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The Incentive of Cournot Duopolists to Reveal Cost Information After Costs are Realized


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Abstract

This is a study of the incentive of firms to disclose private information about their costs to competitors (when firms compete by setting quantities). This paper expands on previous contributions by analyzing a model in which firms decide whether to disclose their cost information to their rivals after they observe their own costs. I calculate the levels of profit to the firm, benefit to the consumers, and welfare to society when the competitors do not disclose such information, when a firm unilaterally discloses private cost information to its competitor and when firms exchange such information. The results show that risk-neutral Cournot competitors have an incentive to disclose firm-specific cost information *ex post* if their costs are below the expected mean cost. Disclosure reduces consumer surplus when the disclosing firm's costs are below the expected mean cost. The effect of disclosure on social welfare depends on the parameters of the problem. Finally, I analyze the incentive of firms to agree to exchange information when disclosure exposes a firm to the risk of antitrust liability.

Acknowledgements

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The Incentive of Cournot Duopolists to Reveal Cost Information After Costs are Realized

Introduction

This study concerns the incentive and welfare implications of disclosure of private cost information by Cournot competitors. A series of papers has examined the topic of incentives for information disclosure. Novshek and Sonnenschein (1982), Clark (1983), Fried (1984), Vives (1984), Li (1985), Gal-Or (1986), Shapiro (1986), Kirby (1988), Hwang and Lee (1992), and Kao and Hughes (1993). The papers of Fried, Li, Gal-Or, and Shapiro analyze the incentives of firms to share cost information in Cournot competition. However, in these papers, firms decide whether to disclose cost information ex ante, i.e. before their cost are realized. The incentive to disclose ex post has received little attention. This study expands on previous contributions by analyzing a model in which firms decide whether to disclose private cost information to their rivals after they observe their own costs.

This paper begins with a discussion of the incentive to disclose information ex ante. I replicate the findings of Fried (1984) whose duopoly model assumes that demand is linear, marginal costs are constant and firms act noncooperatively in Cournot competition. Reproducing his findings elucidates the changes in results that occur with changes in the timing of the decision to disclose cost information. Then, I analyze the incentive to disclose information ex post. The model used is similar to Fried's except in terms of the timing of the decision to disclose information. I compare the results obtained when no firm discloses its cost information, when one firm unilaterally discloses its cost information and when both firms disclose their cost information. For each of these situations, I compare the levels of profit to the firm, benefit to consumers, and welfare to society. The results show that risk-neutral Cournot competitors have
an incentive to disclose firm-specific cost information \textit{ex post} if their costs are below the expected mean cost. If a firm's costs are below mean costs, the disclosure reduces consumer surplus. The effect of disclosure on social welfare depends on the parameters of the problem. Finally, this paper analyzes the incentive of firms to agree to exchange information when disclosure exposes a firm to the risk of antitrust liability. I characterize the conditions on expected profit which ensure that firms do not cooperatively disclose private cost information \textit{ex ante} and \textit{ex post}. A study of this topic has significant implications in antitrust law.

A series of papers has established that sharing of firm-specific information about costs is a dominant strategy of risk-neutral Cournot competitors. Fried (1984), Li (1985), Gal-Or (1986), and Shapiro (1986).\textsuperscript{1} In a homogeneous good, Cournot duopoly, Fried showed that when demand is linear and marginal costs are constant, risk-neutral firms are better off \textit{ex ante} committing to individually disclose their private cost information or agreeing to collectively disclose their private cost information. Shapiro showed that disclosing cost information increases expected profit and expected social welfare but reduces expected consumer surplus. In these models, firms decide whether to disclose cost information before realizing their costs. Changing the timing of the decision to disclose information makes the results more interesting. When demand is linear and marginal costs are constant, a risk-neutral firm is better off disclosing its private cost information \textit{ex post} if its realized cost is below the mean cost. Also, consumer surplus decreases if and only if the disclosing firm's realized cost is below the mean cost. In terms of welfare, the effect of disclosure depends on the parameters of the problem.

The purpose of sharing cost information is "to allow the opponents to make the necessary

\textsuperscript{1}See Kao and Hughes (1993) on the preferences of risk-averse Cournot duopolists not to share firm-specific cost information.
and mutually beneficial 'counter' adjustment." (Fried, p. 378) Gal-Or finds an incentive to reveal private information when each duopolist observes its own costs with noise and may send noisy signals to its rival. Shapiro considers an oligopoly (rather than a duopoly) and concludes that when oligopolists compete in Cournot competition, expected profits and welfare rise, but expected consumer surplus falls. "Cost sharing increases efficiency by raising the market shares of lower cost firms and reducing the variability of aggregate output." (Shapiro, p. 433) "By sharing costs, the low cost firms induce their rivals to reduce their output by making their own relatively 'aggressive' reaction curves known. This leads to a more efficient pattern of production than arises in the absence of information exchange." (Shapiro, pp. 439-40).

