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Bayesian Nash Equilibria in a Public-Goods Problem with a Provision Point:
An Experimental Test of a Two-Person Game.

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Abstract

This paper extends both theory and experimentation in the context of two parties in a group who contribute to a public good with a provision point. This study analyzes the voluntary contributions game in which a public good is provided if and only if the sum of contributions meets a threshold. I present several Bayesian Nash equilibria in this game and examine their efficiency implications. In an experimental test of a public-goods problem with a threshold and rebate, the behavior of some subjects was consistent with a linear bidding strategy. Further, the behavior of some subjects was sensitive to changes in the prior distribution of valuations and the relative cost of the public good.

Key Words: public goods, voluntary contributions, provision points, experiments, Bayesian Nash equilibria.

JEL classification: H41, C92

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Bayesian Nash Equilibria in a Public-Goods Experiment with a Provision Point

Introduction

This paper reports the results of a public-goods experiment in which the public good was provided if and only if the contributions were sufficient to pay for it. If the sum of the subjects' contributions was less than the provision point, the contributions were refunded. The public-goods problem with a provision point arises in a variety of contexts. For example, Dawes, Orbell, Simmons, and van de Kragt (1986) reported that the Association of Oregon Faculty successfully funded the salary for a lobbyist by soliciting contributions from all faculty in the state. "The desired lobbyist--a public good for all faculty members because any pay increases he produced would go to all faculty in the system, not just to those who contributed--required an annual retainer of \$30,000 for his work.... The request contained an explicit promise that all money would be returned if less than the \$30,000 was raised." (Dawes et al., p. 1172) Bagnoli and McKee (1991) report similar successes in obtaining voluntary contributions to the provision of a public good. When soliciting pledges, broadcasters affiliated with National Public Radio sometimes state that public radio will be discontinued unless listeners' pledges meet a certain threshold.

The public-goods problem with a provision point can exist in a Coase theorem environment. The Coase theorem is the cornerstone of laissez-faire legal and economic policy governing contract and property law. The Coase theorem implies that distortions associated with externalities will be resolved by voluntary bargains among the parties without government

intervention. If there are multiple parties jointly purchasing a right from the source of an externality and the injured parties do not have an opportunity to coordinate their offers, they confront a public-goods problem with a threshold.

In this experiment, members of a group jointly attempted to buy property rights from a hypothetical firm by submitting offers. If the group bought the property rights from the firm, each member was compensated in the amount of his or her valuation minus his or her respective bid (with a formula for excess contributions). Payment to all members of the group regardless of contribution makes the property a nonexcludable public good. Each member of the group was assigned randomly a valuation which was private information. Incomplete information causes members to face a trade-off between the amount of a contribution and the probability that the sum of the bids exceeds the value of the property right to the firm. A member who thinks that other members of the group will contribute enough to meet the firm's demand has an incentive to "free ride," to submit a bid of zero. However, each member who submits a low bid increases the risk that the members will not raise enough money to meet the firm's demand. If the members do not raise enough money to meet the firm's demand, no public good in the form of joint ownership of property by the group would accrue to any member. In this experiment, the members have (common) prior knowledge of the distribution of the valuations, the cost of the public good, and the number of members in each group.

This paper extends both theory and experimentation in the context of two parties in a group who contribute to a public good with a provision point. I formalize a public-goods problem mathematically and develop three Bayesian Nash equilibria of this game: a linear equilibrium, a one-price equilibrium and an equilibrium with asymmetric strategies. Briefly, in

the one-price equilibrium, each player bids one half the level of the provision point if that amount exceeds the player's realized value for the public good and zero otherwise. Under the equilibrium strategy with asymmetric strategies, the individually rational bids of the players differ but together equal the provision point. I calculated the linear strategies for the formalized public-goods problem following a technique used by Chatterjee and Samuelson (1983) and Myerson and Satterthwaite (1983). Also, I derived interesting efficiency properties of the linear equilibrium. To understand the linear strategy, suppose player i whose realized value for the public good is V_i makes his or her bid based on the strategy $B_i(V_i) = d_i V_i + c_i$. I show that if players' valuations are distributed uniformly and one player plays a linear strategy, then the other player's best response is also linear. I calculate the linear strategies such that the players' linear strategies are best responses to each other. I show that the bid predicted by the linear equilibrium is directly proportional to the level of the provision point and inversely proportional to the upper limit of the distribution of values. To test the linear equilibrium, I conducted multiple sessions of the experiment in which I manipulated the upper limit of the distribution of members' valuations and the provision point, the relative value of the property right to the firm. Further, the design of this experiment included a combination of features not found in any previous experimental tests of the public-good problem with a provision point. In this experiment, private valuations and payoffs were heterogeneous. The amount of each subject's offer was not restricted to be either all or no part of the subject's private valuation. In fact, individual contributions were not restricted to integral units. The threshold was not an integral multiple of the number of players in each group. Also, aggregate contributions in excess of the provision point were returned to the players. Finally, the message space was restricted: The

parties negotiated anonymously by submitting bids electronically at a computer terminal.

In the next section, I review the results of pertinent public goods experiments. With mild assumptions regarding the negotiation process, experimental tests of the Coase theorem can be reduced to a public goods problem with a provision point and refunds. Therefore, I describe the Coase theorem and previous experiments designed to test the behavioral predictions of the Coase theorem. Next, I develop several Bayesian Nash equilibria in the voluntary contribution game in this experimental design and examine their efficiency implications. Then, I will describe the subjects and design of this experiment. Finally, I will report the experimental results. I tested whether the members' bids were individually rational, whether the final results were efficient *ex post*, and whether the bids conformed to predictions of Bayesian equilibria. The results provide insight into how contributions to a public good are influenced by changes in prior beliefs about the contributors' valuations for the public good and the relative cost of the public good. Briefly, the behavior of several subjects was sensitive to changes in the prior distribution of valuations and the relative cost of the public good. The behavior of some subjects is consistent with a linear bidding strategy. However, the predictions of the Bayesian Nash equilibrium with linear strategies hold for only one subject.

Literature review

Public-goods experiments

In a typical public goods experiment, participants allocate 'tokens' to a private good or a public good. Tokens invested in the public good yield a lower return to the individual, but provide an additional return to each participant in the group. The public-goods problem arises

when joint income is maximized if all players contribute to the public good, but each player has an individual incentive to contribute to the private good. The marginal rate of substitution of the private for the public good is sometimes called the marginal per capita return on the contribution to the group exchange. Contributions to the private exchange are individually optimal if the marginal per capita return of a contribution to the group exchange is less than the marginal return to the private exchange.

Several scholars have conducted experiments with public goods with a targeted minimum-aggregate-contributions condition (also referred to as a provision point or threshold) and a "give-back" option (also referred to as a refund, rebate or money-back guarantee). With a provision point, the public good is provided if and only if the sum of the contributions equals or exceeds a certain threshold. Under a "give-back" option, contributions are returned in the event the provision point was not met. "A provision point creates additional equilibria by breaking the continuity of rewards for unilateral reductions in contributions." (Davis and Holt, 1993, p. 338) When the provision point and number of members in a group is common knowledge, the provision point divided by the number of members can serve as a focal point.

Isaac, Schmitz and Walker (1989) varied the threshold to determine its effect. They found that increases in the threshold increase contributions. They also found the addition of a money-back guarantee improved the provision of the public good. Suleiman and Rapoport (1992) obtained similar results. In an experiment by Dawes, Orbell, Simmons, and van de Kragt (1986), subjects were allowed to contribute or keep \$5. The threshold was contributions by K of seven subjects. If at least K people out of seven subjects contributed, everyone received at least \$10: Contributors received \$10, and non-contributors received \$15. In a test of the standard

dilemma, they found that for $K=3$ of 7, 51% contribute; for $K=5$ of 7, 64% contribute. When they added a "no fear" condition under which all contributors receive their \$5 back if less than K contribute, there was no significant difference in contributions. With the "no fear" condition, they found that for $K=3$ of 7, 61% contribute; for $K=5$ of 7, 65% contribute. They noted that the contribution rates were somewhat higher in the experiment in which five contributions were required to activate the bonus than in the experiment in which three contributions were required. In their decision environment, the contribution choice was binary and aggregate contributions in excess of the provision point produced no additional payoff.

Suleiman and Rapoport (1992) conducted an experiment in which twelve groups of five subjects each participated in a game in which each member of a group received the same endowment and decided how much of it to contribute to the group benefit. Each member received the same reward (regardless of the size of his or her contribution) if, and only if, the sum of contributions was equal to or larger than a prespecified provision threshold. Their results showed that subjects contributed larger proportions of their (fixed) endowment as the provision threshold increased. Further, the level of contribution increases when contributions are not restricted to be all-or-none.

Davis and Holt (1993) note that instituting a give-back option adds stability to the provision-point equilibria by reducing the risk of contributing to the group exchange. "In game-theoretic terminology, the give-back option causes any contribution level below [a provision point equilibria] to be weakly dominated by [the provision point equilibria.]" (Davis and Holt, p. 341) A behavioral and theoretic complication can occur when the provision point does not require full contribution by all participants.

