Duong, Diep; Swanson, Norman R.

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Empirical Evidence on the Importance of Aggregation, Asymmetry, and Jumps for Volatility Prediction*

Diep Duong and Norman R. Swanson
Rutgers University

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Abstract

Many recent modelling advances in finance topics ranging from the pricing of volatility-based derivative products to asset management are predicated on the importance of jumps, or discontinuous movements in asset returns. In light of this, a number of recent papers have addressed volatility predictability, some from the perspective of the usefulness of jumps in forecasting volatility. Key papers in this area include Andersen, Bollerslev, Diebold and Labys (2003), Corsi (2004), Andersen, Bollerslev and Diebold (2007), Corsi, Pirino and Reno (2008), Barndorff, Kinnebrock, and Shephard (2010), Patton and Shephard (2011), and the references cited therein. In this paper, we review the extant literature and then present new empirical evidence on the predictive content of realized measures of jump power variations (including upside and downside risk, jump asymmetry, and truncated jump variables), constructed using instantaneous returns, i.e., $|r_t|^q$, $0 \leq q \leq 6$, in the spirit of Ding, Granger and Engle (1993) and Ding and Granger (1996). Our prediction experiments use high frequency price returns constructed using S&P500 futures data as well as stocks in the Dow 30; and our empirical implementation involves estimating linear and nonlinear heterogeneous autoregressive realized volatility (HAR-RV) type models. We find that past "large" jump power variations help less in the prediction of future realized volatility, than past "small" jump power variations. Additionally, we find evidence that past realized signed jump power variations, which have not previously been examined in this literature, are strongly correlated with future volatility, and that past downside jump variations matter in prediction. Finally, incorporation of downside and upside jump power variations does improve predictability, albeit to a limited extent.

JEL Classification: C58, C53, C22.
Keywords: realized volatility, jump power variations, downside risk, semivariances, market microstructure, volatility forecasts, jump test.

*Diep Duong, Department of Economics, Rutgers University, 75 Hamilton Street, New Brunswick, NJ 08901, USA, dduong@econ.rutgers.edu. Norman R. Swanson, Department of Economics, Rutgers University, 75 Hamilton Street, New Brunswick, NJ 08901, USA, nswanson@econ.rutgers.edu. The authors wish to thank the editor, Michael McAleer, for organizing this special issue on finnancial derivatives, and for useful comments on our paper. Additionally, we wish to thank Valentina Corradi, Roger Klein, John Landon-Lane, Richard McLean, George Tauchen and participants at workshops given at the Bank of Canada, East Carolina University, Rutgers University, and Utica College for many useful comments and discussions.

1 Introduction

Many recent modelling advances in asset pricing and management are predicated on the importance of jumps, or discontinuous movements in asset returns. Indeed, if jumps are found to be present in the data, the economic implications of including jump processes in dynamic asset pricing exercises are substantial. For example, the incorporation of jumps leads to break-downs in typical market completeness conditions needed for portfolio replication strategies in derivatives valuations. Additionally, jumps complicate the implementation of "state of the art" change of risk measure in risk neutral pricing. As a result, asset allocation and risk management, which heavily depend on risk measures and underlying asset return dynamics, are affected. In volatility measurement, it is necessary to separate out the volatility due to jumps or construct variables that appropriately summarize information generated by jumps. The above considerations are of particular importance, given the evidence presented in Huang and Tauchen (2005), suggesting that there are discrete large jumps in 7% of daily S&P500 cash and future (log) returns, during the period 1997 to 2002. In a related paper, Andersen, Bollerslev and Diebold (ABD: 2007) find that separating out the volatility jump component results in improved out-of-sample volatility forecasting, and find that jumps are closely related to macroeconomic announcements. Aït-Sahalia and Jacod (2009b) consider "small" instead of "large" jumps, and develop methods for testing for "infinite activity jumps" - those jumps that are tiny and look similar to continuous movements, but whose contribution to the jump risk of the process is not negligible. Cont and Mancini (2007) implement their method of testing for the existence of infinite activity jumps using foreign exchange rate data, and find no evidence of such jumps. Aït-Sahalia and Jacod (2009b), on the other hand, estimate that the degree of activity of jumps in Intel and Microsoft log returns is approximately 1.6, which implies evidence of infinite activity jumps for these, and possibly many other stocks. In summary, it is now generally accepted that many return processes contain jumps.[1]

In this paper, we add to the empirical literature on volatility prediction with jumps, building on key papers including Andersen, Bollerslev, Diebold and Labys (2003), Corsi (2004), ABD (2007), Corsi, Pirino and Reno (2008), Barndorff, Kinnebrock, and Shephard (BKS: 2010), Patton and Shephard (2011), and the references cited therein. We begin with a review of the literature and in particular of key recent theoretical advances in the areas of jump testing and the characterization of continuous time processes useful for isolating and examining jumps with magnitudes larger than a fixed level, γ. This examination is based on methodology developed by Huang and Tauchen (2005), Barndorff-Nielsen and Shephard (BNS: 2006), Jacod (2007), and Aït-Sahalia and Jacod (2009a). The idea underlying their methods is to measure the difference between the variation of the continuous component and the overall quadratic variation of a given log return process. Of

[1] For other examples of work in this area, see Aït-Sahalia (2002), Carr, Geman, Madan, Yor (2002), Carr and Wu (2003), Barndorff-Nielsen and Shephard, Woerner (2006), Jacod (2008), Jiang and Oomen (2008), Lee and Mykland (2008), Tauchen and Todorov (2009), Aït-Sahalia and Jacod (2009a) and the references cited therein.

note is that BNS (2006) develop methodology appropriate for processes with finite activity jumps. In our analysis, we also allow for infinite activity jumps, as we take advantage of the limit theory developed for this purpose in Jacod (2008) and Aït-Sahalia and Jacod (2009b). Once jumps are found, we truncate the process in order to isolate those jumps with size larger than γ, and construct realized measures of the variational contribution of large and small jumps to total variation.

One potential use of our jump decomposition approach is in jump risk assessment and management. For example, financial managers may be interested in knowing not only the probability of jumps, but also the probability that jumps of certain pre-defined "large" magnitudes will occur. This is an important distinction, particularly given that, as shown by Aït-Sahalia and Jacod (2009b, 2012), infinite activity jumps are present in the dynamics of some asset returns. However, such jumps, when of small magnitude, may not only be difficult to distinguish (in practice) from the continuous component of the process, but may not be of as serious concern to financial planners as "large" jumps. In this sense, it is empirical interest not only to test for jumps in general, but also to check for jumps of varying magnitudes, and to characterize the contribution of such jumps to total variation. In particular, the partitioning of jumps into those that are "small" and "large" allows us to uncover empirical evidence concerning what type of jumps are contributing to overall jump variation. This is also potentially of interest in macroeconomics, for example, as it may turn out that larger but less frequent jumps characterize periods of economic recession, while smaller jumps characterize expansionary periods, say. More generally, jump frequency and magnitude (i.e. jump risk) may play an important role in dating business cycle turning points. Moreover, it is already known from ABD (2007) that many significant jumps are associated with specific macroeconomic news announcements, and our approach provides a simple framework from within which this finding can be further explored.

In volatility forecasting, once jumps are detected, understanding the role of variables that capture jump information is potentially important for applied practitioners, especially in the construction of hedging strategies.[2] In general, volatility predictability is important in numerous areas ranging from the pricing of volatility-based derivative products to asset management. In light of this, a number of recent papers (see above) have addressed volatility predictability, some from the perspective of the usefulness of jumps in forecasting volatility. However, although there is strong evidence of the importance of jumps in pricing, investment and risk management, there is mixed evidence concerning whether information extracted from jumps is useful for volatility forecasting. In an important paper, ABD (2007) show that almost all of the predictability in daily, weekly, and monthly return volatilities comes from the non-jump component for DM/$ exchange rates, the S&P500 market index, and the 30-year U.S. Treasury bond yield. Corsi, Pirino and Reno (2008) find that jumps are positively correlated with, and have a significant impact on future volatility of the S&P500 index, various individual stocks and U.S. bond yields. Patton and Shephard (2011)

[2] See ABD (2007) and Aït-Sahalia and Jacod (2011) for further discussion.

point out that the impact of a jump on future volatility critically depends on the sign of the jump, for both the S&P500 index, as well as 105 individual stocks. In this paper we add to the empirical literature on this topic by providing results on volatility forecasting using a variety of "new" variables that capture information generated by jumps.

There are two ingredients used in the experiments that we carry out to examine the usefulness of jumps in volatility prediction. The first ingredient involves the choice of volatility estimator. One available estimator is based on "backing out" volatility from parametric ARCH, GARCH, stochastic volatility, or derivatives pricing models. Another estimator, which we use, is "model free". Examples include realized volatility (RV) (see the seminal work of Andersen, Bollerslev, Diebold and Laby (2001)), and variants thereof such as bipower variation, tripower variation, multipower variation, semivariance, and various others.[3] One reason for the use of these "model free" realized measures (RMs), is that they allow us to treat volatility as if it is observed, when we subsequently fit regressions in order to assess jump predictability. Modeling and forecasting RMs is important not only because RMs are a natural proxy for volatility, but also because of the many practical applications and uses of RMs in constructing synthetic measures of risk in the financial markets. For example, since shortly after the inception in 1993 of the VIX (index of implied volatility), a variety of volatility-based derivative products have been engineered using RV as an input. These include variance swaps, caps on variance swaps, corridor variance swaps, covariance swaps, options on RV overshooters, and up and downcrossers. The key here is that investors worry about future volatility risk, and hence often opt for this type of contract in order to hedge against risk, adding to the traditional volatility "vega".[4] In light of the above uses of RV, several authors have advocated forecasting RV (and more generally RMs) using extensions of ARMA models (see e.g., Andersen, Bollerslev, Diebold and Labys (2003), Corsi (2004), and ABD (2007)). In related work, Corradi, Distaso and Swanson (2009, 2011) develop model-free conditional predictive density estimators and confidence intervals for integrated volatility.

The second ingredient involves which variables we use to measure jumps. Our approach is to examine various different realized measures of jump power variations, all formed on the basis of power transformation of the instantaneous return, i.e., $|r_t|^q$. The analysis of power transformations of returns is not new. Ding, Granger and Engle (1993) and Ding and Granger (1996) develop long memory Asymmetric Power ARCH models based on power transformations of daily absolute returns. They find that the autocorrelations of power transformations of S&P500 returns are the strongest for $q < 1$. In the context of high frequency data, Liu and Maheu (2005) and Ghysels and Sohn (2009) study the predictability of future realized volatility using past absolute power

[3] See e.g., Barndorff-Nielsen and Shephard (2004), Aït-Sahalia, Mykland and Zhang (2005), Zhang (2006), Barndorf-Nielsen, Hansen, Lunde, and Shephard (2008), Jacod (2008), BKS (2010), and the references cited therein.

[4] Volatility and variance swaps are newer hedging instruments, adding to the traditional volatility "vega", which is derived from options data. See Hull (1997, pp. 328) for a definition of vega. For example, as noted in Carr and Lee (2008), the UBS book was short many millions of vega in 1993, and they were the first to use variance swaps and options on realized volatility to hedge against volatility risk. See Duong and Swanson (2011) for further discussion.

variations and multipower variations. Ghysels and Sohn (2009) find that the optimal value of q is approximately unity. However, their empirical evidence considers the continuous class of models, and does not account for jumps. ABD (2007) develop an interesting framework for separating jump and continuous components of RV, and carry out predictability experiments indicating that jumps play a small but notable role in forecasting volatility. In related recent work, BKS (2010) construct new estimators of downside (and upside) risk (i.e., so-called realized semivariances), using square transformations of positive and negative intra-daily return, and find that downside risk measures are important when attempting to model and understand risk. They note, as quoted from Granger (2008), that: *'It was understood that risk relates to an unfortunate event occurring, so for an investment this corresponds to a low, or even negative, return. Thus getting returns in the lower tail of the return distribution constitutes this "downside risk." However, it is not easy to get a simple measure of this risk.'* This point is noteworthy, since it is argued in the literature (see e.g., Ang, Chen and Xing (2006)), that investors treat downside losses differently than upside gains. As a result, agents who put higher weight on downside risk demand additional compensation for holding stocks with high sensitivity to downside market movements. Most authors in this literature pay attention to co-skewness as a measure of downside risk, and use daily data for estimation thereof. Patton and Shephard (2011) build on this idea and use semivariance estimators to forecast volatility. In the parametric framework, some authors also develop approaches to modeling time-varying higher order conditional moments (see e.g. Hansen (1994), Harvey and Siddique (1999), Timmermann (2000), Perez-Quiros and Timmermann (2001), and Premaratne and Bera (2001)). Maheu and Curdy (2004) take this sort of analysis one step further and incorporate past jumps as a new source of asymmetry; and find improved volatility forecastability.

In our experiments, we add to the work of the above authors, and in particular BKS (2010). Our jump power variation type measures are constructed using power transformations of absolute intra-daily returns, and are predicated on recent limit theory advances due to Jacod (2008) and BKS (2010). Theoretically, our measures do not require the use of a jump test in order to "pre-test" for jumps. Although construction of the measures is closely related to the work of GS (2009), our approach differs in that we focus on jump power variations with $q > 2$. Furthermore, the limit theory that we adopt allows us to construct estimators of downside and upside jump power variations using intra-daily positive and negative returns. These estimators are suggested by BKS (2010) as alternatives to the semivariances implemented in Patton and Shephard (2011). We also examine jump asymmetry (i.e., realized signed jump power variation). Of note is that the role of the size of jumps that are most useful for forecasting can be inferred (to some extent) through examination of the order of q. For this reason, we consider jump power variations with $0 \leq q \leq 6$. While previous authors have focused on $q \leq 2$,allowing for a wider range of values for q is sensible, given that convergence to jump power variation occurs only when $q > 2$ (see e.g. Todorov and

Tauchen (2010) and BKS (2010)). [5] Finally, our prediction experiments are designed to separately analyze "large" and "small" jumps.

The dataset used in our empirical investigation includes high frequency price returns constructed using S&P500 futures index data for the period 1993-2009, as well as stocks in the Dow 30, for the period 1993-2008; and our empirical implementation involves estimating linear and nonlinear extended heterogeneous autoregressive realized volatility (HAR-RV) type models. Our findings can be summarized as follows. First, we find evidence that jumps characterize the structure of S&P500 futures and the individual stocks that we examine. Moreover, the prevalence of jumps is dependent upon sample period; and is also dependent upon truncation level. This is consistent with "clustering" occurring during "bad" times; but, just as importantly, it suggests that jump information aggregation might be of relevance in financial applications, and in particular in forecasting exercises. Second, our prediction experiments show improvements, both in- and out-of-sample, when RMs of jump power variations are used as additional predictors in volatility forecasting. However, past "large" jump power variations help less in the prediction of future realized volatility, than past "small" jump power variations. This in turn suggests the "larger" jumps might help less in the prediction than "smaller" jumps. In a related finding, we note that seemingly rare and possibly *iid* jumps do not help in prediction, while smaller, less rare and possibly serially correlated jumps do help. Third, the continuous component dominates in all prediction experiments, which is consistent with previous findings in the literature on volatility forecasting using high frequency data. Fourth, incorporation of downside and upside jump power variations does improve predictability, albeit to a limited extent. Fifth, comparing "no jump test" cases with "jump test" cases indicates that findings do change, to some degree, when jump tests are used in the construction of jump variation variables. Additionally, the power of q associated with our R^2-"best" model is higher when S&P500 index returns are predicted, than when individual DOW components are predicted. This suggests that aggregation plays a crucial role in risk prediction. Finally, values of q less than 2 dominate under individual stocks, while values greater than 2 dominate under our index variable. Taken together, these results suggest that what's best for in-sample analysis is far from best for out-of-sample analysis. Moreover, jumps do play a role, at least when modelling aggregate (index) data such as S&P500 futures returns; and while modelling jump risk power variations may not be important for in-sample fit, it clearly plays an important role in out-of-sample volatility prediction.

The rest of the paper is organized as follows. Section 2 discusses volatility and jumps, while Section 3 discusses the various realized measures of price jump variation that we examine. Section 4 outlines our experimental setup, and Section 5 gathers our empirical findings. Concluding remarks are contained Section 6.

[5] In our implementation, for $q > 6$, the prediction results are almost the same as the case $q = 6$ and therefore are not presented.

2 Volatility and Price Jump Variations

2.1 Set-up

We adopt a general semi-parametric specification for asset prices. Following Todorov and Tauchen (2010), the log-price of asset, $p_t = \log(P_t)$, is assumed to be an Itô semimartingale process,

$$p_t = p_0 + \int_0^t b_s ds + \int_0^t \sigma_s dB_s + J_t, \tag{1}$$

where $p_0 + \int_0^t b_s ds + \int_0^t \sigma_s dB_s$ is a Brownian semi-martingale and J_t is a pure jump process which is the sum of all "discontinuous" price movements up to time t,

$$J_t = \sum_{s \leq t} \Delta p_s.$$

J_t is assumed to be finite[6] and a jump at time s is defined as $\Delta p_s = p_s - p_{s-}$.