Shapiro also noted the incentive of low-cost firms to report their cost realizations \textit{ex post}. (Shapiro, p. 443) Shapiro expects full disclosure when firms have the ability to costlessly report accurate information after realization of their costs if verification and reporting costs are low. "The reason is that low-cost firms will report their cost realization \textit{ex post}, and this will induce higher cost firms to do so. This unravelling process only stops when all firms report their costs \textit{ex post}." He cites Grossman (1981) as an example of a similar argument. The argument assumes that a firm's failure to disclose cost information will cause the firm's competitors to believe that the nondisclosing firm has the highest possible costs, because only a producer with the highest cost would find it profit maximizing to disclose nothing. Therefore, the firm will find it optimal to disclose the lowest possible cost consistent with the truth; it discloses its cost if its costs are known. Shapiro states that this unravelling argument applies when reporting is costless, deception is ruled out and all parties have rational expectations in the event a party fails to report. In the analysis below, I assume that the failure to disclose cost information does not signal high costs. It is possible that a nondisclosing producer will be treated as if it had costs which varied
over the entire distribution. Sometimes disclosure of cost information may be costly. Even if reporting is costless, disclosure may not be beneficial if rivals are unable to verify the accuracy of the reports. Indeed, the cost of verifying a report is likely to increase with the level of costs. Also, it is possible that producers with higher costs have higher degrees of risk aversions. The incentive to disclose has been shown to depend on a variety of characteristics including the degree of risk aversion and whether the firms are engaged in Cournot or Bertrand competition.

**Incentives for information disclosure *ex ante***

An early example of the models in which firms decide whether to disclose cost information prior to realizing their costs is Fried (1984). The timing structure in Fried's model is as follows: First, each duopolist decides whether to disclose its private information about its cost to its rival. Second, each firm observes its costs determined by nature. Third, each firm discloses information in accordance with its previous decision. Fourth, each duopolist chooses its quantity output to maximize its expected profit based on the information available. Figure 1 is a time line for this model.

![Figure 1: Time line (*ex ante*).](image)

As with the papers of Fried, Gal-Or and Shapiro cited above, I assume that the firms can verify each other's report. Disclosure of private information is assumed to be costless. A firm's 

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\(^2\) Shapiro points out that this assumption has the same effect as the assumption that firms disclose the true value of their realized costs. Shapiro has noted that in the absence of the ability to verify, each firm would always report low costs in order to induce its rivals to contract their
decision to disclose private information is assumed to be binding upon the participants (i.e., a firm cannot renege \textit{ex post} on a prior commitment to disclose information).

In Fried's duopoly model, demand is linear, marginal costs are constant and each firm acts noncooperatively in production in Cournot competition. The model consists of a two-firm industry with the linear demand given by $P=A-X$ where $A$ is a known constant and the total quantity produced is $X=x_1+x_2$. Both duopolists have linear cost functions, $c_i x_i$. The firms' marginal costs are denoted $c_1$ and $c_2$, respectively. The parameters $c_1$ and $c_2$ are assumed to be independently distributed random variables having known mean $\bar{c}$ and known variance, $\text{Var}(c_i) > 0$. I assume that $A > \bar{c}$ and that $A$ is greater than the upper limit of the distribution of $c_i$.

Following Shapiro (1986), I avoid boundary problems by assuming all firms produce positive amounts for all cost realizations. The profit function of duopolist $i$ is

$$\pi_i = (A - x_i - x_j - c_j)x_i \quad i = 1,2; j = 1,2; i \neq j$$

(1)

Firms are risk neutral and seek to maximize their expected profits. Specifically, duopolist $i$ picks $x_i$ to solve the following expression:

$$\max_{x_i} E[(A-x_i-x_j-c_j)x_i] \quad i = 1,2; j = 1,2; i \neq j$$

(2)

where $E[\cdot]$ is the expected value operator and the expectation is taken over the distribution of the rival's costs, $c_j$. Setting the first derivative equal to zero yields

$$x_i = \frac{1}{2}(A - E[c_j] - c_i) \quad i = 1,2; j = 1,2; i \neq j$$

(3)

The expected value of $x_i$ over the distribution of $c_i$ is

$$E_{c_i}[x_i] = \frac{1}{2}(A - E[c_j] - \bar{c}) \quad i = 1,2; j = 1,2; i \neq j$$

(4)

A solution to the system of equations in this stage will be a Bayesian equilibrium. In this model, neither firm observes nature's choice for its opponent's cost. The equilibrium notion
appropriate for games of incomplete information is Bayesian equilibrium. A Bayesian equilibrium specifies an action or strategy for each type of firm such that each firm maximizes its expected profit contingent on its type and taking the other firm's type-contingent strategies as given (By definition, a firm's type subsumes all its private information when it chooses its action). Specifically, the set of strategies \( \{x^*_i(c_1), x^*_2(c_2)\} \) is a Bayesian equilibrium if the expected profit for firm \( i \) \((i = 1, 2)\) is maximized at \( x^*_i(c_i) \) when the other firm's strategy is to produce \( x^*_i(c_i) \).\(^3\) Then, \( x^*_i(c_i) \) is a best response to \( x^*_i(c_i) \).

The equilibrium output quantities for the four possible arrangements are contained in Table 1 below.

<table>
<thead>
<tr>
<th>Firm 1\Firm 2</th>
<th>No disclosure</th>
<th>Disclosure</th>
</tr>
</thead>
<tbody>
<tr>
<td>No disclosure</td>
<td>( x_1 = (2A + c - 3c_1) / 6 ) ( x_2 = (2A + c - 3c_2) / 6 )</td>
<td>( x_1 = (2A - c - 3c_1 + 2c_2) / 6 ) ( x_2 = (A + c - 2c_2) / 3 )</td>
</tr>
<tr>
<td>Disclosure</td>
<td>( x_1 = (A + c - 2c_1) / 3 ) ( x_2 = (2A - c + 2c_1 - 3c_2) / 6 )</td>
<td>( x_1 = (A - 2c_1 + c_2) / 3 ) ( x_2 = (A - 2c_2 + c_1) / 3 )</td>
</tr>
</tbody>
</table>

Taking the expected value of each quantity in each quadrant of Table 1 over the distributions of the cost of each firm, one can easily verify that expected price and expected quantities produced are the same across all quadrants of Table 1. Disclosure of cost information does not effect expected output or expected price.