In many instances, each combination of contributions that satisfy the provision-point will be a Nash equilibrium. These multiple equilibria create formidable coordination problems, because players will generally have differing preferences for the equilibrium selected. For example, suppose the provision-point is twelve in [a two-person game], and that any combination of contributions that totals twelve units is a Nash equilibrium. Player X would prefer the equilibrium where Y contributed ten units to the public exchange and X contributed two units. Player Y would prefer just the reverse. (Davis and Holt, p. 342-43)

Isaac, Schmidt and Walker use the term "cheap-riding" to refer to the incentive to coordinate on a more favorable provision-point equilibrium. They found that cheap-rider incentives can frustrate cooperation, even in environments with a give-back option.

In summary, there is widespread agreement that when the sum of the endowments exceeds the provision point, multiple Nash equilibria can exist. Further, there is a consensus that, when the sum of the endowments exceeds the provision point, there exists equilibria which can Pareto dominate the equilibrium in which individuals contribute their entire endowment to the private exchange. Finally, the majority of experiments have found that contributions increase when the provision point increases.

The Coase theorem

Public-goods experiments with provision points and refunds are similar to experimental tests of the Coase theorem. In his paper "The Problem with Social Cost," Ronald Coase posited that a change in a liability has no efficiency implications where the costs of concluding a transaction are very low. The efficient outcome will occur regardless of the rule of law. In the context of property rights, one can consider any beginning assignment of private ownership. If (1) property rights are fully defined, (2) there are no transaction costs, and (3) each party acts in her own self-interest, then the most efficient users will buy (or lease) property from the original owners (when the original owners are not the most efficient users). The Coase theorem environment is reduced to a public-goods problem with a threshold and refund when there are multiple buyers seeking to purchase the property rights of a firm and the only significant transaction cost is the cost of disseminating information about members' offers.

The Coase theorem has undergone theoretical investigation under a variety of models. In one example, Myerson and Satterthwaite (1983) showed that under asymmetric information, a buyer and a seller may not reach agreement about price. Thus, they may refrain from entering into mutually beneficial trades.

Experimental tests of the Coase theorem

Numerous experiments have been conducted to test the behavioral relevance of the Coase theorem. Hoffman and Spitzer (1982) conducted experiments in which there were either two or three parties to a bargain. In some of the experiments, the subjects were provided with both their own and the other subject's payoff schedules. In other experiments, the subjects were shown only

their own payoff schedules. The results for all the two-person experiments and the three-person experiments with full information provided strong support for the Coase Theorem. The Pareto-optimal outcome was chosen in about 90 percent of these experimental treatments. However, their experimental results rejected the hypothesis that the agreements would be attained by a mutually advantageous bargain between the parties. In about 61 percent of all bargains over all their experiments, the parties essentially agreed to split the total payoff equally, even though this often represented a disadvantageous bargain for one of the parties relative to the payoff attainable without bargaining. The results of the three-person experiments with limited information gave less support for the Coase theorem.

Harrison and McKee (1985) developed an experimental design to test the Coase theorem in the context of two parties, full information concerning each other's payoffs and nonsequential bargaining. Using a slightly different set of preliminary questions to determine whether the subject understood the property owner's right to unilaterally choose any number, with or without the opponent's agreement, Harrison and McKee found that about 76 percent of the decisions involved individually rational outcomes. When they replicated the results of Hoffman and Spitzer, only 40% of the outcomes were individually rational for the controller. They concluded that the Hoffman-Spitzer results that are inconsistent with individual rationality were attributable to a lack of understanding by certain subjects of the meaning of unilateral property right. They also found that increasing the social surplus significantly reduced the number of divisions in which the controller received less than his or her individual maximum.

Experimental tests of the Coase theorem differ slightly from public goods experiments. In tests of the Coase theorem, participants bargain "face-to-face." The parties to a potential

bargain interact during a negotiation period. In public-goods experiments, subjects typically submit bids without knowledge of the bids of other members of the group. Also, individually irrational outcomes appear frequently in experimental tests of the Coase theorem. A possible explanation for this is the desire of participants in the Coase theorem environment to cooperate to reach an agreement, even though the resulting bargain is disadvantageous for one of the parties relative to the payoff attainable without bargaining.

Bayesian Nash equilibria

In this experiment, each subject observes only his or her own valuation for the public good. Therefore, each player has imperfect information about nature's choice for his or her opponent's valuation. The equilibrium notion appropriate for games of imperfect information is Bayesian equilibrium. The set of strategies $\{B_1^*(V_1), B_2^*(V_2)\}$ is a Bayesian equilibrium point if the expected utility for player i ($i = 1, 2$) is maximized at $B_i^*(V_i)$ when the other member's strategy is to bid $B_{-i}^*(V_{-i})$.¹ Then, $B_i^*(V_i)$ is a best response to $B_{-i}^*(V_{-i})$. I will describe below several Bayesian Nash equilibria in the voluntary contribution game. I will show that the Bayesian equilibria are sensitive to the relative value of the property rights to the firm, V_{firm} , and that the linear equilibrium described below also is sensitive to the distribution of the individuals' valuations.

1. Linear equilibrium

¹As usual, $B_{-i}^*(V_{-i})$ refers to the strategy for the player other than i .

Consider the case in which there are two players denoted $i = 1, 2$ and each player has private information about his or her valuation, V_i . Suppose that the players' valuations are independently and uniformly distributed on the closed interval $[L, H]$ where $0 < L < H$. Further, assume that the value of the firm is such that $H < V_{\text{firm}} < 2H$. The players simultaneously submit their (non-negative) bids. Then, player i 's action is to submit a bid, $B_i(V_i)$. The action space is $A_i = [0, \infty]$, and the type space is the closed interval $[L, H]$. Because the valuations are independent, player i believes that V_{-i} is uniformly distributed on $[L, H]$, regardless of the value of V_i .

Assuming there is no reimbursement for excess contributions, player i 's payoff function is

$$U_i = \begin{cases} V_i - B_i & \text{if } \sum_j B_j \geq V_{\text{firm}} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

Formally, a set of strategies $\{B_1^*(V_1), B_2^*(V_2)\}$ is a Bayesian Nash equilibrium if for each V_i in $[L, H]$, $B_i(V_i)$ solves

$$\max_{B_i} (V_i - B_i) \text{prob}\{B_i + B_{-i}^* \geq V_{\text{firm}}\} \quad (2)$$

Following Chatterjee and Samuelson (1983) and Myerson and Satterthwaite (1983),² I looked for linear bidding strategies: $B_i(V_i) = d_i V_i + c_i$ for $i = 1, 2$. This does not restrict the players' strategy spaces to include only linear strategies. The players may choose arbitrary strategies. If the players are risk-neutral, the following is a Bayesian Nash equilibrium of this game (see appendix I for details):

$$B_i^*(V_i) = \begin{cases} \frac{1}{2} V_i + \frac{(2V_{\text{firm}} - H)}{6} & \text{if } V_i \geq \frac{(2V_{\text{firm}} - H)}{3} \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

²Myerson and Satterthwaite (1983) demonstrated that linear bidding strategies maximize the expected sum of profits of the two players in a sealed-bid mechanism.

Bids are increasing in V_i and V_{firm} and decreasing in H . To test whether subjects react to changes in V_{firm} and H , I conducted multiple sessions. In each session, the computer assigned random valuations to each member of the group. In the second session, I decreased H , the upper limit of the distribution of valuations. In the third session, I reduced V_{firm} , the level of the provision point.

Individual rationality (or elimination of dominated strategies) requires that $B_i \leq V_i$ for $i = 1, 2$. Hence, player i should play the linear strategy described above only if $\frac{1}{2} * V_i + (2V_{\text{firm}} - H)/6 \leq V_i$ (i.e., when $(2V_{\text{firm}} - H)/3 \leq V_i$). If $V_i < (2V_{\text{firm}} - H)/3$, then any bid less than V_i yields an expected payoff of zero if the other player is playing the linear strategy.³ If player 2 makes a bid of zero, the threshold will not be met if $B_1 + 0 < V_{\text{firm}}$. Because it is irrational for player 1 to make a bid greater than V_1 and the highest possible valuation is H , the threshold will not be met by a rational bid if $H < V_{\text{firm}}$. Therefore, if player 2 makes a bid of zero, player 1's payoff cannot be increased by any individually rational B_1 .

