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AN EXPERIMENTAL TEST FOR STABILITY OF THE PROBABILITY TRANSFORMATION FUNCTION IN RANK-DEPENDENT EXPECTED UTILITY THEORY

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Abstract

We conduct an experiment in an attempt to (i) measure the structure of preferences over lotteries and (ii) test for stability of the probability transformation functions over different choice sets. The design is based on manipulations of the “probability triangle” A disaggregated nonparametric analysis in which we classify subjects according to which transformation function is most consistent with their revealed choice behavior shows that a linear and a strictly concave transformation function are the most common for risky choice. We find essentially no evidence of an S-shaped transformation function for choice under risk. Formal econometric estimation clearly rejects the S-shaped function in favor a strictly concave function. A formal econometric analysis exploiting the ordered discrete nature of the data leads us to infer a strictly concave probability transformation function, consistent with the nonparametric analysis. The difference between our results and those of previous studies can be attributed to the choice of functional forms used in estimating the transformation function, to the limited space of lotteries upon which estimates have been based.

1 Introduction

Anticipated Utility theory (Quiggin (1982)) and the many “rank dependent” generalizations of expected utility that followed (Chew, Karni and Safra (1985), Segal (1989), Wakker (1990), to name just a few) provides a theoretically coherent and testable structure within which violations of expected utility theory

can be interpreted and specific remedies (e.g., the nature of nonlinear weighting of probabilities) can be tested. Empirical studies of the rank dependent model (e.g., Wu and Gonzalez, Prelec, Camerer and Ho) have generally concluded that the best-fitting specification for the probability weighting function is an inverted S-shaped function. We find, contrary to previous studies, little evidence for the S-shaped function. We believe that the fact that we have explored choice behavior over the entire probability triangle and that we have used a flexible functional form for the probability transformation function in our analysis should lend credence to our findings. A more detailed exploration of the differences in our experiment and previous studies is clearly warranted, but it appears that the main difference between our approach and others is that we have focused on estimating the transformation function for a specific 3-outcome triangle, while others have estimated the transformation function based on binary choices over a broader range of monetary outcomes. What we present is clear evidence for a strictly concave transformation function in the context of a given probability triangle. We also find that estimates of the transformation function are stable when the middle-sized outcome is varied.

Most studies that attempt to estimate the transformation function have assumed that all subjects have the same preferences, but that they make errors in decision making. One contribution of this paper is to provide a disaggregated analysis of choice under risk without this homogeneity assumption. We also estimate the transformation function directly in the context of an ordered discrete choice specification, with the homogeneity assumption. Unlike previous studies, however, we are able to estimate the transformation function without making any assumption about the functional form of the elementary utility, or value, function. For the transformation function, we use a flexible polynomial specification that allows the function to take essentially any shape, and we test for the correct order of the polynomial.

The organization of the paper is as follows. We first outline the probability triangle and examine the nature of rank dependent expected utility preferences in the triangle. Next, we outline the experimental design and procedures, and follow with a discussion of the main empirical results. Finally, we outline the econometric model that we use to estimate the transformation function and to test for stability of the function to shifts in the middle outcome in the triangle.

The probability triangle is based on the assumption that probabilities change in a constrained fashion within the triangle. For example, moving from the origin along a ray towards the hypotenuse, the probability of winning the middle prize falls, and the probability of winning the large and small prizes rise in fixed proportions. The utility of dollar outcomes does not change unless the dollar outcomes change. Many experimental studies using the MM triangle have shown that subjects may have variable levels of risk aversion as the probability of winning the middle prize changes, and that simple expected utility theory cannot explain this phenomenon.