In this model, each firm evaluates its expected profits over the distributions of both firms' costs. If firm \( i \)'s equilibrium output is \( x^*_i \), one can show from equations (2) and (3) that its profits

\(^3\)As usual, \( x^*_i(c_i) \) refers to the strategy for the player other than \( i \).
will be equal to \((x^*_i)^2\). Its expected profit level is \(E_{c_i}[(x^*_i)^2]\) where expectation is taken over the distribution of \(c_i\). Because the expected profit levels for firm 1 and firm 2 are symmetric, I present in Table 2 only the expected payoffs for firm 1 resulting from the equilibrium output quantities.

<table>
<thead>
<tr>
<th>Firm 1 \ Firm 2</th>
<th>No disclosure</th>
<th>Disclosure</th>
</tr>
</thead>
<tbody>
<tr>
<td>No disclosure</td>
<td>(k + \frac{1}{4}\text{Var}(c_1))</td>
<td>(k + \frac{1}{4}\text{Var}(c_1) + (1/9)\text{Var}(c_2))</td>
</tr>
<tr>
<td>Disclosure</td>
<td>(k + (4/9)\text{Var}(c_1))</td>
<td>(k + (4/9)\text{Var}(c_1) + (1/9)\text{Var}(c_2))</td>
</tr>
</tbody>
</table>

In this table and several that follow, \(k = (A - \bar{c})^2/9\). When the decision to disclose private cost information precedes the realization of costs, this model assumes that each firm considers its expected profit over the entire distribution of both firms' costs.\(^4\) For each strategy that firm 2 could choose, the expected payoff to firm 1 from disclosing is greater than the expected payoff to firm 1 from not disclosing. In other words, nondisclosure is strictly dominated by disclosure for firm 1. Because the expected profits for firm 2 are symmetric, disclosure is the dominant strategy for both firms. Therefore, bilateral disclosure is a subgame-perfect Bayesian equilibrium. Increases in the variability of firm 1's private costs increase its expected profits. This occurs because the profit function is concave due to the convexity of firm costs. Shapiro (1986) has noted that variability of industry output, \(ceteris paribus\), reduces expected profits. "The firms receive lower prices when they produce more output, so the expected price per unit is reduced when output is variable." (Shapiro, p. 437)

The subgame-perfect Bayesian equilibrium, bilateral disclosure, provides the lowest level

\(^4\)In each instance, the expectation is taken over the entire distribution of \(c_i\). The failure to reveal cost information does not, by assumption, signal that costs are high.
of expected consumer surplus and the highest level of expected social welfare. Consumer surplus is the difference between the total amount consumers are willing to pay and the amount they actually pay. The expected consumer surplus is equal to

\[ E_{c_1,c_2}[CS] = \frac{1}{2}E_{c_1,c_2}[(x_1 + x_2)^2] \]  

where expectation is taken over the distributions of \( c_1 \) and \( c_2 \). The levels of expected consumer surplus are presented in Table 3.

Table 3: Levels of expected consumer surplus.

<table>
<thead>
<tr>
<th>Firm 1 \ Firm 2</th>
<th>No disclosure</th>
<th>Disclosure</th>
</tr>
</thead>
<tbody>
<tr>
<td>No disclosure</td>
<td>( 2k + \frac{1}{6}\text{Var}(c_1) + \frac{1}{6}\text{Var}(c_2) )</td>
<td>( 2k + \frac{1}{6}\text{Var}(c_1) + (1/18)\text{Var}(c_2) )</td>
</tr>
<tr>
<td>Disclosure</td>
<td>( 2k + (1/18)\text{Var}(c_1) + \frac{1}{6}\text{Var}(c_2) )</td>
<td>( 2k + (1/18)\text{Var}(c_1) + (1/18)\text{Var}(c_2) )</td>
</tr>
</tbody>
</table>

Expected consumer surplus is highest when neither firm discloses its cost information and lowest when both firms disclose their cost information. When one or both firms disclose their cost information \textit{ex ante}, expected consumer surplus decreases because disclosure reduces the variance of output, and consumer surplus is a convex function of output. The market price declines when firms produce more output. So output variability increases expected consumer surplus by reducing the expected price. Also, expected consumer surplus is increasing in cost variance in each quadrant.

The expected social welfare is

\[ E_{c_1,c_2}[W] = E_{c_1,c_2}[\int_0^x p(z)dz - \Sigma_{i=1}^2 c_i x_i] \]

This is equivalent to the sum of the expected profits of the firms and expected consumer surplus. The levels of the expected social welfare are indicated in Table 4.
Table 4: Levels of expected welfare.

<table>
<thead>
<tr>
<th>Firm 1\Firm 2</th>
<th>No disclosure</th>
<th>Disclosure</th>
</tr>
</thead>
<tbody>
<tr>
<td>No disclosure</td>
<td>$4k+(27\text{Var}(c_i)+27\text{Var}(c_j))/72$</td>
<td>$4k+(27\text{Var}(c_i)+44\text{Var}(c_j))/72$</td>
</tr>
<tr>
<td>Disclosure</td>
<td>$4k+(44\text{Var}(c_i)+27\text{Var}(c_j))/72$</td>
<td>$4k+(44\text{Var}(c_i)+44\text{Var}(c_j))/72$</td>
</tr>
</tbody>
</table>

The expected social welfare increases when one or both firms disclose their cost information. The subgame-perfect Bayesian equilibrium, bilateral disclosure, provides the highest level of social welfare. Also, increases in the variance of either firm's private costs increase the expected social welfare.

