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**The CAPM and the Duration of Poorly Performing Mutual Funds**

by

Alexander J. Keenan

## Abstract

Using duration analysis and CAPM, this paper seeks to estimate the length of time performance measures affect the probability of a mutual fund liquidating. Data was collected on small cap growth funds from 1980-Nov. 2000 using the Sharpe Ratio to estimate the probability that a mutual fund closes due to poor performance. Using a parametric approach the results show that a fund with a lower Sharpe ratio as well as overall strong performance by the market increases the probability of a fund's failure. The results also show the existence of positive duration implying older funds face a higher probability of failure. The results are then compared to other models to test the appropriateness of the model.

## I. Introduction

The existence of performance persistence in mutual funds has recently emerged as a popular field of study in finance. Several articles have been written suggesting evidence of positive performance persistence, implying that a fund earning excess returns in one period will also perform well in later periods. As Malkiel states, “good performing funds tend to continue to perform well, at least over the near term” (Malkiel 1995). For mutual funds that underperform the market, there is even stronger evidence to suggest that they will continue to underperform (Peterson 1998). These results are surprising because it implies that investors do not punish poorly performing mutual funds.

It has been suggested that investors tend to over-emphasize the importance of recent performance when weighing investment decisions. They tend to chase past performance and flock to top rated mutual funds. If that were the case, however, one might also conclude that there would be a significant drain away from underperforming mutual funds as people shift their portfolios to chase the winners. Barber and Odean (2000) find, however, that investors appear to be loss averse and often unwilling to realize any losses. This suggests they will continue to hold losing mutual funds even in the face of poor performance persistence.

Investors are not willing to hold a losing portfolio indefinitely, however, and will eventually adjust his or her portfolio to weed out the worst performing mutual fund. Therefore, there needs to be a better understanding of how quickly investors process

information regarding a mutual fund's returns, and the length of time to react to the returns. As Lunde, Timmerman, and Blake (1998) write, "If poorly performing funds tend to close down after only a short period, this suggests that investors possess good information about the fund performance, whereas a longer average time before closure might indicate that performance signals are weak, so that investors need more time before they can identify a fund's genuine performance."

There are several reasons why a mutual fund may close, including managerial changes or deciding that two of a firm's mutual funds are too closely related. Or, as Brown (1995) states, "Fund disappearance is a management decision which is presumable based upon the funds profitability and ultimately on consumer demand."

Using duration analysis and portfolio choice theory, this paper seeks to estimate the length of time performance measures affect the probability of a mutual fund liquidating.

The paper is organized as follows: the next section describes recent literature, followed by the dataset, then a layout of the model in section four. In section five, the model is estimated and the results are discussed, and section six concludes.

## II. Related Literature

While much has been written about the performance of mutual funds (Sharpe 1966), and the performance persistence in mutual fund returns Carpenter (1999), Carhart (1997),

Hendricks, et al (1993), Brown (1992), and Grinnblatt (1992) and whether mutual funds persist for several periods; less has been written about investor behavior in response to poor performance. Harless and Peterson construct two models to compare investor behavior by estimating growth in assets. In one model the investor chooses funds on the basis of risk adjusted returns and another assumes a representativeness heuristic investor who responds only to the most recently available returns, not taking into account the risk adjusted return or the validity or recent returns with respect to future performance. They find “that investors do not combine returns with risk in accord with this [Jensen’s alpha] performance measure. Rather intuitive judgements about fund performance are overly influenced by extreme recent returns unadjusted for differential risk” (Peterson 1998). Additionally they find that investors ignore differences in expense ratios between funds that tend to have a high predictive power for future returns. Although the Peterson paper discusses the flow of assets into a mutual fund, it does not analyze the life of the mutual fund as to whether it stays in operation.

Continuing with the representativeness heuristic model, Barber and Odean (2000) state that investor’s response to recent poor performance is “more than offset by investors’ reluctance to realize losses (disposition effect).” In this model they will hold losing mutual funds too long and sell winners too quickly. Again this implies that fund survival does not depend of the return a fund generates (at least in the immediate future).

Brown and Goetzmann (1995), on the other hand, estimate the probability of a mutual fund closing. They estimate a probit on fund disappearance using relative return that year

(fund's return that year minus average fund return), relative size, expense ratio, and age to explain the probability of a fund closing. Their results return consistent coefficients with respect to previous research. Low relative returns and higher expense ratios increase the probability of fund closure. The larger the fund, and the older the fund, the lower the probability of closing, although they state that it is difficult to separate size from past performance because well-performing funds tend to attract customers. An interesting observation about the model set-up is that even though there was a negative coefficient on relative return, the returns were not estimated on a risk-adjusted basis. In this case one could not distinguish between a mutual fund with a historically greater variance (riskier investment) and a mutual fund with a one-year aberration. Although Brown's estimation highlights the importance of returns in the attrition process, it treats all returns as equal.

Lastly, Lunde, Timmermann, and Blake (1998) estimate a semi-parametric Cox proportional hazards model<sup>1</sup> to capture the probability of a fund closing using monthly data from UK open-ended mutual funds from 1973-1995. Their sample contains 973 mutual funds that die and 1402 surviving mutual funds (59% of the mutual funds). They model both relative fund return and risk adjusted fund return as covariates finding an inverse U-shape hazard rate. Negative performance by the mutual fund relative to its peer group leads to a higher hazard rate in 19 out of the 20 sectors of funds analyzed (8 were significant). They also find performance over the long-term (using three year performance) matters for survival. The reason given for the shape of the hazard rate is that investors need time to analyze a start-up mutual fund and allow the fund to establish a track record. Once a fund clears the initial hurdle, however, then performance seems to

dictate which of the funds survive. Eventually, those funds with a proven track record, given the performance persistence of the mutual funds, face a decreasing probability of failure. As Lunde (1998) states:

[The] estimates imply that the cumulative effect of a one- percent abnormal underperformance in a given month is to more than double a fund's hazard rate relative to a scenario of zero relative underperformance<sup>2</sup>. Similarly, the cumulative effect of a decrease in the return on UK equities of one percent in a given month is to increase the hazard rate by 17 percent. These estimates suggest that both relative and absolute performance are important determinants of the UK equity funds' hazard rate, with relative performance being particularly important.