2 The Probability Triangle and Rank-Dependent Expected Utility

Consider a N-outcome lottery L , denoted by

$$L = [x_1, p_1; x_2, p_2; \dots x_N, p_N],$$

with utility function $u(L)$. Since the probabilities add to 1, $p_i = 1 - \sum_{n \neq i} p_n$ for some i . For a three-outcome lottery we can illustrate and analyze lottery choice in a triangle. The expected utility of a lottery, L , is

$$EU(S) = u(x_1)p_1 + u(x_2)p_2 + u(x_3)p_3.$$

We suppose that x_1 is the largest prize, and x_2 and x_3 are the middle and smallest prizes. Taking the total derivative and setting equal to zero, we can derive the following useful expression:

$$\frac{dp_1}{dp_3} = \frac{u(x_2) - u(x_3)}{u(x_1) - u(x_2)}$$

This is the (constant) slope of a constant-expected utility in the probability triangle. Rank dependent expected utility augments the basic expected utility model with a transformation or weighting function for the probabilities. Letting

$$L = [x_1, p_1; x_2, p_2; \dots x_N, p_N]$$

denote a lottery, the rank-dependent expected utility of L is given by:

$$RDEU(L) = u(x_1)f(p_1) + u(x_2)(f(1 - p_2) - f(p_3)) + u(x_3)(1 - f(1 - p_3))$$

where f is the transformation function. The only conditions placed on f are that $f(0) = 0$, $f(1) = 1$, and f is monotonic. If f is smooth and differentiable, then the slope of the $RDEU$ function in the triangle is given, through total differentiation, by

$$\frac{dp_1}{dp_3} = \frac{u(x_2) - u(x_3)}{u(x_1) - u(x_2)} \frac{f'(1 - p_3)}{f'(p_1)}$$

This expression can be used to derive implications for the shape of constant rank-dependent expected utility contours in the probability triangle for different assumed forms of the transformation function. An illustration of several possible transformation functions and the implied preference maps are shown in Figure 1.

The purpose of the experiment reported on here is to determine (i) the form of the transformation function within a given triangle and (ii) whether this form is robust to changes in the triangle. The main experimental manipulation is to

vary the middle prize (from near the largest prize to near the smallest prize). Such a manipulation, leaving the transformation function unchanged, change the slope of constant preference contours in a predictable way, namely, they make the contours flatter in the relevant triangle, as illustrated in Figure 2. This implies that choices should move towards the hypotenuse of the triangle as the middle prize is shifted towards the smallest prize. This places restrictions on which types of observed choice patterns can be associated with which possible transformation functions. We now proceed to the details of this exercise.

3 Design of the Experiment

Figure 3 summarizes the experimental design. In the experiment there are twenty choice questions. A question consists, in essence, of asking the individual which point on a chord in a triangle he or she most prefers. Figure 3 illustrates the precise location of these chords in the choice triangle. The numbers above the hypotenuse correspond to the chords emanating from that part of the hypotenuse in descending order, according to the slope of the chord. Thus, for example, chord a1 is the dashed line with slope = $1/3$ intersecting the vertical axis at .67, chord b1 is the solid line with slope = 1 intersecting the vertical axis at .5, and chord c1 is the dotted line with slope=3 intersecting the origin. The same 9 chords appear in both Triangle I and Triangle II in the experiment.

The under the triangle indicates that the only difference between Triangle I and II is in the size of the middle prize. The labels on the vertical and horizontal axis indicate that the probability of a prize is measured on these axes. The main analysis consists of comparing choice patterns over each group of chords with the same slope in Triangle I to choice patterns over the corresponding group of chords from Triangle II. The key fact that we are exploiting in this is that changes in the slope of preference contours (equation 2.5) must be accounted for through changes in the middle prize while the transformation function remains unchanged. This allows us to make inferences about the shape of the transformation function.

3.1 Method

The experiments were conducted by computer. Subjects were recruited from economics classes at Rutgers University. Subject arrived at an appointed time and were seated at individual computer terminals. When all subjects were present, instructions were read aloud and questions answered. The instructions are included as an appendix to the paper. Subjects were then free to work through the questions at their own pace. There were twenty questions in the experiment. The first two questions in each experiment were warm-up questions which gave subjects choices among sets of choices where there was a strictly dominant choice.

Payments in the experiment were denominated in U.S. dollars. At the end

of the experiment, one of the lotteries chosen by each individual was randomly selected, played out, and the result paid to the subject. The average payoff in the experiments was approximately \$20. The average time spent was approximately 45 minutes, including the instruction period. There were a total of 27 subjects in the experiment.