The analysis above shows that, when demand is linear and marginal costs are constant, the subgame-perfect Bayesian equilibrium is bilateral disclosure. Also, from Table 2 and the symmetry of the expected profits, both firms prefer *ex ante* their expected profit levels when both firms disclose their cost information to their expected profit levels when neither firm discloses its cost information. Bilateral disclosure provides the lowest level of expected consumer surplus but the highest level of expected social welfare. Because consumer surplus is a convex function of output, Shapiro (1986) noted that expected consumer surplus decreases with disclosure because consumers benefit from output variability. Industry output variability increases expected consumer surplus by reducing the expected price. However, industry output variability reduces the firms' aggregate expected profits. Society benefits from information sharing because the expected benefit to producers exceeds the expected harm to consumers. The expected profits to producers increase with information sharing because it shifts greater output to low-cost firms.

**Incentives for information disclosure *ex post***

In the model used above, firms decide whether to disclose cost information prior to
observing their costs. The model above may be appropriate when the decision to undertake some project precedes realization of the costs of the project (e.g., R&D). However, for an established firm, realization of the firm's costs may precede the firm's decision to disclose its cost information. In the model below, the timing structure is as follows: First, each firm observes its costs. Second, each duopolist decides whether to disclose its private cost information to its rival. Third, each firm discloses information in accordance with its previous decision. Fourth, each duopolist chooses its quantity output to maximize its expected profit based on the information available. Figure 2 is a time line for this model.

Figure 2: Time line (ex post).

| Costs are observed | Firms decide whether to disclose costs | Firms disclose costs | Firms choose quantities |

As before, I assume demand is linear, marginal costs are constant and each firm acts noncooperatively in production in Cournot competition. The equilibrium output quantities in this environment are those contained in Table 1 above.

As before, if equilibrium output is $x_i^*$, profits will be equal to $(x_i^*)^2$. In this model, each firm evaluates its expected profits over the distribution of its rival's costs only (rather than expected profits over the distributions of both firms' costs as in Fried's model). However, if $A$ is greater than the upper limit of the distribution of $c_i$, then a solution to the normal-form representation of this game is the same as the solution to the game in which the payoffs are the actual levels of profits. For example, the profits of firm 1 are shown in Table 5.
Unlike the *ex ante* model, this model has no dominant strategy for all realized costs. For all $x_i^* > 0$, firm 1's profits increase with disclosure if and only if $\bar{c} > c$, regardless of whether firm 2 discloses its cost information. Similarly, firm 2's profits increase with disclosure if and only if firm 2's realized cost is below the mean cost, regardless of whether firm 1 discloses its cost information. Firms with costs greater than the mean will find their profits reduced by information sharing while firms with costs less than the mean will find their profits increased by information disclosure. Therefore, a subgame-perfect Bayesian equilibrium specifies that each firm will disclose its cost information if its costs are less than the mean and not disclose its cost information otherwise (assuming a firm's failure to disclose does not signal high costs).

Therefore, all four combinations of strategies in Table 5 can be the subgame-perfect Bayesian equilibrium.

The preceding analysis indicates when duopolists gain by sharing private information about their costs. What is the effect of the equilibria on consumer surplus? In summary, the subgame-perfect Bayesian equilibria described above do not guarantee the highest level of consumer surplus. The level of consumer surplus is equal to

$${\text{CS}} = \frac{1}{2}(x_1 + x_2)^2 \quad (7)$$

The level of consumer surplus in each situation is indicated in Table 6.
To compare the levels of consumer surplus, let $CS_{n,n}$ be the level of consumer surplus when neither firm discloses its information (so $CS_{n,n} = (4A+2\bar{c}-3c_1-3c_2)^2/72$). Also, let $CS_{n,d}$ be the level of consumer surplus when firm 1 does not disclose its cost information but firm 2 discloses its cost information (so $CS_{n,d} = (4A+\bar{c}-3c_1-2c_2)^2/72$). Table 7 provides a comparison of the levels of consumer surplus.

<table>
<thead>
<tr>
<th>Firm 1</th>
<th>Firm 2</th>
<th>No disclosure</th>
<th>Disclosure</th>
</tr>
</thead>
<tbody>
<tr>
<td>No disclosure</td>
<td>$CS_{n,n}$</td>
<td>$CS_{n,d}$</td>
<td></td>
</tr>
<tr>
<td>Disclosure</td>
<td>$CS_{n,n} + <a href="c_i-%5Cbar%7Bc%7D">8A-2\bar{c}+5(\bar{c}-c_1)-6c_2</a>/72$</td>
<td>$CS_{n,d} + <a href="c_i-%5Cbar%7Bc%7D">8A-4\bar{c}+5(\bar{c}-c_1)-4c_2</a>/72$</td>
<td></td>
</tr>
</tbody>
</table>

If firm 1’s realized cost is less than the mean, firm 1’s disclosure of its cost information in accordance with the subgame-perfect equilibrium reduces consumer surplus, regardless of whether firm 2 discloses its cost information. Symmetrically, if firm 2’s realized cost is less than the mean, firm 2’s disclosure of its cost information in accordance with the subgame-perfect equilibrium reduces consumer surplus, regardless of whether firm 1 discloses its cost information. Therefore, for example, if both firms have costs less than the mean cost, the level of consumer surplus generated by the subgame-perfect Bayesian equilibrium strategy, bilateral disclosure, is less than the level of consumer surplus that would occur if neither firm discloses its cost information.