In most of the current literature available, the existence of performance persistence of the fund raises questions regarding what measure of performance the investor is using when deciding to sell a mutual fund.

### III. Data

The data for this paper was taken from the tradeline.com mutual fund database<sup>3</sup>. There are approximately 13,096 currently active mutual funds that fall into 44 categories.

Additionally, data is maintained on about 3,897 closed mutual funds<sup>4</sup> (about 22.9% of all mutual funds) from 1973 to the present (the database contains all frequencies; monthly data is used in this paper). From the mutual fund universe, this paper focuses on funds

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<sup>1</sup> Please see Appendix for description of Cox proportional hazards model.

<sup>2</sup> Abnormal return in the paper is defined as the funds average return relative to the average return of the fund's sector during the same period.

<sup>3</sup> The author is extremely grateful to Brian Tietje at tradeline.com for providing the data as well as technical support.

<sup>4</sup> Closed mutual funds in this paper refer to liquidated mutual funds rather than close-ended mutual funds.



that fall under the small-cap growth category according to the Wall Street Journal. Small cap growth funds are described as:

Funds that invest in small companies with long-term earnings that are expected to grow significantly faster than the earnings of stocks in major indexes. Funds normally have above-average price-to-earnings ratio, price-to-book ratio, and three year earnings growth (WSJ 12/04/00).

There are several reasons for singling out a particular group, such as small-cap growth funds. Most importantly, looking at funds within the same category with similar investment goals removes the need to estimate a fund's "beta" and use risk adjusted returns when looking at performance. A priori, as a fund enters the market at  $t=0$ , funds operating with similar selection criteria have the same probability of success or failure. Only after the portfolio has been selected, and the fund manager takes an active role in the performance of the fund, do the mutual funds begin to distinguish themselves. Within the category, one cannot say that any particular fund or group of funds is biased towards success or failure over any other. Second, small cap growth funds have consistently underperformed standard benchmarks like the S&P 500 for the past several years. While the underperformance as a group continues, many individual funds persist and thrive, while others fail. Narrowing the focus to a group of underperforming funds could shed some light on the importance relative or absolute benchmarks returns play in the survival of the fund.

Table 1 shows the births and deaths of funds contained in the sample. The sample contains 255 currently active mutual funds and 81 liquidated funds (24.1% of all funds). It is obvious that growth in the small-cap market did not accelerate until the mid-to-late

Table 1: Small Cap Growth Mutual Funds, 1980 - Nov. 2000

	Active Funds	Births	Deaths	Average Age 1/
< 1980	9	9	0	6.5
1981	9	0	0	18.5
1982	9	0	0	30.5
1983	10	2	1	36.9
1984	14	4	0	36.1
1985	15	2	1	40.8
1986	22	7	0	40.8
1987	27	6	1	42.9
1988	30	3	0	46.5
1989	32	4	2	54.8
1990	38	7	1	52.7
1991	47	9	0	56.4
1992	58	13	2	55.0
1993	78	24	1	49.8
1994	109	35	4	45.3
1995	138	30	1	44.3
1996	160	29	7	46.0
1997	171	16	5	51.2
1998	201	43	12	53.8
1999	221	40	21	55.3
2000	255	56	22	54.4

Source: Tradeline.com  
1/ In months.

types of funds has lagged significantly behind the S&P 500 which large and mid-cap mutual funds usually track more closely.

There are several reasons for the increase in the number of small-cap funds in the 1990's. The small-cap market has several characteristics that make it unique. First, given the business cycles in the economy, small-cap stocks tend to be counter-cyclical. That is, when the U.S. economy is weakest, small cap stocks perform best. A quick glance at Table 2 shows that four of the five years that the small-cap funds out-performed the S&P 500 are years in or immediately following a recession. This is the main reason that there

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large increase in the

failure rate. The

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of small-cap funds is

actually a bit of a

surprise as over the

last five years the

performance of these

has been a large surge in the births of small-cap mutual funds in the early 90's. Coming out of the 1990-91 recession, investors saw the returns being generated by small-cap growth funds at the same time the whole mutual fund industry was beginning to grow. Due to superior performance of the small-caps and the growth of the mutual fund industry as a whole, the small-cap growth mutual funds grew quickly during 1991-93. Growth in the small caps slowed considerably in the mid 1990's as the economy boomed and the larger capitalized stocks generated higher returns.

Another reason for an increase in the births of small-cap mutual funds is the nature of the investment. According to Morningstar, a stock is considered a small-cap if the firm has a market capitalization of less than \$1bn (stock price multiplied by shares outstanding). This creates an interesting issue for the fund manager to consider. First, the firm (and therefore the mutual fund that holds that stock in its portfolio) that generates fast growth finds its market capitalization rising quickly as interest in the company grows. A mutual fund manager must decide whether to keep that company in the portfolio and reclassify the fund, or divest from that company<sup>5</sup>. Second, as the mutual fund generates high returns because of the high growth stock assets, will quickly flow into that mutual fund. A substantial increase in the fund's assets impedes the fund's ability to invest in small-cap companies, as the fund's assets are too big. In this situation the mutual fund would have to decide whether to keep that fund open to new investors or to close it to new investors. In both cases there would likely be the launch of a new mutual fund with the same asset