The program is Windows-based and written in Visual Basic. For each question, the program would initially display a point of a chord in one of the triangles. The lottery was displayed in both tabular format and in a graphical format (pie chart). The subject could use a mouse to move to the right or left on a bar at the bottom of the screen to change the lottery displayed. Movements to the right and left corresponded to moving up and down a chord in a triangle. The subject could, at any time, select and confirm the currently displayed lottery. The initial point displayed was always the midpoint of the chord.

4 Empirical Results

Table 1 contains a summary statistic for the choices in the experiment, the mean normalized choice on each chord in each triangle. The normalized choice ranges from 0 to 1, with 0 indicating a choice on one of the axes of a triangle, 1 indicating a choice on the hypotenuse, and numbers strictly between 0 and 1 indicating choices strictly in the interior of a triangle. The lines in the table labeled “horizontal dominant choice.” and “vertical dominant choice.” refer to the first two choices in the experiment, each of which has a dominant choice. The horizontal dominance question corresponds to a horizontal chord intersecting the vertical axis of the choice triangle at .75, and calls for a choice of 0, since any other choice involves a direct trade of probability of the largest prize for probability of the smallest prize. The vertical dominance question corresponds to a vertical chord intersecting the horizontal axis of the choice triangle at .25, and calls for a choice of 1, since any other choice involves trading probability of the largest for probability of the middle prize, or payment in the earliest period for payment in the middle period. The other lines in the table refer to the chords described in the design section above, and illustrated in Figure 3.

Table 1: Means of Normalized Choices			
	Triangle	I	II
Chord			
Horizontal Dominant		.05	
Vertical Dominant		.86	
a1		.25	.23
b1		.25	.30
c1		.29	.51
a2		.16	.22
b2		.21	.43
c2		.24	.42
a3		.11	.25
b3		.13	.22
c3		.35	.50

The main thing to note in this table is that there is generally an increase in the normalized choice from Triangle I to Triangle II in each experiment, consistent with RDEU. That is, the average choices are consistent with the expression for the slope of constant preference contours (equation (2.5)) that is separable into a part due to the utility and a part due to the transformation function.

Table 2 contains a detailed classification of choice patterns. Specifically, the data is organized into triples of choices, with each triple containing choices along chords of the same slope within a given triangle. For example, column Ia refers to the “a” chord choices in Triangle I (those with a slope of $1/3$), Ib refers to the “b” chord choices in Triangle I (those with a slope of 1), and Ic refers to the “c” chord choices in Triangle I (those with a slope of 3). Each triple of choices is classified into one of 27 patterns according to whether the choices in the triple were at the axis end of a chord, in the interior of the chord, or at the hypotenuse of the chord. In the “description” column of the table there is a triple of numbers for each possible pattern, ($\#1, \#2, \#3$), where the entries correspond to choices as one sweeps from the northeast to the southwest of the triangle, and each entry is either a 1 (axis choice), a 2 (interior choice) or a 3 (hypotenuse choice). A choice is classified as at the end of a chord if it was within .01 of the length of the chord from that end. The first eight patterns (1-8) in the table all have at least 2 end choices at the same end of the chord and at most one interior choice. The next 12 patterns (9-20) all have at least one end choice at each end of a chord, and at most one interior choice. The last seven patterns (21-27) all have at least two interior choices. In the next part of the analysis we aggregate choice patterns into the just described three groups of patterns, and refer to them as “linear,” “inflected,” and “concave” patterns, with the name of each group indicating the shape of transformation function consistent with such a choice pattern. The main thing to note in Table 2 is the prominence of the strictly linear (patterns 1 and 5) and the strictly concave (pattern 27) choice patterns. Together these three pattern account for just under 50% of choices in the Experiment.