When the jump component is a compound poisson process (CPP) - i.e. a finite activity jump process - then,

$$J_t = \sum_{i=1}^{N_t} Y_i, \tag{2}$$

where N_t is number of jumps on $[0, t]$. N_t follows a Poisson process, and the jump magnitudes, i.e. the $Y_i's$ are iid random variables. The CCP assumption has been widely used in the literature on modeling, forecasting, and testing for jumps. However, jumps may arise in other model setups, such as when infinite activity jumps are specified (see Todorov and Tauchen (2010)).

The empirical evidence discussed in this paper involves examining the variation of the log-price jump component using an equally spaced path of historically observed prices, i.e. $\{p_0, p_{1\Delta_n}, p_{2\Delta_n} ..., p_{n\Delta_n}\}$, where the sampling frequency, $\Delta_n = \frac{t}{n}$, is deterministic[7]. The intra-daily return or increment of p_t is

$$r_{i,n} = p_{i\Delta_n} - p_{(i-1)\Delta_n}.$$

Returns are observed at various frequencies. However, volatility of log-prices is often treated as an unobserved variable. The "true" price variance (risk) is defined in this paper by the quadratic variation of the process p_t, i.e.,

$$V_t = [p, p]_t = \int_0^t \sigma_s^2 ds + QJ_t,$$

where the variation of the continuous component (integrated volatility) is

$$IV_t = \int_0^t \sigma_s^2 ds,$$

[6] See, for example Jacod (2008) or Todorov and Tauchen (2009) for the conditions for the finiteness of jumps.

[7] For instance, if we use a 5 minute sampling frequency to calculate daily measures in our application, then $t = 1$, $n = 78$, and $\Delta_n = \frac{1}{78}$.

and the variation of the price jump component is

$$QJ = \sum_{s \leq t} (\Delta p_s)^2.$$

The realized volatility (RV), constructed by simply summing up all successive intra-daily squared returns, converges to the quadratic variation of the process, as $n \to \infty$. Andersen, Bollerslev, Diebold and Labys (2001) use realized volatility as an estimator of volatility of the price process. In particular, they use

$$RV_t = \sum_{i=1}^{n} r_{i,n}^2 \xrightarrow{ucp} V_t = IV_t + QJ_t, \tag{3}$$

where ucp denotes uniform convergence in probability.

2.2 Jump Tests and Jump Decompositions

In this section, we review results on jump tests and the jump decomposing technique used in Duong and Swanson (2011) and Aït-Sahalia and Jacod (2012).

2.2.1 Testing for Jumps

First, we review some theoretical results on testing for jumps; namely testing whether $J_t \neq 0$. In pioneering work, BNS (2006) propose a robust and simple test for a class of Brownian Itô-semimartingales plus compound poisson jump processes. In recent work, Aït-Sahalia and Jacod (2009a), among others, develop a different test which applies to a large class of Itô-semimartingales, and allows the log price process to contain infinite activity jumps - small jumps with infinite concentrations around 0. In this paper, we follow the jump test methodology of Huang and Tauchen (2005) as well as BNS (2006), which looks at the difference between the continuous component and total quadratic variation in order to test for jumps. However, we make use of the limit theorems developed and used by Jacod (2008) and Aït-Sahalia and Jacod (2009a) in order to implement the BNS (2006) type test under the presence of both infinite activity and finite activity jumps.

A simplified version of the results of the above authors applied to (1) for the one-dimensional case is as follows. If the process is continuous, let $f(x) = x^n$, let ρ_{σ_s} be the law $N(0, \sigma_s^2)$, and let $\rho_{\sigma_s}(f)$ be the integral of f with respect to this law. Then:

$$\sqrt{\frac{1}{\Delta_n}} \left(\Delta_n \sum_{i=1}^{n} f(\frac{r_{i,n}}{\sqrt{\Delta_n}})^2 - \int_0^t \rho_{\sigma_s}(f) ds \right) \xrightarrow{L-S} \int_0^t \sqrt{\rho_{\sigma_s}(f^2) - \rho_{\sigma_s}^2(f)} dB_s \tag{4}$$

Here, $L-S$ denotes stable convergence in law, which also implies convergence in distribution. For $n = 2$, the above result is the same as BNS (2006). More generally:

$$\sqrt{\frac{1}{\Delta_n}}\left(\sum_{i=1}^{n}(r_{i,n})^2-\int_0^t\sigma_s^2ds\right)\xrightarrow{D}N(0,\int_0^t\vartheta\sigma_s^4ds) \tag{5}$$

or

$$\frac{\sqrt{\frac{1}{\Delta_n}}\left(\sum_{i=1}^{n}(r_{i,n})^2-\int_0^t\sigma_s^2ds\right)}{\sqrt{\int_0^t\vartheta\sigma_s^4ds}}\xrightarrow{D}N(0,1), \tag{6}$$

where ϑ is constant and where $\int_0^t\sigma_s^2dt$ is known as the integrated volatility or the variation of the continuous component of the model. Additionally, $\int_0^t\sigma_s^4dt$ is the integrated quarticity. From the above result, notice that if the process does not have jumps, then $\sum_{i=1}^{n}(r_{i,n})^2$, which is an approximation of the quadratic variation of the process, should be "close" to the integrated volatility. This is the key idea underlying the BNS (2006) jump test. A crucial issue in this jump test is the estimation of $\int_0^t\sigma_s^2dt$ and $\int_0^t\sigma_s^4dt$ in the presence of both finite and infinite activity jumps. As remarked in BNS (2006), in order to ensure that tests have power under the alternative, intergrated volatility and integrated quarticity estimators should be consistent under the presence of jumps. The authors note that robustness to jumps is straightforward so long as there are a finite number of jumps, or in cases where the jump component model is a Lévy or non-Gaussian OU model (see Barndorff-Nielsen, Shephard, and Winkel (2006)). Moreover, under infinite activity jumps, note that as pointed out in Jacod (2007), there are available limit results for volatility and quarticity estimators for the case of semimartingales with jumps.

Turning again to our discussion of volatility and quarticity, note that in a continuation of work initiated by Barndorff-Nielsen and Shephard (BNS: 2004), Barndorff-Nielsen, Graverson, Jacod, Podolskij, and Shephard (2005) develop so-called multipower variation estimators of $\int_0^t\sigma_s^qds$, in the case of continuous semimartingales and semimartingales with jumps. These estimators are defined as follows.

$$V_{m_1,m_2...,m_j}=\sum_{i=2}^{n}|r_{i,n}|^{m_1}|r_{i-1,n}|^{m_2}....|r_{i-j,n}|^{m_j},$$

where $m_1,m_2,...,m_j$ are positive, such that $\sum_1^j m_i=q$. Regardless of the estimator of $\int_0^t\sigma_s^2dt$ that is used, the appropriate test hypotheses are:

$$H_0: p_t \text{ is a continuous process in the interval } [0,t]$$
$$H_1: \text{the negation of } H_0 \text{ (there are jumps)}$$

If we use multipower variation, under the null hypothesis the test statistic which directly follows from the CLT mentioned above is:

$$LS_{jump}=\frac{\sqrt{\frac{t}{n}}\left(\sum_{i=1}^{n}(r_{i,n})^2-\widehat{IV}\right)}{\sqrt{\vartheta\widehat{IQ}}}\xrightarrow{D}N(0,1),$$

The the so-called jump ratio test statistic is:

$$RS_{jump} = \frac{\sqrt{\frac{t}{n}}}{\sqrt{\vartheta \widehat{IQ}/(\widehat{IV})^2}} \left(1 - \frac{\widehat{IV}}{\sum_{i=1}^{n}(r_{i,n})^2}\right) \xrightarrow{D} N(0,1).$$

where $\widehat{IV}$ and $\widehat{IQ}$ are (multipower variation) estimators of integrated volatility $\int_0^t \sigma_s^2 ds$ and of $\int_0^t \sigma_s^4 ds$. BNS (2006) use $V_{1,1}$ (bipower variation) and $V_{1,1,1,1}$. In our empirical analysis, we also use tripower variation, $V_{\frac{2}{3},\frac{2}{3},\frac{2}{3}}$, instead of bipower variation, $V_{1,1}$, as it more robust to clustered jumps. In particular, we set:

$$\widehat{IV} = V_{\frac{2}{3},\frac{2}{3},\frac{2}{3}} \mu_{\frac{2}{3}}^{-3} \tag{7}$$

and

$$\widehat{IQ} = \Delta_n^{-1} V_{\frac{4}{3},\frac{4}{3},\frac{4}{3}} \mu_{\frac{4}{3}}^{-3}, \tag{8}$$

where $\mu_r = E(|Z|^r)$ and Z is a $N(0,1)$ random variable. Andersen, Dobrev, Schaumburg (2008) suggest a different estimator that is robust in the case of consecutive jumps. This estimator is also more robust to occurrence of zero-returns, as is constructed as follows.

$$\widehat{IV} = MedRV_n = \frac{\pi}{6 - 4\sqrt{3} + \pi} \left(\frac{n}{n-2}\right) \sum_{i=2}^{n-1} med\left(|r_{i-1,n}|\,|r_{i-2,n}|\,|r_{i-3,n}|\right)^2.$$

Of note is that a related "adjusted" jump ratio statistic has been shown by extensive Monte Carlo experimentation in Huang and Tauchen (2005), in the case of CCP jumps, to perform better than the two above statistics, being more robust to jump over-detection. This adjusted jump ratio statistic is:

$$AJ_{jump} = \frac{\sqrt{\frac{n}{t}}}{\sqrt{\vartheta max(t^{-1}, \widehat{IQ}/(\widehat{IV})^2)}} \left(1 - \frac{\widehat{IV}}{\sum_{i=1}^{n}(r_{i,n})^2}\right) \xrightarrow{D} N(0,1).$$

In general, given a daily test statistic, say $Z_{t,n}(\alpha)$, where n is the number of observations per day and α is the test significance level, we reject the null hypothesis if $Z_{t,n}(\alpha)$ is in excess of the critical value Φ_α, leading to a conclusion that there are jumps during the day. The converse holds if $Z_{t,n}(\alpha) < \Phi_\alpha$. In our empirical application, $Z_{t,n}(\alpha)$ is the adjusted jump ratio statistic.

2.2.2 Price Jump Decompositions

For a given level of γ, $\gamma > 0$, equation (1) can be written as:

$$p_t = p_0 + \int_0^t b_s ds + \int_0^t \sigma_s dB_s + \sum_{s \le t} \Delta p_s I_{|\Delta p_s| \le \gamma} + \sum_{s \le t} \Delta p_s I_{|\Delta p_s| > \gamma}, \tag{9}$$

where $I_{|\Delta p_s \geq \gamma|}$ is an indicator which equals 1 for $|\Delta p_s| \geq \gamma$ and 0 otherwise. Thus, once the process is found to have jumps, the jumps process can be decomposed into 2 components. One contains jumps with size larger than γ (large jumps) and the other contains jumps with size smaller than γ (small jumps).

This decomposition of jumps into "large" and "small" components allows us to assess relative contributions to the overall variation of the price process. In particular, for some fixed level γ, define large and small jump components as follows, respectively:

$$LJ_t(\gamma) = \sum_{s \leq t} \Delta p_s I_{|\Delta p_s| \geq \gamma} \text{ and } SJ_t(\gamma) = \sum_{s \leq t} \Delta p_s I_{|\Delta p_s| < \gamma}.$$

The choice of γ may be data driven, but scenarios where there is prior knowledge concerning the magnitude of γ are also of interest. For example, under various regulatory settings, capital reserving and allocation decisions may be based to a large extent on the probability of jumps or shocks occurring that are of a magnitude greater than some known value, γ. In such cases, planners may be interested not only in knowledge of jumps of magnitude greater than γ, but also in characterizing the nature of the variation associated with such large jumps. The procedure discussed in this section can readily be applied to uncover this sort of information.

As jumps are often linked to abnormal or tail behavior of returns, the assessment of different RMs of jump variations is also important. One way is to decompose price jumps, Δp_s, using a pre-fixed truncation level γ, $\gamma \geq 0$, is to define

$$QJ_{t,\gamma} = \sum_{0 < s \leq t} (\Delta p_s)^2 I_{\Delta p_s > \gamma} + \sum_{0 < s \leq t} (\Delta p_s)^2 I_{\Delta p_s < -\gamma}, \tag{10}$$

where $I(\cdot)$ is an indicator taking 1 if jump size is larger than γ (upside truncated jumps) or less than $-\gamma$ (downside truncated jumps).

In summary, once jumps are detected, it should be of interest to examine realized measures of the above jump variations. We do this by following and building on ABD (2007). Namely, we construct:

$RVJ_t = Variation\ of\ the\ jump\ component = max\{0, RV_t - \widehat{IV_t}\} * I_{jump,t}$

$RVC_t = Variation\ of\ continuous\ component = RV_t - VJ_t,$

where RV_t and $\widehat{IV_t}$ are the daily realized volatility measures (defined above), $I_{jump,t}$ is an indicator taking the value 0 if there are no jumps and 1 otherwise, and n is the number of intra-daily observations. One can then calculate daily jump risk. Note that in these formulae, the variation of the continuous component has been adjusted using the *max* operator (i.e. the variation of the continuous component equals realized volatility if there are no jumps and equals $\widehat{IV_t}$ if there are jumps). In addition, note that $\sum_{i=1}^{n} r_{i,n}^2 I_{|r_{i,n}| \geq \gamma}$ converges uniformly in probability to $\sum_{s \leq t} (\Delta p_s)^2 I_{|\Delta p_s| \geq \gamma}$, as n goes to infinity [8]. Thus, the contribution of the variation of jumps with magnitude larger than γ and smaller than γ are denoted and calculated as follows:

[8] See Jacod (2008), Aït-Sahalia and Jacod (2011) for further details.

Realized measure of large jump variation: $VLJ_{t,\gamma}$=min$\{RVJ_t, (\sum_{i=1}^{n} r_{i,n}^2 I_{|r_{i,n}|\geq\gamma} * I_{jump,t})\}$,
Realized measure of small jump variation: $VSJ_{t,}\gamma = RVJ_t - VLJ_{t,\gamma}$,
where I_{jump} is defined above and $I_{jump,\gamma}$ is an indicator taking the value 1 if there are large jumps and 0 otherwise. This condition simply implies that large jump risk is positive if the process has jumps and has jumps with magnitude greater than γ.

Finally, we can write the relative contribution of the variation of the different jump components to total variation in a variety of ways:
Relative contribution of continuous component $= \frac{RVC_t}{RV_t}$
Relative contribution of jump component $= \frac{RVJ_t}{RV_t}$
Relative contribution of large jump component $= \frac{VLJ_{t,\gamma}}{RV_t}$
Relative contribution of small jump component $= \frac{VLS_{t,\gamma}}{RV_t}$
Relative contribution of large jumps to jump variation $= \frac{VLJ_{t,\gamma}}{RVJ_t}$
Relative contribution of small jumps to jump variation $= \frac{VLS_{t,\gamma}}{RVJ_t}$

3 Jump and Signed Jump Power Variations

In previous section, we discussed jump variation decompositions using arbitrary truncation levels. We can also assess jump variations using jump power variations formulated by power transformation of absolute log-price jumps ($|\Delta p_s|^q$). In particular, define the jump power variation as follows.

$$JP_{q,t} = \sum_{0<s\leq t} |\Delta p_s|^q, \tag{11}$$

with "upside" jump power variation defined as

$$JPV_{q,t}^{+} = \sum_{0<s\leq t} |\Delta p_s|^q I_{\Delta p_s>0}, \tag{12}$$

and "downside" jump power variation defined as

$$JPV_{q,t}^{-} = \sum_{0<s\leq t} |\Delta p_s|^q I_{\Delta p_s<0}. \tag{13}$$

Finally, jump asymmetry can be measured using so-called signed jump power variation, defined as follows.

$$JA_{q,t} = \sum_{0<s\leq t} |\Delta p_s|^q I_{\Delta p_s>0} - \sum_{0<s\leq t} |\Delta p_s|^q I_{\Delta p_s<0}. \tag{14}$$

In the above expressions, we are particularly interested in the case where $q \geq 2$. Note that for large values of q, $JP_{q,t}, JPV_{q,t}^{+}, JPV_{q,t}^{-}$, $JA_{q,t}$ are dominated by large jumps. For $q < 2$, the jump variations are not always guaranteed to be finite. One of our main goals in this paper is to construct and examine realized measures (RMs) of jump power variations including $JP_{q,t}$, $JPV_{q,t}^{+}$, $JPV_{q,t}^{-}$, $JA_{q,t}$, for a wide range of values of q, and to use them in prediction experiments.