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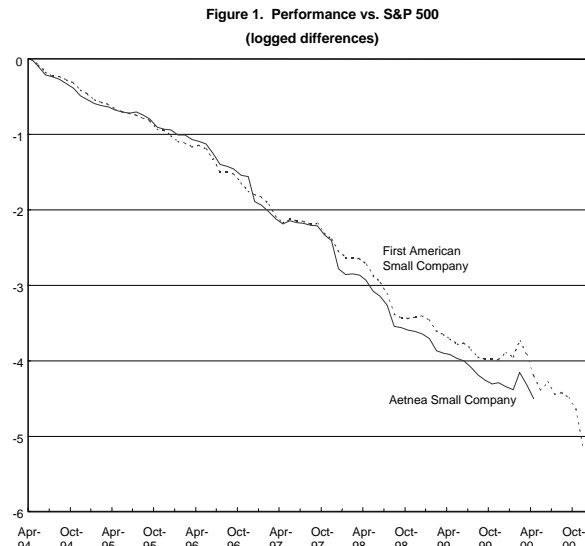
<sup>5</sup> In this paper mutual funds that are re-classified are not included in the sample as they have not died, nor have they had consistent investment criteria.

classification as the previous one. One would therefore expect increased small-cap growth mutual fund births.

Figure 1 shows two mutual funds' relative performance against the S&P 500. Both of the mutual funds began in the same month, and show relatively dismal performance against the market.

But the First American Mutual Funds continues to attract new money and currently has assets over \$330 million (the average for active

small-cap funds is about \$260 million in assets), whereas Aetnea ceased operation in April 2000.



The similar performance of these two funds against the S&P suggest that investors do not strictly analyze the return of the mutual fund or create absolute benchmarks when deciding divest from a fund.

Finally, Table 2 highlights the returns of the mutual funds over the sample period. The return of a mutual fund is calculated according to Brown (1995) as:

$$Ret = \frac{NAV_t - NAV_{t-1}}{NAV_{t-1}} + \frac{Div_t}{NAV_{t-1}}$$

The Net Asset Value (NAV) is the mutual fund's equivalent to a stock price<sup>6</sup>. Dividend disbursements, however, come infrequently and can cause large swings in one month's returns. In order to smooth the return data, average monthly returns are used in the model rather than total returns or annualized returns. The data is broken out into active and

Table 2:  
Average Monthly Return  
(1980 - Nov. 2000)

S & P	Mutual Funds	
	Alive	Dead
< 1980	2.50	3.22
1980	-0.36	-0.47
1981	1.77	1.63
1982	1.75	1.18
1983	0.58	-1.51
1984	2.38	1.59
1985	1.56	-0.10
1986	0.04	-1.44
1987	1.33	1.10
1988	2.38	1.64
1989	-0.13	-0.60
1990	2.33	3.63
1991	0.63	0.65
1992	0.82	1.05
1993	0.15	-0.28
1994	2.70	2.22
1995	1.78	0.95
1996	2.52	0.81
1997	2.30	0.49
1998	1.67	3.53
1999	-0.79	-1.04
2000		...

Source: Tradeline.com

inactive mutual funds to highlight the differences in performance. Only in 1983 and 1984 are the returns of the mutual funds that will eventually liquidate above the returns of active funds.

One question that naturally follows after looking at Table 2 is why have small-cap growth funds underperformed the market so frequently. In fact, in only five of the twenty-one years in the sample did small

<sup>6</sup> "A mutual fund calculates its NAV by adding up the current value of all the stocks, bonds, and other securities (including cash) in its portfolio, adjusting for expenses, and then dividing that figure by the fund's total number of shares" (morningstar.com). There are differences between a company's stock price and the NAV. For example while there is only a certain amount of stock for a given company, there are not limits on the number of shares available in a mutual fund. Additionally, while one can only buy shares of stock in units, there is no restriction to owning fractions of shares of mutual funds.

caps outperform the S&P 500. There are three major reasons why small-cap growth funds have been among the lowest performers in the mutual fund universe. First, small-cap growth funds face two liquidity issues. The small-cap market itself is illiquid because of the nature of the stocks, which creates disincentives to invest in the small-cap funds. As Robert Markam explains, "If [an institutional investor] has \$10 million to invest, he's not going to put that money into an illiquid sector. Instead you are probably going to buy larger companies . . . .At least he knows he can get out if he has to. In a small-cap market that's not so liquid, that puts tremendous pressure on the sector."

So in the small-cap market fewer, individuals can cause larger swings in performance of the market due to liquidity issues. Additionally, small-cap managers have devoted more of their holdings to cash than their large-cap competitors because of the liquidity issues in that market. Since larger firms are more liquid, managers are able to invest a higher percent of their assets in the market increasing the return of the fund over the long term.

Second, small-cap stocks underperform because they are counter-cyclical. The sample time period contains only one recession, when small-caps are expected to perform best. Given the record expansion of the economy over the last decade, it is perhaps not surprising that small-caps have suffered by comparison. The implication is that the larger companies stocks are hurt more during a recession as they are forced to go through a painful cost-cutting and restructuring as profits tank. The stocks will then rebound as the economy starts to grow, and the larger companies emerge more efficient and profitable and see large increases in the stock price.

Finally small cap growth have performed poorly due to “losers bias”. Growth stocks are expected to have a high P/E ratio that suggests high future growth and profits. A successful company will not remain a small-cap stock for long. As the company grows, the mutual fund must be reclassified if it is going to hold that particular stock or the portfolio must be rebalanced. What remains are either poorly performing portfolios that are still considered small cap or a mutual fund that is now a mid or large cap fund and not contained in the sample. As the selection of small-cap growth funds become slimmer there is a downward bias on the performance, of those funds, because of the investment criteria.

When comparing the returns to the S&P 500, it is important to note the survivorship bias in existence when comparing returns. Historical estimates of sectoral returns often calculate only the returns of those mutual funds that are currently active, thereby overstating the true performance of the sector. Brown and Goetzman (1995) calculate the survivorship bias for mutual funds and found that equal-weighting the funds, including funds that closed, lowered the performance by about 0.8% in 1988, while value-weighting the mutual funds lowered the performance by only 0.2%. This suggests, of course, that larger funds are less likely to close, and therefore stay within the sample. In this sample, equal-weighting the returns lowers the overall return for small cap mutual funds by about .25% in 1999. Using returns to measure a fund’s performance one can estimate how that performance influences a fund’s survival.