This linear equilibrium can miss some mutually beneficial trades. A trade occurs if and only if $\sum_i B_i \geq V_{\text{firm}}$. Substituting from (3) and manipulating, a trade will occur if and only if and only if the following condition is met:

$$\sum V_i \geq 2(V_{\text{firm}} + H)/3 \quad (4)$$

A trade is efficient *ex post* if $\sum V_i \geq V_{\text{firm}}$. If both $\sum V_i \geq V_{\text{firm}}$ and (4) are met, a mutually

³Because $V_j \sim U[L, H]$, player j 's maximum bid under the linear strategy will be $\frac{1}{2} * H + (2V_{\text{firm}} - H)/6$. If player j plays this maximum bid, the minimum bid, $B_{i(\text{min})}$, necessary to meet the threshold, V_{firm} , is defined implicitly as

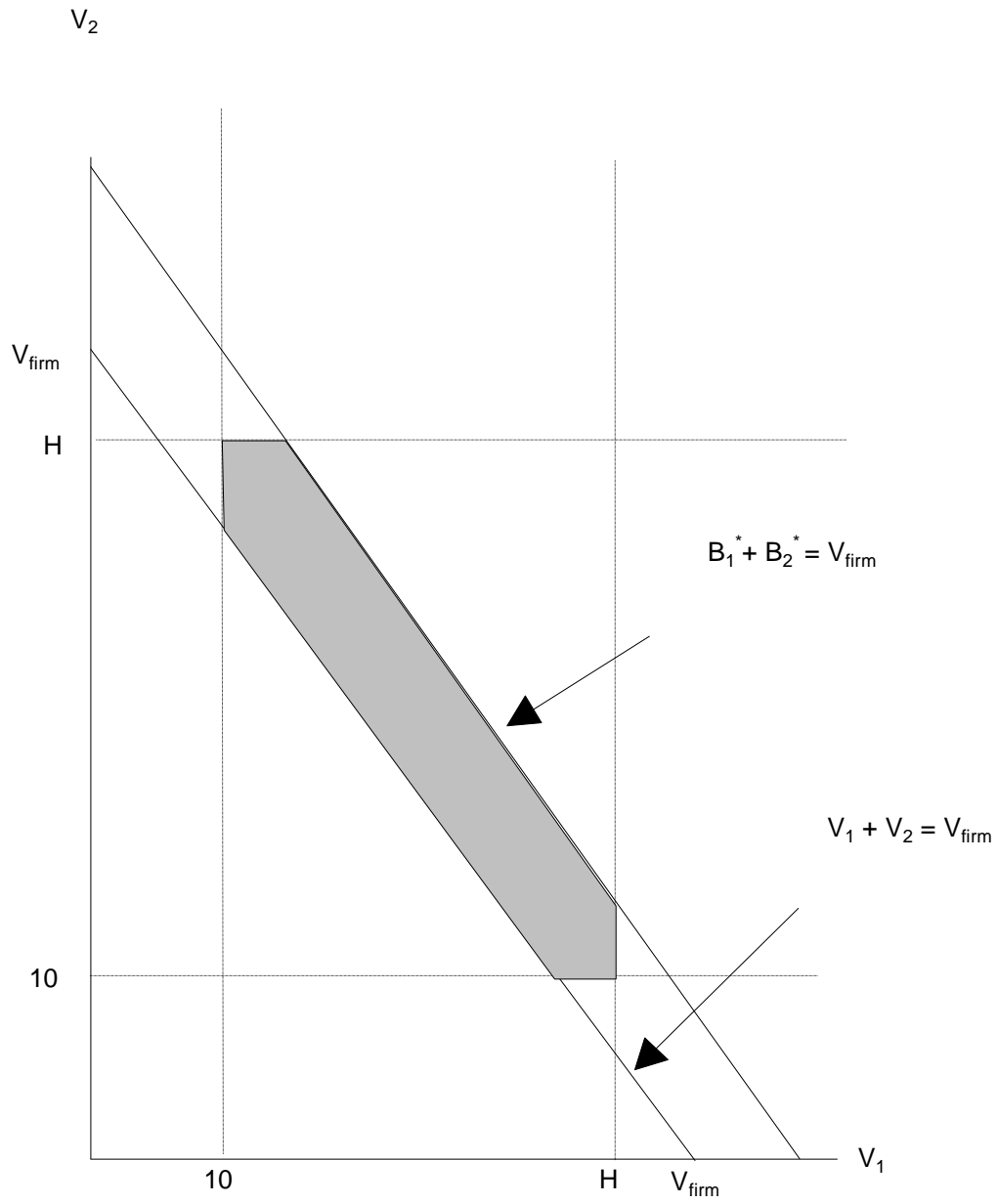
$$B_{i(\text{min})} + \frac{1}{2} * H + (2V_{\text{firm}} - H)/6 = V_{\text{firm}}$$

Manipulation yields this minimum bid to be $B_{i(\text{min})} = (2V_{\text{firm}} - H)/3$.

beneficial trade will occur. However, if the realized valuations are such that $V_{\text{firm}} \leq \Sigma V_i < 2(V_{\text{firm}} + H)/3$, then no trade will occur and the outcome will be inefficient. Figure 1 depicts the valuation pairs which induce inefficient outcomes when both players use the linear equilibrium. The linear equilibrium achieves all trades in which both players have high realized valuations. However, the linear equilibrium misses all trades in the shaded region of Figure 1.⁴

⁴If the subjects played the linear equilibrium, the *ex ante* predicted levels of efficiency whenever the sum of realized valuations exceeds V_{firm} would be 48%, 44% and 50% for sessions 1, 2, and 3 of the experiment, respectively.

Figure 1: Valuations which induce inefficient outcomes under linear



2. One-price equilibrium

Consider the case in which there are two players, the players' valuations are distributed $V_i \sim U[L, H]$, and the firm value is such that $H < V_{\text{firm}} < 2H$. There is a symmetric Bayesian Nash equilibria in which players bid $B_i(V_i) = V_{\text{firm}}/2$ if $V_{\text{firm}}/2 \leq V_i$ and 0 otherwise. Given player j is playing this strategy, player i 's choice amounts to submitting a bid in the range $V_{\text{firm}}/2 \leq B_i \leq V_i$ (in which case the threshold would be met if and only if player j bids $V_{\text{firm}}/2$) or not meeting the threshold. When $V_{\text{firm}}/2 \leq V_i$, player i prefers bidding $V_{\text{firm}}/2$ to obtaining a payoff of 0. Bids higher than $V_{\text{firm}}/2$ decrease player i 's return. If player j is bidding 0, no individually rational bid by player i can increase player i 's profit above 0. A bid in the amount of $V_{\text{firm}}/2$ may be a focal point.

This Bayesian Nash equilibrium will result in a transfer of the property rights to the residents if and only if $V_{\text{firm}}/2 \leq V_i$ for $i = 1, 2$. If $V_{\text{firm}}/2 \leq V_i$ for $i = 1, 2$, then $V_{\text{firm}} \leq \sum_i V_i$, the property rights will be transferred to the residents, and the outcome will be efficient.⁵ However, the valuations *ex post* may be such that $V_{\text{firm}} \leq \sum_i V_i$ even though one V_i is less than $V_{\text{firm}}/2$ (e.g., $V_1 \gg V_{\text{firm}}/2$ and $V_2 = V_{\text{firm}}/2 - \varepsilon$ where ε is small).⁶ In this instance, efficiency may require a transfer of the property rights to the residents, but such a transfer will not occur under this Bayesian Nash equilibrium. This Bayesian Nash equilibrium misses some Pareto-improving trades. Figure 2 depicts the valuation pairs which induce inefficient outcomes when both players use the one-price equilibrium. The one-price equilibrium misses all trades in the shaded region

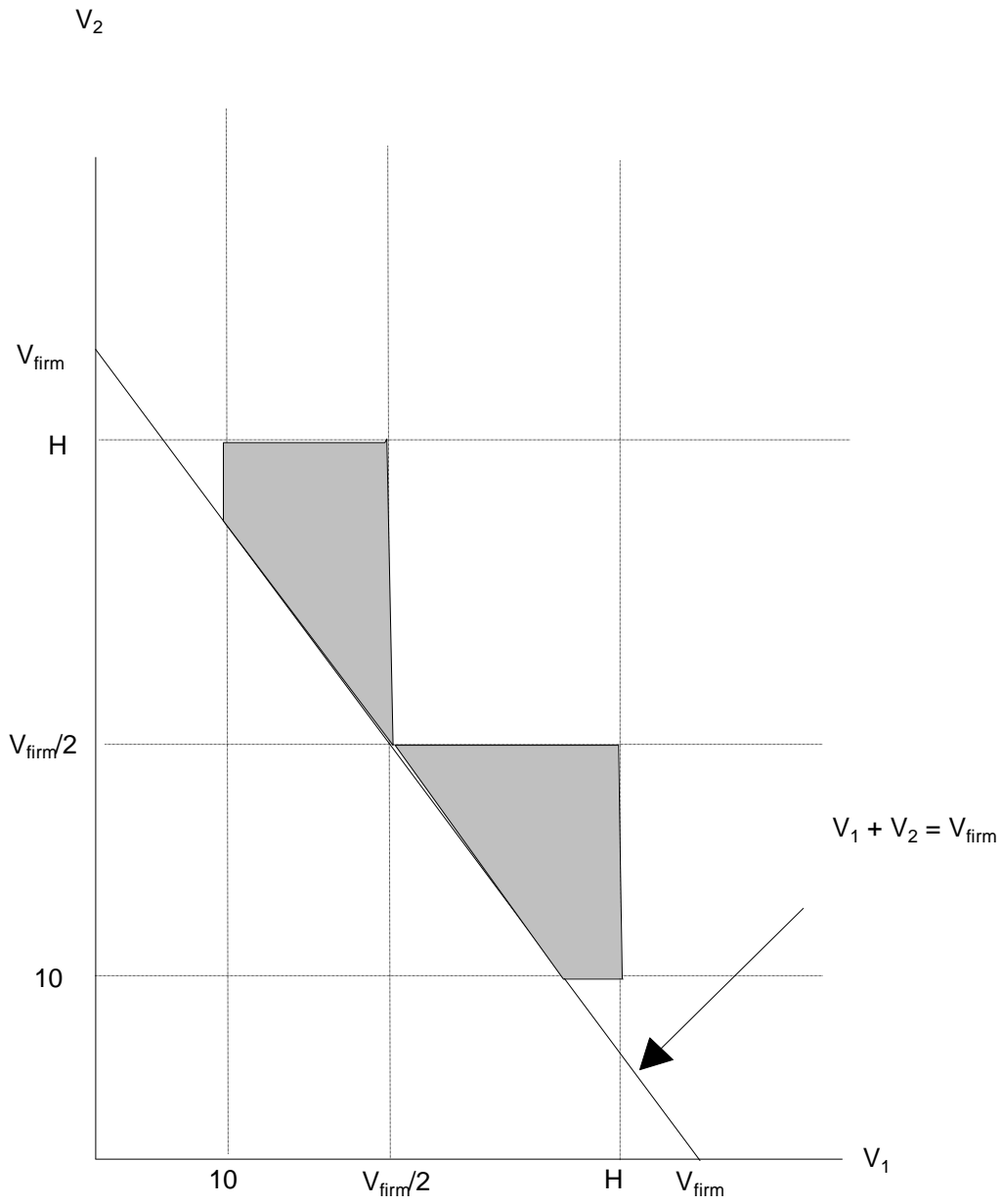
⁵ $V_{\text{firm}}/2 \leq V_i$ for $i = 1, 2$ implies $V_{\text{firm}} \leq \sum_i V_i$.