For the case $q = 2$, BKS (2010) develop so-called realized semivariances which are particular estimators of $JPV^{+}_{q,t}$, $JPV^{-}_{q,t}$. PS (2011) build on these results and make use of realized semivariances to forecast volatility. The realized semivariances of BKS (2010) are defined as follows:

$$RS^{-} = \sum_{i=1}^{n} (r_{i,n})^2 I_{\{r_{i,n}<0\}} \text{ and } RS^{+} = \sum_{i=1}^{n} (r_{i,n})^2 I_{\{r_{i,n}>0\}}.$$

Here, RS^{-} (RS^{+}) contain only negative (positive) intra-daily returns and can serve as measures of downside (upside) risk as pointed out in BKS (2010). They show that RS^{+} and RS^{-} converge uniformly in probability to semi-variances. Namely,

$$RS^{+} \to \frac{1}{2}\int_0^t \sigma_s^2 ds + \sum (\Delta p_s)^2 I_{\Delta p_s>0} \text{ and } RS^{-} \to \frac{1}{2}\int_0^t \sigma_s^2 ds + \sum (\Delta p_s)^2 I_{\Delta p_s<0}. \quad (15)$$

Realized measures of "downside" and "upside" jump variation are thus obtained by replacing $\int_0^t \sigma_s^2 ds$ with $\widehat{IV}$. For example, we see that "downside" variation can be constructed by calculating

$$\sum_{i=1}^{n} r_{i,n}^2 I_{\{r_{i,n}<0\}} - \frac{1}{2}\widehat{IV} \to \sum (\Delta p_s)^2 I_{\Delta p_s \leq 0}. \quad (16)$$

In volatility forecasting experiments, PS (2011) use bipower variation for $\widehat{IV}$. In addition, they construct "signed" jump variation, $\Delta RJ = RS^{+} - RS^{-}$, which captures jump variation asymmetry, since $\Delta RJ \to \sum (\Delta p_s)^2 I_{\Delta p_s>0} - \sum (\Delta p_s)^2 I_{\Delta p_s<0}$. When jumps are not present, ΔRJ converges to 0 and there is no asymmetry in volatility. When the process has jumps, ΔRJ can proxy for jump variation asymmetry.

Turning now to the case of variations with $q \neq 2$, GS (2009) undertake to find the "optimal" realized power variation, $n^{-1+q/2}\sum_{i=1}^{n} |r_{i,n}|^q$, for some q, when forecasting future RV. Recall, however, that they assume that the price process follows a Brownian semi-martingale. Their results are therefore restricted to the case of higher order variations of the continuous component, $\int_0^t \sigma_s^q ds$, involving no jumps. In this case, Ait-Sahalia and Jacod (2012) point out that for all $q > 0$,

$$n^{-1+q/2}\sum_{i=1}^{n} |r_{i,n}|^q \to \mu_q \int_0^t \sigma_s^q ds, \quad (17)$$

where $\mu_q = E(|u|^q)$ and u is a standard normal random variable.

Recent limit theory advances due to Jacod (2008) and BKS (2010) allow us to construct estimators of downside and upside jump power variations, $JPV^{+}_{q,t}$, $JPV^{-}_{q,t}$ for $q > 2$, using intra-daily positive and negative returns. These estimators are suggested by BKS (2010) as alternatives to the semivariances implemented in PS (2011). Namely, define jump power variation as $RPV_{q,t} = \sum_{i=1}^{n} |r_{i,n}|^q$, $q > 0$. Realized downside and upside power variations are defined as:

$$RJ^{+}_{q,t} = \sum_{i=1}^{n} |r^{+}_{i,n}|^q \text{ and } RJ^{-}_{q,t} = \sum_{i=1}^{n} |r^{-}_{i,n}|^q,\ q > 2.$$

Convergence of the above RMs to jump power variations occurs when $q > 2$. Therefore, in our prediction experiments, differentiating our approach from that of previous authors, we are particularly interested a range of q from 2 to 6, and allow the price process to contain jumps.

In their analysis of the limiting behavior of $RPV_{q,t}$, Todorov and Tauchen (2010) summarize selected results from Barndorff-Nielsen et. al. (2005), Barndorff-Nielsen et. al. (2006) and Jacod (2008). In their set-up, the log-price process contains continuous martingale, jump and drift components. The value of q directly affects the limiting behavior of $RPV_{q,t}$. For instance, for $q < 2$, the limit of $RPV_{q,t}$ is determined by the continuous martingale. For $q > 2$, the limit is driven by jump component. When $q = 2$, both continuous and jump components contribute to the limit of $RPV_{q,t}$. The results are summarized as follows:

$$\begin{cases} \Delta_n^{1-q/2} RPV_{q,t} \xrightarrow{ucp} \mu_q \int_0^t \sigma_s^q ds \text{ , if } 0 < q < 2, \\ RPV_{q,t} \xrightarrow{ucp} V \text{ if } q = 2, \\ RPV_{q,t} \xrightarrow{ucp} JP_{q,t} \text{ if } q > 2. \end{cases} \tag{18}$$

BKS (2010) point out that we can go one step further and decompose jump power variations into upside movements and downside movements, i.e.,

$$\begin{cases} RJ_{q,t}^+ \xrightarrow{ucp} JPV_{q,t}^+ \\ RJ_{q,t}^- \xrightarrow{ucp} JPV_{q,t}^- \end{cases} \text{ if } q > 2. \tag{19}$$

As mentioned earlier, for $q < 2$, scaled $RPV_{q,t}$ converges to the power variation of the continuous component, i.e. no jumps. Intuitively, with $q > 2$, scaled $RPV_{q,t}, RJ_{q,t}^+, RJ_{q,t}^-$ eliminate all variations due to the continuous component and keep all "large" jumps. In addition, these realized measures are evidently dominated by larger jumps the higher the value of q. Finally, building on (19), we construct a new RM of jump power variation asymmetry, so-called "signed" jump power variation. It is straightforward to verify that:

$$RJA_{q,t} = RJ_{q,t}^+ - RJ_{q,t}^- \xrightarrow{ucp} JA_{q,t}.$$

In our forecasting experiments, we also examine the usefulness of this new jump asymmetry variable, $RJA_{q,t}$ for a wide range of values of $q > 2$. Of final note is that, as elsewhere in this paper, we use $V_{m_1,m_2...,m_j}$, to estimate $\int_0^t \sigma_s^q ds$ in all calculations of jump variations.

In summary, the (daily) variables that we construct when carrying out our prediction experiments are as follows.

$RPV_{q,t}$ = Realized Measure of qth order power variation at day $t = \sum_{i=1}^n |r_{i,n}|^q$ with $q > 0$,

$RJ_{q,t}^+$ = Realized Measure of qth order upside jump power variation at day $t = \sum_{i=1}^n \left(|r_{i,n}^+|^q\right)$, $q > 2$,

$RJ_{q,t}^-$ = Realized Measure of qth order downside jump power variation at day $t = \sum_{i=1}^n \left(|r_{i,n}^-|^q\right)$, $q > 2$, and

$RJA_{q,t}$ = *Realized Measure of qth order signed jumps power variation at day* $t = RJ^{+}_{q,t} - RJ^{-}_{q,t}$, $q > 2$.

Additionally, we consider variants of all of these variables that are multiplied by an indicator variable, $I_{jump,t}$, where $I_{jump,t} = 1$ if jumps occur at day t and $I_{jump,t} = 0$ otherwise. Thus, for example, we also model $RPV_{q,t} = I_{jump,t} * \{\sum_{i=1}^{n} |r_{i,n}|^q\}$, $RJ^{+}_{q,t} = I_{jump,t} * \{\sum_{i=1}^{n} \left(|r^{+}_{i,n}|^q\right)\}$, $RJ^{-}_{q,t} = I_{jump,t} * \{\sum_{i=1}^{n} \left(|r^{-}_{i,n}|^q\right)\}$, and $RJA_{q,t} = I_{jump,t} * \{RJ^{+}_{q,t} - RJ^{-}_{q,t}\}$.

4 Prediction Models and Methodology

In a classic paper, Ding, Granger and Engle (DGE:1993) find that the auto-correlation of power transformations of daily S&P500 returns is strongest when $q = 1$, as opposed to the value $q = 2$, which was previously widely used in the literature. This led them to formulate the so-called Asymmetric Power ARCH (APARCH) model. The APARCH specification allows for flexibility via use of qth power transformations of absolute returns. GS (2009) point out that this class of models ends up working with volatility that is not measured by squared returns, which is what researchers and practitioners care about the most. Using five-minute intra-daily returns on the Dow Jones composite index for the period 1993-2000, GS (2009) carry out a thorough empirical correlation analysis (using MIDAS) of daily RV and realized power variations, with the forecasting horizon from one to four weeks. They conclude that realized power variation with $q = 1$ and future RV display the strongest cross-correlation over the first 10 lags. Beyond this first 10 lags, the cross-correlation holds for $q = 0.5$. This suggests that the prediction of RV using variables such as realized power variation might yield better results compared to simply using lags of RV. As mentioned in the introduction, our approach is to utilize our "new" power variation variables that capture information generated by jumps by estimating and carrying out prediction experiments using HAR-RV models. The HAR-RV model, initially developed in Corsi (2009), is formulated on the basis of the so-called heterogeneous ARCH, or HARCH class of models analyzed by Müller et al. (1997), in which the conditional variance of discretely sampled returns is parameterized as a linear function of the lagged squared returns over the identical return horizon together with the squared returns over shorter return horizons. Intuitively, different groups of investors have different investment horizons, and consequently behave differently. The original HAR-RV model is a constrained AR(22) model and is convenient in applications, as volatility is treated as if it is observed.

Define the multi-period normalized realized measures for jump and continuous components as the average of the corresponding one-period measures. Namely for daily time series Y_t, construct $Y_{t,t+h}$ such that

$$Y_{t,t+h} = h^{-1}[Y_{t+1} + Y_{t+2} + ... + Y_{t+h}], \tag{20}$$

where h is an integer. $Y_{t,t+h}$ aggregates information between time $t+1$ and $t+h$. The daily time

series Y_t can be any of $RV_t, RVJ_t, RVC_t, RPV_{q,t}, RJ^+_{q,t}, RJ^-_{q,t}$, or $RJA_{q,t}$, with $q = \{0.1k\}_{k=1}^{k=60}$. In standard linear and nonlinear HAR-RV models, future RV depends on past RV. Namely,

$$\phi(RV_{t,t+h}) = \beta_0 + \beta_d\phi(RV_t) + \beta_w\phi(RV_{t-5,t}) + \beta_m\phi(RV_{t-22,t}) + \epsilon_{t+h}, \tag{21}$$

where ϕ is a linear, square root or log function. The incorporation of RMs of jump variations, such RVJ_t can be done as in ABD (2007), using the HAR-RV-J model, specified as follows

$$\phi(RV_{t,t+h}) = \beta_0 + \beta_d\phi(RV_t) + \beta_w\phi(RV_{t-5,t}) + \beta_m\phi(RV_{t-22,t}) + \beta_j\phi(RVJ_t) + \epsilon_{t+h},$$

or the HAR-RV-CJ model,

$$\begin{aligned}\phi(RV_{t+h}) = \beta_0 &+ \beta_d\phi(RVC_t) + \beta_w\phi(RVC_{t-5,t}) + \beta_m\phi(RVC_{t-22,t}) + \beta_{jd}\phi(RVJ_t),\\ &+ \beta_{jw}\phi(RVJ_{t-5,t}) + \beta_{jm}\phi(RVJ_{t-22,t}) + \epsilon_{t+h}.\end{aligned}$$

ABD (2007) find that the class of log HAR-RV, log HAR-RV-J and log HAR-RV-CJ models perform the best for several market indexes. DS (2011) revisit this class of models but focus on the predictive performance of the models for analyzing individual stock returns. PS (2011) assess different predictors, including realized semivariances and realized signed jump measures. Their extended HAR-RV model is,

$$\begin{aligned}\phi(RV_{t,t+h}) = \beta_0 &+ \beta^+_1\phi(RS^+_t) + \beta^-_1\phi(RS^-_t) + \beta^+_5\phi(RS^+_{t-5,t}) + \beta^-_5\phi(RS^-_{t-5,t})\\ &+ \beta^+_{22}\phi(RS^+_{t-22,t}) + \beta^-_{22}\phi(RS^-_{t-22,t}) + \varepsilon_{t+h}.\end{aligned}$$

Building on the above papers, we extend the HAR-RV model to incorporate time series of RMs of jump power variations. In addition, we examine forecasts of RV_{t+h}, rather than $RV_{t,t+h}$, and we carry out both in-sample regression analysis as well as ex ante prediction experiments using both rolling and recursive estimation windows. All estimation is carried out using least squares, and heteroskedasticity and autocorrelation consistent standard errors are used in all inference based on the models. The models, which are re-estimated for each value of q, are as follows:

Specification 1: Standard HAR-RV-C Model (Benchmark Model):

$$\phi(RV_{t+h}) = \beta_0 + \beta_{cd}\phi(RVC_t) + \beta_{cw}\phi(RVC_{t-5,t}) + \beta_{cm}\phi(RVC_{t-22,t}) + \epsilon_{t+h}. \tag{22}$$

In this benchmark case, future RV depends on lags of the variation of the continuous component of the process.

Specification 2: HAR-RV-C-PV(q) Model:

$$\begin{aligned}\phi(RV_{t+h}) = \beta_0 &+ \beta_{cd}\phi(RVC_t) + \beta_{cw}\phi(RVC_{t-5,t}) + \beta_{cm}\phi(RVC_{t-22,t})\\ &+ \beta_{jd}\phi(RPV_{q,t}) + \beta_{jw}\phi(RPV_{q,t-5,t}) + \beta_{jm}\phi(RPV_{q,t-22,t}) + \epsilon_{t+h},\end{aligned} \tag{23}$$

where $RPV_{q,t}$ is the qth order variation of the jump component. $RPV_{q,t-5,t}$ and $RPV_{q,t-22,t}$ are calculated using (20), and $0.1 \leq q \leq 6$.

Specification 3: HAR-RV-C-UJ(q) Model (Upside Jumps):

$$\begin{aligned}\phi(RV_{t+h}) = \beta_0 &+ \beta_{cd}\phi(RVC_t) + \beta_{cw}\phi(RVC_{t-5,t}) + \beta_{cm}\phi(RVC_{t-22,t}) \\ &+ \beta^+_{jd}\phi(RJ^+_{q,t}) + \beta^+_{jw}\phi(RJ^+_{q,t-5,t}) + \beta^+_{jm}\phi(RJ^+_{q,t-22,t}) + \epsilon_{t+h}. \end{aligned} \tag{24}$$

$RJ^+_{q,t}, RJ^+_{q,t-5,t}, RJ^+_{q,t-22,t}$ measure the qth order power variation of positive jumps today, last week, and last month, and are calculated using (20), and $2.1 \leq q \leq 6$.

Specification 4: HAR-RV-C-DJ(q) Model (Downside Jumps):

$$\begin{aligned}\phi(RV_{t+h}) = \beta_0 &+ \beta_{cd}\phi(RVC_t) + \beta_{cw}\phi(RVC_{t-5,t}) + \beta_{cm}\phi(RVC_{t-22,t}) \\ &+ \beta^-_{jd}\phi(RJ^-_{q,t}) + \beta^-_{jw}\phi(RJ^-_{q,t-5,t}) + \beta^-_{jm}\phi(RJ^-_{q,t-22,t}) + \epsilon_{t+h}. \end{aligned} \tag{25}$$

The range of q is $2.1 \leq q \leq 6$.

Specification 5: HAR-RV-C-UDJ(q) Model (Upside and Downside Jumps):

$$\begin{aligned}\phi(RV_{t+h}) = \beta_0 &+ \beta_{cd}\phi(RVC_t) + \beta_{cw}\phi(RVC_{t-5,t}) + \beta_{cm}\phi(RVC_{t-22,t}) \\ &+ \beta^+_{jd}\phi(RJ^+_{q,t}) + \beta^+_{jw}\phi(RJ^+_{q,t-5,t}) + \beta^+_{jm}\phi(RJ^+_{q,t-22,t}) \\ &+ \beta^-_{jd}\phi(RJ^-_{q,t}) + \beta^-_{jw}\phi(RJ^-_{q,t-5,t}) + \beta^-_{jm}\phi(RJ^-_{q,t-22,t}) + \epsilon_{t+h}. \end{aligned} \tag{26}$$

Specification 6: HAR-RV-C-APJ(q) Model (Asymmetric Jumps):

$$\begin{aligned}\phi(RV_{t+h}) = \beta_0 &+ \beta_{cd}\phi(RVC_t) + \beta_{cw}\phi(RVC_{t-5,t}) + \beta_{cm}\phi(RVC_{t-22,t}) \\ &+ \beta_{jd}\phi(RJA_{q,t}) + \beta_{jw}\phi(RJA_{q,t-5,t}) + \beta_{jm}\phi(RJA_{q,t-22,t}) + \epsilon_{t+h}. \end{aligned} \tag{27}$$

This model uses RMs of signed jump power variations, i.e., measures of jump asymmetry, as explanatory variables. These variables, $RJA_{q,t}$, $RJA_{q,t-5,t}$ and $RJA_{q,t-22,t}$, are calculated using (20).