#### IV. The Model

While there may be several reasons why a fund may liquidate, the most interesting analysis comes from investors rebalancing non-efficient portfolios in an effort to seek higher returns. This model seeks to draw a link between the utility maximizing investor who responds to the performance of any particular mutual fund by adjusting his/her mutual fund. If the mutual fund is underperforming by some measure, theory would state that investors would pull money away from the asset and as money left the asset, the probability of the asset liquidating would increase.

There are several current portfolio choice models available including Fama-French model (1993) based on book-to-market factors, and Daniel-Titman model (1997) based on the characteristics of the firm to estimate excess returns. One of the earliest advances in modern portfolio theory, however, was the development of the CAPM model by Sharpe (1964) and Lintner (1965) creating the link between expected utility maximizers and risk and return. Assume a pure risk free interest rate to which all investors have access, with return  $R_0$ . Second, assume all investors have homogeneous expectations with respect to return and variance of any asset and base their investment decision strictly on the expected return and expected risk. Lastly, assume there are no research or transactions costs. Given those assumptions, the investor maximizes his utility according to the following constrained maximization problem (for computational ease, assume three assets in the portfolio, although the results hold for the more general case)<sup>7</sup>.



$$\begin{aligned}
& \text{Max } V(ER_M, \sigma^2(R_M)) \\
& \text{s.t. } \alpha_0 + \alpha_1 + \alpha_2 = 1 \\
& \quad ER_M = \alpha_0 R_0 + \alpha_1 ER_1 + \alpha_2 ER_2 \\
& \quad \text{Var}(R_M) = \alpha_1^2 \text{var}(R_1) + \alpha_2^2 \text{var}(R_2) + 2\alpha_1 \alpha_2 \text{cov}(R_1, R_2).
\end{aligned}$$

Substituting  $\alpha_0 = 1 - \alpha_1 - \alpha_2$  and taking the derivative with respect to  $\alpha_1$  and  $\alpha_2$  yields

$$\begin{aligned}
(ER_1 - R_0)V_1 + 2(\alpha_1 \text{var}(R_1) + \alpha_2 \text{cov}(R_1, R_2))V_2 &= 0 \\
(ER_2 - R_0)V_1 + 2(\alpha_1 \text{var}(R_2) + \alpha_2 \text{cov}(R_1, R_2))V_2 &= 0
\end{aligned}$$

where the first equation can be multiplied by  $\alpha_1$  and the second by  $\alpha_2$  and summed to be rewritten as

$$\begin{aligned}
(ER_M - R_0)V_1 + 2\text{var}(R_M)V_2 &= 0, \text{ using the definitions of } ER_M \text{ and } \text{Var}(R_M). \text{ Lastly using} \\
\text{the two equations above we can write} \\
ER_i - R_0 &= \text{cov}(R_i, R_M) / \text{Var}(R_M) (ER_M - R_0).
\end{aligned}$$

Where  $\beta_i = \text{Cov}(R_i, R_M) / \text{Var}(R_M)$  is the standard OLS regression result of the above equation.

Any portfolio that has the highest expected return for that given level of variance (risk) is said to be on the mean-variance efficient frontier. Additionally, although the  $R_M$  is the return for an individual investor's wealth, defining  $R_M$  as the market portfolio as the weighted sum of all investor's portfolios; then the market portfolio is mean-variance efficient as well. Lastly, the assumption of the ability to borrow at the risk free interest rate implies that any investor can have an efficient portfolio regardless of the investors

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<sup>7</sup> The derivation of the CAPM follows the same derivation used by Jagannathan (1995).

risk aversion, therefore we do not need to assume that all the investors have the same expected utility function.

We have then created the link between the excess return of a particular asset ( $ER_i - R_0$ ) and the excess market return by  $\beta_i$ , which completely captures the variation of cross sectional returns. For a given class of mutual funds (i.e. small cap funds)  $\beta_i$  should be identical, or near identical, as the whole class should have the same risk premium. The CAPM implies that any asset should have the same ratio of expected excess return ( $ER_i - R_0$ ) to risk ( $\beta_i$ ).

The Sharpe ratio is then defined as:

$$(ER_i - R_0)/\sigma_i$$

This ratio measures the amount of expected return per unit of risk. This measure allows comparisons between different asset's performance. The higher the Sharpe ratio the closer the asset is to the mean-variance frontier. One would expect that mutual funds with low Sharpe ratios would have a shorter life than their more efficient partners. While the Sharpe ratio is intended to determine which assets to invest in, it should also be used as an indicator of what assets to divest from.

While some aspects about the validity of the CAPM model have been questioned, the idea that investors balance risk and return has become standard finance theory. The question then becomes what is the probability that a mutual fund closes due to poor

performance. In order to model the probability of a mutual fund closing over time, some concepts over survival analysis must be introduced.

The key concept in understanding the probability of a mutual fund's failure is the fund's hazard rate. The hazard rate measures the probability that a fund will fail in the next period  $(t+d)$  given that the fund has lasted until today  $(t)$ . Mathematically:

$$\lim_{d \rightarrow 0} \lambda(t) = \frac{\text{Prob}(t < T < t+d \mid T > t, x\beta)}{d}$$

The first step in finding the hazard rate is to take the funds out of calendar time and transpose them into duration time. It is not of concern what caused the mutual fund to begin, or even when it began, but rather after the mutual fund starts, what affects its probability of closing. By transforming the data one is able to compare the probability of survival for a fund starting in January 1984 living for two years and a mutual fund starting in May 1990 surviving until present. Therefore, all the mutual funds in the sample have the same birth date of  $t=0$ .

