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# GENETICS, FAMILY STRUCTURE, AND ECONOMIC GROWTH\*

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## GENETICS, FAMILY STRUCTURE, AND ECONOMIC GROWTH

### Abstract

Recent biomedical research shows that roughly three-quarters of cognitive abilities are attributable to genetics and family environment. This paper presents a theory of growth in which human capital is determined by inheritable factors and family size. The distribution of income is shown to affect the number of births, with greater inequality raising the fertility rate and reducing output growth in the transitional dynamics. If human or physical stocks are sufficiently low, the model shows that an economy can be caught in a fertility-caused poverty trap, while countries with more resources will converge to a balanced growth path where the average rate of transmission of human capital from parents to children determines the long-run rate of output growth.

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KEYWORDS: Genetics, Siblings, Growth, Fertility, Human Capital.

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*Man is a glorious and unique species of animal. The species originated by evolution; it is actively evolving, and it will continue to evolve. Future evolution could raise man to superb heights as yet hardly glimpsed, but will not automatically do so.*

G.G. Simpson, *This View of Life: The World of an Evolutionist*, 1947

## 1 INTRODUCTION

INNATE ABILITY, family influences, and environmental effects are the constituents of human capital according to Gary Becker (1993). An individual's genetic endowment, formed over the 200,000 years of *Homo sapiens* evolution, is therefore a constraint on achievable economic outcomes. Because one-half or more of adult intelligence can be traced to genetics (Plomin & Petrill, 1997), "innate ability" is a substantial component of human capital. Home environment also plays a role in determining human capital, especially the nurturing of children by their parents. The distribution of human capital, therefore, depends critically on the transmission of genes across generations as well as on environmental factors that determine the expression of inherited proclivities. If, as argued by Lucas (1988) and Jones (1995, 1997), it is *people* that generate technological advances, better production processes, and new products, then a model that examines both family structure and the inheritability of human capital is fundamental to an understanding of economic growth.

This paper presents a theory of economic growth in which fertility is endogenous, physical and human capital accumulate, and parents pass genetic and social traits to their children. This is a model of nature *and* nurture, for both are required to develop productive members of society. A primary motivation is to provide a biological basis for the "production function" of human capital. A variety of human capital production functions have been used in the growth literature, with these models inheriting

the dynamic properties of the chosen functions.<sup>1</sup> Herein, I build a theory of the production of human capital based on the production of humans. This approach follows Joseph Schumpeter (1954) by utilizing “biological facts and theories...whenever the question of inheritance of physical or mental qualities of the human material is brought in” (p789). This method has also been advocated by Jack Hirshliefer (1977, 1985) and Paul Samuelson (1985).

One of the innovations this model introduces is that the rate of transmission of human capital from parent to child is not uniform for all agents. Indeed, when the number of children in a family is large, per child parental nurturing is low, diluting the transmission of human capital; alternatively, when family size is small, the rate of human capital transmission is high. Further, the theory demonstrates that societies with unequal distributions of income will have higher aggregate birth rates and lower rates of economic growth. Analyzing the dynamics of the model reveals that the interaction of nature and nurture on the accumulation of human capital produces both a poverty trap as well as a balanced growth equilibrium.

The theory developed here is closely related to the seminal work on fertility and child quality by Becker & Tomes (1976), and the subsequent work by Becker & Barro (1988), as well as the models of fertility and economic growth by Becker, Murphy & Tamura (1990), Azariadis (1996), and especially the heterogeneous agent models of Galor & Tsiddon (1997a, 1997b) and Hassler & Rodríguez Mora (1998). However, the present paper departs from these in two primary ways. First, human capital depends stochastically on one’s parents’ human capital, as well as on parental nurturing of children, and therefore varies both across agents and over time. Second, I do not assume that parents are altruistic towards their children, only that they respond to the genetic imperative to reproduce, tempered by economic factors that affect the

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<sup>1</sup>There are many variants of the production function for human capital, e.g. Lucas (1988), Galor & Zeira (1993), Stokey (1996), Bond, Wang & Yip (1996), Tamura (1996), Galor & Tsiddon (1997a).

choice of the number of children. That is, parents do not explicitly evaluate the sequence of their progeny's human capital as in Lucas (1988) or Tamura (1996), nor do they consider the "quantity vs. quality trade-off" associated with resources invested per child (Becker & Tomes, 1976; Hanushek, 1994). A variant of the model in which parents are altruistic is presented in Section 6 and shows that in some cases altruism prevents the complex transitional dynamics of the base model—for example, eliminating the poverty trap. The transitional dynamics of the base model admit a variety of dynamic phenomena, yet the analysis shows that there is a large set of initial conditions that lead to balanced growth. Thus, the model reveals that the interaction between genetics and family structure generates a diverse set of growth experiences.

The foundation of the model is the biology of human genetics, as well as the sociology and psychology of child development. Section 2 briefly reviews the relevant findings from these literatures. Section 3 presents the base model where a continuum of agents heterogeneous in their human capital live three periods in overlapping generations. Section 4 characterizes the effects of changes in the distribution of human capital on aggregate fertility and output growth, while Section 5 investigates the dynamics of the model, both analytically and via numerical simulations. Section 6 examines the robustness of the findings by exploring several variants of the base model. Section 7 concludes with directions for future research linking genetics and family structure to economic growth.

## 2 BIOLOGICAL PRELIMINARIES

By studying twins, McClearn *et al* (1997) estimate that 62% of adult intelligence is due to genetics. Plomin & Petrill (1997) and Hamer & Copeland (1998, Ch. 6) survey a large number of studies of twins, siblings, and adopted children that use a variety of analytical techniques, and report that the heritable proportion of intelligence is

estimated between 48% and 75%. Household environment accounts for between 11% and 25% of the variance of intelligence in estimates by McClearn *et al* (1997), and Plomin & Petrill (1997), respectively. These studies complement research that has begun to identify the specific genes responsible for intelligence.<sup>2</sup> The studies cited above indicate that formal education and learning-by-doing only account for about one-quarter of one's ability. This occurs because highly educated members of society self-select to obtain more education as they are endowed with the ability to do so (Behrman, Rosenzweig & Taubman, 1994; Ashenfelter & Rouse, 1998; Rubinstein & Tsiddon, 1999). Self-selection by innate ability also occurs when workers choose particular types of jobs (Bartel & Sicherman, 1999). Behrman & Taubman (1989) find in their study of twins that genetics determines 81% of educational attainment. Plomin & Bergeman (1991) call self-selection guided by genetics "the nature of nurture."

In addition to genetics, family environment also plays a fundamental role in determining cognitive ability. Humans are distinct among primates in the length of their childhood. Nonhuman primates are adults by their fifth or sixth year, while it takes humans twice as long to mature. The extraordinary length of human childhood is related to the time required for parents to teach children complex skills (Weiss & Mann, 1985).<sup>3</sup> Anthropologist Gladys Reichard (1938) noted that "In many languages, the word for 'teach' is the same as the word for 'show,' and the synonymity is literal." Because per child parental nurturing is affected by the number of children in a family, as family size increases each child's educational attainment falls (Behrman

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<sup>2</sup>See Plomin, McClearn & Rutter (1997) and Brody (1992) on the search for the genes that produce the multiple aspects of intelligence.

<sup>3</sup>For example, Cosmides & Tooby (1992) and Bergstrom (1995) argue that cooperative social exchange enhances survivability and is therefore inculcated into children by parents. Fitch (1985) identifies stimulating play, verbal skills, and support of children's independence as the most important ways that parents influence children's cognitive development. See Harris (1998) on peer group effects vs. parental effects on child development.