Finally, for completeness, we also carry out our empirical analysis using the above models, but with jump variables re-defined as follows, $RJ_q(\gamma) = \sum_{i=1}^n |r_{i,n}|^q I_{|r_{i,n}|<\gamma}$, $RJ^-_q(\gamma) = \sum_{i=1}^n |r_{i,n}|^q I_{-\gamma<r_{i,n}<0}$, and $RJ^+_q(\gamma) = \sum_{i=1}^n |r_{i,n}|^q I_{0<r_{i,n}<\gamma}$. Evidently, in these experiments, we truncated our measures to include only jump variations associated with large (small) jumps, as discussed in Section 2.

The forecast horizons that we examine are $h = 1, 5, 22$, which correspond to one day, one week, and one month ahead, respectively. For each specification (except for Specifications 1 and 2), there are 40 sub-models, corresponding to 40 different values of q. In our forecasting experiments, the entire sample of T observations is divided into two samples, the estimation sample containing R observations and the prediction sample containing $P = T - R$ observations. Both rolling and recursive windows of data are used in model estimation, prior to the construction of each new prediction. In addition to reporting out-of-sample R^2, calculated by projecting RV forecasts on

historical RV, we also report traditional in-sample adjusted R^2, calculated using entire sample of T observations. In our prediction experiments, we also carry out pairwise Diebold and Mariano (DM: 1995). Our DM tests assume quadratic loss, have a null of equal predictive ability, and are asymptotically normally distributed (under a nonnestedness assumption - see Corradi and Swanson (2006) and the references cited therein for a complete discussion). The test statistic is $DM = P^{-1}\sum_{k=1}^{P}(d_t/\widehat{\sigma})$ where $d_t = \widehat{\varepsilon}_{1,t+h}^2 - \widehat{\varepsilon}_{2,t+h}^2$,the $\widehat{\varepsilon}$s are forecast errors from the two competing models, and $\widehat{\sigma}$ is a heteroskedasticity and autocorrelation consistent estimator of the standard error of the mean of d_t.

5 Empirical Findings

5.1 Data Description

Our S&P500 futures index and Dow 30 individual stock datasets (collected for the period 1993-2009 and 1993-2008, respectively) were obtained from the TAQ database. When processing the data, we followed the common practice of eliminating from the sample those days with infrequent trades (less than 60 transactions at our 5 minute frequency). In the literature, two methods are often applied for filtering out an evenly-spaced sample - the *previous tick* method and the *interpolation* method (Dacorogna, Gencay, Müller, Olsen, and Pictet (2001)). As shown in Hansen and Lund (2006), in applications using quadratic variation, the *interpolation* method should not be used, as it leads to realized volatilities with value 0 (see *Lemma 3* in their paper). Therefore, we use the *previous tick* method (i.e. choosing the last price observed during a given interval). We restrict our dataset to regular time and ignore ad hoc transactions outside of this time interval. To reduce microstructure noise effects, the suggested sampling frequency in the literature ranges from 5 minutes to 30 minutes. We choose the 5 minute frequency, yielding 78 observations per day in most cases.[9]

5.2 Basic Analysis of Jumps

All daily statistics are calculated using the formulae in Sections 2 with

$$\Delta_n = \frac{1}{n} = \frac{1}{\#\text{ of 5 minute transactions / day}}.$$

For instance, $\Delta_n = 1/78$ for most of the stocks in the sample. This implies that the time interval $[0,1]$ maps into a beginning time of 9 am (set equal to 0) and an end time of 4:30 pm (set equal to 1), in our setup. In all calculations involving integrated volatility and integrated quarticity, we use multipower variation, as discussed above. Let T denote the number of days in the sample. We construct the time series $\{Z_{t,n}(\alpha)\}_{t=1}^{T}$ and $\left\{\frac{RVC_t}{RV_t}, \frac{RVJ_t}{RV_t}, \frac{VLJ_{t,\gamma}}{RV_t}, \frac{VSJ_{t,\gamma}}{RV_t}\right\}_{t=1}^{T}$. The number of

[9] A main drawback of realized measures constructed using high frequency data is that they are contaminated by mictrostructure noise, and hence our use of a 5 minute data interval. See Aït-Sahalia, Mykland and Zhang (2005) for further dicussion.

days and proportion of days identified as containing jumps can easily be calculated as: *number of days identified as having jumps* $= \sum_{i=1}^{T} I(Z_{i,n}(\alpha) > \Phi_\alpha)$ and *proportion of days identified as having jumps* $= \frac{\sum_{i=1}^{T} I(Z_{t,n}(\alpha) > \Phi_\alpha)}{T}$, where $I(\cdot)$ denotes the indicator function, as usual. The average relative contribution of continuous, jump, and large jump components to the variation of the process is reported using the mean of the sample (i.e., we report the means of $\frac{RVC_t}{RV_t}$, $\frac{RVJ_t}{RV_t}$, $\frac{VLJ_{t,\gamma}}{RV_t}$, and $\frac{VSJ_{t,\gamma}}{RV_t}$). In the sequel, we provide numerical results and figures for S&P500 futures, while only select (representative) results are reported for the Dow 30 components, in cases were brevity becomes and issue, and where qualitative findings remain the same. Complete results are available upon request.

Turning to our findings, a first impression regarding the prevalence of jumps can be obtained by inspecting Figures 1 and 4, where statistics higher than 3.09 (i.e., the 0.001 significance level critical value) are depicted for the entire sample from 1993-2009 for S&P500 futures returns and from 1993-2008 for Citigroup, Home Depot, Intel, and Microsoft returns. It is obvious that jumps are prevalent. The highest statistic values are around 8 from 2006 to 2008, for S&P500 futures, as shown in Figure 1. The highest statistic values for individual stocks are around 7 for Citigroup from 1996 to 1997, around 7 in 1999, 2003 and between 2006 and 2008 for Home Depot, around 9 for Intel in 1994, and finally around 6 in 1998, 2001, and 2007 for Microsoft.

When examining large jumps, an important step is the choice truncation level, γ. If we choose arbitrarily large truncation levels, then clearly we will find no evidence of large jumps. Also, one might imagine proceeding by picking truncation levels based on the percentiles of the entire historical sample of 5 minute returns. However, results will then be difficult to interpret, as the usual choice of $90th$ or $75th$ percentiles leads to virtually no large jumps while the choice of $25th$ or $10th$ percentiles leads to a very large number of large jumps. In addition, large jumps are often thought of as abnormal events that arise at a frequency of one in several months or even years. Therefore, a reasonable way to proceed is to pick the truncation level on the basis of the sample of the monthly maximal increments, i.e., monthly abnormal events. Specifically, we set four levels $\gamma = 1, 2, 3, 4$ to be the $50th, 75th$, $90th$ and $95th$ percentiles of the entire sample of maximal increments from 1993-2009 for S&P500 futures and from 1993-2008 for the Dow 30 components. As an illustration, Figure 2 depicts the monthly largest absolute increments and the jump truncation levels used in our calculations of the variation of large and small jump components at three levels, $\gamma = 1, 2, 3$, for S&P500 futures. It is quite obvious that the monthly maximum increments are dominant for the period from 1998-2002 and for the period from 2006-2008. The truncation level for S&P500 futures ranges from approximately 0.03 to 0.08.

Next, notice that the graphs in Figure 3 depict magnitudes of the variation of continuous, jump, and truncated jump components of S&P500 futures returns. Namely, the plots are of daily realized volatility and realized variances of continuous, jump and large jump components at different truncation levels. As might be expected, inspection of the graphs suggests a close linkage between

the greater number of jumps and the magnitude of jump risk over the same period. For example, in the case of S&P500 futures, the variation of the jump components is clearly dominant in the sample periods from 1998-2002 and from 2006-2008. The highest daily jump risk occurs in 2001, and is above 0.12. Indeed, at jump truncation level 2, we only see large jump risk for the years 1998, 2001, and 2007. Combined with the results of Figure 1, this again strongly suggests that there are notable jumps in S&P500 futures data.

Turning now to our tabulated results, Tables 1 and 2 contain results summarizing the contribution of realized variations of various price components of S&P500 futures and Dow 30 stock components, relative to total variation. Table 1 reports the average percentage of daily variation of the continuous and jump components, at truncation levels $1, 2, 3, 4$, relative to daily realized variances, for the sample period from 1993-2009, across test significance levels, $\alpha = 0.0001, 0.001, 0.005$ and 0.01. For example, at the $\alpha = 0.001$ and 0.0001 levels, the average daily jump variations are 25.3% and 14.4% during the 1993-2009 period, respectively. Corresponding average variations of large daily jumps at truncation level 3 are 1.7% and 0.7% respectively. For individual stocks, Table 2 reports average percentage of days identified as having jumps, and the average percentage of daily variation of continuous, jump, and large jump components, at truncation levels $1, 2, 3$, for significance level, $\alpha = 0.001$, and across 25 stocks in the Dow 30, for the period from 1993-2008. There is clear evidence of "jump-days" for all of these stocks. For instance, as illustrated in Figure 4, and tabulated in Table 2, the proportion of "jump-days" for Citigroup, Home Depot, Intel and Microsoft is 15.4%, 17.5%, 14.7% and 13.9%, respectively. In addition, jumps contribute a significant part of the realized volatility across all stocks. For instance, the average daily jump variations for Citigroup, Home Depot, Intel and Microsoft are 8.3%, 9.2%, 6.8%, and 6.3%. When considering large jumps with $\gamma = 3$, the average daily jump variations of the same stocks are 0.2%, 0.0%, 0.1%, and 0.1%, respectively.

In summary, and not surprisingly, we have strong evidence that jumps characterize the structure of S&P500 futures and Dow 30 returns. Moreover, the prevalence of jumps is dependent upon sample period, and, just as importantly, is dependent upon truncation level. For example, the overall contribution of jumps is quite dissimilar across Dow 30 stocks, ranging from around 3% to over 10%; but when truncation levels are applied, the relative contribution of jumps appears very similar (e.g., when $\gamma = 2$ the range is 0.0% to 0.2%). This certainly suggests that clustering is occurring during "bad" times; but, just as importantly, it suggests that jump information aggregation might be important in financial applications, and in particular in forecasting exercises.

5.3 RV Prediction using Realized Jump Power Variations

We begin by calculating all daily RMs, as discussed above, using our S&P500 dataset; yielding time series with $T = 4123$ observations. In our out-of-sample forecasting experiments, we set

$P = 410$.[10] The models used in our experiments are discussed above and summarized in Table 4. Finally, as a point of reference, recall that the empirical analyses of exchange rates, equity index returns, and bond yields reported in ABD (2007) suggest that the volatility jump component is both highly important and distinctly less persistent than the continuous component, and that separating "rough" jump movements from smooth continuous movements results in significant in-sample volatility forecast improvements (i.e., linear and nonlinear HAR-RV-CJ models perform better than models without "separate" jumps).

We now turn to our analysis of the alternative models presented summarized in Table 4. Consider S&P500 futures. The predictive performance of a model is measured by both in-sample and out-of sample R^2, which is similar to approach taken in ABD (2007). We also carry out DM (1995) predictive accuracy tests to determine whether the choice of q matters when forecasting RV. Table 3 reports regression estimates, as well as in-sample and out-of-sample R^2 values for linear, square root and log HAR-RV-C models at daily ($h = 1$), weekly ($h = 5$) and monthly ($h = 22$) prediction horizons. Entries in brackets are robust t-statistics. When comparing in-sample and out-of-sample R^2 statistics, it is clear that the square root and log models perform much better than their linear counterparts, regardless of prediction horizon. For instance, for $h = 1$, the in-sample and out-of-sample R^2 statistics for square root models are 0.45 and 0.34 while those of their linear counterparts are 0.35 and 0.24, respectively. In addition, the estimates of β_{cd}, β_{cw}, β_{cm}, as well as associated t-statistics confirm the long memory (persistence) property of volatility. For the linear model with $h = 1$, the t-statistic of the monthly forecast parameter is 7.81, implying that the continuous component from the previous month is potentially important for one-day ahead prediction of volatility. This statistical pattern holds for square root and log models, across all forecast horizons. In addition, at prediction horizon $h = 22$, while the in-sample R^2s are large, out-of sample results show deteriorating behavior, as might be expected.

When constructing $RPV_{q,t}$, $RJ^{+}_{q,t}$, $RJ^{-}_{q,t}$, and $RJA_{q,t}$, values of q including $\{2.1, 2.2, ..., 5.8, 5.9, 6.0\}$ were tried.[11] Larger values of q effectively eliminate the effects of the continuous component and of smaller jumps, while magnifying the relevance of large jumps. In Tables 5A-5D, we report results only for $q = 2.5$ and $q = 5$, as these are two good representative cases when distinguishing between small and large jump power variations. Each table contains results for linear, square root and log models. All bracketed entries are t-statistics. Observe first that jump coefficients are not usually statistically significant for $q = 5$ (large jumps). This result holds across all model specifications, and holds for all cases where $q = 5$, except in Table 5B. Here, β_{jw} and β_{jm} associated with the square root model at h=5 have t-statistics of 17.89 and -5.09, respectively. Additionally, in Table 5C, β^{-}_{jd} (linear model and h=1) has a t-statistic of 1.96. For $q = 2.5$, t-statistics are significant for β_{jm} in linear and square root HAR-RV-C-PV(q) models (the t-statistics are 2.37 and 2.10 for $h = 1$, in

[10] We also analyzed alternative out-of-sample periods, including P ={210, 310, 510, 610,710}. Results were qualitatively similar to those reported here, and are available upon request.

[11] For $q > 6$, prediction results are almost the same as when case $q = 6$, and are therefore not discussed.

linear and square root HAR-RV-C-DJ(q) models, respectively. Turning to our "full decomposition" HAR-RV-C-UDJ(q) model, we find that downside jumps rarely have an impact on future RV, such as when $h = 1$. Also, for the linear model, note in Table 5C that the t-statistic associated with β_{jd}^{-} is 2.14, for $h = 22$. Upward jump variations generally have a negligible impact in our prediction models, however. Most interestingly, correlation between past $RJA(q)$ and future RV is rather strong across all forecast horizons (daily, weekly and monthly) for linear and square root models, as indicated by a large number of statistically significant coefficient estimates on this variable (see Table 5D).

Table 6 reports on tests carried out to compare the predictive accuracy of a subset of our prediction models. In particular, and for each model listed in the first column of the table, q_b denotes the value of q that yields the largest out-of-sample R^2 values, while q_s denotes the value of q that yields the smallest R^2 values, for $q = \{2.5 + k * 0.1\}_{k=0}^{k=35}$. The DM statistics in the first row of each panel of the table are based on the comparison of each pair of (q_b, q_s) models, and positive values indicate that the q_b model dominates, in terms of out-of-sample mean square forecast error fit. Since almost all DM statistics are positive, we have evidence that the highest out-of-sample R^2 model is statistically superior to the lowest. Moreover, as we generally see that $q_b = 2.5$, we have strong evidence that large, seemingly rare and possibly *iid* jumps do not help in prediction, while smaller, less rare and possibly serially correlated jumps do help.

Continuing our discussion of predictive performance, note that our prediction experiments show improvements, both in- and out-of-sample, when RMs of jump power variations are used as additional predictors in volatility forecasting. For example, at forecast horizons $h = 1$ and $h = 5$, the out-of sample R^2 values of the benchmark HAR-RV-C square root models are 0.34 for $h = 1$ and 0.24 for $h = 5$. Compare these values with those of 0.37 and 0.26, which obtain when our HAR-RV-C-PV(q) model is used to construct forecasts. This is equivalent to an 8% and 7.5% increase in R^2, when switching from HAR-RV-C to HAR-RV-C-PV models. However, as shown in the table, the continuous component, RVC, dominates in all prediction experiments, which is consistent with previous findings in the literature on volatility forecasting using high frequency data. Moreover, there is little improvements in R^2 when HAR-RV-C-UDJ(q) is used for prediction. Interestingly, results in the table suggest that in- and out-of sample R^2 values are smaller, the larger is q (compare the cases where $q = 2.5$ and $q = 5$). This pattern is clearly depicted in the figures discussed below.