⁶Technically, $\text{prob}\{V_{\text{firm}}/2 \leq V_i \text{ for } i = 1, 2 \mid V_{\text{firm}} \leq \sum_i V_i\} = \text{prob}\{V_{\text{firm}}/2 \leq V_i \text{ for } i = 1, 2 \text{ and } V_{\text{firm}} \leq \sum_i V_i\} / \text{prob}\{V_{\text{firm}} \leq \sum_i V_i\} = \text{prob}\{V_{\text{firm}}/2 \leq V_i \text{ for } i = 1, 2\} / \text{prob}\{V_{\text{firm}} \leq \sum_i V_i\}$.

of Figure 2.⁷

⁷If the subjects played the one-price equilibrium, the *ex ante* predicted levels of efficiency whenever the sum of realized valuations exceeds V_{firm} would be 52%, 50% and 56% for sessions 1, 2, and 3 of the experiment, respectively.

Figure 2: Valuations which induce inefficient outcomes under one-price



3. Equilibrium with asymmetric strategies

Again, suppose valuations are uniformly distributed on $[L, H]$ and that the firm value is in the range $H < V_{\text{firm}} < 2H$. Then, there are many Bayesian Nash equilibria of the following form:

$$B_i^*(V_i) = V_{\text{firm}} - K \quad \text{if } V_{\text{firm}} - K \leq V_i$$

$$B_i^*(V_i) = 0 \quad \text{otherwise}$$

$$B_j^*(V_j) = K \quad \text{if } K \leq V_j$$

$$B_j^*(V_j) = 0 \quad \text{otherwise}$$

where K satisfies the following conditions: $V_{\text{firm}} - K \leq H$ and $K \leq H$. Given player j is playing this strategy, player i 's choice amounts to submitting a bid in the range $V_{\text{firm}} - K \leq B_i \leq V_i$ (in which case the threshold would be met if and only if player j bids K) or not meeting the threshold. When $V_{\text{firm}} - K \leq V_i$, player i prefers bidding $V_{\text{firm}} - K$ to obtaining a payoff of 0. Bids higher than $V_{\text{firm}} - K$ decrease player i 's return. A similar argument can be made to show that player j has no incentive to deviate from the strategy described above. This Bayesian Nash equilibrium misses Pareto-improving trades in a manner similar to the one-price equilibrium above.

Experimental design

The subjects of the experiment were 14 undergraduate students at Rutgers University recruited from economics courses in the spring of 1996. The experiment was conducted on a set of networked personal computers. The subjects were seated at computer terminals. Subjects participated in a sequence of 75 sessions, one immediately following the other. The experiment lasted 75 minutes, including instructions. The average total earnings in the experiment was

approximately \$21, with earnings ranging from \$27 to \$13. An "experimental currency" used in the experiment was converted to dollars at an exchange rate announced at the beginning of the experiment.

The monitor provided the subjects with instructions. The instructions are included below as appendix II. The monitor read the instructions aloud while the subjects followed on their own instruction sheets. The instructions told the subjects that a computer would randomly assign each subject to a group consisting of two participants. The subjects were told that the composition of each group would change every decision period. Therefore, the chance that any pair of subjects would be in the same group in consecutive periods was very small. The subjects were not told the identities of current, future, or past members of their group. Further, the subjects were not allowed to communicate with other participants during the experiment. Therefore, the opportunities for explicit coordination were absent and each decision period approximated a "one-shot" play of the game.

Each subject was endowed with an initial amount of cash. The computer screen displayed the individual's cash holdings, the initial owner of the property rights, and the earnings or losses from previous decision periods. In each decision period, the members of each group submitted bids representing contributions to the group's purchase of the property. In all decision periods, the subjects negotiated by entering an offer at a computer terminal. The amount of each subject's offer was not revealed to the other participants. The subjects were told that offers which are acceptable to both parties in their group would be enforced by appropriate increases or decreases in earnings. Subjects were told that they would be paid in cash at the end of the experiment. At the end of the experiment, each subject received earnings equal to a multiple of

the sum of his or her earnings over all the decision periods and his or her initial endowment.

The computer assigned to each participant a valuation drawn randomly from a distribution (with replacement) at the beginning of each decision period. In each subsequent decision period, subjects' valuations were again assigned randomly by the same procedure, independently from past draws. The prior distribution of the variables was known to all parties. In each session, the value of the property rights to the firm was public information. After each decision period, each subject received a message stating the sum of the offers of members of his or her group, whether a transaction occurred, and the subject's earnings for the just-completed period.

In all sessions, the property initially belonged to a hypothetical firm. The party to whom the property rights were initially assigned has been called the "controller" in experimental tests of the Coase theorem. Each subject could click on "PRIVATE VALUE" on his or her computer screen to obtain his or her valuation. The subject's computer screen briefly displayed the subject's valuation but not the valuation of other group members. The subjects were not informed of the valuations assigned to other members of the group. Thus, prior to play of the decision period, individual valuations were private information.

In all sessions, the property belonged to the firm unless the firm sold the property to the group. In each decision period, each member of the group submitted an offer to pay the firm. If the sum of the offers of the group members was equal to or greater than the value of the property to the firm, each member's earnings for the decision period would be his or her valuation minus the amount of his or her bid (with a formula for distribution of excess contributions, if any). If the sum of the members' bids was less than the firm's value, the members' proffered bids would

be refunded and the earnings to each member for the decision period would be zero.

A "pro-rata reimbursement" rule ensured that the players would be reimbursed for any contribution they made in excess of the firm's value. If V_i is player i 's realized valuation and B_i is player i 's bid, then player i 's payoff is:

$$U_i = \begin{cases} V_i - B_i + (\sum_j B_j - V_{\text{firm}}) * B_i / (\sum_j B_j) & \text{if } \sum_j B_j \geq V_{\text{firm}} \\ 0 & \text{otherwise} \end{cases}$$

If the sum of the players' contributions exceed the firm's value, the players received a pro-rata portion of the excess contribution. The presence of the pro-rata reimbursement rule does not provide incentive for risk-neutral participants to increase their bids. The pro-rata reimbursement rule has the property that increasing one's contribution by \$1 increases one's payoff by less than \$1 provided one's opponent is making a positive bid. Therefore, players prefer a contribution making the sum of the contributions exactly equal to V_{firm} to any larger contribution.⁸ The firm