Table 2 Frequencies of Choice Patterns							
Pattern #	Description	Ia	IIa	Ib	IIb	Ic	IIc
1 (linear x)	1,1,1	21	6	7	1	2	1
2 (alx1)	1,1,2	1	5	1	3	1	0
3 (alx2)	1,2,1	1	1	2	1	0	0
4 (alx3)	2,1,1	1	0	0	0	1	0
5 (linear h)	3,3,3	0	0	0	0	1	1
6 (alh1)	3,3,2	1	0	1	0	1	0
7 (alh2)	3,2,3	0	0	0	0	1	1
8 (alh3)	2,3,3	0	0	0	0	0	1
9 (lnpv1)	1,1,3	0	0	0	0	1	1
10 (lnpv2)	1,3,1	0	0	1	0	0	0
11 (lnpv3)	3,1,1	0	0	0	0	0	1
12 (lnpv4)	3,3,1	0	0	0	0	0	0
13 (lnpv5)	3,1,3	0	0	0	0	0	0
14 (lnpv6)	1,3,3	0	0	0	0	0	1
15 (alnpv1)	1,3,2	0	1	0	2	0	0
16 (alnpv2)	3,1,2	1	0	1	1	0	0
17 (alnpv3)	1,2,3	0	0	0	0	0	0
18 (alnpv4)	3,2,1	1	0	0	0	0	0
19 (alnpv5)	2,1,3	0	0	0	0	0	0
20 (alnpv6)	2,3,1	0	0	0	0	0	1
21 (acon1)	2,2,3	0	0	0	0	1	3
22 (acon2)	2,3,2	0	0	0	2	0	0
23 (acon3)	3,2,2	1	1	2	1	1	3
24 (acon4)	2,2,1	2	1	2	3	2	0
25 (acon5)	2,1,2	1	0	1	0	4	0
26 (acon6)	1,2,2	2	2	3	5	4	0
27 (concave)	2,2,2	3	10	6	8	7	13
Total		27	27	27	27	27	27

Table 3 contains direct comparisons of choice patterns over chord triples of the same slope between the two triangles in the experiment. For example, the matrix at the top of Table 3 compares choice patterns over the chord choices with slope 1/3 in Triangle I of the Experiment with choice patters over the same chord choices in Triangle II. The Stuart-Maxwell Chi-Square test statistic reported below each matrix tests the hypothesis that the distribution of choice patterns is unchanging from Triangle I to Triangle II. We interpret these tests as evidence of stability (or lack thereof) of the transformation function. If choice patterns in Triangle I and Triangle II are consistent with the same general sort of transformation function (linear, inflected, or concave), then we call the transformation function stable. This hypothesis is never rejected statistically. More interesting, perhaps, than the stability of the transformation function is what the distribution of transformation functions implies. The preponderance of choice patterns are consistent with a linear or concave transformation function,

with very few inflected patterns.

Table 3: Nonparametric Test for Stability of the					
	Probability Weighting Function				
(Ia vs IIa)	After	Linear	Inflected	Concave	Total
Before					
Linear		11	1	4	16
Inflected		0	0	2	2
Concave		1	0	8	9
Total		12	1	14	27
Stuart-Maxwell Chi-square Statistic: 3.65 P-value: .16					
(Ib vs IIb)	After	Linear	Inflected	Concave	Total
Before					
Linear		4	3	4	11
Inflected		0	0	2	2
Concave		1	0	13	14
Total		5	3	19	27
Stuart-Maxwell Chi-square Statistic: 4.90 P-value: .09					
(Ic vs IIc)	After	Linear	Inflected	Concave	Total
Before					
Linear		2	1	4	7
Inflected		0	1	0	1
Concave		2	2	15	19
Total		4	4	19	27
Stuart-Maxwell Chi-square Statistic: 3.60 P-value: .17					

Conventional wisdom has it that the probability transformation function in RDEU is inflected, with a crossing point around $p=.35$, but there is simply no evidence in support of this in our experiment.