Finally, the above conclusions are confirmed in Figures 5-8. In these figures, both in- and out-of-sample R^2 values are reported. In all plots, the vertical axis ranges from 0 to 1, and denotes the value R^2. The horizontal axis ranges from 0.1 to 6, representing 60 grid points of values of q, i.e. $q = \{0 + 0.1 * k\}_{k=1}^{60}$. Notice first that there is little to choose between the models, in a majority of cases, confirming our earlier finding that jumps, while prevalent, add relatively little to predictive accuracy. Second, comparing "no jump test" cases with "jump test" cases indicates that findings do change, to some degree, when jump tests are used in the construction of jump

variation variables. In particular, compare Figures 5 and 6 (the case where S&P500 futures are modelled). The maximal in-sample R^2 values that is achieved when no jump tests are used is usually modestly higher, under our log model, regardless of forecast horizon (compare the last column of plots in each figure). Naturally, the R^2-"best" value of q also varies, although to a very small extent, when comparing these two figures. The same broad result holds when comparing out-of-sample R^2 values in Tables 7A (no jump test) and 7B (jump test). In summary, little is gained in our experiments by constructing realized measures that directly incorporate a variable indicating whether our jump test find evidence of jumps during a particular day. Third, Table 8 clearly indicates that the R^2-"best" value of q is higher when S&P500 index returns are predicted, than when individual DOW components are predicted. This suggests that aggregation plays a crucial role in risk prediction. Values of q less than 2 dominate under individual stocks, while values greater than 2 dominate under our index variable. Evidently, jumps matter much more for risk prediction in a return variable that aggregates many jumps from many companies than in isolated companies. Finally, while the in-sample R^2-"best" value of q is always near unity in our log models, when evaluating the S&P500 index (see Figures 5-7), the out–of-sample R^2-"best" value of q is always near or greater than 2 (see Figures 7A-7B). This rather interesting finding suggests that what's best for in-sample analysis is far from best for out-of-sample analysis. In particular, jumps do play a role, at least when modelling aggregate (index) data such as S&P500 futures returns; and while modelling jump risk power variations may not be important for in-sample fit, it clearly plays an important role in out-of-sample volatility prediction.

6 Concluding Remarks

In this paper, we use recent theoretical results of Jacod (2008), BNS (2004, 2006), and BKS (2010) to examine jumps and the usefulness of jumps in forecasting volatility. Our key findings can be summarized as follows. First, we find evidence that jumps characterize the structure of S&P500 futures and the individual stocks that we examine. Moreover, the prevalence of jumps is dependent upon sample period; and is also dependent upon truncation level. This is consistent with "clustering" occurring during "bad" times; but, just as importantly, it suggests that jump information aggregation might be of relevance in financial applications, and in particular in forecasting exercises. Second, our prediction experiments show improvements, both in- and out-of-sample, when RMs of jump power variations are used as additional predictors in volatility forecasting. However, past "large" jump power variations help less in the prediction of future realized volatility, than past "small" jump power variations. This in turn suggests the "larger" jumps might help less in the prediction than "smaller" jumps. In a related finding, we note that seemingly rare and possibly *iid* jumps do not help in prediction, while smaller, less rare and possibly serially correlated jumps do help. Third, the continuous component dominates in all prediction experiments, which is consistent with previous findings in the literature on volatility forecasting using high frequency data.

Fourth, incorporation of downside and upside jump power variations does improve predictability, albeit to a limited extent. Fifth, comparing "no jump test" cases with "jump test" cases indicates that findings do change, to some degree, when jump tests are used in the construction of jump variation variables. Additionally, the power of q associated with our R^2-"best" model is higher when S&P500 index returns are predicted, than when individual DOW components are predicted. This suggests that aggregation plays a crucial role in risk prediction. Finally, values of q less than 2 dominate under individual stocks, while values greater than 2 dominate under our index variable. Taken together, these results suggest that what's best for in-sample analysis is far from best for out-of-sample analysis. Moreover, jumps do play a role, at least when modelling aggregate (index) data such as S&P500 futures returns; and while modelling jump risk power variations may not be important for in-sample fit, it clearly plays an important role in out-of-sample volatility prediction.

Many questions remain for future research. For example, it remains to be seen whether prediction based "gains" associated with modelling jumps translates into improved performance when carrying out real-world derivative pricing, asset allocation, and hedging exercises. Additionally, and although we have presented some evidence tying jump variations to general economic activity, it remains to exhaustively analyze the linkages between jumps, jump variations, market risk, and business cycle activity.

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Table 1: Daily S&P500 Futures Returns:
Ratio of Continuous, Total Jump, Large Jump and Small Jump (Truncation Levels 1,2,3,4) to Total Realized Variation for the Period 1993-2009*

Variation Component\Significance Level	0.0001	0.001	0.005	0.01
Continuous	85.6	74.7	64.3	59.4
Total Jump	14.4	25.3	35.7	40.6
Large Jump (Truncation Level 4)	0.1	0.1	0.20	0.2
Large Jump (Truncation Level 3)	0.7	1.7	2.6	2.8
Large Jump (Truncation Level 2)	2.2	4.3	6.2	6.8
Large Jump (Truncation Level 1)	4.0	8.7	13.3	15.4
Small Jump (Truncation Level 4)	14.3	25.2	35.4	40.4
Small Jump, (Truncation Level 3)	13.6	23.6	33.1	37.8
Small Jump (Truncation Level 2)	12.2	21	29.5	33.8
Small Jump (Truncation Level 1)	10.3	16.6	22.4	25.2

* Entries in rows 2 and 3 denote the average percentage of daily variation of the continuous component and total jump component, relative to daily realized variance. Entries in rows 3 to 8 denote the average percentage of daily variation due to large and small jumps constructed using truncation levels 1, 2, 3,4 relative to the daily realized variance, where Truncation Level 1 corresponds to the median of monthly maximum increments, Truncation Level 2 corresponds to 75th percentile monthly maximum increments, Truncation level 3 corresponds to 90th percentile monthly maximum increments, and truncation level 4 corresponds to 95th percentile monthly maximum increments of (log) prices of S&P500 futures returns for the sample 1993-2009. Entries are caculcated for jump tests carried out using 4 different significance levels, $\alpha = 0.0001, 0.001, 0.005, 0.01$. See Sections 2 and 5 for further details.

Table 2: Daily DOW 30 Component Returns:

Ratio of Continuous, Total Jump, Large Jump, and Small Jump (Truncation Levels 1,2,3) to Total Realized Variation for the Period 1993-2008*

Company	Jump Frequency	Continuous	Total Jumps	Truncation Level 1	Truncation Level 2	Truncation Level 3
Alcoa	17.6	90.8	9.2	0.7	0.3	0.1
American Express	13.8	92.8	7.2	0.5	0.3	0.1
Bank of America	15.2	92.1	7.9	0.7	0.3	0.1
Citigroup	15.4	91.7	8.3	0.5	0.3	0.2
Caterpillar	17.7	90.6	9.4	0.7	0.4	0.2
Dupont	17	91.2	8.8	0.5	0.2	0.0
Walt Disney	18.9	89.8	10.2	0.6	0.2	0.1
General Electric	16	91.8	8.2	0.4	0.1	0.0
GM	18.1	90.3	9.7	0.5	0.2	0.1
Home Depot	17.5	90.8	9.2	0.4	0.2	0.0
IBM	12.7	93.4	6.6	0.6	0.3	0.1
Intel	14.7	93.2	6.8	0.6	0.2	0.1
Johnson &Johnson	18.2	90.6	9.4	0.6	0.3	0.2
JPM	15.3	92.1	7.9	0.5	0.2	0.0
Coca Cola	17.2	91.0	9.0	0.6	0.2	0.1
McDonald's	19.1	89.6	10.4	0.5	0.3	0.1
3M	17.3	90.8	9.2	0.6	0.2	0.1
Microsoft	13.9	93.7	6.3	0.6	0.2	0.1
Pfizer	18	90.6	9.4	0.6	0.3	0.1
Procter &Gamble	16.5	91.3	8.7	0.6	0.3	0.1
AT &T	18.6	90.1	9.9	0.8	0.4	0.2
United Tech.Corp.	15.4	92.3	7.7	0.8	0.3	0.1
Verizon	11.9	94.9	5.1	0.7	0.3	0.1
Walmart	11.6	93.9	6.1	0.6	0.3	0.1
ExxonMobil	7.5	96.9	3.1	0.4	0.2	0.1

* See notes to Table 1. Entries in column 2 of the table denote the percentage of days identified as having jumps based on the calculation of daily statistics, and using the adjusted jump statistic of BNS (2006) and Huang and Tauchen (2005), with significance level α =0.001. Entries in columns 3 and 4 denote the average percentage of daily variation of the continuous and total jump components relative to daily realized variance, based on the use of jump tests. Entries in columns 5-7 denote the average percentage of daily variation due to jumps constructed using truncation levels 1, 2, 3, relative to daily realized variance. All calculation are for the sample period 1993-2008. See Sections 2 and 5 for further details.

Table 3: Daily, Weekly and Monthly HAR-RV-C Prediction Regression Results for S&P500 Futures Returns (Benchmark Model)*

Linear Model				Square Root Model				Log Model			
β_0	β_d	β_w	β_m	β_0	β_d	β_w	β_m	β_0	β_d	β_w	β_m
Forecast Horizon h=1 (Daily)											
0.00	0.09	0.06	1.65	0.00	0.07	0.12	1.00	-0.20	0.17	0.10	0.72
(0.67)	(1.93)	(0.38)	(7.81)	(1.57)	(2.70)	(1.70)	(11.91)	(1.15)	(7.04)	(1.66)	(11.71)
$R^2_{in}(R^2_{out}) = 0.35(0.24)$				$R^2_{in}(R^2_{out}) = 0.45(0.34)$				$R^2_{in}(R^2_{out}) = 0.45(0.39)$			
Forecast Horizon h=5 (Weeky)											
0.00	0.06	-0.08	1.83	0.00	0.06	0.04	1.08	-0.35	0.13	0.14	0.69
(0.71)	(0.51)	(0.43)	(10.31)	(1.00)	(0.94)	(0.40)	(12.62)	(1.84)	(5.80)	(2.40)	(10.85)
$R^2_{in}(R^2_{out}) = 0.35(0.17)$				$R^2_{in}(R^2_{out}) = 0.44(0.24)$				$R^2_{in}(R^2_{out}) = 0.43(0.30)$			
Forecast Horizion h=22 (Monthly)											
0.00	-0.03	0.39	1.38	0.00	0.01	0.14	0.98	-0.77	0.08	-0.01	0.85
(0.32)	(0.89)	(3.47)	(11.86)	(0.18)	(0.41)	(1.92)	(15.04)	(3.08)	(3.24)	(0.19)	(12.39)
$R^2_{in}(R^2_{out}) = 0.33(0.03)$				$R^2_{in}(R^2_{out}) = 0.41(0.04)$				$R^2_{in}(R^2_{out}) = 0.38(0.03)$			

* See notes to Tables 1 and 2. Entries are prediction regression results (i.e., out-of-sample forecast model estimates), as well as both in-sample and out-of-sample R^2 values, for linear, square root, and log HAR-RV-C models at daily (h=1), weekly (h=5) and monthly (h=22) forecast horizons. Entries in brackets are robust t-statistics.

Table 4: Summary of Additional Models Used for Forecasting RV*

Specification 1 (HAR-RV-C)	$\phi(RV_{t+h}) = \beta_0 + \beta_{cd}\phi(RVC_t) + \beta_{cw}\phi(RVC_{t-5,t}) + \beta_{cm}\phi(RVC_{t-22,t}) + \epsilon_{t+h}$
Specification 2 (HAR-RV-C-PV(q))	$\phi(RV_{t+h}) = \beta_0 + \beta_{cd}\phi(RVC_t) + \beta_{cw}\phi(RVC_{t,t-5}) + \beta_{cm}\phi(RVC_{t,t-22}) + \beta_{jd}\phi(RPV_{q,t}) + \beta_{jw}\phi(RPV_{q,t,t-5}) + \beta_{jm}\phi(RPV_{q,t,t-22}) + \epsilon_{t+h}$
Specification 3 (HAR-RV-C-UJ(q))	$\phi(RV_{t+h}) = \beta_0 + \beta_{cd}\phi(RVC_t) + \beta_{cw}\phi(RVC_{t,t-5}) + \beta_{cm}\phi(RVC_{t,t-22}) + \beta^+_{jd}\phi(RJ^+_{q,t-5,t}) + \beta^+_{jw}\phi(RJ^+_{q,t-5,t}) + \beta^+_{jm}\phi(RJ^+_{q,t-22,t}) + \epsilon_{t+h}$
Specification 4 (HAR-RV-C-DJ(q))	$\phi(RV_{t+h}) = \beta_0 + \beta_{cd}\phi(RVC_t) + \beta_{cw}\phi(RVC_{t,t-5}) + \beta_{cm}\phi(RVC_{t,t-22}) + \beta^-_{jd}\phi(RJ^-_{q,t-5,t}) + \beta^-_{jw}\phi(RJ^-_{q,t-5,t}) + \beta^-_{jm}\phi(RJ^-_{q,t-22,t}) + \epsilon_{t+h}$
Specification 5 (HAR-RV-C-UDJ(q))	$\phi(RV_{t+h}) = \beta_0 + \beta_{cd}\phi(RVC_t) + \beta_{cw}\phi(RVC_{t-5,t}) + \beta_{cm}\phi(RVC_{t-22,t}) + \beta^+_{jd}\phi(RJ^+_{q,t-5,t}) + \beta^+_{jw}\phi(RJ^+_{q,t-5,t}) + \beta^+_{jm}\phi(RJ^+_{q,t-22,t}) + \beta^-_{jd}\phi(RJ^-_{q,t-5,t}) + \beta^-_{jw}\phi(RJ^-_{q,t-5,t}) + \beta^-_{jm}\phi(RJ^-_{q,t-22,t}) + \epsilon_{t+h}$
Specification 6 (HAR-RV-C-APJ(q))	$\phi(RV_{t+h}) = \beta_0 + \beta_{cd}\phi(RVC_t) + \beta_{cw}\phi(RVC_{t-5,t}) + \beta_{cm}\phi(RVC_{t-22,t}) + \beta_{jd}\phi(RJA_{q,t}) + \beta_{jw}\phi(RJA_{q,t-5,t}) + \beta_{jm}\phi(RJA_{q,t-22,t}) + \epsilon_{t+h}$

* See notes to Table 3. Entries in this table are for forecast models examined in our prediction experiments.

Table 5A: HAR-RV-C-PV(q) Prediction Regression Results (q=2.5 and 5) for S&P500 Futures Returns*

		Linear Models			Square Root Models			Log Models		
		h=1	h=5	h=22	h=1	h=5	h=22	h=1	h=5	h=22
	$q = 2.5$	0.00	0.00	0.00	0.01	0.01	0.01	-0.52	-0.70	-1.23
β_0		(3.04)	(1.74)	(2.11)	(3.44)	(2.89)	(3.29)	(-1.90)	(-2.37)	(-3.29)
	$q = 5$	0.00	0.00	0.00	0.00	0.00	0.01	-0.38	-0.55	-1.02
		(3.17)	(2.35)	(2.47)	(2.49)	(2.46)	(3.14)	(-1.62)	(-2.17)	(-3.20)
	$q = 2.5$	0.07	0.00	-0.07	0.02	0.02	0.03	0.17	0.14	0.08
β_{cd}		(1.45)	(0.03)	(-1.09)	(0.60)	(0.35)	(0.67)	(6.81)	(5.69)	(2.98)
	$q = 5$	0.07	0.02	-0.08	0.06	0.06	0.03	0.17	0.13	0.07
		(1.77)	(0.17)	(-1.32)	(1.79)	(0.96)	(0.71)	(6.89)	(5.73)	(2.94)
	$q = 2.5$	-0.12	-0.25	0.42	0.02	-0.12	0.11	0.09	0.13	-0.02
β_{cw}		(-0.81)	(-1.42)	(2.79)	(0.18)	(-1.06)	(1.25)	(1.35)	(2.10)	(-0.22)
	$q = 5$	-0.07	-0.20	0.43	0.06	-0.07	0.09	0.10	0.14	-0.01
		(-0.44)	(-1.15)	(2.94)	(0.78)	(-0.62)	(1.02)	(1.59)	(2.29)	(-0.16)
	$q = 2.5$	0.70	1.23	0.69	0.52	0.70	0.37	0.68	0.65	0.79
β_{cm}		(2.47)	(4.93)	(2.22)	(3.63)	(4.69)	(1.93)	(10.06)	(9.15)	(10.09)
	$q = 5$	1.25	1.57	1.03	0.85	0.97	0.78	0.69	0.67	0.82
		(6.84)	(10.87)	(5.28)	(9.24)	(10.03)	(7.70)	(10.76)	(9.92)	(11.08)
	$q = 2.5$	0.07	0.21	0.17	0.11	0.07	-0.04	-16.35	-9.51	-2.19
β_{jd}		(0.43)	(1.32)	(0.70)	(1.94)	(0.88)	(-0.43)	(-1.44)	(-0.79)	(-0.14)
	$q = 5$	18.27	56.89	71.90	0.63	-0.05	-0.84	-2597.00	-1047.00	3180.00
		(0.35)	(1.17)	(1.00)	(0.56)	(-0.04)	(-0.45)	(-0.88)	(-0.30)	(0.89)
	$q = 2.5$	0.79	0.79	-0.19	0.32	0.47	0.08	27.95	30.64	10.90
β_{jw}		(1.63)	(1.78)	(-0.54)	(1.92)	(2.57)	(0.54)	(0.87)	(0.97)	(0.36)
	$q = 5$	194.64	195.39	-76.42	3.98	7.13	3.42	2032.00	6163.00	127.00
		(1.16)	(1.47)	(-0.81)	(1.06)	(1.88)	(1.22)	(0.22)	(0.68)	(0.02)
	$q = 2.5$	1.18	0.46	1.24	0.39	0.20	0.75	32.42	26.80	51.59
β_{jm}		(2.24)	(0.89)	(1.98)	(1.97)	(0.79)	(2.74)	(0.98)	(0.73)	(1.39)
	$q = 5$	114.43	2.21	211.49	0.88	-1.85	2.98	10776.00	6132.00	10073.00
		(0.71)	(0.02)	(1.35)	(0.25)	(-0.45)	(0.99)	(1.07)	(0.59)	(1.06)
R^2_{in}	$q = 2.5$	0.38	0.37	0.33	0.46	0.45	0.42	0.45	0.43	0.38
	$q = 5$	0.37	0.37	0.33	0.46	0.45	0.41	0.45	0.43	0.38
R^2_{out}	$q = 2.5$	0.32	0.20	0.03	0.37	0.26	0.04	0.39	0.30	0.03
	$q = 5$	0.24	0.17	0.03	0.34	0.24	0.04	0.39	0.30	0.03

* See notes to Tables 3 and 4. Prediction model estimates, as well as in-sample and out-of-sample R^2 values, are reported for linear, square root and log HAR-RV-C-PV(q) models, for q=2.5 and q=5, at daily (h=1), weekly (h=5) and monthly (h=22) prediction horizons. Entries in brackets are robust t-statistics.