& Taubman, 1989) as do grades in school (Downey, 1995).

Dawkins (1976) calls the social-cultural norms passed from parents to children “memes,” and argues that they are transmitted much like genes.<sup>4</sup> Thus, children’s intelligence depends on the genetic endowment from parents as well as the memes that they acquire through parental nurturing. The interaction of these two effects and general environmental influences have produced significant increases in average IQ scores during the post-war period. Known as the “Flynn effect” after Flynn (1987), average IQ scores in developed countries have increased between 10% and 25% between the 1950s and 1980s. Neuroscientist Christopher Wills (1998) argues that the evolution of intelligence is accelerating because of the rapid transformation of the social milieu, especially the stimulating effects of visual media, as well as improvements in child health following nutritional advances and vaccines. Wills (1998) states that “Once th[e] brain-body-environment feedback loop was established, it proceeded at an ever-increasing pace as the complexity of culture and the opportunity for new inventions increased... there is no reason to suppose that selection for increased intellectual capacity in our species has slackened” (p252).<sup>5</sup> Becker (1993) also identifies the compounding of genetics and the environment on human capital, writing “[T]he separation of ‘nature from nurture’ or ability from education and other environmental factors is apt to be difficult, for high earnings would tend to signify both more ability and a better environment” (p99). If intelligence—resulting from the combination of genetics and environmental influences—is a significant determinant of human capital, then the Flynn effect indicates that at least in the developed countries surveyed, average human capital is increasing.

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<sup>4</sup>Also see the discussion in Bergstrom (1996).

<sup>5</sup>There is evidence that the genotype-environment feedback occurs through “regulator genes” that turn off and on “structural genes” that code for proteins or enzymes. Edelman (1992) and Gazzaniga (1992) apply the regulator genes model to explain intelligence and consciousness, though the molecular basis for this mechanism is just beginning to be discovered; see Plomin (1994).



This brief overview of the genetics, psychology, and sociology literatures indicates that the intergenerational transmission of traits is primarily due to nature (genetics) and parental nurture (memetics). Based on these studies, the “law of motion” for human capital, which is at the core of this paper, reflects both parent-to-child genetic similarities, as well as parental nurturing.

### 3 THE MODEL

Consider a world with a continuum of agents who vary by their level of human capital. Individuals in this world are identified by  $i \in \mathbb{R}^+$  and live three periods in overlapping generations. The first period of life is childhood, the second is young adulthood, and the third is old age. Reproduction is limited to the second period of life, and, for simplicity, children are produced by parthenogenesis so that a child’s human capital is a function of a single parent’s human capital. This permits us to avoid the issue of marriage matching, and obviates the need to model the cross-over, mutation and linkage between genetic alleles which occurs when two parents supply genetic material to children.<sup>6</sup>

#### 3.1 THE CONSUMER’S PROBLEM

It is convenient to specify the model in units per effective worker so that human capital enters the model in a tractable way. When agents are children (age zero), their consumption is funded by their parent. Since a child’s consumption is not chosen by him or her, no utility flows from consuming goods in childhood. In the second period of life (age one), agents choose how many children  $b^i$  to have, supply labor inelastically to firms, earning labor income  $wh^i$  (the economy-wide wage  $w$  times

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<sup>6</sup>This setup can be thought of as modeling the average genetic material transmitted to children from both parents. See Burdett & Coles (1997) for a model of the search for a marriage partner.

$i$ 's human capital  $h^i$ ), which funds consumption  $c_1^i$ , supports children at a cost  $e^i$  per child, and permits them to save  $a^i$  for old age. The genetically programmed desire to reproduce is captured by having children generate utility for their parents. In old age (age two), agents are retired and consume  $c_2^i$  from the principal and interest on their savings  $Ra^i$ , where  $R \equiv 1 + r - \delta$  is one plus the net interest rate which is the gross interest rate  $r$  less the depreciation rate on capital  $\delta \in [0, 1]$ . Agents die at the end of the third period of their lives.

In order to concretize the analysis, I use functional forms for utility and production. When utility is logarithmic, the lifetime utility maximization problem for agent  $i$  born at time  $t - 1$  is

$$\text{Max}_{c_1^i, c_2^i, b^i} (1 - \beta) \ln(c_{1,t}^i) + \beta \ln(c_{2,t+1}^i) + \gamma \ln(b_t^i) \quad (1)$$

s.t.

$$\begin{aligned}
 c_{1,t}^i &= w_t h_t^i - b_t^i e_t^i - a_{t+1}^i \\
 c_{2,t+1}^i &= R_{t+1} a_{t+1}^i, \\
 b_t^i &\geq 1
 \end{aligned}$$

where  $\beta \in (0, 1)$  is the preference for consumption when middle-aged vs. old-aged, and  $\gamma > 0$  is the preference for children. The budget constraints in (1) relate to the two periods of adulthood during which agents consume. The choice of the number of children is limited to be at or above replacement rate ( $b_t^i = 1$ ) in order to produce a stable population in the long-run. Except for the choice of the number of children, (1) is an otherwise standard overlapping generations model.

The primary cost of children is the time spent raising them (Birdsall, 1988). The cost-of-children function  $e_t^i$  is parameterized so that desired births are decreasing in labor income for low to moderate income levels, and then are constant at high incomes, matching the relationship found in the data (Feng, Kugler & Zak, 2000;

Bongaarts, Mauldin, & Phillips, 1990),

$$e_t^i = \begin{cases} D(w_t h_t^i)^2 & \text{for } w_t h_t^i < \kappa \\ D_1 w_t h_t^i & \text{for } w_t h_t^i \geq \kappa \end{cases} \quad (2)$$

for the constants  $0 < D < \frac{1}{w_t h_t^i} \forall i, t$ , and  $D_1 = \frac{\gamma}{1+\gamma}$ , where  $\kappa \equiv \frac{\gamma}{D(1+\gamma)}$ . The functional form for  $e^i$  is a direct result of a limiting value for births, and has no substantive effect on the model other than keeping the dynamics well-defined in the limit.

Using the above parameterization for the cost of children, the optimal choices made by agent  $i$  at time  $t$  for savings and the number of children are

$$a_{t+1}^{i*} = \beta \left[ \frac{w_t h_t^i}{1+\gamma} \right] \quad (3)$$

$$b_t^{i*} = \text{Max} \left\{ \frac{\gamma}{D(1+\gamma)w_t h_t^i}, 1 \right\}. \quad (4)$$

Optimal savings, (3), is proportional to income, decreasing in the preference for children  $\gamma$  (since children have a cost), and increasing in the patience parameter  $\beta$ . The optimal number of children (4) is decreasing in income due to the cost of raising children, and increases as the preference for children rises. It is straightforward to show that the desired number of children is right continuous at its minimum, one.