After calculating the hazard function to look at the rate of failure, the counterpart of the hazard is the survival function. The survival function measures the probability that a mutual fund has lived at least until age  $t$ .

$$S(t) = \text{Prob}(T > t) = \exp\left(-\int_0^t \lambda(w)dw\right)$$

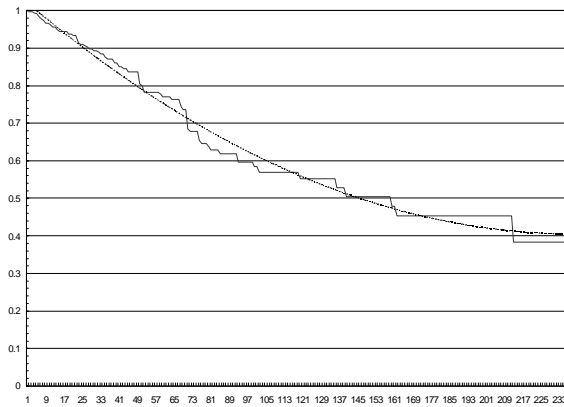
Figure 2 shows the non-parametric Kaplan-Meier estimate for the survivor function:

$$\hat{S}(t) = \prod (n_i - h_i) / n_i \text{ where } h_i \text{ is the number of failures at time } i \text{ and } n_i \text{ is the risk set at time } i$$

i. The risk set in this example is the existing funds in the previous period minus the number of censored funds at  $i$ . For example, if at time  $t=1$  there were four funds in the sample, and one fund fails at  $t=1$  and there were no observations left for another (end of sample), then the risk set for  $t=2$  would be two. The dashed line is a polynomial trend line, creating monotonically decreasing survivor function. The fact that the survivor function is decreasing at a decreasing rate suggests that if this model, estimated parametrically, a Weibull distribution of the hazard function would be most appropriate.

The hypothesis is that positive duration dependence is expected as the probability of a mutual fund failing increases with its age. A positive duration is an upward sloping

Figure 2. Kaplan-Meier Estimate of the Survivor Function



hazard function, implying the probability of leaving any state increases with increases in time. If this were true, it would contradict Brown's (95) results, which indicated older funds have a higher probability of survival.

The Weibull distributions hazard function is:  $\lambda p(\lambda t)^{p-1}$ , and therefore any  $p > 1$  would demonstrate positive duration dependence. The survivor function of the Weibull is  $e^{-(\lambda t)^p}$ . The log-likelihood function then is:

$$\ln L = \sum_{\text{uncensored}} \lambda(t|x)\beta + \sum_{\text{all observations}} \ln S(t|x\beta)$$

The covariates used in the model attempt to capture the degree to which investors shift out of the portfolio given poor performance. Using the CAPM model as the baseline assumption regarding rational investor behavior, any mutual fund with a low Sharpe ratio, with respect to either the market portfolio or some other asset increase the probability of the mutual fund closing. Given the assumptions in the CAPM model investors are assumed to care only about risk and expected return of an asset; and the Sharpe ratio allows comparison among different assets. One can construct the model to explain the probability of a mutual fund closing using the assets Sharpe ratio against the Sharpe ratio of alternative investments. If the CAPM holds, then investors who are keeping vigilant watch over the mutual fund will punish the fund not generating high returns for a given level of riskiness. As Sharpe (1966) states, "The only basis for persistently inferior performance would be the continued expenditure of large amounts of a fund's assets on the relatively fruitless search for incorrectly valued securities." This result is consistent with an efficient market that any fund that does not perform will eventually be forced out of the market.

Therefore, a utility maximizing individual compares Sharpe ratios across investments and sells the mutual fund. Since all investors have the same beliefs about a

mutual fund's expected return and risk, the length of time a poor performing mutual fund stays in the market becomes a function of the fund's Sharpe ratio and differences in the amount of time it takes investors to process the information (time variable). A fully specified model is:

$$(\lambda(t|x)) = p t^{p-1} \exp^{-(c + \beta_1 * (\text{Sharpe Ratio Asset } i) + \beta_2 * (\text{Sharpe Ratio of market portfolio}) + \sum_j \beta_j * (\text{Sharpe Ratio asset } j))}$$

One difficulty that arises in the formulation of the likelihood function is the fact that with time varying covariates the likelihood function is not a joint probability distribution as the distribution of the covariates changes. For example, the log likelihood is written:

$$\ln L = \sum_{\text{uncensored}} \lambda(t|x\beta) + \sum_{\text{all observations}} \ln S(t|x\beta)$$

where  $\lambda(t|x\beta) = f(t|x\beta)/S(t|x\beta)$ , and can therefore be rewritten as:

$$\ln L = \sum_{\text{uncensored}} f(t|x\beta) + \sum_{\text{sensored}} \ln S(t|x\beta)$$

Following Lancaster (90), define a covariate process  $\{x(t)\}$  as exogenous for T if and only if:

$$P(x(t, t+dt)|T>t+dt, x(t)) = P(x(t, t+dt)|x(t))$$

for all  $t>0, dt>0$

Any covariate that is not exogenous is then endogenous. The definition states that information for mutual funds that have survived to  $t+dt$  does not aid to the path of the Sharpe ratio from  $t$  to  $t+dt$ . Since the Sharpe ratio is expected return over the variance, theoretically these values should not change when evaluated at any point in time, given

the true distribution of expected returns. Therefore we can state that the Sharpe ratio does fit the definition above and can be considered time-invariant and exogenous. As Lancaster states:

The likelihood based on the joint density of the exit time and the covariate path . . . are valid probabilities, conditional on the covariate path, if the covariates are exogenous in the sense of definition 1, but not otherwise.

## V. Results

For the first model, only one alternative investment was considered. The Legg Mason Value fund has one of the highest 20 year total returns and falls into the large value category is the proxy for other available investments. In constructing the Sharpe ratios for the market portfolio and the alternative investments, with respect to both currently active and dead funds, the Sharpe ratio was calculated from fund inception until either death of the fund or end of the data for the fund under consideration. This was done in order to allow varying Sharpe ratios for all funds. This implies that investors only consider the reward to variability ratio during the window they remained in the investment. They do not consider all data available for comparison.