Whether consumer surplus increases or decreases may depend on the parameters in each

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5To see this, note that $A > \bar{c}$ and $A > c_2$ imply that $8A - 2\bar{c} - 6c_2 > 0$ and $8A - 4\bar{c} - 4c_2 > 0$. So, if $\bar{c} > c_i$, then $[8A-2\bar{c}+5(\bar{c}-c_i)-6c_2](c_i-\bar{c})/72 < 0$ and $8A-4\bar{c}+5(\bar{c}-c_i)-4c_2](c_i-\bar{c})/72 < 0$. 

---
of the four possible equilibria. For example, if \( c_1 > \bar{c} \) and \( c_2 > \bar{c} \) (so firm 2's dominant strategy is nondisclosure), firm 1's dominant strategy is nondisclosure, and firm 1's nondisclosure results in higher consumer surplus if and only if \( 8A+3\bar{c}-5c_1-6c_2 < 0 \). If \( c_1 > \bar{c} \) and \( c_2 < \bar{c} \) (so firm 2's dominant strategy is disclosure), firm 1's dominant strategy is nondisclosure, and this increases consumer surplus if and only if \( 8A+\bar{c}-5c_1-4c_2 < 0 \). However, if \( c_1 < \bar{c} \) and \( c_2 < \bar{c} \) (so firm 2's dominant strategy is disclosure), firm 1's dominant strategy is disclosure, and firm 1's disclosure always decreases consumer. Similarly, if \( c_1 < \bar{c} \) and \( c_2 > \bar{c} \) (so firm 2's dominant strategy is nondisclosure), firm 1's dominant strategy is disclosure, and firm 1's disclosure always decreases consumer.

The preceding analysis indicates that consumers may not be better off when duopolists gain by sharing private information about their costs. What is the social welfare effect of information disclosure? Social welfare is the sum of the firms' profit levels and the level of consumer surplus. To compare the levels of welfare, let \( W_{n,n} \) be the level of welfare when neither firm discloses its cost information. Let \( W_{n,d} \) be the level of welfare when firm 1 does not disclose its cost information but firm 2 discloses its cost information. Specifically, \( W_{n,n} \) and \( W_{n,d} \) are defined as follows:

\[
W_{n,n} = [(2A+\bar{c}-3c_1)/6]^2 + [(2A+\bar{c}-3c_2)/6]^2 + \frac{1}{2}[(4A+2\bar{c}-3c_1-3c_2)/6]^2
\]

\[
W_{n,d} = [(2A-\bar{c}-3c_1+2c_2)/6]^2 + [(A+\bar{c}-2c_2)/3]^2 + \frac{1}{2}[(4A+\bar{c}-3c_1-2c_2)/6]^2
\]

The levels of welfare are indicated in Table 8.
To make this negative, c_1 must be significantly below the mean (to dominate the positive effects of (A-c_1) and (c_1 - c_2)).

The sign of this condition depends on the parameters. Likewise, if c_1 < c and c_2 > c (so firm 2's dominant strategy is nondisclosure), firm 1's dominant strategy is disclosure, and firm 1's disclosure increases social welfare if and only if 16A-3c_1+17c_1-30c_2 < 0. The sign of this condition also depends on the parameters. Similarly, if c_1 > c and c_2 > c (so firm 2 does not disclose its cost information), firm 1's nondisclosure in accordance with the subgame-perfect equilibrium increases social welfare if and only if 16A-3c_1+17c_1-30c_2 < 0. Finally, if c_1 > c and c_2 < c (so firm 2 discloses its cost information), firm 1's nondisclosure in accordance with the subgame-perfect equilibrium increases social welfare if and only if 16A+11c_1+17c_1-44c_2 > 0. The effect of firm 2's disclosure on welfare is analogous. In conclusion, all four combinations of strategies may be the equilibrium. However, whether the equilibrium play of the firms maximizes consumer surplus or welfare often depends on the parameters.

Antitrust laws may apply to agreements to exchange cost information. Therefore, I will analyze the incentive of firms to agree to exchange information. In the analysis below, I assume

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6To make this negative, c_1 must be significantly below the mean (to dominate the positive effects of (A-c) and (c - c_2)).

7To make this negative, c_1 and c_2 must be significantly below the mean (to dominate the positive effects of (A-c)).
that, if both firms disclose their cost information, they do so pursuant to an agreement to
exchange information. The difference in firm 1’s profit between when both firms disclose and
when neither firm discloses can be expressed as follows:

\[
\Delta \pi_1 = \pi_{1, \text{Both disclose}} - \pi_{1, \text{Neither disclose}} \\
= \frac{2(c_2 - \bar{c}) + (c - c_1)}{36}
\]  

(9)

Taking the expectation of expression (9) over the distribution of \(c_2\) yields:

\[
E_{c_2}[\Delta \pi_1] = E_{c_2}[\pi_{1, \text{Both disclose}} - \pi_{1, \text{Neither disclose}}] \\
= \frac{(c - c_1)}{36}
\]  

(10)

This amount is positive if firm 1’s costs are below the mean. If firm 1’s costs are below the mean,
firm 1 has an incentive to cooperate with its rival \textit{ex post} to exchange cost information.

The difference in consumer surplus between when both firms disclose and when neither
firm discloses can be expressed as follows:

\[
\Delta CS = CS_{\text{Both disclose}} - CS_{\text{Neither disclose}} \\
= \frac{8A - 8\bar{c} + 5(\bar{c} - c_1) + 5(\bar{c} - c_2)}{72}[c_1 - \bar{c} + (c_2 - \bar{c})]/72
\]

(11)

Disclosure by both firms reduces consumer surplus if the realized costs of both firms are below
the mean. If both firms have costs less than the mean cost, consumers would be better off when
firms are barred from disclosing their cost information than when firms disclose their cost
information in accordance with the subgame-perfect equilibrium.