5 Estimating & Testing Transformation Functions

5.1 An Ordered Choice Model

Recall that along a given chord within a triangle, each individual is presented with the same finite number of ordered points, $\mathcal{C} \equiv \{c_1, \dots, c_R\}$, from which a selection is made. In so doing, note that individuals are not permitted to locate anywhere along the chord, but rather at one of the R given chord points. In this restricted problem, suppose that an individual selects c_j , and denote c_{j-1} and c_{j+1} as the adjacent chord points. Now, consider the unrestricted choice problem in which an individual can locate anywhere along the chord. With c^*

as the optimum chord location for the unconstrained problem, we then infer that:

$$c^* \in [c_{j-1}, c_{j+1}].$$

From the above discussion, we do not know the optimum (unrestricted) chord location but rather an interval that contains it. Consequently, for estimation purposes, we formulate an ordered choice model. To this end, for a "typical" individual, let

$$\alpha \left[\frac{f'(1-p_3)}{f'(p_1)} \right] \equiv \bar{s}(c)$$

be the slope of the RDEU function as given in (2.5), where:

$$\alpha \equiv \frac{u(x_2) - u(x_3)}{u(x_1) - u(x_2)}.$$

Clearly, all individuals are not identical, as evidenced by the fact that they make different choices. To allow for individual differences (without having observed demographic information), we assume that individual i has an RDEU function with slope at chord point c given by:

$$s(c) = \alpha \left[\frac{f'(1-p_3(c))}{f'(p_1(c))} \right] + \varepsilon \equiv \bar{s}(c) + \varepsilon,$$

where ε denotes an unobserved individual specific component. We assume that ε is identically distributed over individuals and over chords for a given individual. Further, along a given chord, we assume that the ε_i are independently distributed over individuals. For any specific individual, the choice made along one chord may or may not depend on that made along another. If each individual behaves as if ε is redrawn before making each decision, then independence is plausible; otherwise it is not. If we incorrectly assume that ε is independently distributed over individuals, then following an argument in Robinson [] for Tobit models, it can be shown that the resulting quasi-maximum likelihood estimator is still consistent. Accordingly, since we do not have a sample size sufficiently large to explore alternative correlation structures, here we proceed under the assumption that ε is i.i.d. distributed chords and individuals.

As we do not want to make strong assumptions about the transformation function, we model it flexibly by a series (polynomial) approximation. The degree of this approximation will be examined and subjected to tests below. Employing this series approximation, and a normality assumption for the unobserved factor, we then formulated an ordered model for which we obtain a maximum likelihood estimator.

We begin by approximating the transformation function with a polynomial of degree p :

$$f(x) = \sum_{i=0}^p \gamma_i X^i$$

We can now approximate the slope of an indifference curve for a "typical" individual:

$$\bar{s}(c) = \alpha \frac{\sum_{i=1}^p \left[i \gamma_i p_3(c)^{i-1} \right]}{\sum_{i=1}^p \left[i \gamma_i p_1(c)^{i-1} \right]} = \alpha \frac{\sum_{i=2}^p \left[1 + i \beta_i p_3(c)^{i-1} \right]}{\sum_{i=2}^p \left[1 + i \beta_i p_1(c)^{i-1} \right]},$$

where $\beta_i \equiv \gamma_i / \gamma$, $i = 2, \dots, p$, denote the identifiable parameters for the transformation function. Denote β as the $(p-1) \times 1$ vector with i th element β_i and denote σ^2 as the error variance. Then, with $\theta = [\beta', \alpha, \sigma^2]'$ as the parameter vector of interest, it remains to specify the likelihood for θ .

For individual i and chord d , let c_{d1}, \dots, c_{dK} denote the choice set ordered along a chord in a north-east direction. With $m(d)$ as the slope of cord d and with $c^*(d)$ as the unconstrained optimal choice:

$$\begin{aligned} c^*(d) &< c_1 \iff m < \bar{s}(c_1) + \varepsilon_i \\ c_{j-1} &\leq c^*(d) < c_j \iff \bar{s}(c_{j-1}) + \varepsilon_i \leq m(d) < \bar{s}(c_j) + \varepsilon_i, \quad j = 1, \dots, R \\ c^*(d) &\geq c_R \iff m(d) \geq \bar{s}(c_R) + \varepsilon_i. \end{aligned}$$

With ε_i distributed as $N(0, \sigma^2)$ and $\Phi(\bullet)$ as the distribution function for a $N(0, 1)$ random variable, probabilities for the above three events are given as:

$$\begin{aligned} P_{1d}(\theta) &= \Phi([\bar{s}(c_1) - m(d)] / \sigma) \\ P_{jd}(\theta) &= \Phi([\bar{s}(c_j) - m(d)] / \sigma) - \Phi([\bar{s}(c_j) - m] / \sigma), \quad j = 1, \dots, R-1 \\ P_{Rd}(\theta) &= 1 - \Phi([\bar{s}(c_R) - m(d)] / \sigma) \end{aligned}$$