Table 5B: HAR-RV-C-DJ(q) Prediction Regression Results (q=2.5 and 5) for S&P500 Futures Returns*

		Linear Models			Square Root Models			Log Models		
		h=1	h=5	h=22	h=1	h=5	h=22	h=1	h=5	h=22
β_0	$q=2.5$	0.00	-0.70	-1.23	0.01	0.01	0.01	-0.53	-0.71	-1.24
		(3.02)	(-2.37)	(-3.29)	(3.46)	(2.89)	(3.28)	(-1.92)	(-2.39)	(-3.30)
	$q=5$	0.00	-0.55	-1.02	0.00	2.49	0.01	-0.38	-0.55	-1.03
		(3.18)	(-2.17)	(-3.20)	(2.53)	(0.00)	(3.15)	(-1.63)	(-2.18)	(-3.20)
β_{cd}	$q=2.5$	0.07	0.14	0.08	0.02	0.03	0.03	0.17	0.14	0.07
		(1.47)	(5.69)	(2.98)	(0.68)	(0.36)	(0.65)	(6.81)	(5.65)	(2.96)
	$q=5$	0.08	0.13	0.07	0.06	0.97	0.03	0.17	0.13	0.07
		(1.78)	(5.73)	(2.94)	(1.81)	(0.06)	(0.67)	(6.89)	(5.72)	(2.93)
β_{cw}	$q=2.5$	-0.12	0.13	-0.02	0.02	-0.11	0.11	0.09	0.13	-0.02
		(-0.80)	(2.10)	(-0.22)	(0.22)	(-1.04)	(1.23)	(1.35)	(2.13)	(-0.20)
	$q=5$	-0.07	0.14	-0.01	0.06	-0.63	0.09	0.10	0.14	-0.01
		(-0.46)	(2.29)	(-0.16)	(0.80)	(-0.06)	(0.98)	(1.60)	(2.29)	(-0.16)
β_{cm}	$q=2.5$	0.66	0.65	0.79	0.50	0.68	0.37	0.68	0.65	0.79
		(2.21)	(9.15)	(10.09)	(3.44)	(4.54)	(1.97)	(10.05)	(9.12)	(10.04)
	$q=5$	1.23	0.67	0.82	0.85	10.02	0.79	0.69	0.67	0.82
		(6.49)	(9.92)	(11.08)	(8.96)	(0.98)	(7.67)	(10.74)	(9.90)	(11.06)
β_{jd}	$q=2.5$	0.12	-9.51	-2.19	0.14	0.09	-0.06	-33.78	-16.72	-2.73
		(0.35)	(-0.79)	(-0.14)	(1.80)	(0.86)	(-0.41)	(-1.49)	(-0.73)	(-0.09)
	$q=5$	21.14	-1047.00	3180.00	0.72	-0.08	-1.05	-5414.00	-2223.00	7005.00
		(0.20)	(-0.30)	(0.89)	(0.45)	(-0.42)	(-0.41)	(-0.92)	(-0.34)	(0.99)
β_{jw}	$q=2.5$	1.55	30.64	10.90	0.44	0.65	0.13	56.90	57.26	17.94
		(1.64)	(0.97)	(0.36)	(1.92)	(2.52)	(0.62)	(0.89)	(0.90)	(0.30)
	$q=5$	390.00	6163.00	127.00	5.42	1.89	5.19	3453.00	12940.00	-688.00
		(1.18)	(0.68)	(0.02)	(1.06)	(17.89)	(1.39)	(0.18)	(0.71)	(-0.04)
β_{jm}	$q=2.5$	2.53	26.80	51.59	0.58	0.30	1.04	65.74	56.05	106.02
		(2.37)	(0.73)	(1.39)	(2.10)	(0.86)	(2.75)	(1.00)	(0.76)	(1.42)
	$q=5$	263.00	6132.00	10073.00	1.72	-0.45	3.73	22752.00	12109.00	20505.00
		(0.82)	(0.59)	(1.06)	(0.35)	(-5.09)	(0.86)	(1.10)	(0.58)	(1.05)
R^2_{in}	$q=2.5$	0.38	0.37	0.33	0.46	0.45	0.42	0.45	0.43	0.38
	$q=5$	0.37	0.37	0.33	0.46	0.45	0.41	0.45	0.43	0.38
R^2_{out}	$q=2.5$	0.32	0.20	0.03	0.36	0.26	0.04	0.39	0.30	0.03
	$q=5$	0.24	0.17	0.03	0.35	0.24	0.04	0.39	0.30	0.03

* See notes to Tables 3, 4, and 5A. Prediction model estimates, as well as in-sample and out-of-sample R^2 values, are reported for linear, square root and log HAR-RV-C-DJ(q) models, for q=2.5 and q=5, at daily (h=1), weekly (h=5) and monthly (h=22) prediction horizons. Entries in brackets are robust t-statistics.

Table 5C: HAR-RV-C-UDJ(q) Prediction Regression Results (q=2.5 and 5) for S&P500 Futures Returns*

		Linear Models			Square Root Models			Log Models		
		h=1	h=5	h=22	h=1	h=5	h=22	h=1	h=5	h=22
	$q = 2.5$	0.00	0.00	0.00	0.01	0.01	0.01	-0.57	-0.75	-1.28
β_0		(2.95)	(1.65)	(2.06)	(3.55)	(2.90)	(3.28)	(-2.04)	(-2.47)	(-3.36)
	$q = 5$	0.00	0.00	0.00	0.00	0.00	0.01	-0.42	-0.58	-1.03
		(3.22)	(2.33)	(2.61)	(2.73)	(2.56)	(3.13)	(-1.75)	(-2.26)	(-3.16)
	$q = 2.5$	0.07	0.00	-0.07	0.02	0.02	0.03	0.17	0.14	0.07
β_{cd}		(1.48)	(0.03)	(-1.14)	(0.58)	(0.34)	(0.67)	(6.76)	(5.65)	(2.95)
	$q = 5$	0.07	0.02	-0.07	0.06	0.05	0.03	0.17	0.13	0.07
		(1.85)	(0.18)	(-1.30)	(1.78)	(0.94)	(0.71)	(6.86)	(5.73)	(2.96)
	$q = 2.5$	-0.10	-0.24	0.41	0.02	-0.11	0.11	0.09	0.13	-0.01
β_{cw}		(-0.63)	(-1.44)	(2.76)	(0.22)	(-1.03)	(1.29)	(1.46)	(2.17)	(-0.14)
	$q = 5$	-0.04	-0.19	0.41	0.07	-0.06	0.08	0.10	0.14	-0.01
		(-0.24)	(-1.13)	(2.76)	(0.83)	(-0.56)	(0.98)	(1.67)	(2.33)	(-0.17)
	$q = 2.5$	0.58	1.15	0.71	0.51	0.68	0.36	0.67	0.64	0.78
β_{cm}		(1.71)	(3.75)	(2.47)	(3.55)	(4.64)	(1.94)	(9.92)	(8.89)	(9.76)
	$q = 5$	1.16	1.50	1.06	0.83	0.95	0.79	0.68	0.66	0.82
		(5.21)	(8.90)	(6.02)	(8.65)	(10.12)	(8.49)	(10.53)	(9.60)	(10.98)
	$q = 2.5$	-1.12	0.42	1.84	-0.30	-0.11	0.08	-144	89.00	75.00
β_{jd}^{-}		(-0.66)	(0.40)	(2.14)	(-1.00)	(-0.32)	(0.26)	(-1.04)	(0.78)	(0.64)
	$q = 5$	-526.00	8.00	532.00	-5.81	-4.14	3.52	-16818	-7675	39133
		(-0.79)	(0.03)	(1.96)	(-0.83)	(-0.70)	(0.67)	(-0.43)	(-0.31)	(1.64)
	$q = 2.5$	0.52	-0.58	0.40	0.00	0.59	0.61	214.00	-130.00	-173.00
β_{jw}^{-}		(0.11)	(-0.24)	(0.08)	(0.00)	(0.76)	(0.70)	(0.45)	(-0.34)	(-0.41)
	$q = 5$	400.00	245.00	328.00	-2.50	16.94	11.89	-22222	58161	-39700
		(0.29)	(0.35)	(0.22)	(-0.14)	(1.48)	(0.95)	(-0.20)	(0.75)	(-0.43)
	$q = 2.5$	14.00	9.03	-3.69	2.77	1.64	0.10	946	989	1155
β_{jm}^{-}		(1.68)	(0.80)	(-0.50)	(1.44)	(0.74)	(0.05)	(1.22)	(1.06)	(1.26)
	$q = 5$	3597.00	2303.00	-1813.00	47.31	18.59	-22.76	277860	154197	28836
		(1.69)	(0.83)	(-0.92)	(1.69)	(0.57)	(-0.72)	(1.53)	(0.74)	(0.12)
	$q = 2.5$	1.27	0.00	-1.54	0.46	0.21	-0.14	113.35	-109.98	-81.23
β_{jd}^{+}		(0.76)	(0.00)	(-1.55)	(1.60)	(0.57)	(-0.48)	(0.82)	(-0.88)	(-0.69)
	$q = 5$	567.86	103.84	-397.29	6.77	4.20	-4.74	11699	5426	-33271
		(0.83)	(0.33)	(-1.31)	(1.02)	(0.66)	(-0.90)	(0.29)	(0.19)	(-1.37)
	$q = 2.5$	0.89	2.09	-0.69	0.45	0.06	-0.50	-172.66	185.57	187.45
β_{jw}^{+}		(0.18)	(0.79)	(-0.13)	(0.37)	(0.09)	(-0.55)	(-0.36)	(0.49)	(0.43)
	$q = 5$	-79.64	108.40	-441.16	7.85	-7.33	-6.75	22772	-48575	40677
		(-0.06)	(0.14)	(-0.27)	(0.44)	(-0.79)	(-0.51)	(0.21)	(-0.66)	(0.43)
	$q = 2.5$	-11.31	-7.91	6.10	-2.23	-1.36	0.97	-875	-934	-1049
β_{jm}^{+}		(-1.44)	(-0.73)	(0.78)	(-1.17)	(-0.62)	(0.46)	(-1.13)	(-1.01)	(-1.16)
	$q = 5$	-3255.07	-2218.65	2196.22	-45.62	-20.66	26.62	-252245	-138328	-8874
		(-1.63)	(-0.80)	(1.07)	(-1.65)	(-0.63)	(0.81)	(-1.47)	(-0.68)	(-0.04)
R_{in}^2	$q = 2.5$	0.38	0.37	0.34	0.46	0.45	0.42	0.48	0.43	0.38
	$q = 5$	0.37	0.37	0.34	0.46	0.45	0.04	0.45	0.43	0.38
R_{out}^2	$q = 2.5$	0.34	0.21	0.03	0.37	0.26	0.04	0.39	0.30	0.03
	$q = 5$	0.25	0.17	0.03	0.35	0.25	0.04	0.39	0.30	0.03

*See notes to Tables 3, 4, and 5A. Prediction model estimates, as well as in-sample and out-of-sample R^2 values, are reported for the linear, square root and log HAR-RV-C-UDJ(q) models, for q=2.5 and q=5, at daily (h=1), weekly (h=5) and monthly (h=22) prediction horizons. Entries in brackets are robust t-statistics.

Table 5D: HAR-RV-C-APJ(q) Prediction Regression Results (q=2.5 and 5) for S&P500 Futures Returns*

		Linear Models			Square Root Models			Log Models		
		h=1	h=5	h=22	h=1	h=5	h=22	h=1	h=5	h=22
β_0	$q = 2.5$	0.00	0.00	0.00	0.01	0.01	0.01	-0.52	-0.70	-1.23
		(3.04)	(1.74)	(2.11)	(3.45)	(2.89)	(3.29)	(-1.90)	(-2.37)	(-3.29)
	$q = 5$	0.00	0.00	0.00	0.00	0.00	0.01	-0.38	-0.55	-1.02
		(3.17)	(2.35)	(2.47)	(2.48)	(2.45)	(3.14)	(-1.62)	(-2.17)	(-3.20)
β_{cd}	$q = 2.5$	0.07	0.00	-0.07	0.02	0.02	0.03	0.17	0.14	0.08
		(1.45)	(0.03)	(-1.09)	(0.58)	(0.34)	(0.68)	(6.81)	(5.69)	(2.98)
	$q = 5$	0.07	0.02	-0.08	0.06	0.06	0.03	0.17	0.13	0.07
		(1.77)	(0.17)	(-1.32)	(1.78)	(0.95)	(0.71)	(6.89)	(5.73)	(2.94)
β_{cw}	$q = 2.5$	-0.12	-0.25	0.42	0.01	-0.12	0.11	0.09	0.13	-0.02
		(-0.81)	(-1.42)	(2.79)	(0.16)	(-1.05)	(1.24)	(1.35)	(2.10)	(-0.22)
	$q = 5$	-0.07	-0.20	0.43	0.06	-0.06	0.09	0.10	0.14	-0.01
		(-0.44)	(-1.15)	(2.94)	(0.74)	(-0.60)	(1.01)	(1.59)	(2.29)	(-0.16)
β_{cm}	$q = 2.5$	0.70	1.23	0.69	0.52	0.70	0.37	0.68	0.65	0.79
		(2.47)	(4.93)	(2.22)	(3.65)	(4.68)	(1.93)	(10.06)	(9.15)	(10.09)
	$q = 5$	1.25	1.57	1.03	0.86	0.97	0.78	0.69	0.67	0.82
		(6.84)	(10.87)	(5.28)	(9.29)	(9.99)	(7.69)	(10.76)	(9.92)	(11.08)
β_{jd}	$q = 2.5$	0.07	0.21	0.17	0.08	0.05	-0.03	-16.14	-9.32	-2.04
		(0.43)	(1.32)	(0.70)	(1.97)	(0.90)	(-0.43)	(-1.43)	(-0.78)	(-0.13)
	$q = 5$	18.27	56.89	71.90	0.45	0.00	-0.59	-2596.87	-1046.81	3180.15
		(0.35)	(1.17)	(1.00)	(0.57)	(0.00)	(-0.45)	(-0.88)	(-0.30)	(0.89)
β_{jw}	$q = 2.5$	0.79	0.79	-0.19	0.23	0.33	0.06	27.70	30.42	10.64
		(1.63)	(1.78)	(-0.54)	(1.93)	(2.59)	(0.55)	(0.87)	(0.97)	(0.36)
	$q = 5$	194.64	195.39	-76.42	2.94	4.93	2.42	2031.94	6162.49	127.11
		(1.16)	(1.47)	(-0.81)	(1.11)	(1.90)	(1.23)	(0.22)	(0.68)	(0.02)
β_{jm}	$q = 2.5$	1.18	0.46	1.24	0.27	0.14	0.53	32.37	26.71	51.50
		(2.24)	(0.89)	(1.98)	(1.95)	(0.81)	(2.74)	(0.99)	(0.73)	(1.39)
	$q = 5$	114.43	2.21	211.49	0.50	-1.22	2.10	10776.18	6131.78	10073.20
		(0.71)	(0.02)	(1.35)	(0.20)	(-0.44)	(0.99)	(1.07)	(0.59)	(1.06)
R^2_{in}	$q = 2.5$	0.38	0.37	0.34	0.46	0.45	0.42	0.45	0.43	0.38
	$q = 5$	0.37	0.37	0.34	0.46	0.45	0.42	0.45	0.43	0.38
R^2_{out}	$q = 2.5$	0.32	0.20	0.03	0.37	0.26	0.04	0.39	0.30	0.03
	$q = 5$	0.24	0.17	0.03	0.34	0.24	0.04	0.39	0.30	0.03

*See notes to Tables 3, 4, and 5A. Prediction model estimates, as well as in-sample and out-of-sample R^2 values, are reported for linear, square root and log HAR-RV-C-APJ(q) models, for q=2.5 and q=5, at daily (h=1), weekly (h=5) and monthly (h=22) prediction horizons. Entries in brackets are robust t-statistics.