⁸As noted by Bagnoli and Lipman (1989), in a one-shot game, any vector of strategies $\{B_1^*, B_2^*\}$ such that $\sum_i B_i^* = V_{\text{firm}}$ and $V_i - B_i^* \geq 0$ for $i = 1, 2$ is a Nash equilibrium point. In the event that the sum of the players' contributions equals or exceed the firm's value, the payoff to individual i can be expressed as $U_i = (V_i * \sum_j B_j - V_{\text{firm}} * B_i) / (\sum_j B_j)$ using the pro-rata reimbursement rule described above. Because the first derivative of this function with respect to B_i , $(-1)V_{\text{firm}} * B_j / (B_i + B_j)^2$, is negative; an individual's utility decreases as his or her bid increases provided $\sum_j B_j \geq V_{\text{firm}}$. Therefore, if the sum of the players' contributions equals or exceeds the firm's value, no player has an incentive to increase his or her bid. If $\sum_j B_j < V_{\text{firm}}$, player i will receive earnings of zero. If $\sum_j B_j = V_{\text{firm}}$, player i will receive $(V_i - B_i)$, and a small reduction in any person's bid will cause all players to have earnings of zero. Therefore, whenever $\sum_j B_j = V_{\text{firm}}$ and $V_i - B_i > 0$, individual i has no incentive to decrease his or her bid. Further, all players' bids are pivotal because $\sum_j B_j = V_{\text{firm}}$. If any player reduces his or her contribution, then $\sum_j B_j^* < V_{\text{firm}}$ and all players will receive \$0. Finally, the vector of strategies $\{B_1^*, B_2^*\}$ having the property that $\sum_j B_j^* < V_{\text{firm}}$ and the sum of the bids would still be less than V_{firm} if any individual i chose to contribute V_i is a Nash equilibrium. No player can benefit by contributing less. Player i cannot increase his or her utility unless he or she can make a bid $B_i \leq V_i$ that can cause $\sum_j B_j \geq V_{\text{firm}}$. However, in such instance, it is not sensible for any player to bid $B_i = 0$ where $B_i \leq V_i$ because there is no penalty for increasing one's bid to an intermediate value between 0 and V_i as long as $\sum_j B_j^* < V_{\text{firm}}$.

value, V_{firm} , is the provision point of the experiment.

Experimental design for session one

In session one consisting of 25 decision periods, the computer assigned to each participant a valuation drawn randomly and uniformly (with replacement) from the set of consecutive positive integers $\{10, 11, 12, \dots, 40\}$ at the beginning of each decision period. Thus, an approximation of uniform prior beliefs was induced upon the subjects. The value of the property rights to the firm was $V_{\text{firm}} = \$45$.

Experimental design for session two

In session two, the computer assigned to each participant a valuation drawn randomly and uniformly (with replacement) from the set of consecutive positive integers $\{10, 11, 12, \dots, 35\}$ at the beginning of each decision period. Thus, the upper limit of the distribution of valuations was reduced. The purpose of this manipulation was to test whether bids are inversely proportional to the upper limit of the distribution of valuations as predicted by the linear equilibrium. As before, the value of the property rights to the firm was $V_{\text{firm}} = \$45$.

Experimental design for session three

In session three, the value of the property rights to the firm was reduced to $V_{\text{firm}} = \$41$. The purpose of this manipulation was to test whether bids are directly proportional to the level of the provision point as predicted by the linear equilibrium. As in session one, the computer assigned to each participant a valuation drawn randomly and uniformly (with replacement) from the set of consecutive positive integers $\{10, 11, 12, \dots, 40\}$ at the beginning of each decision period.

Experimental results

Individual rationality

If the property rights are traded from the firm to the group, the maximum benefit member i can obtain is V_i , and V_i can be obtained only if $B_i = 0$. Therefore, individual rationality requires that each subject contribute no more than his or her valuation for the property.⁹ There were 183 individually irrational bids out of the 1050 total bids (17.4%). The following table shows the number of individually irrational bids made by each participant:

Table 1: Irrational bids per participant.

Participant	Number of bids in which $B_i > V_i$
1	0
2	8
3	0
4	3
5	23
6	37
7	53
8	6
9	7
10	15
11	2
12	29
13	0
14	0

In fact, four participants (identified in the experiment as participants 5, 6, 7, and 12) were responsible for 142 of the 183 irrational bids (78%). A possible explanation for the irrational bids is a desire among members to "split the difference" to meet the threshold. This may explain

⁹However, it is possible to obtain positive net earnings by the pro-rata distribution rule when $B_i > V_i$ provided V_{firm} is small relative to $\sum_j B_j$.

participant 7's bids. Participant 7 bid rationally whenever the realized value was 33 or higher and bid irrationally whenever the realized value was 23 or below. The number of individually irrational bids in this experiment also might reflect lack of understanding by certain subjects of the payoff structure. Some of the irrational bids can be attributed to learning behavior. The irrational bids of participants 2, 8, 9 and 11 occurred only in the first session. However, the following table shows the number of irrational bids in each session:

Table 2: Irrational bids per session.

Session	Number of bids in which $B_i > V_i$
1	28
2	111
3	44

In about 28% of the situations over all the sessions in which a trade occurred and firm ownership was inefficient; that is, the valuations of the members were such that a payment scheme that compensates the firm and leaves every resident satisfied could have been devised; the payoff represented a loss for at least one of the parties. In six situations in which firm ownership was inefficient, the property rights were traded to a group whose members both made individually irrational bids. An example of this occurred in period 3 when one member with realized valuation of \$31 offered \$33 and the other member with valuation of \$23 bid \$24.

These results exhibit a higher degree of individual rationality than the results obtained in experimental tests of the Coase theorem. In the experiment conducted by Hoffman and Spitzer (1982), in about 46% (31 out of 67) of those situations in which the controller agreed to an allocation within \$1.00 of an equal split, the controller obtained less than his or her individual

maxima. In terms of individual rationality, the results of Harrison and McKee (1985) were not significantly different from the results of Hoffman and Spitzer. The higher degree of individual rationality in my results may be due to the anonymous negotiation process in this experiment. Sharing buys little "good will" when the subjects do not know the identity of the other member in the group, the subjects submit bids electronically at a computer terminal and the composition of each group changes after every decision period.

Ex post efficiency

Efficiency *ex post* requires that the property rights be traded from the firm to the group whenever the sum of realized valuations exceeds V_{firm} . For example, suppose a group consists of two participants, the valuation assigned to individual i is V_i and the value of the property to the firm is V_{firm} . If the sum of the valuations of the members of a group *ex post* exceed the property's value to the firm (i.e., $\sum_{i=1}^2 V_i > V_{\text{firm}}$), the final outcome is efficient if the group purchases the property. An inefficient outcome occurs when the members do not buy the property and the valuations of the members *ex post* were such that a payment scheme that compensates the firm and leaves every member satisfied could have been devised. If the sum of the valuations of members of a group *ex post* is less than the property's value to the firm, the joint profits are maximized when the firm retains ownership of the property. In this circumstance, the failure of the parties to reach an agreement does not violate *ex post* efficiency. If the sum of the realized valuations equals the property's value to the firm, both outcomes are Pareto optimal.

In each session, seven groups of players made 25 joint decisions. Therefore, subjects made a total of 175 joint decisions each session. The absolute numbers of efficient results in the

three sessions are indicated in the following table:

Table 3: Efficient outcomes in each session.

Session	Property rights traded to group and $\sum_i V_i \geq V_{\text{firm}}$ (efficient outcomes)	Property rights not traded to group and $\sum_i V_i \geq V_{\text{firm}}$ (inefficient outcomes)	Percentage of times rights traded to group and $\sum_i V_i \geq V_{\text{firm}}$
1	77	43	63%
2	60	31	66%
3	97	37	72%
Total	234	111	67%

The property rights were traded to the group in about 67% of those instances in which the sum of members' realized valuations was at least as large as V_{firm} . If the subjects played the linear equilibrium, the predicted levels of *ex post* efficiency whenever the sum of realized valuations exceeds V_{firm} would be 47%, 46% and 50% for sessions 1, 2, and 3, respectively. Although the number of joint profit-maximizing decisions exceeded the inefficient outcomes, many efficient outcomes were obtained even though members offered less than their private valuation. In 14 situations in which firm ownership was efficient; that is, the valuations of the members were such that a payment scheme that compensates the firm and leaves every member satisfied could not have been devised; the property rights nevertheless were traded to the group. In each case, at least one member made an individually irrational bid. An example of this occurred in period 3 when one member with realized valuation of \$18 offered \$29 and the other member with valuation \$11 bid \$25.

Bidding strategies

Only a few subjects appeared to adopt the one-price equilibrium strategy of bidding a constant amount regardless of realized valuation. Participant 3 bid 25 on 25 occasions. Participant 5 bid 25 on 13 occasions. The behavior of several participants appears consistent with a linear bidding strategy in which bids increased with valuation. Scatter plots of the observed relation between bids and values for each subject in each session are available from the author. Bids above the 45° line are individually irrational. As one can see from the plots of bids against valuation, a linear pattern is evident for many subjects. However, players appeared to underbid more when they received realized valuations above 25, precisely those realizations for which trade is most likely. Under the linear equilibrium described above, participants are expected to bid 0 if the realized valuation is less than $(2V_{\text{firm}} - H)/3$ and otherwise bid the linear strategy $B_i = \alpha + \beta_1 V_i$ where α and β_1 are coefficients. There were no bids of zero during the experiment, and only five bids were below 5. When valuations were less than $(2V_{\text{firm}} - H)/3$, the average bid was 13.8 and the standard deviation was 5.1. Therefore, one can reject the hypothesis that players bid 0 when their realized valuations were less than this critical value. The coefficients predicted by the linear equilibrium are $\alpha = (2V_{\text{firm}} - H)/6$ and $\beta_1 = 1/2$. The predicted value for α for each session is indicated in the following table:

Table 4: Predicted values for intercept of linear bidding function.