### 3.2 HUMAN CAPITAL

The crux of this paper is the transmission of human capital from parents to their offspring. Section 2 provides a biological foundation for the effects of nature and nurture on children which is formalized here. The discussion in Section 2 indicates that a parent with human capital  $h_t^i$  will produce children who each have human capital  $h_{t+1}^i$ , where

$$h_{t+1}^i = \frac{\tilde{\omega} h_t^i}{(b_t^i)^\theta}. \quad (5)$$

In equation (5),  $\tilde{\omega}$  is a random variable that determines the inherited portion of human capital,  $b_t^i$  is the number of births in the family, and  $\theta > 0$  specifies the dilution effect on parental nurturing from having multiple children. Let  $\tilde{\omega} \sim G$ , where  $G$  has strictly positive support and  $E\{\tilde{\omega}\} = \omega > 1$ . The factor  $\tilde{\omega}$  includes both genetic traits as well as social and cultural factors (memes) that parents inculcate into children. Specifying the mean of  $\tilde{\omega}$  to exceed unity captures the Flynn effect discussed in Section 2. Relation (5) models the dilution of parental nurturing for children in large families by proportional reductions of their human capital relative to children in smaller families. Note that the “production function” for human capital (5) is similar to that used by Lucas (1988) when each family has a single child.

### 3.3 THE FIRM’S PROBLEM

I close the model by specifying the problem faced by firms and then defining a competitive equilibrium. Assume that there is a large number of firms operating in a competitive environment and that agents of all human capital types are necessary to produce output. Let  $\mu$  be an appropriately defined measure over working agents,  $\int_0^\infty d\mu_t = N_t$ .

The profit maximization problem for a representative firm at time  $t$  is

$$Max_{K,H} = Y_t - r_t K_t - w_t H_t, \quad (6)$$

where aggregate human capital  $H_t = \int_0^\infty h_t^i d\mu_t$ . Let the production function be Cobb-Douglas

$$Y_t = K_t^\alpha H_t^{1-\alpha}, \quad (7)$$

for  $\alpha \in (0, 1)$ . Solving for the firm’s profit maximizing conditions using (6) and (7), the labor income paid to a type  $i$  agent is

$$w_t h_t^i = (1 - \alpha) K_t^\alpha H_t^{-\alpha} h_t^i, \quad (8)$$

and the rate of return on capital is its marginal product,

$$r_t = \alpha K_t^{\alpha-1} H_t^{1-\alpha}. \quad (9)$$

### 3.4 MARKETS, AGGREGATES, AND EQUILIBRIUM

There are three markets in this model: goods, labor (all types), and capital. The labor market clears for agents with human capital  $h_t^i$  for some value of  $w_t$  by the concavity of the production function. The capital market clears when, for some value of  $R_{t+1}$ <sup>7</sup>

$$K_{t+1} = \int_0^\infty a_{t+1}^{i*} d\mu_t. \quad (10)$$

Aggregate fertility can be found by integrating over all agents  $i$ ,

$$B_t = \int_0^\infty b_t^{i*} d\mu_t, \quad (11)$$

where  $b^{i*}$  is given by (4). Since agents work for a single period, the law of motion for the working population,  $N$ , is

$$N_{t+1} = B_t. \quad (12)$$

That is, next period's working population is the aggregate number of births in the current period.

A *competitive equilibrium* for the model above is a set of prices  $\{w_t, R_{t+1}\}_{t=0}^\infty \forall i$ , given initial conditions for the distribution of physical capital,  $\int_0^\infty a_0^i d\mu = K_0 > 0$ , and of human capital, a law of motion for human capital (5), such that taking wages and the return to capital as given, consumers maximize lifetime utility by solving (1), firms maximize profits by solving (6), and prices clear all markets.

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<sup>7</sup>The goods market clears by Walras' Law.

## 4 IMPLICATIONS OF THE MODEL

The model shows that the distribution of human capital and the level of physical capital jointly determine output. The genetic and social “draw,”  $\tilde{\omega}$ , that one receives partially determines human capital and productivity. An agent with a high draw for  $\tilde{\omega}$  and not too many siblings will have a high level of human capital, will generally earn a high labor income, and will therefore desire few children. An agent with a poor draw will have low human capital, low labor income and as a result will have a large family in which the children, on average, have low levels of human capital. Now, consider an agent who receives the average draw,  $\omega$ . If she is a single child, then her human capital will exceed her parent’s (since  $\omega > 1$ ) and her parental “tutoring” will be high. Therefore, holding physical capital constant, such a child will earn a wage that exceeds her parent’s wage and will choose to have a single child in her family. If, on the other hand, this child with an average draw is born into a household in which income is low and therefore the number of children is high, she may have less human capital than her parent. Thus, the distribution of inheritable traits via the stochastic factor  $\tilde{\omega}$  and parental nurturing have a fundamental impact on fertility rates and the dynamics of the distribution of human capital.

Lemma 1 shows that the intergenerational transmission of human capital is monotonically increasing in parents’ human capital, with the maximal growth rate obtaining when family size is at its minimum.<sup>8</sup>

**Lemma 1** *When  $b^i > 1$ ,  $h_{t+1}$  increases at an increasing rate in  $h_t$ , with the maximal rate of increase in  $h_{t+1}$  obtaining when  $b^i = 1$ .*

Lemma 1 shows that the higher an agent’s human capital, the faster his progeny are expected to acquire human capital, up to the maximal value  $\tilde{\omega}$ . Lemma 2 shows if

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<sup>8</sup>Proofs are contained in the Appendix.

a parent has a low level of human capital she will, on average, produce children who also have little human capital, leading to an intergenerational poverty trap.

**Lemma 2** *If the human capital of agent  $i$  is sufficiently low relative to physical capital,*

$$h_t^i < \xi K_t^{\frac{-\alpha}{1-\alpha}},$$

*then the human capital of agent  $i$ 's children is less than that of agent  $i$ , where  $\xi \equiv [\tilde{\omega}^{\frac{1}{\theta}} D(1 + \gamma)(1 - \alpha)\gamma^{-1}]^{\frac{-1}{1-\alpha}}$ .*

This lemma shows that the threshold for negative human capital growth falls as  $K$  increases. This is the result of income affecting family size decisions. Parents with high levels of human capital, and therefore high incomes, are unlikely have children whose human capital is less than their own. The parameters that make up  $\xi$  reveal that human capital within a family is more likely to decline when children receive a poor inheritable draw ( $\tilde{\omega}$  small).

Lemmas 1 and 2 predicts that poverty within families persists over generations, which is supported by the empirical evidence surveyed by Mulligan (1997). The theory here offers a nature and nurture explanation for this finding: parents who have low human capital earn low wages, have more children, and invest less time per child in teaching them; these children, therefore, have less human capital, on average, than their parents. In the context of the model, this cycle is only broken by children who receive extraordinarily good genetic/memetic draws,  $\tilde{\omega}$ , that permit them to escape poverty, or by sufficient growth in the physical capital stock,  $K$  that raises parents' incomes. Extending the results of Lemmas 1 and 2 to the aggregate, the model predicts that even if growth leads to uniform increases in household incomes, the distribution of income will widen during development. This occurs because human capital rapidly accumulates in rich households, while poor households continue to

have large families which dilutes the transmission of human capital. Nevertheless, if the economy is growing, in the long-run all households reach replacement fertility and invest maximal nurturing in each child.<sup>9</sup>

We can, in fact, characterize the relationship between the number of births and the shape of the distribution of human capital (equivalently, the distribution of labor income). To derive this result, I use the notion of a *mean preserving spread* (Rothschild & Stiglitz, 1971) in which one distribution is constructed from another by moving mass from the middle of the distribution to the tails, keeping the mean constant and increasing the variance.

**Theorem 1** *A simple mean preserving spread of the distribution of labor income increases the aggregate number of births.*

The intuition for this result is straightforward: since high income agents choose small families, raising the proportion of these agents has little effect on aggregate births, while increasing the proportion of low income agents significantly raises aggregate births since low income agents' fertility choices are sensitive to income changes. This theorem does not depend on the minimum number of births being one, but follows simply from the increasing opportunity cost of children as labor income rises.