The market portfolio was constructed creating an equal-weighted index for the NYSE and NASDAQ composites base month December 1979. One of the shortcomings of the

CAPM is empirically finding the true market portfolio, creating this proxy, however, accounts for the majority of publicly traded stocks available for investment.

The model returned:

$$\ln(\lambda) = -8.38475 - 1.69874*(\text{Own SR}) + 5.1013*(\text{Market SR}) + .8412*(\text{LMV SR})$$

(-12.22)
(-1.922)
(4.967)
(.722)

Where LMV SR is the Legg Mason Value Sharpe ratio. The insignificance of the alternative investment should not be surprising, because by definition the market portfolio should be the most mean-variance efficient; and therefore and other single investment should have a lower Sharpe ratio and be inferior to the market portfolio.

Taking out the Legg Mason Sharpe Ratio yields:

$$\ln(\lambda) = -8.2357 - 2.02496*(\text{Own SR}) + 5.82926*(\text{Market SR})$$

(-13.27)
(-2.194)
(9.261)

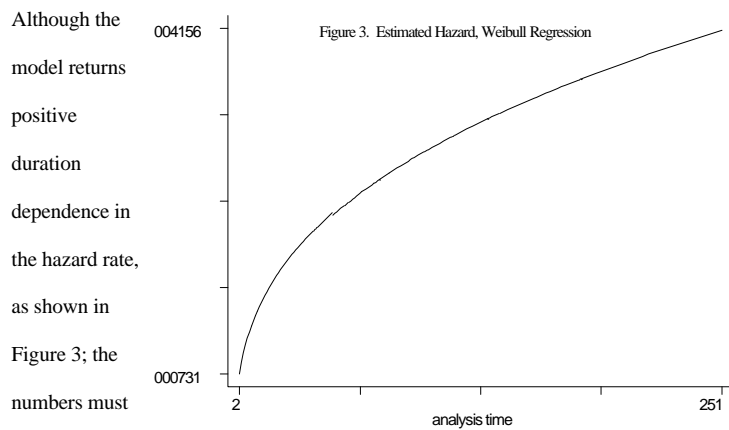
Or as a hazard function:

$$\lambda(t) = 1.35t^{-35} \exp(-8.2357 - 2.02496\text{SR} + 5.82926\text{SM})$$

The t-statistics are in parenthesis. In the second specification all coefficients are significant at the 95 percent level.  $P = 1.359545$  suggests positive duration dependence with respect to the failure of mutual funds. Positive duration dependence means that the probability a mutual fund will close increase as the fund gets older. The null  $H_0 p=1$  is



rejected with a t-statistic of 3.471. The results appear to be consistent with CAPM theory. The coefficient on the fund's Sharpe ratio states that a one percent lower Sharpe ratio increases the probability of a mutual fund closing by about 2%. Likewise if the market portfolio generated a one- percent higher return given no change in risk the hazard rate increases by almost six percent. This implies that investors do in fact watch the risk adjusted performance of the mutual fund and compare it to other investment opportunities that offer a higher reward after adjusting for risk. Given the relatively low coefficient on the fund's own Sharpe ratio it appears investors do not punish the mutual fund immediately, however. As Lunde (98) stated investors may have poor information about the true characteristics causing them to more slowly readjust their portfolios.

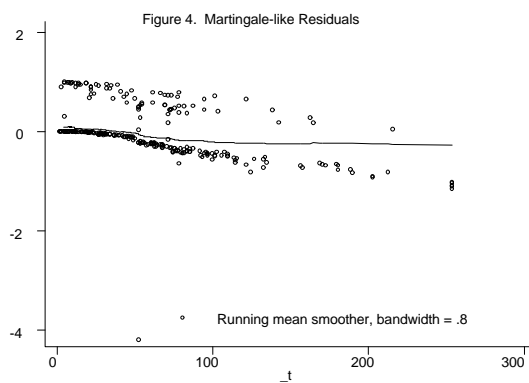


the duration dependence, as one cannot state the true duration dependence for 76% of the mutual funds. It is possible that a dataset with a higher percentage of failures could change the shape of the hazard function. The Lunde (1998) paper indicates that older funds have a decreased probability of failure with an inverted U shaped hazard rate. The Lunde paper contains 2375 UK mutual funds where 41% of them fail.

After having run the regression and imposing a Weibull as the hazard rate, it is important to test the appropriateness of that assumption. Theoretically the justification of an increasing hazard is: given the same Sharpe ratio for two funds, the older the fund the higher the probability of closure of that mutual fund. Its track record has already been noted, and investors would then leave the funds in search of higher risk adjusted returns. Therefore, one would expect (and finds) positive duration dependence. Statistically, however, it cannot be assumed that the Weibull distribution describes the true hazard. Two tests were constructed to check the appropriateness of the Weibull assumption. The first was the nested Wald test. A generalized log gamma regression, which is a three parameter distribution (Weibull is a one parameter distribution), was estimated allowing the highest flexibility for the shape of the hazard function. The Weibull is a special case of the generalized gamma, and the Wald test can be used to establish whether the exponential, Weibull, or lognormal distributions can be rejected. The first test on the lognormal distribution with a  $\chi^2(1)$  had a test statistic of 12.48 and rejected the lognormal at the 99.5% confidence level. The second test statistic for the Weibull scored a 1.02 and could not be rejected. Lastly, testing the exponential hazard (memoryless process) with  $\chi^2(2)$  scored a 13.32, which could also be rejected with 99.5% confidence. Therefore as

far as parametric modeling of the hazard, it appears as though a Weibull distribution is the most appropriate form of the hazard<sup>8</sup>.