Session	α
1	8.33
2	9.17
3	7

Separate dummy variables were created for changes in the prior distribution and changes

in firm value. The dummy variable High takes on the value of 0 if the upper limit of the distribution of valuations was 40 and 1 if the upper limit was 35. The dummy variable Threshold takes on the value of 1 if $V_{\text{firm}} = 41$ and 0 if $V_{\text{firm}} = 45$. Pooling the bids of each session, I estimated the following regression for each participant:

$$B_i = \alpha + \beta_1 V_i + \beta_2(\text{High}) + \beta_3(\text{Threshold}) + \mu_i$$

where μ_i is the disturbance term endowed with the usual properties. A t-test of the null hypothesis that $\beta_2 = 0$ is a test of the hypothesis that there is no difference associated with changes in the upper limit of the distribution. The OLS estimate of the coefficient associated with High represents the difference in bid associated with a reduction in the upper limit of the distribution from 40 to 35.¹⁰ Likewise, a t-test of the null hypothesis that $\beta_3 = 0$ is a test of the hypothesis that there is no difference associated with changes in the threshold. The OLS estimate of the coefficient associated with Threshold represents the difference in bid associated with a reduction in the threshold from 45 to 41.¹¹ The results of the regressions are contained in Table 5. The first column in Table 5 is the estimation results when all subjects are pooled.

¹⁰Note that $E[B]$ equals $\alpha + \beta_1 E[V_i] + \beta_3 E[\text{Threshold}]$ if the upper limit is 40 and $\alpha + \beta_1 E[V_i] + \beta_2 + \beta_3 E[\text{Threshold}]$ if the upper limit is 35.

¹¹Note that $E[B]$ equals $\alpha + \beta_1 E[V_i] + \beta_2 E[\text{High}]$ if $V_{\text{firm}} = 45$ and $\alpha + \beta_1 E[V_i] + \beta_2 E[\text{High}] + \beta_3$ if $V_{\text{firm}} = 41$.

Table 5: OLS estimates of each subject's bids over all sessions.

Explanatory variables	Player:							
	all	1	2	3	4	5	6	7
intercept	6.96 (10.4*) _{$\alpha=0$} (2.1*) _{$\alpha=8.33$}	0.71 (0.43) (4.7*)	3.11 (1.53) (2.6*)	5.88 (3.2*) (3.0*)	8.97 (6.3*) (0.45)	22.8 (10.*) (6.4*)	6.90 (3.0*) (0.62)	11.7 (7.4*) (2.1*)
valuation	0.61 (27.9*) _{$\beta=0$} (4.90*) _{$\beta=1=\frac{1}{2}$}	0.70 (13*) (3.8*)	0.76 (12*) (4.0*)	0.55 (10*) (0.93)	0.45 (8.7*) (0.91)	-0.02 (-0.3) (7.0*)	0.78 (11*) (3.8*)	0.57 (10.3*) (1.23)
High 0 if H=40 1 if H=35	0.36 (0.95) _{$\beta=0$}	2.38 (2.6*)	2.50 (2.0)	1.85 (1.91)	0.85 (1.00)	-10.8 (-7.4*)	-0.36 (-0.3)	0.62 (0.77)
Threshold 0 if $V_{\text{firm}} = 45$ 1 if $V_{\text{firm}} = 41$	-0.43 (-1.23) _{$\beta=0$}	1.74 (1.99)	2.74 (2.3*)	0.11 (0.13)	-0.37 (-0.5)	-4.74 (-3.9*)	-3.5 (-3.0*)	1.57 (2.01)
adj. R ²	0.49	0.75	0.73	0.70	0.60	0.51	0.72	0.65
F	267	59.7	49.5	39.8	29.9	18.5	49.6	36.1
sample size	805	60	54	52	58	52	58	57

* indicates significance at the 0.05 level.

Table 5: OLS estimates of each subjects' bids over all sessions (continued).

Explanatory variables	Player:	8	9	10	11	12	13	14
intercept		9.80 (6.2*) (0.9)	8.92 (8.2*) (0.5)	2.76 (1.4) (2.9*)	2.91 (2.2*) (4.1*)	2.11 (1.4) (4.1*)	2.09 (1.75) (5.2*)	6.40 (5.48*) (1.65)
valuation		0.53 (9.9*) (0.6)	0.65 (18*) (4.1*)	0.68 (11*) (3.0*)	0.75 (17*) (5.9*)	0.84 (17*) (6.8*)	0.67 (17*) (4.4*)	0.60 (18.2*) (3.08*)
High		-0.22 (-0.2)	-3.05 (-5.7*)	4.38 (4.1*)	0.63 (0.9)	0.67 (0.77)	2.62 (3.5*)	0.12 (0.23)
	0 if H=40 1 if H=35							
Threshold		-1.93 (-2.5*)	-3.79 (-6.9*)	4.74 (4.6*)	-1.4 (-2.1*)	0.67 (0.8)	1.66 (2.4*)	-0.30 (-0.62)
	0 if $V_{\text{firm}} = 45$ 1 if $V_{\text{firm}} = 41$							
adj. R ²		0.65	0.89	0.68	0.83	0.84	0.85	0.89
F		33.8	169	45	108	97	107	158
sample size		54	58	64	67	55	57	59

* indicates significance at the 0.05 level.

The first number in parenthesis is the t-statistic for null hypothesis that the coefficients are 0 and the second number in parenthesis is the t-statistic for the null hypothesis that $\alpha = 8.33$ or $\beta_1 = \frac{1}{2}$ (the coefficients in the linear equilibrium in session 1).

Players 4 and 8 have estimated coefficients for the intercept and the valuation consistent with the predicted coefficients for the Bayesian equilibrium which employs linear strategies. Under the linear strategy calculated above, the expected sign of the coefficient for High is positive and the expected sign of the coefficient for Threshold is negative. The results indicate that the strategies employed by several participants are sensitive to changes in the prior distribution of valuations and the relative cost of the public good. In the pooled regression over all the subjects, a reduction in the upper limit of the distribution enters positively into the equation and a reduction in the threshold enters negatively, as predicted as predicted by the linear equilibrium. However, the coefficients were not significant. For most players, the estimated coefficient for High was positive. For players 1, 10 and 13, a reduction in the upper limit of the distribution enters positively and significantly into the equation, as predicted. For players 5, 6, 8, 9 and 11, a reduction in the threshold enters negatively and significantly into the equation, as predicted. Surprisingly, the estimated coefficient for Threshold was positive and significant for four players. One possible explanation for the failure of these four players to modify their behavior optimally when the threshold changed is that these individuals did not understand the implications of the environment change caused by a reduction in the threshold.

To test whether the slope parameter remained constant, the dummy variables VHigh and VThreshold were created. The dummy variable VHigh is the product of V_i and the dummy variable High. The dummy variable VThreshold is the product of V_i and the dummy variable Threshold. Pooling the bids of each session, I estimated the following regression for each participant:

$$B_i = \alpha + \beta_1 V_i + \beta_2(\text{High}) + \beta_3(\text{Threshold}) + \beta_4(\text{VHigh}) + \beta_5(\text{VThreshold}) + \mu_i$$

The OLS estimate of the coefficient associated with VHigh represents the difference in the slope parameter associated with a reduction in the upper limit of the distribution from 40 to 35. A t-test of the null hypothesis that $\beta_4 = 0$ is a test of the hypothesis that the change in the slope in the second session is statistically significant.¹² Likewise, a t-test of the null hypothesis that $\beta_5 = 0$ is a test of the hypothesis that there is no difference in the slope parameter associated with changes in the threshold.¹³ The regression results indicate that the dummy variable VHigh, VThreshold or both were significant for players 2, 9, 10, 12 and 13. The results of the regressions for these players are contained in Table 6. The first column in Table 6 is the estimation results when all subjects are pooled.

¹²Note that $E[B]$ equals $\alpha + \beta_1 E[V_i] + \beta_3 E[\text{Threshold}] + \beta_5 E[\text{VThreshold}]$ if the upper limit is 40 and $\alpha + (\beta_1 + \beta_4) E[V_i] + \beta_2 + \beta_3 E[\text{Threshold}] + \beta_5 E[\text{VThreshold}]$ if the upper limit is 35.

¹³Note that $E[B]$ equals $\alpha + \beta_1 E[V_i] + \beta_2 E[\text{High}] + \beta_4 E[\text{VHigh}]$ if $V_{\text{firm}} = 45$ and $\alpha + (\beta_1 + \beta_5) E[V_i] + \beta_2 E[\text{High}] + \beta_3 + \beta_4 E[\text{VHigh}]$ if $V_{\text{firm}} = 41$.

Table 6: OLS estimates when slope and intercept are allowed to change.