Table 6: Diebold-Mariano Predictive Accuracy Tests Results for Various Values of q, and for S&P500 Futures Returns*

Panel A: Recursive Scheme

		Linear Models			Square Root Models			Log Models		
		h=1	h=5	h=22	h=1	h=5	h=22	h=1	h=5	h=22
HAR-C-PV(q)	DM Stat	5.30	2.75	-3.04	3.42	2.60	3.25	2.08	2.84	2.29
	q_b	2.50	2.50	2.50	2.50	2.50	2.50	3.20	2.50	2.50
	q_s	4.40	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00
HAR-C-UJ(q)	DM Stat	4.05	1.61	-2.40	3.20	2.90	3.41	2.51	3.20	2.64
	q_b	2.50	2.50	2.50	2.50	2.50	2.50	3.10	2.50	2.50
	q_s	4.40	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00
HAR-C-DJ(q)	DM Stat	6.16	2.89	-1.75	3.51	2.31	3.16	2.19	2.28	1.63
	q_b	2.50	2.50	2.50	2.50	2.50	2.50	3.20	2.50	2.50
	q_s	4.30	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00
HAR-C-UDJ(q)	DM Stat	5.80	2.36	-1.69	3.20	2.05	3.30	2.20	1.49	1.31
	q_b	2.50	2.50	2.50	2.50	2.50	2.50	3.10	2.50	2.50
	q_s	4.40	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00

Panel B: Rolling Scheme

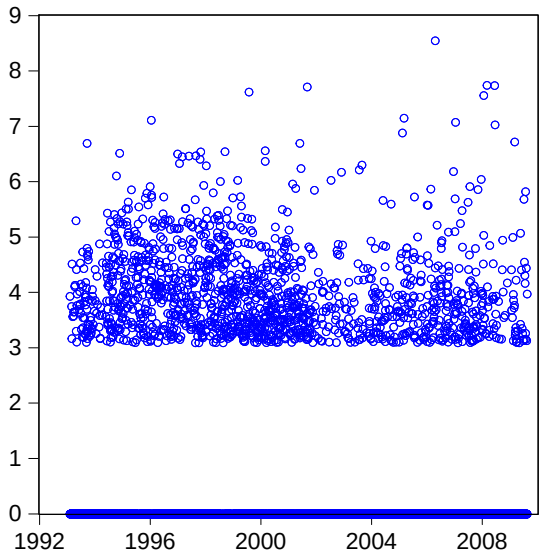
HAR-C-PV(q)	DM Stat	6.17	3.19	-3.41	3.29	2.55	3.19	0.99	2.87	2.49
	q_b	2.50	2.50	2.50	2.50	2.50	2.50	3.40	2.50	2.50
	q_s	4.20	4.70	5.40	6.00	6.00	6.00	6.00	6.00	6.00
HAR-C-UJ(q)	DM Stat	4.28	2.15	-2.67	3.09	2.84	3.32	-3.18	3.30	2.83
	q_b	2.50	2.50	2.50	2.50	2.50	2.50	3.30	2.50	2.50
	q_s	4.20	4.70	5.40	6.00	6.00	6.00	6.00	6.00	6.00
HAR-C-DJ(q)	DM Stat	7.13	3.40	-1.75	3.35	2.26	3.11	2.11	2.24	1.88
	q_b	2.50	2.50	2.50	2.50	2.50	2.50	3.40	2.50	2.50
	q_s	4.20	4.70	5.40	6.00	6.00	6.00	2.50	6.00	6.00
HAR-C-UDJ(q)	DM Stat	6.56	2.86	-1.89	3.04	1.99	3.20	0.92	1.46	1.53
	q_b	2.50	2.50	2.50	2.50	2.50	2.50	3.10	2.50	2.50
	q_s	4.20	4.70	5.30	6.00	6.00	6.00	6.00	6.00	6.00

Panel C: Fixed Scheme

HAR-C-PV(q)	DM Stat	6.17	3.11	-3.23	4.27	3.15	3.41	-4.82	3.32	2.35
	q_b	2.50	2.50	2.50	2.50	2.50	2.50	3.50	2.50	2.50
	q_s	4.30	4.80	5.80	6.00	6.00	6.00	2.50	6.00	6.00
HAR-C-UJ(q)	DM Stat	4.23	1.98	-2.46	3.67	3.25	3.52	-3.75	3.46	2.66
	q_b	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50	2.50
	q_s	4.30	5.80	5.80	6.00	6.00	6.00	6.00	6.00	6.00
HAR-C-DJ(q)	DM Stat	7.13	3.05	-1.80	4.37	2.87	3.34	1.73	3.19	1.85
	q_b	2.50	2.50	2.50	2.50	2.50	2.50	3.50	3.50	3.50
	q_s	4.30	4.30	4.30	6.00	6.00	6.00	6.00	6.00	6.00
HAR-C-UDJ(q)	DM Stat	6.82	2.76	-1.84	3.86	2.43	3.37	0.21	1.80	1.23
	q_b	2.50	2.50	2.50	2.50	2.50	2.50	3.20	2.50	2.50
	q_s	4.30	4.90	3.40	6.00	6.00	6.00	6.00	6.00	6.00

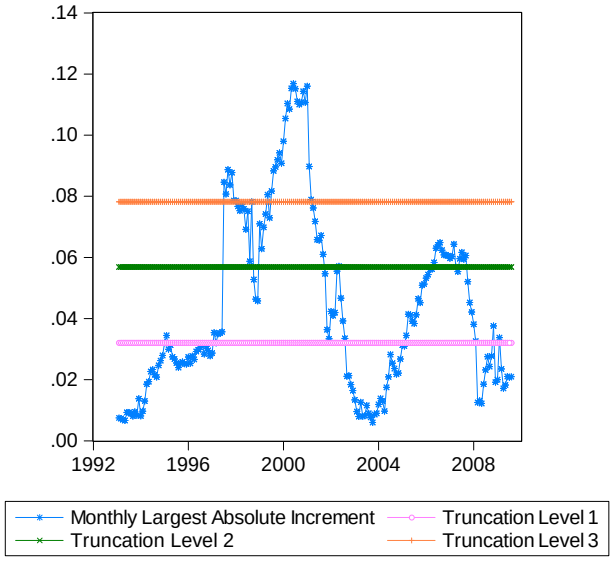
* Table 6 reports Diebold-Mariano (1995) test statistics, carried out to compare the predictive accuracy of a subset of our prediction models, including linear, square root and log HAR-C-PV(q), HAR-C-DJ(q), HAR-C-UDJ(q), and HAR-C-APJ(q) models at daily (h=1), weekly (h=5) and monthly (h=22) prediction horizons. For each model listed in the first column of the table, q_b denotes the value the value of q that yields the largest out-of-sample R^2, while q_s denotes the value of q that yields the smallest R^2, for $q = \{2.5+k*0.1\}_{k=0}^{k=35}$. The DM statistics in the first row of each panel of the table are based on the comparison of each pair of q_b,q_s models, and positive values indicate that the q_b model dominates, in terms of out-of-sample forecast mean square error fit. The statistics are calculated using robust t-statistics (using up to 44 autoregresive lags in estimation of the denominator of the statistics). See Sections 4 and 5 for further details.

Figure 1: Jump Test Statistics on Days Identified as Having Jumps Using S&P500 Futures Returns: Sample Period 1993-2009 *



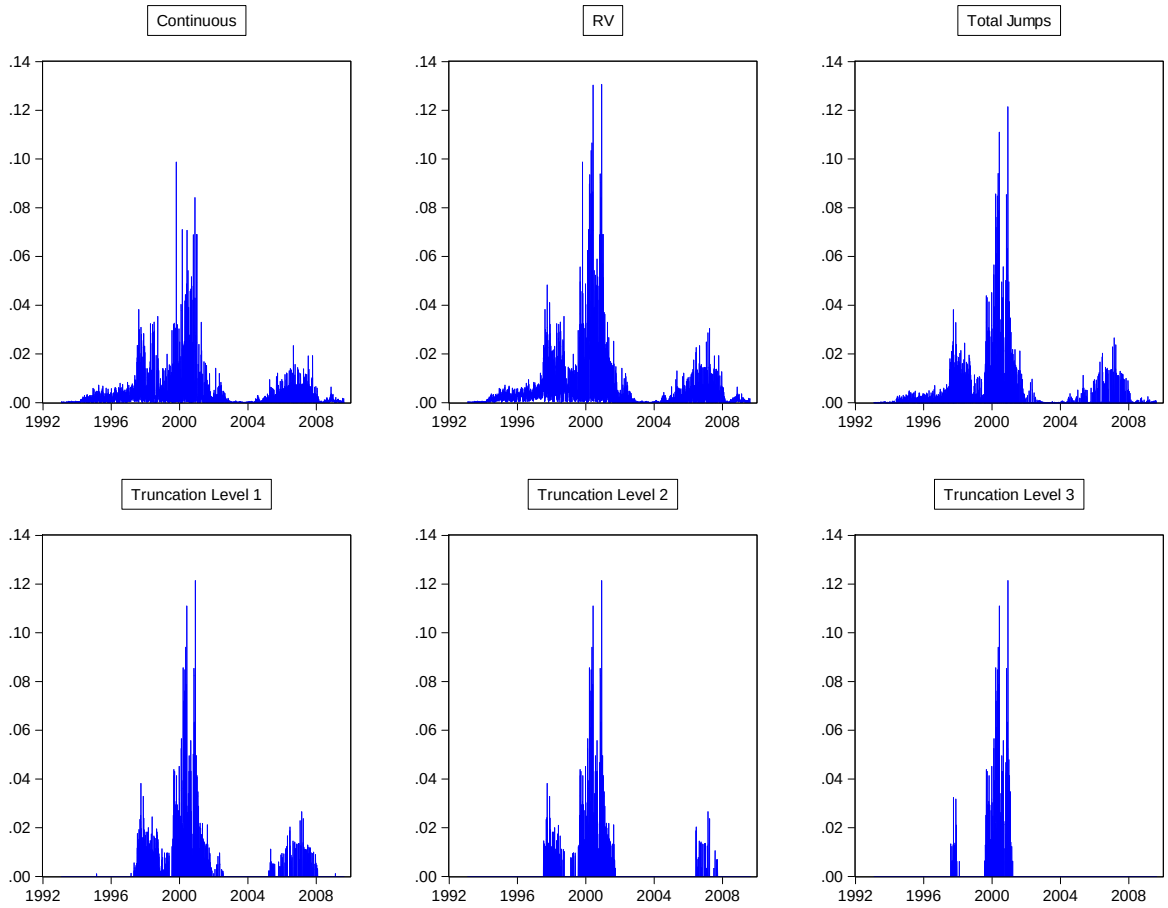
* Daily test statistics are plotted for days identified as having jumps using S&P500 futures (log) price returns; and using a 0.001 jump test significance level.

Figure 2: Monthly Largest Increments and Truncation Levels 1,2,3 Using S&P500 Futures Returns: Sample Period 1993-2009 *



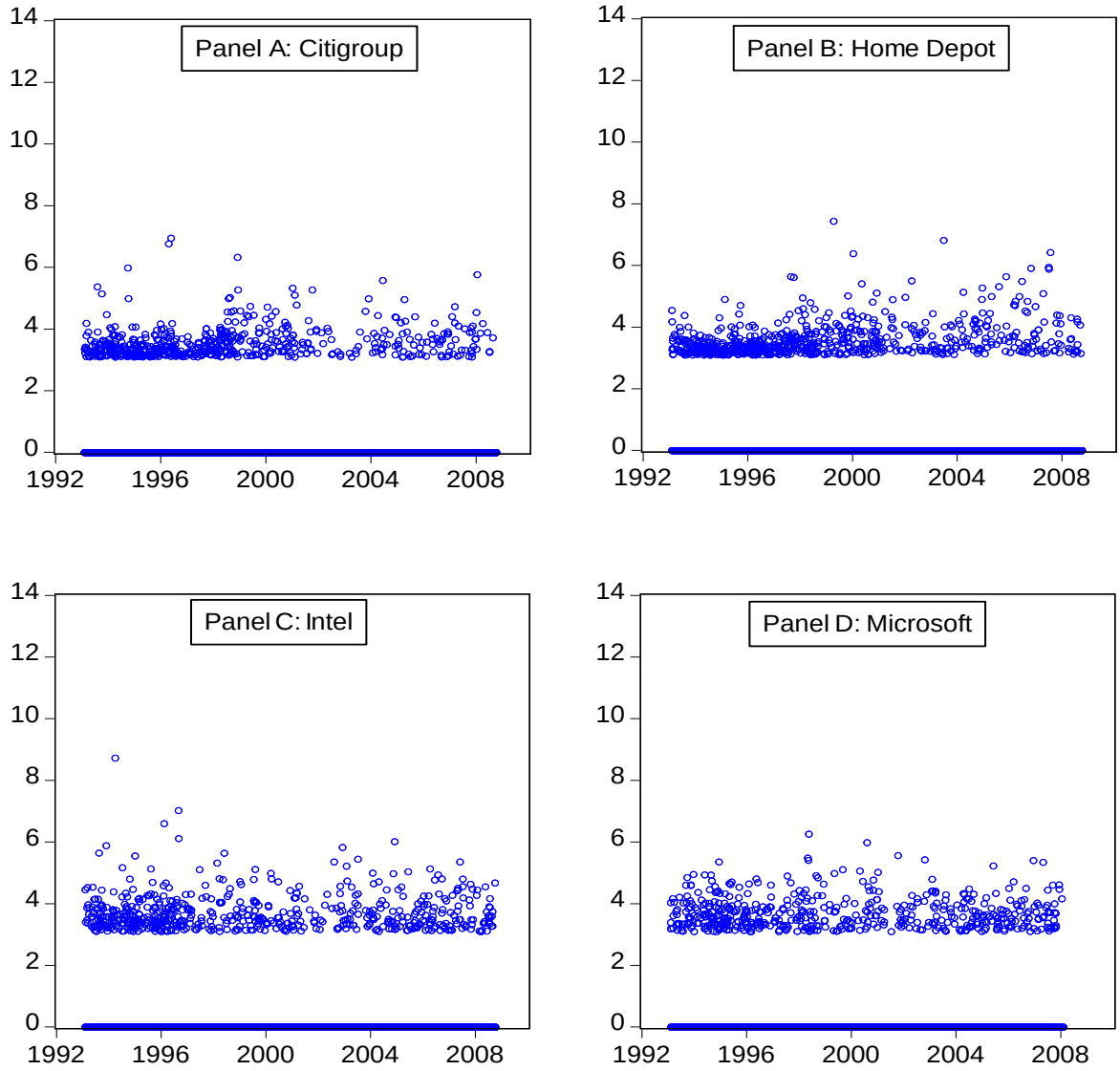
* Monthly "largest" absolute increments and jump truncation levels used as thresholds in our calculations of the variations of large and small jump components are plotted, where Truncation Level 1 corresponds to the median monthly maximum increments, Truncation Level 2 corresponds to 75th percentile monthly maximum increments, and Truncation Level 3 corresponds to 90th percentile monthly maximum increments of S&P500 futures price returns.