Employing these probabilities, we can now write the log probability associated with individual i 's choice on chord $d = 1, \dots, D$. Define the indicator $I(E)$ as 1 if E occurs and 0 otherwise. Let:

$$\begin{aligned} Y_{1i}(d) &= I(c_i^*(d) < c_1) \\ Y_{ji}(d) &= I(c_j \leq c_i^*(d) < c_{j+1}), \quad j = 1, \dots, R-1 \\ Y_{Ri}(d) &= I(c_i^*(d) \geq c_R) \end{aligned}$$

Along chord c , the log likelihood contribution for individual i is then:

$$L_{id}(\theta) = \sum_{j=1}^R Y_{ji}(d) \ln(P_{jd}(\theta)).$$

The averaged Log-likelihood (over individuals and chords) can now be defined as:

$$\text{Log}L \equiv \sum_{d=1}^D L_{id}(\theta).$$

5.2 Results

Beginning with Triangle 1, transformation functions were estimated with approximating polynomials of degrees 3,4,5, and 6. Based on a likelihood ratio test, the P-value for comparing degrees 3:4 is .13, which is marginal evidence that a third degree polynomial is satisfactory. The Pvalue increases to a value in excess of .4 in comparing polynomial degrees 4:5 and 5:6. Not surprisingly, given these results, in Figure 1 the transformation function for polynomial degrees 4,5, and 6 are virtually identical. Based on these results, a fourth degree polynomial provides a satisfactory approximation to the transformation function. As is evidenced in Figure 4, while no restrictions were placed on the shape of this function, the estimate is strictly concave.

Turning to Triangle 2, we found very similar results to those obtained above. Namely, based on high P-values, a polynomial degree of four or higher provides a good approximation to the transformation function. As shown in Figure 5, transformation functions based in approximating polynomials of degrees 4-6 are virtually identical. Moreover, as in Triangle 1, the transformation function is also strictly concave for Triangle 2.

In theory, the transformation function for Triangle 1 should be the same as that for Triangle 2. To provide a test this proposition, we estimated a "combined" model in which the transformation function was restricted to be the same over both triangles. For the case of a fourth degree approximating polynomial, Figure 6 compares this combined transformation function with that for the separate triangles. As is evidenced in this figure, these transformation functions are close to eachother. As the P-value for this test is in excess of .9, we (strongly) are unable to reject the null hypothesis that the transformation function for Triangle 1 is the same as that for Triangle 2. Accordingly, all subsequent results are based on the pooled (over triangles) sample with the transformation function being approximated. by a fourth degree polynomial.

Table 4 provides the estimation results for the pooled sample.

Table 4: Parameter Estimates			
Parameter	Estimate	Standard Error	Uncombined
β_1	-1.49	(.17)	[-1.52, -1.44]
β_2	1.62	(.22)	[1.80, -1.47]
β_3	-.71	(.09)	[-.81,-.63]
σ_I^2	3.50	(.77)	[3.03]
σ_{II}^2	1.94	(.26)	[2.04]
α_I	7.16	(1.32)	[6.04]
α_{II}	4.26	(4.26)	[4.65]

Note that the parameter estimates reported in Table 4 must be normalized in order to get the coefficients for the transformation function, properly speaking. The estimates come from estimating the first-order condition for maximization, which is a ratio of derivatives of the transformation function.

6 Conclusions

In this paper we have presented a disaggregated analysis of experimental data which allows us to make inferences about the nature of preferences under risk in the relevant choice triangles. Contrary to many previous studies of the rank dependent model, we find little evidence of an S-shaped probability transformation function for choice under risk. For choice under risk we find that the majority of subjects choose in a manner consistent with either a linear or a strictly concave transformation function. The difference between our results and those of previous studies can be attributed to the restricted choice of functional forms used in those studies for estimating the transformation function, and to the different (more restricted) space of lotteries upon which those estimates have been based.