The second test conducted was for non-nested models concerning the proportional hazards (Cox) model and a parametric Weibull model<sup>9</sup>. The Akaike Information Criteria allows for non-nested testing calculating  $AIC = -2(\log \text{likelihood}) + 2(c + p)$ , where  $c$  is the number of covariates and  $p$  the specific parameter distribution (the Weibull has 2 for the constant and the time dependence, while a Cox model has 0). The AIC for the Weibull is 378.865, while for the Cox model it is 722.3929, suggesting the Weibull for a better fit.



Lastly there is an attempt to capture the degree to which the model is well specified using a martingale approach to the residuals. Following the results in McCall (94) we can apply

martingale theory to a parametric hazard model with time-varying covariates and right-censoring. The residuals should follow a martingale difference sequence where the one

<sup>8</sup> A Wald test was also rejected a model incorporating the interaction between age of the fund and performance.

step ahead forecast should have zero mean and uncorrelated with past prediction errors. Therefore defining  $X_i(k) = N(k) - P_k(z_i(1), z_i(2), \dots)$  where  $N(k)$  is the actual event outcome at  $k$ , and  $P_k()$  is the expected outcome at time  $k$  conditional on the covariates  $z_i()$  available at  $k-1$ ,  $X_i(k)$  should be pure white noise. As Figure 4 shows, a well-specified model should show residual plots with mean zero and uncorrelated with other prediction errors.

## VI. Conclusion

Using the CAPM theory to determine how investors analyze mutual fund performance, a duration model was constructed to determine the probability of closing for a poorly performing mutual fund within a specific mutual fund category. Investors do seem to adjust their portfolios based on risk adjusted returns. A mutual fund has a low Sharpe Ratio increases the probability that the fund will close by two percent and higher performance by the market as a whole raises the hazard by about six percent. Investors do seem willing to divest in search of higher returns for a given level of risk. As the investment community engages in this activity, the probability of survival for the fund is severely decreased. Older funds also seem to have a lower probability of survival as successful small-cap stocks will not remain available in their portfolio for long periods of time. This paper hopefully highlights some issues as to why and when a mutual fund may fail. Further areas of research include incorporating all equity funds for an overall hazard rate of the mutual fund market and testing to see whether focusing on small cap funds caused any sample selection bias.

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<sup>9</sup> See appendix for Cox proportional hazards model

## References

- Atkinson, A.B. and Micklewright, **John Unemployment Benefits and Unemployment Duration**, Suntory-Toyota International Centre for Economics and Related Disciplines, The London School of Economics and Political Science, 1985.
- Barber, Brad and Odean, Terrance **The Behavior of Mutual Fund Investors**, Working Paper Sept 2000.
- Black, Fischer **Beta and Return**, Streetwise: The Best of the Journal of Portfolio Management, 10-13, 1997
- Brown, Stephen and Goetzmann, William, **Performance Persistence**, Journal of Finance Vol 50, 670 – 698, June 1995.
- Brown, Stephen, Goetzmann, William, Ibbotson, Roger, and Ross Stephen, **Survivorship Bias in Performance Studies**, Review of Financial Studies, Volume 5, pp 553 – 580, 1992.
- Carhart, Mark, **On Persistence in Mutual Fund Performance**, Journal of Finance, Volume 52, 57-82, March 1997.
- Carpenter, Jennifer and Lynch Anthony **Survivorship Bias and Attrition Effects in Measures of Performance Persistence**, Journal of Financial Economics, Vol. 54, 337-374, 1999.
- Dunkin, Amy and Gutner Toddi **Time to Size up Small Caps?** Business Week, Sep. 28, 1998.
- Fama, Eugene and French, Kenneth **The CAPM is Wanted, Dead or Alive**, Journal of Finance, Vol. 51 No. 5, 1947-1958, Dec. 1996.
- Fama, Eugene and French, Kenneth **Common Risk Factors in the Returns of Stocks and Bonds**, Journal of Financial Economics, Vol. 33, 3-56, 1993.
- Grinblatt, Mark and Titman, Sheridan, **The Persistence of Mutual Fund Performance**, Journal of Finance, Volume 47, 1977 – 1984, Dec 1992.
- Harless, David and Peterson, Steven, **Investor Behavior and the Persistence of Poorly –Performing Mutual Funds**, Journal of Economic Behavior and Organization, Vol 37, 257 – 276, 1998.
- Harvey, Sharon **Portfolio Strategy: Small Caps, Big Problem**, Institutional Investor, Feb. 1991.

Heckman, James and Singer, **Burton** **Econometric Duration Analysis**, *Journal of Econometrics*, Vol. 24, 63-132, 1984.

Hendricks, Darryll, Patel Jayendu, and Zeckhauser Richard **Hot Hand in Mutual Funds: Short-Run Persistence of Relative Performance, 1974 – 1988**, *Journal of Finance*, Volume 48, 93 – 130, March 1993.

Imbens, G.W. **Transition Models on a Non-Stationary Environment**, *Review of Economics and Statistics*, Volume 76, 703-720, Nov. 1994.

Jagannathan, Ravi and McGrattan, Ellen **The CAPM Debate**, *Federal Reserve Bank of Minneapolis Quarterly Review*, Vol. 19 No. 4, 2-17, Fall 1995.

Jensen, Michael **The Performance of Mutual Funds in the Period 1945-1964**, *Journal of Finance*, Vol. 23 No. 2, 389-416, May 1968.

Kahneman, Daniel and Tversky, Amos, **Prospect Theory: An Analysis of Decision under Risk**, *Econometrica*, Volume 47 Issue 2, 263-292, March 1979

Kiefer, Nicholas **Economic Duration Data and Hazard Functions**, Vol. 26, 646-679, June 1988.

Lancaster, **Tony** **The Econometric Analysis of Transition Data**, Cambridge University Press, 1990.