Figure 3: Daily Realized Volatility (RV) and Realized Variation of Continuous, Jump and Truncated Jump Components of S&P500 Futures Returns for Truncation Levels 1, 2, 3: Sample Period 1993-2009*



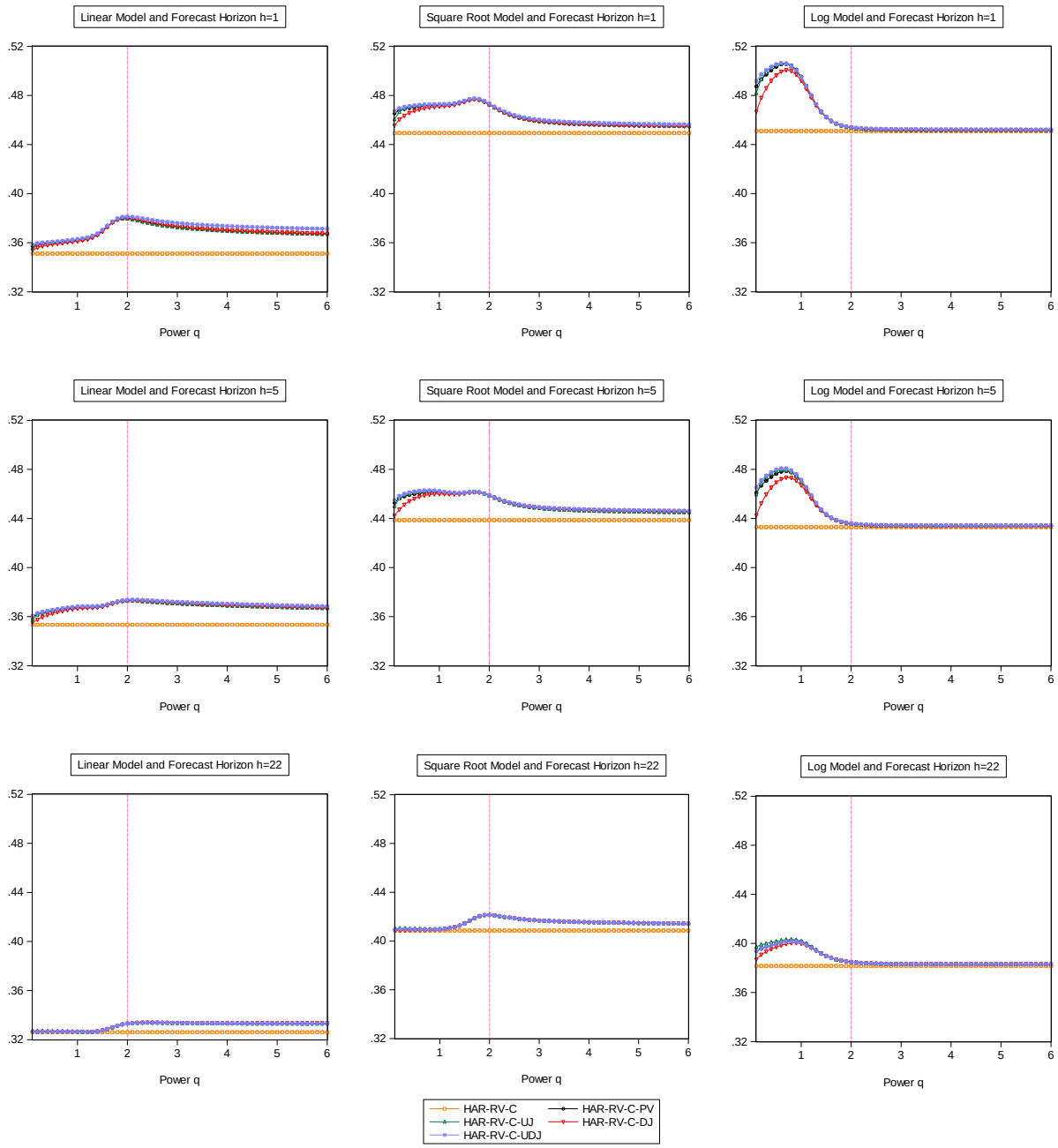
* See notes to Figure 2 for details about the jump truncation levels. The above panels plot daily realized volatility, as well as realized measures of the variation of continuous, jump and large jump components at truncation levels 1,2,3 for S&P500 futures returns, for the period 1993-2009.

Figure 4: Selected DOW 30 Jump Test Statistics for Days Identified as Having Jumps: Sample Period 1993-2008 *



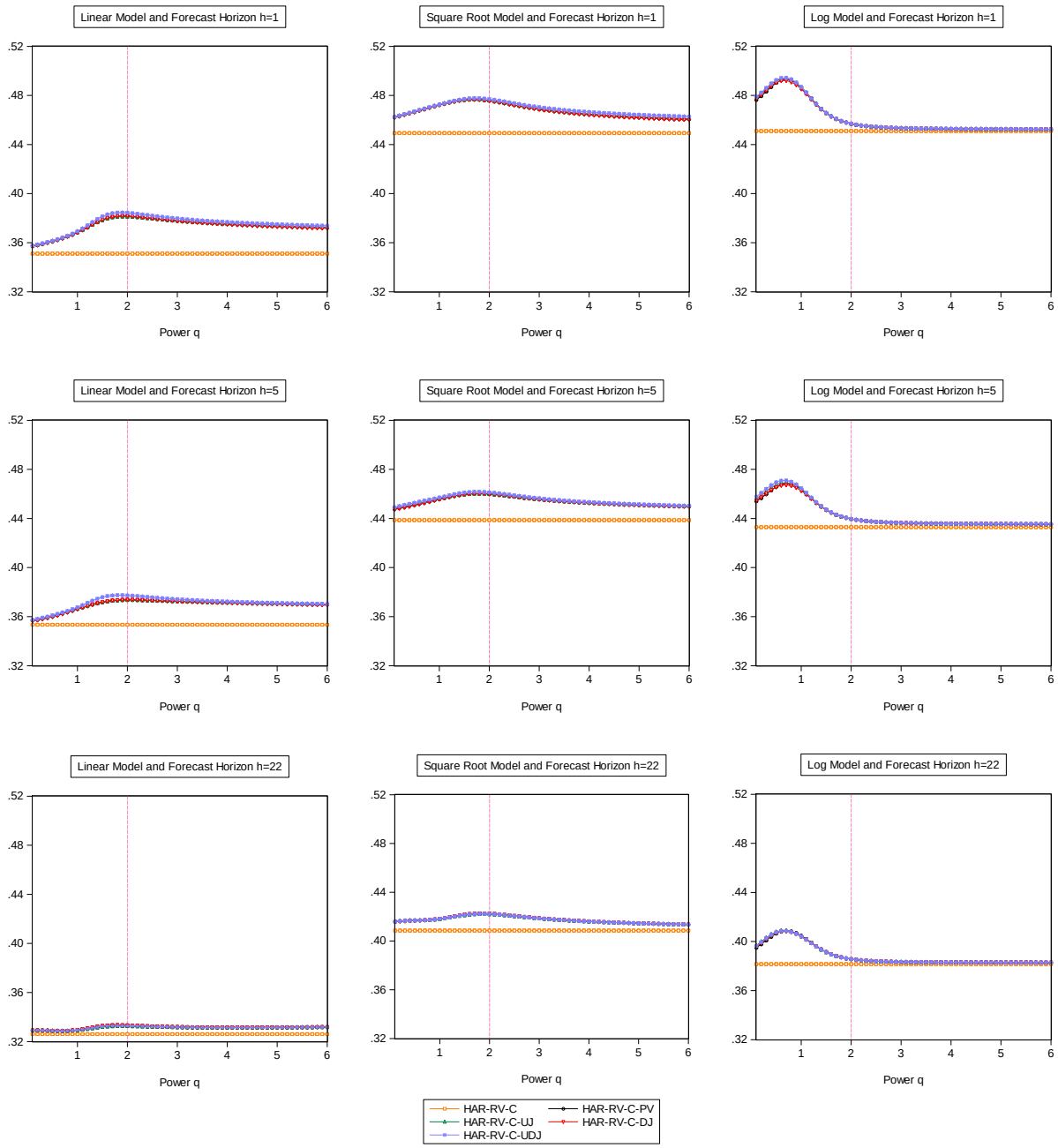
* See notes to Figure 1. Plots depict selected DOW 30 component daily test statistic for days identified as having jumps, using 0.001 significance level.

Figure 5: In-sample R^2 Values for S&P500 Futures, No Jump Test*



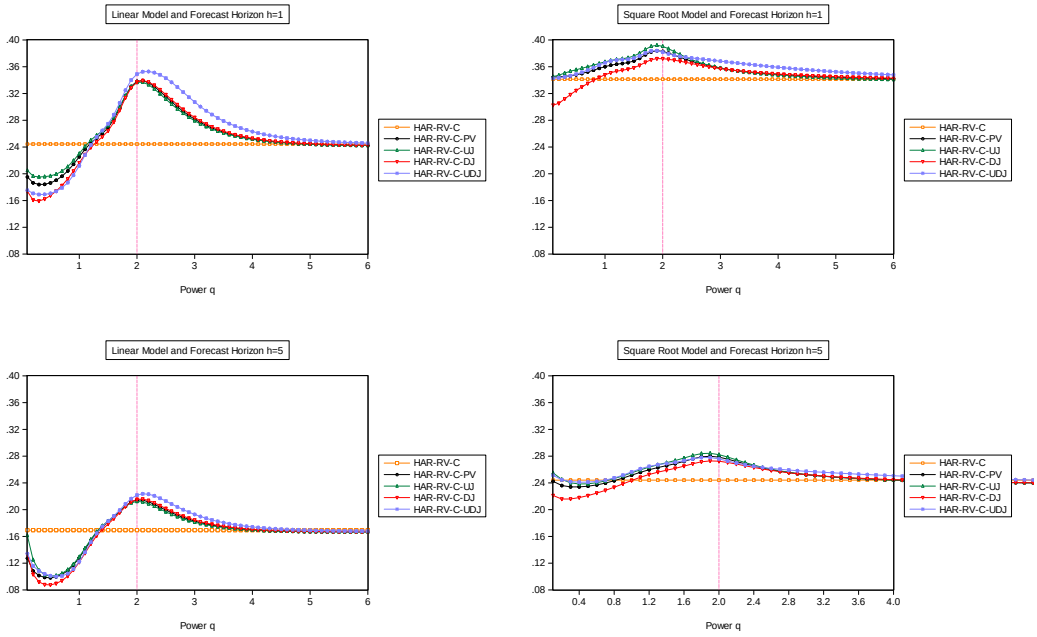
* Figure 5 contains plots of in-sample R^2 values for linear, square root and log HAR-RV-C, HAR-RV-C-PV(q), HAR-RV-C-UJ(q), HAR-RV-C-DJ(q), HAR-RV-C-UDJ(q) models at daily (h=1), weekly (h=5) and monthly (h=22) prediction horizons, for the case where jumps tests are not used when calculating realized measures of jumps for S&P500 futures returns, for the sample period 1993-2009. In each plot, the vertical axis ranges from 0 to 1, and denotes R squared statistic value. The horizontal axis ranges from 0.1 to 6, representing 60 grid points of values of q, i.e. $q = \{0 + 0.1 * k * 0.1\}_{k=0}^{k=60}$.

Figure 6: In-sample R^2 Values for S&P500 Futures, with Jump Test*



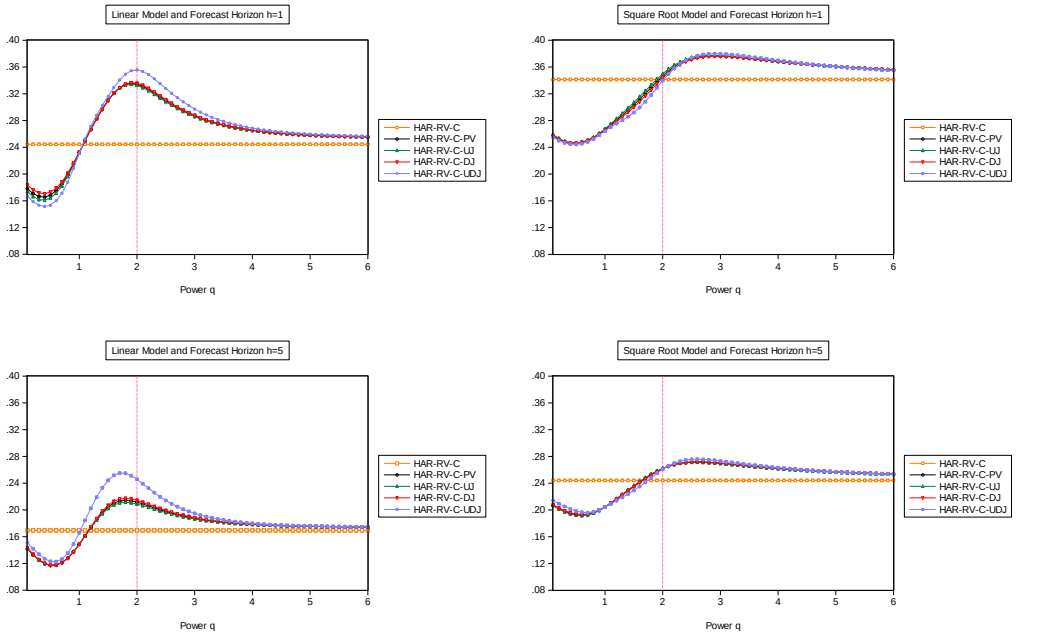
* See notes to Figure 5. All plotted values are based on use of jumps tests when calculating realized measures of jumps for S&P500 futures returns, for the sample period 1993-2009.

Figure 7A: Out-of-sample R^2 Values for S&P500 Futures, No Jump Test*



* See notes to Figure 5. This figure corresponds to Figure 5, except that all reported results are for out-of-sample experiments. See Sections 4 and 5 for further details.

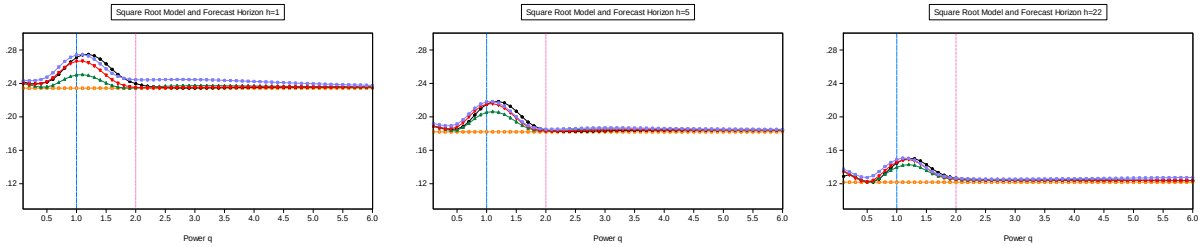
Figure 7B: Out-of-sample R^2 Values for S&P500 Futures, with Jump Test



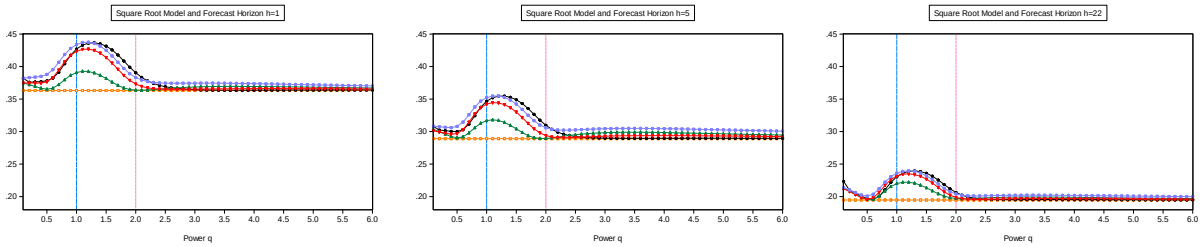
* See notes to Figure 6. This figure corresponds to Figure 6, except that all reported results are for out-of-sample experiments. See Sections 4 and 5 for further details.

Figure 8: In-Sample R^2 Values for Dow 30 Components for Square Root Models, No Jump Test*

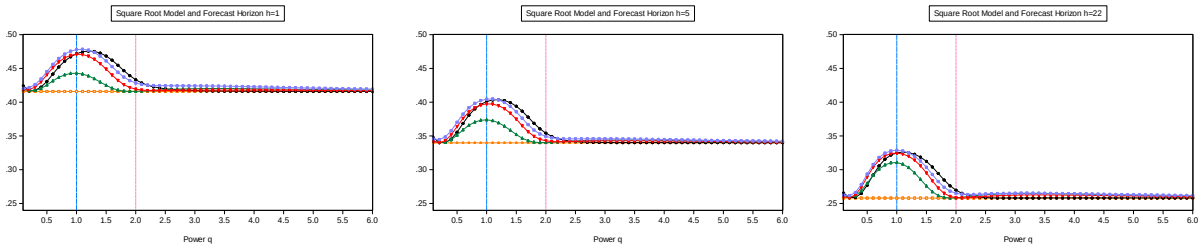
Panel A: Citigroup



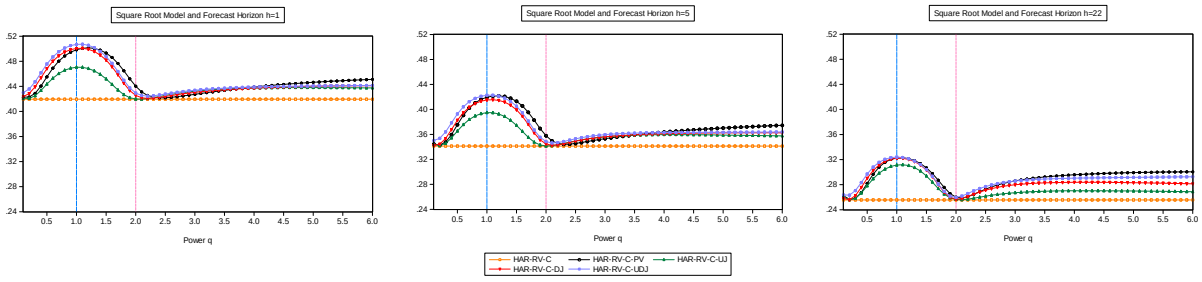
Panel B: Home Depot



Panel C: Intel



Panel D: Microsoft



* See notes to Figure 5. This figure corresponds to Figure 5, except that reported results are for selected DOW 30 components. See Sections 4 and 5 for further details.