Explanatory variables	Player:					
	all	2	9	10	12	13
intercept	8.68 (8.1*) _{α=0} (0.33) _{α=8.33}	10.3 (3.6*) (0.7)	5.61 (3.0*) (1.44)	9.62 (3.3*) (0.44)	6.37 (2.8*) (0.86)	5.85 (3.0*) (1.3)
valuation	0.54 (15*) _{β=0} (1.27) _{β1=1/2}	0.51 (5.1*) (0.05)	0.77 (11*) (4.0*)	0.45 (4.7*) (0.5)	0.68 (8.6*) (2.3*)	0.54 (7.8*) (0.51)
High 0 if H=40 1 if H=35	-2.52 (-1.3) _{β2=0}	-8.85 (-1.9)	-0.23 (-0.1)	-8.88 (-1.8)	-4.9 (-1.0)	-5.4 (-1.4)
Threshold 0 if V _{firm} = 45 1 if V _{firm} = 41	-2.97 (-2.2*) _{β3=0}	-8.87 (-2.2*)	1.10 (-0.5)	-4.33 (-1.2)	-5.7 (-2.0)	-3.4 (-1.4)
VHigh 0 if H = 40 V _i if H = 35	0.10 (1.52) _{β4=0}	0.43 (2.4*)	-0.10 (-0.9)	0.47 (2.8*)	0.29 (1.6)	0.28 (2.1*)
VThreshold 0 if V _{firm} = 45 V _i if V _{firm} = 41	0.09 (1.92) _{β5=0}	0.41 (3.1*)	-0.19 (-2.3)	0.32 (2.5*)	0.24 (2.3*)	0.18 (2.2*)
adj. R ²	0.50	0.77	.90	.71	0.85	0.86
F	162	37	108	32	64	71
sample size	805	54	58	64	55	57

* indicates significance at the 0.05 level.

Now, the aggregate estimate is consistent with many of the predictions of the linear equilibrium.

In the estimation when all subjects are pooled, one cannot reject the null hypothesis that the intercept is 8.33, the predicted value for the intercept of the linear bidding function in session 1.

Further, one cannot reject the null hypothesis that the slope coefficient is $\beta_1 = 1/2$, the slope

coefficient predicted by the linear equilibrium. Also, when all subjects are pooled, the reduction in the threshold is associated with a statistically significant decrease in the intercept, as predicted by the linear equilibrium. Finally, in the aggregate estimate, the change in the slope coefficient corresponding with a reduction in the upper limit of the distribution or a reduction in the threshold is not statistically different from zero.

In the estimations for players 2, 9, 10, 12 and 13, one cannot reject the null hypothesis that the intercept is 8.33, the predicted value for the intercept of the linear bidding function in session 1. In the estimations for players 2, 10, and 13, one cannot reject the null hypothesis that $\beta_1 = \frac{1}{2}$, the slope coefficient predicted by the linear equilibrium. However, player 9 had a statistically significant decrease in the slope coefficient when the threshold decreased. This may be due to learning behavior. Also, players 2, 10 and 13 had statistically significant increases in the slope coefficient when the threshold or the upper limit of the distribution decreased. Player 12 had a statistically significant increase in the slope coefficient when the threshold decreased. The increases in the slope coefficient may be due to increased risk aversion. The bids of risk-averse players are likely to be higher than the bids of risk-neutral players.

To test more rigorously the assumption that the participants employed linear strategies, I ran a regression on the following extended model for each participant:

$$B_i = \alpha + \beta_1 V_i + \beta_2 V_i^2 + \beta_3 V_i^3 + \beta_4(\text{High}) + \beta_5(\text{Threshold}) + \mu_i$$

An F-test was run on the results of the regressions to test whether nonlinearities were present.

The test involves a comparison of the error sum of squares associated with the linear model and the extended model. The OLS model which includes the parameter restriction (that the coefficients $\beta_2 = \beta_3 = 0$) is expected to have a higher expected squared forecast error. If the

increase in the sum of squared residuals is not significant when the restriction is added, the restriction is considered proper. If the sum of squared residuals changes substantially, the restriction is not proper. The F-test is conducted on the null hypothesis $H_0: \beta_2 = \beta_3 = 0$. The appropriate test statistic is

$$S = \{(RSS_0 - RSS_1)/2\} / \{RSS_1/(T - k)\}$$

where RSS_0 is the sum of squared residuals in the OLS linear model, RSS_1 is the sum of squared residuals in the unrestricted model, and k is the number of parameters in the unrestricted model. If S is greater than the 5% critical value for an $F(2, T - k)$ distribution, then we reject the null hypothesis that the linear model is appropriate; that is, if S is sufficiently large, we conclude that the unrestricted model is appropriate.¹⁴

The results are as follows:

Table 7: Summary of OLS linear and unrestricted regressions.

Regression	F for independent variables	adj. R^2	Standard error of estimates	Degrees of freedom
Player 1:				
Linear model	59.7	0.75	7.47	56
Unrestricted model	39.5	0.77	6.73	54
Player 2:				
Linear model	49.5	0.73	12.23	50
Unrestricted model	29.3	0.73	11.99	48
Player 3:				
Linear model	39.8	0.70	6.16	48
Unrestricted model	48.0	0.82	3.46	46

¹⁴However, if the error term is serially correlated, the F-tests based on estimated coefficients are invalid.

Player 4:					
Linear model	29.9	0.60	5.89	54	
Unrestricted model	20.7	0.63	5.25	52	
Player 5:					
Linear model	18.5	0.51	13.7	48	
Unrestricted model	11.9	0.52	12.9	46	
Player 6:					
Linear model	49.6	0.72	13.3	54	
Unrestricted model	32.8	0.74	12.1	52	
Player 7:					
Linear model	36.1	0.65	5.77	53	
Unrestricted model	24.3	0.68	5.21	51	
Player 8:					
Linear model	33.8	0.65	5.67	50	
Unrestricted model	20.5	0.65	5.48	48	
Player 9:					
Linear model	168.8	0.90	2.47	54	
Unrestricted model	101.1	0.90	2.39	52	
Player 10:					
Linear model	45.0	0.68	10.9	60	
Unrestricted model	39.2	0.75	8.09	58	
Player 11:					
Linear model	107.8	0.83	4.98	63	
Unrestricted model	101.3	0.88	3.29	61	
Player 12:					
Linear model	97.5	0.84	6.97	51	
Unrestricted model	57.4	0.84	6.85	49	
Player 13:					
Linear model	106.6	0.85	4.52	53	
Unrestricted model	71.2	0.86	3.99	51	

Player 14:					
Linear model	158.4	0.89	1.84	55	
Unrestricted model	146.0	0.93	1.19	53	

The F-statistics for the coefficients are reported below:

Table 8: F-tests for additional coefficients in unrestricted model.

Player	F
1	3.08
2	0.49
3	18.73*
4	3.30*
5	1.55
6	2.84
7	2.88
8	0.88
9	0.91
10	10.44*
11	16.24*
12	0.47
13	3.55*
14	14.66*

*Significant at 0.05 level (assumed distributed as F(2, 40)).

These results support the conclusion that the bidding functions of players 1, 2, 5, 6, 7, 8, 9, and 12 were linear. Of these players, only player 8 had an estimated linear bidding function consistent with the linear strategy predicted by the Bayesian Nash equilibrium described above.

Conclusion

About 17% of all bids were individually irrational. The number of individually irrational bids might be attributable to learning behavior or a desire among members to "split the difference" to meet the threshold. Nevertheless, these results exhibit a higher degree of

individual rationality than the results obtained in experimental tests of the Coase theorem. On average, the property rights were traded to the group in about 67% of those instances in which the sum of members' realized valuations was at least as large as V_{firm} . That many efficient outcomes represented a disadvantageous bargain for one of the parties relative to the payoff attainable without bargaining is consistent with the results of experimental tests of the Coase theorem. The results provide little support for the one-price equilibrium strategy of bidding a constant amount regardless of realized valuation. The results suggest that some subjects employ a linear bidding function in which bids increased with valuation. The strategies employed by several participants are sensitive to changes in the prior distribution of valuations and the relative cost of the public good. For several participants, a reduction in the upper limit of the distribution enters positively into the equation and a reduction in the threshold enters negatively, as predicted by the linear equilibrium. In other words, several subjects modified their strategy in the direction predicted by the linear equilibrium when the threshold or the upper limit of the distribution was reduced.

The linear equilibrium described above assumes that players are risk-neutral. Risk-neutral players who employ linear strategies may understate the maximum amount they are willing to pay more than do risk-averse players. In other words, the bids of risk-averse players are likely to be higher than the bids predicted by the linear strategy. Those players who consistently bid closer to the realized valuation than predicted by the linear equilibrium may exhibit risk aversion. Relaxing the assumption that players are risk-neutral may lead to interesting theoretical results.

Typical of most experiments, this experiment raises many questions. This experimental

design could be modified to determine whether bidding behavior changes when the following occur:

- Participants remain in the same group in consecutive decision periods.
- Realized valuations are public information.
- A different method is employed to distribute excess contributions.
- Property rights are initially owned by the participants who attempt to sell them to a third party.
- The underlying prior distribution is changed.
- Group size increases.
- An efficient bargaining mechanism is used.
- Cheap talk is allowed.