Lintner, John **The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets**, *Review of Economics and Statistics*, Vol. 47, 13-37, Feb. 1965

Lunde, Asger, Timmerman, Allan, and Blake, **David** **The Hazards of Mutual Fund Underperformance: A Cox Regression Analysis**, University of San Diego Discussion Paper 98-11, April 1998

Malkiel, Burton **Returns in Equity Mutual Funds 1971 to 1991**, *Journal of Finance*, Volume 50, Issue 2, 549 – 572, June 1995.

McCall, Brian **Specification Diagnostics for Duration Models: A Martingale Approach**, *Journal of Econometrics*, Vol. 60, 293-312, 1994.

O'Connell, Brian **Small-Cap Viability: Dismal Performance has left some Investors Wondering whether Small-Cap Funds will ever live up to Their Potential**, *Financial Planning*, Nov. 1, 1998.

Pastor, Lubos **Portfolio Selection and Asset Pricing Models**, *Journal of Finance*, Vol. 55 No. 1, 179-223, Feb. 2000.

Pastor, Lubos and Stambaugh, Robert **Comparing Asset Pricing Model: An Investment Perspective**, Journal of Financial Economics, Vol. 56, 335-381, 2000.

Petersen, Trond **Fitting Survival Models with Time-Dependent Covariates**, Applied Statistics, Vol. 35 No. 3, 281-288, 1986.

Sharpe, William **Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk**, Journal of Finance, Vol. 19, 425-442, Sep. 1964.

Sharpe, William **Mutual Fund Performance**, Journal of Business, Vol. 39 No. 1 Part 2, 119-138, Jan. 1966.

Sharpe, William **The Sharpe Ratio**, Streetwise: The Best of the Journal of Portfolio Management, 169-178, 1997

Simons, Katerina **Risk-Adjusted Performance of Mutual Funds**, Federal Reserve Bank of Boston, New England Economic Review, 33-48, Fall 1998.

Sueyoshi, Glenn **Semiparametric Proportional Hazards Estimation of Competing Risks Models with Time-Varying Covariates**, Journal of Econometrics, Volume 51, 25-58, 1992

## Appendix: The Proportional Hazards Model

The proportional hazards (PH) model introduced by Cox (1972) was designed to be a “representation of failure-time that is convenient, flexible, and yet entirely empirical.”

The basis for the PH model was the hazard function  $\lambda(t, x, \beta) = \lambda_0(t)\phi(x\beta)$ , where  $\beta$  is a vector of unknown parameters and  $\lambda_0(t)$  is some function giving a standard behavior for condition  $x=0$ .  $\phi(x\beta)$  is usually defined as  $\exp(x\beta)$  because of its non-negativity and therefore imposes no constraints on  $\beta$ . The first thing to understand about the PH model is how it differs from parametric models.

Parametric models calculate  $\Pr(\text{subject } j \text{ failed at } t_j | x_j \beta)$  and from that probability, one can calculate a likelihood function and maximize it with respect to  $\beta$ . The PH model, on the other hand, makes comparisons at the time when failures happen to occur. Therefore each mutual fund has a probability of failure of:

$$\frac{\lambda_0(t)\exp(x_i\beta)}{\sum_{j \in \mathfrak{R}(t)} \lambda_0(t)\exp(x_j\beta)}$$

where  $\mathfrak{R}(t)$  is the risk set at time  $t$ . This reduced to

$$\frac{\exp(x_i\beta)}{\sum_{j \in \mathfrak{R}(t)} \exp(x_j\beta)}$$

where each failure contributes that factor creating a conditional log-likelihood:

$$\text{Ln } L(\beta) = \sum_{i=1}^k x_i\beta - \sum_{i=1}^k (\ln \sum_{j \in \mathfrak{R}(t)} \exp(x_j\beta))$$



maximized to obtain the estimates of  $\beta$ .

The second important distinction about the PH model is understanding the role of  $\lambda_0(t)$ .

In parametric modeling  $\lambda_0(t)$  is forced to take on a certain shape, while in the Cox model

no assumptions are made about its shape, and is treated more as a nuisance parameter.

In fact, “no information can be contributed about  $\beta$  by time intervals in which no failures

have occurred because the component  $\lambda_0(t)$  might conceivably be identically equal to

zero in such intervals” (Cox 1972). What Cox is also stating is that no information

about  $\lambda_0(t)$  can help give more information about the probability of a specific mutual

fund failing or help produce better and more efficient estimates of  $\beta$  (in fact  $\lambda_0(t)$  drops

out in the estimation procedure).

The effect of the explanatory variables, then, is to multiply  $\lambda_0$  by a factor that does not

depend on duration  $t$ .  $\hat{\beta}$  can, therefore, be interpreted as the proportional effect on  $x$  on

the conditional probability of completion of a spell (Kiefer, 1988).

After the MLE estimate of  $\beta$  is calculated the baseline hazard can be found by  $\lambda_0(t) = 1 - \xi$ . Where  $\xi$  solves

$$\sum_{l \in D(t)} \frac{\exp(\beta x_l)}{1 - \xi^{\exp(\beta x)}} = \sum_{l \in \mathcal{N}} \exp(\beta x)$$

and  $D(t)$  indexes the deaths at time  $t$ . This is the estimation procedure used by Lunde (et al, 1998) on U.K. mutual funds looking at excess returns and the hazard function..

Using the Cox PH model on the small-cap mutual funds the results are:

$$\lambda(t|x\beta) = \lambda_0(t)\exp(-1.875225 \text{ SR} + 5.8529 \text{ SM})$$

(-1.95)                      (8.50)

with the t-statistics in parentheses. The baseline hazard is left undefined as it is relevant only when  $x=0$ , which given this dataset is not a meaningful condition.

The overall fit of the model is worse than the parametric Weibull model noted from the AIC test in the Results section. There is no reason to believe that the failure of mutual funds move proportionally, but one would expect increased probability of failure with poorly performing and older mutual funds. Therefore it seems that modeling the hazard parametrically is the most appropriate format.