We augmented the nonparametric analysis with a formal econometric analysis which exploits the detailed information that our experimental procedure provides. Since subjects were able to make choices in discrete intervals along any given chord in the probability triangle, the choice data leads naturally to an ordered discrete choice formulation which provides more precise information than would be the case with binary choice data. The parameter estimates for our flexible polynomial parameterization of the probability transformation function imply a strictly concave function, consistent with our findings from the nonparametric analysis.

We believe that the rank dependent model is a natural extension of developments in choice since Bernoulli. Expected utility theory, for example, enriched the expected value model by capturing the essential notion that risk attitude is important in choice under risk. The rank dependent model similarly enriches expected utility theory by quantifying, through the probability transformation function, departures from linearity due to psychological factors that depend on the structure of risk prospects one faces.

As Quiggin (1993) has noted, the rank dependent model is based on the requirement that the entire probability distribution, and not the individual probabilities in isolation, must be transformed to fully represent an individual's decision making. This general approach to modelling decision making seems to us to be very useful in that it allows us to quantify departures from rationality, through the transformation function. A special case of these models is the linear transformation, which corresponds to expected utility. Our particular experimental implementation allows subjects to express their preferences more precisely, by giving them a large menu of possible choices, instead of the more usual pairwise choice format employed in other experimental studies. This, in turn, allows us to make more precise inferences about the nature of the transformation function without strong functional form assumptions.

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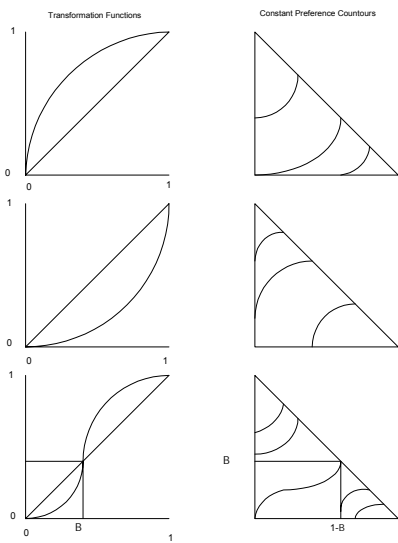


Figure 1: Transformation Functions and Associated Preference Maps

Figure 1:

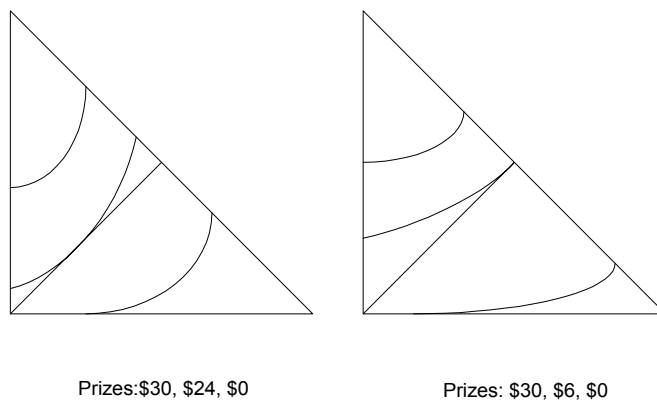


Figure 2: Flattening of Preference Contours Due to Reduction in Middle Prize

Figure 2:

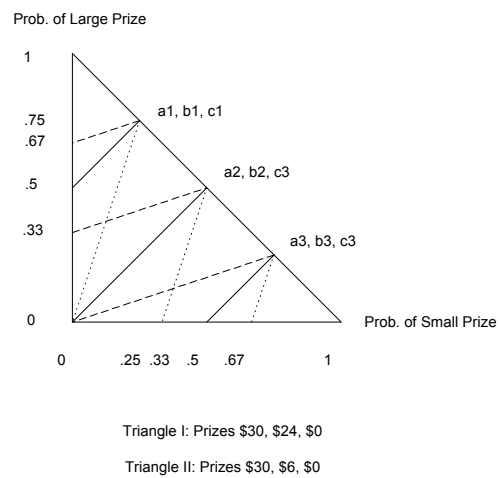


Figure 3: Experimental Design

Figure 3: