$ is shown on the $x$-axis and the effect on firm $i$’s relative bargaining position $\Delta D_i - \Delta D_j$ is shown on the $y$-axis, so the diagonal represents $\Delta S = -(\Delta D_i - \Delta D_j)$. Actions in the NE quadrant are efficient, $\Delta S > 0$, and always taken, while actions in the SW quadrant are inefficient, $\Delta S < 0$, and never taken. Problems occur in two cases. In the SE quadrant, an inefficiency arises when an action would increase joint profits, $\Delta S > 0$, but not enough to overcome the damage to firm $i$’s relative bargaining position, $\Delta S < -(\Delta D_i - \Delta D_j)$. In the NW quadrant, an inefficiency arises when an action decreases joint profits, $\Delta S < 0$, but is sufficiently beneficial to firm $i$’s relative bargaining position that it is taken anyway, $\Delta S > -(\Delta D_i - \Delta D_j)$.

We can further differentiate the types of inefficiencies by which firm’s assets are affected. In many cases a beneficial or detrimental action by firm $i$ will only affect firm $i$’s assets, in which case $\Delta D_j = 0$, or only affect firm $j$’s assets, in which case $\Delta D_i = 0$. We therefore concentrate on the following four types of inefficiencies, although in our analysis of the impact of equity stakes we will also consider actions that simultaneously affect both firms’ assets.

**Definition 1** Underinvestment: *Firm $i$ does not take an action that increases joint profits, decreases its own disagreement profits, and has no impact on $j$’s disagreement profits, $\Delta S > 0$, $\Delta D_i < 0$, $\Delta D_j = 0$.*

This is the classic hold-up problem. Even though an investment in relationship-specific capital would increase joint profits, the investment is not taken because the expense of the investment undermines the firm’s bargaining position. Without equity stakes this occurs when $\Delta S < -\Delta D_i$.

have different discount rates, the conclusions do not change qualitatively though the size of the equity stake needed to induce a particular effect will vary.
Definition 2 Undercooperation: Firm $i$ does not take an action that increases joint profits, increases firm $j$’s disagreement profits, and has no impact on its own disagreement profits, $\Delta S > 0$, $\Delta D_j > 0$, $\Delta D_i = 0$.

For instance, a firm does not help its partner improve its production processes if it will make the partner’s capital less relationship-specific. Without equity stakes this occurs when $\Delta S < \Delta D_j$.

Definition 3 Overinvestment: Firm $i$ takes an action that decreases joint profits, increases its own agreement profits, and has no impact on firm $j$’s disagreement profits, $\Delta S < 0$, $\Delta D_i > 0$, $\Delta D_j = 0$.

For instance, a firm builds an unprofitable second plant just to increase its bargaining position. Without equity stakes, this occurs when $\Delta S > -\Delta D_i$.

Definition 4 Sabotage: Firm $i$ takes an action that decreases joint profits, decreases firm $j$’s disagreement profits, and has no impact on its own disagreement profits, $\Delta S < 0$, $\Delta D_j < 0$, $\Delta D_i = 0$. 

Figure 1: Inefficient behavior by firm $i$
For instance, a firm acts to undermine its partner’s outside options at a cost to joint profits by choosing to produce a good that is less profitable for the partner to sell, but that makes the partner more dependent on it. Without equity stakes this occurs when $\Delta S > \Delta D_j$.

### 3 Game Structure

To evaluate the impact of equity stakes, we consider a three-stage game that is an extension of the basic game without equity stakes. First, a target firm $t$ sells rights to $e$ share of its profits to a stakeholder firm $s$. Second, the stakeholder and target take actions affecting both sides’ profitability. Third, the two firms bargain over the division of the joint profits. In this final stage there are two cases corresponding to whether or not the stakeholder can resell the equity stake before negotiations begin. The game is one-shot, the firms have complete information, and they are risk neutral. To ensure that the solution is subgame perfect we use backward induction, starting with calculating the payoffs that each side will receive in the final stage taking variables determined earlier as given.

### 4 Bargaining over Joint Profits (Stage 3)

We first solve for the division of joint profits generated by the two firms’ business dealings. At this stage the stakeholder has already acquired an equity stake and both sides have already taken any investments or other actions in anticipation of bargaining over the joint profits. We consider both the case where equity resales are restricted and the case where the stakeholder may choose to sell the equity before bargaining begins.
4.1 Equity Resales are Restricted

If equity sales are restricted the stakeholder receives share $e$ of the target’s profits whether or not agreement on how to divide joint profits is reached. In agreement, the stakeholder receives payoff $A_s + eA_t$ and the target receives payoff $(1 - e)A_t$. Likewise, if agreement is not reached the stakeholder receives payoff $D_s + eD_t$ and the target receives payoff $(1 - e)D_t$. Regardless of the equity stake, any distribution of joint profits is still feasible through adjustment of $A_s$ and $A_t$. For instance, if the firms have an upstream-downstream relationship, the price of the product sold by the upstream firm is still open to negotiation. The equity stake does not force a particular distribution of joint profits, but indirectly affects bargaining by changing the firms’ bargaining positions.

The Nash solution is now the pair of agreement profits $A_s$ and $A_t$ that maximizes

$$((A_s + eA_t) - (D_s + eD_t))^2 ((1 - e)A_t - (1 - e)D_t)^2$$

subject to the same constraint that $A_s + A_t \leq S$. The agreement profits are then

$$A_s(e) = \frac{\left(\frac{1}{2} - e\right) S + \frac{1}{2} D_s - \left(\frac{1}{2} - e\right) D_t}{1 - e},$$

$$A_t(e) = \frac{\frac{1}{2} S - \frac{1}{2} D_s + \left(\frac{1}{2} - e\right) D_t}{1 - e},$$

and the agreement payoffs are,

$$A_s(e) + eA_t(e) = D_s + eD_t + \frac{1}{2} (S - D_s - D_t)$$

$$= \frac{1}{2} S + \frac{1}{2} D_s - \left(\frac{1}{2} - e\right) D_t,$$

$$A_t(e) = (1 - e)D_t + \frac{1}{2} (S - D_s - D_t)$$

$$= \frac{1}{2} S - \frac{1}{2} D_s + \left(\frac{1}{2} - e\right) D_t.$$
Note that the agreement profits are a function of \( e \) directly as above, and also indirectly in that \( e \) will affect \( S \), \( D_s \), and \( D_t \) as seen in the next section. In this section we only consider the direct impact. From equation (4), \( \partial A_s/\partial e = -S-D_s-D_t \frac{e}{2(1-e)^2} \) < 0, while from equation (6) \( \partial (A_s(e) + eA_t(e))/\partial e = D_t \), implying the following:

**Proposition 1** If resale of the equity stake is restricted, then for given \( S \), \( D_s \), and \( D_t \), an increase in the equity stake (i) decreases the stakeholder’s agreement profits \( A_s(e) \) and (ii) increases (decreases) the stakeholder’s agreement payoff \( A_s(e) + eA_t(e) \) if the target’s disagreement profits \( D_t \) are positive (negative).

An increase in the equity stake is not necessarily a blessing for the stakeholder. Although dividend income \( eA_t(e) \) rises, the gain displaces income that would have been bargained for anyway, and agreement profits \( A_s(e) \) fall.\(^{13}\) Since the feasible set of the bargaining game is unchanged by \( e \), the agreement payoff \( A_s(e) + eA_t(e) \) is affected by an equity stake only if it changes bargaining positions.\(^{14}\) When the target is unprofitable in disagreement, the stakeholder’s bargaining position is undermined since it will also share in this loss. Conversely, when the target is still profitable even if agreement is not reached, the stakeholder’s bargaining position is strengthened since it will share in the profits.\(^{15}\)

Figure 3 shows the effects of an equity stake when target profits in disagreement, \( D_t \), are negative as may be the case if the stakeholder is the primary market for the target’s product. Without an equity stake the disagreement point \( (D_t, D_s) \) leads to agreement payoffs that split the difference above each firm’s disagreement payoff. If an equity stake \( e = \frac{1}{3} \) is introduced the firms still split the difference, but from the new disagreement point \( (\frac{2}{3}D_t, D_s + \frac{1}{3}D_t) \). Since \( D_t < 0 \), the stakeholder’s bargaining position worsens and the stakeholder is less able to benefit from the target’s poor bargaining position. Consistent with Proposition 1(ii), owning an equity stake actually lowers the stakeholder’s payoff. Note that, regardless of whether \( D_t \) is positive or negative, the equity stake makes the payoffs to each side, \( A_s = \frac{1}{2}S + \frac{1}{2}D_s - \frac{1}{6}D_t \)

\(^{13}\)This displacement effect was analyzed by Weitzman (1987) in the context of union-firm bargaining.

\(^{14}\)There is no effect in the knife-edge case where the disagreement payoff of the target firm is zero. Aghion and Tirole (1994) first normalize the disagreement payoffs to zero before considering the impact of an equity stake so they find that the bargaining outcome does not change. To allow for the possibility of a bargaining position effect, any normalization should occur after the disagreement payoffs are first adjusted for the impact of the equity stake.

\(^{15}\)In the limit where neither firm’s assets are relationship-specific so that \( S = D_s + D_t \), there is no displacement effect and the stakeholder benefits by exactly the dividend income.
Figure 3: Restricted equity stake can lower stakeholder payoff, $e = 1/3$

and $A_t = \frac{1}{2}S - \frac{1}{2}D_s + \frac{1}{6}D_t$, less dependent on $D_t$. It is this effect that will allow equity stakes to resolve the various hold-up problems.

### 4.2 Equity Resales are Permitted

Holding an equity stake lowers the stakeholder’s profits for given $S$, $D_s$, and $D_t$, suggesting that the stakeholder might want to sell the equity. Since anonymous sales by a major equity holder are unlikely, we will assume that the stakeholder sells the stock publicly on the open market. Suppose the stakeholder plans to reduce its equity stake from $e$ to $e'$. In a competitive capital market where it is known that the stake is being sold, the price will reflect the new profits of the target once the equity has been sold, so the equity will sell for a total price of $(e - e')A_t(e')$. The total gain to the stakeholder from selling the equity is then

$$(A_s(e) + eA_t(e) + (e - e')A_t(e')) - (A_s(e') + eA_t(e')) = \frac{e - e'}{1 - e'^2}(S - D_s - D_t) > 0$$

(8)
so the stakeholder should sell all of the equity. Of course, since any threat to sell the equity is therefore credible, the mere ability to sell the stock provides the stakeholder with a powerful threat that may obtain the same result.\textsuperscript{16} Given that the stakeholder is expected to sell all the equity, the stakeholder’s total agreement payoff incorporating proceeds from sale of the entire stake $e$ is

$$A_s(0) + eA_t(0) = (1 + e)\frac{1}{2}S + (1 - e)\frac{1}{2}(D_s - D_t),$$

(9)

and the target’s total agreement payoff is

$$(1 - e)A_t(0) = (1 - e)\frac{1}{2}S - (1 - e)\frac{1}{2}(D_s - D_t).$$

(10)

Since $\partial(A_s(0) + eA_t(0))/\partial e = A_t(0)$ we get the following:

**Proposition 2** If resale of the equity stake is unrestricted, then for given $S$, $D_s$, and $D_t$, an increase in the equity stake increases the stakeholder’s total agreement payoff $A_s(0) + eA_t(0)$ if target firm agreement profits $A_t(0)$ are positive.

The ability to resell equity therefore leads to very different outcomes than when sales are restricted. As shown in the following section, the types of actions affected by equity stakes also differ in the two cases.

## 5 Choice of Relationship-Specific Actions (Stage 2)

We now consider the actions that the two firms will take in knowledge of the bargaining outcomes derived above for the two cases.

\textsuperscript{16}Consider the following alternating offers game where both firms have equal discount factors $\delta$. The target firm first makes an offer to the stakeholder and then, before making a counteroffer, the stakeholder has the choice to sell the equity stake $e$. The game then proceeds as a normal stationary bargaining game with the firms alternating offers. If the stakeholder sells the equity it receives $\frac{1 - \delta}{1 - \delta^2}e$ of the “pie” discounted by one period, or $A_s = D_s + \delta\frac{1 - \delta}{1 - \delta^2}(S - D_s - D_t)$, plus $e$ share of the target’s payoffs, $A_t = D_t + \delta\frac{1 - \delta}{1 - \delta^2}(S - D_s - D_t)$. If it holds on to the equity it receives $A_s = D_s + eD_t + \delta\frac{1 - \delta}{1 - \delta^2}(S - D_s - D_t)$ which is always lower. Given that the stakeholder receives more by selling the equity, the threat to sell the equity is therefore credible, and the target should immediately offer the stakeholder the payoff that the stakeholder would receive if it sold the equity.
5.1 Equity Resales are Restricted

If the stakeholder must hold on to the equity, the firms’ payoffs are given by equations (6) and (7). The stakeholder will take an action if

\[
\Delta (A_s(e) + eA_t(e)) = \frac{1}{2} \Delta S + \frac{1}{2} \Delta D_s - \left( \frac{1}{2} - e \right) \Delta D_t \geq 0
\]

(11)

and similarly the target will take an action if

\[
\Delta ((1-e)A_t) = \frac{1}{2} \Delta S - \frac{1}{2} \Delta D_s + \left( \frac{1}{2} - e \right) \Delta D_t \geq 0.
\]

(12)

For the stakeholder, an equity stake

\[
e \geq -\frac{1}{2} \frac{\Delta S + \Delta D_s - \Delta D_t}{\Delta D_t}
\]

(13)

ensures that an action will be taken if \( \Delta D_t > 0 \) and will not be taken if \( \Delta D_t < 0 \). For the target firm, an equity stake

\[
e \geq \frac{1}{2} \frac{\Delta S - \Delta D_s + \Delta D_t}{\Delta D_t}
\]

(14)

ensures that an action will be taken if \( \Delta D_t < 0 \) and will not be taken if \( \Delta D_t > 0 \). Using our definitions of the four types of inefficiencies, we can identify the particular hold-up problems that are resolved by equity stakes.

**Proposition 3** If equity resales are restricted, there exists \( \tau \in (0, \frac{1}{2}) \) such that all equity stakes \( e \in [\tau, 1] \) (i) solve the stakeholder’s undercooperation and sabotage problems and (ii) solve the target’s underinvestment and overinvestment problems.

Proof: (i) For both undercooperation and sabotage by the stakeholder, \( \Delta D_s = 0 \), implying from (13) that for \( e \geq \frac{1}{2}(\Delta D_t - \Delta S)/\Delta D_t \) an action will be taken if \( \Delta D_t > 0 \) and will not be taken if \( \Delta D_t < 0 \). Let \( \tau = \frac{1}{2}(\Delta D_t - \Delta S)/\Delta D_t \). For undercooperation to occur for \( e = 0 \) it must be true that the action is efficient but not worthwhile to the stakeholder, or \( \Delta S > 0 \) and \( \Delta D_t > \Delta S \). These imply \( 0 < (\Delta D_t - \Delta S)/\Delta D_t < 1 \) and \( \Delta D_t > 0 \) so \( \tau \in (0, \frac{1}{2}) \) and the action will be taken for \( e \in [\tau, 1] \). For sabotage to occur for \( e = 0 \) it must be true that the action is inefficient but worthwhile to the stakeholder, or \( \Delta S < 0 \) and \( \Delta D_t < \Delta S \). These imply \( 0 < (\Delta D_t - \Delta S)/\Delta D_t < 1 \) and \( \Delta D_t < 0 \) so \( \tau \in (0, \frac{1}{2}) \) and the action will not be taken for \( e \in [\tau, 1] \).
(ii) For both underinvestment and overinvestment by the target $\Delta D_s = 0$, implying from (14) that for $e \geq \frac{1}{2}(\Delta S + \Delta D_t) / \Delta D_t$ an action will be taken if $\Delta D_t < 0$ and will not be taken if $\Delta D_t > 0$. Let $\tau = \frac{1}{2}(\Delta S + \Delta D_t) / \Delta D_t$. For underinvestment to occur for $e = 0$ it must be true that the action is efficient but not worthwhile to the target, or $\Delta S > 0$ and $\Delta S < -\Delta D_t$. These imply $0 < (\Delta S + \Delta D_t) / \Delta D_t < 1$ and $\Delta D_t < 0$ so $\tau \in (0, \frac{1}{2})$ and the action will be taken for $e \in [\tau, 1]$. For overinvestment to occur for $e = 0$ it must be true that the action is inefficient but worthwhile to the target, or $\Delta S < 0$ and $\Delta S > -\Delta D_t$. These imply $0 < (\Delta S + \Delta D_t) / \Delta D_t < 1$ and $\Delta D_t > 0$ so $\tau \in (0, \frac{1}{2})$ and the action will not be taken for $e \in [\tau, 1]$.

An equity stake makes the stakeholder’s bargaining position partially dependent on the target’s bargaining position. Since any contributions to the target’s profitability also strengthen the stakeholder’s bargaining position, the stakeholder is assured of benefitting. The equity stake therefore works as an effective form of “gainsharing” that encourages cooperative actions. Similarly, the interdependence of bargaining positions gives the stakeholder less incentive to sabotage the target’s bargaining position to try to get a larger share of the surplus.

This same alignment of interests via bargaining positions makes the target more willing to invest even if the investment is specific to the stakeholder and therefore undermines the target’s bargaining position. Since the stakeholder’s bargaining position will also be undermined, the target can still bargain for a share of the larger surplus. Similarly, strategic overinvestment to get a larger share of a shrunken surplus is no longer so appealing to the target because such investments concurrently strengthen the stakeholder’s bargaining position.

Of course an equity stake is not a panacea. The stakeholder’s investment decisions are unaffected because the displacement effect of the equity stake on bargained profits prevents the stakeholder from benefiting more from its investment. Similarly, the target’s cooperation and sabotage decisions are unaffected by the equity stake. Since these decisions do not affect the target’s bargaining position, there is no interaction with the equity stake on the bargaining outcome. In some cases, an equity stake can even be detrimental if an action simultaneously affects the disagreement payoffs of both firms. Consider sabotage by the target which is not costless as assumed above but is expensive so that it undermines the target’s disagreement payoffs. By linking the stakeholder’s disagreement payoff to the target’s, an equity stake reduces the target’s losses hurting its own bargaining position. As a result the target is more willing to incur the cost of wastefully hurting the stakeholder’s bargaining position.
Figure 4: Restricted equity stake strengthens target firm investment incentives, $e = 1/3$

The effect of an equity stake on target underinvestment is shown in Figure 4 for the case where $e = 1/3$, $\Delta S > 0$, and $\Delta S < -\Delta D_t$. If the firm does not invest the payoffs are the same as in Figure 3. Without an equity stake, investment would move the disagreement point to $(D'_t, D_s)$, representing a substantial deterioration in the target’s bargaining position. This would be the case if the investment is costly and is so specific to the stakeholder that it has little value in disagreement. Although investment would contribute to joint profits, it damages the target’s bargaining position so much that the target would be worse off as seen in the shift of the agreement point to $(\frac{1}{3}S' - \frac{1}{2}D_s + \frac{1}{2}D'_t, \frac{1}{2}S' + \frac{1}{2}D_s - \frac{1}{2}D'_t)$. With equity stake $e = 1/3$, the stakeholder’s bargaining position depends on the target’s disagreement payoff as already shown in Figure 3. The deterioration in the target’s disagreement payoff is therefore shared by the stakeholder, moving the disagreement point to $(\frac{2}{3}D'_t, D_s + \frac{1}{3}D'_t)$. Since the stakeholder’s
bargaining position also weakens, the target receives a large enough share of joint profits that it benefits from the investment, as seen from the shift to \((\frac{1}{2}S' - \frac{1}{2}D_s + \frac{1}{6}D_t', \frac{1}{2}S' + \frac{1}{2}D_s - \frac{1}{6}D_t')\). In this example the stakeholder also does slightly better from having the equity stake. Although the bargaining position effect on the stakeholder’s payoff is negative from Proposition 1(ii) for given \(S, D_s, \) and \(D_t,\) the stakeholder benefits because the equity stake induces the target to invest in enlarging joint profits from \(S\) to \(S'.\)

That efficient actions can be induced with a share less than one may be surprising. A lower share is sufficient because the firms’ incentives are not completely opposed to begin with. Considering the target’s actions, in the bargaining process the target receives a share of joint profits regardless of the equity stake. An equity stake supplements this existing incentive by diminishing the impact of bargaining positions on the division of joint profits. For \(e = \frac{1}{2}\) the firms’ bargaining positions move exactly in concert as \(D_t\) changes and the relative bargaining positions of the firms have no role. In Figure 4 this would be represented by the disagreement point shifting up or down along a line with slope of 45°. Any increase in joint profits would therefore increase the payoffs to both sides, ensuring efficient investment by the target firm.

5.2 Equity Resales are Unrestricted

When the stakeholder can resell the equity the types of action affected are different. The stakeholder will take an action if

\[ \Delta A_s(0) + e \Delta A_t(0) = (1 + e)^\frac{1}{2} \Delta S + (1 - e)^\frac{1}{2} (\Delta D_s - \Delta D_t) \geq 0 \]  \hspace{1cm} (15)

and the target will take an action if

\[ (1 - e) \Delta A_t(0) = (1 - e)^\frac{1}{2} \Delta S - (1 - e)^\frac{1}{2} (\Delta D_s - \Delta D_t) \geq 0. \]  \hspace{1cm} (16)

Note that an equity stake never affects whether an action increases or decreases the target’s payoff, so all of the impact is on the stakeholder’s behavior. As \(e\) approaches 1, the effect of bargaining positions on the stakeholder’s payoff disappears. The stakeholder therefore acts to maximize joint profits, implying the following proposition:

**Proposition 4** If equity resales are unrestricted, there exists \(\tau \in (0, 1)\) such that all equity stakes \(e \in [\tau, 1]\) solve the stakeholder underinvestment, overinvestment, undercooperation, and sabotage problems.
When underinvestment is a problem, selling the equity allows the stakeholder to capture part of the target’s higher payoff from any deterioration in the stakeholder’s bargaining position. Since the equity is valued more highly in the market if the stakeholder’s bargaining position is weakened by extra investment, the stakeholder can recapture part of the loss, increasing the incentive to invest. When there is initially overinvestment, this process works in reverse. Since the equity will be valued less if the stakeholder’s bargaining position is strengthened by the overinvestment, the return to unproductive investment falls.

The effect is similar for actions which affect the target’s profits. Regarding the undercooperation problem, if the stakeholder has an equity stake it will benefit from the strengthening of the target’s bargaining position when it sells the equity. Similarly, sabotage is no longer so rewarding because the deterioration in the target’s bargaining position results in a lower price for the equity.

6 Purchase of Equity Stake (Stage 1)

An equity stake may increase efficiency, but the payoffs to each side do not necessarily increase. To ensure that both sides benefit and will agree to the equity stake, the price at which the stakeholder buys the equity from the target can serve as a side payment. Ideally this price could be derived as an outcome of a bargaining game, but the bargaining problem has many ambiguities that make an exact solution difficult to defend. For instance, if the firms disagree over the equity price, will this disagreement continue until bargaining over the input price begins? If so, does disagreement in one stage imply disagreement in the other? Despite these modeling difficulties, some general statements can still be made without deriving an exact solution for the equity price. If the stakeholder takes an $e^*$ stake in the target, then as shown above the two firms’ investment and other strategies may change. In the following, profits and payoffs for the case where actions may have changed because of an equity stake $e^*$ are designated by a star. For instance $S^*$ represents joint profits with equity stake $e^*$ while $S$ represents joint profits with a zero stake. We will assume $S^* - S > 0$ since otherwise there is no basis for agreement.

6.1 Equity Resales are Restricted

We are interested in three different prices for the equity. First is the maximum price that the stakeholder would ever be willing to pay, which is the total gain in the stakeholder’s payoff
from owning the equity, \( \bar{q} = A_s^*(e^*) + e^* A_t^*(e^*) - A_s(0) \).\textsuperscript{17} Second is the market price given that market participants expects the stakeholder to end up with \( e^* \) of the target. This price will incorporate any expected changes in the firm’s profitability due to the purchase (Grossman and Hart, 1980) so \( \hat{q} = e^* A_t^*(e^*) \). Third is the minimum price the target would be willing to sell equity at. This is the total change in the target’s payoff from the stakeholder owning the equity, \( q = A_t(0) - (1 - e^*) A_t^*(e^*) \).