The next result shows that the variance of the distribution of income affects output growth by impacting fertility choices.

**Theorem 2** *If  $wh^i < \kappa \forall i$ , then a simple mean preserving spread of the distribution of labor income decreases output growth.*

Theorem 2 demonstrates that “inequality reduces growth,” a proposition that has robust empirical support (Perotti, 1996). The model presented in this paper identifies fertility choices as a pathway through which inequality negatively impacts

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<sup>9</sup>The evolution of the distribution of human capital is characterized in Section 5.3.



growth. Fertility decisions, coupled with the inheritable and nurturing constituents of human capital, result in both intrafamily persistence of poverty and variations in aggregate output.<sup>10</sup>

## 5 THE DYNAMICS OF FERTILITY AND GROWTH

In order to examine the dynamics of the model, I begin by characterizing the special case where all agents within a generation have the mean level of human capital. That is, the dynamics lie in the space of per worker physical capital,  $k_t \equiv \frac{K_t}{N_t}$ , and average human capital,  $h_t = \frac{\int_0^\infty h_t^i d\mu_t}{N_t}$ . In Section 5.3, I investigate the dynamics of the full heterogeneous model.

When all agents have the mean level of human capital, and the factor relating parents' to children's human capital is set to its expected value,  $\omega$ , the dynamics of the model are given by

$$k_{t+1} = \frac{\beta(1-\alpha)k_t^\alpha h_t^{1-\alpha}}{(1+\gamma)b_t^*}, \quad (13)$$

$$h_{t+1} = \frac{\omega h_t}{(b_t^*)^\theta}, \quad (14)$$

where  $b_t^*$  is given by (4) when human capital is at its mean,  $h_t$ . Since with sufficient income growth the birth rate reaches its replacement value,  $b_t^* = 1$ , after which human capital grows at a constant rate, there is a difference between the dynamics on the balanced growth path and the transitional dynamics.

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<sup>10</sup>Galor & Zang (1997) also find that inequality reduces growth via fertility, with this result following from borrowing constraints on educational expenditures. Other explanations that relate inequality to growth include transfers chosen by the median voter (Alesina & Rodrik, 1994; Persson & Tabellini, 1994), credit constraints on education (Bénabou, 1996; Galor & Zeira, 1993), increasing returns to scale and a small middle class (Murphy, Shleifer & Vishny, 1989), and imperfect contract enforcement that reduce trust between transacting parties (Zak & Knack, 2001).

## 5.1 TRANSITIONAL DYNAMICS

In the transitional dynamics, population growth is nonconstant, i.e.  $b_t^* > 1$ . As a result, the transitional dynamics are, in per worker terms,

$$k_{t+1} = Ak_t^{2\alpha} h_t^{2(1-\alpha)} \quad (15)$$

$$h_{t+1} = Bk_t^{\alpha\theta} h_t^{1+\theta(1-\alpha)}, \quad (16)$$

where  $A \equiv \frac{\beta D(1-\alpha)^2}{\gamma}$ , and  $B \equiv [\omega^{\frac{1}{\theta}} D(1+\gamma)(1-\alpha)\gamma^{-1}]^\theta$ . The model admits two steady states, the trivial one ( $k_t = h_t = 0, \forall t$ ), and a unique interior steady state  $(\bar{k}, \bar{h})$  given by

$$\bar{k} = AB^{\frac{-2}{\theta}} \quad (17)$$

$$\bar{h} = [A^\alpha B^{\frac{1-2\alpha}{\theta}}]^{-\frac{1}{1-\alpha}}. \quad (18)$$

The dynamics of system (15), (16), are robust to variations in parameter values as the next result shows.

**Theorem 3** *For all admissible parameter values, the steady state given by (17), (18), is locally saddle-point stable.*

Figure 1 depicts the phase portrait for the transitional dynamics when capital's share of output  $\alpha < \frac{1}{2}$ . The phase space in the figure is partitioned by the curves where physical capital is constant, denoted by KK, and where average human capital is constant, HH. The unique interior steady state has, as Lemma 3 demonstrates, a saddle path leading to it.<sup>11</sup> Thus, for a given value of initial physical capital  $k_0$  in a

<sup>11</sup>The phase portrait when  $\alpha > \frac{1}{2}$  has the same HH curve, but the KK curve is strictly downward sloping and flatter than the HH curve. When  $\alpha = \frac{1}{2}$ , the KK partition is a horizontal line. I am focusing on the  $\alpha < \frac{1}{2}$  case since the share of output paid to capital is typically measured as being near one-third (Stokey & Rebelo, 1995; Christiano, 1988).

[Figure 1 about here]

Figure 1: The transitional dynamics when  $\alpha < \frac{1}{2}$ .

neighborhood of the steady state, there is a unique value of initial human capital  $h_0$  that puts the dynamics on the saddle path leading to the interior steady state  $(\bar{k}, \bar{h})$ .

For initial values of  $k_0$  and  $h_0$  that are not on the saddle path, the phase arrows in Figure 1 suggest that there are dynamic pressures that lead the economy towards the origin for initial conditions in regions I and II. As a result, the origin is a poverty trap in this model, while the interior steady state is a “middle-income trap.”<sup>12</sup> For initial conditions in regions II and IV, there appear to be pressures for growth in both  $k$  and  $h$ . It is this issue to which we next turn.

## 5.2 BALANCED GROWTH DYNAMICS

In this section I first characterize the balanced growth dynamics and then I combine the transitional and balanced growth dynamics into a complete depiction of the model’s evolution. Since balanced growth requires that all endogenous variables grow at constant rates, this does not obtain for the growth in human capital until  $b^* = 1$ . As a result, the values for  $h$  and  $k$  at which  $b^* = 1$  provides a lower bound on the balanced growth path (BGP).

**Lemma 3** *A lower bound for the balanced growth path is*

$$k_t = \left( \frac{\gamma}{D(1+\gamma)(1-\alpha)} \right)^{\frac{1}{\alpha}} h_t^{\frac{\alpha-1}{\alpha}}.$$

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<sup>12</sup>See Azariadis (1996) on the interpretation of various steady states as poverty traps.

Lemma 3 shows that the BGP is reached more rapidly when  $h$  or  $k$  is higher, and the preference for children,  $\gamma$ , is lower. The latter result obtains because families that have more children transmit less human capital on average intergenerationally, slowing the time until the maximal rate of human capital transmission is reached. The opposite holds when the proportionality constant of the cost of children,  $D$ , is raised—fewer children are chosen and the BGP is reached more rapidly. This lemma is useful because it permits us to separate the transitional dynamics from the balanced growth dynamics.

On a BGP, the equilibrium dynamics are given by

$$k_{t+1} = \frac{\beta(1-\alpha)k_t^\alpha h_t^{1-\alpha}}{1+\gamma} \quad (19)$$

$$h_{t+1} = \omega h_t, \quad (20)$$

which is similar to the model of Lucas (1988) with a generational structure. On a BGP, as in Lucas's model, all the endogenous variables grow at the rate of growth in human capital.