The difference in social welfare between when both firms disclose and when neither firm
discloses can be expressed as follows:

---

8To see this, note that \(A > \bar{c}\) implies that \(8A - 8\bar{c} > 0\). So, if \(\bar{c} > c_1\) and \(\bar{c} > c_2\), then \([8A - 8\bar{c} + 5(\bar{c} - c_1) + 5(\bar{c} - c_2)][(c_1 - \bar{c}) + (c_2 - \bar{c})]/72 < 0\).

15
\[ \Delta W = W_{\text{Both disclose}} - W_{\text{Neither discloses}} \]
\[ = \frac{16(A-\bar{c})(c_1+c_2-2\bar{c}) + 17(c_2-c_1)^2 - 40(c_1-c_2)^2}{72} \]  \hfill (12)

When \( A > \bar{c} \), \( c_1 < \bar{c} \) and \( c_2 < \bar{c} \), the sign of this expression depends on the parameters. For example, if \( c_1 < \bar{c} \), \( c_2 < \bar{c} \) and \( c_1 = c_2 \), then \( \Delta W < 0 \). Under these conditions, the firms participate in a prisoner's dilemma. In other words, although social welfare is higher when these firms are barred from disclosing their cost information, each firm's dominant strategy is disclosure.

Further analysis of expression (12) reveals that an increase in the difference between \( c_1 \) and \( c_2 \), \textit{ceteris paribus}, increases \( \Delta W \). Also, if \( c_1 < \bar{c} \) and \( c_2 < \bar{c} \), then an increase in \( A \) decreases \( \Delta W \).

In the model described above, firms decide whether to disclose cost information after realizing their costs. In summary, when demand is linear and marginal costs are constant, a risk-neutral firm is better off disclosing its private cost information \textit{ex post} if its realized cost is below the mean. Disclosure by a firm with low costs discourages production by the firm's competitor which raises the residual demand for the low-cost firm. If the costs of both firms are below the mean, they have incentive to agree to exchange cost information. However, disclosure by low-cost firms reduces consumer surplus. The effect of such disclosure on social welfare depends on the parameters.

\textbf{Incentives for collusive information exchange with risk of antitrust liability}

Regardless of the timing of the decision to disclose cost information, firms may have an incentive to collude to disclose their cost information. In models in which each firm decides whether to disclose its cost information prior to realizing its costs, both firms are better off \textit{ex ante} when they exchange cost information than when only one of them discloses or none of them discloses. In the model in which firms decide whether to disclose cost information after realizing...

15 U.S.C. § 15 states as follows: "Any person who shall be injured in his business or property by reason of anything forbidden in the antitrust laws may sue therefore in any district court of the United States in the district in which the defendant resides or is found or has an agent, without respect to the amount in controversy, and shall recover threefold the damages by him sustained, and the cost of suit, including a reasonable attorney's fee."

Proof by a private plaintiff of antitrust violation is greatly simplified by a prior successful government prosecution because "[a] final judgment or decree ... rendered in any civil or criminal proceeding brought by ... the United States under the antitrust laws to the effect that a defendant has violated said laws [is] prima facie evidence against such defendant in any action or proceeding brought by any other party against such defendant." 15 U.S.C.A. § 15, Sect. 4. Thus, successful government prosecution leads, indirectly, to private damages.

The preceding analysis assumes that firms ignore possible penalties--criminal and/or civil--associated with disclosure of cost information. Gary S. Becker posited that an individual would engage in criminal activity only if the private gains exceed the costs including the cost of a fine. Because enforcement is costly and the same level of deterrence can be achieved at less cost by raising the fine and lowering the probability of detection, Becker argued that the enforcement authority should set the probability very low and the fine correspondingly high.

When the penalty is fixed, the level of deterrence is determined by the probability of enforcement. Antitrust laws are an example. Any violation of the antitrust laws exposes the firm to the risk of being held liable under section 4 of the Clayton Act (15 U.S.C. § 15) for payment of treble damages to injured private plaintiffs. Public prosecution may act as a trigger for private antitrust litigation. Antitrust authorities are concerned about the disclosure of firm-specific

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11Proof by a private plaintiff of antitrust violation is greatly simplified by a prior successful government prosecution because "[a] final judgment or decree ... rendered in any civil or criminal proceeding brought by ... the United States under the antitrust laws to the effect that a defendant has violated said laws [is] prima facie evidence against such defendant in any action or proceeding brought by any other party against such defendant." 15 U.S.C.A. § 15, Sect. 4. Thus, successful government prosecution leads, indirectly, to private damages.
information because it may facilitate coordination among firms.\textsuperscript{12}

In the model presented below, the timing is as follows: First, the antitrust authority picks the probability of enforcement. Each firm observes the probability of enforcement and then decides whether to agree to disclose its private cost information. Cournot competition follows. While economists often seek to maximize social welfare defined as the sum of consumer surplus and the aggregate profit of all firms, the antitrust agencies often prefer to maximize consumer surplus only. As demonstrated above, in the model in which firms decide whether to disclose cost information after realizing their costs, if both firms' realized costs are less than the mean cost, each firm has an incentive to cooperate with its rival \textit{ex post} to exchange cost information. However, nondisclosure by both firms will always provide a higher level of consumer surplus and may provide a higher level of social welfare than an information exchange agreement. The antitrust agency will seek to discourage information exchange agreements if its goal is to maximize consumer surplus or to maximize social welfare and the parameters are such that nondisclosure provides a higher level of social welfare than an information exchange agreement. The antitrust agency can choose the probability of successful antitrust action such that an information exchange agreement is not a perfect Bayesian Nash equilibrium. Specifically, the antitrust agency can raise the probability of successful antitrust action until it would not be profit