Comparing the maximum and minimum prices,

\[
\bar{q} - q = A_s^*(e^*) + e^* A_t^*(e^*) - A_s(0) - A_t(0) = S^* - S, \tag{17}
\]

so there is room for negotiation as long as the equity stake does in fact lead to larger joint profits. Consider now whether the maximum price \( \bar{q} \) is necessarily larger than the market price \( \hat{q} \).

\[
\bar{q} - \hat{q} = A_s^*(e^* - A_s(0) = (A_s^*(e^* - A_s(0)) + (A_s^*(0) - A_s(0)). \tag{18}
\]

Since \( A_s^*(e^*) < A_s^*(0) \) from the displacement effect of Proposition 1(i), \( \bar{q} > \hat{q} \) only if \( A_s^*(0) \) is sufficiently larger than \( A_s(0) \). Since the stakeholder acts to maximize its profits anyway even without the equity stake, \( A_s^*(0) > A_s(0) \) only if the target firm’s behavior becomes more efficient.

The stakeholder should therefore be wary of acquiring equity at the market rate. The loss in profits due to the displacement effect must be offset by a sufficient gain in efficiency, and this efficiency gain must come from the target’s behavior rather than the stakeholder’s behavior. Of course, when efficiency gains are large the target firm also benefits, so the stakeholder might still be in a good position to demand a lower than market price.

### 6.2 Equity Resales are Unrestricted

The same analysis can be applied when the stakeholder can resell the equity. In this case the maximum price the stakeholder will pay is \( \bar{q} = A_s^*(0) + e A_t^*(0) - A_s(0) \), the market price is

\textsuperscript{17}Previously we calculated equity prices after firms had already taken any actions. Here we are considering prices before such actions so the calculations must consider the impact of the equity on investment and other incentives.
\[ \hat{q} = e^* A_t^* (0), \text{ and the minimum price the target will accept is } \underline{q} = A_t (0) - (1 - e^*) A_t^* (0), \text{ so} \]

\[
\underline{q} - q = A_s^* (0) + A_t^* (0) - A_s (0) - A_t (0) = S^* - S. \tag{20}
\]

As before, there is a positive spread between the maximum and minimum prices as long as there are any efficiency gains from an equity stake. Comparing again the maximum price and the market price, the difference is

\[
\underline{q} - \hat{q} = A_s^* (0) - A_s (0) \leq 0, \tag{21}
\]

implying the stakeholder should not acquire the equity at the market price, but insist on a lower price from the target. The inequality follows because only the stakeholder, not the target, changes investment and other actions when equity resales are unrestricted. As argued above, if \( A_s^* (0) > A_s (0) \) this would imply that the stakeholder was not maximizing profits for \( e = 0 \).

### 7 Conclusion

Equity stakes emerge from this analysis as an attractive strategy when relationship-specific investments and other strategic actions are important and contracts cannot determine ex ante the division of joint profits generated by such actions. In a bargaining framework, parties to a transaction consider the effects of their actions not just on the joint profits the two firms share, but also on their ability to capture a sufficient share of the profits. This paper shows that an equity stake can align conflicting interests so that both sides are more interested in maximizing joint profits. For many strategic actions, total alignment of interests can be achieved with only a partial ownership share.

Interest alignment is also the key to other models in which sharing profits increases efficiency. Such alignment is beneficial when players have actions which are for some reason not fully contractible, preventing pareto-improving trades and driving a wedge between pursuit of individual interests and maximization of joint payoffs. While interest alignment plays a role in other models, it is unique in this model because the alignment works via disagreement payoffs. When equity resales are restricted, the party owning the equity stake must share in the other side’s profits or losses in disagreement. When equity resales are unrestricted, the party owning the equity stake appropriates through the resale price some of the losses or gains that the other party receives due to changes in bargaining positions. In both cases the gains from investment
and related actions become less dependent on each side’s bargaining position, freeing each firm to put more emphasis on maximizing joint profits and less emphasis on maximizing their share of the profits.

8 Bibliography