In previous tests of the Coase theorem, the players had incomplete information but bargained face to face. Another experiment could be conducted in which the players have a different message space. For example, after the members learn their valuations (but before they submit their bids), the members in each group could be allowed to communicate either in writing or face to face (but not exhibit their valuations). One potential benefit of such pre-play communication is to select one of the equilibria that exists without pre-play communication. These topics will require further research. This paper is a step in the continuing effort to find more efficient solutions to the public-goods problem with a threshold.

Appendix I

In this appendix, I derive the linear equilibrium. Suppose that player 2 adopts the strategy $B_2(V_2) = d_2V_2 + c_2$ where $d_2 > 0$. For a given value of V_1 , player 1's best response solves

$$\max_{B_1} (V_1 - B_1)\text{prob}\{B_1 + (d_2V_2 + c_2) \geq V_{\text{firm}}\} \quad (1)$$

This problem can be expressed equivalently as

$$\max_{B_1} (V_1 - B_1)\text{prob}\{V_2 \geq (V_{\text{firm}} - B_1 - c_2)/d_2\} \quad (2)$$

Because $V_2 \sim U[L, H]$,

$$\text{prob}\{V_2 \geq (V_{\text{firm}} - B_1 - c_2)/d_2\} = (d_2H - V_{\text{firm}} + B_1 + c_2)/\{d_2(H - L)\} \quad (3)$$

Therefore, player 1's maximization problem can be expressed as

$$\max_{B_1} (V_1 - B_1)(d_2H - V_{\text{firm}} + B_1 + c_2)/\{d_2(H - L)\} \quad (4)$$

The first derivative of this function with respect to B_1 is

$$(V_1 - d_2H + V_{\text{firm}} - 2B_1 - c_2)/\{d_2(H - L)\} \quad (5)$$

The second derivative with respect to B_1 is less than zero where $d_2 > 0$. Setting the first derivative equal to zero yields

$$B_1^*(V_1) = \frac{1}{2}V_1 + (V_{\text{firm}} - d_2H - c_2)/2 = d_1V_1 + c_1 \quad (6)$$

Thus, if player 2 plays a linear strategy, player 1's best response is also linear. If the players' responses are to be best responses to each other, then player 2's strategy is

$$B_2^*(V_2) = \frac{1}{2}V_2 + (V_{\text{firm}} - d_1H - c_1)/2 = d_2V_2 + c_2 \quad (7)$$

Solving these equations simultaneously yields

$$d_i = \frac{1}{2} \quad c_i = (2V_{\text{firm}} - H)/6 \quad i = 1, 2 \quad (8)$$

Hence, the linear strategies can be expressed as

$$B_i(V_i) = \frac{1}{2} * V_i + (2V_{\text{firm}} - H)/6 \quad i = 1, 2 \quad (9)$$

Appendix II

EXPERIMENTAL INSTRUCTIONS

General

You are about to participate in an experiment in decision making. The purpose of the experiment is to gain insight into certain features of complex economic processes. If you follow the instructions carefully, you might earn a considerable amount of money. In this experiment, you will make a series of decisions. For each decision, you will be placed in a group with other participants. Your cash earnings for each decision will depend on the decisions that you and the other members of your group make. During the experiment, you will earn experimental dollars. You will begin the experiment with an initial cash value of 40 experimental dollars. At the end of the experiment, your initial cash value and your total earnings will be exchanged according to the exchange rate: 1 experimental dollar = 0.07 U.S. dollars. At the end of the experiment, you will be paid this amount in cash.

This experiment will be conducted using a computer which will serve as a means for transacting your decisions. Do not communicate with other participants at any time during the experiment. If you have any questions, please direct them to the monitor.

In the following decision periods, the computer will randomly assign you to a group consisting of two participants. The composition of your group will change every decision period. After each decision period you will be reassigned randomly to a new group of two participants. Therefore, the chance that any other participant will be in a group with you in consecutive periods is very small. At no point in the experiment will the identity of the other member of your group be made known to you, nor will your identity be made known to the other member of your

group.

Determining the Ownership of the Property

The right to own certain property will initially belong to a firm. In each decision period in this session, the firm is willing to sell the property for 45 experimental dollars (i.e., the value of the property to the firm, denoted V_{firm} , is \$45). The members of your group jointly will attempt to purchase the property from the firm.

Determination of Valuations

At the beginning of each decision period, the computer will independently assign random valuations for the property to each member of your group. The valuation assigned to you is the amount the property is worth to you if your group purchases the property from the firm. At the beginning of each of the following decision periods, the computer will independently assign to each participant a valuation for the property drawn randomly from the set of consecutive positive integers $\{10, 11, 12, \dots, 40\}$. Each integer in the set is equally likely to be drawn. For example, suppose the participants in a group are Player 1 and Player 2 and that the randomly determined valuations for these players are denoted V_1 and V_2 , respectively. The value V_1 will be drawn randomly from the set $\{10, 11, 12, \dots, 40\}$. Simultaneously and independently, the computer will randomly assign the value V_2 from the same set.

The valuations of each member of the group will be private information. Hence, you are the only member of your group who will know your valuation. You will not be informed of the valuations of other members of your group.

In each decision period, you and the other members of your group will each submit an offer to buy the property owned by the firm. If the *sum* of the offers of members of your group is

less than the value of the property to the firm (V_{firm}), ownership of the property will remain with the firm, the amount of your offer will be refunded to you, and your earnings and the earnings of the other member of your group for that decision period will be zero. If the *sum* of the offers of members of your group *equals or exceeds* the value of the property to the firm (V_{firm}), ownership of the property will be transferred to your group. If the property is transferred to your group, each member will receive earnings in the amount of his or her valuation minus his or her respective bid (with a formula for excess contributions, if any). In the example above, suppose Player 1 and Player 2 submit offers to contribute 15 experimental dollars and 30 experimental dollars, respectively, and the firm values the property at 45 experimental dollars. Then, ownership of the property would be transferred to the group, Player 1 would earn $V_1 - 15$ experimental dollars and Player 2 would earn $V_2 - 30$ experimental dollars. By making a small offer, you stand the chance that the property will be transferred to your group without your having to pay so much. But, if the sum of the offers of the members of your group is less than V_{firm} , your earnings for the decision period will be zero.

The amounts the members of the group actually pay to buy the right may be different from the submitted offers when the sum of the players' offers exceeds the amount the firm is willing to accept, V_{firm} . A "pro-rata reimbursement" rule will determine the trading price from the players' offers when there are excess contributions. When a transaction occurs and there are excess contributions, the pro-rata reimbursement rule directs that the firm receive the firm's value for the property (V_{firm}) and that the excess be reimbursed to the members of the group in proportion to their offers. In the example above, if Player 1 contributes more than Player 2 and the sum of the contributions exceeds V_{firm} , then Player 1 receives a proportionately larger

reimbursement than Player 2. Formally, suppose that Player 1 has valuation V_1 and submits an offer of B_1 . The payoff to Player 1 can be expressed as follows:

$$\begin{aligned} V_1 - B_1 + (B_1 + B_2 - V_{\text{firm}}) * B_1 / (B_1 + B_2) & \quad \text{if } B_1 + B_2 \geq V_{\text{firm}} \text{ (i.e., trade occurs)} \\ 0 & \quad \text{otherwise (i.e., trade does not occur)} \end{aligned}$$

The payoff to Player 2 is similar. The pro-rata reimbursement rule will ensure that the sum of the payments of the members of the group will not exceed the firm's value for the property.

Procedures for Submitting an Offer

Each participant shall submit an offer at his or her computer terminal. To determine the valuation assigned to you if the group purchases the property, click on PRIVATE VALUE. To make an offer, type the amount representing the *maximum* amount you are willing to pay at the prompt which asks you for YOUR OFFER. Then, click on SUBMIT YOUR OFFER. Once you have typed an offer and submitted it, you can not change your offer. After each participant has submitted an offer, the computer will determine whether the sum of the offers from members of your group equals or exceeds the firm's value for the property, V_{firm} . If so, the property is purchased by the group. If not, the firm retains the property. Finally, the computer will report the sum of the offers made by members of your group, whether a transaction occurred, your earnings (or losses) for the decision period and your new cash holdings.

Session two (change in distribution of valuations)

At the beginning of each of the following decision periods, the computer will independently assign random valuations for the property to each member of your group from the set of consecutive positive integers $\{10, 11, 12, \dots, 35\}$. Each integer in the set is equally likely to be drawn. In other words, the value V_1 will be drawn randomly from the set $\{10, 11, 12, \dots, 35\}$. Simultaneously and independently, the computer will randomly assign the value V_2 from the same set.

As before, after each decision period, the computer will randomly assign you to a new group consisting of two participants. As before, the firm is willing to accept offers totalling 45 experimental dollars to sell to the group the property.

Session three (change in firm value)

For the following decision periods in this session, the firm is willing to accept offers totalling \$41 experimental dollars to sell to the group the property (i.e., $V_{\text{firm}} = \$41$).

As before, after each decision period, the computer will randomly assign you to a new group consisting of two participants. Again, the computer will independently assign random valuations to each member of your group from the set of consecutive positive integers $\{10, 11, 12, \dots, 40\}$. Each integer in the distribution is equally likely to occur.

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