**Theorem 4** *On a balanced growth path, output, physical capital, and human capital all grow at rate  $\omega$ .*

**Theorem 5** *The balanced growth path is attracting.*

Figure 2 presents the transitional and balanced growth dynamics in the same phase portrait, where, by Theorem 5, the BGP is attracting and lies in region III. The figure suggests that there are ranges for the initial values of  $k$  and  $h$  that lead to balanced growth, while other initial conditions cause the economy to be mired in a poverty trap. Specifically, for initial conditions in Figure 2 in regions III and IV the economy is generally attracted to the BGP. This holds “generally” because the region of attraction to the BGP is quite difficult to determine, and in addition, there are a

[Figure 2 about here]

Figure 2: The complete dynamics when  $\alpha < \frac{1}{2}$ .

set of initial conditions in region IV that place the dynamics on the one-dimensional manifold leading to the interior steady state and therefore for these initial conditions balanced growth does not obtain.<sup>13</sup>

More broadly, Figure 2 suggests that the HH curve acts as a boundary leading the dynamics for most initial conditions to the right of the HH curve toward the BGP. Thus, if this economy begins with sufficient human and physical capital, then both  $h$  and  $k$  will grow in the transitional dynamics (i.e. the economy eventually enters region III of Figure 2. Alternatively, initial conditions in regions I and II, where the economy is capital poor, cause human capital to decumulate over time, leading the economy into a fertility-led poverty trap.

### 5.3 DYNAMICS WITH INTERGENERATIONAL HETEROGENEITY

In order to characterize the dynamics of the full heterogeneous agent model, simulations are presented in which there are 100 agents and the initial distribution of human capital varies. Of particular interest is the evolution of the distribution of human capital and its relationship to output growth. Theorem 2 demonstrates that inequality restrains growth. In this section I show that sufficient initial inequality in the distribution of human capital leads the economy into a poverty trap, extending Theorem 2. For less inequality in initial human capital, balanced growth obtains.

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<sup>13</sup>For the case  $\alpha \geq \frac{1}{2}$ , initial conditions in the same regions as for  $\alpha < \frac{1}{2}$  lead the dynamics either to the origin or toward the BGP, with the exception of the one-dimensional manifold leading to the interior steady state.

The first panel of Figure 3 depicts an initial distribution of human capital which is fairly uniform, except for a large group of very low human capital agents, and a relatively large mass of high human capital agents.<sup>14</sup> Mean human capital for this distribution is 1.064, with standard deviation .015. Simulating the economy for twenty generations, the second panel of Figure 3 shows that the distribution of human capital compresses and shifts downward until all agents possess very little capital, consistent with Lemma 2. The final distribution of human capital has mean  $2E^{-11}$  and maximum  $6.6E^{-5}$ . The third panel of the figure plots physical capital and average human capital, both of which monotonically approach zero indicating that the economy is caught in a poverty trap. Thus, sufficient inequality in human capital, coupled with low initial physical capital and/or low mean human capital, induce a sustained contraction in output.<sup>15</sup> This simulation reveals the critical interaction between fertility and inequality in determining a country's growth trajectory.

A different scenario is illustrated in Figure 4. For this economy, initial human capital, shown in the first panel, is skewed toward the high side, being a  $\beta(5, 1)$ . Nevertheless, the mean remains low at 1.088, and the standard deviation is .005. Holding all other values the same as in the previous simulation, the third panel of the figure indicates that the economy begins to contract, nearing a poverty trap in period fourteen, and then growing, eventually reaching a balanced growth path. In this case, even a distribution of human capital which contains a large mass of

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<sup>14</sup>The initial distribution of human capital is a  $\beta(.5, 1)$ . All simulations use the following values which were chosen for their reasonableness, following the real business cycle literature (Cooley, 1995) when possible:  $\gamma = .333$ ,  $\alpha = .35$ ,  $D = .6$ ,  $\beta = .333$ ,  $\theta = 1.02$ , the transmission of human capital  $\tilde{\omega} \sim N(2, .000055)$ , and initial capital  $k_0 = .04$ . Lastly, maximum fertility in these simulations is limited to five.

<sup>15</sup>For the same initial distribution of human capital, the economy grows rather than contracts if  $k_0 \geq .043$ ; or, keeping initial physical capital at its original value, if mean initial human capital is greater than 1.09, showing the veracity of Lemma 3.

[Figure 3 about here]

Figure 3: Inequality leading to a poverty trap.

relatively high human capital agents is unable to produce uniformly positive growth. The poverty trap was avoided in this simulation because a sufficient number of good genetic/memetic draws,  $\tilde{\omega}$ , arose for children, raising productivity and output growth. Without this good luck, having a large number of skilled individuals living in an economy with low initial physical capital or low mean human capital is insufficient to generate positive output growth.

The second panel of Figure 4 shows that after 20 generations, the distribution of human capital is dominated by mostly average human capital agents, though there are a few of “geniuses” ( $h^i > 400$ ). The final distribution of human capital for this simulation has mean 12.7 and maximum 457. Growth eventually causes agents to have, on average, more human capital than their progenitors (though minimum human capital for this distribution is  $6.9\text{E}^{-7}$ ).

The two reported simulations show that having a large mass of very productive agents in an economy is insufficient to sustain growth if average human capital or physical capital is low. On the other hand, if the economy’s initial position is not too impoverished, most agents in the economy experience increasing incomes, causing a demographic transition to low birth rates. In low birth rate economies, the nurturing of children is high, and balanced growth obtains.

To conserve space, simulations with a higher average initial human capital or a higher initial physical capital are not shown. Each of these cases is consistent with Lemma 3 and Theorems 4 and 5, with positive growth rates in the transitional dynamics and rapid convergence to a BGP.

Modeling the transmission of human capital as a function of inherited traits and

[Figure 3 about here]

Figure 4: Less inequality and growth.

nurturing received by children produces dynamics that explain a large variety of growth experiences. Quah (1997) documents the “twin peaks” pattern of the cross-country data showing that there are countries massed about a no-growth equilibrium. The model here shows that one cause of this phenomenon is high fertility which retards the intergenerational transmission of human capital. If, on the other hand, initial conditions lead to rising incomes and declining fertility, the rate of growth accelerates, causing initially poor countries to grow rapidly ( $\beta$ -convergence). Barring outside influences, such countries will become “information economies” where growth on a BGP is driven by the accumulation of human capital. On a BGP, the model predicts that countries will converge in growth rates, as shown in the empirical studies by Barro & Sala-i-Martin (1997), and Razin & Yuen (1994). The results here demonstrate that the effect of fertility choices on human capital accumulation can account for both the abysmal growth experiences of poor countries as well as the knowledge-based growth in developed countries.

## 6 VARIANTS OF THE BASE MODEL

This section briefly examines two variants of the model in order to determine the robustness of the results. The first case considered here posits that parents receive utility not from children themselves, but from the human capital of their children, as in Lucas (1988), Tamura (1996), and Zilcha (1996). This variant internalizes the dilution effect of parental nurturing when the desired number of children is greater than one that is an externality from the parent’s point of view in the base model. Parents in this variant of the model are altruistic in that they care about their children’s labor



market outcomes. An agent born at time  $t - 1$  solves,

$$Max_{c_1^i, c_2^i, b^i} E(1 - \beta) \ln(c_{1,t}^i) + \beta \ln(c_{2,t+1}^i) + \gamma \ln(h_{t+1}^i) \quad (21)$$

s.t.

$$\begin{aligned} c_{1,t}^i &= w_t h_t^i - b_t^i e_t^i - a_{t+1}^i \\ c_{2,t+1}^i &= R_{t+1} a_{t+1}^i \\ b_t^i &\geq 1 \\ h_{t+1}^i &= \frac{\tilde{\omega} h_t^i}{(b_t^i)^\theta}. \end{aligned}$$

In this version of the model, the optimality conditions are

$$a_{t+1}^{i*} = \frac{\beta(1 - \alpha)k_t^\alpha (h_t^i)^{1-\alpha}}{1 - \gamma\theta} \quad (22)$$

$$b_t^{i*} = 1 \quad \forall t, \quad (23)$$

subject to the parameter restriction  $\gamma\theta < 1$ . Since parents are choosing the level of human capital of their children, the optimal number of children is simply a single child, for all parents and for all time periods.