\textsuperscript{12}See, Maple Flooring Mfrs. Ass'n v. United States, 268 U.S. 563, 45 S.Ct. 578, 69 L.Ed. 1093 (1925)(Allowing a trade association to circulate mean cost data among its members); United States v. Container Corp., 393 U.S. 333, 89 S.Ct. 510, 21 L.Ed.2d 526 (1969)(Prohibiting an agreement to exchange price information); United States v. Machine Chain Manufacturers Ass'n, (D.R.I. 1955), CCH 1955 Trade Cases, ¶68, 009; cf. Chain Institute, Inc. v. Federal Trade Commission, 246 F. 2d 231 (8th Cir. 1957)(An association of manufacturers of machine-made chain and individual members of the association consented to a judgment enjoining them from exchanging any statistics representing costs of operation for the purpose or with the effect of fixing prices or otherwise restraining trade). A series of volumes published by Commerce Clearing House and known popularly as the "Bluebook" reports that 61 cases of horizontal conspiracies involving the exchange of information arose between 1890 and 1969.
maximizing for the firms to collude to exchange information. I discuss below the firms' incentives, \textit{ex ante} and \textit{ex post}, and compare the two extreme situations: no information disclosure and collusive information sharing.

The expected penalty is the product of the probability of a successful antitrust action and the judgment amount. The judgment amount is a multiple of the overcharge.\textsuperscript{13} The overcharge is defined as the difference between the price that occurs when information is disclosed and the price which would have prevailed when the firms compete without disclosing private cost information, \((A-x_i-x_j)-P_c\).\textsuperscript{14} Therefore, \(3\{(A-x_i-x_j)-P_c\}x_i\) is the fine that firm \(i\) must pay if held liable for triple damages under the Clayton Act.\textsuperscript{15} The firms' subjective probability of a successful antitrust lawsuit is \(\gamma\).\textsuperscript{16} For simplicity, I assume that the probability of a successful

\textsuperscript{13}This multiple is 1 when the government is the victim or when the seller is participating in a certified research joint venture or certified production joint venture.

\textsuperscript{14}\(P_c\) can be interpreted as the price that occurs under competitive conditions. \(P_c\) is the reference price if nondisclosure is the noncooperative equilibrium because it represents the price that occurs when no firm discloses its private information. If disclosure is the noncooperative equilibrium, then \(P_c\) is the reference price for calculation of damages if firms do not disclose their cost information to avoid accidental or erroneous conviction of an information exchange agreement in violation of the antitrust laws.

\textsuperscript{15}For example, \textit{Reiter v. Sonotone Corp.}, 442 U.S. 330, 60 L.Ed. 931, 99 S.Ct. 2326 (1979). The court in \textit{City of Atlanta v. Chattanooga Foundry & Pipe Co.}, 101 Fed. 900, 901 (E.D. Tenn. 1900), \textit{affd.}, 203 U.S. 390 (1906), calculated the amount of damages by multiplying the quantity purchased by the plaintiff(s) by the "estimated difference between the just and fair market price of the goods and the price actually paid...."

Block, Nold, and Sidak (1981) use the letter \(t\) as the anticipated damage multiplier to capture the combined criminal penalty and civil treble damages. The multiple must be at least as great as 1 because a fine equal to the overcharge alone would not deter would-be violators. The justification for the use of multiple damages rather than single damages is that multiple damages can deter would-be violators even when there is a chance that violations will escape successful prosecution.

\textsuperscript{16}The probability of a successful antitrust lawsuit, \(\gamma\), can have several meanings: For example, the government may detect only a fraction of the violations and prosecute all those firms whose violations are detected. Alternatively, the government may detect all violations but have the resources to prosecute only a fraction of the violators. It is also possible that the
suit is independent of the size of the award. With probability $\gamma$, a successful antitrust lawsuit is brought and the firm must pay treble damages. With probability $(1-\gamma)$, disclosure does not result in an adverse judgment.

If there is a risk of antitrust liability for triple damages, duopolist $i$ picks $x_i$ to solve the following expression:

$$\max E_{\gamma} \{ (A-x_i-x_j-c_i)x_i - 3\gamma \{ (A-x_i-x_j) - P_j \}x_j \} \quad i = 1,2; \ j = 1,2; \ i \neq j \quad (13)$$

The maximization problem in the absence of risk of antitrust liability, expression (2) above, is the special case of expression (13) in which $\gamma = 0$. The analysis below assumes that firms that disclose cost information bear a risk of antitrust liability. Setting the first derivative of expression (13) equal to zero yields

$$x_i = \frac{1}{2}(A-E[x_j]-c_i-3\gamma(A-x_i-E[x_j]-E[P_j])/(1-3\gamma) \quad i = 1,2; \ j = 1,2; \ i \neq j \quad (14)$$

If equilibrium output is $x^*_i$, one can show from equations (13) and (14) that profits will be equal to $(1-3\gamma)(x^*_i)^2$. The second derivative of the expected profit function (13) is less than zero for all $\gamma < \frac{1}{6}$. If $\gamma > \frac{1}{6}$, the firm will not disclose its cost information because it would expect to pay more in treble damages than it receives in profits (i.e., the fine discounted by the probability of conviction is greater than the expected benefit from disclosing information). More formally, if the product of the probability of conviction times the damage multiple of the overcharge exceeds 1, information disclosure is totally deterred.