To study the dynamics of this version of the model, let agents within a generation have identical levels of human capital, as above, and set  $\tilde{\omega}$  to its mean value  $\omega$ . Then, the equilibrium dynamics of this economy are given by

$$k_{t+1} = \frac{\beta(1 - \alpha)k_t^\alpha h_t^{1-\alpha}}{1 - \gamma\theta} \quad (24)$$

$$h_{t+1} = \omega h_t. \quad (25)$$

Since parents care about their child's human capital, the transmission of human capital is always at a maximum and, as a result, the poverty trap in the base model disappears. The transitional dynamics lead to rapid growth when  $k$  is low so that poor countries catch-up to wealthier countries growing on the BGP. The BGP in this version of the model is globally attracting and all countries eventually reach it. Once

a country is on the BGP, output grows at the rate of human capital transmission,  $\omega$ , and countries show no further convergence in levels of per worker output. Because this version of the model internalizes the dilution effect of multiple siblings, the rate of growth in the transitional dynamics exceeds the transitional growth rate of the base model.

A second variant of the base model includes a choice by parents to invest both time and goods in raising the human capital of their children, as in Becker & Tomes (1976), Boldrin (1993), Zilcha (1996), and Dahan & Tsiddon (1998). Human capital can be raised for all children in a family by investing in a tutor at cost  $\sigma^i > 0$ .<sup>16</sup> In this version of the model, parents receive utility from both the number of children they have as well as the human capital of their children. By assumption, increasing expenditures on a tutor have a positive effect on the human capital of children, but does so with diminishing marginal productivity as measured by the parameter  $\zeta \in (0, 1)$ . Since the genetic transmission of human capital is stochastic and, by assumption, parents choose  $\sigma^i$  prior to the realization of  $\tilde{\omega}$  for each child, spending on tutors is based on the expected human capital of children and is therefore uniform for all children within a family.

An agent born at time  $t - 1$  chooses her own consumption, how many children to have, and an investment  $\sigma^i$  that raises her children's human capital by solving,

$$\text{Max}_{c_1^i, c_2^i, b^i, \sigma^i} E(1 - \beta) \ln(c_{1,t}^i) + \beta \ln(c_{2,t+1}^i) + \gamma_1 \ln(b_t^i) + \gamma_2 \ln(h_{t+1}^i) \quad (26)$$

s.t.

$$c_{1,t}^i = w_t h_t^i - b_t^i e_t^i - \sigma_t^i - a_{t+1}^i$$

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<sup>16</sup>The cost of a tutor is equivalent to a tax paid for public education, where the tax paid varies across households. Educational expenditures are an *inter vivos* bequest from parents to children. This variant of the model can be transformed into one with a uniform educational tax,  $\sigma$ , paid by all agents if, for example, the median voter determines the value of  $\sigma$ .

$$\begin{aligned} c_{2,t+1}^i &= R_{t+1} a_{t+1}^i \\ b_t^i &\geq 1 \\ h_{t+1}^i &= \frac{\tilde{\omega} h_t^i (\sigma_t^i)^\zeta}{(b_t^i)^\theta}, \end{aligned}$$

for  $\gamma_1, \gamma_2 > 0$ . The optimality conditions are given by

$$a_{t+1}^{i*} = \frac{\beta w_t h_t^i}{1 + \gamma_1 + \gamma_2(\zeta - \theta)} \quad (27)$$

$$b_t^{i*} = \text{Max}\left\{\frac{\gamma_1 - \gamma_2 \theta}{D(1 + \gamma_1 + \gamma_2(\zeta - \theta)) w_t h_t^i}, 1\right\} \quad (28)$$

$$\sigma_t^{i*} = \frac{\zeta \gamma_2 w_t h_t^i}{1 + \gamma_1 + \gamma_2(\zeta - \theta)} \quad (29)$$

with the parameter restriction  $\gamma_1 > \gamma_2 \theta$ . In this model, a proportion of income  $\sigma^{i*}$  is spent on tutors, and, as above, the desired number of children is decreasing in income. The optimality conditions for this version of the model match those for the base model given in Section 2 when parameters are redefined, although the optimal number of children is always below that in the base model because parents care about their children's human capital. Thus, the birth rate in this version of the model lies between that of the base model and the first variant of the model where parents only care about their children's human capital. By Lemma 1, per worker output is negatively related to the birth rate, indicating that this version of the model produces faster growth and more rapid convergence to the BGP than the base model.

Overall, the major predictions of the base model—a poverty trap for low values of  $k$  or  $h$ , rapid growth in the transitional dynamics for countries above the poverty trap threshold, and convergence to a balanced growth path—are all preserved when parents care about their children's human capital and the number of children, and can invest resources to raise human capital. Interestingly, parents' investment in their children's human capital in this version of the model does not eliminate the poverty trap. The poverty trap persists because of the short time-horizon of parents.

## 7 CONCLUSION

This paper presented a heterogeneous agent endogenous growth model where, following the biology, psychology, and sociology literatures, the intergenerational transmission of human capital depends on genetics and parental nurturing. The dynamics of the model show that fertility choices and household structure contribute to our understanding of the variety of growth experiences, from less developed economies caught in poverty traps, to rapid catch-up by industrializing countries, to balanced growth in developed societies. The model's flexibility derives from relaxing the assumption of many growth models that human capital always accumulates. By focusing on familial influences, the model permits human capital to decumulate when parental nurturing is sufficiently diluted by large family size.

One of the noteworthy aspects of the model is that there are identifiable bounds on the initial conditions that lead to balanced growth. The model shows that countries that begin with sufficient physical or average human capital are attracted to the balanced growth path, while countries that have a paucity of capital of either type remain at a low level of income per worker permanently. Further, the model shows that the distribution of income affects aggregate fertility, the rate of human capital accumulation, and, therefore, the prospects for economic growth. Interestingly, the analysis demonstrates that in developed economies, the genetically-driven female mating preference of fewer higher fitness offspring is the utility maximizing choice, rather than the male mating preference of maximizing the quantity of children so that some proportion of these will survive (Ridley, 1993, Ch. 6). The model shows that the male impulse inhibits the transmission of human capital and will only occur in economies that are capital poor.

In order to keep the dynamics of the model tractable, several simplifying assumptions were made that should be relaxed in future research to gain a fuller understanding of the effect of fertility and the distribution of human capital on growth. First,

the model shows that variance of human capital affects the economy's growth rate as shown in Theorem 2. This issue should be explored further, especially in the context of subsidizing public education when some households may be credit constrained, as in Galor & Zeira (1993). The model shows that a sufficiently wide variance in human capital slows or even stops growth. This stands in contrast to the finding of Galor & Tsiddon (1997a) who show that increasing inequality may move a country away from a poverty trap when there are local and global externalities. Second, permitting children to be produced by the mating of agents identified by their sex, rather than by parthenogenesis, would add realism to the model and would permit an exploration of how the mixing of genes affects growth. Nevertheless, as long as most mating is assortative vis-à-vis levels of human capital, the basic results here will be maintained.

## APPENDIX

This Appendix provides proofs for lemmas and theorems that are either instructive or novel. Other proofs are omitted to save space but are available from the author upon request.

PROOF. [Lemma 1] Substituting the optimal value for  $b^i$  from (4) and the value of the wage (8) into the law of motion for human capital (5) produces

$$h_{t+1}^i = \omega D^\theta (1 + \gamma)^\theta \gamma^{-\theta} (1 - \alpha)^\theta K_t^{\alpha\theta} (h_t^i)^{1+\theta(1-\alpha)}. \quad (30)$$

One can immediately see from (30) that  $\frac{dh_{t+1}}{dh_t}$  is convex for all admissible parameter values. It remains to show that the rate of increase in human capital for all  $b^i > 1$  is less than that when  $b^i = 1$ . Since  $\frac{dh_{t+1}}{dh_t}$  is strictly convex, the result obtains if  $\frac{dh_{t+1}}{dh_t} < \omega$  for  $b^i$  close to, but less than one. Solving for the value of  $K$ , call it  $\hat{K}$ , such that  $b^i = 1$  using (4), one finds that  $\hat{K} = [\frac{\gamma}{D(1+\gamma)(1-\alpha)(h^i)^{1-\alpha}}]^\frac{1}{\alpha}$ . Let  $\epsilon > 0$  be some small value and substitute  $\hat{K}(1 - \epsilon)$  into (30). Then,  $\frac{dh_{t+1}}{dh_t} = \omega(1 - \epsilon)^{\alpha\theta} < \omega$  and the result is proved. ■

PROOF. [Theorem 1] Let  $\mathcal{F}$  be the nondegenerate distribution of labor income at a particular point in time for a given level of the capital stock,  $K$ , and let  $\mathcal{G}$  be a distribution derived from  $\mathcal{F}$  via a simple mean preserving spread (MPS). By (4), the desired number of children decreases monotonically in labor income  $wh^i$ , and is concave, with  $\lim_{wh^i \rightarrow 0} b(wh^i) = \infty$ , and  $\lim_{wh^i \rightarrow \infty} b(wh^i) = 1$ . Theorem 1 of Diamond & Stiglitz (1974) characterizes the effects of a mean preserving spread of a distribution defined over a concave function. By that theorem,  $\int_0^\infty b^i d\mathcal{F} < \int_0^\infty b^i d\mathcal{G}$ . Therefore, a mean preserving spread in labor income increases aggregate births. ■

PROOF. [Theorem 2] Using the production function (7), next period's output is  $Y_{t+1} = K_{t+1}^\alpha H_{t+1}^{1-\alpha}$ . The method of proof is to show that a MPS has no effect on  $K_{t+1}$  and decreases  $H_{t+1}$ , thus reducing  $Y_{t+1}$ . Using first order conditions (3) and (4) and assuming  $wh^i < \kappa \forall i$ , savings by young adult  $i$  at time  $t$  can be written as  $a_{t+1}^{i*} = \beta[w_t h_t^i - e_t^i b_t^{i*}] = \beta[w_t h_t^i - D(w_t h_t^i)^2 b_t^{i*}] = \frac{\beta w_t h_t^i}{1+\gamma}$ . The capital market clearing condition (10) can therefore be written  $K_{t+1} = \frac{\beta w_t}{1+\gamma} \int_0^\infty h_t^i d\mu_t$ . Since this is linear in physical capital, a MPS has no effect on its accumulation. Next, we examine human capital. Substituting the optimality condition (4) into the law of motion of human capital (5), results in  $h_{t+1}^i = \nu \tilde{\omega} (h_t^i)^{1+\theta} w_t^\theta$ , where  $\nu \equiv (\frac{1+\gamma D}{\gamma})^\theta$ . Setting the random variable  $\tilde{\omega}$  to its mean  $\omega$ , and integrating this equation over all agents to find aggregate human capital,  $H_{t+1} = \omega \nu w_t^\theta \int_0^\infty (h_t^i)^{1+\theta} \mu_t$ . Clearly, the function to be integrated,  $(h_t^i)^{1+\theta}$ , is convex. Therefore, by Theorem 1 of Diamond & Stiglitz (1974) applying a MPS to the measure  $\mu$ , reduces  $H_{t+1}$ . As a result, an MPS reduces next period's output  $Y_{t+1}$ . ■

Next, we proceed to prove Theorem 3. The proof utilizes the following lemma,

**Lemma 4** *A first order approximation of the transitional dynamical system given by (15) and (16) has only real eigenvalues.*

PROOF. [Theorem 3] Since this is a planar dynamical system, the two eigenvalues can be characterized using the trace ( $TR$ ) and determinant ( $DET$ ) of the Jacobian of the local approximation of the system of transitional dynamics given by (15) and (16), noting that the characteristic equation  $h(\lambda)$  can be written as  $h(\lambda) = \lambda^2 - TR\lambda + DET = 0$ .

Direct calculation reveals that  $DET = 2\alpha$ , and  $TR = 2\alpha + 1 + \theta(1 - \alpha)$ . Clearly  $DET$  and  $TR$  are strictly positive, with  $TR > DET$ , and by Lemma 4, the roots to the characteristic equation are real. The eigenvalues near the steady state satisfy  $\lambda_1 < 1 < \lambda_2$  if  $h(1) < 0$  or equivalently,  $DET < TR - 1$ . Using the values for  $DET$  and  $TR$  given above,  $DET < TR - 1$  obtains if  $\theta(1 - \alpha) > 0$  which is true for all admissible parameter values. Therefore, the steady state is a saddle. ■

PROOF. [Theorem 4] Observe that the growth in output can be written as

$$g \equiv \frac{y_{t+1}}{y_t} = \left(\frac{k_{t+1}}{k_t}\right)^\alpha \left(\frac{h_{t+1}}{h_t}\right)^{1-\alpha}$$

which is equivalent on a BGP to

$$\frac{y_{t+1}}{y_t} = \left(\frac{y_t}{y_{t-1}}\right)^\alpha \omega^{1-\alpha}$$

by noting that  $k_{t+1} = \frac{\beta(1-\alpha)}{1+\gamma}y_t$  by (19) and using (20). Since output growth is constant on a balanced growth path,  $\frac{y_{t+1}}{y_t} = \frac{y_t}{y_{t-1}} = g$ . Therefore,  $g = \omega$ . ■

PROOF. [Theorem 5] following Barro & Sala-i-Martin (1995), we analyze the dynamics at a point on the BGP by transforming the system into one in which there is a steady state. Defining the new variable  $z_t \equiv \frac{k_t}{h_t}$ , the dynamical system on the BGP can be written as

$$z_{t+1} = \frac{\beta(1-\alpha)z_t^\alpha}{\omega(1+\gamma)}.$$

This new system has the steady state  $\bar{z} = [\frac{\beta(1-\alpha)}{\omega(1+\gamma)}]^{\frac{1}{1-\alpha}}$ . Taking the Jacobian of the new system and evaluating it at  $\bar{z}$  one can show that the eigenvalue is  $\alpha \in (0, 1)$ . Therefore, the BGP is stable. ■

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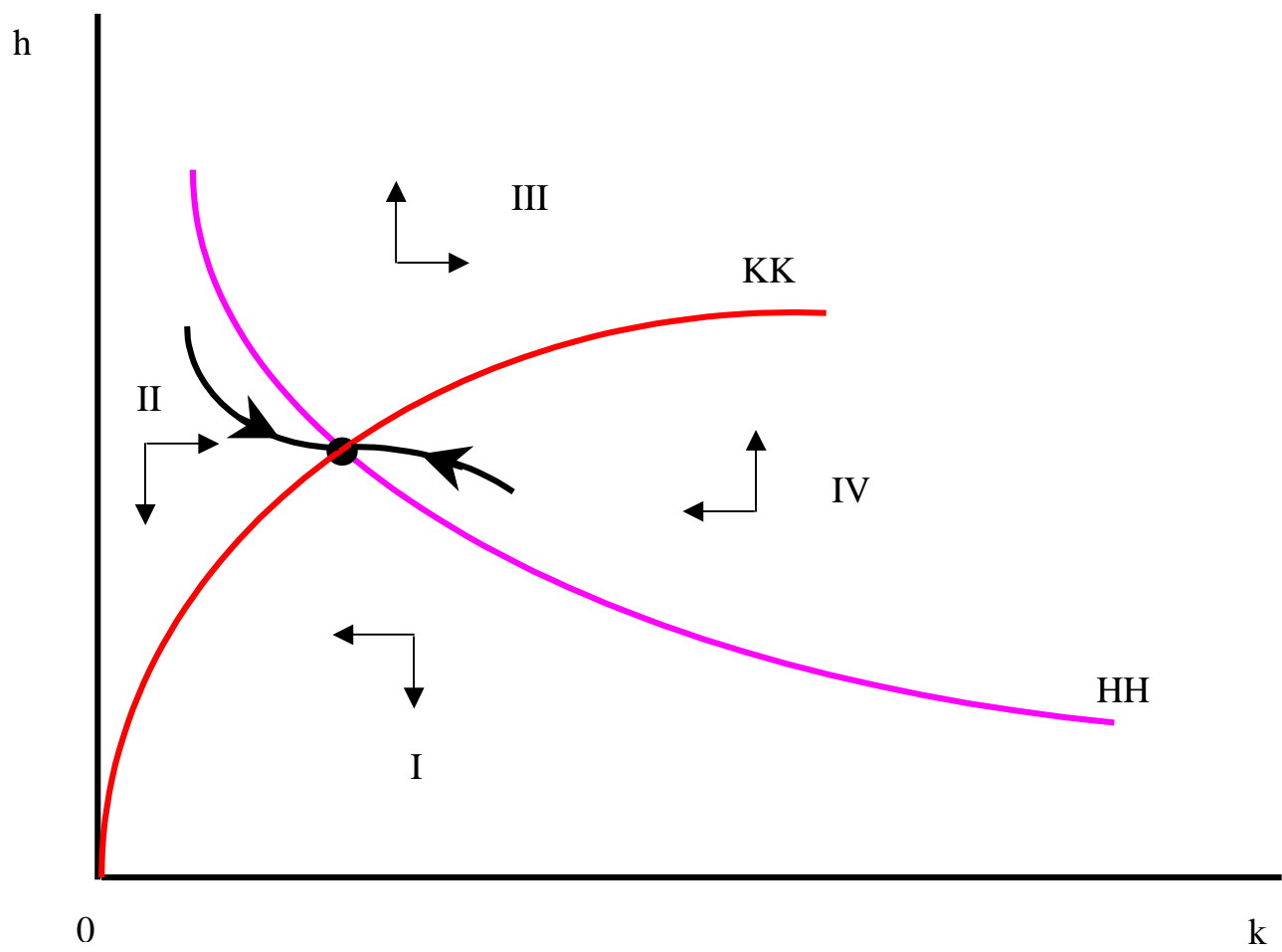


Figure 1

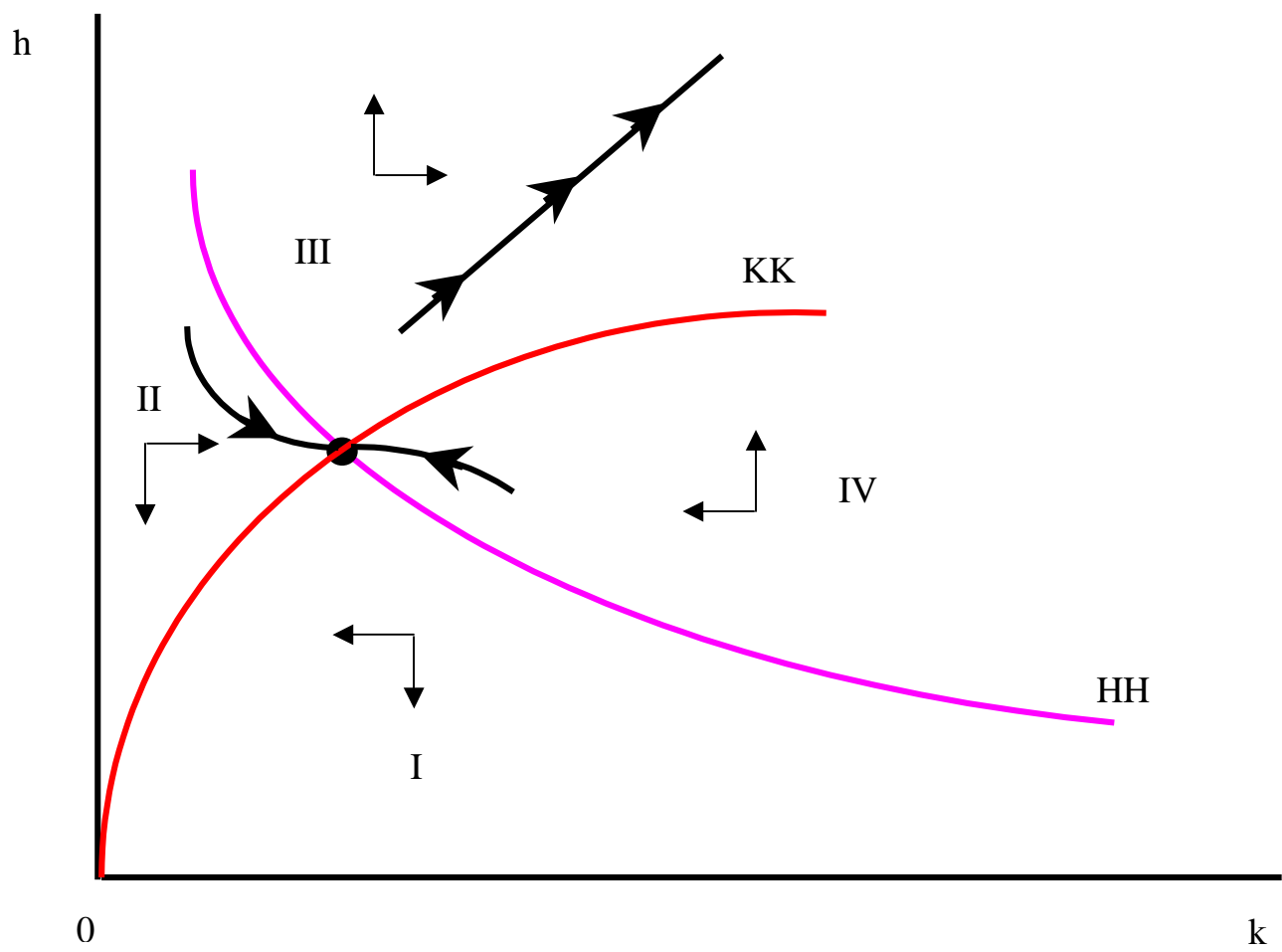


Figure 2