From the demand function and the equilibrium output quantities when neither firm

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17 Of course, a private plaintiff will sue a defendant for damages if the plaintiff’s expected award, the probability of success times the award, exceeds expected costs.
discloses, $P_\varepsilon$ can be expressed as

$$P_\varepsilon = \frac{(2A - 2c + 3c_1 + 3c_2)}{6}$$  \hspace{1cm} (15)

The equilibrium output quantities are indicated in Table 9.

<table>
<thead>
<tr>
<th>Firm 1\Firm 2</th>
<th>No disclosure</th>
<th>Disclosure</th>
</tr>
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<tbody>
<tr>
<td>No disclosure</td>
<td>$x_1 = (2A + \bar{c} - 3c_1)/6$</td>
<td>$x_1 = \frac{[2(1-\gamma)A-(1-2\gamma)\bar{c} - (3\gamma-2)c_2-3(1-3\gamma)c_1]}{(6(1-3\gamma))}$</td>
</tr>
<tr>
<td></td>
<td>$x_2 = (2A + \bar{c} - 3c_2)/6$</td>
<td>$x_2 = \frac{[(1-\gamma)A+(1-2\gamma)\bar{c} + (3\gamma-2)c_2]/(3(1-3\gamma))}{6(1-3\gamma)}$</td>
</tr>
<tr>
<td>Disclosure</td>
<td>$x_1 = \frac{[(1-\gamma)A+(1-2\gamma)\bar{c} + (3\gamma-2)c_2]/(3(1-3\gamma))}{6(1-3\gamma)}$</td>
<td>$x_1 = \frac{[2(1-2\gamma)A-2\gamma\bar{c}+(3\gamma-4)c_1+(3\gamma+2)c_2]}{(6(1-3\gamma))}$</td>
</tr>
<tr>
<td></td>
<td>$x_2 = \frac{[2(1-2\gamma)A-2\gamma\bar{c}-(3\gamma-2)c_1-3(1-3\gamma)c_2]}{(6(1-3\gamma))}$</td>
<td>$x_2 = \frac{[2(1-2\gamma)A-2\gamma\bar{c}-(3\gamma-4)c_2+(3\gamma+2)c_1]}{(6(1-3\gamma))}$</td>
</tr>
</tbody>
</table>

In the absence of the risk of antitrust liability, an information exchange agreement is a perfect Bayesian Nash equilibrium if the realized costs of both firms are less than the mean. However, the antitrust agency can choose the probability of successful antitrust action such that an information exchange agreement is not a perfect Bayesian Nash equilibrium. To do so, the antitrust agency must set the probability of successful antitrust action high enough so that it would not be optimal for firms to agree to exchange cost information. If information exchange agreements are to be discouraged, it is necessary to determine the probability of enforcement that discourages such agreements ex ante and ex post. The increase in profit that firm 1 receives when it joins an information exchange coalition is

$$\Delta \pi_1 = \pi_{1,\text{Both disclose}} - \pi_{1,\text{Neither discloses}}$$  \hspace{1cm} (16)
\[= (1-3\gamma)[(2(1-2\gamma)A-2\gamma \bar{c}+(3\gamma-4)c_1+(3\gamma+2)c_2)/(6(1-3\gamma))]^2 - \{(2A+\bar{c}-3c_i)/6\}^2\]

To discourage information exchange agreements *ex post*, the probability of enforcement must be such that \(E_{c_2}[\Delta \pi_1] < 0\) where expectation is taken with respect to \(c_2\). If \(E_{c_2}[\Delta \pi_1] < 0\), then an information exchange agreement is strictly dominated by nondisclosure for firm 1. To deter information exchange agreements *ex ante*, the probability of enforcement must be such that \(E_{c_1}[E_{c_2}[\Delta \pi_1]] < 0\) where expectation is taken with respect to both \(c_1\) and \(c_2\). If \(E_{c_1}[E_{c_2}[\Delta \pi_1]] < 0\), then an agreement to exchange information is strictly dominated by nondisclosure for each firm.

For a given \(A\) and a distribution of firm costs, one can find the minimum \(\gamma\) that satisfies these conditions by solving tedious quadratic expressions in \(\gamma\).

**Conclusion**

In previous models, firms realize their costs after they decide whether to disclose their cost information. The results showed that when demand is linear and marginal costs are constant, sharing cost information increases expected profit and expected social welfare but reduces expected consumer surplus. If firms realize their costs before they decide whether to disclose information, the results are modified. A risk-neutral firm is better off disclosing its private cost information *ex post* if its realized cost is below the mean cost. Also, consumer surplus decreases if the disclosing firm's realized cost is below the mean cost. The effect of disclosure on social welfare depends on the parameters.

The antitrust agency may choose the probability of successful antitrust action such that an information exchange agreement is not a perfect Bayesian Nash equilibrium. However, if reporting and verification costs are low and if unilateral disclosure of cost information creates no risk of antitrust enforcement, low-cost firms unilaterally will report their cost realization *ex post.*
Ironically, if all firms have costs below mean, full disclosure will be the noncooperative result.

Changes in the timing of the decision to disclose information may cause changes in incentives and welfare implications in other models. Further research is required to determine the effect of changes in the timing of the decision to disclose information when the assumptions of linear demand, constant marginal costs and risk-neutrality are relaxed. Also, the model may be analyzed under the assumption of Bertrand competition or other behavior. Finally, the incentive to disclose information about a common parameter such as a common demand intercept may change with changes in the timing structure.
References


Studies. 53 (1986), pages 433-446.